PHY644 Problem set 1

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August 28, 2025

Problem 1: Redshifts

This problem concerns determining redshifts using photometric techniques with filters to identify the Lyman break. At the rest-frame Lyman limit, $\lambda_0 = 912, \text{Å}^1$, the flux drops to nearly zero due to absorption by stellar atmospheres and the interstellar medium (ISM) causing a step like feature. This is the basis of the "dropout technique." However, at high z this feature can be confused with absorption by the intergalactic medium (IGM), which affects the spectrum over the range $912, \text{Å} < \lambda_0 < 1216, \text{Å}$. The Lyman series corresponds to electronic transitions in hydrogen where electrons fall to the ground state ($n = 1$).

The photometric technique is less precise than spectroscopic redshifts, which are obtained by taking a full spectrum and fitting spectral lines. However, it is much cheaper and faster. This method relies on fitting template spectra to the observed data through the filters used.

A.

We estimate a photometric redshift using relative fluxes (magnitudes) from different filters (Figure). By comparing the template spectrum and filter responses in the video, we find $z \sim 4.6$ (Figure 2).

Filter	Δ mag
b	0.0
v	1.5
i	0.1
z	0.0

Figure 1: Photometric magnitude differences by filter.

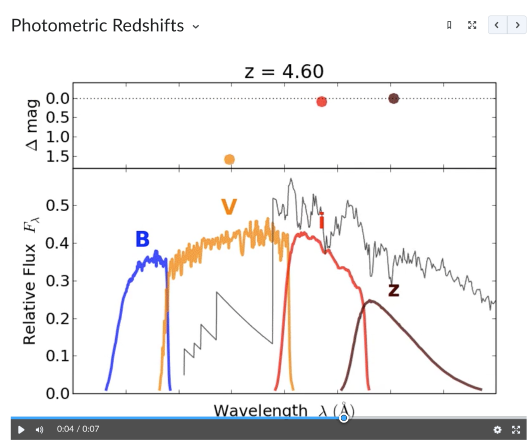

Figure 2: Screen shot of the photometric z simulator, from https://mycourses2.mcgill.ca/d2l/le/content/802628/viewContent/8637994/View

B.

This time we are asked to look at Figure 1 of the homework (not reproduced), and and reflect on what redshifts can be most cleanly identified with the Lyman dropout technique before it gets

[1]where λ_0 is the rest frame wavelength.

confused with absorption from the IGM.

Using the filter response, we can eyeball when the Lyman break z, and IGM absorption z, at the egde of the filters. Using this I think, below $z \sim 6$, photometric z are alright because the Lyman break moves through the optical bands.

Beyond $z \sim 6 - 7$, it starts to get hard / degeneracy with IGM absorption. (Excluding effects of dusty galaxies).

Problem 2: The Plummer Potential

The Plummer gravitational potential is:

$$\Phi(r) = \frac{-GM}{(r^2 + r_0^2)^{1/2}} \tag{1}$$

where r is the distance from the center, M is the total mass of the galaxy cluster, r_0 is the characteristic radius, and G is Newton's gravitational constant.

A.

We are asked to derive $\rho(r)$ — the mass density of the Plummer potential. My idea here is to use Gauss's law for gravity in differential form! It looks like this (It's in Griffith's EM):

$$\nabla \cdot \mathbf{g} = -4\pi G\rho \tag{2}$$

Recall that $\mathbf{g}(r) = -\nabla\Phi(r)$, putting this into Equation 2 we have:

$$\nabla \cdot (-\nabla\Phi) = -4\pi G\rho \quad \Rightarrow \quad \nabla^2\Phi = 4\pi G\rho \tag{3}$$

Recall for a spherically symmetric potential, the Laplacian in spherical coordinates is:

$$\nabla^2\Phi = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) \tag{4}$$

Now we just need to take a few derivatives and rearrange! I wonder if there is a faster way. Anyway lets start with $\frac{d\Phi}{dr}$:

$$\frac{d\Phi}{dr} = \frac{GMr}{(r^2 + r_0^2)^{3/2}} \tag{5}$$

The term in () in Equation 4 is then:

$$\Rightarrow \frac{GMr^3}{(r^2 + r_0^2)^{3/2}} \tag{6}$$

Taking the next derivative we get:

$$\Rightarrow GM\frac{3r^2r_0^2}{(r^2 + r_0^2)^{5/2}} \tag{7}$$

Tossing in the factor of $1/r^2$, the Laplacian aka Equation 4 is:

$$\nabla^2\Phi = \frac{3GMr_0^2}{(r^2 + r_0^2)^{5/2}} \tag{8}$$

Finally, using Equation 3, we solve for $\rho(r)$ - the mass density profile of the Plummer potential.

$$\boxed{\rho(r) = \frac{3Mr_0^2}{4\pi(r^2 + r_0^2)^{5/2}}} \tag{9}$$

Factoring the r_0 we get the form from the problem set.

$$\boxed{\rho(r) = \frac{3M}{4\pi r_0^3 \left(1 + \left(\frac{r}{r_0}\right)^2\right)^{5/2}}} \tag{10}$$