PHYS644 Problem Set 8

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Problem 1: Quadratic inflation during reheating

Some refrence equations

$$V(\phi) = \frac{1}{2}\alpha^2\phi^2 \tag{1}$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \tag{2}$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{3}$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{4}$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] \tag{5}$$

Problem 1A:

Equation 3 from the equation block 1, has units of

$$[\rho] = [ML^1t^2] = \tag{6}$$

For this to be true, the units of $\dot{\phi}^2$, and V must also be energy density. There for, $[\dot{\phi}^2] = [ML^1t^2] \Rightarrow [\phi] = [M^{1/2}L^{-1/2}]$, and $[V] = [ML^1t^2]$. From 1, we know $[\alpha^2][ML^1] = [ML^1t^2] \Rightarrow [\alpha] = [t^{-1}]$, these are the same units as H.

I'm not sure what to box here.

Problem 1B:

Assuming that $\alpha >> H$, we are told that we discard the H term in equation 2. We are asked with finding the solution. With $H << \alpha$ we have:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \Rightarrow \ddot{\phi} + +V'(\phi) = 0 \tag{7}$$

Writing this out we have:

$$\frac{d^2\phi}{dt^2} = -\frac{dV}{d\phi} = -2\alpha^2\phi \tag{8}$$

And we can directly solve, its a classical sinusoidal solution.

$$\boxed{\phi(t) = A\cos(\alpha\, t) + B\sin(\alpha\, t) = A\cos\left(\alpha t + \delta\right)} \tag{9}$$

And we are free to choose A, B to meet the boundary conditions. I like cos.

Problem 1C

We can write the equation of state parameter w as:

$$w(t) = \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \tag{10}$$

We know $\dot{\phi} = A\alpha t \sin(\alpha t + \delta) \Rightarrow \dot{\phi}^2 = A^2\alpha^2 t^2 \sin(\alpha t + \delta)$, and $V(\phi) = \frac{1}{2}\alpha^2 A^2 \cos^2{(\alpha t + \delta)}$.

The time averaged kinetic like, and potential like energies are (averaged over many oscillations since $\alpha >> H$):

$$\left\langle \frac{1}{2}\dot{\phi}^2 \right\rangle = \frac{1}{4}\alpha^2\phi^2, \tag{11}$$

$$\langle V(\phi) \rangle = \frac{1}{4}\alpha^2\phi^2 \tag{12}$$

if $w = 0$, then $< p >= 0$, so we can test this:

$$\frac{1}{2}\dot{\phi}^2 - V(\phi) = \frac{1}{4}\alpha^2 - \frac{1}{4}\alpha^2 = 0 \tag{13}$$

This means that $\boxed{w = 0}$

Problem 2: Silk Damping

The first half of this problem isn't physics :((I was looking forward to doing a random walk problem.

The problem says:

$$r_{\text{silk,phy}} = \sqrt{\frac{|\Delta z|}{z} H^{-1} \frac{c}{\sigma_t n_e}} \tag{14}$$

And we should use $\Delta z = 200, z = 1000$, $n_e = 0.25\,\mathrm{m}^{-3} a_{decouple}^{-3}$.

We can convert to the comoving scale using $(1+z) = a$:, and $n_e = n_{e,0}(1+z)^3$. This is physics.

$$r_{\text{silk,comoving}} = (1+z)\sqrt{\frac{|\Delta z|}{z} H^{-1} \frac{c}{\sigma_t n_{e,0}(1+z)^3}} = (1+z)^{-1/2}\sqrt{\frac{|\Delta z|}{z} H^{-1} \frac{c}{\sigma_t n_{e,0}}} \tag{15}$$

We should use matter dominated H because this is the in the regime of a matter dominate universe. This is also physics.

$$H^{-1} = \frac{1}{H_0\sqrt{\Omega_m(1+z)^3}} = H_0^{-1}\Omega_m^{-1/2}(1+z)^{-3/2} \tag{16}$$

$$r_{\text{silk,comoving}} = (1+z)^{-5/4}\sqrt{\frac{|\Delta z|}{z} H_0^{-1}\Omega_m^{-1/4} \frac{c}{\sigma_t n_{e,0}}} \tag{17}$$

For our numerical calculation, we will use the `astropy` value of H_0, and instead of approximating the universe as matter dominated, we can use the `astropy` model to simply compute $H(z = 1000)$

Our solution is Silk damping scale (comoving): $\boxed{9.27\,\mathrm{Mpc}}$. Code is available on the github for the class.