PHYS 644 Lecture #9: Cosmological Parameters and Distances

What we did last time was to examine the Friedman equations that govern the expansion of our Universe

$\Rightarrow$ **Review Slides**

What we've solved is the evolution for single-component universes, but in reality, everything is mixed in for our actual Universe!

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{r,0}\left(\frac{1}{a}\right)^4 + \frac{8\pi G}{3}\rho_{m,0}\left(\frac{1}{a}\right)^3 - \frac{\kappa}{a^2} + \frac{8\pi G}{3}\rho_\Lambda$$

Because the different constituents have different dependencies on "a", we have different components dominating as "a" increases with our Universe's expansion. We go from radiation domination to matter domination to curvature domination (although our Universe happens to have $\kappa = 0$, so we skip this phase, as far as we can tell) to dark energy domination.

Thinking about multi-component universes is more than just because our Universe is multi-component; it's also of formal theoretical interest. Today we are deep into the dark energy-dominated era, so sometimes we say $\rho = \rho_\Lambda = \text{const.}$ and $a \propto \exp(H_0 t)$, but...

$\Rightarrow$ **de Sitter slicing slides**

A constant energy density means $\rho(t)$ doesn't evolve with time, so I can slice however I want in spacetime and still respect isotropy and homogeneity, because it doesn't matter if I mix different times. In reality, we still have $\rho_m \neq 0$, which breaks the symmetry and gives us a preferred slicing where we have homogeneity and isotropy.

Which we saw from the slides gives any spatial curvature we want!

We therefore want to describe multi-component universes, which means we need to specify exactly what mix we have.

Take the Friedman equation and evaluate it today

Recall the "0" notation for present day but ~~ρ_Λ~~ ρ_Λ is constant, so no need for subscript.

$$H_0^2 = \frac{8\pi G}{3}\left(\rho_{m,0} + \rho_{r,0} + \rho_\Lambda\right) - \frac{k}{a_0^2}$$

Divide both sides by H_0^2, so that.....

$$1 = \underbrace{\frac{8\pi G \rho_{m,0}}{3H_0^2}}_{\equiv \Omega_{m,0}} + \underbrace{\frac{8\pi G \rho_{r,0}}{3H_0^2}}_{\equiv \Omega_{r,0}} + \underbrace{\frac{8\pi G \rho_\Lambda}{3H_0^2}}_{\equiv \Omega_\Lambda} - \underbrace{\frac{k}{a_0^2 H_0^2}}_{\equiv \Omega_{k,0}}$$

$$\Omega_0 \equiv \Omega_{m,0} + \Omega_{r,0} + \Omega_\Lambda + \ldots$$

("Ω total" without a subscript means the sum of everything except $\Omega_{k,0}$)

The Friedman equation is now a sum rule that must add up to 1. Another way to phrase this is that all the densities are now expressed ~~as~~ as ratios to the critical ~~den~~ density:

$$\rho_{crit,0} \equiv \frac{3H_0^2}{8\pi G} \simeq 1.877 \times 10^{-26}\, h^2 \ \mathrm{Kg/m^3}$$

Recall $H_0 \equiv 100h \,\frac{\mathrm{Km/s}}{\mathrm{Mpc}}$.

This is approximately 1 atom per m^3.

In general, then for the i^{th} constituent of stuff, we have

$$\Omega_{i,0} \equiv \frac{\rho_{i,0}}{\rho_{crit}}.$$

The critical density is interesting because it says that if we add up the different sources of energy density in our Universe and we get ρ_{crit}, then Ω_k must be equal to zero, and we can infer that our Universe is spatially flat ($k=0$). If it falls on either side of ρ_{crit}, we would interpret it as saying we have a closed or open spatial geometry.

What do we find for our Universe?

Mostly from Planck CMB results with some input from others

	Parameter	Symbol	Current belief
1	Spatial curvature	$\Omega_{k,0}$	0.001 ± 0.002
2	Matter density	$\Omega_{m,0}h^2$	0.14240 ± 0.00087
	Baryon density	$\Omega_{b,0}h^2$	0.02242 ± 0.00014
5	Hubble constant	H_0	$(67.9 \pm 0.7)\ \frac{\text{km/s}}{\text{Mpc}}$
3	Amplitude of matter fluctuations	A_s	$(2.10 \pm 0.03) \times 10^{-9}$
	Scalar spectral index	n_s	0.966 ± 0.005
4	CMB optical depth	τ	0.0561 ± 0.0071
	CMB temperature	$T_{CMB,0}$	(2.7255 ± 0.0006) K
	Dark energy w	w	-1.04 ± 0.06

Sum of neutrino masses	$\sum m_\nu$	< 0.12 eV (95% credibility)
Effective # of neutrino species	N_{eff}	$2.99^{+0.34}_{-0.33}$ (95% cred)
Tensor to scalar ratio	r	< 0.106 (95% cred)

A few notes about this:

1. Our universe is spatially flat — more on this when we study inflation

2. Why are we quoting things like $\Omega_{m,0} h^2$ rather than Ω_m?

Notice how $\Omega_{m,0} \equiv \frac{8\pi G \rho_{m,0}}{3H_0^2}$, which means that $\rho_{m,0} \propto \Omega_{m,0} h^2$ is ~~the~~ proportional to the physical density (ie the what our Universe actually cares about, rather that the Ω's, which were designed for human mathematical convenience).

