8 FORMULARIUM

Note: numbers in the right column refer to equations in the textbook. In some cases, equations are not explicitly given in the textbook, but are part of the text or a footnote. In those cases, they are numbered according to the closest numbered equation.

8.1 GASLAWS, WATER VAPOUR AND THERMODYNAMICS (APPENDIX B)

(T in Kelvin, unless otherwise stated; some definitions are given in table B.2: they are referred to as TB2.x)

Equation of state	$p = \rho \frac{R^*}{M} T = \rho R T$	B.1
Hydrostatic equilibrium	$dp = -g\rho dz$ $g = 9.8 \text{ m s}^{-2}$	B.11
Thermodynamics	$dQ = c_v dT + p d\alpha$ $dQ = c_p dT - \frac{dp}{\rho}$ $c_p - c_v = R$	B.10 B.7
Dry air (occurs in text)	$M_d = 29 \text{ kg kmol}^{-1}$	B.3a
Dry-adiabatic	$\frac{dT}{T} = \frac{R}{c_p}\frac{dp}{\rho}$ $\theta = T\left(\frac{p_0}{p}\right)^{\frac{R}{c_p}}$	B.13 B.14
Relative humidity RH	$RH = \frac{e}{e_{\text{sat}}}$	TB2.6
Specific humidity q	$q = \frac{\rho_v}{\rho}$ $q \approx \frac{R_d}{R_v}\frac{e}{p}$ $q = \frac{r}{1+r}$	TB2.4 B.19 B.17
Mixing ratio r	$r = \frac{\rho_v}{\rho_d}$	TB.3
Dew point T_d	$e = e_{\text{sat}}(T_d)$	TB2.7
Virtual temperature T_v	$T_v = \frac{T}{1-\left(1-\frac{R_d}{R_v}\right)\frac{e}{p}} \approx T(1+0.61q)$	TB2.5
Wet bulb temperature T_w and psychrometer-equation	$e = e_{\text{sat}}(T_w) - \frac{R_v}{R_d}\frac{c_p}{L_v}p(T-T_w) = e_{\text{sat}}(T_w) - \gamma(T-T_w)$ with $\gamma = \frac{R_v}{R_d}\frac{c_p}{L_v}p$	TB2.8

Thermodynamic properties of dry and moist air (note: L_v is given in the text, section B.5)

	depends on					
property	T	q	p	value	unit	
R^*				8314	J kmol^{-1}K^{-1}	B.1a
R_v				462	J kg^{-1}K^{-1}	B.4a
R_d				287	J kg^{-1}K^{-1}	B.3b
R		●		$287(1+0.61q)$	J kg^{-1}K^{-1}	B.6
c_{pd}				1004	J kg^{-1}K^{-1}	B.7a
c_{vd}				717	J kg^{-1}K^{-1}	B.7b
c_p		●		$1004(1+0.84q)$	J kg^{-1}K^{-1}	B.8
c_v		●		$717(1+0.37q)$	J kg^{-1}K^{-1}	B.9
L_v	●			$2501000(1-0.00095(T-273.15))$	J kg^{-1}	B.23b

	γ	●	●	●	$65.5\dfrac{1+0.84q}{1-0.00095(T-273.15)}\dfrac{p}{101300}$	Pa K^{-1}	B.23
	e_{sat} (over water)	●			$611.2\,exp\left[\dfrac{17.62(T-273.15)}{-30.03+T}\right]$	Pa	B.20a
	s (over water)	●			$e_{sat}(T)\frac{4284}{(-30.03+T)^2}$ (with e_{sat} over water)	Pa K^{-1}	B.21a
	e_{sat} (over ice)	●			$611.2\,exp\left[\dfrac{22.46(T-273.15)}{-0.53+T}\right]$	Pa	B.20b
	s (over ice)	●			$e_{sat}(T)\frac{6123}{(-0.53+T)^2}$ (with e_{sat} over ice)	Pa K^{-1}	B.21b

