PHYS 644 Lecture # 13 : BBN

We have so far talked about some of the key events of our Universe when $t \simeq 1\,\text{sec}$, $T \sim 1\,\text{MeV}$, and $T \sim 10^{10}\,\text{K}$

$\Rightarrow$ $\boxed{\text{Review slides}}$

We will now press on with the story and talk about Big Bang Nucleosynthesis. This is where the hot cauldron of our Universe is an excellent oven for nuclear reactions that bake a lot of ^4He and small amounts of light elements up to ^7Li.

Let's set the context. Big Bang Nucleosynthesis (BBN) begins around $T \sim 0.1$ MeV, so not long after e^+/e^- annihilation. At this point, $g_* \simeq 3.38$ (photons and neutrinos), which means that

$$t \simeq 132\,\text{s} \left(\frac{0.1\,\text{MeV}}{T} \right)^2,$$

which means BBN takes place starting a few seconds to a few minutes after the big bang.

Neutrons are the main character

Our goal now is to work out the elemental abundances. Conceptually, the most important thing to keep in mind to understand BBN is "follow the neutrons".

Why are neutrons the main determinant of what BBN products are?

(1) To make heavy nuclei, we need both neutrons and protons.

I can't make a nucleus entirely out of protons or neutrons! For nuclear physics stability reasons, we need specific mixes of protons and neutrons to be stable. Can't do it with just protons!

In principle unstable, but long-lived enough to be considered stable for BBN

Hydrogen isotopes:	protons	${}^1_1H$	stable
	deuterium	${}^2_1H$	~~$t_{1/2}$ = 12 yr~~ stable
	tritium	${}^3_1H$	$t_{1/2} = 12$ yr
Helium isotopes:	helium-3	${}^3_2He$	stable
	helium-4	${}^4_2He$	stable
Lithium isotope:	lithium-7	${}^7_3Li$	stable
Beryllium isotope:	beryllium-7	${}^7_4Be$	$t_{1/2} = 52$ days

But why do I say that neutrons are the main character and not protons and neutrons?

(2) Neutrons ~~are~~ slightly heavier than protons ($Q \equiv m_n - m_p = 1.29\ \text{MeV}$) and this disfavors them, limiting their ~~supplies~~ supply.

Neutrons and protons can convert into one another via weak interactions:

$$n + \nu_e \rightleftharpoons p + e^-$$

$$n + e^+ \rightleftharpoons p + \bar{\nu}_e$$

Starting prior to BBN (say, $T \gtrsim 10$ MeV), the neutrons and protons are in equilibrium via these reactions. Plus, recalling that $m_p, m_n \sim 1$ GeV, they are definitely non-relativistic. We therefore use the non-relativistic limit of the equilibrium distributions:

$$n_p = g_p \left(\frac{m_p T}{2\pi}\right)^{3/2} \exp\left(-\frac{m_p - \mu_p}{T}\right) \quad ; \quad n_n = g_n \left(\frac{m_n T}{2\pi}\right)^{3/2} \exp\left(-\frac{m_n - \mu_n}{T}\right)$$

Therefore,

$$\frac{n_n}{n_p} = \left(\frac{m_n}{m_p}\right)^{3/2} \exp\left(\frac{\mu_n - \mu_p}{T}\right) \exp\left(-\frac{m_n - m_p}{T}\right) \approx \exp\left(-\frac{Q}{T}\right)$$

Informally, the neutron and protons are like two "states" of a nucleon, with a Boltzmann factor governing the relative number in each state.

What we see is that as time goes on and we cool to $T \sim 1\,\text{MeV}$ and $T \sim 0.1\,\text{MeV}$ appropriate for BBN, the equilibrium shifts toward more protons than neutrons.

As with every interaction we've studied, ~~is~~ this one needs to compete with the Hubble rate. We found in the last lecture that neutrinos decouple from e^+/e^- around $T \sim 1\,\text{MeV}$. The cross-section for nucleon-neutrino interactions is higher than for e^+/e^-, so the decoupling of ~~neutrons~~ nucleons from neutrinos / neutron freezeout happens later at $T_{\text{neutron freezeout}} \sim 0.8\,\text{MeV}$.

$$\Rightarrow \frac{n_n}{n_p} = \exp\left(-\frac{Q}{T_{\text{neutron freezeout}}}\right) = \exp\left(-\frac{1.29\,\text{MeV}}{0.8\,\text{MeV}}\right) = 0.2$$

Therefore, when this conversion of protons to neutrons (and vice versa) stops, only 1/6 of nucleons are neutrons.

The neutrons, though, are even more limited in supply than this because ...

(3) Free neutrons are not stable and decay.

Hilroy

Beta decay: $n \rightarrow p + e^- + \bar{\nu}_e$.

This has a decay lifetime of $\tau_n \sim 890\,s \approx 15$ minutes. It's therefore relevant to BBN.

Why so hard? Neutrons are neutral and are hard to trap and study.

Incidentally, this number is not easy measure and is one of the key uncertainties in BBN codes. The precision is 0.1%, which sounds impressive, but that's quite bad by particle physics standards (and is large enough to matter)

What we find, then, is that neutrons determine the outcome of BBN. To a good approximation, all the neutrons end up in ^4He, so this is how we predict helium abundance.

~~To know exactly~~ BBN is therefore a bit of a race against time. We need the relevant nuclear reactions to turn on before all the neutrons decay away. To figure out how long we have, we now need to examine the actual reactions...

