DIFFERENTIATION

LIST OF DEVRIVATIVES

1] $\frac{d}{dx}(x^n) = nx^{n-1}$, 2]. $\frac{d}{dx}(x) = 1$, 3] $\frac{d}{dx}(x^2) = 2x^{2-1} = 2x$, 4] $\frac{d}{dx}(x^3) = 3x^{3-1} = 3x^2$

5] $\frac{d}{dx}(x^4) = 4x^{4-1}$ 6] $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$,7] $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$,8] $\frac{d}{dx}(e^x) = e^x$

9] $\frac{d}{dx}(\log x) = \frac{1}{x}$ 10] $\frac{d}{dx}(\sin x) = \cos x$, 11] $\frac{d}{dx}(\cos x) = -\sin x$,

12] $\frac{d}{dx}(\tan x) = \sec^2 x$, 13] $\frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$,14] $\frac{d}{dx}(\sec x) = \sec x \tan x$

15] $\frac{d}{dx}(\mathrm{cosec}\, x) = -\mathrm{cosec}\, x \cot x$, 16] $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$,

17], $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$, 18] $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$ 19] $\frac{d}{dx}(any\ number) = 0$

20] $\frac{d}{dx}(ax) = a\frac{d}{dx}(x)$

1) Evaluate $\lim_{x\to 0}\frac{2x-3}{3x+5} = \frac{2\times 0-3}{3\times 0+5} = \frac{0-3}{0+5} = -\frac{3}{5}$

2) **State product rule of differentiation**

$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ = First function$\times\frac{d}{dx}(second) + second \times \frac{d}{dx}(first)$

3) Differentiate x logx w. r. to x

to find derivatives of terms in y , multiply $with\ \frac{dy}{dx}$

ANS: $x^2+y^2=25$, Taking derivatives, $\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(25)$

$2x + 2y \times \frac{dy}{dx} = 0$, $2y \times \frac{dy}{dx} = -2x$, $\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$

5)If y = sin x, find $\frac{d^2y}{dx^2}$

$\frac{dy}{dx} = \frac{d}{dx}(\sin x) = \cos x$

$\frac{d^2y}{dx^2}$ = derivative of $\frac{dy}{dx} = \frac{d}{dx}(\cos x) = -\sin x$

6) **Evaluate** $\lim_{\theta\to 0}\frac{\sin 3\theta}{3\theta}$

$$\lim_{\theta\to 0}\frac{\sin a\theta}{a\theta} = 1$$

$\lim_{\theta\to 0}\frac{\sin 3\theta}{3\theta} = 1$, $a = 3$

7) Differentiate $e^x \sin x$ $with\ respect\ to\ x$

$\frac{d}{dx}(e^x) = e^x$, $\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(e^x \sin x)$ = First function $\times \frac{d}{dx}(second) + second \times \frac{d}{dx}(first)$

$= e^x \times \frac{d}{dx}(\sin x) + \sin x \times \frac{d}{dx}(e^x) = e^x \times \cos x + \sin x \times e^x$

8) **Differentiate** $\frac{\sin x}{1+\cos x}$ **with respect to** x

Quotient Rule

$$\frac{\text{Denominator} \times \frac{d}{dx}(numerator) - numerator \times \frac{d}{dx}(denominator)}{denominator^2}$$

$y = \frac{\sin x}{1+\cos x}$, $Numerator = \sin x,\ \ Denominator = 1 + \cos x$

$$\frac{dy}{dx} = \frac{(1+\cos x) \times \frac{d}{dx}(\sin x) - \sin x \times \frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}$$

$$= \frac{(1+\cos x) \times \cos x - \sin x \times (0+-\sin x)}{(1+\cos x)^2} = \frac{(1+\cos x) \times \cos x - \sin x \times (-\sin x)}{(1+\cos x)^2}$$

$$= \frac{(1+\cos x) \times \cos x + \sin^2 x}{(1+\cos x)^2}$$

9) $x = a\sec\theta, y = b\ \tan\theta, find\ \frac{dy}{dx}$

$\frac{d}{dx}(\tan x) = sec^2x$, $\frac{d}{d\theta}(\tan\theta) = sec^2\theta$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$\frac{dy}{d\theta}$, $y = b\ \tan\theta, \frac{dy}{d\theta} = bsec^2\,\theta$

