Problem 4 Angel Flump

Part a

```
for(int i=n-1 ; i?=0 ; i--){
    for(int k=0; k<i*n ; k++){
        // something O(1)
    }
}
```

inner loop times it will run

$$\sum_{i=0}^{n-1} \sum_{k=0}^{in-1} c$$

$$= \sum_{i=0}^{n-1} cin = cn \sum_{i=0}^{n-1} i$$

$$= cn \frac{(n-1)(n)}{2} = \theta(n^3)$$

loop does not start at 0
but will always reach it b/c
it is decrementing from n-1

$n=1$ $i=0$; $i?=0$
for ($k=0$; $k<0 \cdot 1$; $k++$) → does not run
$k=0$ $k<0$

$n=2$ $i=1$ $i?=0$ $i--$ runs twice
2 4 times
$k=0$; $k<2$, $k++$ → runs 2 times

4

$n=3$ $i=2$ $i \geq 0$ $i--$ runs 3 times
6 18 times
$k=0$ $k<6$ $k++$ runs 6 times

6

$n=4$ $i=3$; $i \geq 0$ $i--$ runs 4 times
12 48 times
$k=0$ $k<12$ $k++$ runs 12 times

8

$n=5$ $i=4$ $i?=0$ $i--$ runs 5 times
20 100 times
$k=0$, $k<20$, $k++$ runs 20 times

10

$n=6$ $i=5$ → 6 times
30 180 times $k=0$ $k<30$ → 30 times

Part (b)

```
for (int i=1 ; i<n; i++) {
    for (int k=1; k<=n ; k++) {
        if (A[k]==i) {
            for (int m=1; m<=n ; m=m+m) {
                // something
            }
        }
    }
}
```

outer loop $n-1$
middle loop n
if statement: n
inner loop : $\log n$
∴ the inner for loop only runs $n\log n$ times

$n=2$ $i=1$ $i<2$ → 1 time
$k=1$ $k \leq 2$ → 2 times

$n=3$ $i=1$ $i<3$
$k=1$ $k \leq 3$

worst-case scenario : $A[1]=1$, $A[2]=2$, $A[3]=3$
when $i=k$, it will cause the IF statement to always happen once when i increments therefore happens n times

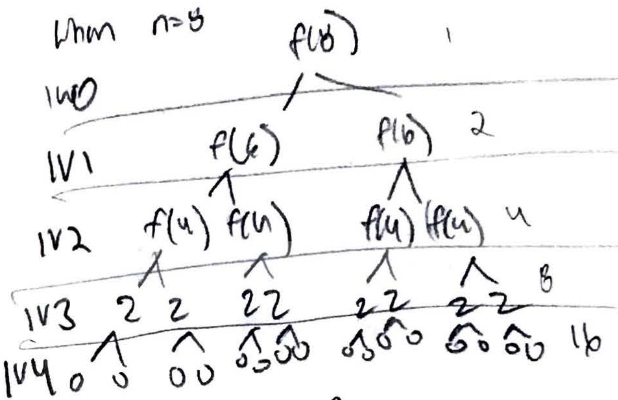
We know that the 2 outer loops will always run. The inner outer loop will run $n-1$ times and the most outer loop will run at most $n-1$ times

∴ runtime will be $n^2 + n\log n$ → $\boxed{\theta(n^2)}$
since the for loop will not run all n^2 times

Part(c)

```
void f3(int*A, int n){   T(n)
    if (n≤1) return;  → Base case only ran once
    else{
        f3(A, n-2)  = happens n/2 times
          // O(1)
        f3(A, n-2)  = happens n/2 times
    }
}
```

n is not modified inside the if statement, therefore it calls the same recursion twice with the same value of (n-2).


when n=8
f(8) 1
lvl0
lvl1 f(6) f(6) 2
lvl2 f(4) f(4) f(4) f(4) 4
lvl3 2 2 2 2 2 2 2 2 8
lvl4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 16

we are summing the work it takes per each level. At max, there are $n/2$ levels each level costs $2^{\frac{n}{2}}$

$\therefore \sum_{i=0}^{n/2} 2^i = 2^{n/2}$ → as runtime analysis → $O(2^{n/2})$

Problem 4d.

Angel Flores

```
int* a = new int [10]    O(1)
int size = 10;    O(1)
  for (i=0 to n) {
      if (i = ~~s~~size)
          int newsize = 3/2 size;    O(1)
          int* b = new int [newsize];    O(1)
          for (j=0 to size) { θ(1) }
          delete[] a;    O(1)
          a = b;    O(1)
          size = newsize;
      }
  }
```

The most times we will need to resize is $\log_{3/2} n$ since every time you reach the max size, it makes a new array that is 3/2 times bigger

original size   # of times it will need to be resized

$$\sum_{i=0}^{\log_{3/2} n} 10\left(\tfrac{3}{2}\right)^i = 10\sum_{i=0}^{\log_{3/2} n}\left(\tfrac{3}{2}\right)^i$$

$$= 10\left(\tfrac{3}{2}\right)^{\log_{3/2} n} = 10n =$$ how long it would take if the if statement ran.

$(n - \log_{3/2} n)$ time it would take if the if statement was not triggered (total time - # of resize required)

$\therefore$ Total runtime $= 10n + n - \log_{3/2} n = 11n - \log_{3/2} n = \theta(n)$.