Cosmic Microwave Background Part 1

Today we begin to discuss one of the most successful probes of cosmology — the Cosmic Microwave Background (CMB)

First, let us remind ourselves why the CMB exists

⇒ $\boxed{\text{Slides with qualitative CMB}}$ → $\boxed{\text{And CMB beach ball!}}$

So when we look at the CMB, we get to see early in our Universe. Now, why do we see these anisotropies? Sure, ~~this~~ we can say that there are fluctuations, but let's be more precise.

In talking about the matter power spectrum and all that, we've talked about inhomogeneities. These get imprinted as anisotropies in the CMB.

⇒ $\boxed{\text{Slide with last scattering movie}}$ → (In the last frame of the movie, why doesn't this imply we should see blobs of different sizes in different parts of the sky?)

This tells us that to predict what the CMB, we have to worry about several things:

1. Primary CMB fluctuations (connected to, say, density fluctuations) → must be consistent with galaxy surveys!

2. Projection effects from Fourier modes to sphere.

3. Secondary effects that affect photons as they travel to our telescopes.

We'll give a bit of a treatment of each contribution, although because of time we will necessarily have to keep our discussion semi-qualitative.

But first, let's figure out how we describe these fluctuations. Just as with density fluctuations we saw that if we looked at things in harmonic space, we got some useful insights. Same here.

	Inhomogeneities	Anisotropies
Field	$\delta(\vec{r})$ (overdensity)	$\Theta(\hat{r}) \equiv \frac{\delta T}{T}$ (fractional temp. difference)
Harmonic space	$\tilde{\delta}(\vec{k}) = \int d^3\vec{r}\, e^{-i\vec{k}\cdot\vec{r}} \delta(\vec{r})$	$a_{lm} = \int d\Omega\, \underbrace{Y^*_{lm}}_{\text{Spherical harmonics}}(\hat{r})\, \Theta(\hat{r})$
	$\delta(\vec{r}) = \int \frac{d^3\vec{k}}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \tilde{\delta}(\vec{k})$	$\Theta(\hat{r}) = \sum_{lm} Y_{lm}(\hat{r})\, a_{lm}$
Length / angular scales	$\lambda \sim \frac{2\pi}{k}$	$\theta \sim \frac{180°}{l}$
Power spectrum	$\langle \tilde{\delta}(\vec{k}) \tilde{\delta}(\vec{k}')^* \rangle = (2\pi)^3 \delta^D(\vec{k}-\vec{k}')\, P(k)$	$\langle a_{lm} a^*_{l'm'} \rangle = \delta_{ll'} \delta_{mm'} C_l$
"Practical" estimator	$\hat{P}(k) = \frac{\sum_{\vec{k}\in k} \lvert\tilde{\delta}(\vec{k})\rvert^2}{V}$	$\hat{C}_l = \frac{1}{2l+1} \sum_{m=-l}^{l} \lvert a_{lm}\rvert^2$
"Dimensionless" power spectrum	$\Delta^2(k) \equiv \frac{k^3 P(k)}{2\pi^2}$	$D_l = \frac{l(l+1)}{2\pi} C_l$.

Just as with the matter power spectrum there was some really good physics governing the shape of $P(k)$, here we want to look at the shape of C_ℓ to learn about our Universe.

Let's first look at our target

$\Rightarrow$ $\boxed{\text{CMB power spectrum from 2018}}$

Notice how this peaks at $\ell \sim 200$, which corresponds to $\theta \simeq 1$ deg. So even though the CMB beach ball has power on all sorts of angular scales, your eyes predominantly sees fluctuations on $\simeq 1$ deg scales.

Alright. Let's dive into this!

If we forget the projection and secondary effects, the CMB fluctuations can be written as

grav. potential   peculiar velocity   overdensity

$$\Theta(\hat{r}) \equiv \frac{\delta T(\hat{r})}{T} = \delta\Phi(\vec{r}) - \hat{r}\cdot\vec{v}(\vec{r}) + \frac{1}{3}\delta(\vec{r})$$

Direction vector to a particular part of sky

Position vector to surface of last scattering.

The $\delta\Phi$ term arises because if a photon finds itself in a deep gravitational well, it loses energy as it climbs out. (Recall that a potential well has $\delta\Phi < 0$, so we see a lower temperature)

These two effects are in opposition with each other!