8.2 Radiation (partly from appendix A) and soil heat flux

Planck curve	$M_{b\lambda} = \frac{C_1\lambda^{-5}}{\left[exp\left(\frac{C_2}{\lambda T}\right)-1\right]}$ W m^{-2} μm^{-1} C_1 = 3.74 10^8 W m^{-2} μm^4, C_2 = 1.439 10^4 μm K	A.1
Stefan-Boltzmann	$M_b = \sigma T^4$ W m^{-2} with σ = 5.67 10^{-8} W m^{-2}K^{-4}	A.2
Wien	$\lambda_m T = 2897.8$ μm K	A.3
Lambert-Beer	$dI_\lambda = -I_{\lambda 0}\, k_\lambda q\, \rho\, ds$	2.9
Radiation at top of atmosphere	$K_0^{\downarrow} = \overline{I_0}\left(\dfrac{\overline{d_{\text{Sun}}}}{d_{\text{Sun}}}\right)^2 cos(\theta_z)$ $\overline{I_0}$ = 1365 W m^{-2} $\cos(q_z) = \sin(\delta)\sin(\phi) + \cos(\delta)\cos(\phi)\cos(w)$ $\overline{K_0^{\downarrow}}^{24} = \dfrac{\overline{I_0}}{\pi}\left(\dfrac{\overline{d_{\text{Sun}}}}{d_{\text{Sun}}}\right)^2 [\omega_S \sin(\delta)\sin(\phi) + \cos(\delta)\cos(\phi)\sin(\omega_s)]$ (θ_z: zenit angle, δ: declination angle, ϕ: latitude, ω: hour angle, ω_s : daylength (radians))	2.3 A.10 A.15
Global radiation (empirical estimate daily sum)	$\overline{K^{\downarrow}}^{24} = \overline{K_o^{\downarrow}}^{24}\left(a + b\dfrac{n}{N_d}\right)$ (N_d: daylength (hours) $= 2\frac{24}{2\pi}\omega_s$)	2.17
Extinction of radiation	$I_\lambda = I_{\lambda 0}\, exp\left[-m_r\int_0^\infty k_{\lambda,i} q_i \rho dz\right] = I_{\lambda 0}\tau_{\lambda\theta,i}$	2.10
Global radiation as sum of direct and diffuse	$K^{\downarrow} = S + D = I\cos(\theta_z) + D$	2.4
Optical thickness	$\delta_{\lambda,i} \equiv \int_0^\infty k_{\lambda,i} q_i \rho dz$	2.11
Optical mass (vertical, relative)	$m_a = \int_\infty^0 \rho ds$ $m_v = \int_\infty^0 \rho dz$ $m_a = m_v m_r$ $m_r \approx [\cos(\theta_z)]^{-1}$	2.5 2.6 2.7 2.8
Transmissivities	$\tau_{b,\theta} = \dfrac{I}{I_0}$ $\tau_b = \dfrac{K^{\downarrow}}{K_0^{\downarrow}}$	2.15 2.16

