Hilbert Spaces in Quantum Mechanics

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November 6, 2019

1 Hilbert Spaces

In quantum mechanics, we do physics in terms of 'states', which one may have seen denoted by a ket vector which looks like $|\psi\rangle$, and then extract probabilities of interest by taking the square of the modulus. For example, one may be familar with calculating the probability that a particle exists in some region of time by integrating over $|\psi(x)|^2$, which is a probability distribution function.

But what exactly is the mathematical formulation behind these states, and how do we put them on a slightly more general mathematical footing? This is where the idea of a Hilbert space comes in. A Hilbert space is a vector space, whose underlying field is the complex numbers $\mathbb{C}$. In addition, we have a well defined notion of an 'inner product', and in particular take note that this vector space is not necessarily finite in dimension. Our final condition is that this space is complete

In quantum mechanics, states are vectors in an infinite dimensional Hilbert space, and the inner product is defined to satisfy the following. The inner product is a map from states in the Hilbert space and its dual, to the complex numbers.

$$\mathcal{H}^* \times \mathcal{H} \to \mathbb{C}$$

The inner product between states $|\psi\rangle, \langle\phi|$ is denoted $\langle\phi|\psi\rangle$, which is a complex number. This somewhat represents an amplitude, and we can get a real number by taking the modulus of this. p> ;l

- Conjugate

1.1 Position and momentum representation

1.2 Things to cover

Classical versus quantum formalisms Harmonic oscillator.