A **B3A005** **Pages:2**

Reg. No._____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

THIRD SEMESTER B.TECH DEGREE EXAMINATION, MARCH 2017

MA 201: LINEAR ALGEBRA AND COMPLEX ANALYSIS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer any 2 questions

1. a. Check whether the following functions are analytic or not. Justify your answer.

   i) $f(z)=z+\bar{z}$ (4)

   ii) $f(z)=|z|^2$ (4)

   b. Show that $f(z)=\sin z$ is analytic for all z. Find $f'(z)$ (7)

2. a. Show that $v=3x^2y-y^3$ is harmonic and find the corresponding analytic function $f(z)=u(x,y)+iv(x,y)$ (8)

   b. Find the image of $0<x<1, \frac{1}{2}<y<1$ under the mapping $w=e^z$ (7)

3. a. Find the linear fractional transformation that carries $z_1=-2, z_2=0$ and $z_3=2$ on to the points $w_1=\infty$, w_2 =1/4 and w3=3/8. Hence find the image of x-axis.(7)

   b. Find the image of the rectangular region -π<=x<= π, $a\le y<b$ under the mapping $w=\sin z$ (8)

PART B

Answer any 2 questions

4. a. Evaluate $\int_C |z|dz$ where

   i) C is the line segment joining -*i* and *i* (3)

   ii) C is the unit circle in the left of half plane (4)

   b. Verify Cauchy's integral theorem for z^2 taken over the boundary of the rectangle with vertices -1, 1, 1+*i*, -1+*i* in the counter clockwise sense. (8)

5. a. Find the Laurent's series expansion of $f(z)=\frac{1}{1-z^2}$ which is convergent in

   i) $|z-1|<2$ (4)

   ii) $|z-1|>2$ (4)

   b. Determine the nature and type of singularities of

   i) $\frac{e^{-z^2}}{z^2}$ (3)

ii) $z \sin\left(\frac{1}{z}\right)$ (4)

6. a. Use residue theorem to evaluate $\int_C \frac{30z^2 - 23z + 5}{(2z-1)^2(3z-1)} dz$ where C is $|z| = 1$ (7)

b. Evaluate $\int_0^\infty \frac{1}{(1+x^2)^2} dx$ using residue theorem. (8)

PART C

Answer any 2 questions

7. a. Solve the following by Gauss elimination

$$y + z - 2w = 0, \quad 2x - 3y - 3z + 6w = 2, \quad 4x + y + z - 2w = 4$$ (6)

b. Reduce to Echelon form and hence find the rank of the matrix

$$\begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$$ (6)

c. Find a basis for the null space of $\begin{bmatrix} 2 & -2 & 0 \\ 0 & 4 & 8 \\ 2 & 0 & 4 \end{bmatrix}$ (8)

8. a. i) Are the vectors (3 -1 4),(6 7 5) and (9 6 9) linearly dependent or independent? Justify your ans (5)

ii) Is all vectors (x, y, z) in $\mathbb{R}^3$ with $y - x + 4z = 0$ form a vector space over the field of real numbers? Give reasons for your answer. (5)

b. i) Find a matrix **C** such that $Q = \boldsymbol{x}^T \boldsymbol{C}\, \boldsymbol{x}$ where

$$Q = -3x_1^2 + 4x_1x_2 - x_2^2 + 2x_1x_3 - 5x_3^2$$ (4)

ii) Obtain the matrix of transformation

$$y_1 = \cos\theta\, x_1 - \sin\theta\, x_2, \quad y_2 = \sin\theta\, x_1 + \cos\theta\, x_2$$

Prove that it is orthogonal. Obtain the inverse transformation. (6)

9. a. Find the eigenvalues, eigenvectors and bases and dimensions for each Eigen space of

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$ (10)

b. Find out what type of conic section, the quadratic form $17x_1^2 - 30x_1x_2 + 17x_2^2 = 128$ and transform it to principal axes. (10)