Date: 02.04.2023 $\nabla \cdot A = \frac{1}{h_1 h_2 h_3}$

Divergence & Curl

3 ①

$\bar{A} = \hat{x} z y^3 + \hat{y} 2y \sin(xy) + \hat{z} 3x^2 \ln z$

$\text{div}(\bar{A}) = \bar{\nabla} \cdot \bar{A} = \frac{1}{1.1.1}\left[\frac{\partial}{\partial x}(zy^3 . 1 . 1) + \frac{\partial}{\partial y}(1 . 2y\sin(xy) . 1) + \frac{\partial}{\partial z}(1 . 1 . 3x^2 \ln z)\right]$

$= 0 + 2y . \cos(xy) x + \sin(xy) . 2 + 3x^2 \frac{1}{z}$

$= 2xy \cos(xy) + 2\sin(xy) + \frac{3x^2}{z}$

$\therefore$ A is not solenoidal

Now,

$\text{curl} \bar{A} = \bar{\nabla} \times A$

$$= \frac{1}{1.1.1} \begin{vmatrix} h_1 \hat{u}_1 & h_2 \hat{u}_2 & h_3 \hat{u}_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{vmatrix}$$

$$= \frac{1}{1.1.1} \begin{vmatrix} 1\hat{x} & 1 . \hat{y} & 1\hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 1 . zy^3 & 1 . 2y\sin(xy) & 1 . 3x^2 \ln z \end{vmatrix}$$

$= \hat{x}(0-0) - \hat{y}(6x\ln z - y^3) + \hat{z}(2y^2\cos(xy) - 3y^2 z)$

$\therefore$ A is not conservative.

(6) $\bar{A} = \hat{r}\,\frac{\sin\varphi}{r} + \hat{\varphi}\,\frac{\cos\varphi}{r^2}$, $A_1 = \frac{\sin\varphi}{r}$, $A_2 = \frac{\cos\varphi}{r^2}$, $A_3 = 0$

$$\nabla\cdot\bar{A} = \frac{1}{1.r.1}\left[\frac{\partial}{\partial r}\left(\sin\varphi/\cancel{r}\,.\,r.1\right) + \frac{\partial}{\partial\varphi}\left(1.\frac{\cos\varphi}{r^2}.1\right) + \frac{\partial}{\partial z}(1.r.0)\right]$$

$$= \frac{1}{r}\left[0 + \frac{1}{r^2}(-\sin\varphi)\right]$$

$$= -\frac{\sin\varphi}{r^3}$$

$\therefore$ A is not solenoidal

Curl, $\bar{A} = \bar{\nabla}\times\bar{A}$

$$= \frac{1}{1.r.1}\begin{vmatrix} 1.\hat{r} & r.\hat{\varphi} & 1.\hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ \frac{\sin\varphi}{r} & r.\frac{\cos\varphi}{r^2} & 1.0 \end{vmatrix}$$

$$= \frac{1}{r}\left[\cancel{\hat{r}(\ldots 0) - \hat{\varphi}(0}\right.$$

$$= \frac{1}{r}\left[\hat{z}\left(-\frac{1}{r^2}.\cos\varphi - \frac{1}{r}\cos\varphi\right)\right]$$

$$= \frac{-\cos\varphi}{r^3}\,\hat{z}\,(1+r)$$

$\therefore$ A is not conservative.

(c) $\bar{A} = \hat{R}\cos\theta + \hat{\theta}(R - \sin\theta)$

$$\bar{\nabla}\cdot\bar{A} = \frac{1}{1.R.R\sin\theta}\left[\frac{\partial}{\partial R}(\cos\theta . R . R\sin\theta) + \frac{\partial}{\partial\theta}(1.(R-\sin\theta). R\sin\theta) + \frac{\partial}{\partial\varphi}(1.R.0)\right]$$

$$= \frac{1}{R^2\sin\theta}\left[2R\cos\theta\sin\theta + \frac{\partial}{\partial\theta}(R^2\sin\theta - R\sin^2\theta)\right]$$

$$= \frac{1}{R^2\sin\theta}\left[2R\cos\theta\sin\theta + R^2\cos\theta - R.2\sin\theta\cos\theta\right]$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta$$

