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# Cryptography

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# Information and Contacts

Personal notes and summaries collected as part of the *Cryptography* course offered by the degree in Computer Science of the University of Rome "La Sapienza".

Further information and notes can be found at the following link:

<https://github.com/aflaag-notes>. Anyone can feel free to report inaccuracies, improvements or requests through the Issue system provided by GitHub itself or by contacting the author privately:

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The notes are constantly being updated, so please check if the changes have already been made in the most recent version.

## Suggested prerequisites:

TODO

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# 1

## TODO

### 1.1 TODO

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duction

#### Definition 1.1: Perfect secrecy

Given any distribution  $M$  over  $\mathcal{M}$ , and  $k$  chosen UAR on  $\mathcal{K}$ , we say that  $\Pi = (\text{Enc}, \text{Dec})$  is **perfectly secret** if

$$\forall m \in \mathcal{M}, c \in \mathcal{C} \quad \Pr[M = m] = \Pr[M = m \mid C = c]$$

TODO In other words, this definition requires the encrypted text  $c$  to *not reveal* anything about the plaintext  $m$ . The following lemma shows some properties about perfect secrecy.

de sta  
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tribuzione

#### Lemma 1.1

The following three conditions are equivalent:

1. perfect secrecy
2. independence of  $M$  and  $C$
3.  $\forall m, m' \in \mathcal{M}, c \in \mathcal{C} \quad \Pr_{k \in \mathcal{K}}[\text{Enc}(k, m) = c] = \Pr_{k \in \mathcal{K}}[\text{Enc}(k, m') = c]$

*Proof.* We will prove the statements cyclically.

- $1 \implies 2$ . By perfect secrecy, we have that

$$\Pr[M = m] = \Pr[M = m \mid C = c] = \frac{\Pr[M = m \wedge C = c]}{\Pr[C = c]}$$

therefore, by rearranging the terms we get that

$$\Pr[M = m \wedge C = c] = \Pr[M = m] \cdot \Pr[C = c]$$

- 2  $\implies$  3. Fix  $m \in \mathcal{M}$  and  $c \in \mathcal{C}$ ; we have that

$$\begin{aligned} \Pr_{k \in \mathcal{K}}[\text{Enc}(k, m) = c] &= \Pr_{k \in \mathcal{K}}[\text{Enc}(K, M) \mid M = m] \\ &= \Pr_{k \in \mathcal{K}}[C = c \mid M = m] && \text{(by definition)} \\ &= \Pr[C = c] && \text{(by independence of } M \text{ and } C) \end{aligned}$$

Now fix another message  $m' \in \mathcal{M}$ ; we can repeat the same steps and obtain that  $\Pr_{k \in \mathcal{K}}[\text{Enc}(k, m') = c] = \Pr[C = c]$  which concludes the proof.

- 3  $\implies$  1. Fix  $c \in \mathcal{C}$ .

**Claim:**  $\Pr[C = c] = \Pr[C = c \mid M = m]$

*Proof of the Claim.* By assuming property 3, we get that

$$\begin{aligned} \Pr[C = c] &= \sum_{m' \in \mathcal{M}} \Pr[C = c \mid M = m'] \cdot \Pr[M = m'] && \text{(by the L.T.P.)} \\ &= \sum_{m' \in \mathcal{M}} \Pr_{k \in \mathcal{K}}[\text{Enc}(k, M) = c \mid M = m'] \cdot \Pr[M = m'] \\ &= \sum_{m' \in \mathcal{M}} \Pr_{k \in \mathcal{K}}[\text{Enc}(k, m') = c] \cdot \Pr[M = m'] \\ &= \sum_{m' \in \mathcal{M}} \Pr_{k \in \mathcal{K}}[\text{Enc}(k, m) = c] \cdot \Pr[M = m'] && \text{(by property 3)} \\ &= \Pr_{k \in \mathcal{K}}[\text{Enc}(k, m) = c] \cdot \sum_{m' \in \mathcal{M}} \Pr[m = m'] \\ &= \Pr_{k \in \mathcal{K}}[\text{Enc}(k, m) = c] \\ &= \Pr_{k \in \mathcal{K}}[\text{Enc}(k, M) = c \mid M = m] \\ &= \Pr_{k \in \mathcal{K}}[C = c \mid M = m] \end{aligned}$$

□

Finally, by Bayes' theorem we have that

$$\begin{aligned} \Pr[M = m] &= \frac{\Pr[M = m \mid C = c] \cdot \Pr[C = c]}{\Pr[C = c \mid M = m]} \\ &= \Pr[M = m \mid C = c] && \text{(by the claim)} \end{aligned}$$

which is precisely perfect secrecy.



TODO

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