The $\hat{r}\cdot\vec{v}(\vec{r})$ term is due to peculiar velocities causing photons to be blueshifted or redshifted.

The $\frac{1}{3}\delta(\vec{r})$ term is because overdense regions are hotter.

Why the factor of $\frac{1}{3}$? Recall that up until recombination happens, the photons and baryons are tightly coupled.

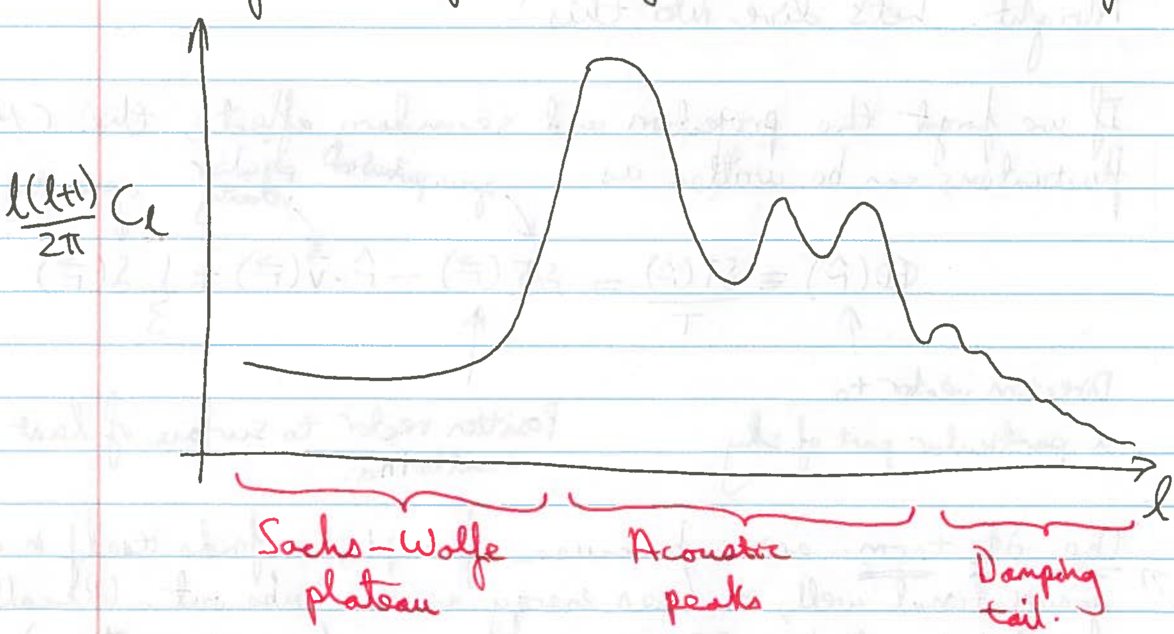
$$\Rightarrow n_\gamma \propto n_b \propto \rho_b \propto \rho \quad \text{and} \quad n_\gamma \propto T^3$$

photons tracing baryons

from Bose-Einstein stats

$$\Rightarrow T \propto \rho^{1/3} \Rightarrow dT \propto \frac{1}{3} d\rho\, \rho^{-2/3} \Rightarrow \frac{dT}{T} = \frac{1}{3}\frac{d\rho}{\rho} = \frac{1}{3}\delta.$$

The first and third terms are in opposition with one another. Which one wins? It turns out this depends on the angular scales involved. In the discussion that follows, it will be helpful to segment things into large, medium, and small angular scales.



Let's start with the largest angular scales, the Sachs-Wolfe region of the angular power spectrum.

Just as with the matter paper, these scales correspond to the largest scales that are only barely beginning to cluster $\rightarrow$ not much movement so cross out the Doppler term.