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Models for broadband transmissivities	$\tau_{b,\theta} = \left[\left(\tau_{bv,cda}\right)^{m_r}\right]^{T_L} = \exp\left(\delta_{b,cda} m_r T_L\right)$ $\frac{K^{\downarrow}}{K_0^{\downarrow}} = \tau_b = 0.868\, e^{-0.0387\, m_r T_{L,2}}$ (with T_L the Linke turbidity and $\delta_{\mathrm{b,cda}}$ the broadband optical thickness of a clean, dry atmosphere).	2.18 2.19
Brunt	$\varepsilon_{\mathrm{a,clear}} = c_1 + c_2\sqrt{e_a}$ with e_a in mbar and $c_1 = 0.52$ and $c_2 = 0.065$ hPa$^{-0.5}$	2.25
Incoming longwave radiation (empirical estimate)	$L^{\downarrow} = \varepsilon_a \sigma T_a^4$ W m^{-2}, with $\varepsilon_a = f_{\mathrm{cloud}} + (1 - f_{\mathrm{cloud}})\varepsilon_{\mathrm{a,clear}}$	2.24 2.26
Upwelling longwave radiation	$L^{\uparrow} = L_e{}^{\uparrow} + (1 - \varepsilon_s)L^{\downarrow}$ with $L_e{}^{\uparrow} \approx \varepsilon_s \sigma T_s^4$	2.28 2.27
Soil heat flux	$G = -\lambda_s \frac{\partial T}{\partial z_d}$ $\frac{\partial G}{\partial z_d} = -\rho_s c_s \frac{\partial T}{\partial t}$ $\frac{\partial T}{\partial t} = \kappa_s \frac{\partial^2 T}{\partial {z_d}^2}$	2.29 2.30 2.31
Sine-wave forcing of T at surface	$T(z_d, t) = \overline{T} + A(0)e^{-\frac{z_d}{D}} sin\left(\omega\, t - \frac{z_d}{D}\right)$ with $\omega = \frac{2\,\pi}{P}$ (P is period of sine wave) Damping depth (penetration depth): $D = \sqrt{\frac{2\lambda}{\omega \rho_s c_s}} = \sqrt{\frac{2\kappa_s}{\omega}}$ Amplitude at depth z_d: $A(z_d) = A(0)e^{-\frac{z_d}{D}}$ Flux density at depth z_d and time t: $G(z_d, t) = A(0)e^{-\frac{z_d}{D}}\sqrt{\omega \rho_s c_s \lambda_s}\, sin\left[\omega\, t - \frac{z_d}{D} + \frac{\pi}{4}\right]$	2.35 2.37 2.36 2.38
Force-restore method	$\frac{\partial T_{top}}{\partial t} = \frac{1}{C_s d_{top}}\{\underbrace{(Q^* - H - L_v E)}_{\text{force}} - \underbrace{\Lambda_s(T_{top} - T_{bot})}_{\text{restore}}$ with: $\Lambda_s = C_s d_{top}\omega$ and $d_{top} = \sqrt{\kappa_s/2\omega}$	2.40
Frost penetration depth	$z_f = a\sqrt{-I_n}$	2.51

8.3 TURBULENT FLUXES

K-theory (equation 3.1 in the book is the example for temperature)	$H = -\overline{\rho} c_p K_h \frac{\partial \overline{\theta}}{\partial z}$ $E = -\overline{\rho} K_e \frac{\partial \overline{q}}{\partial z}$ $\tau = \overline{\rho} K_m \frac{\partial \overline{u}}{\partial z}$	3.1a 3.1b 3.1c
Flux densities (do not occur as numbered equations in the book, see page 88)	$H = \overline{\rho} c_p \overline{w'\theta'}$	3.13a
	$E = \overline{\rho}\ \overline{w'q'}$	3.13b
	$\tau = -\overline{\rho}\ \overline{u'w'}$	3.13c
Turbulent scales (moisture, temperature, velocity (friction velocity), virtual temperature)	$u_* = \sqrt{\frac{\tau}{\overline{\rho}}}$	3.17a

