Inflation!

Today we are going to take things to even higher redshifts. This will also lead into discussing a universe with perturbations — notice that up until now, the quantities that we have written down have only been functions of time / scale factor / redshift. There has been no position dependence $\vec{r}$ on quantities like the density of matter! We'll fix this.

But first, let's review where we stand up until now.

Previously on PHYS 644™....

- We have a framework for describing the expansion of our Universe, given its constituents

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{\kappa}{a^2} \qquad (c \equiv 1).$$

With right mix of stuff in "ρ", can fit the data that we have on our expansion history.

We think that the main constituents are

Universe decelerating

Radiation $\quad \rho \propto \frac{1}{a^4}$

$a \propto t^{1/2}$

Matter $\quad \rho \propto \frac{1}{a^3}$

$a \propto t^{2/3}$

Accelerating

Dark energy $\quad \rho = \text{const.}$

$a \propto e^{H_0 t}$

time

Since each scales with "a" differently, different parts of our energy budget take turns being the dominant part of ρ.

First radiation dominated, then matter, then dark energy.

There could've been curvature domination here, but we seem to have $\kappa = 0$.

- Within this framework, we took known laws of nuclear physics and thermodynamics and pressed the rewind button. Successfully predicted:
  i) The Cosmic Microwave Background (CMB), with a temperature of $T = 3K$ today (or $\approx 10^{-13}$ GeV).

  ii) Big Bang Nucleosynthesis, which gives the right primordial abundances of elements.

It would be fair to say that this "Hot Big Bang model" is incredibly successful in these ways, and well-tested until $t \sim 1$ second (ie when energies were at nuclear scales). Things are a lot murkier prior to that point. Besides, the Hot Big Bang model has some problems....

Problems with the Hot Big Bang Model

① The Flatness Problem

Let's look at the Friedman equation again.

$$H^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2}$$

$$1 = \underbrace{\frac{8\pi G\rho}{3H^2}}_{\equiv \Omega_{tot}} - \underbrace{\frac{k}{a^2H^2}}_{\equiv \Omega_k.}$$

Planck 2018 + Galaxy Surveys

Today, Ω_k is measured to be very close to zero:

$$\boxed{\Omega_k = 0.0007 \pm 0.0019}$$

It is actually really weird to see this! Why? Let's see how Ω_k evolves as time goes on.

Don't care about the sign, just whether it's close to zero

$$|\Omega_k|_0 = |\Omega_k|_{\text{earlier}} \left(\frac{a_e^2 H_e^2}{a_0^2 H_0^2}\right)$$

Today

$$= |\Omega_k|_{\text{earlier}} \left(\frac{a_e^2/t_e^2}{a_0^2/t_0^2}\right)$$

$$= |\Omega_k|_{\text{earlier}} \left(\frac{\dot{a}_e}{\dot{a}_0}\right)^2$$

Good to factors of order unity.

Now, for most of our Universe's history the expansion has been decelerating. So the rate of change of the scale factor (ie the expansion rate) today, $\dot{a}_0$, is lower than $\dot{a}_{\text{earlier}}$.

$\Rightarrow \left(\frac{\dot{a}_e}{\dot{a}_0}\right) > 1$, so if our Universe is seen as flat today, it must've been really flat earlier.

Just how much flatter are we talking?

Well, earlier I mentioned how we want to push to higher redshifts. Let's say that buoyed by our success earlier, we just extrapolate known physics to earlier times, assuming radiation domination.

So $a \sim t^{1/2} \Rightarrow \dot{a} \sim t^{-1/2} \sim a^{-1}$. But $T \sim \frac{1}{a}$, so $\dot{a} \sim T$.

$$\Rightarrow \left(\frac{\dot{a}_e}{\dot{a}_0}\right)^2 = \left(\frac{T_e}{T_0}\right)^2 = \left(\frac{10^{15}\,\text{GeV}}{10^{-13}\,\text{GeV}}\right)^2 = 10^{56}$$

Grand Unified Theory (GUT) scale. Just a placeholder for when we might extrapolate back to.

CMB temperature today

Today we see that our Universe is flat to within $\sim 10^{-3}$. But to achieve this, it must ~~be~~ have been flat to $\sim 10^{-59}$ (or flatter!) in our early universe!

