PROBLEM SET 3

16822 GEOMETRY-BASED METHODS IN VISION (FALL 2022)
https://piazza.com/cmu/fall2022/16822

OUT: Oct. 04, 2022
DUE: Oct. 11, 2022 11:59 PM
Instructor: Shubham Tulsiani
TAs: Mosam Dabhi, Kangle Deng, Jenny Nan

START HERE: Instructions

- **Collaboration policy:** All are encouraged to work together BUT you must do your own work (code and write up). If you work with someone, please include their name in your write up and cite any code that has been discussed. If we find highly identical write-ups or code without proper accreditation of collaborators, we will take action according to university policies, i.e. you will likely fail the course. See the Academic Integrity Section detailed in the initial lecture for more information.

- **Late Submission Policy:** There are **no** late days for Problem Set submissions.

- **Submitting your work:**

    - We will be using Gradescope (https://gradescope.com/) to submit the Problem Sets. Please use the provided template. Submissions can be written in LaTeX. Regrade requests can be made, however this gives the TA the opportunity to regrade your entire paper, meaning if additional mistakes are found then points will be deducted. Each derivation/proof should be completed on a separate page. For short answer questions you **should** include your work in your solution.

- **Materials:** The data that you will need in order to complete this assignment is posted along with the writeup and template on Piazza.

For multiple choice or select all that apply questions, replace \choice with \CorrectChoice to obtain a shaded box/circle, and don't change anything else.

Instructions for Specific Problem Types

For "Select One" questions, please fill in the appropriate bubble completely:

Select One: Who taught this course?

- ● Shubham Tulsiani
- ○ Deepak Pathak
- ○ Fernando De la Torre
- ○ Deva Ramanan

For "Select all that apply" questions, please fill in all appropriate squares completely:

Select all that apply: Which are scientists?

- ■ Stephen Hawking
- ■ Albert Einstein
- ■ Isaac Newton
- □ None of the above

For questions where you must fill in a blank, please make sure your final answer is fully included in the given space. You may cross out answers or parts of answers, but the final answer must still be within the given space.

Fill in the blank: What is the course number?

16-822

1 Camera Models [6 pts]

1. **[4 pts]** Are these statements true or false?

   (a) Any camera projection matrix $\mathbf{P}$ can be decomposed as $\mathbf{KR}[\mathbf{I}| - \tilde{\mathbf{C}}]$.

   (b) A camera with centre at infinity is an affine camera.

   (c) If a 3×4 matrix $\mathbf{P}$ represents an affine camera, then $rank(\mathbf{P}) = 2$.

   (d) $\mathbf{P}^+\mathbf{u}$ corresponds to a point at infinity given a image location $\mathbf{u}$ and a camera matrix $\mathbf{P}$, where $\mathbf{P}^+ = \mathbf{P}^\mathsf{T}(\mathbf{PP}^\mathsf{T})^{-1}$.

   a) True.   P156

   b) False. can be affine / non-affine camera.   P166

   c) False rank(P)=3   P172

   d) True.   P161

2. **[2 pts]** What is the direction of surface normal to the principal plane?

   (a) when expressed in terms of the elements of $\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}$

   (b) when expressed in terms of the elements of the intrinsic matrix $\mathbf{K}$, rotation $\mathbf{R}$, and camera centre $\tilde{\mathbf{C}}$.

   For both the above, your answer need not be a normalized vector.

   a) $\vec{v} = \det(M)\vec{m}^3 = \begin{vmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{vmatrix} \begin{bmatrix} P_{31} \\ P_{22} \\ P_{33} \end{bmatrix}$

   b) $\vec{v} = \det(KR)(KR)^{3\mathsf{T}}$

          $(KR)^3$ is the third row of KR

2 Single-view Geometry [6 pts]

3. **[4 pts]** How many 3D to 2D correspondences would be required to determine the camera matrix $\mathbf{P}$ in each of these cases? Which of these cases can be solved by reducing the constraints on the unknown variables to equations of the form $\mathbf{Ax} = \mathbf{b}$, or $\mathbf{Ax} = \mathbf{0}$ s.t. $||x|| = 1$?

   (a) $\mathbf{P}$ matrix given all unknowns.

   (b) Known camera rotation and intrinsics, but unknown camera translation.

(c) Known camera extrinsics, and unknown camera intrinsics but with the information that pixels are square.

(d) **P** matrix given **P** corresponds to an affine camera.

a). $11/2 \geq 6$ c) $3/2 = 2$

b). $3/2 = 2$ d). $3/2 = 2$

4. **[2 pts]** Given a line l in an image, what is the equation of the plane that projects to l? You may assume **P** is known.

$$\pi = P^T \ell. \quad \text{proof } P.197$$

3   Single-view Reconstruction [8 pts]

A

5. **[2 pts]** Show that the absolute conic in $\mathbb{P}^3$ is unchanged under a 3D euclidean transform.

since absolute conic Ω_∞ is on π_∞, we can compute its image under $H = KR$ which is considered as a 3D euclidean transform.

Mapping of a absolute conic: $\pi'_\infty = H^{-T}\pi_\infty H^{-1} = (KR)^{-T} I (KR)^{-1} = K^{-T}R R^{-1}K^{-1}$

$$= K^{-T}K^{-1}$$

6. **[4 pts]** Are these statements true or false?

   (a) The image of absolute conic ω only comprises of imaginary points in the image plane.

   (b) The rays corresponding to pixels $\mathbf{u}_1$ and $\mathbf{u}_2$ are orthogonal if and only if $\mathbf{u}_1^T \omega \mathbf{u}_2 = 0$.

(c) Assuming zero skew, the IAC can be identified given 4 sets of vanishing points for line pairs in orthogonal directions.

(d) The homography between the image plane and the plane at infinity does not depend on the camera location.

a). True P240

b) True.

c) True. $w = \begin{bmatrix} w_1 & 0 & w_2 \\ 0 & w_3 & w_4 \\ w_2 & w_4 & w_5 \end{bmatrix}$

d. True. $x = P X_\infty = KR[I|-\hat{C}]\begin{pmatrix}\vec{d}\\0\end{pmatrix}$

$= KR\vec{d}$

$H = KR$

7. **[2 pts]** Given $\mathbf{K} = diag(2, 2, 1)$, find the normal direction of a plane whose vanishing line is given by $[1, 2, 3]^T$.

P248 $\vec{n} = K^T \ell = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}$

Collaboration Questions Please answer the following:

1. Did you receive any help whatsoever from anyone in solving this assignment?

    ○ Yes

    ○ No

   - If you answered 'Yes', give full details:

   - (e.g. "Jane Doe explained to me what is asked in Question 3.4")

2. Did you give any help whatsoever to anyone in solving this assignment?

    ○ Yes

    ○ No

   - If you answered 'Yes', give full details:

   - (e.g. "I pointed Joe Smith to section 2.3 since he didn't know how to proceed with Question 2")

3. Did you find or come across code that implements any part of this assignment ?

    ○ Yes

    ○ No

   - If you answered 'Yes', give full details: <u>No</u>

   - (book & page, URL & location within the page, etc.).