PHYS 644 Lecture #20: Matter Power Spectrum

We've been building the pieces for understanding the form of the matter power spectrum

$\Rightarrow$ $\boxed{\text{Review Slides}}$

Shape of the matter power spectrum

Recall that we had written:

Transfer fct.

$$\tilde{\delta}(\vec{k}, a) \propto \tilde{\delta}_{\text{primordial}}(\vec{k})\, T(k)\, D_1(a)$$

Initial conditions

Growth factor.

To get to $T(k)$, I need to refine our discussion of what happens to modes as they enter / exit the horizon.

Previously, I had carelessly said that modes outside the horizon are <u>frozen</u>. But I need to be more precise. What is frozen = the fluctuations in gravitational potential Φ, <u>not density</u>.

comoving k!

The Poisson eqn. gives: $\nabla^2 \delta\Phi = 4\pi G \bar{\rho}\delta \Rightarrow \frac{k^2 \delta\tilde{\Phi}}{a^2} \propto \bar{\rho}\,\tilde{\delta}$.

If $\delta\tilde{\Phi}$ is frozen for superhorizon modes, then $\tilde{\delta}$ isn't because $\bar{\rho}$ evolves with time (Eg $\bar{\rho} \propto \frac{1}{a^3}$ during matter domination and $\frac{1}{a^4}$ during radiation domination).

$\Rightarrow$ For superhorizon perturbations:

$$\tilde{\delta} \propto a \qquad\qquad \tilde{\delta} \propto a^2$$

(matter domination) (radiation domination)

Once the modes reenter the horizon, they evolve according to the Newtonian expressions we found before.

What we need to know is the amplitude as I enter the horizon:

$$\langle \delta^2_{rms,R_H} \rangle \sim \int_0^{\frac{1}{R_H}} \frac{dk\, k^2 (4\pi)}{(2\pi)^3} P(k)$$

r.m.s. on horizon scales

Cut off @ $k_H \sim \frac{1}{R_H}$ $\because$ don't care about fluctuations on smaller scales.

Let $u \equiv k R_H \Rightarrow du = R_H\, dk$. Also let $P(k) \propto k^{n_s}$ from inflation

$$\Rightarrow \langle \delta^2_{rms,R_H} \rangle \sim R_H^{-(n_s+3)} \Rightarrow \delta \sim R_H^{-(n_s+3)/2}$$

When a mode enters the horizon,

(Amplitude) $\sim$ (Superhorizon growth) (primordial r.m.s.)

Therefore:

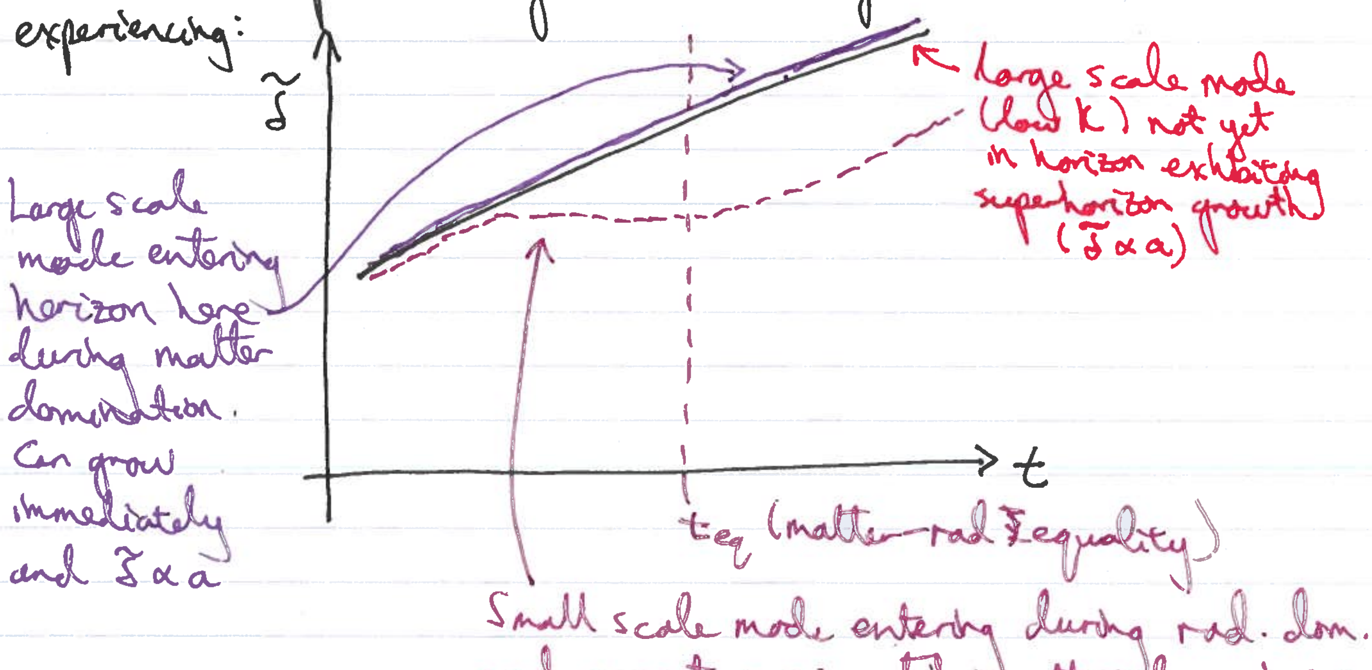
Epoch	Comoving horizon scale	Superhorizon growth	Amplitude at horizon entry.
Radiation domination	$R_H \sim \frac{ct}{a} \propto a$ $a \sim t^{1/2}$	$\tilde{\delta} \propto a^2 \propto R_H^2$	$\tilde{\delta}_{entry} \sim R_H^2 R_H^{-\frac{(n_s+3)}{2}}$
Matter domination	$R_H \sim \frac{ct}{a} \propto a^{1/2}$ $a \sim t^{2/3}$	$\tilde{\delta} \propto a \propto R_H^2$	$\tilde{\delta}_{entry} \sim R_H^2 R_H^{-(\frac{n_s+3}{2})}$

Regardless of when we enter the horizon, the amplitude of fluctuations goes like

$$\tilde{\delta}_{entry} \sim R_H^{\left(\frac{1-n_s}{2}\right)}.$$

Notice that if n_s were perfectly $=1$, all density modes enter the horizon at the same amplitude. This is another sense in which people call an $n_s = 1$ spectrum a "scale invariant spectrum".

What happens after horizon entry? Recall from our Newtonian results that during radiation domination, growth is negligible (logarithmic). Thus a mode that enters during radiation domination will miss out on the superhorizon growth that larger modes are still experiencing:



Since the superhorizon growth goes like $\tilde{\delta} \propto R_H^2$, if the large k modes miss out on this because they entered the horizon, their amplitude is suppressed by

$$\frac{\tilde{\delta}(k)}{\tilde{\delta}(k_{eq})} = \left(\frac{R_{H,k}}{R_{H,eq}}\right) = \left(\frac{k}{k_{eq}}\right)^{-2}$$

Mode that entered at ?

This means

$$T(k) \propto \begin{cases} 1 & \text{for } k \ll k_{eq} \\ k^{-2} & \text{for } k \gg k_{eq}. \end{cases}$$

And since $\tilde{\delta}(\vec{k}, a) \propto \tilde{\delta}_{primordial}(\vec{k})\, T(k)$

