PHYS 644 Lecture # 12: Neutrinos

Last time we finished discussing the basic thermodynamic principles and started discussing thermal relics.

$\Rightarrow$ $\boxed{\text{Review slides.}}$

We applied this an example cold relic — the hypothetical WIMP dark matter. Now let's do a hot relic.

Neutrinos (hot relic application)

At $t \lesssim 1\,\text{sec}$, $T \gtrsim 1\,\text{MeV}$ (or $10^{10}\,\text{K}$), neutrino interactions are active:

I'm using "neutrinos" as shorthand for "neutrinos and antineutrinos"

$$e^- + \nu_e \rightleftharpoons e^- + \nu_e$$

$$e^+ + e^- \rightleftharpoons \nu_e + \bar{\nu}_e$$

etc.

These types of weak nuclear interactions are also felt by muons $\mu^\pm$ (talking to muon neutrinos $\nu_\mu, \bar{\nu}_\mu$), but with ~~≠~~ $m_\mu \sim 105\,\text{MeV}$, they have long annihilated by now.

To figure out when these interactions "turn off" (and therefore when ~~we~~ neutrinos decouple from being in thermal equilibrium from everyone else), we do our usual comparison of Γ and H.

This means knowing σ for these interactions $\Rightarrow$ $\boxed{\text{Slide Q on } \sigma \text{ scaling}}$

Now we have:

$v \approx c = 1$

$$\Gamma = n_\gamma \langle \sigma v \rangle \approx T^3 G_F^2 T^2 = G_F^2 T^5$$

$n_\gamma \propto T^3$ because relativistic!

With our radiation-dominated universe, we have $H \sim \frac{T^2}{m_{pl}}$

$$\Rightarrow \quad \frac{\Gamma}{H} \approx \frac{G_F^2 T^5}{T^2/m_{pl}} \approx \left(\frac{T}{1\,\mathrm{MeV}}\right)^3.$$

Thus neutrinos decouple at $\simeq 1$ sec or $T \sim 1$ MeV. Once they decouple, their number density simply dilutes as $1/a^3$. Their distribution function is a snapshot of what it looked like at decoupling, except their temperature goes like $1/a$:

$$T_\nu(a) = T_{\nu,\mathrm{decoupling}}\left(\frac{a_{dec}}{a}\right)$$

However, it's important to note that if I evaluate T_ν at $a=1$ (today), I do not get $T_\gamma = 2.73$ K. This is because soon after neutrinos decouple, we get...

Electron-Positron Annihilation

Even though by $T \sim 1$ MeV we have lost $e^- + \nu_e \rightleftarrows e^- + \nu_e$, the electron can keep interacting because it interacts with electromagnetism (since it's electrically charged) and has

$$e^- + e^+ \rightleftarrows 2\gamma.$$

But remember that $m_e \approx 0.5$ MeV, so not long after neutrino decoupling ($T = 1$ MeV vs 0.5 MeV; $t = 1$ sec vs 4 sec), the electrons become non-relativistic. Since they're still in thermal equilibrium, their abundances gets exponentially suppressed as annihilation dominates.

This causes the e^+/e^- energy density and entropy to be transferred to the photons, heating them up!

Let's work out the new photon temperature.

Recall that the entropy density $s = \frac{2\pi^2}{45} g_{*s} T^3$ where

$$g_{*s} \equiv \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8}\sum_{\text{fermions}} g_i \left(\frac{T_i}{T}\right)^3$$

and that ~~g_{*s}~~ $sa^3 \propto g_{*s} a^3 T^3$ is conserved because total entropy is conserved. If g_{*s} changes $\because$ e^+/e^- annihilate, then T must change in response.

