MAP55616-02 - Cylindrical Radiator

Finite Differences model

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Task 1 - calculation in the cpu

The CPU version simulates heat propagation on a 2D matrix over p iterations.
Two functions were implemented:

- `propagate_step`: updates the matrix for one time step, considering periodic boundary conditions.
- `compute_averages`: calculates the average temperature of each row.

A double-buffering approach was used (two matrices swapping roles) to avoid overwriting data during updates.

```
liuy41@cuda01:~$ ./radiator_cpu
Completed 10 iterations on 32x32 matrix.
Overall average temperature: 0.179616
liuy41@cuda01:~$
```

```
liuy41@cuda01:~$ ./radiator_cpu -n 10 -m 10 -p 20 -a
Row averages:
  row 0: 0.00913908
  row 1: 0.0365563
  row 2: 0.0822517
  row 3: 0.146225
  row 4: 0.228477
  row 5: 0.329007
  row 6: 0.447815
  row 7: 0.584901
  row 8: 0.740265
  row 9: 0.913908
liuy41@cuda01:~$ ./radiator_cpu -n 5 -m 5 -p 5 -a
Row averages:
  row 0: 0.032884
  row 1: 0.131536
  row 2: 0.295956
  row 3: 0.526144
  row 4: 0.8221
```

Task 2 - parallel implementation

The GPU version follows the same main steps as the CPU version: matrix initialization, p-step propagation, and row average calculation. The default block size is 16x16.

Two CUDA kernels were used:

- `propagate_kernel`: one thread updates one matrix element with periodic boundaries.
- `average_kernel`: one thread computes one row's average.

Memory copies and synchronization were managed properly. CUDA events measured GPU allocation, transfer, and kernel times. Options `-c` and `-t` were added.

Result

```
liuy41@cuda01:~$ ./radiator_gpu -c -t -n 64 -m 64 -p 20
GPU alloc: 0.085056 ms
GPU H2D: 0.030176 ms
GPU propagate: 0.321472 ms
GPU average: 0.022912 ms
GPU D2H: 0.046848 ms
liuy41@cuda01:~$ ./radiator_gpu -t -n 64 -m 64 -p 20
CPU compute: 0.863016 ms
GPU alloc: 0.086016 ms
GPU H2D: 0.026624 ms
GPU propagate: 0.286112 ms
GPU average: 0.030592 ms
GPU D2H: 0.047648 ms
Matrix mismatches (>1e-4): 0, max diff: 4.17233e-07
Average mismatches (>1e-4): 0, max diff: 1.78814e-07
Speedup: 2.72499
liuy41@cuda01:~$ ./radiator_gpu -t -n 32 -m 32 -p 20
CPU compute: 0.206836 ms
GPU alloc: 0.092128 ms
GPU H2D: 0.019712 ms
GPU propagate: 0.30224 ms
GPU average: 0.031584 ms
GPU D2H: 0.047968 ms
Matrix mismatches (>1e-4): 0, max diff: 4.17233e-07
Average mismatches (>1e-4): 0, max diff: 2.08616e-07
Speedup: 0.619596
liuy41@cuda01:~$ ./radiator_gpu -t -n 64 -m 128 -p 20
CPU compute: 1.76643 ms
GPU alloc: 0.084256 ms
GPU H2D: 0.045056 ms
GPU propagate: 0.284224 ms
GPU average: 0.024832 ms
```

```
GPU D2H: 0.072096 ms
Matrix mismatches (>1e-4): 0, max diff: 4.17233e-07
Average mismatches (>1e-4): 0, max diff: 1.49012e-07
Speedup: 5.71556
liuy41@cuda01:~$ ./radiator_gpu -t -n 128 -m 64 -p 20
CPU compute: 1.7249 ms
GPU alloc: 0.088192 ms
GPU H2D: 0.046464 ms
GPU propagate: 0.289248 ms
GPU average: 0.022528 ms
GPU D2H: 0.047552 ms
Matrix mismatches (>1e-4): 0, max diff: 4.17233e-07
Average mismatches (>1e-4): 0, max diff: 1.78814e-07
Speedup: 5.53248
liuy41@cuda01:~$ ./radiator_gpu -t -n 64 -m 64 -p 100
CPU compute: 4.30844 ms
GPU alloc: 0.08432 ms
GPU H2D: 0.02816 ms
GPU propagate: 0.780288 ms
GPU average: 0.023008 ms
GPU D2H: 0.048224 ms
Matrix mismatches (>1e-4): 0, max diff: 1.43051e-06
Average mismatches (>1e-4): 0, max diff: 7.7486e-07
Speedup: 5.36346
liuy41@cuda01:~$ ./radiator_gpu -t -n 70 -m 70 -p 20
Error: block (16x16) must divide matrix (70x70)
```