$derivative\ of, secx = secxtanx\ , derivative\ of\ a\ secx = a \times secxtanx$

$\frac{dx}{d\theta}$, $x = a\sec\theta,\ \ \frac{dx}{d\theta} = a\ sec\theta\ tan\theta,$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}},\ \frac{dy}{dx} = \frac{bsec^2\theta}{asec\theta tan\theta} = \frac{b\ sec\theta}{a\ tan\theta} = \frac{b}{a} \times \frac{\frac{1}{cos\theta}}{\frac{sin\theta}{cos\theta}}$$

$$= \frac{b}{a} \times \frac{1}{sin\theta} = \frac{b}{a}\ cosec\theta$$

$$\frac{1}{cos\theta} = sec\theta,\ \frac{sin\theta}{cos\theta} = tan\theta,\ \frac{1}{sin\theta} = cosec\theta$$

10) Find the second derivative of $x\ log\ x$

$x\ log\ x$ is a product, First function x,second function log x

First derivative = $\frac{dy}{dx} = \frac{d}{dx}(x\ log\ x)$

$\frac{d}{dx}(x\ log\ x)$ = First function$\times\frac{d}{dx}(second) + second \times \frac{d}{dx}(first)$

$= x \times \frac{d}{dx}(logx) + \ logx \times \frac{d}{dx}(x) = x \times \frac{1}{x} + logx \times 1 = 1 + log\ x,$

- $x \times \frac{1}{x} = 1$

11) $lim_{x\to a}\frac{x^n - a^n}{x-a} = na^{n-1}$,Evaluate $lim_{x\to 3}\frac{x^4-81}{x^2-9}$, 81 = 3×3×3×3 =$3^4$, 9 = 3^2

$lim_{x\to3}\frac{x^4-3^4}{x^2-3^2}$,divide numerator and denominator with $x-3$, $lim_{x\to3}\frac{\frac{x^4-3^4}{x-3}}{\frac{x^2-3^2}{x-3}}$

$$lim_{x\to3}\frac{\frac{x^4-3^4}{x-3}}{\frac{x^2-3^2}{x-3}} = \frac{4\times3^{4-1}}{2\times3^{2-1}} = \frac{4\times3^3}{2\times3^1} = \frac{4\times3\times3\times3}{2\times3} = 18$$

12) Differentiate with respect to x, $y = x^2\,secx$,

$\frac{d}{dx}(x^2) = 2x$, $\frac{d}{dx}(secx) = secxtanx$

$y = x^2\,secx$,use product rule

$\frac{d}{dx}(x^2\,secx)$ = First function$\times\frac{d}{dx}(second) + second \times \frac{d}{dx}(first)$

$= x^2 \times \frac{d}{dx}(secx) + secx \times \frac{d}{dx}(x^2) = x^2 \times secxtanx + secx \times 2x$

13) Differentiate with respect to x, $y = \frac{1-x^2}{1+x^2}$, use quotient rule

$\frac{d}{dx}(x^2) = 2x$, $\frac{d}{dx}(1) = 0$

$$\frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right) = \frac{\text{Denominator}\times\frac{d}{dx}(numerator) - numerator\times\frac{d}{dx}(denominator)}{denominator^2}$$

$$Numerator = 1 - x^2, \ \ Denominator = 1 + x^2$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right) = \frac{(1+x^2)\times\frac{d}{dx}(1-x^2) - (1-x^2)\times\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$$

$$= \frac{(1+x^2)\times(0-2x) - (1-x^2)\times(0+2x)}{(1+x^2)^2} = \frac{(1+x^2)\times(-2x) - (1-x^2)\times(2x)}{(1+x^2)^2}$$