	$\theta_* = -\frac{H}{\bar{\rho} c_p u_*}$	3.17b
	$q_* = -\frac{E}{\bar{\rho} u_*}$	3.17c
	$\theta_{v*} = -\frac{H_v}{\bar{\rho} c_p u_*}$	3.17d
TKE and TKE equation	$\bar{e} = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right)$	3.9
	$\frac{d\bar{e}}{dt} = -\overline{u'w'}\frac{\partial \bar{u}}{\partial z} + g\frac{\overline{w'\theta_v'}}{\overline{\theta_v}} - \varepsilon + \cdots$	3.10
Stability parameter and Obukhov length	Stability parameter $\frac{z}{L} = -\frac{\kappa z g}{\overline{\theta_v}} \frac{H_v}{\bar{\rho} c_p u_*^3}$	3.19
	Obukhov length: $L = \frac{\overline{\theta_v}}{\kappa g}\frac{u_*^2}{\theta_{v*}} = -\frac{\bar{\rho} c_p \overline{\theta}_v u_*^3}{\kappa g H_v}$	3.19b
Aerodynamic resisistance	$H = -\bar{\rho} c_p \frac{\bar{\theta}(z_2) - \bar{\theta}(z_1)}{r_{\mathrm{ah}}}$	3.24a
	$E = -\bar{\rho}\frac{\bar{q}(z_2) - \bar{q}(z_1)}{r_{\mathrm{ae}}}$	3.24b
	for neutral conditions:	3.25a
	$r_{\mathrm{ae}} = r_{\mathrm{ah}} = r_{\mathrm{am}} = \frac{ln\left[\frac{z_2}{z_1}\right]}{\kappa u_*}$ (with u_*)	
	$r_{\mathrm{ah}} = \frac{ln\left[\frac{z_{\theta 2}}{z_{\theta 1}}\right] ln\left[\frac{z_{u2}}{z_{u1}}\right]}{\kappa^2[\bar{u}(z_2) - \bar{u}(z_1)]}$ (with windspeed)	3.25b
	(κ=0.4, von Karman constant)	
Richardson numbers	$Ri_f = \frac{g}{\overline{\theta_v}}\frac{\overline{w'\theta_v'}}{\overline{u'w'}\frac{\partial \bar{u}}{\partial z}}$	3.11
	$Ri_g = \frac{g}{\overline{\theta_v}}\frac{\frac{\partial \overline{\theta_v}}{\partial z}}{\left[\frac{\partial \bar{u}}{\partial z}\right]^2}$	3.33
	$Ri_f = \frac{K_h}{K_m} Ri_g$	3.34a
Buoyancy flux (these are hidden in footnote 6 (page 85) and footnote 16 (page 101))	Surface flux (energy): $H_v = H(1 + 0.61\bar{q}) + 0.61 c_p \bar{\theta} E$	3.10b
	Turbulent flux (kinematic): $\overline{w'\theta_v'} = \overline{w'\theta'}(1 + 0.61\bar{q}) + 0.61\bar{\theta}\,\overline{w'q'}$	3.19b
Flux-gradient relationships	$\frac{\partial \bar{\theta}}{\partial z}\frac{\kappa z}{\theta_*} = \phi_h\left(\frac{z}{L}\right)$	3.20a
	$\frac{\partial \bar{q}}{\partial z}\frac{\kappa z}{q_*} = \phi_e\left(\frac{z}{L}\right)$	3.20b
	$\frac{\partial \bar{u}}{\partial z}\frac{\kappa z}{u_*} = \phi_m\left(\frac{z}{L}\right)$	3.20c
	$Ri_g = \frac{\phi_h}{\phi_m^2}\frac{z}{L}$	3.34b
	$\phi_h = \phi_e = \phi_m^2 = \left(1 - 16\frac{z}{L}\right)^{-\frac{1}{2}}$ (if $L < 0$)	3.21a
	$\phi_h = \phi_e = \phi_m = \left(1 + 5\frac{z}{L}\right)$ (if $L > 0$)	3.21b

