PHYS 644 Lecture #14: Recombination + Reionization

We have so far talked about various important events in the early universe:

$\Rightarrow$ $\boxed{\text{Review slides}}$

After BBN, we are left with a big soup of electrons, protons, helium nuclei, and photons. (The neutrinos are still around, but they have decoupled by now and are ignoring everyone else!)

What happens to this soup? That's today's story, and the answer is "not much for a while, and then some pivotal events:

- Matter-radiation equality

  ($z \sim 3250$, $t \simeq 50{,}000$ yrs, $T \simeq 9000$ K)

- Recombination (protons and e^- form neutral hydrogen)

  ($z \sim 1275$, $t \simeq 290{,}000$ yrs, $T \simeq 3500$ K)

- Decoupling and last scattering (photons free to roam universe)

  ($z \sim 1090$, $t \simeq 380{,}000$ yrs, $T \simeq 3000$ K)

- Reionization (first galaxies separate p and e^- again)

  Highly uncertain! ($z \overset{?}{\simeq} 7$, $t \simeq 800$ Myr, $T \simeq 25$ K)

The middle events are particularly important because they give rise to the Cosmic Microwave Background (CMB), where photons can free stream all the way to our telescopes.

⇒ Cartoon of what happens

Tight Coupling and Decoupling

At $z \gg 1000$, we have photons and baryons in a tightly coupled photon-baryon fluid.

This coupling is driven by Thomson scattering of photons off electrons:

$$\gamma + e^- \rightleftharpoons \gamma + e^-$$

where the cross-section is

$$\sigma_T = \frac{8\pi}{3}\left(\frac{\alpha \hbar}{m_e c}\right)^2 = 6.65 \times 10^{-29}\ m^2 \quad (\text{SI units!}).$$

The e^- then drag the nuclei around via Coulomb attraction, so this is all tightly coupled.

In this era, the average distance a photon travels between collisions (the mean free path) is short:

$$\lambda = \frac{1}{n_e \sigma_T}$$

and the interaction rate is high:

$$\Gamma = c\, n_e \sigma_T$$

Think of this as "$n\langle\sigma v\rangle$" from before with $v=c$ for photons, or the fact that the time between collisions is λ / c.

Now define the ionization fraction $x_e \equiv \frac{n_e}{n_B} = \frac{n_p}{n_B}$ 

free protons, not the total

charge neutrality

This goes from $x_e = 1$ early on to $x_e \ll 1$ later. With this definition we have

$$n_e = n_B x_e = \underbrace{n_{B,0}}_{\cancel{a^3}} (1+z)^3 \, x_e(z)$$

$\frac{1}{a^3}$ dilution of matter

This gives

$$\Gamma = c \sigma_T n_{B,0} x_e(z) (1+z)^3 = 5 \times 10^{-21} \, s^{-1} \, x_e(z) \, (1+z)^3.$$

Now we do the usual thing and ask when the interaction (in this case Thomson scattering) turns off by comparing to H:

$$H = H_0 \sqrt{\Omega_{m,0} (1+z)^3 + \Omega_{r,0} (1+z)^4} \approx 10^{-18} \, s^{-1} \, (1+z)^{3/2}$$

no longer in radiation-dominated universe!

We see that Γ varies more sharply than $(1+z)^3$, whereas H varies $\sim (1+z)^{3/2}$. Therefore, $\Gamma \gg H$ at high redshifts (interaction "on") and $\Gamma \ll H$ at low redshifts (interactions "off").

It is crucial, though, that we account for $x_e(z)$ because it will vary by orders of magnitude.

The redshift of decoupling can then be computed by imposing our usual condition:

$$\Gamma(z_{dec}) = H(z_{dec}).$$

This gives:

$$1 + z_{dec} = \frac{38.3}{x_e(z_{dec})^{2/3}}$$

This expression gives us the fun fact that if hydrogen just magically stayed ionized, decoupling would happen at $z \sim 37$ and photons would only free-stream to our telescopes then and the CMB would be pictures of our Universe from $z \sim 37$!