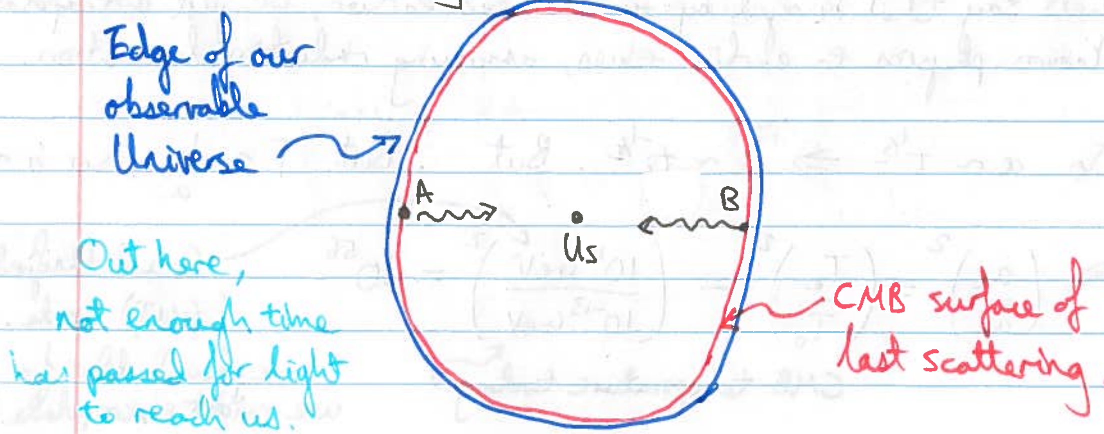
This is incredibly fine-tuned, which is not a hallmark of a good theory. We either want a theory that is robust to the exact values of parameters OR a theory that explains why a parameter has such a fine-tuned value.

② The Horizon Problem

This is essentially a statement of homogeneity. Why is our Universe so homogeneous?

We look at the CMB and we see an incredibly uniform thing. The fluctuations are incredibly tiny in amplitude (1 part in 10^5)

Let's think about it this way. Recombination — and therefore when the CMB was released — happened at around $t_{rec} \sim 400{,}000$ years. Our Universe is about 14 Gyr old. So the CMB photons have been travelling towards us for essentially the age of our Universe.



By definition, in the age of our Universe, photons coming to us from points A and B have just arrived today. They're too far away to have influenced each other. How do they "know" to be at the same temperature?

Let's formalize this a little and work out some numbers. First, let's work out the radius of our observable Universe. This is the total distance that light can travel in the whole age of our Universe.

FRW metric: $ds^2 = -c^2 dt^2 + a^2(t)\, d\chi^2$ (Suppressing θ and φ)

Light travels on null geodesics $ds^2 = 0 \Rightarrow \chi = \int \frac{c\,dt}{a(t)}$

This is the comoving distance. To get the proper distance we need to multiply by $a(t)$:

This is the technical name for it

$$\text{Particle horizon} = a(t) \int_0^t \frac{c\,dt'}{a(t')} \sim \frac{c}{H_0}$$ up to factors of order unity today.

At any time t, I can plug things into this formula to figure out how big of a region could be in causal contact.

Here's the strategy. I'm going to take the size of our observable Universe and trace it back in time to some earlier time t_e. This size will be smaller because our Universe is expanding. This will essentially tell me how far apart points A and B are at earlier times.

And then what I'm going to do is to compare this to the size of the particle horizon at the earlier time. If this particle horizon is larger than the distance between A and B, then maybe they could've been in causal contact after all.

$$(*) = \frac{\begin{pmatrix}\text{Today's radius of obs. universe} \\ \text{scaled back to time } t_e\end{pmatrix}}{\begin{pmatrix}\text{Particle horizon at} \\ \text{time } t_e\end{pmatrix}} \sim \frac{\left(\frac{a_e}{a_0}\right)\frac{c}{H_0}}{a_e \int_0^{t_e} \frac{c\,dt}{a(t)}}$$

Now, in the various scenarios we've looked at (radiation dominated, matter dominated, etc.), we've had $a(t) \sim t^\gamma$ where $0 < \gamma < 1$.

$$\text{So } a_e \int_0^{t_e} \frac{c\,dt}{a(t)} \propto t_e^\gamma \int_0^{t_e} c\,t^{-\gamma}\,dt = c\,t_e^\gamma \left[\frac{t^{1-\gamma}}{1-\gamma}\right]_0^{t_e} \sim c\,t_e$$