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	With the above functions: $\frac{z}{L} = Ri_g$ (if $L < 0$) $\frac{z}{L} = \frac{Ri_g}{1-5Ri_g}$ (if $L > 0$)	3.36a 3.36b
	Neutral conditions: $\frac{\partial\bar{\theta}}{\partial z}\frac{\kappa z}{\theta_*} = 1$ $\frac{\partial\bar{q}}{\partial z}\frac{\kappa z}{q_*} = 1$ $\frac{\partial\bar{u}}{\partial z}\frac{\kappa z}{u_*} = 1$ (these are not explicitly included in the book, but the consequence of 3.20)	3.20d 3.20e 3.20f
Integrated flux-gradient relationships	$\bar{\theta}(z_{\theta 2}) - \bar{\theta}(z_{\theta 1}) = \frac{\theta_*}{\kappa}\left[ln\left(\frac{z_{\theta 2}}{z_{\theta 1}}\right) - \Psi_h\left(\frac{z_{\theta 2}}{L}\right) + \Psi_h\left(\frac{z_{\theta 1}}{L}\right)\right]$ $\overline{u}(z_{u2}) - \overline{u}(z_{u1}) = \frac{u_*}{\kappa}\left[ln\left(\frac{z_{u2}}{z_{u1}}\right) - \Psi_m\left(\frac{z_{u2}}{L}\right) + \Psi_m\left(\frac{z_{u1}}{L}\right)\right]$	3.29a 3.29b
	$\Psi_m\left(\frac{z}{L}\right) = 2\ln\left(\frac{1+x}{2}\right) + \ln\left(\frac{1+x^2}{2}\right) - 2\arctan(x) + \frac{\pi}{2}$ (if $L< 0$) with $x = \left(1 - 16\frac{z}{L}\right)^{\frac{1}{4}}$ $\Psi_h\left(\frac{z}{L}\right) = 2\ln\left(\frac{1+x^2}{2}\right)$ (if $L< 0$) with $x = \left(1 - 16\frac{z}{L}\right)^{\frac{1}{4}}$ $\Psi_m\left(\frac{z}{L}\right) = \Psi_h\left(\frac{z}{L}\right) = -5\frac{z}{L}$ (if $L> 0$)	3.30a 3.30b 3.30c
	Neutral: $\overline{u}(z_{u2}) - \overline{u}(z_{u1}) = \frac{u_*}{\kappa} ln\left(\frac{z_{u2}}{z_{u1}}\right)$ $\bar{\theta}(z_{\theta 2}) - \bar{\theta}(z_{\theta 1}) = \frac{\theta_*}{\kappa} ln\left(\frac{z_{\theta 2}}{z_{\theta 1}}\right)$	3.23a 3.23b
General equations for H and u_* (these equations do not occur explicitly in the book, but are directly related to 3.43)	$H = \frac{-\bar{\rho}c_p\kappa^2\Delta\bar{\theta}\Delta\overline{u}}{\left[\ln\left(\frac{z_{\theta 2}}{z_{\theta 1}}\right) - \Psi_h\left(\frac{z_{\theta 2}}{L}\right) + \Psi_h\left(\frac{z_{\theta 1}}{L}\right)\right]\left[ln\left(\frac{z_{u2}}{z_{u1}}\right) - \Psi_m\left(\frac{z_{u2}}{L}\right) + \Psi_m\left(\frac{z_{u1}}{L}\right)\right]}$ $u_* = \frac{\kappa\Delta\overline{u}}{\left[\ln\left(\frac{z_{u2}}{z_{u1}}\right) - \Psi_m\left(\frac{z_{u2}}{L}\right) + \Psi_m\left(\frac{z_{u1}}{L}\right)\right]}$	3.43a 3.43b
Analytical solutions (L<0)	$H = \frac{-\bar{\rho}c_p\kappa^2\Delta\bar{\theta}\Delta\overline{u}}{\left[ln\left(\frac{z_2}{z_1}\right)\right]^2}(1 - 16Ri_{b*})^{\frac{3}{4}}$ $u_* = \frac{\kappa\Delta\overline{u}}{\left[ln\left(\frac{z_2}{z_1}\right)\right]}(1 - 16Ri_{b*})^{\frac{1}{4}}$ with: $Ri_{b*} = \sqrt{z_1 z_2}\left[ln\left(\frac{z_2}{z_1}\right)\right]\frac{g}{\overline{\theta_v}}\frac{\Delta\overline{\theta_v}}{(\Delta\overline{u})^2}$ (as a consequence: $L = \frac{\overline{\theta_v}}{g}\frac{(\Delta\overline{u})^2}{\Delta\overline{\theta_v}\, ln\left(\frac{z_2}{z_1}\right)}$)	3.47 3.48 3.49 3.49b
Analytical solutions (L>0)	$H = \frac{-\bar{\rho}c_p\kappa^2\Delta\bar{\theta}\Delta\overline{u}}{\left[ln\left(\frac{z_2}{z_1}\right)\right]^2}(1 - 5Ri_b)^2$ (provided that $Ri_b < 0.2$) $u_* = \frac{\kappa\Delta\overline{u}}{\left[ln\left(\frac{z_2}{z_1}\right)\right]}(1 - 5Ri_b)$ (provided that $Ri_b < 0.2$)	3.50 3.51

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	with: $Ri_b = (z_2 - z_1)\frac{g}{\overline{\theta_v}}\frac{\Delta\overline{\theta_v}}{(\Delta\overline{u})^2}$ (as a consequence: $L = \frac{(z_2-z_1)}{ln\left(\frac{z_2}{z_1}\right)}\frac{(1-5Ri_b)}{Ri_b}$)	3.52 3.52b

