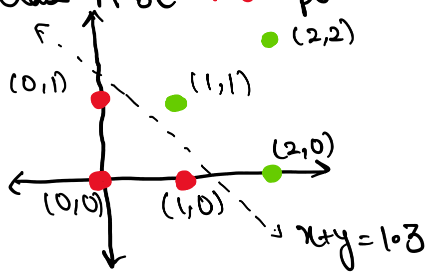
ML Assignment Section A

Monday, 14 November 2022 2:55 PM

(1) Let class A be red points & B be green points



we see that the line $y+x=1.3$ correctly classifies all points as the red points lie on one side while the other green points are on the other side

(2) Let $\beta_1=(0,1)$, $\beta_2=(1,0)$, $\beta_3=(1,1)$, $\beta_4=(2,0)$, $\beta_5=(0,0)$, $\beta_6=(2,2)$

To maximize margin $\frac{\|w\|^2}{2}$ with the constraint $y_i[w^T x_i + b_i]$

$y_1=-1$, $y_2=-1$, $y_3=1$, $y_4=1$, $y_5=-1$, $y_6=1$

We maximize,

$$J(w,b,\alpha)=\frac{\|w\|^2}{2}-\sum_{i=1}^{6}\alpha_i\left[y_i(w^T x_i+b)\right]$$

No line passing via $(2,2)$ & $(0,0)$ can correctly classify the data.

Hence $\alpha_5=\alpha_6=0$

So now our loop only goes for 4 iterations

So our dual form which we maximize is

$$Q(\alpha)=\sum_{i=1}^{4}\alpha_i-\frac{1}{2}\left[\sum_{i=1}^{4}\sum_{j=1}^{4}\alpha_i\alpha_j y_i y_j \beta_i\beta_j\right]$$

Subject to the condition $\sum_{i=1}^{4}\alpha_i y_i=0$ & $\alpha_i \geq 0 \ \forall i$

Now, $\sum_{i=1}^{4}\alpha_i y_i=0 \Rightarrow \alpha_1+\alpha_2=\alpha_3+\alpha_4$

$$Q(\alpha)=(\alpha_1+\alpha_2+\alpha_3+\alpha_4)-\frac{1}{2}\left[\alpha_1^2+\alpha_2^2+2\alpha_3^2+4\alpha_4^2 -2\alpha_1\alpha_3-2\alpha_2\alpha_3 -4\alpha_2\alpha_4+4\alpha_3\alpha_4\right]$$

Now we use $\alpha_1=\alpha_3+\alpha_4-\alpha_2$

$\Rightarrow -2\alpha_1\alpha_3=(-2\alpha_3^2-2\alpha_3\alpha_4+2\alpha_2\alpha_3)$

So $Q(\alpha)$ will now become

$$Q(\alpha)=\alpha_1+\alpha_2+\alpha_3+\alpha_4-\frac{1}{2}\left[\alpha_1^2+\alpha_2^2-4\alpha_4^2-4\alpha_2\alpha_4+2\alpha_3\alpha_4\right]$$

$$\frac{\partial Q(\alpha)}{\partial\alpha_1}=0 \Rightarrow 1-\alpha_1=0$$
$$\Rightarrow \alpha_1=1$$

$$\frac{\partial Q(\alpha)}{\partial\alpha_2}=0 \Rightarrow 1-\frac{1}{2}\left[2\alpha_2-4\alpha_4\right]$$
$$\Rightarrow \alpha_2-2\alpha_4=1$$

$$\frac{\partial Q(\alpha)}{\partial\alpha_3}=0 \Rightarrow 1-\frac{1}{2}\left[8\alpha_4-4\alpha_2+2\alpha_3\right]=0$$
$$\Rightarrow \alpha_3=3$$

$$w=\sum_{i=1}^{4}\alpha_i y_i\beta_i=-1\begin{bmatrix}0\\1\end{bmatrix}+(-3)\begin{bmatrix}1\\0\end{bmatrix}+3\begin{bmatrix}1\\1\end{bmatrix}+1\begin{bmatrix}2\\0\end{bmatrix}$$

$$\omega = \sum_{i=1} \alpha_i y_i \beta_i = -1\begin{bmatrix}0\\1\end{bmatrix} + (-0)\begin{bmatrix}1\\0\end{bmatrix} + 0\begin{bmatrix}1\\1\end{bmatrix} + 1\begin{bmatrix}2\\0\end{bmatrix}$$
$$= \begin{bmatrix}2\\2\end{bmatrix}$$