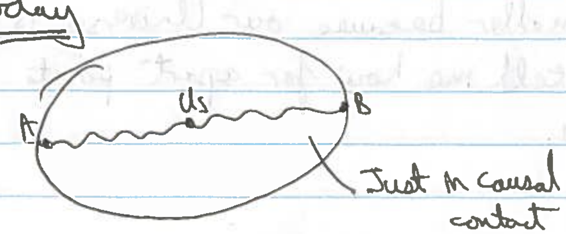
Then:

$t_0 \sim \frac{1}{H_0}$

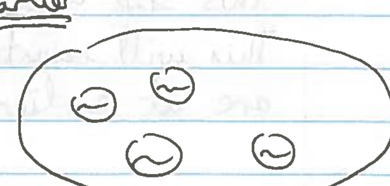
$$(*) = \frac{\left(\frac{a_e}{a_0}\right)\frac{1}{H_0}}{t_e} \sim \frac{a_e/t_e}{a_0/t_0} \sim \frac{\dot{a}_e}{\dot{a}_0} \sim 10^{28} \quad \text{from before!}$$

This is really bad! This means that if $\dot{a}_e > \dot{a}_0$ — which is the case for a decelerating universe — then points A and B were definitely <u>not</u> within a particle horizon of each other.

<u>Today</u>



<u>Earlier</u>



Our Universe consists of more and more causally disconnected islands as we trace back in time → So how do these islands "coordinate" to give us a homogeneous universe?

③ The Monopole Problem

If we are interested in pushing physics to even earlier and even hotter / higher energy times, we eventually hit the GUTS scale, and lots of theories predict the existence of magnetic monopoles.

This is a problem. Why?

- We don't "see" any monopoles / they live for long times (by particle physics standards) and can be around even @ $t \sim 1$ sec, interfering with BBN.
- They are massive ($M_{mono} \sim 10^{17}$ GeV) and are produced copiously enough to cause our Universe to be matter dominated to early.

Magnetic monopoles are not particles in the usual sense. They are examples of topological defects. Let me explain what I mean by this. As our Universe descends from high energies, it undergoes various phase transitions (you might've heard of fancy terms like electroweak symmetry breaking @ $\sim 10^2$ GeV)

Let's look at a picture of a phase transition that we're a little more familiar with — crystallization.

⟹ $\boxed{\text{Picture of crystal}}$

If different parts of something start to crystallize from seed crystals, they don't "coordinate", and when they grow enough to meet, there are defects in the crystal.

It turns out that under the right circumstances, with our Universe some of these defects would be considered magnetic monopoles!

How many such defects might we produce? Well, how coordinated can phase transitions in our early universe be? ~~the~~ The causally disconnected patches we talked about earlier are... well, causally disconnected! So they can't possibly coordinate.

$$\Rightarrow \underset{\text{in observable universe}}{(\#\text{ of magnetic monopoles})} \sim (\text{fudge factor}) \left(\frac{\text{Today's obs. univ radius scaled back}}{\text{Particle horizon back then}} \right)^3$$

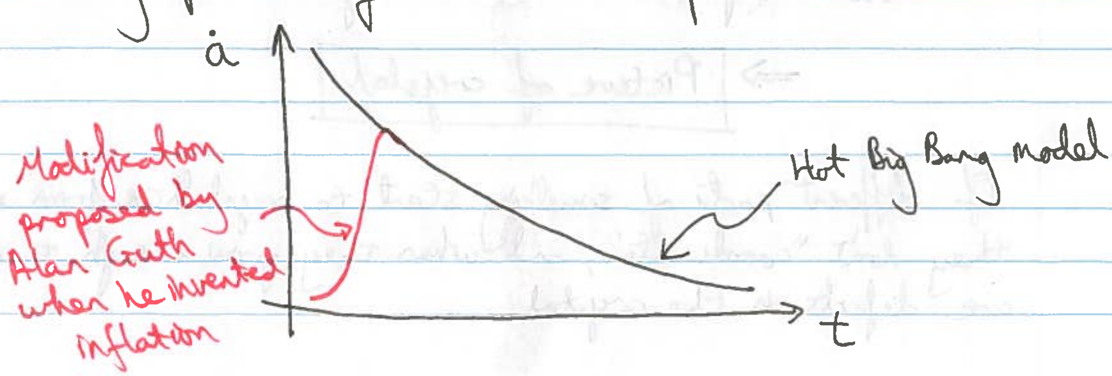
$$\sim \left(\frac{\dot{a}_e}{\dot{a}_0} \right)^3$$

Huge number. Like before. Not good.