Test	(n, m, p)	CPU compute (ms)	GPU propagate (ms)	Max matrix diff	Max avg diff	Speedup
1	64×64, p=20 (GPU only)	N/A	0.3215	N/A	N/A	N/A
2	64×64, p=20	0.8630	0.2861	4.17×10^{-7}	1.79×10^{-7}	2.725
3	32×32, p=20	0.2068	0.3022	4.17×10^{-7}	2.09×10^{-7}	0.620
4	64×128, p=20	1.7664	0.2842	4.17×10^{-7}	1.49×10^{-7}	5.716
5	128×64, p=20	1.7249	0.2892	4.17×10^{-7}	1.79×10^{-7}	5.532
6	64×64, p=100	4.3084	0.7803	1.43×10^{-6}	7.75×10^{-7}	5.363
7	128×128, p=20	3.4860	0.3339	4.77×10^{-7}	1.49×10^{-7}	9.769
8	256×256, p=20	19.8525	0.3571	5.36×10^{-7}	2.68×10^{-7}	51.881
9	70×70, p=20	—	—	—	—	—

speedup = CPU_compute / GPU_compute

- The GPU results were compared against the CPU. No mismatches larger than 10^{-4} were found.
- Significant speedup was achieved for larger matrix sizes. The table summarizes key results.
  If the matrix size is not a multiple of the block size (e.g., 70x70), an error will occur.

Task 3 - performance improvement

The test was conducted at the reference scale with different block sizes using `benchmark_task3.sh`. Due to the large scale, only the GPU was tested, and the test was performed twice.
For the small scale, 16x4 was selected for the CPU/GPU comparison.
To keep it short, not all the output is included below.

```
liuy41@cuda01:~$ chmod +x benchmark_task3.sh
./benchmark_task3.sh
Done. Results in bench.txt
liuy41@cuda01:~$ cat bench.txt
bx,by,Propagate_ms,Average_ms,Total_ms
16,16,5086.71,11.225,5097.94
16,8,4694.81,11.1712,4705.98
16,4,4528.65,11.1849,4539.83
16,32,5856.71,11.2311,5867.94
32,16,5362.82,11.203,5374.02
32,8,4742.91,11.1964,4754.11
32,4,4540.25,11.2284,4551.48
32,32,6529.69,11.1989,6540.89
64,16,6092.64,11.2312,6103.87
64,8,4923.74,11.296,4935.04
64,4,4567.81,11.3395,4579.15
64,32,0.764896,11.2067,11.9716
8,16,5339.41,11.3716,5350.78
8,8,5063.87,11.3938,5075.26
8,4,7324.81,11.316,7336.13
8,32,6081.1,11.3583,6092.46
```

```
./radiator_gpu_task3 -t -n 64 -m 64 -p 20 -bx 16 -by 4
./radiator_gpu_task3 -t -n 128 -m 128 -p 20 -bx 16 -by 4
./radiator_gpu_task3 -t -n 256 -m 256 -p 20 -bx 16 -by 4
./radiator_gpu_task3 -t -n 512 -m 512 -p 20 -bx 16 -by 4
./radiator_gpu_task3 -t -n 1024 -m 1024 -p 20 -bx 16 -by 4
```

```
// Reference size test
liuy41@cuda01:~$ ../radiator_gpu_task3 -c -t -n 15360 -m 15360 -p 1000 -bx 16 -by 4
GPU alloc: 0.314976 ms
GPU H2D: 301.574 ms
GPU propagate: 4529.31 ms
GPU average: 11.272 ms
GPU D2H: 412.849 ms
```