14) Find the derivative of tan$x$ using quotient rule

$tanx = \frac{sinx}{cosx}$, $\frac{d}{dx}(sinx) = cosx$, $\frac{d}{dx}(cosx) = -\ sinx$, $cos^2x + sin^2x = 1$

$$Numerator = sinx, \ \ Denominator = cosx$$

$$\frac{d}{dx}(\ tanx) = \frac{d}{dx}\left(\frac{sinx}{cosx}\right) = \frac{\text{Denominator}\times\frac{d}{dx}(numerator) - numerator\times\frac{d}{dx}(denominator)}{denominator^2}$$

$$= \frac{(cosx)\times\frac{d}{dx}(sinx) - (sinx)\times\frac{d}{dx}(cosx)}{(cosx)^2} = \frac{(cosx)\times(cosx) - (sinx)\times(-sinx)}{(cosx)^2}, \ (-)\times(-) = +$$

$$= \frac{cos^2x + sin^2x}{(cosx)^2} = \frac{1}{cos^2x} = sec^2x \ , \ \frac{1}{cosx} = secx, (cosx)^2 = cos^2x$$

15)Find the derivative of cot x using quotient rule

$cotx = \frac{cosx}{sinx}$, $\frac{d}{dx}(sinx) = cosx$, $\frac{d}{dx}(cosx) = -\ sinx$, $sin^2x + cos^2x = 1$

$$\frac{d}{dx}(\ cotx) = \frac{d}{dx}\left(\frac{cosx}{sinx}\right) = \frac{\text{Denominator}\times\frac{d}{dx}(numerator) - numerator\times\frac{d}{dx}(denominator)}{denominator^2}$$

$$\frac{(sinx)\times\frac{d}{dx}(cosx) - (cosx)\times\frac{d}{dx}(sinx)}{(sinx)^2} = \frac{(sinx)\times(-sinx) - (cosx)\times(cosx)}{(sinx)^2}, \ \ (+)\times(-) = -$$

$$= \frac{-sin^2x - cos^2x}{(sinx)^2} = \frac{-(sin^2x + cos^2x)}{(sinx)^2} = \frac{-1}{sin^2x} = -cosec^2x$$

16) $x = a\,(\theta - \sin\theta), y = a(1 - \cos\theta)$, find $\frac{dy}{dx}$, $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

$\frac{d}{dx}(\sin x) = \cos x$, $\frac{d}{d\theta}(\sin\theta) = \cos\theta$, $\frac{d}{d\theta}(\cos\theta) = -\sin\theta$, $\frac{d}{dx}(1) = 0$, $\frac{d}{d\theta}(1) = 0$

$y = a(1 - \cos\theta)$, $\frac{dy}{d\theta} = a(0 - -\sin\theta), a(- - \sin\theta) = a\sin\theta$, $- - = +$

$\frac{d}{dx}(x) = 1, \frac{d}{d\theta}(\theta) = 1$, $x = a\,(\theta - \sin\theta)$, $\frac{dx}{d\theta} = a\,(1 - \cos\theta)$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a\,(1-\cos\theta)}$$

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17) $x = a\,t^2, y = 2at$, find $\frac{dy}{dx}$, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$, $\frac{d}{dx}(x) = 1$, , $\frac{d}{dt}(t) = 1$,

$\frac{d}{dx}(x^2) = 2x, \frac{d}{dx}(t^2) = 2t$

$y = 2at$, $\frac{dy}{dt} = \frac{d}{dt}(2at) = 2a \times, \frac{d}{dt}(t) = 2a \times 1 = 2a$

$x = a\,t^2$, $\frac{dx}{dt} = \frac{d}{dt}(at^2) = a \times 2t = 2at$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$$

18) If $y = x\sin x$, prove that $\frac{d^2y}{dx^2} + y = 2\cos x$

$y = x\sin x$,product ,use product rule

$\frac{d}{dx}(x\sin x)$ = First function$\times\frac{d}{dx}(second) + second \times \frac{d}{dx}(first)$

$= x \times \frac{d}{dx}(\sin x) + \sin x \times \frac{d}{dx}(x) = x \times (\cos x) + \sin x \times 1 = x\,(\cos x) + \sin x$

$\frac{d^2y}{dx^2} = \frac{d}{dx}[x\,(\cos x) + \sin x] = \frac{d}{dx}[x\,(\cos x)] + \frac{d}{dx}(\sin x) =$

First function$\times\frac{d}{dx}(second) + second \times \frac{d}{dx}(first) + \frac{d}{dx}(\sin x)$