8.4 SOIL WATER FLOW

Warrilow model	$\frac{d\theta}{dt} = \frac{1}{D_r}[P - E(\theta) - D(\theta)]$ $E(\theta) = \beta_w(\theta)E_p \quad \text{with} \quad \beta_w = \begin{bmatrix} 1 & \text{for} & \theta_c < \theta < \theta_s \\ \frac{\theta - \theta_w}{\theta_c - \theta_w} & \text{for} & \theta_w < \theta < \theta_c \\ 0 & \text{for} & \theta < \theta_w \end{bmatrix}$ $D = k(\theta) = k_s\left(\frac{\theta - \theta_w}{\theta_s - \theta_w}\right)^n$	4.4 4.2 4.3
Vapour pressure	$\ln\left(\frac{e}{e_{\text{sat}}}\right) = 7.5x10^{-7}\ \text{cm}^{-1}(h + \pi)$	4.10
Effective conductivity	$k_{\text{eff}} = \frac{\sum_{j=1}^{N} L_j}{\sum_{j=1}^{N}\frac{L_j}{k_j}}$	4.15
Continuity eq.	$\frac{\partial\theta}{\partial t} = -\frac{\partial q}{\partial z} - S$	4.21
Richards' equation	$\frac{\partial\theta}{\partial t} = C(h)\frac{\partial h}{\partial t} = \frac{\partial\left[k(h)\left(\frac{\partial h}{\partial z} + 1\right)\right]}{\partial z} - S(z)$	4.22
Soil hydraulic functions	$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{(1 + \lvert\alpha h\rvert^n)^{\frac{n-1}{n}}}$ $k(\theta) = k_s S_e^{\lambda}\left[1 - \left(1 - S_e^{\frac{n}{n-1}}\right)^{\frac{n-1}{n}}\right]^2$ with $S_e = \frac{\theta-\theta_r}{\theta_s-\theta_r}$	4.23 4.24 4.25
Infiltration	Horton: $I = I_f + (I_0 - I_f)e^{-\beta t}$ $I_{\text{cum}} = I_f t + \frac{I_0 - I_f}{\beta}[1 - e^{-\beta t}]$ Green Ampt horizontal: $I_{\text{cum}} = [-2k_t h_f(\theta_t - \theta_i)]^{½} t^{½} = St^{½}$ Green Ampt vertical: $t = \frac{\theta_t-\theta_i}{k_t}\left[s_f + h_f\ln\left(\frac{s_f-h_f}{-h_f}\right)\right]$	4.26 4.27 4.35 4.40
Capillary rise	$z_c = \frac{2\sigma\cos\varphi}{\rho g r}$ $Z = \int_0^Z dz = -\int_0^h \frac{k(h)}{q + k(h)}dh$	4.7 4.43

8.5 SOLUTE TRANSPORT

Solute flux	$J = J_{dif} + J_{con} + J_{dis} = qC_l - \theta(D_{dif} + D_{dis})\frac{\partial C_l}{\partial z}$	5.5
Mass balance	$\frac{\partial C_T}{\partial t} = -\frac{\partial J}{\partial z} - S_s$	5.6
General transport equation	$\frac{\partial}{\partial t}(\rho_b C_a + \theta C_l) = \frac{\partial}{\partial z}\left(\theta D_e\frac{\partial C_l}{\partial z}\right) - \frac{\partial}{\partial z}(qC_l) - S_s$	5.8

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Pulse breakthrough	$C(z,t) = \frac{-zC_0}{2\sqrt{\pi D_e t^3}} \exp\left(-\frac{(z-vt)^2}{4D_e t}\right)$	5.12
Retardation factor	$R = 1 + \frac{\rho_b S_d}{\theta}$	5.16a
First order decay	$M(t) = M_0 \mathrm{e}^{-\mu t}$	5.20
Salinization root zones	$C_l(z) = \frac{C_0}{1 + (L_f - 1)\frac{\lvert z\rvert}{D_r}}$ $\frac{\bar{C}}{C_0} = \frac{1}{1-L_f} \ln\left(\frac{1}{L_f}\right)$	5.26 5.30
Residence time in groundwater	$T_{res} = \frac{\phi H}{R} \ln\left(\frac{L}{2x}\right)$ $C_{out} = C_{in} + (C_{orig} - C_{in}) e^{\frac{-Rt}{\phi H}}$	5.32 5.37