The Inflationary Solution

We have these problems with the Hot Big Bang model, and they all seem to have a common element to them — decelerating expansion, ie the fact that $\frac{\dot{a}_e}{\dot{a}_0} \gg 1$.

The inflationary solution is to postulate that there was an early period of accelerated expansion:



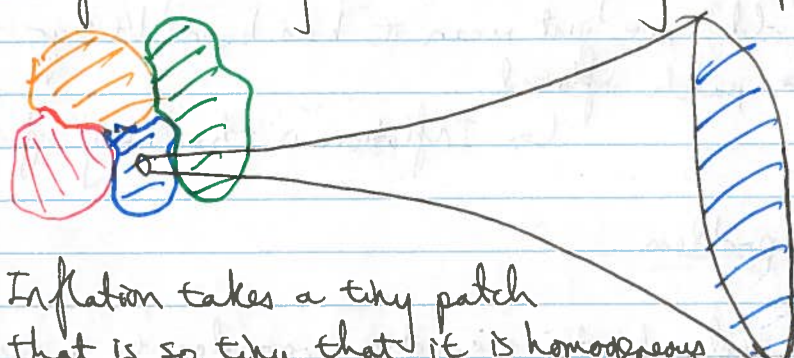
In detail, how does this solve our problems?

Flatness problem

Our Universe is flat because ~~[illegible]~~ everything we see today was stretched out from a really small patch.

Horizon / homogeneity problem

Imagine starting out with a really lumpy, inhomogeneous universe

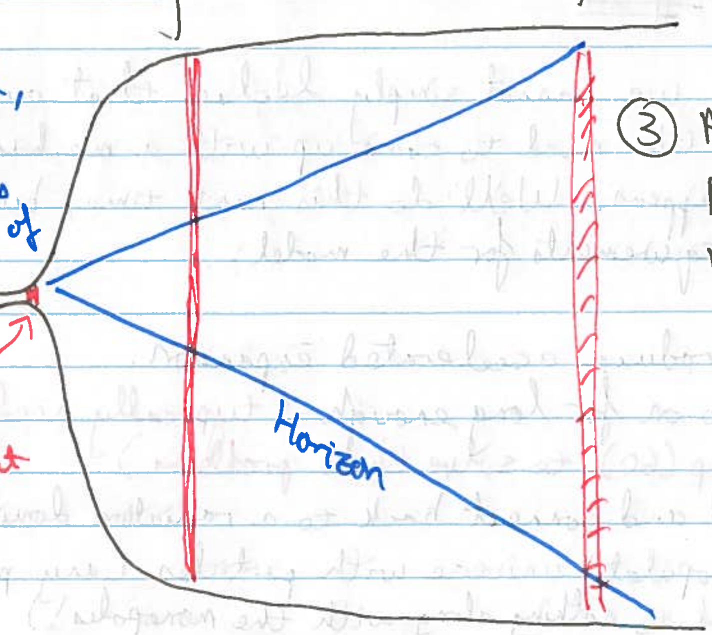


Inflation takes a tiny patch that is so tiny that it is homogeneous and stretches it out.

Another way to think about this, in terms of horizons

② Exponential, accelerated stretch pulls the patch out of causal contact

① Start with a tiny patch in causal contact with itself



③ As time goes on, different parts of our Universe come into causal contact as the horizon size grows

So the solution to the horizon problem is that because all this stuff was close together before, they could confer and "get their stories straight" before they were yanked out of causal contact.

Note that this picture dramatically increases the vastness of what we might think of as the universe.

The pre-inflationary "wild west" universe can be lumpy and inhomogeneous and look like anything. It could also have been around for a long time. When we say "our Universe is 14 Gyr old", we just mean it has been 14 Gyr since our little microscopic patch inflated.

↳ Inflation is the "Bang" of the "Big Bang"

Monopole problem

Inflation simply dilutes magnetic monopoles to a negligible density. We ~~are~~ come from a small patch that had $n_{monopole} \ll 1$.

Realizing inflation

Of course, we cannot simply declare that our Universe behaves like this! We need to come up with a mechanism by which all this happens. We'll do this next time, but for now, let's list our requirements for the model:

(Not a precise number)

So a cosmological constant wouldn't do! That gives unending accelerated expansion

- Must produce accelerated expansion.
- Must go on for long enough (typically need $a(t)$ to grow by $\exp(60)$ to solve these problems)
- Must end and connect back to a radiation dominated Universe by "1 sec"
- Must repopulate universe with particles (any pre-inflation relic gets diluted to nothing along with the monopoles!)