$= x \times \frac{d}{dx}(\cos x) + \cos x \times \frac{d}{dx}(x) + \cos x = x \times -\sin x + \cos x \times 1 + \cos x$

$= -x\sin x + \cos x + \cos x = -x\sin x + 2\cos x$, $y = x\sin x$

$\frac{d^2y}{dx^2} + y = -x\sin x + 2\cos x + x\sin x = 2\cos x$

19) $y = \log(\sec x + \tan x)$, find $\frac{dy}{dx}$, $\frac{d}{dx}(\log x) = \frac{1}{x}$

$\frac{d}{dx}(\sec x) = \sec x\tan x$, $\frac{d}{dx}(\tan x) = \sec^2 x$

18) If $y = x \sin x$, prove that $\frac{d^2y}{dx^2} + y = 2\cos x$

$y = x \sin x$,product ,use product rule

$\frac{d}{dx}(x \sin x)$ = First function$\times\frac{d}{dx}(second) + second \times \frac{d}{dx}(first)$

$= x \times \frac{d}{dx}(\sin x) + \sin x \times \frac{d}{dx}(x) = x \times (\cos x) + \sin x \times 1 = x(\cos x) + \sin x$

$\frac{d^2y}{dx^2} = \frac{d}{dx}[x(\cos x) + \sin x] = \frac{d}{dx}[x(\cos x)] + \frac{d}{dx}(\sin x) =$

First function$\times\frac{d}{dx}(second) + second \times \frac{d}{dx}(first) + \frac{d}{dx}(\sin x)$

$= x \times \frac{d}{dx}(\cos x) + \cos x \times \frac{d}{dx}(x) + \cos x = x \times -\sin x + \cos x \times 1 + \cos x$

$= -x\sin x + \cos x + \cos x = -x\sin x + 2\cos x$, $y = x \sin x$

$\frac{d^2y}{dx^2} + y = -x\sin x + 2\cos x + x\sin x = 2\cos x$

19) $y = \log(\sec x + \tan x)$, find $\frac{dy}{dx}$, $\frac{d}{dx}(\log x) = \frac{1}{x}$

$\frac{d}{dx}(\sec x) = \sec x \tan x$, $\frac{d}{dx}(\tan x) = \sec^2 x$

Ans: $y = \log(\sec x + \tan x)$. $\frac{dy}{dx} = \frac{1}{(\sec x + \tan x)} \times \frac{d}{dx}(\sec x + \tan x)$

$$= \frac{1}{(\sec x + \tan x)}(\sec x \tan x + \sec^2 x)$$

20) $y = \sin 2x$, find $\frac{dy}{dx}$, $\frac{d}{dx}(\sin x) = \cos x$, $\frac{d}{dx}(2x) = 2$,

$\frac{d}{dx}(\sin 2x) = \cos 2x \times 2$

21) $y = e^{\log x}$, find $\frac{dy}{dx}$, $\frac{d}{dx}(e^x) = e^x$, $\frac{d}{dx}(\log x) = \frac{1}{x}$,

$\frac{d}{dx}(e^{\log x}) = (e^{\log x}) \times \frac{d}{dx}(\log x) = (e^{\log x}) \times \frac{1}{x}$

22) $y = \frac{\sin 2x}{1+\cos 2x}$, find $\frac{dy}{dx}$ (use quotient rule), $\frac{d}{dx}(\sin 2x) = \cos 2x \times 2$

$\frac{d}{dx}(\cos 2x) = -\sin 2x \times 2$, $\frac{d}{dx}(1) = 0$

$$\frac{d}{dx}\left(\frac{\sin 2x}{1+\cos 2x}\right) = \frac{\text{Denominator}\times\frac{d}{dx}(numerator) - numerator\times\frac{d}{dx}(denominator)}{denominator^2}$$

$$= \frac{(1+\cos 2x)\times\frac{d}{dx}(\sin 2x) - (\sin 2x)\times\frac{d}{dx}(1+\cos 2x)}{(1+\cos 2x)^2} =$$

$$\frac{(1+\cos 2x)\times \cos 2x \times 2 - (\sin 2x)\times(0 + -\sin 2x \times 2)}{(1+\cos 2x)^2}$$