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Question 1

a. pigeons - students

pigeonholes - scores on scale from 0 to 100 points.

let set X be the scores on scale from 0 to 100 points,

$X = \{x \in N \mid 0 \leq x \leq 100\}$ while N is natural numbers.

$|X| = 101$

Number of students in a class to guarantee that at least two students received the same score.

$|X| + 1 = 101 + 1$

$= 102$ students

b. pigeons - students

pigeonholes - grade

let set x be the grade for Discrete Structure class.

$X = \{A, B, C, D, F\}$ $\quad |X| = 5$

Minimum number of students required in Discrete structure class so that at least 6 students received the same grade, N :

$\left\lceil \frac{n}{m} \right\rceil = k$

$m(k-1) < m\frac{n}{m} = n$

$\left\lceil \frac{N}{5} \right\rceil = 6$

$5(6-1) < N$

$5(5) < N$

$N > 25$

$N = 25 + 1$

$= 26$ students

STANDARD

Question 2

a) $P(B1) = 0.70$

b) $P(B2) = 0.30$

c) $P(W|B1) = 0.20$

d) $P(B1 \cap W) = P(B1) \times P(W|B1)$

$= 0.70 \times 0.20$

$= 0.14$

e) $P(B2 \cap W) = P(B2) \times P(W|B2)$

$= 0.30 \times 0.40$

$= 0.12$

f) $P(W) = P(B1 \cap W) + P(B2 \cap W)$

$= 0.14 + 0.12$

$= 0.26$

g) $P(B1|W) = \dfrac{P(B1 \cap W)}{P(W|B1)P(B1) + P(W|B2)P(B2)}$

$= \dfrac{0.14}{0.20(0.70) + 0.40(0.30)}$

$= 0.5385$

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Question 3

a. Vertices - Vertices is plural for vertex. Vertex represents a point in the graph.

b Edges - Edges is the connection between vertices in the graph. An edge can connects two vertices or to itself.

c. Adjacent vertices - For edge that connect two vertices, these vertices are called adjacent. For loop, edge that start and end in the same vertex, is called adjacent to itself.

d. Incident edge - when a vertex have multiple edges, these edges are called incident edges.

e. Isolated vertex - Vertex that does not have any edges. This vertex is not connected to itself or another vertices.

f. Loop - Loop is a vertex that is connected to itself.

g. Parallel edges - multiple edges that connect the same pair of vertices

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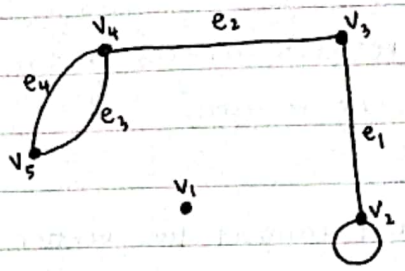
Graph:



Example:

1. Vertices, $V = \{v_1, v_2, v_3, v_4, v_5\}$

2. Edges, $E = \{e_1, e_2, e_3, e_4\}$

3. v_3 and v_4 are adjacent v_2 and v_3 are adjacent
   v_2 are adjacent to itself v_5 and v_4 are adjacent

4. For vertex 3, incident edges are (v_3, v_4) and (v_3, v_2)

5. v_1 is the isolated vertex

6. v_2 is loop

7. e_3 and e_4 are parallel edges

Question 4.

