1 Introduction

Speaker: Bidisha from ICTS Bangalore

This talk is about non-linear Langevin dynamics in Holography.

2 Papers

Effective quantum field theories for the Thermodynamic Bethe Ansatz - Ivan Kostov

Confinement as Analytic Continuation Beyond Inifinty - Masahito Yamazaki

Renormalon problems?

Techniques - Place things on a cylinder - when coupling constants switch sign

Partition function on different kinds of worldsheets.

3 Papers

Loganajan, Chakravanty, Sivakuman

4 Motivation

In non equilibrium processes, how do we compute real time correlation functions? In the path integral framework, we usually study equilibrium processes, but we should extend this to non-equilibrium states.

This framework is known as the Schwinger-Keldysh path integral framework. (We draw up the time contour) looks like a hook. We initialise the density matrix $\rho_i = |\psi\rangle \langle\psi|$ on the time contour. We're splitting the parts of the Lagrangian by doubling the degrees of freedom along Q_1 and Q_2 as

$$\mathcal{L}_{SK} = \mathcal{L}_1 Q_1 - \mathcal{L}_2(Q_2)$$

Suppose we have a quantum brownian particle with degrees of freedom denoted by q, with interacting degrees of freedom X. Our density matrix is given by

$$\rho_0 = \rho_{OP} \otimes \rho_{BO}$$

Let's initialise our density matrix at t_0 as ρ_0. We double the degrees of freedom on legs of the contour as q_i and X_i. Our microscopic lagrangian is given by

$$\mathcal{L}_{\text{micro}} = \mathcal{L}_P + \mathcal{L}_B + \lambda q X, \quad \mathcal{L}_{SK} = \mathcal{L}_1 - \mathcal{L}_2$$

Our evolution is given by

$$\rho_f = (Q_{1f}, Q_{2f}) = \int dQ_{10} dQ_{20} \rho_0(Q_{10}Q_{20}) \int [\mathcal{D}Q_1][\mathcal{D}Q_2] \exp\left(i \int_{Q_1(t_1)=Q_{10}}^{Q_{1f})=Q_{1f}} dt \left(L_1(Q_1) - L_2(Q_2)\right)\right)$$

What is the reduced density matrix of the particles? To do this, we need to integrate out some things. Let's say that $X_{1f} = X_{2f} = X_f$. representing the bath degrees of freedom are integrated out. Then, computing the reduced density matrix we get that

$$\rho_{pf}(q_{1f}, q_{2f}) = \int dq_{10} \int dq_{20} \rho_{p_0}(q_{10}, q_{20}) \int_{q_1(t_0)=q_{10}, q_2(t_0)=q_{20}}^{q_1(t_f)=q_{1f}, q_2(t_f)=q_{2f}} [\mathcal{D}q_1] [\mathcal{D}q_2] e^{i\left[\frac{1}{2}mp_0 \int_{t_0}^{t_f} dt\left[(\dot{q}_1^2 - \mu_0^2 q_1^2) - (\dot{q}_2^2 - \mu 0^2 q_2^2) + W_{SK}\right]\right]}$$

The part W_{SK} is known as the influence phase of the particle. We expand W_{SK} as

$$W_{SK} = \sum_{n=1}^{\infty} \lambda^n W_{SK}^{(n)}$$

with each of the components being

$$W_{SK}^{(n)} = i^{n-1} \sum_{i_1 \dots i_n = 1}^{2} \int_{t_1}^{t_f} dt_1 \cdots \int_{t_1}^{t_{n-1}} dt_n \left\langle T_c O_{i_1}(t_1) \dots O_{i_n}(t_n) \right\rangle q_{i_1}(t_1) q_{i_2}(t_2) \dots q_{i_n}(t_n)$$

Here, we observe that the effective action is non-local in time. For a class of baths, the bath correlations decay exponentially fast. For example, we have that

$$W^{(2)} = \int dt_1 dt_2 \left\langle T_c O_{i_1}(t_1) O_{i_2}(t_2) \right\rangle q_{i_1}(t_1) q_{i_2}(t_2)$$

Our correlated part enclosed in angle brackets exponentially decays as $e^{-\alpha t_{12}}$. We also can Taylor expand out $q_{i_2}(t_2)$ in from t_1. Referencing Feynman-Vernon.

4.1 Quadratic Quantum Effective Theory of q

If we focus on the saddle point of this integral, then we get an equation of motion

$$\frac{d^2 q}{dt^2} + \gamma \frac{dq}{dt} + \overline{\mu}^2 q = \left\langle f^2 \right\rangle \mathcal{N}$$

We have that our noise has distribution

$$\rho(\mathcal{N}) = \exp\left[-\int dt \frac{\left\langle f^2 \right\rangle}{2} \mathcal{N}^2\right]$$

Our equation is called the Linear Langevin equation. We have that

$$\left\langle f^2 \right\rangle = \frac{2}{\beta} \gamma \sim \left\langle \{O(t_1), O(t_2)\} \right\rangle$$

The above is the statement of equilibrium. This is how we derive the equation of motion from the path integral. We define the helper variables

$$q_1, q_2 \to q_a = \frac{q_1 + q_2}{2}; \quad q_d = q_1 - q_2$$

Consider the following stochastic path integral

$$\int [dq_a] [d\mathcal{N}]$$

P $[N]\,\delta(\langle f^2\rangle \mathcal{N} - E(q_a, \mathcal{N}))$this can be written as

$$\int \mathcal{D}q_a \mathcal{D}\mathcal{N} e^{-i\int dt q_d (E(q_a,\mathcal{N}) - \langle f^2\rangle \mathcal{N})} P(\mathcal{N})$$

This can be rewritten as a path integral as

$$\int \mathcal{D}q_a \mathcal{D}q_d e^{i\int dt \mathcal{L}_{\text{ eff}}}$$

Our Lagrangian for the effective action is

$$\mathcal{L}_{\text{ eff}} = \frac{i\,\langle f^2\rangle}{2} q_d^2 + \dot{q}_d\dot{q}_a - \gamma q_d \dot{q}_a - \overline{\mu}^2 q_d q_a$$

Thus, we've computed non linear corrections to the Linear langevin equation How to get this big effective action $\mathcal{L}_{\text{ eff}}$ Our most general Lagrangian for the effective action is

$$\mathcal{L}_{\text{ eff}} = \frac{i\,\langle f^2\rangle}{2} (\text{ insert most general terms here })$$

If we take the classical limit using saddle points of our effective Lagrangian, we get that

4.2 Non-Linear FDR

We have that

$$\zeta_N = -\frac{12}{\beta}\zeta_\gamma$$

from OTO extension of effective theory. ζ_N comes from time-reversal invariance and thermality of bath.

5 Holographic problem

We'll now talk about the Holographic SK prescription. The system we'll talk about is a quark moving in a thermal CFT d-dimensional plasma. In AdS / CFT, this quark is dual to an open dtring from the boundary of an AdS d + 1 dimensional black brane.

The generating function for real time CFT correlators when initial state was thermal. It has also been done that we verified non-linear FDR for quark using holographic SK. Check out Hong Liu, et al.

The string is in the black brane, the black brane has Hawking radiation. Modes captured by diffferent Green's function.

Fluctutation and dissipation are measurable quantities. This can be tested!