Note also that Ω_m is the total matter density, including baryons and cold dark matter.

Important to remember that $\Omega_b \approx \frac{1}{6}$ to $\frac{1}{7}$ of Ω_m.

3. These are parameters governing the "lumpiness" of the matter distribution. We'll discuss them later in the course!

(4) This is the (small but non-zero) probability that a CMB photon is ~~~~ scattered on its way to our telescope. We will also discuss this later.

(5) Sometimes people will say that our Universe is well-fit by a "6-parameter ΛCDM model".

Constant vacuum energy $w=-1$

Cold dark matter

There are two points I want to raise about this. The first is that there is some freedom as to which parameters we consider independent. For example, notice how Ω_Λ is not a parameter in the table. That's because we can compute it:

$$1 = \Omega_{m,0} + \Omega_\Lambda + \underbrace{\Omega_k}_{=0 \text{ assuming flat universe}}$$

$$\Rightarrow \Omega_\Lambda = 1 - \frac{(\Omega_{m,0} h^2)}{h^2}$$

← independent param from table

← know this from H_0.

Ends up being $\approx 0.6889 \pm 0.0056$

The other point is that it's not really 6 parameters! It's only 6 if we fix certain parameters.

Eg theory motivations to fix $\Omega_{k,0} = 0$

or independent experiments that measure T_{CMB} exquisitely.

The last four are also common extensions to ΛCDM. With increasing precision in cosmological probes, a lot of these – such as $\sum m_\nu$ – are becoming non-optional.

Aside from deriving other parameters that appear in the Friedman equation (like Ω_Λ), there are other things we can derive. Here are some important ones:

Photon density: if we know T_{CMB} and we expect that the ambient bath of CMB photons has a blackbody spectrum, then:

Energy density of photons

$$\rho_{\gamma,0} = 4\sigma T^4_{CMB,0} \Rightarrow \Omega_{\gamma,0} h^2 \sim 2.47 \times 10^{-5}$$

Neutrinos are light and at early times (when our Universe was very hot) they also travelled relativistically and add to an overall energy density of "radiation".

But what's T_ν? Stay tuned for next time!

Similar logic gives us $\Omega_{\nu,0} h^2 \sim 1.68 \times 10^{-5}$.

Then we like to say $\Omega_{r,0} \equiv \Omega_{\gamma,0} + \Omega_{\nu,0} \sim 8.5 \times 10^{-5}$

Matter–radiation equality: The energy density of radiation may be small today, but since $\rho_r \sim \frac{1}{a^4}$ whereas $\rho_\Lambda = \text{const.}$ and $\rho_m \sim \frac{1}{a^3}$, at early times when $a(t)$ was small it used to dominate!

Cross-over point: $\rho_m(z_{eq}) = \rho_r(z_{eq})$

$$\cancel{\rho_{m,0}(1+z)} \quad \rho_{m,0}\left(\frac{a_0^3}{a_{eq}^3}\right) = \rho_{r,0}\left(\frac{a_0^4}{a_{eq}^4}\right)$$

$$\Rightarrow \rho_{m,0} = \rho_{r,0}(1+z_{eq})$$

$$\Rightarrow z_{eq} = \frac{\rho_{m,0}}{\rho_{r,0}} - 1 = \frac{\Omega_{m,0}}{\Omega_{r,0}} - 1 \simeq 3250.$$

Matter - dark energy equality: Similarly, there is a crossover point for matter domination and dark energy domination.

$$\rho_\Lambda(z_{DE}) = \rho_m(z_{DE})$$

$$\Rightarrow \rho_\Lambda = \rho_{m,0}(1+z_{DE})^3 \Rightarrow z_{DE} = \left(\frac{\Omega_\Lambda}{\Omega_{m,0}}\right)^{1/3} - 1$$

This gives $z_{DE} \sim 0.5$. Note that this is different from $z_{acc} \sim 0.86$, when our Universe began to accelerate (ie it's not necessary for dark energy to be dominant for acceleration to begin).