$b = 1 - \omega^T \beta_i$ s.t. $y_i = 1$

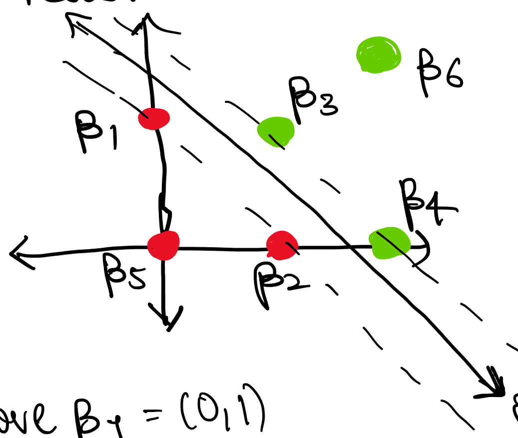
Let $\beta_i = \beta_3$

$b = 1 - [2\ 2]\begin{bmatrix}1\\1\end{bmatrix} = 1 - 4 = -3$

$\therefore$ Optimal line is $2x_1 + 2x_2 - 3 = 0$

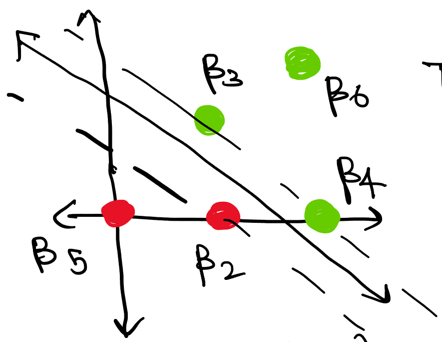
Length of optimal margin $= \frac{2}{\|\omega\|} = \frac{2}{\sqrt{2^2+2^2}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

No α_i for $i = 1,2,3,4$ is 0 so all of them are support vectors



③ Case I $\rightarrow$ Remove $\beta_1 = (0,1)$

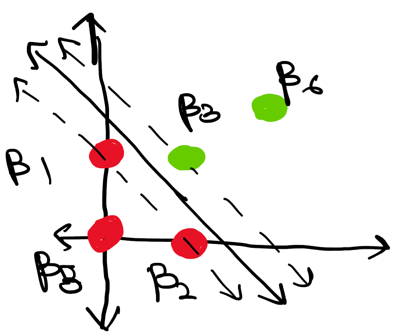
NO CHANGE



The support vectors are now (1,0), (1,1), (2,0)

Case II $\rightarrow$ Remove $\beta_4 = (2,0)$.

NO CHANGE



The support vectors are now (1,0), (0,1), (1,1)

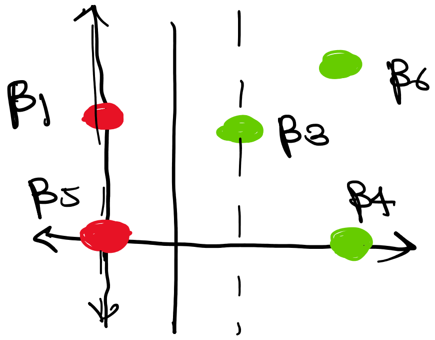
Case III $\rightarrow$ Remove $\beta_2 = (1,0)$

MARGIN CHANGES

NEW MARGIN HAS TOTAL LENGTH = 1

The support vectors are

NEW MARGIN HAS TOTAL LENGTH = 1

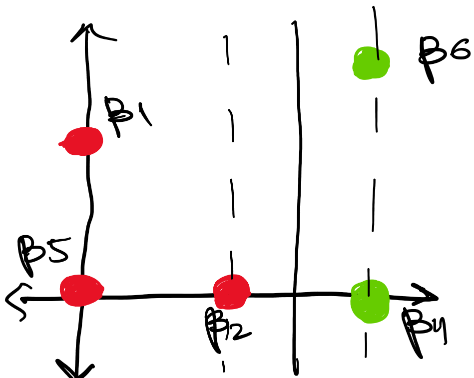


The support vectors are now $(0,0)$, $(0,1)$, $(1,1)$

Case IV $\rightarrow$ Remove $\beta_3 = (1,1)$

MARGIN CHANGES

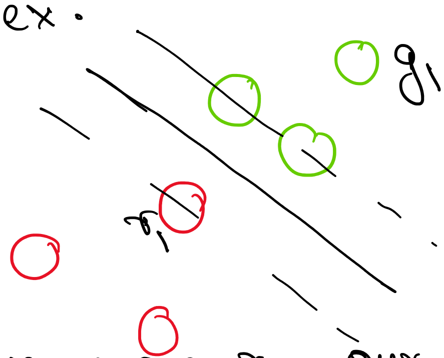
NEW MARGIN HAS TOTAL LENGTH = 1



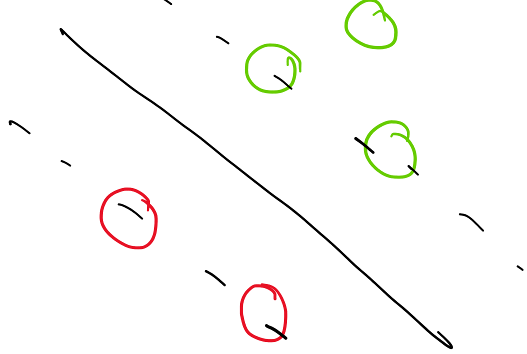
The support vectors are $(1,0)$, $(2,0)$, $(2,2)$

(4) The margin may or may not change as we saw in part (3)
If some new point gets introduced which is a support vector then the margin changes

For ex.



If we remove r_1 our margin and support vectors change



If we remove g_1 there is no change in margin