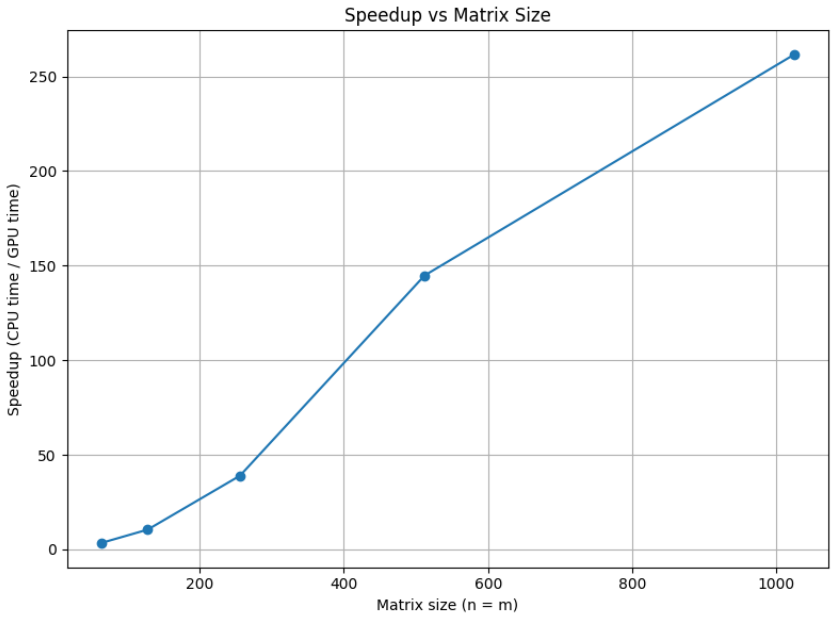
bx	by	Propagate (ms)	Average (ms)	Total (ms)
16	16	5086.1 – 5086.7	11.22	5097.4 – 5098.0
16	8	4694.8 – 4696.5	11.17	4706.0 – 4707.8
16	4	**4528.6 – 4530.3**	11.18	**4539.8 – 4541.6**
16	32	5855.9 – 5867.9	11.23	5867.2 – 5867.9
32	16	5362.8 – 5367.9	11.19	5374.0 – 5379.1
32	8	4735.1 – 4743.5	11.19	4754.1 – 4754.7
32	4	4540.3 – 4542.6	11.21	4551.5 – 4553.8
32	32	6529.7 – 6532.4	11.20	6540.9 – 6543.6
64	16	5404.7 – 6101.4	11.22	5415.9 – 6112.6
64	8	4923.7 – 4955.9	11.24	4935.0 – 4967.2
64	4	4550.1 – 4577.1	11.19	4561.3 – 4588.4
64	32	~~0.76 – 0.78~~*	11.20	~~11.97 – 11.95~~*
8	16	5339.4 – 5369.5	11.35	5350.8 – 5380.8
8	8	5063.9 – 5109.1	11.39	5075.3 – 5120.5
8	4	5544.6 – 7378.8	11.32	5555.8 – 7390.2
8	32	6081.1 – 6108.5	11.36	6092.5 – 6119.8

- invalid: 64x32 resulted in Propagate ≈ 0.7 ms, which is unrealistic and should be excluded.

Blocksize

The test was conducted at the reference scale with different block sizes using `benchmark_task3.sh`. Due to the large scale, only the GPU was tested, and the test was performed twice.

For large-scale GPU-only testing (15360×15360×1000) at the reference scale, the total GPU compute time for block size **(16×4)** is the lowest (≈4.54 s), making it the fastest.



For the small-scale matrix, we set m=n and tested both GPU and CPU using the optimal block size (16×4) found earlier. The speedup graph shows a significant improvement in GPU acceleration as the matrix size increases.

In Assignment 1, splitting the GPU reduction into a parallel rowSumKernel and a serial reduceKernel added overhead and slowed performance.

In Tasks 2/3, the average_kernel calculates row averages in a single kernel, without using intermediate storage. This reduces the number of kernel launches and memory accesses. The average time dropped from 20–30 ms to 11 ms, showing a clear performance boost.