Age of our Universe: let us rewrite the Friedman equation at $z \neq 0$ as follows:

Recall that $1+z = \frac{1}{a}$

$$H^2(z) = \frac{8\pi G}{3}\left(\rho_{m,0}(1+z)^3 + \rho_{r,0}(1+z)^4 + \rho_\Lambda\right) - \kappa(1+z)^2$$

But since $H_0^2 = 8\pi G \rho_{crit,0}/3$, we can say

$$H^2(z) = H_0^2 \underbrace{\left[\Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4 + \Omega_\Lambda + \Omega_{k,0}(1+z)^2\right]}_{\equiv E(z)^2 \text{ for convenience.}}$$

Now, $(1+z) = \frac{1}{a}$, so $-\frac{\dot{a}}{a^2} = \frac{dz}{dt} \Rightarrow -H(z)(1+z) = \frac{dz}{dt}$

$$\Rightarrow \boxed{t(z) = \frac{1}{H_0}\int_{-\infty}^{z} \frac{dz'}{(1+z')E(z')}}$$

Hubble time

Setting $z = 0$ gives $t_0 \sim 13.8$ Gyr.

Distance measures: as we've mentioned before, if I see a high redshift object, its an object that's very far away.

Can now get this from FLRW metric $\Rightarrow$ $\boxed{\text{Show slide with metric}}$

If I imagine a photon travelling radially ~~out~~ inward towards me from the distant universe, I get:

Photons travel along null geodesics

$$ds^2 = 0 \Rightarrow d\chi = \frac{dt}{a(t)}$$

$$\Rightarrow \chi = \int \frac{dt}{a(t)} = \int \frac{da}{a^2 H} = -\int_z^0 \frac{dz'}{H(z')}$$

$$\Rightarrow \boxed{\text{Comoving dist. to } z \equiv \chi(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')}}$$

All distances in cosmology proportional to c/H_0.

The comoving distance isn't directly observable, so it's not great for cosmological tests (though very useful for theory!)

One way we might define an observable distance is the luminosity distance d_L.

Flux ~~luminosity~~ of distant object

$$f = \frac{L}{4\pi d_L^2}$$

Luminosity of distant object

As photons spread out from the source, they spread themselves over area

$$A = 4\pi S_k(\chi)^2 .$$

But there are a couple of other effects to account for:

① the photons redshift, and since $E = \frac{hc}{\lambda}$, they're less energetic by a factor of $(1+z)$ when received.

② the physical distance stretches by a factor of $(1+z)$, so successive photons take longer to arrive ~~by (1+z)~~.

$$\Rightarrow \quad L_0 = \frac{\Delta E_0}{\Delta t} = \frac{\Delta E_e/(1+z)}{\Delta t_e (1+z)} = \frac{L}{(1+z)^2}$$

Observed today

$$S_k(x) = \begin{cases} \sinh x, & k<0 \\ x, & k=0 \\ \sin x, & k>0 \end{cases}$$

is the dimensionless version of S_K

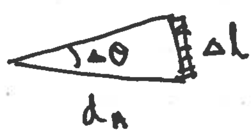
Therefore, $f = \frac{L_0}{A} = \frac{L}{4\pi S_K(\chi)^2 (1+z)^2}$

$$\Rightarrow \quad \boxed{d_L(z) = (1+z) S_K(\chi) = \frac{c}{H_0} \frac{(1+z)}{\sqrt{|\Omega_{k,0}|}} S_k\left[\sqrt{|\Omega_{k,0}|} \int_0^z \frac{dz'}{E(z')}\right]}$$

If we have a standard candle where we know L, then measuring f at a number of different redshifts lets us plot $d_L(z)$, which we can fit cosmological parameters using this formula (recall that Ω_m, Ω_Λ etc. hidden in $E(z)$)

An alternative to measuring fluxes is to measure the sizes of objects. This gives us the angular diameter distance d_A.

$$d_A \equiv \frac{\Delta l}{\Delta \theta} \quad \begin{matrix} \leftarrow \text{proper size} \\ \leftarrow \text{angular extent} \end{matrix}$$



From the metric, get arc length $\Delta l = a(t) S_K(\chi) \Delta\theta$

$$\Rightarrow \quad \boxed{d_A(z) = \frac{S_K(\chi)}{(1+z)} = \frac{d_L(z)}{(1+z)^2}}$$

Analogously, if I have a standard ruler where I know the physical size of something, I can measure its angular size as a function of z to constrain cosmological parameters.

Note how d_A and d_L differ by $(1+z)^2$. They both come from the same geometric considerations of the metric, but d_L has those two extra effects we mentioned earlier

$\Rightarrow$ $\boxed{\text{Show plot of } d_A \text{ and } d_L}$

At high redshifts, d_A is not monotonic! At high enough z, our Universe was small enough that photons emitted from the two ends of the standard ruler were really close together.