we have

$$P(k) \propto P_{primordial}\, T^2(k)$$

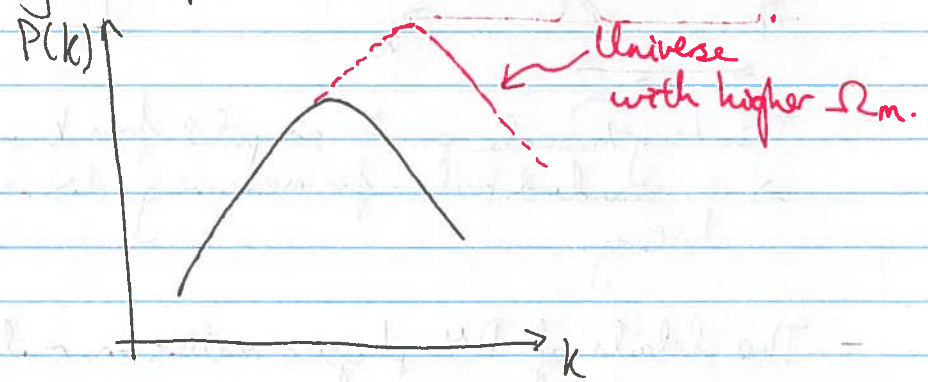
$\Rightarrow$ We finally have

$$P(k) \propto \begin{cases} k^{n_s} & \text{for } k \ll k_{eq} \\ k^{n_s - 4} & \text{for } k \gg k_{eq}. \end{cases}$$

$\Rightarrow$ Show matter power spectrum and ask slide Q

What physics might we learn from measuring the matter power spectrum?

- Primordial power spectrum at low k's $\Rightarrow n_s$.
- The turnover wavenumber depends on when matter-radiation equality took place $\Rightarrow \Omega_m$



- If we zoom into the high-k region, we notice some wiggles

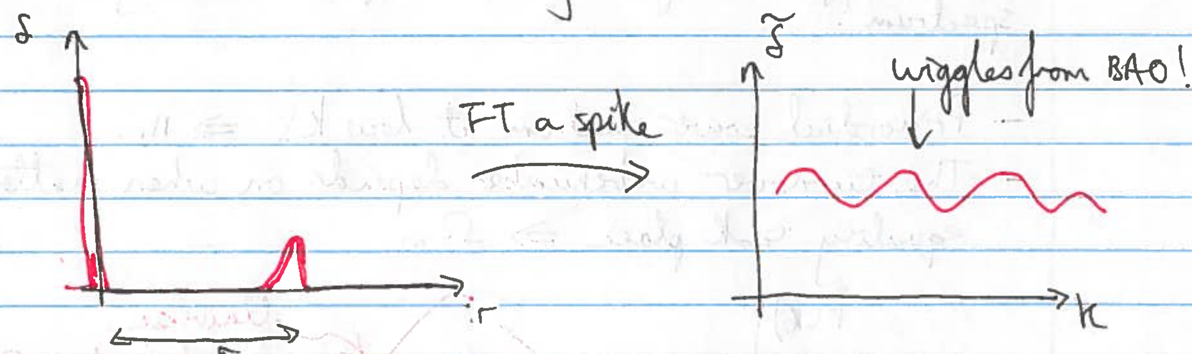
$\Rightarrow$ Show BAO pictures

What's going on here? Recall that last time I said that after recombination, the baryons decouple from the photons and essentially follow the CDM. This is mostly true, but it needs refinement — the CDM pulls on the baryons, sure, but the baryons also tug back on the CDM! So the whatever the baryons were doing with the photons before recombination does get imprinted on the matter pspec a tiny bit.

And what is going on? Remember that before recombination, the pressure from the photon-baryon fluid pushes out against clumping.

$\Rightarrow$ Show BAO movie

This means we have something like this:



This length scale can be computed from known physics
$\Rightarrow$ a standard ruler for measuring distances + expansion history.

- The details of DM physics matters on really small scales.

$\Rightarrow$ Cold dark matter clumps very easily and forms lots of structure on small scales.

$\Rightarrow$ Warm dark matter is more able to stream out of shallow potential wells, inhibiting small scale structure.

$\Rightarrow$ $\boxed{\text{Slides}}$

So looking at possible suppression on small scales can be very helpful. But <u>careful</u> – eventually one gets to high enough k values that nonlinear effects become important, with nonlinear baryonic galaxy feedback processes setting in.

