"Sapienza" Università di Roma
Ingegneria dell'Informazione,
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Dipartimento di Informatica

Discrete Mathematics

TODO non so se scriverò qualcosa qui idk

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Informazioni e Contatti

Segnalazione errori ed eventuali migliorie:

Per segnalare eventuali errori e/o migliorie possibili, si prega di utilizzare il **sistema di Issues fornito da GitHub** all'interno della pagina della repository stessa contenente questi ed altri appunti (link fornito al di sotto), utilizzando uno dei template già forniti compilando direttamente i campi richiesti.

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Number Theory

1.1 TODO

1.1.1 TODO

Definizione 1.1.1.1: Peano axioms

The **Peano axioms** are 5 axioms which define the set $\mathbb{N}$ of the **natural numbers**, and they are the following:

i) $0 \in \mathbb{N}$

ii) $\exists \operatorname{succ} : \mathbb{N} \to \mathbb{N}$, or equivalently, $\forall x \in \mathbb{N} \quad \operatorname{succ}(x) \in \mathbb{N}$

iii) $\forall x, y \in \mathbb{N} \quad x \neq y \implies \operatorname{succ}(x) \neq \operatorname{succ}(y)$

iv) $\nexists x \in \mathbb{N} \mid \operatorname{succ}(x) = 0$

v) $\forall S \subseteq \mathbb{N} \quad (0 \in S \land (\forall x \in S \quad \operatorname{succ}(x) \in S)) \implies S = \mathbb{N}$

Principio 1.1.1.1: Induction principle

Let P be a property which is true for $n = 0$, thus $P(0)$ is true; also, for every $n \in \mathbb{N}$ we have that $P(n) \implies P(n+1)$; then $P(n)$ is true for every $n \in \mathbb{N}$.

With symbols, using the formal logic notation, we have that

$$\frac{P(0) \quad P(n) \implies P(n+1)}{\forall n \quad P(n)}$$

Osservazione 1.1.1.1: The fifth Peano axiom

Note that the fifth Peano axiom is equivalent to the induction principle, since, it states that for every subset S of $\mathbb{N}$ containing 0 and closed under succ must be equal to $\mathbb{N}$ itself.

Problema 1.1.1.1: Cardinality of the power set

Show that for every given set S such that $n := |S|$ it holds true that $|\mathcal{P}(S)| = 2^n$.

Dimostrazione. The statement will be shown by induction over n, the number of elements contained into S.

Caso base. $n = 0 \implies S = \varnothing \implies \mathcal{P}(S) = \mathcal{P}(\varnothing) = \{\varnothing\} \implies |\mathcal{P}(S)| = 1 = 2^0 = 2^n$.

Ipotesi induttiva. Assume that the statement is true for some fixed integer n.

Passo induttivo. It must be shown that the statement is true for $n + 1$ as well.

□

Definizione 1.1.1.2: Integers

TODO

Definizione 1.1.1.3: Divisor

TODO

Esempio 1.1.1.1 (Divisors)**.** TODO

Proposizione 1.1.1.1: $\mathbb{P}$ is infinite

There are infinitely many primes. With symbols

$$|\mathbb{P}| = +\infty$$

Dimostrazione. By way of contradiction, assume that $\mathbb{P}$ is finite, thus

$$\exists n \in \mathbb{N} \mid \mathbb{P} = \{p_1, \ldots, p_n\}$$

and let $x = p_1 \cdot \ldots \cdot p_n$. Since $x \neq p_1, \ldots, p_n$, then $x \not\mathbb{P}$, so x is not a prime number; but x can't be divided by any of the $p_1, \ldots, p_n$ either, because the remainder will always be 1. This means that x is neither prime nor non-prime, which is a contradiction $\lightning$. □

Problema 1.1.1.2: $n^2 + n$ is even

Show that $\forall n \in \mathbb{N} \quad n^2 + n$ is an even number.

Dimostrazione. Note that $n^2 + n = n \cdot (n+1)$, hence:

- if n is even, then
$$\exists k \in \mathbb{N} \mid n = 2k \implies n(n+1) = 2k(2k+1) = 4k^2 + 2k = 2(k^2 + k)$$
which is an even number;
- if n is odd, then
$$\exists k \in \mathbb{N} \mid n = 2k+1 \implies n(n+1) = (2k+1)(2k+2) = 4k^2+6k+2 = 2(2k^2+3k+1)$$
which is an even number.

$\square$

Problema 1.1.1.3: $4n - 1$ is not prime

Show that there are infinitely many numbers of the form $4n-1$ that are not prime.

Dimostrazione. Note that $\forall x^2 \in \mathbb{N} - \{0\} \quad 4x^2 - 1 = (2x+1)(2x-1)$ which is a proper factorization of $4x^2 - 1$, hence every perfect square yields a number of the form $4n - 1$ which is not a prime number. Note that the number of perfect squares is infinite since the set of perfect square has the same cardinality of $\mathbb{N}$ since it's possibile to construct a bijective function as follows:
$$f : \mathbb{N} \to \mathbb{N} : x \mapsto x^2$$

Also, note that this proof does not show *every non-prime number of the form* $4n - 1$, since that is outside the scope of the problem. $\square$