PHYS644 Final Problem Set

Maxwell A. Fine: SN 261274202
maxwell.fine@mail.mcgill.ca
December 10, 2025

Problem 1: Peculiar Velocities

Problem 2: Useful expressions for low-redshift cosmology ($z << 1$)

Problem 2A:

We are asked to find an expression for the commoving distance d as a function of redshift z, and then to comment on if it matters what type of distance.

We can start from the equation for commoving distance in natural units is.

$$d(z) = \int_0^z \frac{1}{H(z)}\, dz \tag{1}$$

For the case that $z << 1$, The Hubble parameter is a constant, $H(z) \approx H_0$. So we can take it out of the integral, and then the integral is trivial.

$$\boxed{d(z) = \frac{1}{H_0}\int_0^z dz = \frac{z}{H_0}} \tag{2}$$

Does the requested kind of distance matter? To first order in z not really: comoving, proper/physical, luminosity and angular-diameter distances differ by factors of $(1 + z)$. But to first order those are now 1.

Problem 2B:

Now we are asked to write the recession velocity at redshift z.

$$\boxed{v = H_0 d = v = H_0 \frac{z}{H_0} = z} \tag{3}$$

This is pretty simple, we can just directly substitute into our earlier expression.

Problem 2C:

We are asked to show that H_0^{-1} is close to a round number when expressed in units of $h^{-1}\,\mathrm{Mpc}$. h here is dimensionless, by definition is $H_0 = 100h\mathrm{kms}^1\mathrm{Mpc}^1$. Given that it says this will be usefull for expressions like in problem 1A, it is assumed that this expression has a factor of c possibly mixed in.

Using $c \approx 3 \times 10^5\,\mathrm{km\,s^{-1}}$, we obtain

$$\frac{c}{H_0} = \frac{3\times 10^5}{100h}\,\mathrm{Mpc} = \frac{3000}{h}\,\mathrm{Mpc} = 3000\,h^{-1}\,\mathrm{Mpc}.$$

To one significant digit, H_0^{-1} to $\boxed{c/H_0 \approx 3\times 10^3\,h^{-1}\,\mathrm{Mpc}}$.

Problem 2D:

We are asked to now express the comoving volume per solid angle per redshift interval. The comoving volume element per unit solid angle and redshift is given by:

$$\frac{dV}{d\Omega\, dz} = d^2(z)\,\frac{dd}{dz}. \tag{4}$$

This comes from $dV = d^2 d\Omega dd$ which is the area multiplied by the thickness of our solid angle "slice", and divide by $d\Omega$, and change from dd to dz using the chain rule $dd = \frac{dd}{dz}dz$

Since we are in $z << 1$, we can use the low-redshift approximation.

$$d(z) \approx \frac{z}{H_0}, \tag{5}$$

$$\frac{dd}{dz} \approx \frac{1}{H_0}. \tag{6}$$

We can substitute these into the volume element expression above giving:

$$\frac{dV}{d\Omega\, dz} \approx \left(\frac{z}{H_0}\right)^2 \frac{1}{H_0} = \frac{z^2}{H_0^3}. \tag{7}$$

Rewriting:

$$\boxed{\frac{dV}{d\Omega\, dz} \approx \frac{z^2}{H_0^3}} \tag{8}$$

Problem 2E:

We are asked to compute the following quantities without a calculator or a computer[1].

We are using in natural units $H^{-1} \approx c/H_0 \approx 3 \times 10^3\, h^{-1}\,\text{Mpc}$.

- The comoving distance to $z = 0.1$. Answer: $\frac{z}{H_0} = 0.1 * 3 \times 10^3\, h^{-1}\,\text{Mpc} = \boxed{3 \times 10^2\, h^{-1}\,\text{Mpc}}$.
- The comoving volume out to $z = 0.1$. Answer: $\frac{z^2}{H_0^3} = 0.1^2 * (3 \times 10^3\, h^{-1}\,\text{Mpc})^3 = \boxed{3 \times 10^1\, (h^{-1}\,\text{Mpc})^3}$. I am assuming it is asking about the comoving volume per solid angle per redshift interval, as that makes more sense. I'm happy to be wrong.
- What redshift does a galaxy with a peculiar velocity of $v_p \approx 300\,\text{km/s}$ have this peculiar velocity be 10% of its recession velocity? How far away is such a galaxy? Answer: $v_p = 0.1 v_{rec}$, then $\boxed{v_{rec} = 3000\,\text{km/s}}$. Then $\boxed{z \approx \frac{v_{rec}}{c} \approx 0.01}$

[1] There is no rule against using slide rules!!

Problem 3: CMB power spectrum in a different universe