$G = \{V, E\}$

$V = \{v1, v2, v3, v4\}$

$E = \{a, b, c, d, e, f\}$

$a = \{v1, v2\}$

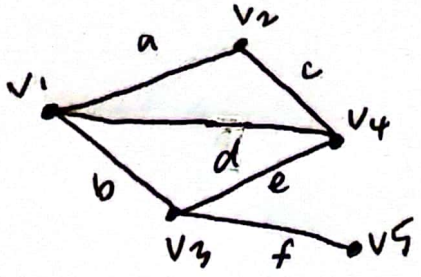
$b = \{v1, v3\}$

$c = \{v2, v4\}$

$d = \{v1, v4\}$

$e = \{v3, v4\}$

$f = \{v3, v5\}$



vertex	v1	v2	v3	v4	v5
Degree	3	2	3	3	1

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Question 5

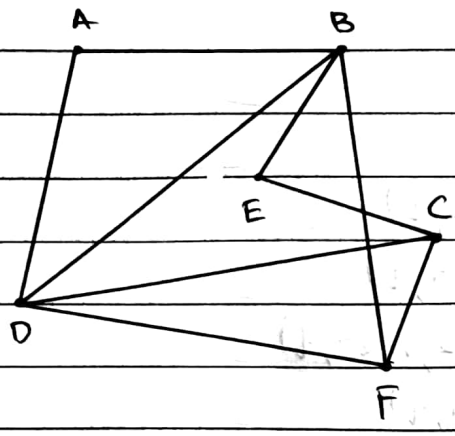
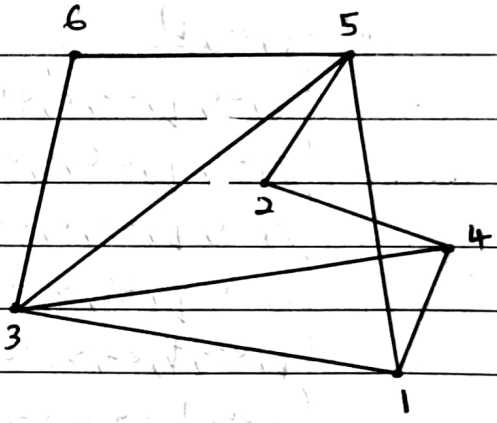
i. Incidence matrix

	a	b	c	d	e	f	g	h	i	k
1	1	2	1	1	0	0	0	0	0	0
2	0	0	0	0	1	0	0	0	0	0
3	1	0	1	0	0	1	1	1	0	0
4	0	0	0	1	1	1	0	0	1	0
5	0	0	0	0	0	0	1	1	0	1
6	0	0	0	0	0	0	0	0	1	1

ii. Adjacency matrix

	1	2	3	4	5	6
1	1	0	2	1	0	0
2	0	0	0	1	0	0
3	2	0	0	1	1	1
4	1	1	1	0	0	1
5	0	0	1	0	0	1
6	0	0	1	1	1	0

6.

	Graph Y	Graph Z
Number of vertices	6	6
Number of edges	9	9
Degree of each vertex	$d(A) = 2$ $d(B) = 4$ $d(C) = 3$ $d(D) = 4$ $d(E) = 2$ $d(F) = 3$	$d(1) = 3$ $d(2) = 2$ $d(3) = 4$ $d(4) = 3$ $d(5) = 4$ $d(6) = 2$
Number of loops	0	0
Number of parallel edges	0	0
Graph type	Connected	Connected
Pairs of connected vertices		
Conclusion	Graph Y and Z are isomorphic.	

Adjacency matrix

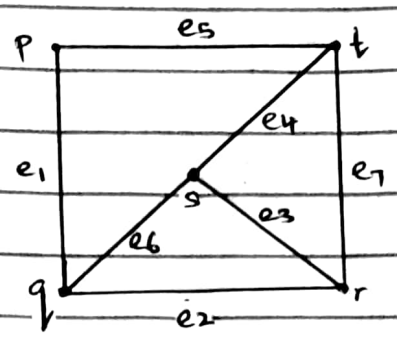
$A_Y =$

	A	B	C	D	E	F
A	0	1	0	1	0	0
B	1	0	0	1	1	1
C	0	0	0	1	1	1
D	1	1	1	0	0	1
E	0	1	1	0	0	0
F	0	1	1	1	0	0

$A_Z =$

	6	5	4	3	2	1
6	0	1	0	1	0	0
5	1	0	0	1	1	1
4	0	0	0	1	1	1
3	1	1	1	0	0	1
2	0	1	1	0	0	0
1	0	1	1	1	0	0

7.



i) (p, e5, t)
(p, e1, q, e2, r, e7, t)
(p, e1, q, e6, s, e4, t)
(p, e1, q, e2, r, e3, s, e4, t)
(p, e1, q, e6, s, e3, r, e7, t)

ii) (p, e5, t)
(p, e1, q, e2, r, e7, t)
(p, e1, q, e6, s, e4, t)
(p, e1, q, e2, r, e3, s, e4, t)
(p, e1, q, e6, s, e3, r, e7, t)
(p, e5, t, e4, s, e6, q, e2, r, e7, t)
(p, e5, t, e4, s, e3, r, e7, t)
(p, e5, t, e7, r, e3, s, e4, t)
(p, e5, t, e7, r, e2, q, e6, s, e4, t)

iii) Shortest path from vertex p to vertex t = (p, e5, t)
Longest path from vertex p to vertex t = (p, e1, q, e2, r, e3, s, e4, t) or
(p, e1, q, e6, s, e3, r, e7, t)

iv) Shortest trail from vertex p to vertex t = (p, e5, t)
Longest trail from vertex p to vertex t = (p, e5, t, e4, s, e6, q, e2, r, e7, t) or
(p, e5, t, e7, r, e2, q, e6, s, e4, t)