Now, recall that we can think of the different hot and cold spots as being due to our Universe having different amounts of time to cool via expansion. In particular,

$\Rightarrow$ Inside a potential well due to GR time dilation
$\Rightarrow$ "Universe younger" inside well.
$\Rightarrow$ Spot hasn't had as much time to expand $\Rightarrow$ denser $\Rightarrow$ hotter

$$\frac{dt}{t} = \delta\Phi \quad ; \quad a \sim t^{2/3} \quad ; \quad T \sim \frac{1}{a} \sim t^{-2/3}$$

$$\Rightarrow \frac{\delta T}{T} = \frac{-2}{3}\frac{dt}{t} = \frac{-2}{3}\delta\Phi .$$

Add this to the gravitational redshift to get

$$\Theta(\hat{r}) = \frac{\delta T}{T} = \delta\Phi - \frac{2}{3}\delta\Phi = \frac{1}{3}\delta\Phi .$$

Recall that overdensities have $\delta\Phi < 0$. This predicts that on large scales, overdense regions actually give rise to cold spots in the CMB!!!

Although in practice one uses more than just the Sachs-Wolfe plateau - the modelling is just harder

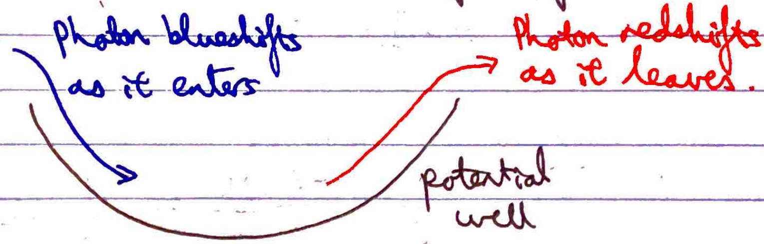
Except for this weird sign flip, the low l really is just probing the density (or equivalently, up to some conversion factors, the gravitational potential) fluctuations. It's therefore not surprising that we can probe the matter power spectrum with the CMB

$\Rightarrow$ Slide with matter power spectrum

It turns out, though, that to really accurately account for the shape of the low-ℓ region, I need to account for something called the Integrated Sachs-Wolfe (ISW) effect.

This is our first example of a secondary effect, where the CMB photons are affected on their way to us.

What happens is that as the CMB photons come to us, they have to go through the potential wells of the inhomogeneities in matter between us and the surface of last scattering.



If the gravitational potential well were static, the two effects would cancel out. So it's really just time-dependent gravitational potentials that ~~make~~ make a difference:

ISW effect: $$\frac{\delta T}{T} = \int \underbrace{\delta\dot{\Phi}}_{\text{time derivative}}[\underbrace{\vec{r}(t)}_{\text{where the photon is at time } t}, t]\, dt$$

How does the grav. potential change with time? Surprisingly, it doesn't during matter domination (at least in linear theory)!

Recall that during matter domination, we have $\delta \propto a$.
Use the Poisson equation to relate to grav. potential:
$$\nabla^2 \delta\Phi = 4\pi G\bar{\rho}\delta \implies k^2 \delta\Phi = 4\pi G\bar{\rho}\delta.$$

Now, k is in physical coords. We want to translate to comoving coordinates! So $k_{phys} = a^{-1} k_{co}$, and

$$k_{co}^2 \, \delta\Phi \sim 4\pi G a^2 \bar{\rho} \, \delta = \text{constant in } a(t), \text{ i.e. time-independent!}$$

$\bar{\rho} \propto \frac{1}{a^3}$ during matter domination

$\delta \propto a$ during matter domination

This means that $\delta\dot{\Phi} \neq 0$ only if we violate one of our assumptions. Each way of violating our assumptions gives a different type of ISW effect

⇒ Early-time ISW effect occurs because even though recombination happens during the matter-dominated epoch, the energy density in radiation is not completely negligible.

⇒ Late-time ISW effect occurs because eventually we get into the dark-energy dominated epoch.

⇒ Rees-Sciama effect is a small effect that comes about when we remember that linear perturbation theory is just an approximation and that there is some non-linearity involved.

All these effects happen slightly differently along different lines of sight ⇒ extra source of anisotropy ⇒ extra power in C_ℓ.