$\therefore \bar{A}$ is not solenoidal

$$\bar{\nabla}\times\bar{A} = \frac{1}{1.R.R\sin\theta}\begin{vmatrix} 1.\hat{R} & R.\hat{\theta} & R\sin\theta\,\hat{\varphi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\varphi} \\ 1.\cos\theta & R(R-\sin\theta) & 0 \end{vmatrix}$$

$$= \frac{1}{R^2\sin\theta}\left[\hat{R}(0-0) - R\hat{\theta}(0-0) + R\sin\theta\,\hat{\varphi}\left(\frac{\partial}{\partial R}(R^2 - R\sin\theta) - \frac{\partial}{\partial\theta}(\cos\theta)\right)\right]$$

$$= \frac{1}{R^2\sin\theta}\left\{R\sin\theta\,\hat{\varphi}\,(2R - \cancel{\sin\theta} + \cancel{\sin\theta})\right\}$$

$$= \frac{1}{R}\hat{\varphi}\, 2R$$

$$\Rightarrow 2\hat{\varphi}$$

$\therefore \bar{A}$ is not conservative

Q-2 Monday

Coordinate transformation → 2

Linear dependency

Divergence / curl

$d\bar{l}$ = differential length

$d\bar{l} = \hat{u}_1 h_1 du_1 + \hat{u}_2 h_2 du_2 + \hat{u}_3 h_3 du_3$

④ conservative

$\bar{\nabla} \times \bar{A} = 0$
$\neq 0$

$T = \int \bar{A} \cdot d\bar{l}$

$\frac{d}{dx}(T) = A$

$\Rightarrow T = A\,dx$

4 Ⓐ $\bar{A} = \hat{x}(\sin y + 1) + \hat{y}(2yz + x\cos y) + \hat{z}(y^2 - 3)$

$\therefore \bar{\nabla} \times \bar{A} = \frac{1}{1.1.1} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (\sin y + 1) & 2yz + x\cos y & y^2 - 3 \end{vmatrix}$

$= \hat{x}[2y - 2y] - \hat{y}[0 - 0] + \hat{z}[(0 + \cos y) - \cos y]$

$= 0$

$\therefore \bar{A}$ is a conservative vector field.

Now,

$\bar{\nabla} T = \bar{A}$

$\Rightarrow T = \int \bar{A} \cdot d\bar{l}$

$= \int [\hat{x}(\sin y + 1) + \hat{y}(2yz + x\cos y) + \hat{z}(y^2 - 3)] \cdot [\hat{x}dx + \hat{y}dy + \hat{z}dz]$

$= \int [(\sin y + 1)dx + (2yz + x\cos y)dy + (y^2 - 3)dz]$

$= \int (\sin y\,dx + dx + 2yz\,dy + x\cos y\,dy + y^2 dz - 3dz)$

$= \int dx - \int 3dz + \int d(x\sin y) + \int d(y^2 z)$

$= x - 3z + x\sin y + y^2 z + C$

$\int \sin y\,dx = \sin y\, x$ | $d(x\sin y) = x\cos y\,dy + \sin y\,dx$

1/ (a) continuation of last class

$T = \int \bar{A} \cdot d\bar{l}$

$= x - 3z + x\sin y + y^2 z + c$

Workdone in moving from, $(1,-1,5)$ to $(2, \frac{\pi}{2}, 1)$ is

$$\left[x - 3z + x\sin y + y^2 z + c\right]_{(1,-1,5)}^{(2,\frac{\pi}{2},1)}$$

$$= \left(2 - 3.1 + 2\sin\frac{\pi}{2} + \frac{\pi^2}{4}.1\right) - \left(1 - 3.5 + \sin(-1) + (-1)^2.5\right)$$

$$= 1 + \frac{\pi^2}{4} + 9 + \sin 1$$

$$= 10 + \frac{\pi^2}{4} + \sin 1$$

(b) $\bar{A} = \hat{r}(2rz - \cos\varphi) + \hat{\varphi}\sin\varphi + \hat{z}(r^2)$

