String Theory Lecture Notes

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Contents

1 Introduction

If you'd like to read about string theory, check out

- String Theory, Vol 1, Polchinksi, CUP
- Superstrings, Vol 1, Green et al, CUP
- A String Theory Primer, Schomerus, CUP
- David Tong's Notes, at arXiv: 09080333
- 'Why String Theory' Conlon, CRC

What is string theory? We don't really know what string theory is. The question itself requires a little more fleshing out to make sense of it. String theory is a work in progress. A final version of string theory would be a theory which we understand physically and mathematically. It's a work in progress - the final form will be quite different from what we have now.

What do we know? Well, in some sense, string theory is an attempt to answer the question of how we quantise the gravitational field. A theory of **quantum gravity**. There are however, a number of obstacles. In particular, naive quantisation of the Einstein Hilbert action presents a number of problems.

There are deep conceptual problems associated with this.

- There's a question about the nature of time in quantum gravity. If we think about time in quantum mechanics, time is treated as a fixed clock in which the Hamiltonian governs the evolution. In GR, space and time are on the same footing. Thus, the descriptions are on a different footing in QM versus GR. This is not neseccarily a technical problem, just something to think about.
- How do we quantise things without a pre-existing causal structure? What do we mean by this? We we quantise things in QFT, it's important to know whether two operators are timelike or spacelike separated. We have a notion that for all operators which are spacelike separated, commute. (One should not be able to influence the other). If we try to quantise GR, the metric is the object which we would like to quantise. But, this determines the casual structure. So, it's not immediately obvious what the algebra of operators should look like.
- GR has a very big symmetry - diffeomorphism symmetry (symmetry under coordinate reparametrisations). This is diffeomorphism invariance. This is something we'll discuss a bit later on. This is considered to be a gauge symmetry.
- One thing we'll discuss is that there are no local diffeomorphism invariant observables in GR. It's not even clear what the observables should be.

More importantly, there are technical obstacles. The previous issues are hard, but can we make some assumptions which help us make progress? We can look at perturbation theory; we can take our metric an expand it around some classical solution

$$g_{\mu\nu}(X) = \eta_{\mu\nu} + h_{\mu\nu}(x)$$

for our purposes, this classical solution will be Minkowski space-time. This means we can use the causal structure of the background metric (Minkowski) to learn about the causal structure of the perturbation, and quantise. This immediately neutralises the first two conceptual problems. We can call the fluctuations $h_{\mu\nu}$ as gravitons. So, if we take a background static spacetime, add a field, then quantise.

There are some unsatisfactory things about this. When we split the metric into two, we hide a lot of the deep structure that we want. Nonetheless, we can take our Einstein-Hilbert action and expand it out

$$S\left[g\right] = \frac{1}{K_0}\int d^D X\sqrt{-g}R(g)$$

Choose a gauge and expand out

$$S\left[h\right] = \frac{1}{K_0}\int d^D X\left(h_{\mu\nu}\Box h^{\mu\nu}\right)$$

Since the Ricci scalar contains inverses, this expansion goes on forever. This is called a non-polynomial action. The first quadratic term gives the propagator, and the higher order term gives us our interactions.

The propagator is represented by a wiggly line. The interaction term gives us vertices. (Three or four wiggly lines coming together) These lead to Feynman rules.

However, when we compute loops, we get divergences. But, in physical QFT, we can absorb these divergences into coupling constants. These can be dealt with using standard techniques. From advanced quantum field theory, this technique is called renormalisation.

Thus, the difficulties run deeper than conceptual ones. We simply don't know how to calculate. So, string theory provides a way to do quantum perturbation theory of the gravitational 'field' We put field in quotation marks because it's not really a field which we'll be dealing with.

String theory answers some of the questions, in the sense that it gives us a framework to ask meaningful questions in quantum gravity. But not all of these questions are questions that we can answer.

