PHYS 644 Lecture #2: Galaxies as Gravitational Systems

What we've done so far is that we've looked at how we observe galaxies. In the next few lectures we'll talk about theoretical models of galaxies.

First, let me provide an operational definition for a galaxy:

A gravitationally bound system of stars embedded in a dark matter halo and exhibiting sustained star formation over cosmological time periods

The next two lectures will focus on the gravitational part of the modelling.

First, let's remind ourselves.... Self-gravitating systems are weird slides!

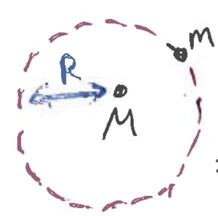
All of this is to remind you that self-gravitating systems are strange. Part of this comes from the way these systems obey the virial theorem:

In equilibrium, $U = -2T$

U: Total grav. potential energy

T: Total kinetic energy.

Simple example from 2 bodies:



$$\frac{mv^2}{R} = \frac{GMm}{R^2}$$

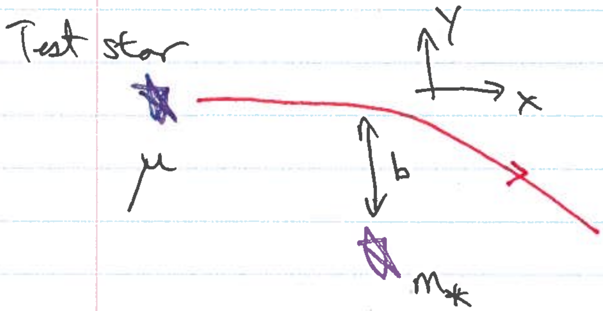
$$\Rightarrow \frac{1}{2}mv^2 - \frac{GMm}{2R} = 0$$

$$\Rightarrow T + U/2 = 0.$$

Stellar Scattering

In trying to understand these systems, it's useful to figure out what is the more appropriate model for the movement of stars in a galaxy: should I think of individual star-star interactions or think of stars as experiencing the global potential.

Let's start by seeing how important nearby star-star interactions are. Suppose I asked how a "test star" with mass μ is affected by a different star with mass m_*:



$b \equiv$ "impact parameter"

As a result of this encounter, the star gets a speed in the vertical direction:

$$\Delta v_y = \frac{F_y}{\mu} \Delta t = \frac{G m_* b \Delta t}{(x^2+b^2)^{3/2}}$$

Of course, this doesn't just happen as a single impulse, so we take the continuous limit:

$$\Delta v_y = \frac{G m_*}{b^2} \int_{-\infty}^{+\infty} \frac{dt}{\left[1 + \left(\frac{vt}{b}\right)^2\right]^{3/2}} = \frac{2 G m_*}{b v}$$

$x = vt$ if $t=0$ @ closest approach.

We can define a "strong encounter" that substantially changes the direction as one where $\Delta v_y \approx v$. This means

$$v = \frac{2 G m_*}{b_{strong} v} \Rightarrow \sigma \equiv \begin{array}{c}\text{cross-section of}\\ \text{"interaction"}\end{array} = \pi b_{strong}^2 = \frac{4\pi G^2 m_*^2}{v^4}.$$

But recall from the virial theorem that $v^2 \sim \frac{G N_* m_*}{R}$

total # of stars in galaxy

which means that $\sigma_{strong} = \frac{4\pi R^2}{N_*^2}$.

With this, I can work out the mean free path of "collisions" of stars in a galaxy:

average length travelled between "collisions"

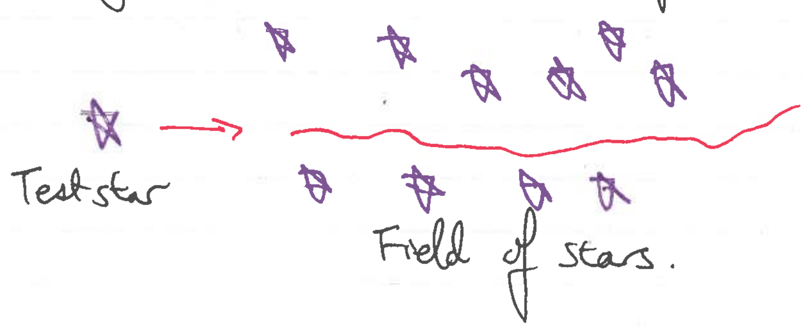
$$\ell = \frac{1}{n_* \sigma_{strong}} = \frac{V}{N_* \sigma_{strong}} = \frac{\frac{4}{3}\pi R^3}{N_*} \frac{N_*^2}{4\pi R^2} \Rightarrow \boxed{\ell = \frac{R N_*}{3}}$$

density of stars

Weird! Since $N_* \sim 10^{11}$, this is saying the mean free path is much larger than the galaxy! These strong encounters basically don't happen! Even stranger, this says that the more stars there are, the longer the mean free path, ie the rarer the collisions?!