$(1,0,-1)$ to $(1,\pi,0)$

$$\bar{\nabla} \times \bar{A} = \frac{1}{1.\pi.1}\begin{vmatrix} 1.\hat{r} & r.\hat{\varphi} & 1.\hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ 1.(2rz-\cos\varphi) & r\sin\varphi & 1.r^2 \end{vmatrix}$$

$$= \frac{1}{\pi}\left[\hat{r}\left(\frac{\partial}{\partial\varphi}r^2 - \frac{\partial}{\partial z}r\sin\theta\right) - \hat{\varphi}r\left(\frac{\partial}{\partial r}r^2 - \frac{\partial}{\partial z}(2rz-\cos\varphi)\right) + \hat{z}\left(\frac{\partial}{\partial r}r\sin\varphi - \frac{\partial}{\partial\varphi}(2rz-\cos\varphi)\right)\right]$$

$$= \frac{1}{\pi}\left[\hat{r}(0 - 0) - \hat{\varphi}r(2r - 2r) + \hat{z}(\sin\varphi - (0+\sin\varphi))\right]$$

$$= \frac{1}{\pi}.0 = 0$$

$\therefore$ A is conservative.

$\therefore T = \int \bar{A} \cdot d\bar{l}$

$= \int \left[\hat{r}(2rz - \cos\varphi) + \hat{\varphi}\sin\varphi + \hat{z}r^2\right] \cdot \left[\hat{r}\,dr + \hat{\varphi}\,r\,d\varphi + \hat{z}\,dz\right]$

$= \int \left[(2rz - \cos\varphi)\,dr + r\sin\varphi\,d\varphi + r^2\,dz\right]$

$= \int \left[2rz\,dr - \cos\varphi\,dr + r\sin\varphi\,d\varphi + r^2\,dz\right]$

$= \int d(r^2 z) + \int d(-r\cos\varphi)$

$= r^2 z - r\cos\varphi + C$

$\therefore$ Workdone $= \left[r^2 z - r\cos\varphi\right]_{(1,0,-1)}^{(1,\pi,0)}$

$= (1.0 - 1.\cos\pi) - (-1 - 1.\cos 0)$

$= (1.0 - 1(-1)) - (-1 - 1.1)$

$= 3$

Ex. 3.4

$$\bar{A} = \hat{R}(\sin\theta\cos\varphi) + \hat{\theta}(\cos\theta\cos\varphi) + \hat{\varphi}(-\sin\varphi)$$

$$\nabla \times \bar{A} = \frac{1}{1.R.R\sin\theta}\begin{vmatrix} 1.\hat{R} & R\hat{\theta} & R\sin\theta.\hat{\varphi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\varphi} \\ \sin\theta\cos\varphi & R\cos\theta\cos\varphi & -R\sin\theta\sin\varphi \end{vmatrix}$$

$$= \frac{1}{R^2\sin\theta}\left[\hat{R}(-R\cos\theta\sin\varphi + R\cos\theta\sin\varphi) + R\hat{\theta}(-\sin\theta\sin\varphi + \sin\theta\sin\varphi) + R\sin\theta\hat{\varphi}(\cos\theta\cos\varphi - \cos\theta\cos\varphi)\right]$$

$$= 0$$

$\therefore$ A is conservative

$$\therefore T = \int \bar{A}.d\bar{T}$$

$$= \int\left[\hat{R}(\sin\theta\cos\varphi) + \hat{\theta}(\cos\theta\cos\varphi) + \hat{\varphi}(-\sin\theta)\right]\cdot\left[\hat{R}dR + \hat{\theta}Rd\theta + \hat{\varphi}R\sin\theta d\varphi\right]$$

$$= \int(\sin\theta\cos\varphi dR + R\cos\theta\cos\varphi d\theta - R\sin\theta\sin\varphi d\varphi)$$

$$= \int d(R\sin\theta\cos\varphi)$$

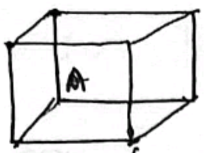
$$= R\sin\theta\cos\varphi + c$$

$$\therefore \text{Workdone} = \left[R\sin\theta\cos\varphi\right]_{(0,\frac{\pi}{4},\frac{\pi}{3})}^{(1,\frac{\pi}{2},\pi)}$$