The viewpoint that we're going to take for this course will be from perturbation theory. But we'll always try to understand what this tells us about the non-perturbative physics.

1.1 Worldsheets and Embeddings

Let's try to put together some sort of language to get started. From any popular science book, you may find that particles are described as vibrating strings. The starting point is to consider a worldsheet Σ which is a 2-dimensional surface swept out by a string. This is analogous to a worldline swept out by a particle.

(Insert diagram of line parametrised by τ and pointlike object, and diagram of cylinder object with surface called Σ, with two axes called τ and σ. This diagram is in 3d Minkwoski space). We put coordinates (σ, τ) on Σ, at least locally. And, we can define an embedding of the worldsheet Σ in the background spacetime, $\mathcal{M}$ (Minkowski space), by the functions $X^\mu\left(\sigma, \tau\right)$, where the X^μ

are coordinates on $\mathcal{M}$. So if you like, we have that

$$X : \Sigma \to \mathcal{M}$$

Why is this an embedding? A choice σ and τ on Σ gives us a location in Minkwoski space. There are rules (which we shall investigate), for gluing such worldsheets together in a way which is consistent with the symmetries of the theory.

So, we can not only describe the embeddings of a single string propagating through space-time, but multiple strings coming together.

(Draw a diagram of two tubes merging into one tube, then splitting back again into two tubes - this looks like a three point vertex)

We shall see that such diagrams like the above are in one-to-one correspondence with correlation functions in some quantum theory. It is natural to interpret such diagrams as Feynman diagrams in a perturbative expansion of some theory about a given vacuum.

So what we have is a way of understanding writing down Feynman rules, and performing successively better approximations to an exact result in field theory which we don't have.

So what we have here will turn out to be Feynman rules for a theory which we don't yet have. Where do these Feynman rules come from? Quantising is tremendously restrictive.

Summary

Worldline actions

- Our action is
$$S = -m \int_{s_1}^{s_2} ds = -m \int_{\tau_1}^{\tau_2} d\tau \sqrt{-\eta^{\mu\nu} \dot{x}^\mu \dot{x}^\nu}$$
- We have conjugate momenta with on-shell mass condition
$$P^\mu = -\frac{m\dot{x}^\mu}{\sqrt{-\dot{x}^2}}, \quad P^2 + m^2 = 0$$
- It makes more sense to work with Einbeins, since we can work in the $m \to 0$ limit
$$S = \frac{1}{2} \int d\tau \, \left(e^{-1} \eta_{\mu\nu} \dot{x}^\mu \dot{x}^\nu - em^2\right)$$
- Two equations of motion come from the Einbein action
$$\frac{d}{d\tau}\left(e^{-1}\dot{x}^\mu\right) = 0, \quad \dot{x}^2 + e^2 m^2 = 0$$
- This has symmetries
$$\delta x^\mu = \xi \dot{x}^\mu, \quad \delta e = \frac{d}{d\tau}\left(\xi \dot{e}\right)$$
- In the massless limit, if we replace our Minkowski metric with a general metric, we recover the geodesic equations
$$S = \frac{1}{2} \int d\tau e^{-1} g_{\mu\nu} \dot{x}^\mu \dot{x}^\mu, \quad \ddot{x}^\mu + \Gamma^\mu_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta = 0$$

Strings

- A string is two dimensional, embedded with parameters σ, τ

$$X^{\mu}(\sigma, \tau) = X^{\mu}(\sigma + 2\pi n, \tau), \quad n \in \mathbb{Z}$$

- Our associated action is the Nambu-goto action

$$S[X] = -\frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{-\det\left(\eta_{\mu\nu}\partial_a X^{\mu} \partial_b X^{\beta}\right)}$$

- We add an extra degree of freedom h_{ab} to introduce the Polyakov action

$$S[X, h] = -\frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{-h} h^{ab} \eta_{\mu\nu} \partial_a X^{\mu} \partial_b X^{\nu}$$

Example Sheet 1