↳ Self-gravitating systems are weird. The reason here is that $v^2 \sim \frac{G N_* m_*}{R}$, so more stars ⇒ typically faster ⇒ harder to deflect.

What about a series of weak encounters? Maybe a bunch of stars together have a cumulative effect.



Each star randomly kicks the test star up or down ⇒ ~~a~~ a random walk in velocity:

$$\Rightarrow \Delta v_{y,tot} = \sqrt{\sum_i (\Delta v_{y,i})^2}$$

In the continuous limit:

$$\sum_i (\Delta y_{,i})^2 \longrightarrow \int \underbrace{2\pi b\, db\ vt}_{\text{cylinder of field of stars}} \cdot \overset{\#\text{ density}}{n} \cdot \underbrace{\left(\frac{2Gm_*}{vb}\right)^2}_{(\Delta v_{y,i})^2 \text{ from before}}$$

* Our expression for b_{min} here is off from b_{strong} by a factor of 2 ∴ this is often how $\ln \Lambda$ is defined.

"Coulomb logarithm"

$$\Rightarrow \Delta V_{y,tot}^2 = \frac{8\pi G^2 m_*^2 n t}{V} \int_{b_{min}}^{b_{max}} \frac{db}{b}$$

$b_{max} \to R$, $b_{min} \to b_{strong}^*$

$$\equiv \ln \Lambda = \ln\left(\frac{b_{max}}{b_{min}}\right) = \ln\left(\frac{R V^2}{G M_*}\right) \sim \ln\left(\frac{M}{M_*}\right) \sim \ln N_*$$

Define τ (relaxation time) to be when $\Delta V_{y,tot}^2 \simeq V^2$. This gives:

$$\boxed{\tau = \frac{V^3}{8\pi G^2 m_*^2 n \ln N_*}}$$

"Chandrasekhar dynamical relaxation time"

How many orbits of the galaxy does it take to dynamically relax? Can say $t_{cross} \sim \frac{R}{V}$ (crossing time, which is comparable to orbital time)

$$\frac{t_{relax}}{t_{cross}} \sim \frac{V^2 \frac{4\pi}{3} R^3}{8\pi G^2 m_*^2 R N_* \ln N_*} \sim \frac{N_*}{6 \ln N_*} \sim 10^9.$$

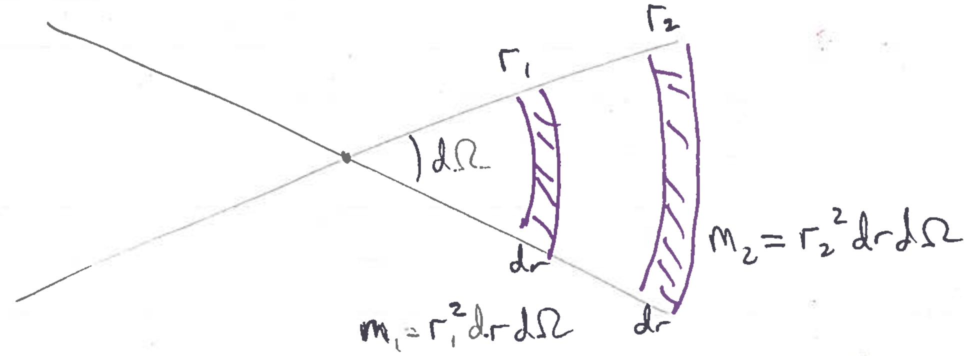
In the entire age of our Universe, we haven't even had one relaxation time! $\Rightarrow$ Weak encounters also insignificant.

This means that the correct way to think about the "gas of stars" is not as a series of "collisions". Instead, the stars form a collisionless fluid that feel the gravitational potential well generated by the entire collective of stars (and dark matter).

Next lecture we will build some of the formalism for describing this fluid. For now, I want to provide some more intuition for how this is different from a gas of particles.

Ideal gas: extremely violent short-range collisions between atoms that happen quickly, followed by long periods with no interactions.