Task 4 - double precision version

The code was modified from single precision (`float`) to double precision (`double`), then the same tests as in task3 were performed and compared with the single precision results

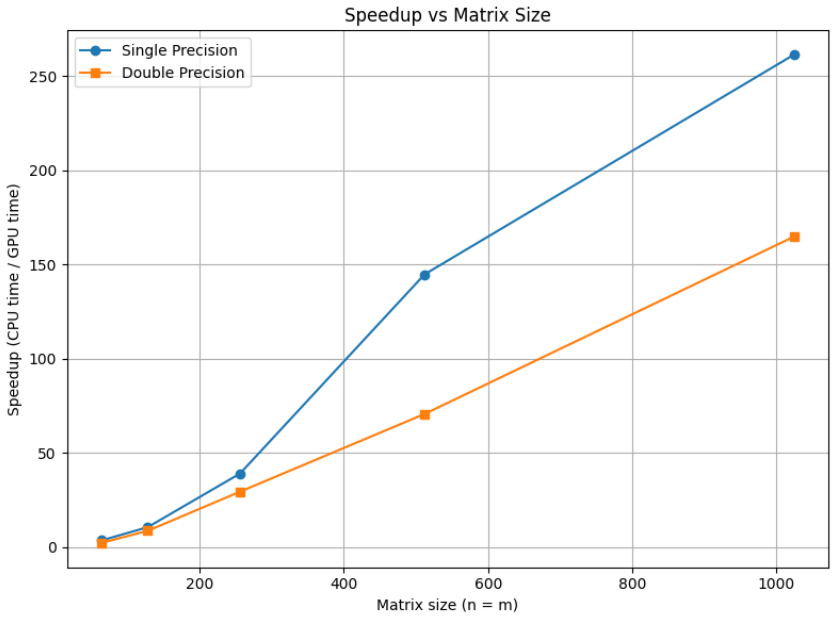
Reference size test

```
liuy41@cuda01:~$ ./radiator_gpu_dp -c -t -n 15360 -m 15360 -p 1000 -bx 16 -by 4
GPU alloc: 0.388448 ms
GPU H2D: 600.307 ms
GPU propagate: 9099.79 ms
GPU average: 48.3792 ms
GPU D2H: 839.998 ms
liuy41@cuda01:~$
```

Blocksize test

```
liuy41@cuda01:~$ chmod +x benchmark_task4.sh
./benchmark_task4.sh
```

```
liuy41@cuda01:~$   cat bench_dp.txt
bx,by,Propagate_ms,Average_ms,Total_ms
16,16,9412.89,48.3537,9461.24
16,8,9138.45,48.3472,9186.8
16,4,9182.73,48.7256,9231.46
16,32,10372.2,48.7484,10420.9
32,16,9971.72,49.1107,10020.8
32,8,9324.66,49.4966,9374.16
32,4,9399.83,51.2613,9451.09
32,32,13701.5,49.4337,13750.9
64,16,13669.8,50.552,13720.4
64,8,9755.11,52.1623,9807.27
64,4,9578.43,52.2142,9630.64
64,32,1.07485,48.5055,49.5803
8,16,9679.02,52.9034,9731.92
8,8,9780.97,52.5641,9833.53
8,4,10590.9,53.2037,10644.1
8,32,10201,53.8603,10254.9
```



(The speedup and comparison graphs were generated using Python.)

For comparison, we used the same 16x4 block size as in task3 for the small-scale tests.

Double precision GPU is much slower than single precision, with a lower speedup (CPU vs GPU).

While double precision offers higher accuracy, no significant mismatches were observed in this problem.

n	m	Total_ms (float)	Total_ms (double)
16	16	5097.4	9461.24
16	8	4706.0	9186.8
16	4	4539.8	9231.46
16	32	5867.2	10420.9
32	16	5374.0	10020.8
32	8	4754.1	9374.16
32	4	4551.5	9451.09
32	32	6540.9	13750.9
64	16	5415.9	13720.4
64	8	4935.0	9807.27
64	4	4561.3	9630.64
8	16	5350.8	9731.92
8	8	5075.3	9833.53
8	4	5555.8	10644.1
8	32	6092.5	10254.9

- As shown in the table, single precision is fastest with 16x4, while double precision is fastest with 16x8.
- At the reference size (n=15360, m=15360, p=1000) and blocksize(16x4),single precision GPU propagate time is ≈ 4529 ms, while double precision is ≈ 9099 ms, making double precision about 2 times slower than single precision at the compute stage.

Summary

Double precision is slower as hardware is optimized for single precision, but memory bandwidth (H2D/D2H) and allocation times remain almost the same for both. The main slowdown comes from the compute kernel, not memory transfers.