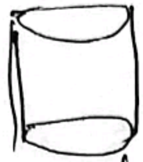
$$= (1.\sin\tfrac{\pi}{2}.\cos\pi) - (0.\sin\tfrac{\pi}{4}.\cos\tfrac{\pi}{3}$$

4 HW

Gauss's Divergence Theorum:



8 surface

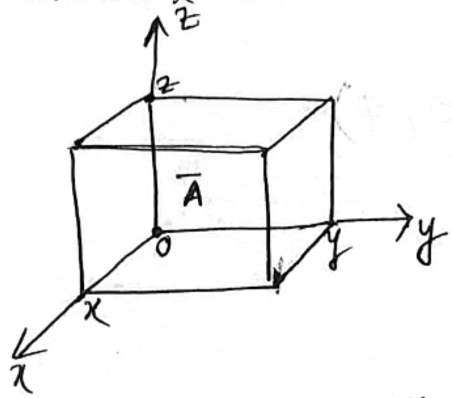


3 surface

$$\oint_C \bar{A} \cdot dS = \int_V (\nabla \cdot \bar{A})\, dV$$

⇒ It is used for closed surface

~~The~~



Top Area : $S = xy$

$~~= xy~~$

$\Rightarrow dS = dx\,dy$

$\boxed{\Rightarrow d\bar{S} = \hat{z}\, dx\, dy}$

Bottom: $d\bar{S} = -\hat{z}\, dx\, dy$

Right Area : $d\bar{S} = \hat{y}\, dx\, dz$

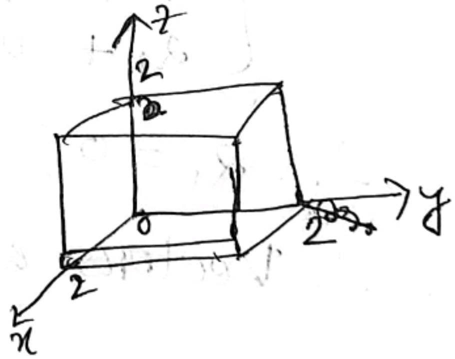
Left: $d\bar{S} = -\hat{y}\, dx\, dz$

Front Area : $d\bar{S} = \hat{x}\, dy\, dz$

Back: $d\bar{S} = -\hat{x}\, dy\, dz$

1. $A = \hat{x}\, xy + \hat{y}\, y^2 z + \hat{z}\, xz$

$\oint \bar{A} \cdot d\bar{S} = ?$



Top surface: $z = 2$, and $d\bar{S} = \hat{z}\, dx\, dy$

$\therefore \bar{A} = \hat{x}\, xy + \hat{y}\, 2y^2 + \hat{z}\, 2x$

Now, $\bar{A} \cdot dS = 2x\, dx\, dy$

$$\therefore \int \bar{A} \cdot d\bar{S} = \int_0^2 \int_0^2 2x\, dx\, dy$$

$$= \int_0^2 [x^2]_0^2\, dy$$

$$= 4\, [y]_0^2 = 8$$

① $(\overset{x}{1}, \overset{y}{-1}, \overset{z}{\sqrt{2}}) \longrightarrow (r, \varphi, z)$

$r = \sqrt{1^2+(-1)^2} = \sqrt{2}$

$\varphi = \tan^{-1}\left(\frac{-1}{1}\right) = -\frac{\pi}{4} + 2\pi$
$= \frac{3\pi}{4}$

