

Part 1: Proposal Logic

Facts about this world:

- (1) $\text{says}(A,S) \wedge \text{knave}(A) \Rightarrow \sim S$
- (2) $\text{says}(A,S) \wedge \text{knight}(A) \Rightarrow S$
- (3) $\sim \text{knight}(A) \Rightarrow \text{knave}(A)$
- (4) $\sim \text{knave}(A) \Rightarrow \text{knight}(A)$

Q1:

- Suppose B is a Knight
 - $\text{Knight}(B)$
 - $\text{Says}(B, \sim(\text{Knave}(B) \wedge \text{Knave}(A)))$
 - By (2) and MP we can conclude: $\sim \text{Knave}(B) \wedge \sim \text{Knave}(A)$
 - By AE: $\sim \text{Knave}(B)$
 - there is no contradiction here
 - But we need to prove that: $\sim \text{Knave}(A)$ for $\text{Knight}(B)$ to be true
- What is A?
 - We know by (2) and from above that $\text{Says}(A,S)$ must be true for $\text{Knight}(B)$ to be true
 - $\text{Says}(A, \text{Knave}(B))$
 - This is a contradiction, so our assumption that $\text{Knight}(B)$ was false
 - By (3), and the resolution rule we conclude $\text{Knave}(B)$ and $\text{Knight}(A)$

Q2:

- Suppose A is a Knight
 - $\text{Knight}(A)$
 - $\text{Says}(A, \text{Knight}(A) \wedge \text{Knight}(B))$
 - By AE: $\text{Knight}(A)$
 - No contradictions here, but $\text{Knight}(B)$ must also be true for $\text{Knight}(A)$ to be true
- Is B a Knight?
 - We know by (2) and from above that $\text{Says}(B,S)$ must be true for $\text{Knight}(A)$ to be true
 - $\text{Says}(B, \text{Knight}(B) \wedge \text{Knave}(A))$
 - By (2), MP, and AE we conclude: $\text{Knight}(B)$
 - This is a contradiction for $\text{Says}(A,S)$
 - By (3) and the resolution rule we conclude $\text{Knave}(A)$

Q3:

Answer is yes.

We know that: $(\text{Freedom}(\text{Left Road}) \vee \text{Freedom}(\text{Right Road}))$ and KB

What we also know which is critical (new fact): **(5) Knight(A) \vee Knave(A)**

Question can be: Would the other member answer yes if asked that the road to freedom is the left road?

- Suppose A is a Knight and $\text{Freedom}(\text{Left Road})$
 - Knight(A)
 - By (2) and (5) and KB, we can conclude: $\sim \text{Freedom}(\text{Left Road})$
 - Therefore the Knight would answer no
 - The knight will always tell the truth and inform us that the response from the Knave would be no
- Now suppose A is a Knave with same setting: $\text{Freedom}(\text{Left Road})$
 - Knave(A)
 - By (1) and (5) and KB, we can conclude: $\sim \text{Freedom}(\text{Left Road})$
 - Therefore the Knave will lie about the Knight and say no
 - The knave will also not tell the truth about the Knight
 - We know that regardless if A is Knight or A is Knave the response is the same
 - Based on this tautology we can conclude: $\text{Response}(A,S) \leftrightarrow \text{Response}(B,S)$
 - $\sim (\text{Response}(A,S) \wedge \text{Response}(B,S))$
 - By DeMorgan's law: $\sim \text{Response}(A,S) \vee \sim \text{Response}(B,S)$
 - Hence, the opposite of what A or B says will be the true road to freedom

Part 2: FOL

Q4:

1. $\forall x,y \text{ Creature}(x) \wedge \text{Creature}(y) \wedge \text{CanEat}(x,y) \rightarrow \text{CanClobber}(x,y)$
2. $\forall x, \exists y \text{ Monster}(x) \wedge \text{Creature}(y) \wedge \neg \text{Monster}(y) \rightarrow \text{CanEat}(x,y)$
3. $\forall x, y, z \text{ CanClobber}(x, y) \wedge \text{CanClobber}(y, z) \rightarrow \text{CanClobber}(x, z)$
4. $\forall x \forall y \text{ IsOgre}(x) \wedge \text{IsDwarf}(y) \rightarrow \text{CanEat}(x, y)$
5. $\forall x \forall y \text{ IsDwarf}(x) \wedge \text{IsGoblin}(y) \rightarrow \text{CanClobber}(x,y)$
6. $\forall x \text{ IsGoblin}(x) \wedge \text{IsMonster}(x)$