↳ State of the art $\simeq$ people trying to see if they can model $k \sim 0.3\ h\,\mathrm{Mpc}^{-1}$ at the precision necessary for upcoming surveys.

How do we measure the matter power spectrum in practice?

The most conventional way is to do galaxy surveys. We measure the locations of galaxies, and point-by-point we build up a picture of the matter distribution.

⇒ $\boxed{\text{Pointilism slide}}$

where galaxies are concentrated, so is matter

Central assumption: Galaxies are biased tracers of δ.

Galaxies only form at peaks of density.

Some fct

$$\delta_g(\vec{r}) = b\left(\delta(\vec{r})\right) \approx b_0 + b_1\,\delta(\vec{r}) + \ldots\ldots$$

Assume $=0$ $\because$ if $\delta=0$, then so should δ_g.

constant

Often assume a linear biasing model: $\delta_g = b\,\delta$

But depending on the sophistication of the application, people often have things like scale-dependent bias $b(k)$ or quadratic bias etc.

Let's make a list (incomplete!) of some things to worry about with observations

① Bias parameter — extra nuisance parameter.

② Shot noise: discrete counting doesn't always capture the underlying distribution ⇝ mitigated by having lots of galaxies

⇒ $\boxed{\text{Shot noise slides}}$

③ Cosmic variance: our Universe is random, so what if I get unlucky and happen to survey an unrepresentative part?

↳ mitigated by doing surveys with large volume.

Cosmic variance can be thought of in the following way. The true power spectrum involves an ensemble average

True P → NOT random

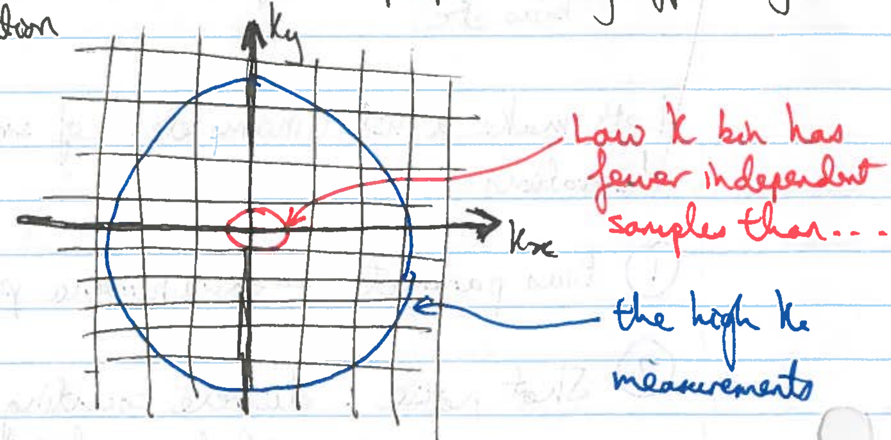
$$P(k) = \frac{\langle|\tilde{\delta}(\vec{k})|\rangle}{V}$$

We can't do an ensemble average in IR life! So we replace this ensemble average with an average over different directions, assuming isotropy.

Estimator of P → Has a random error

$$\hat{P}(k) = \frac{\langle|\tilde{\delta}(\vec{k})|\rangle_{\vec{k}\in k}}{V}$$

But if you imagine binning in bins of constant k like this, the small k's are disproportionately affected by this approximation



Effects (2) and (3) are ~~#~~ can often be traded off for one another.

→ If I use dim galaxies → More common ⇒ lower shot noise

→ Hard to see to large distances ⇒ higher cosmic variance because survey volume is limited

To be fair to quasars, modern samples are quite large, plus quasars also illuminate structure along sightlines

→ If I use bright objects (Eg quasars) → Rarer ⇒ higher shot noise

→ Easy to see over large volumes ⇒ lower cosmic variance

Traditionally, a good choice = luminous red galaxies

They are a "Goldilocks" galaxy type that balances these competing demands, although with more sensitive surveys these days people are expanding the sorts of galaxies being used.