$z = z = \sqrt{2}$

$\therefore (r, \varphi, z) = \left(\sqrt{2}, \frac{3\pi}{4}, \sqrt{2}\right)$

② $(\overset{R}{1}, \overset{\varphi}{\pi}, \overset{z}{0}) \longrightarrow (R, \theta, \varphi)$

$R = \sqrt{1^2+0^2} = 1$

$\theta = \tan^{-1}\left(\frac{1}{0}\right)$
$= 0$

$\varphi = \varphi = \pi$

① $v_1 = (2, 0, 1)$, $v_2 = (3, 1, 0)$, $v_3 = (-1, 0, -2)$

Let, $\alpha_1(2, 0, 1) + \alpha_2(3, 1, 0) + \alpha_3(-1, 0, -2)$

$$\Rightarrow \begin{cases} 2\alpha_1 + 3\alpha_2 - \alpha_3 = 0 \\ 0 + \alpha_2 + 0 = 0 \\ \alpha_1 + 0 - 2\alpha_3 = 0 \end{cases}$$

$\therefore \alpha_1 = 0$, $\alpha_2 = 0$, $\alpha_3 = 0$

$\therefore$ Vectors are linearly independent.

1

(i) $T(x,y,z) = x^2y + xy + xz^2$ at $(1,-1,0)$, $d = \hat{x} - 2\hat{y} + \hat{z}$

$$\vec{\nabla} T = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)(x^2y + xy + xz^2)$$

$$\cancel{= \hat{x}(x^2y + xy + xz^2) + \hat{y}(x^2}$$

$$= \hat{x}(2xy + y + z^2) + \hat{y}(x^2 + x + 0) + \hat{z}(0 + 0 + 2zx)$$

$$= \hat{x}(2xy + y + z^2) + \hat{y}(x^2 + x) + \hat{z}(2xz)$$

at point $(1,-1,0)$,

$$\vec{\nabla} T = \hat{x}\{2.1.(-1) + (-1) + (0)^2\} + \hat{y}(1^2 + 1) + \hat{z}(2.1.0)$$

$$= \hat{x}(-2-1) + 2\hat{y} + 0$$

$$= -3\hat{x} + 2\hat{y}$$

Now, $\hat{d} = \dfrac{\vec{d}}{|\vec{d}|} = \dfrac{\hat{x} - 2\hat{y} + \hat{z}}{\sqrt{1^2 + (-2)^2 + 1^2}}$

$$= \frac{\hat{x} - 2\hat{y} + \hat{z}}{\sqrt{6}}$$

$$= \frac{\hat{x}}{\sqrt{6}} - \frac{2\hat{y}}{\sqrt{6}} + \frac{\hat{z}}{\sqrt{6}}$$

$\therefore$ Directional derivative,

$$(\vec{\nabla} T).(\hat{d}) = (-3\hat{x} + 2\hat{y})\left(\frac{\hat{x}}{\sqrt{6}} - \frac{2\hat{y}}{\sqrt{6}} + \frac{\hat{z}}{\sqrt{6}}\right)$$

$$= -\frac{3}{\sqrt{6}} - \frac{4}{\sqrt{6}}$$

(ii) $A = \hat{r}\, 5r\sin\varphi + \hat{\varphi}\, 10r^2\cos\varphi + \hat{z}\, z$

$$\nabla \cdot A = \frac{1}{\cancel{1.R.R\sin}\ 1.r.1}\left[\frac{\partial}{\partial r}(5r\sin\varphi . r . 1) + \frac{\partial}{\partial \varphi}(1 . 10r^2\cos\varphi . 1) + \frac{\partial}{\partial z}(1 . r . z)\right]$$

$$= \frac{1}{r}\left(10r\sin\varphi\ \cancel{\varphi} - 10r^2\sin\varphi + r\right)$$

$$\cancel{= \frac{1}{r}\left\{10r\sin\varphi\right.}$$

$$= \cancel{10}\frac{1}{r}\, 10r\sin\varphi\left(1 - r + \frac{1}{10\sin\varphi}\right)$$

$$= 10\sin\varphi\left(1 - r + \frac{1}{10\sin\varphi}\right) \text{ Ans}$$

(iii) $B = \hat{R}\, e^R + \hat{\theta}\, R\sin\theta$

$$\nabla \times B = \frac{1}{\cancel{R}1.R.R\sin\theta}\begin{vmatrix} 1.\hat{R} & R\hat{\theta} & R\sin\theta\,\hat{\varphi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ 1.e^R & R.R\sin\theta & 0 \end{vmatrix}$$

