String Theory Lecture Notes

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1 Introduction

Summary

Worldline actions

- Our action is

$$S = -m \int_{s_1}^{s_2} ds = -m \int_{\tau_1}^{\tau_2} d\tau \sqrt{-\eta^{\mu\nu} \dot{x}^\mu \dot{x}^\nu}$$

- We have conjugate momenta with on-shell mass condition

$$P^\mu = -\frac{m\dot{x}^\mu}{\sqrt{-\dot{x}^2}}, \quad P^2 + m^2 = 0$$

- It makes more sense to work with Einbeins, since we can work in the $m \to 0$ limit

$$S = \frac{1}{2} \int d\tau \left(e^{-1} \eta_{\mu\nu} \dot{x}^\mu \dot{x}^\nu - em^2 \right)$$

- Two equations of motion come from the Einbein action

$$\frac{d}{d\tau} \left(e^{-1} \dot{x}^\mu \right) = 0, \quad \dot{x}^2 + e^2 m^2 = 0$$

- This has symmetries

$$\delta x^\mu = \xi \dot{x}^\mu, \quad \delta e = \frac{d}{d\tau} (\xi \dot{e})$$

- In the massless limit, if we replace our Minkowski metric with a general metric, we recover the geodesic equations

$$S = \frac{1}{2} \int d\tau e^{-1} g_{\mu\nu} \dot{x}^\mu \dot{x}^\mu, \quad \ddot{x}^\mu + \Gamma^\mu_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta = 0$$

Strings

- A string is two dimensional, embedded with parameters σ, τ

$$X^\mu (\sigma, \tau) = X^\mu (\sigma + 2\pi n, \tau), \quad n \in \mathbb{Z}$$

- Our associated action is the Nambu-goto action

$$S[X] = -\frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{-\det \left(\eta_{\mu\nu} \partial_a X^\mu \partial_b X^\beta \right)}$$

- We add an extra degree of freedom h_{ab} to introduce the Polyakov action

$$S[X, h] = -\frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{-h} h^{ab} \eta_{\mu\nu} \partial_a X^\mu \partial_b X^\nu$$

Example Sheet 1