Steps in BBN

There are a handful of important reactions $\Rightarrow$ Show slide with reactions

Deuterium production is what kicks this off, so I want to focus on that a little:

$$p + n \rightleftharpoons D + \gamma$$

This gives a lightly bound nucleus with binding energy of

$$B_D \equiv (m_n + m_p - m_D) = 2.22\ MeV.$$

Hilroy

Using similar non-relativistic equilibrium distribution math, we have

$$\frac{n_D}{n_p n_n} = \frac{g_D}{g_p g_n}\left(\frac{m_D}{m_p m_n}\right)^{3/2}\left(\frac{T}{2\pi}\right)^{-3/2} \exp\left(\frac{B_D}{T}\right)$$

$$\simeq 6\left(\frac{m_n T}{\pi}\right)^{-3/2} \exp\left(\frac{B_D}{T}\right)$$

$g_D = 3$ (spin 1)
$g_p = g_n = 2$
and $m_n \simeq m_p \simeq \frac{m_D}{2}$.

It is customary to express this in terms of the baryon-to-photon ratio η:

$$\eta \equiv \frac{n_B}{n_\gamma} = (6.12 \pm 0.04) \times 10^{-10}$$

The fact that this is tiny is another way to say that the matter-antimatter asymmetry is tiny

Now, recalling that after neutron freezeout, 5/6 of baryons are protons, and therefore...

$$n_p \simeq 0.83\, n_B \simeq 0.83\, \eta\, n_\gamma \simeq 0.2\, \eta\, T^3$$

$n_\gamma \simeq 0.243\, T^3$ using $g_\gamma = 2$.

$$\Rightarrow \frac{n_D}{n_n} \simeq 6.7\, \eta \left(\frac{T}{m_n}\right)^{3/2} \exp\left(\frac{B_D}{T}\right)$$

This ratio starts out $\ll 1$ at the beginning of BBN when there is barely any deuterium. It is $\gg 1$ as BBN proceeds and the number of free neutrons n_n plummets as neutrons get incorporated into nuclei.

When does a significant amount of D get produced? This is actually approximately the same as asking by when a significant amount of ^{4}He is produced, because the later reactions are very quick.

A reasonable mark for the transition to nucleosynthesis might be when the ratio $n_D / n_n \sim 1$. With our expression, this happens at

$$T_{BBN} \sim 0.07\,\mathrm{MeV} \quad \text{and} \quad t_{BBN} \sim 300\,\mathrm{s}.$$

This is why I said at the beginning that $T_{BBN} \sim 0.1\,\mathrm{MeV}$

This might be a later time / cooler temperature than one might expect from naively examining a nuclear reaction with $B_D \sim 2\,\mathrm{MeV}$.

The reason is the very small value of η. There are so many photons around that the reverse reaction (photodissociation of D) is heavily favoured.

A slightly more detailed calculation gives $t_{BBN} \sim 200\,\mathrm{s}$.

With this number as the time when BBN happens, we can now figure out how many neutrons are around (after decays) to be bound up into nuclei

Neutron # fraction

$$X_n(t) \equiv \frac{n_n}{n_p + n_n} = X_n(t_{\substack{\text{neutron}\\ \text{freezeout}}})\, e^{-\frac{t_{BBN}}{\tau_n}}$$

$$= \left(\frac{1}{6}\right) \exp\left(-\frac{200}{890}\right) \simeq 0.13 \,.$$

To an excellent approximation, all of this ends up in ^{4}He. It is conventional to express the amount of helium as a mass fraction:

Stands for "primordial", not "proton"!

$$Y_p \equiv \frac{\rho_{He}}{\rho_B} = \frac{m_{He} n_{He}}{m_H n_H + m_{He} n_{He}} = \frac{4 n_{He}}{n_H + 4 n_{He}} = \frac{4 n_{He}}{n_p + n_n}$$

$m_{He} \simeq 4 m_H$

$n_H \equiv$ protons that remain free

$n_p \equiv$ total # of protons

$n_{He} = \frac{n_n}{2}$ if $\approx$ all neutrons end up in ^4He.

$$= \frac{2n_n}{n_p + n_n} = 2X_n \simeq 0.26.$$

A more precise calculation gives $\simeq 0.24$. And that's it! The heavier elements (up to lithium) exist only in trace amounts. Our Universe is 75% hydrogen and 25% helium.

$\Rightarrow$ $\boxed{\text{Show abundance patterns}}$

Why don't we get heavier elements than lithium? There aren't ~~many~~ any stable elements that are just slightly heavier that could be a next step. In stars, there is the triple alpha process:

$$3(^4He) \longrightarrow {}^{12}C$$

But a 3-body process like this can only happen in the dense environment of a star, not the relatively dilute environment of the universe at large (even at early times).

Testing BBN

One of the lovely things about BBN theory is that it just relies on one free parameter: η, the baryon-to-photon ratio.

$\Rightarrow$ $\boxed{\text{Show elemental abundances as fct. of } \eta}$

A very stringent test of BBN is then to ~~to~~ try to measure elemental abundances in our Universe (vertical axis of plot) and then to see if the predicted η values for each element line up. There can only be one value of η! A highly non-trivial test!

BBN generally does very well, maybe with the exception of lithium (see plot). But lithium is created and destroyed in stars, so there may be unaccounted for systematics.

BBN as a probe of standard cosmology

If instead of testing BBN we accept that it happened the way we described it, we can use it to measure $\Omega_b h^2$. Measuring some elemental abundances, we can infer η. And since we know the # density of photons very well, we can convert this to a measurement of baryon density.

⟹ Slide Q on which element

BBN as a probe of new physics

BBN can also place stringent constraints on the presence of new particles Beyond the Standard Model ("BSM"). A new particle increases g_* during BBN, which changes Y_p ....

⟹ Slide Q on how Y_p changes