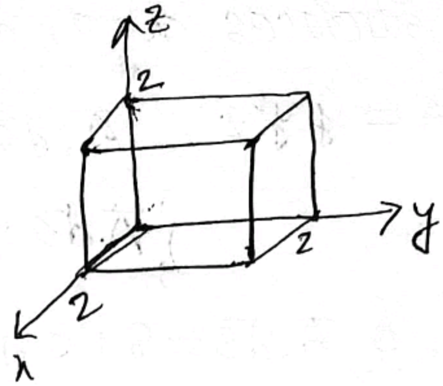
$$= \frac{1}{R^2\sin\theta}\left[\hat{R}(0-0) - R\hat{\theta}(0-0) + R\sin\theta\,\hat{\varphi}\,(2R\sin\theta - 0)\right]$$

$$= \frac{1}{R^2\sin\theta}\ 2R^2\sin^2\theta\,\hat{\varphi}$$

$$= \frac{1}{\sin\theta}\ 2\sin\theta\,\varphi$$

6.2

(1) $\bar{A} = \hat{x}xy + \hat{y}y^2z + \hat{z}xz$



Top: $\int_s \bar{A}.ds = 8$

Bottom: $z=0,\ d\bar{s} = -\hat{z}\,dxdy$

$\bar{A} = \hat{x}\,xy$

$\therefore \bar{A}.d\bar{S} = 0$

$\therefore \int_s \bar{A}.dS = 0$

Right surface: $y=2,\ d\bar{s} = \hat{y}\,dxdz$

$\bar{A} = \hat{x}\,2x + \hat{y}\,4z + \hat{z}\,xz$

$\therefore \bar{A}.ds = 4z\,dx\,dz$

$\therefore \int_s \bar{A}.d\bar{s} = \int_0^2\int_0^2 4z\,dx\,dz$

$= 4\int_0^2 z\,[x]_0^2\,dz$

$= 8\left[\frac{z^2}{2}\right]_0^2$

$= 16$

Left surface: $y=0,\ d\bar{s} = -\hat{y}\,dxdz$

$\bar{A} = \hat{z}\,xz \quad \therefore \bar{A}.d\bar{S} = 0$

$\therefore \int_s \bar{A}.d\bar{S} = 0$

Front surface: $x=2,\ d\bar{s} = \hat{x}\,dydz$

$\bar{A} = \hat{x}\,2y + \hat{y}\,y^2z + \hat{z}\,2z$

$\bar{A}.d\bar{s} = 2y\,dy\,dz$

$\therefore \int_s \bar{A}.d\bar{s} = \int_0^2\int_0^2 2y\,dy\,dz = \int_0^2 [y^2]_0^2\,dz = 4[z]_0^2 = 8$

Back surface: $x=0$, $d\bar{s}=-\hat{x}\,dy\,dz$

$\bar{A}=\hat{y}y^2z$, $\bar{A}.d\bar{s}=0$

$$\int_s \bar{A}.d\bar{s}=0$$

$$\therefore \oint_s \bar{A}.d\bar{s}=8+0+16+0+0+8+0=32$$

(b) $\int_v (\bar{\nabla}.\bar{A})\,dV=?$

Now, $\bar{\nabla}.\bar{A}=\frac{1}{1.1.1}\left[\frac{\partial}{\partial x}(xy)+\frac{\partial}{\partial y}(y^2z)+\frac{\partial}{\partial z}(x-z)\right]$