Right before e^+/e^- ~~freezeout~~ annihilation $(T \gtrsim m_e)$:

$$g_{*s}^{\text{before}} = 2 + \frac{7}{8} \times 2 \times 2 = \frac{11}{2}$$

(No $\frac{T_i}{T}$ $\because$ all T_i the same before annihilation)

2 photon polarizations

e^+ and e^-

2 spin states for spin-$\frac{1}{2}$ particles

Why don't we include neutrinos? They're decoupled now, so they just separately conserve their contribution to entropy.

Right after e^+/e^- ~~freezeout~~ annihilation $(T \lesssim m_e)$:

$$g_{*s}^{\text{after}} \doteq 2$$

(e^+/e^- is gone, just photons!)

$$\Rightarrow \left(g_{*s}^{\text{before}}\right) a\, T_{\gamma,\text{before}}^3 = \left(g_{*s}^{\text{after}}\right) a\, T_{\gamma,\text{after}}^3$$

$$\left(\frac{11}{2}\right) T_{\gamma,\text{before}}^3 = (2)\, T_{\gamma,\text{after}}^3 \Rightarrow T_{\gamma,\text{after}} = \left(\frac{11}{4}\right)^{1/3} T_{\gamma,\text{before}}$$

Now, let's relate this back to the neutrino temperature T_ν

$\Rightarrow$ $\boxed{\text{Slide Q on } T_\nu \text{ vs } T_\gamma}$

Because T_ν and T_γ were both scaling as $1/a$ prior to e^+/e^- annihilation, $T_\nu = T_{\gamma,\text{before}}$, so we usually write

$$\boxed{T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma} \quad \text{Secretly } T_\gamma^{\text{after}}$$

It's a little unfair that we write this as $T_\nu = (\dots)T_\gamma$ rather than $T_\gamma = (\dots)T_\nu$ since the photons are the ones that changed! But observationally we can measure the CMB temperature, so we're usually computing T_ν from T_γ.

In other words, my careless words before, saying that $T \sim \frac{1}{a}$ for photons is only roughly true! Entropy conservation says it's actually

$$T_\gamma \propto \frac{1}{g_{*S}^{1/3}\, a},$$

and we're so used to dealing with $z \lesssim 10^9$ (≈ 1 MeV) when nothing else annihilates (after e^+/e^-) ~~that~~ such that g_{*S} doesn't change any more

$\Rightarrow$ $\boxed{\text{Show plots}}$

We can work out the neutrino temperature T_ν today:

$$T_{\nu,0} = \left(\frac{4}{11}\right)^{1/3} \underbrace{T_{\gamma,0}}_{2.725\,\text{K}} = 1.96 \text{ K}.$$

So there exists a relic Cosmic Neutrino Background of $T_{\nu,0} = 1.96$ K around today, having originated from much higher redshifts than the CMB.

Let's test our understanding by thinking about another hypothetical relic:

$\Rightarrow$ $\boxed{\text{Slide Qs on gravitons!}}$

With neutrinos, we have a hot relic that still exists today. We've figured out its temperature .... what else do we know?

One basic fact that we have is that there are ~~the~~ three types of them! Electron neutrinos $\nu_e, \bar{\nu}_e$
Muon neutrinos $\nu_\mu, \bar{\nu}_\mu$
Tau neutrinos $\nu_\tau, \bar{\nu}_\tau$

~~In today's universe, then~~, After e^+/e^- annihilation then, the effective # of relativistic degrees of freedom is then:

Why do we not include the two (spin states of $\nu, \bar{\nu}$ if they're spin ½? Recall the peculiarity of Nature — there seem to only be left-handed neutrinos and only right-handed anti-neutrinos

$$g_* = \underbrace{2}_{\text{2 photon polarizations}} + \underbrace{\frac{7}{8}}_{\text{Neutrinos are fermions}} \times \underbrace{2}_{\text{Two species } (\nu, \bar{\nu})} \underbrace{N_{eff}}_{\text{Effective \# of families/types}} \underbrace{\left(\frac{4}{11}\right)^{4/3}}_{\left(\frac{T_\nu}{T_\gamma}\right)^4} \approx 3.38$$

The number N_{eff} is not precisely 3 because of subtle quantum effects (Fermi-Dirac distribution changes slightly $\because$ high energy neutrinos interact more strongly).