8.6 PLANT TRANSPORT PROCESSES

Root water uptake (macroscopic)	$S_p(z) = \frac{L_{\mathrm{root}}(z)}{\int_{-D_r}^{0} L_{\mathrm{root}}(z)\partial z} T_p$ $S(z) = \alpha_{\mathrm{rw}}(z)\ \alpha_{\mathrm{rs}}(z)\ S_p(z)$ with $T_a = \int_{-D_r}^{0} S(z)\partial z$	6.15 6.17 6.18
Flow in vessels	$v = -\frac{r^2}{8\eta}\frac{\partial H_p}{\partial x}$	6.19
Fluxes at stomate level	Transpiration: $E = -\rho \frac{q_e - q_i}{r_s}$ Photosynthesis and net CO_2 exchange: $A_n = -F_c = \rho \frac{q_{\mathrm{ce}} - q_{\mathrm{ci}}}{r_{\mathrm{s,c}}} = \rho \frac{q_{\mathrm{ce}} - q_{\mathrm{ci}}}{1.6 r_s} = \frac{g_s}{1.6} \rho (q_{\mathrm{ce}} - q_{\mathrm{ci}})$	6.21 6.23
Empirical dependence of internal CO_2 concentration (q_{ci}) on external vapour pressure deficit (D_e)	$\frac{q_{ci} - \Gamma}{q_{ce} - \Gamma} = f_{max} - a_d D_e$ with $f_{max} \approx 0.9$ and $a_d \approx 0.07$ $(\mathrm{kPa})^{-1}$ (C_3 plants) or $a_d \approx 0.15$ $(\mathrm{kPa})^{-1}$ (C4 plants)	6.28
Reaction of stomatal conductance (for CO_2) on assimilation and transpiration	$g_{s,c} \approx \frac{a_1}{\rho(q_{ce} - \Gamma)} A_n - \frac{a_2}{1.6 \rho a_3 D_0} T$ where a_1, a_2, a_3 and D_0 are plant-dependent constants (some of which are not dimensionless)	6.30
Ecosystem level CO_2 exchange	$NEE = A_n - R_s = A_g - R_d - R_s$	6.32
Crop yield (dry matter production DM_a), water productivity (WP_T)	$WP_T = \frac{DM_a}{T_a} = 1000 \frac{\mu}{D_a}$ (note that the factor 1000 has units of kg m^{-3}; this implies that DM_a is in kg m^{-2} and T_a in m; μ and D_a should have identical units, usually Pa)	6.35
Leaf temperature	$T_{\mathrm{leaf}} = T_a + \frac{r_b}{\rho c_p}(Q^*_{\mathrm{leaf}} - L_v E_{\mathrm{leaf}})$	6.41

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8.7 COMBINATION METHODS

Energy balance method	$\beta \equiv \frac{H}{L_v E} = \frac{c_p}{L_v}\frac{\Delta\overline{\theta}}{\Delta\overline{q}} = \gamma\frac{\Delta\overline{\theta}}{\Delta\overline{e}}$ $H = \beta\frac{Q*-G}{1+\beta}$ $L_v E = \frac{Q*-G}{1+\beta}$	7.1 7.2a 7.2b
Penman or Penman-Monteith	$L_v E = \frac{s(Q^* - G) + \frac{\rho c_p}{r_a}\left[e_{\mathrm{sat}}(\overline{T}_a) - \overline{e}_a\right]}{s+\gamma}$ $L_v E = \frac{s(Q^* - G) + \frac{\rho c_p}{r_a}\left[e_{\mathrm{sat}}(\overline{T}_a) - \overline{e}_a\right]}{s+\gamma\left(1+\frac{r_c}{r_a}\right)}$	7.13 7.16
Equilibrium evaporation and Priestley-Taylor	$L_v E_{eq} = \frac{s}{s+\gamma}(Q^* - G)$ $L_v E = \alpha_{\mathrm{PT}}\frac{s}{s+\gamma}(Q^* - G)$ with $\alpha_{PT} \approx 1.26$	7.17 7.18
Makkink	$L_v E = 0.65\frac{s}{s+\gamma}K^{\downarrow}$	7.20