$=y+2xy+x$

$$\therefore \int_v(\bar{\nabla}.\bar{A})\,dv=\int_0^2\int_0^2\int_0^2(y+2yz+x)\,dx\,dy\,dz$$

$$=\int_0^2\int_0^2\left[xy+2xyz+\frac{x^2}{2}\right]_0^2 dy\,dz$$

$$=\int_0^2\int_0^2(2y+4yz+2)\,dy\,dz$$

$$=\int_0^2\left[y^2+2y^2z+2y\right]_0^2 dz$$

$$=\int_0^2(4+8z+4)\,dz$$

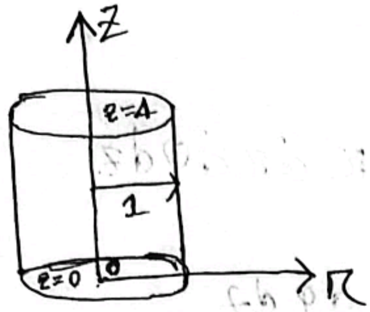
$$=\left[8z+4z^2\right]_0^2$$

$=16+16$

$=32$

6.2

2. $\bar{A} = \hat{r}\, r^2 + \hat{z}\, 3z$, $\quad \underset{\text{radius}}{r=1}, \underset{\text{bottom}}{z=0}, \underset{\text{top}}{z=4}$



Top surface: $z=4$, $d\bar{S} = \hat{z}\, h_1 dr\, h_2 d\varphi$
$= \hat{z}\, r\, dr\, d\varphi$

$\bar{A} = \hat{r}\, r^2 + \hat{z}.3.4$
$= \hat{r}\, r^2 + \hat{z}12$

$\therefore \bar{A}.d\bar{S} = 12\, dr d\varphi$

$\therefore \int_S \bar{A}.d\bar{S} = \int_0^{2\pi}\int_0^1 12 r\, dr\, d\varphi$

$= \int_0^{2\pi} [6r^2]_0^1\, d\varphi$

$= 6[\varphi]_0^{2\pi}$

$= 12\pi$

Bottom surface: $z=0$, $d\bar{S} = -\hat{z}\, r\, dr\, d\varphi$

$\bar{A} = \hat{r}\, r^2 \quad \therefore \int_S A.d\bar{S} = 0$

Circular surface: $r=1, d\bar{S} = \hat{r}\, r\, d\varphi\, dz$
$= \hat{r}\cdot 1 d\varphi\, dz$

$\bar{A} = \hat{r}.1 + \hat{z}\, 3z \quad \therefore \bar{A}.d\bar{S} = d\varphi dz$

$\therefore \int_S \bar{A}.d\bar{S} = \int_0^3 \int_0^{2\pi} d\varphi dz$

$= [\varphi]_0^{2\pi} [z]_0^4$

$= 8\pi$

$\therefore \oint_S \bar{A}.d\bar{S} = 12\pi + 8\pi$
$= \boxed{20\pi}$

Now, ~~A~~

$$(\bar{\nabla}.\bar{A}) = \frac{1}{1.r.1}\left[\frac{\partial}{\partial r}(r^2 r.1) + \frac{\partial}{\partial z}(1.r.3z)\right]$$

$$= \frac{1}{r}(3r^2 + 3r)$$

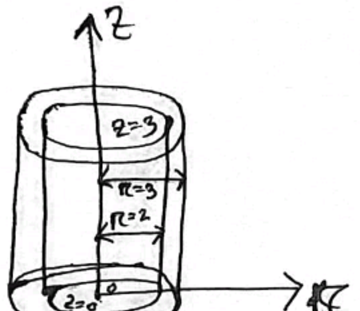
$$\therefore \int_v (\bar{\nabla}.\bar{A})dv = \int_0^4\int_0^{2\pi}\int_0^1 \frac{1}{r}(3r^2 + 3r)\, r\, dr\, d\varphi\, dz$$

