

PHYS644 Problem Set 8

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Problem 1: Quadratic inflation during reheating

Some reference equations

$$V(\phi) = \frac{1}{2}\alpha^2\phi^2 \quad (1)$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \quad (2)$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad (3)$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad (4)$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] \quad (5)$$

Problem 1A:

Equation 3 from the equation block 1, has units of

$$[\rho] = [ML^1t^2] = \quad (6)$$

For this to be true, the units of $\dot{\phi}^2$, and V must also be energy density. Therefore, $[\dot{\phi}^2] = [ML^1t^2] \Rightarrow [\phi] = [M^{1/2}L^{-1/2}]$, and $[V] = [ML^1t^2]$. From 1, we know $[\alpha^2][ML^1] = [ML^1t^2] \Rightarrow [\alpha] = [t^{-1}]$, these are the same units as H .

I'm not sure what to box here.

Problem 1B:

Assuming that $\alpha \gg H$, we are told that we discard the H term in equation 2. We are asked with finding the solution. With $H \ll \alpha$ we have:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \Rightarrow \ddot{\phi} + V'(\phi) = 0 \quad (7)$$

Writing this out we have:

$$\frac{d^2\phi}{dt^2} = -\frac{dV}{d\phi} = -2\alpha^2\phi \quad (8)$$

And we can directly solve, it's a classical sinusoidal solution.

$$\phi(t) = A \cos(\alpha t) + B \sin(\alpha t) = A \cos(\alpha t + \delta) \quad (9)$$

And we are free to choose A, B to meet the boundary conditions. I like cos.

Problem 1C

We can write the equation of state parameter w as:

$$w(t) = \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \quad (10)$$

We know $\dot{\phi} = A\alpha t \sin(\alpha t + \delta) \Rightarrow \dot{\phi}^2 = A^2 \alpha^2 t^2 \sin^2(\alpha t + \delta)$, and $V(\phi) = \frac{1}{2}\alpha^2 A^2 \cos^2(\alpha t + \delta)$.

The time averaged kinetic like, and potential like energies are (averaged over many oscillations since $\alpha \gg H$):

$$\left\langle \frac{1}{2}\dot{\phi}^2 \right\rangle = \frac{1}{4}\alpha^2 \phi^2, \quad (11)$$

$$\langle V(\phi) \rangle = \frac{1}{4}\alpha^2 \phi^2 \quad (12)$$

if $w = 0$, then $\langle p \rangle = 0$, so we can test this:

$$\frac{1}{2}\dot{\phi}^2 - V(\phi) = \frac{1}{4}\alpha^2 - \frac{1}{4}\alpha^2 = 0 \quad (13)$$

This means that $w = 0$

Problem 2: Silk Damping

The first half of this problem isn't physics :((I was looking forward to doing a random walk problem.

The problem says:

$$r_{\text{silk,phy}} = \sqrt{\frac{|\Delta z|}{z} H^{-1} \frac{c}{\sigma_t n_e}} \quad (14)$$

And we should use $\Delta z = 200$, $z = 1000$, $n_e = 0.25 \text{ m}^{-3} a_{\text{decouple}}^{-3}$.

We can convert to the comoving scale using $(1+z) = a$, and $n_e = n_{e,0}(1+z)^3$. This is physics.

$$r_{\text{silk,comoving}} = (1+z) \sqrt{\frac{|\Delta z|}{z} H^{-1} \frac{c}{\sigma_t n_{e,0}(1+z)^3}} = (1+z)^{-1/2} \sqrt{\frac{|\Delta z|}{z} H^{-1} \frac{c}{\sigma_t n_{e,0}}} \quad (15)$$

We should use matter dominated H because this is the in the regime of a matter dominate universe. This is also physics.

$$H^{-1} = \frac{1}{H_0 \sqrt{\Omega_m(1+z)^3}} = H_0^{-1} \Omega_m^{-1/2} (1+z)^{-3/2} \quad (16)$$

$$r_{\text{silk,comoving}} = (1+z)^{-5/4} \sqrt{\frac{|\Delta z|}{z} H_0^{-1} \Omega_m^{-1/4} \frac{c}{\sigma_t n_{e,0}}} \quad (17)$$

For our numerical calculation, we will use the `astropy` value of H_0 , and instead of approximating the universe as matter dominated, we can use the `astropy` model to simply compute $H(z=1000)$

Our solution is Silk damping scale (comoving): 9.27 Mpc. Code is available on the github for the class.

We can get the angular size θ by $\theta = \frac{r_{\text{silk}}}{D_M(z)}$, where the demonitor is the transverse comoving distance — for a flat universe, it's the same as the line-of-sight comoving distance.

Angular Silk scale: 0.0383 deg = 2.30 arcmin