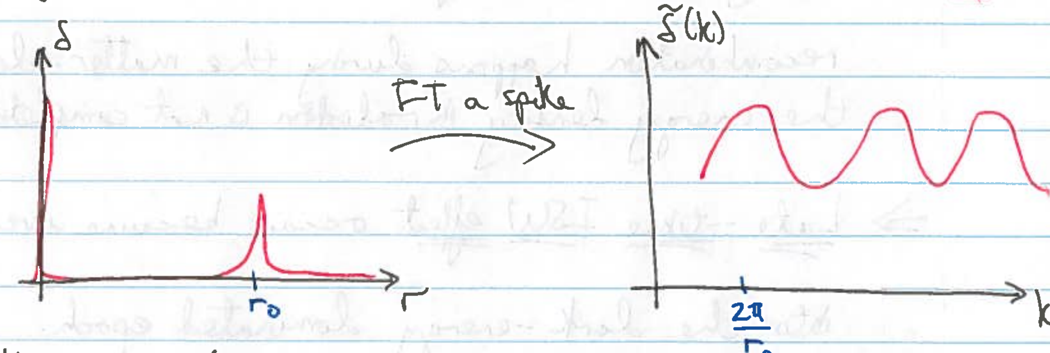
⇒ Show slide with ISW contributions

Acoustic peaks

Why are there these acoustic peaks at intermediate ℓ? There are two very handy ways to think about this. Today we will focus on the angular locations of the peaks, for which one of the ways will be helpful. Next time we'll talk about the amplitudes of the peaks, and a different way will be helpful.

Recall that prior to recombination we have the photons and baryons in a coupled photon-baryon fluid. When gravity tries to pull this coupled fluid together, it resists and a sound wave propagates. Travels until recombination. $\Rightarrow$ $\boxed{\text{Show movie}}$

So



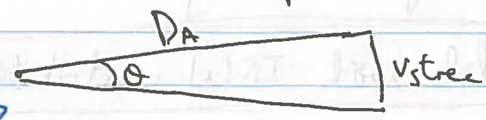
These wiggles are the Baryon Acoustic Oscillations. You get a first peak at the fundamental mode, then harmonics.

(In reality, we have many bumps $\Rightarrow$ $\boxed{\text{Show movie}}$)

Now, r_0 is given by $v_s t_{rec}$. $\rightarrow$ known physics!

If we assume a flat universe ($\Omega_k = 0$), then

Because inhomogeneities get translated into anisotropies



$\Rightarrow \theta = \dfrac{v_s t_{rec}}{D_A}$

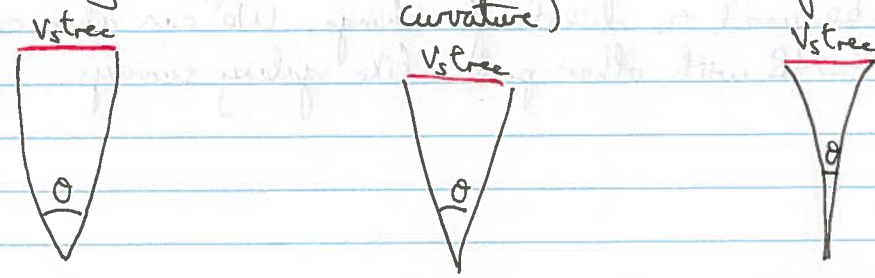
But $\ell \sim \frac{180^\circ}{\theta}$, so $\ell_{acoustic} \sim 180^\circ \frac{D_A}{v_s t_{rec}} \sim 200$.

$\Rightarrow$ First acoustic peak @ $\ell \sim 200$ and harmonics appear @ multiples of this } Show C_ℓ again

However! This assumes a flat universe!

Suppose we didn't! Recall that $\Omega_{tot} + \Omega_k = 1$ and that if

$\Omega_k < 0$ (Positively curved)	$\Omega_k = 0$ (no spatial curvature)	$\Omega_k > 0$ (Negatively curved)



Subtends larger $\theta \Rightarrow$ shifts to lower ℓ

Subtends smaller θ $\Rightarrow$ Shifts to higher ℓ.

$\Rightarrow$ Show slide with different Ω_{tot}.

The location of the first acoustic peak tells us about spatial curvature.

Projection effects are actually useful!

The location of the peaks can also teach us about dark energy.

If I hold the matter density constant, increasing Ω_{DE} causes our Universe to be younger $\Rightarrow$ less time for CMB surface of last scattering to "run away" from us $\Rightarrow$ angles are larger $\Rightarrow$ shift to lower ℓ.

$\Rightarrow$ || Show slide with different Ω_{DE} ||

Note that these effects become degenerate with one another when I vary more than one parameter at a time. Fortunately, these parameters often have other effects on the CMB that can be used to disentangle things. We can also complement the CMB with other probes like galaxy surveys.