Weekly Challenge 01: Comparison

CS/MATH 113 Discrete Mathematics

Spring 2024

1. How about them apples?

The replicator aboard USS Enterprise has developed a fault—synthesized apples have insufficient nutrition but are otherwise identical to regular apples. Doctor Beverly Crusher is on the case. Scanning a bunch of apples, her tricorder can indicate if the bunch contains any faulty apples, but it cannot identify them.

(a) Dr. Crusher is investigating a bunch of 5 apples out of which 1 is known to be faulty. Describe how she can identify the faulty apple in no more than 3 tricorder scans.

(b) What is the minimum number of scans that Dr. Crusher needs to perform in order to guarantee finding the single faulty apple in a bunch of size n? Justify your answer.

Solution: a.

- Divide the apples into 3 and 2 pcs
- Then use the tricorder on either set
- If it detects a faulty apple in the 2 pcs, then the other 3 are eliminated and now we have to scan these 2 further
- We still have 2 scans left, so check one apple and find out
- If the tricorder does not detect a faulty apple in the 2 pcs set, then we come to the 3pcs with 2 scans left knowing that the faulty apple is in there
- Divide these 3 apples into 1 and 2 pcs
- Now use the tricorder again on the 2pcs, if it does not detect anything then the other apple is faulty
- If the tricorder detects a faulty apple in these 2pcs, use your last scan to determine the faulty apple by dividing these 2pcs into 1-1 and repeating step 4

b. The n apples will always break down into 2 (then 1-1) when we recursively divide them into halves (if even or half ± 1 if odd). For example there are 16 apples, they will be halved 4 times till we reach 1 apple, so we need 4 scans. For 15 apples we will proceed as 8-7, 4-4/3-4, 2-2/2-1, then 1-1 and the number of scans still sum up to be 4. This logic is using powers of 2 in reverse i.e. log. If n is odd then we add 1 to it before putting in this formula where n is the number of apples, s is the minimum scans, and 2 is the base.

log2n=s or log2(n+1)=s for odd n