"Sapienza" University of Rome
Faculty of Information Engineering, Informatics and Statistics
Department of Computer Science

Cryptography

Author
Alessio Bandiera

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Contents

Information and Contacts

Personal notes and summaries collected as part of the *Cryptography* course offered by the degree in Computer Science of the University of Rome "La Sapienza".

Further information and notes can be found at the following link:
https://github.com/aflaag-notes. Anyone can feel free to report inaccuracies, improvements or requests through the Issue system provided by GitHub itself or by contacting the author privately:

- Email: **alessio.bandiera02@gmail.com**
- LinkedIn: **Alessio Bandiera**

The notes are constantly being updated, so please check if the changes have already been made in the most recent version.

Suggested prerequisites:

TODO

Licence:

1

TODO

1.1 TODO

TODO

missing introduction

Definition 1.1: Perfect secrecy

Given any distribution M over $\mathcal{M}$, and k chosen UAR on $\mathcal{K}$, we say that $\Pi = (\text{Enc}, \text{Dec})$ is **perfectly secret** if

$$\forall m \in \mathcal{M}, c \in \mathcal{C} \quad \Pr[M = m] = \Pr[M = m \mid C = c]$$

TODO In other words, this definition requires the encrypted text c to *not reveal* anything about the plaintext m. The following lemma shows some properties about perfect secrecy.

de sta definizione non me torna la distribuzione

Lemma 1.1

The following three conditions are equivalent:

1. perfect secrecy
2. independence of M and C
3. $\forall m, m' \in \mathcal{M}, c \in C \quad \Pr_{k \in \mathcal{K}}[\text{enc}(k, m) = c] = \Pr_{k \in \mathcal{K}}[\text{enc}(k, m') = c]$

Proof. We will prove the statements cyclically.

- $1 \implies 2$. By perfect secrecy, we have that

$$\Pr[M = m] = \Pr[M = m \mid C = c] = \frac{\Pr[M = m \wedge C = c]}{\Pr[C = c]}$$

therefore, by rearranging the terms we get that

$$\Pr[M = m \wedge C = c] = \Pr[M = m] \cdot \Pr[C = c]$$

- $2 \implies 3$. Fix $m \in \mathcal{M}$ and $c \in \mathcal{C}$; we have that

$$\begin{aligned}
\Pr_{k \in \mathcal{K}}[\mathrm{enc}(k, m) = c] &= \Pr_{k \in \mathcal{K}}[\mathrm{enc}(K, M) \mid M = m] \\
&= \Pr_{k \in \mathcal{K}}[C = c \mid M = m] && \text{(by definition)} \\
&= \Pr[C = c] && \text{(by independence of } M \text{ and } C\text{)}
\end{aligned}$$

  Now fix another message $m' \in \mathcal{M}$; we can repeat the same steps and obtain that $\Pr_{k \in \mathcal{K}}[\mathrm{enc}(k, m') = c] = \Pr[C = c]$ which concludes the proof.

- $3 \implies 1$. Fix $c \in \mathcal{C}$.

  Claim: $\Pr[C = c] = \Pr[C = c \mid M = m]$

  Proof of the Claim. By assuming property 3, we get that

$$\begin{aligned}
\Pr[C = c] &= \sum_{m' \in \mathcal{M}} \Pr[C = c \mid M = m'] \cdot \Pr[M = m'] && \text{(by the L.T.P.)} \\
&= \sum_{m' \in \mathcal{M}} \Pr_{k \in \mathcal{K}}[\mathrm{enc}(k, M) = c \mid M = m'] \cdot \Pr[M = m'] \\
&= \sum_{m' \in \mathcal{M}} \Pr_{k \in \mathcal{K}}[\mathrm{end}(k, m') = c] \cdot \Pr[M = m'] \\
&= \sum_{m' \in \mathcal{M}} \Pr_{k \in \mathcal{K}}[\mathrm{end}(k, m) = c] \cdot \Pr[M = m'] && \text{(by property 3)} \\
&= \Pr_{k \in \mathcal{K}}[\mathrm{enc}(k, m) = c] \cdot \sum_{m' \in \mathcal{M}} \Pr[m = m'] \\
&= \Pr_{k \in \mathcal{K}}[\mathrm{enc}(k, m) = c] \\
&= \Pr_{k \in \mathcal{K}}[\mathrm{enc}(k, M) = c \mid M = m] \\
&= \Pr_{k \in \mathcal{K}}[C = c \mid M = m]
\end{aligned}$$

  □

  Finally, by Bayes' theorem we have that

$$\begin{aligned}
\Pr[M = m] &= \frac{\Pr[M = m \mid C = c] \cdot \Pr[C = c]}{\Pr[C = c \mid M = m]} \\
&= \Pr[M = m \mid C = c] && \text{(by the claim)}
\end{aligned}$$

  which is precisely perfect secrecy.

□