PHYS644 Problem Set 9

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Problem 1: A more practical expression for the power spectrum

Problem 1A:

Let's start by recalling the definition of the forward Fourier transform in this notation.

$$\tilde{\delta}_{obs}(k) = \int d^3\mathbf{r}\delta_{obs}(\mathbf{r})\exp(-i\mathbf{k}\cdot\mathbf{r}) \tag{1}$$

The inverse transform is:

$$\delta(\mathbf{r}) = \frac{1}{8\pi^3}\int d^3\tilde{k}\tilde{\delta}(\mathbf{k})\exp(i\mathbf{k}\cdot\mathbf{r}) \tag{2}$$

With our value of $\delta_{obs}(\mathbf{r}) = \gamma(\mathbf{r})\delta(\mathbf{r})$. We can write the inverse Fourier[1] transform of $\delta(\mathbf{r})$, and then plug into the forward one.

Step 1:

$$\delta_{obs}(\mathbf{r}) = \frac{\gamma(\mathbf{r})}{8\pi^3}\int d^3\tilde{k}'\tilde{\delta}(\mathbf{k}')\exp(i\mathbf{k}'\cdot\mathbf{r}) \tag{3}$$

Now we put this entire expression into the forward transform for $\delta(\mathbf{r})$.

$$\tilde{\delta}_{obs}(k) = \int d^3\mathbf{r}\frac{\gamma(\mathbf{r})}{8\pi^3}\int d^3\tilde{k}'\tilde{\delta}(\mathbf{k}')\exp(i\mathbf{k}'\cdot\mathbf{r})\exp(-i\mathbf{k}\cdot\mathbf{r}) \tag{4}$$

Now we want to combine the exponential s terms, and recognize the Fourier transform of $\gamma(\mathbf{r})$. The Forward transform is $\tilde{\gamma}(\aleph) = \int d^3\mathbf{r}\gamma(\mathbf{r})\exp\left(-i\aleph\cdot r\right)$.

$$\tilde{\delta}_{obs}(k) = \int d^3\mathbf{r}\frac{\gamma(\mathbf{r})}{8\pi^3}\int d^3\tilde{k}'\tilde{\delta}(\mathbf{k}')\exp(-i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{r}) \tag{5}$$

In our case $\aleph = k - k'$.

$$\boxed{\tilde{\delta}_{obs}(k) = \int\frac{\tilde{\gamma}(\mathbf{k}-\mathbf{k}')}{8\pi^3}d^3\tilde{k}'\tilde{\delta}(\mathbf{k}')} \tag{6}$$

I think this is a convolution?

Problem 1B:

We want to compute:

$$\langle|\tilde{\delta}_{\rm obs}(\mathbf{k})|^2\rangle = \langle\tilde{\delta}_{\rm obs}(\mathbf{k})\tilde{\delta}^*_{\rm obs}(\mathbf{k})\rangle \tag{7}$$

This is the definition, and the * is the complex conjugate because we are dealing with complex valued stuff. We know from Problem 1A what $\tilde{\delta}_{\rm obs}$ is, so we can write:

[1] Fourier is a persons name and should be capitalized.

$$\langle|\tilde{\delta}_{\rm obs}(\mathbf{k})|^2\rangle = \int \frac{\tilde{\gamma}(\mathbf{k}-\mathbf{k}_1)}{8\pi^3} d^3\tilde{k}'\tilde{\delta}(\mathbf{k}_1) \int \frac{\tilde{\gamma}^*(\mathbf{k}-\mathbf{k}_2)}{8\pi^3} d^3\tilde{k}'\tilde{\delta}^*(\mathbf{k}_2) \tag{8}$$

From Eqn 1 in the handout / the definition of the power spectrum we know

$$\langle\tilde{\delta}(\mathbf{k}_1)\tilde{\delta}^*(\mathbf{k}_2)\rangle = (2\pi)^3\,\delta^D(\mathbf{k}_1-\mathbf{k}_2)\,P(\mathbf{k}_1) \tag{9}$$

Why do we write the Dirac with a top exponet? This is horrible, I might switch to a subscript. Anyway, we can use this to replace our $\delta(k)$'s in the integral form.

$$\langle|\tilde{\delta}_{\rm obs}(\mathbf{k})|^2\rangle = \int \frac{\tilde{\gamma}(\mathbf{k}-\mathbf{k}_1)}{8\pi^3} d^3\tilde{k}' \int \tilde{\gamma}^*(\mathbf{k}-\mathbf{k}_2) d^3\tilde{k}'\,\delta^D(\mathbf{k}_1-\mathbf{k}_2)\,P(\mathbf{k}_1) \tag{10}$$

One of the for factors also cancels!, Now we can use the Dirac function to reduce one of the integrals.

$$\langle|\tilde{\delta}_{\rm obs}(\mathbf{k})|^2\rangle = \int \frac{|\tilde{\gamma}(\mathbf{k}-\mathbf{k}_1)|^2}{8\pi^3} d^3\tilde{k}' P(\mathbf{k}_1) \tag{11}$$

And $k_1 = k'$

$$\boxed{\langle|\tilde{\delta}_{\rm obs}(\mathbf{k})|^2\rangle = \int \frac{|\tilde{\gamma}(\mathbf{k}-\mathbf{k}')|^2}{8\pi^3} d^3\tilde{k}' P(\mathbf{k}')} \tag{12}$$

Problem 1C:

Oki, so the first half of this is easy, we say $\gamma \approx$ a Dirac function, for the second half we will have to look up Parseval's theorem.

We can pull the P factor out of the integral.

$$\langle|\tilde{\delta}_{\rm obs}(\mathbf{k})|^2\rangle = P(\mathbf{k}) \int \frac{|\tilde{\gamma}(\mathbf{k}-\mathbf{k}')|^2}{8\pi^3} d^3\tilde{k}' \tag{13}$$

Now our hope lies in Parseval. Parseval's theorem says:

$$\int d^3r|f(\mathbf{r})|^2 = \int \frac{d^3k}{(2\pi)^3}|\tilde{f}(\mathbf{k})|^2 \tag{14}$$

We can change the γ from Fourier space - the tilde to normal space with an r, and inside the survey space it is equal to 1.

$$\int d^3r|\gamma(\mathbf{r})|^2 = V \tag{15}$$

So we can say:

$$\langle|\tilde{\delta}_{\rm obs}(\mathbf{k})|^2\rangle = P(\mathbf{k})V \tag{16}$$

Or matching the form on the handout

$$\boxed{P(\mathbf{k}) = \frac{\langle|\tilde{\delta}_{\rm obs}(\mathbf{k})|^2\rangle}{V}} \tag{17}$$

Problem 2: Silk Damping