Detailed theory calculation: $N_{eff} \simeq 3.046$ ~~Observations: $N_{eff} \simeq 2.9^{+0.3}$~~

Observations: $N_{eff} \simeq 2.99^{+0.34}_{-0.33}$

↳ where we artificially let N_{eff} be a free parameter when fitting cosmological datasets in order to see if there are other particle species that we don't know about.

Another thing that we know about neutrinos from particle physics experiments is that they have mass

We don't know what this mass (or rather set of three masses) is! But we do have some clues:

Neutrino particle physics experiments (Eg neutrino oscillations): $\sum_i m_{\nu,i} \gtrsim 0.05$ to $0.10\ \mathrm{eV}$

summing over three types

Actually Gershtein-Zel'dovich (1966)

Cowsik - McClelland bound (1972): $\Omega_\nu h^2 \approx \frac{\sum m_\nu}{94\ \mathrm{eV}}$

Where does this come from? First, note that if m_ν is, then today they are non-relativistic because $T_{\gamma,0} = 2.725\ \mathrm{K} \sim 0.24\ \mathrm{meV}$

"milli"!

Thus ρ_ν is dominated by rest mass energy and $\rho_\nu = \sum_{i=1}^{3} m_\nu n_\nu$.

Just the # density at freezeout diluted by $1/a^3$!

Then $\Omega_\nu \equiv \frac{\rho_{\nu,0}}{\rho_{crit,0}}$.

gives the constraint above.

Now, we know that $\Omega_{total} > 1$ would close the universe, and we live in a flat universe. Thus to avoid neutrinos from "overclosing the universe", we cannot have $\sum m_\nu$ too large

$$\Rightarrow \sum_\nu m_\nu \lesssim 50\ \mathrm{eV}$$

Modern CMB, galaxy surveys, and lensing: $\sum m_\nu \lesssim 0.06$ to $0.12\ \mathrm{eV}$

(depending on which experiments / analyses you trust).

We will go into this more later in the course, but roughly speaking, neutrinos are light and travel (relatively) quickly and free-stream out of overdensities, inhibiting structure formation

$\Rightarrow$ $\boxed{\text{Show simulations.}}$

Dark matter

Thus far, we have discussed two examples of thermal relics: neutrinos (hot relic) and WIMPs (cold relic). Could either be the dark matter? The latter was proposed as a DM candidate, so they're a possibility but we haven't seen them yet. The former cannot be ~~most~~ a substantial fraction of DM because such a hot relic ("hot dark matter") would wash out too much structure (as explained above).

This is generically an issue for low-mass candidates if they started their lives in thermal equilibrium, so one possible loophole is if they were "born cold".

Right now, the particle nature of DM is highly uncertain, but we do know some things:

- Basic cosmological properties (Eg Ω_m, Ω_b)
- Locally, $\rho_{DM} \sim 0.4 \pm 0.1$ GeV/cm^3.
- Should be stable, such that lifetime $\tau \gg H_0^{-1}$.
- Upper bound on interaction rates (or we would've seen interactions).
- Upper bound on DM velocity / free streaming (see above).

Then there are some interesting constraints on m_{DM}, the mass of the DM particle...

- An upper bound on mass, $m_{DM} \ll$ Observed DM halo masses.
- Lower bounds on mass
  - ↳ If bosons, de Broglie wavelength must fit inside smallest DM halos $\Rightarrow m_{DM} \gtrsim 10^{-22}$ eV.
  - ↳ If fermions, Pauli exclusion cannot prevent existence of smallest DM halos (which have limited size and escape velocity) $\Rightarrow m_{DM} \gtrsim$ keV

    ("Tremaine-Gunn bound")

But there are still 90 orders of magnitude allowed!

$\Rightarrow$ Show slide with OOMs