DSC Assigment 2

Sunday, 29 October 2023 6:30 PM

1)

a) If A & B are independent events

$P(A|B) = P(A)$

Since die rolls are independent

Let X be the R.V. denoting second roll has a multiple of first roll.

$P(X) = \sum_{i=1}^{6} P(X|i)\,P(i)$

$P(X|i) \rightarrow$

i	$P(X\|i)$
$i=1$	1
2	1/2
3	1/3
4	1/6
5	1/6
6	1/6

$\therefore P(X) = 0.388$

b) Die rolls are independent

$P(A) = \sum_{i=1}^{K} P(A|K) \cdot P(K)$

For i b/w 1 & K/2

$P(i) \cdot P(A|i) = \frac{1}{K} \times \frac{(K/i)}{K}$ } K is div. by 1 to K/2

For i b/w K/2 to K

$P(i) \cdot P(A|i) = \frac{1}{K} * \frac{1}{K}$ } Only same number is multiple

$P(X) = \frac{1}{K} \sum i^{-1} + \frac{1}{K} \cdot \frac{1}{K} \cdot \frac{K}{2}$

$= \frac{1}{K} \sum i^{-1} + \frac{1}{2K}$

As $k \rightarrow \infty$ sum tends to log k

$\leq \frac{1}{K} \log K + \frac{1}{2K}$

As $K \rightarrow \infty$

$P(X) \leq 0$

$\therefore P(X) = 0$ as P cannot be < 0

2) a) Let X be the r.v. that 2 people don't have same b.day.

$\therefore P(X) = \frac{364}{365}$

For n people, let Y be the event that no 2 people share a bday

$P(Y) = P(X)^{^nC_2} = P(X)^{\frac{n(n-1)}{2}}$

$\therefore P(Y) < 1/2$

$\frac{n(n-1)}{2} \log \frac{364}{365} < \log 1/2$

$n(n-1) > 505$

$n = 23$

b) From a $\rightarrow$ $P(Y) = P(X)^{\frac{n(n-1)}{2}}$

Now we want

$P(Y) < 1/4$

$\therefore \frac{n(n-1)}{2} \log\left(\frac{364}{365}\right) = \log(1/4)$

$n(n-1) > 1008$

$\therefore n = 33$

c) Let there be n people s.t. R_i or F_i equals R_j or F_j with Probability $\geq p$ $\forall 1 \leq i < j \leq n$

Now due to the bug:

$P(R_i \neq F_i) = 1$

$\therefore P((R_j \neq F_i) \cap (R_j \neq R_i) \cap (R_i \neq F_j) \cap (F_i \neq F_j))$

$\alpha = \underbrace{\frac{363}{365}}_{R_j \neq R_i \;\&\; R_j \neq F_i} \times \frac{362}{364} \rightarrow F_j \neq R_i \;\&\; F_j \neq F_i$

Hence for n people:

$P(Y) \rightarrow$ event that no 2 people have 4 dates common

$P(Y) = \alpha^{\frac{n(n-1)}{2}}$

$\Rightarrow 1 - P(Y) \rightarrow$ Probability that 4 dates common

$\therefore 1 - P(Y) <= p$

$1 - \alpha^{\frac{n(n-1)}{2}} <= p$

$1 - p <= \alpha^{\frac{n(n-1)}{2}}$

$2\log(1-p) <= n(n-1)\log\alpha$

$\log(0.989)n^2 - \log(0.989)n - 2\log(1-p) \geq 0$

$\therefore \text{roots} = \frac{\log(0.989) \pm \sqrt{(\log(0.989))^2 + 8\frac{\log(1-p)}{\log(0.989)}}}{2(\log(0.989))}$ } $\rightarrow D$

We take the +ve root only as $D > -b$

3)

a) Hypothesis $\rightarrow$ 50% spam

Spam recvd $\rightarrow$ 55

Level of significance $\rightarrow \alpha = 0.05$

$p_0 = 0.5$

$H_0: p < p_0$ (At most 50% spam)

$H_1: p \geq p_0$ (Atleast 50% spam)

Let number of mails needed for H_0 to not be rejected = N

$Z = \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{N}}}$

$Z_{\substack{\text{critical}\\ \text{at } \alpha\\ =0.05}} = Z_{0.05} = 1.64$

$\therefore 1.64 > \frac{\frac{55}{n} - 0.5}{\frac{0.5}{\sqrt{N}}}$

$n + 1.64\sqrt{n} - 110 > 0$

$\therefore \sqrt{n} = 9.6 \Rightarrow n \approx 94$

b) $p_0 = 1/3$

$H_0: p < p_0$ (At most 1/3 times card is correct)

$H_1: p \geq p_0$ (Atleast 1/3 times card is correct)

$\alpha = 5/100$

Sample Probability = 28/x

$Z_{0.05} = 1.64$

$1.64 < Z$ (To reject Null Hypothesis)

$1.64 < \left(\frac{28}{x} - \frac{1}{3}\right) \cdot \sqrt{x} \cdot \frac{1}{\sqrt{2}/3}$

$x + 2.32\sqrt{x} - 84 < 0$

$\sqrt{x} = \frac{-2.32 \pm \sqrt{341.41}}{2}$

Since $\sqrt{x} > 0$

$\sqrt{x} = 8.0755$

$\therefore x \approx 65$