Q5:

1.
 - a. $\forall x,y \text{ Creature}(x) \wedge \text{Creature}(y) \wedge \text{CanEat}(x,y) \rightarrow \text{CanClobber}(x,y)$

- b. Eliminate \rightarrow and drop universals: $\neg(\text{Creature}(x) \wedge \text{Creature}(y) \wedge \text{CanEat}(x,y)) \vee \text{CanClobber}(x,y)$
- c. Move in Negation: $\neg\text{Creature}(x) \vee \neg\text{Creature}(y) \vee \neg\text{CanEat}(x, y) \vee \text{CanClobber}(x,y)$
- d. Distribute \wedge over \vee : $(\neg\text{Creature}(x) \vee \neg\text{Creature}(y)) \wedge (\neg\text{Creature}(x) \vee \neg\text{CanEat}(x, y)) \wedge (\neg\text{Creature}(x) \vee \text{CanClobber}(x,y))$
- e. Split Conjunction: $(\neg\text{Creature}(x) \vee \neg\text{Creature}(y)), (\neg\text{Creature}(x) \vee \neg\text{CanEat}(x, y)), (\neg\text{Creature}(x) \vee \text{CanClobber}(x,y))$

2.

- a. $\forall x, \exists y \text{ Monster}(x) \wedge \text{Creature}(y) \wedge \neg\text{Monster}(y) \rightarrow \text{CanEat}(x,y)$
- b. $\neg(\text{Monster}(x) \wedge \text{Creature}(\text{sk}(y)) \wedge \neg\text{Monster}(\text{sk}(y))) \vee \text{CanEat}(x,\text{sk}(y))$
- c. $\neg\text{Monster}(x) \vee \neg\text{Creature}(\text{sk}(y)) \vee \text{Monster}(\text{Sk}(y)) \vee \text{CanEat}(x,\text{sk}(y))$
- d. $(\neg\text{Monster}(x) \vee \neg\text{Creature}(\text{sk}(y))) \wedge (\neg\text{Monster}(x) \vee \text{Monster}(\text{sk}(y))) \wedge (\neg\text{Monster}(x) \vee \text{CanEat}(x, \text{sk}(y)))$
- e. $(\neg\text{Monster}(x) \vee \neg\text{Creature}(\text{sk}(y))), (\neg\text{Monster}(x) \vee \text{Monster}(\text{sk}(y))), (\neg\text{Monster}(x) \vee \text{CanEat}(x, \text{sk}(y)))$

3.

- a. $\forall x, y, z \text{ CanClobber}(x, y) \wedge \text{CanClobber}(y, z) \rightarrow \text{CanClobber}(x, z)$
- b. $\neg(\text{CanClobber}(x, y) \wedge \text{CanClobber}(y, z)) \vee \text{CanClobber}(x, z)$
- c. $(\neg\text{CanClobber}(x, y) \vee \neg\text{CanClobber}(y, z)) \vee \text{CanClobber}(x, z)$
- d. $(\neg\text{CanClobber}(x, y) \vee \neg\text{CanClobber}(y, z)) \wedge (\neg\text{CanClobber}(x, y) \vee \text{CanClobber}(x, z))$
- e. $(\neg\text{CanClobber}(x, y) \vee \neg\text{CanClobber}(y, z)), (\neg\text{CanClobber}(x, y) \vee \text{CanClobber}(x, z))$

4.

- a. $\forall x \forall y \text{ IsOgre}(x) \wedge \text{IsDwarf}(y) \rightarrow \text{CanEat}(x, y)$
- b. $\neg(\text{IsOgre}(x) \wedge \text{IsDwarf}(y)) \vee \text{CanEat}(x, y)$
- c. $\neg \text{IsOgre}(x) \vee \neg\text{IsDwarf}(y) \vee \text{CanEat}(x,y)$
- d. $(\neg\text{IsOgre}(x) \vee \neg\text{IsDwarf}(y)) \wedge (\neg\text{IsOgre}(x) \vee \text{CanEat}(x,y))$
- e. $(\neg\text{IsOgre}(x) \vee \neg\text{IsDwarf}(y)), (\neg\text{IsOgre}(x) \vee \text{CanEat}(x,y))$

5.