Galaxy: star-star interactions negligible. Need to consider far away contributions because even though the forces drop as $1/r^2$, the # of objects grows as r^2:



Of course, if there's a uniform distribution of mass in the galaxy then the shells on the other side will result in no net force. It's differences in the way the mass is distributed in the galaxy — and therefore the gravitational potential of the galaxy as a whole — that determine the dynamics.

Dynamical Friction

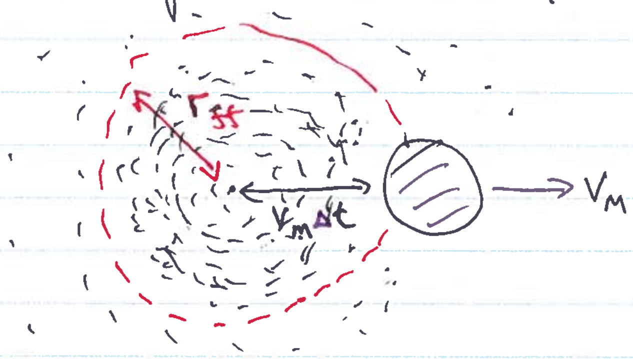
There are more surprises for us in store with these gravitating systems! For example, imagine two galaxies that are merging. Most galaxies have supermassive black holes at their centres. After the merger, the black holes also merge. But the probability that the two black holes are on a direct collision course is miniscule! How do the black holes "find" each other?!

Let's imagine a mass M travelling through a field of objects each of mass m, where $M \gg m$ (unlike before!)

What happens is that the small objects get gravitationally deflected ~~and~~ and form an overdensity (a "wake") behind the big object. The grabitational pull of this overdensity exerts a ~~force~~ force backward, resulting in dynamical friction.

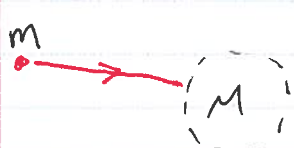
$\Rightarrow$ $\boxed{\text{Show cartoons of setup.}}$

We can get a rough estimate of this effect by imagining that the field of stars is at rest and of constant density ρ. Consider a small displacement $\Delta r = V_M \Delta t$ of the big mass.



There will be a certain amount of material (within a sphere of radius r_{ff}) that can free-fall within Δt. This will form the overdensity.

How long does it take for some mass ~~to~~ to free-fall a distance r_{ff} when attracted by a mass M?



A radial trajectory like this is just a limiting case of an orbit with extremely high eccentricity, so Kepler's 3rd Law holds!

$$\Rightarrow \frac{r_{ff}^3}{(\Delta t)^2} \sim \frac{G(M+m)}{4\pi^2} \quad \Rightarrow \quad \cancel{r_{ff}} \; \Delta M = \frac{4}{3}\pi r_{ff}^3 \rho$$

$$\propto GM\rho(\Delta t)^2$$

Therefore, the backward pointing force is given by

$$F_{df} = -\frac{GM\Delta M}{(\Delta r)^2} \propto -\frac{G^2M^2\rho}{V_M^2}$$

dynamical friction

A more precise calculation (non-radial trajectories, moving background objects) gives the Chandrasekhar dynamical friction formula:

$$\frac{d\vec{V}_M}{dt} = -16\pi^2 \ln\Lambda \, G^2 m(M+m) n_0 \frac{\int_0^{V_M} p(v_m) v_m^2 \, dv_m}{V_M^3} \vec{V}_M .$$

where $n_0 \equiv$ local # density of small objects
$m \equiv$ mass of small objects.
$p(v_m) \equiv$ probability distribution of small objects [Dimensions: $1/(\text{velocity})^3$]

Another intuitive way to think about it is to think about the system as it comes to equilibrium. There will be equipartition of energy, so $\langle \frac{1}{2} m v_m^2 \rangle$ should be comparable to $\langle \frac{1}{2} M V_M^2 \rangle$. Thus, if $M \gg m$, the big masses need to slow down.

To return to our original problem, what happens when two galaxies merge is that the two supermassive black holes sink to the new centre because dynamical friction causes them to slow down and their orbits to decay.

However, there is the final parsec problem. Dynamical friction requires there to be enough stuff around. By the time the black holes are $O(1\text{pc})$ from each other, there isn't enough volume between them for there to be enough stuff. If you work out the timescales involved, it's longer than the age of ~~the~~ our Universe!

(Gravitational wave emission eventually allows the BHs to lose energy and come together, but they need to be much closer for that)