This also ~~tells~~ tells us that we need to understand the ionization history $x_e(z)$ better....

Should really be called "combination" since neutral atoms were formed for the first time then!

Recombination

This is the process whereby the reaction

$$H + \gamma \rightleftharpoons p + e^-$$

gets pushed far to the left. Believe it or not, we can get a fairly good approximation by ignoring helium, even though it's 25% of the baryonic mass.

Let's get some intuition for He recombination though $\Rightarrow$ Slide Q

Before we do the calculation, let's also guess the redshift at which H recombines $\Rightarrow$ Slide Q

Once again, we use equilibrium non-relativistic physics:

$$n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[\frac{-(m_i - \mu_i)}{T}\right]$$

In chemical equilibrium, $\mu_H + \mu_\gamma = \mu_p + \mu_e$

$\mu_\gamma = 0$ as always for photons.

This gives

$$\frac{n_H}{n_p n_e} = \frac{g_H}{g_p g_e} \left(\frac{m_H}{m_p m_e}\right)^{3/2} \left(\frac{T}{2\pi}\right)^{-3/2} \exp\left(\frac{m_p + m_e - m_H}{T}\right)$$

The binding energy of hydrogen is $Q \equiv m_p + m_e - m_H = 13.6\,\mathrm{eV}$, and with this notation we get

Saha Eqn: $$\frac{n_H}{n_p n_e} = \left(\frac{m_e T}{2\pi}\right)^{-3/2} \exp\left(\frac{Q}{T}\right)$$ [Using $m_p \simeq m_H$ and $g_e = g_p = \frac{g_H}{2} = 2$]

Now, recall that our goal was to find

$$x_e \equiv \frac{n_e}{n_B} = \frac{n_p}{n_B} = \frac{n_p}{n_p + n_H}$$ Ignoring helium!

This can be rewritten as

$$\frac{1-x_e}{x_e} = \frac{n_H}{n_p} = n_e \left(\frac{m_e T}{2\pi}\right)^{-3/2} \exp\left(\frac{Q}{T}\right).$$

For n_e, we have $\eta = \frac{n_B}{n_\gamma} = \frac{n_e}{x_e n_\gamma}$.

$$\Rightarrow \quad \frac{1-x_e}{x_e^2} = n_\gamma \eta \left(\frac{m_e T}{2\pi}\right)^{-3/2} \exp\left(\frac{Q}{T}\right)$$

Finally, using the fact that $n_\gamma \propto T^3$ and plugging in the numbers, we have

$$\frac{1-x_e}{x_e^2} \simeq 3.84\,\eta \left(\frac{T}{m_e}\right)^{3/2} \exp\left(\frac{Q}{T}\right)$$

Since we know the CMB temperature today, T_0, and we know $T = T_0(1+z)$, we can solve this equation numerically for $x_e(z)$!

(This is just $T \propto \frac{1}{a}$)

The results look like this $\Rightarrow$ $\boxed{\text{Show plot}}$

What the Saha eqn gets right is that recombination is actually a fairly drawn out process! It doesn't have the final details right, though, which involve modelling all the different energy levels of H and the different paths an electron can cascade down to the ground state.

Now that we know $x_e(z)$, we can find out when $z_{\text{decoupling}}$ happens and the photons decouple. Plugging these curves into our earlier relation gives

$$\boxed{z_{dec} \simeq 1090.}$$

At this point, the ~~CMB~~ photons scatter one last time before free streaming to our telescopes. This is why we call the CMB the "surface of last scattering". It's actually a (slightly) crinkly surface with different parts scattering at different times, roughly $\Delta z \sim \pm 100$.