- a. $\forall x \forall y \text{ IsDwarf}(x) \wedge \text{IsGoblin}(y) \rightarrow \text{CanClobber}(x,y)$
- b. $\neg(\text{IsDwarf}(x) \wedge \text{IsGoblin}(y)) \vee \text{CanClobber}(x,y)$
- c. $(\neg\text{IsDwarf}(x) \vee \neg\text{IsGoblin}(y)) \vee \text{CanClobber}(x,y)$
- d. $(\neg\text{IsDwarf}(x) \vee \neg\text{IsGoblin}(y)) \wedge (\neg\text{IsDwarf}(x) \vee \text{CanClobber}(x,y))$
- e. $(\neg\text{IsDwarf}(x) \vee \neg\text{IsGoblin}(y)), (\neg\text{IsDwarf}(x) \vee \text{CanClobber}(x,y))$

6.

- a. $\forall x \text{ IsGoblin}(x) \wedge \text{IsMonster}(x)$
- b. $\text{IsGoblin}(x), \text{IsMonster}(x)$

Q6:

Knowledge Base:

- 1. $(\neg\text{Creature}(x) \vee \neg\text{Creature}(y))$

2. $(\neg \text{Creature}(x) \vee \neg \text{CanEat}(x, y))$
3. $(\neg \text{Creature}(x) \vee \text{CanClobber}(x, y))$
4. $(\neg \text{Monster}(x) \vee \neg \text{Creature}(\text{sk}(y)))$
5. $(\neg \text{Monster}(x) \vee \text{Monster}(\text{sk}(y)))$
6. $(\neg \text{Monster}(x) \vee \text{CanEat}(x, \text{sk}(y)))$
7. $(\neg \text{CanClobber}(x, y) \vee \neg \text{CanClobber}(y, z))$
8. $(\neg \text{CanClobber}(x, y) \vee \text{CanClobber}(x, z))$
9. $(\neg \text{IsOgre}(x) \vee \neg \text{IsDwarf})$
10. $(\neg \text{IsOgre}(x) \vee \text{CanEat}(x, y))$
11. $(\neg \text{IsDwarf}(x) \vee \neg \text{IsGoblin}(y))$
12. $(\neg \text{IsDwarf}(x) \vee \text{CanClobber}(x, y))$
13. $\text{IsGoblin}(x)$
14. $\text{Monster}(x)$
15. $\text{isDwarf}(x)$

Prove: $\text{CanClobber}(\text{Ogre}, \text{Goblin}) \wedge \text{IsMonster}(\text{goblin})$

16. $\neg \text{CanClobber}(x, y) \wedge \neg \text{IsMonster}(\text{goblin})$
17. (16, 12) $\neg \text{IsDwarf} \wedge \neg \text{IsMonster}(\text{goblin})$ $y = \text{goblin}$
18. (17, 5) $\neg \text{IsMonster}(x) \wedge \neg \text{IsDwarf}(x)$
19. (18, 15) $\neg \text{Monster}(x)$
20. (18, 14) NIL

Therefore $\neg(\neg \text{CanClobber}(x, y))$

Q7:

16. $\neg \text{CanEat}(\text{Ogre}, \text{dwarf})$ $x = \text{ogre}, y = \text{dwarf}$
17. (16, 6) $\neg \text{Monster}(x)$
18. (17, 14) NIL

Therefore $\neg(\neg \text{CanEat}(x, y))$

Part 3: Prog

Q8:

- Q8a. true
- Q8b. $X = \text{mia}$
- Q8c. $X = \text{mia}$
- Q8d. false
- Q8e. $X = \text{Vincent}, X = \text{marsellus}$
- Q8f. $X = \text{Vincent}, X = \text{Marsellus}$

Q8g. X = honey_bunny

Q9:

Code:

```
car(bmw).  
car(civic).  
motorcycle(harley).  
fast(bmw).  
slow(civic).  
fast(harley).  
fast(theFlash).
```

```
fun(X):- car(X), fast(X).  
fun(X):- motorcycle(X), fast(X).
```

Query:

1. fun(bmw). //true, false
2. fun(Harley). //true
3. fun(civic). //fales
4. fun(theFlash). //false

Q10:

Code:

```
creature(dwarf).  
creature(ogre).  
monster(goblin).  
eats(ogre,dwarf).  
eats(dwarf, goblin).  
creature(X):- monster(X).  
eats(X, otherCreatures):- monster(X).  
clobbers(X,Y):- eats(X,Y), creature(X), creature(Y).  
clobbers(X,Y):- clobbers(X,Z), clobbers(Z,Y).
```

Query:

1. Clobbers(ogre, goblin). //true
2. Eats(ogre,X). //X = dwarf