Reionization

Except that's not quite true, for about 5% of the photons, they have been scattered once more since $z \sim 1090$. This is because of reionization — a pivotal event in our Universe's history when first-generation ~~stars~~ galaxies systematically ionized the neutral hydrogen, returning the hydrogen in the intergalactic medium (IGM) back to an ionized state.

$\Rightarrow$ $\boxed{\text{Show reionization movie and light cone}}$

This releases free electrons back into the IGM for further Thomson scattering

In principle it's $P \sim 1 - e^{-\tau}$, but τ is small, so Taylor expanding gives $P \sim \tau$

The probability of more scattering is given by the optical depth parameter τ.

If we assume instantaneous reionization, then τ is given by

$$\tau = \int_{t_{reion}}^{t_0 \;\leftarrow\,\text{today!}} \Gamma(t)\,dt = 0.054 \pm 0.008 \quad (\text{from Planck CMB})$$

This number has been notoriously hard to measure!

$\Rightarrow$ $\boxed{\text{Show } \tau \text{ history plot and slide Q on interpretation}}$

Another way to write this is to remember that $\Gamma = c n_e \sigma_T$, so

$$\tau = \int \underbrace{\sigma_T n_e}_{1/\lambda} \underbrace{c\,dt}_{dl} \quad \Rightarrow \tau \text{ is also "distance in units of mean free path".}$$

$\leftarrow$ mean free path

Writing it in this way also ~~to~~ allows us to account for the fact that reionization is gradual. Recalling that $n_e = x_e n_B$, we can write:

$$\tau = \int \sigma_T n_{B,0} (1+z)^3 \, x_e(z) \, \frac{dl}{dz} \, dz.$$

Up to geometric factors, constants, and simple fcts of $(1+z)$, this is essentially the integral of the ionization fraction $x_e(z)$.

Once again we have to model the ionization history.

Once again, this is a competition between ionizations and recombinations. But there are some differences! Because galaxies are involved, this is now an astro modelling problem, and it's

Often, people that work on reionization prefer this volume-filling fraction, but for the purposes of today's discussion it's conceptually ok to think of this like x_e.

also not an equilibrium problem, so we don't have set expressions for the various number densities n_i.

Define $Q_{HII} \equiv$ Fraction of universe's volume that is ionized

$$\frac{dQ_{HII}}{dt} = \underbrace{\frac{f_{esc.} N_{ion} \dot{\rho}_*}{n_H}}_{\text{Ionizations}} - \underbrace{\alpha C n_e Q_{HII}}_{\text{Recombinations}}$$

The easiest way to understand this is to define the symbols!

$\dot{\rho}_*$: cosmic star formation rate density

$$\int \dot{M}_* \underbrace{\frac{dn}{dM_h}}_{\text{halo mass fct.}} dM_h$$

Star formation rate for one galaxy.

$\dot{M}_*$ is where galaxy formation physics can come in. For example, recall from Lecture 8 than in an accretion based model we have

$$\dot{M}_* = f_* f_b \dot{M}_h$$

accretion rate

$M_* - M_h$ relation

baryon fraction $\frac{\Omega_b}{\Omega_m}$

$\boxed{\text{Show plot on slide}}$

N_{ion}: # of ionizing UV photons per unit stellar mass (get from stellar evolution codes)

f_{esc} : Escape fraction — the fraction of ionizing photons that escape from the ISM to get to the IGM

n_H : # density of hydrogen

α : "Case A recombination coefficient"

n_e : Electron # density.

C : "Clumping factor" $\frac{\langle n_H^2 \rangle}{\langle n_H \rangle^2}$. A fudge factor to account for the fact that recombination is a 2-body process.

We now numerically integrate this and get $Q_{HII}(z)$.

To compare to τ, we then integrate $Q_{HII}(z)$ and since we have embedded this in a galaxy formation model, we can also relate it to other observables like UV luminosity functions!

$\Rightarrow$ $\boxed{\text{Brant Robertson figure}}$