$$= \int_0^4\int_0^{2\pi}\left[r^3 + \frac{3}{2}r^2\right]_0^1 d\varphi\, dz$$

$$= \frac{5}{2}\cdot[\varphi]_0^{2\pi}[z]_0^4$$

$$= \boxed{20\pi}$$

3 $\bar{A} = \hat{r}r^2$



$r=2$
$\hat{r}=-\hat{r}$

Top surface: $z=3$, $d\bar{S} = \hat{z}\, r\, dr\, d\varphi$

$\bar{A} = \hat{r}r^2 \quad \therefore \int_S \bar{A}\cdot d\bar{S} = 0$

Bottom surface: $z=0$, $d\bar{S} = -\hat{z}\, r\, dr\, d\varphi$

$A = \hat{r}r^2 \quad \therefore \int_S \bar{A}\cdot d\bar{S} = 0$

Outer circular surface:

$$r=3,\ d\bar{S} = \hat{r}\, r\, d\varphi\, dz$$
$$= \hat{r}\, 3\, d\varphi\, dz$$

$$\therefore \bar{A}\cdot d\bar{S} = 27\, d\varphi\, dz \quad \therefore \int_S \bar{A}\cdot d\bar{S} = \int_0^3\int_0^{2\pi} 27\, d\varphi\, dz$$

$$= 27[\varphi]_0^{2\pi}[z]_0^3$$

$$= 162\pi$$

Inner circular surface:

$r=2,\ d\bar{S}=-\hat{r}\, r\, d\varphi\, dz$ $\qquad A=\hat{r}\cdot 2^2=4$

$=-\hat{r}\, 2\, d\varphi\, dz$

$\therefore \bar{A}\cdot d\bar{S}=27\, d\varphi\, dz=\int_S \bar{A}\cdot d\bar{S}=\int_0^2\int_0^{2\pi}-8r\, d\varphi\, dz$

$=-\int_0^2 [8\varphi]_0^{2\pi}\cdot 2r\, dz$

$=-\int_0^2 16\pi\, dz$

$=-48\pi$

$\therefore \oint_S \bar{A}\cdot d\bar{S}=162\pi-48\pi$

$\boxed{=114\pi}$

Now. $\bar{\nabla}\cdot\bar{A}=\frac{1}{1.r.1}\left[\frac{\partial}{\partial r}(r^2\cdot r\cdot 1)+\right]$

$=\frac{1}{r}\cdot 3r^2$

$=3r$

$\therefore \int_V(\bar{\nabla}\cdot\bar{A})dv=\int_0^3\int_0^{2\pi}\int_2^3 3r\ r\, dr\, d\varphi\, dz$

$=\int_0^3\int_0^{2\pi}[r^3]_2^3\, d\varphi\, dz$

$=(27-8)\,[\varphi]_0^{2\pi}\cdot[z]_0^3$

$=19\cdot 2\pi\cdot 3$

$\boxed{=114\pi}$

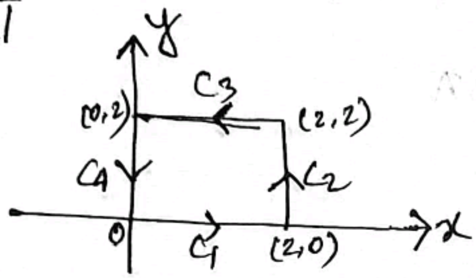
$\therefore$ Divergence theorum verified.

$$\boxed{\oint_C \bar{A}\cdot d\bar{l} = \int_s (\bar{\nabla}\times\bar{A})\, d\bar{s}}$$

6.3

3. $\bar{A} = \hat{x}(x^2-y^2)+\hat{y}(x^2-xy)$

ⓐ $\oint_C \bar{A}\cdot d\bar{l}$ ⓑ $\int_s (\bar{\nabla}\times A)\, ds$



$d\bar{l} = \hat{u}_1 h_1 du_1 + \hat{u} \ldots$

ⓐ ~~$\bar{A}\cdot d\bar{l}$~~

For C_1: $y=0$, $dy=0$ / $\boxed{z=0 \ \& \ dz=0}$ optional

Here, $\bar{A} = \hat{x}(x^2-y^2)+\hat{y}(x^2-xy)$

$d\bar{l} = \hat{x}\,dx+\hat{y}\,dy+\boxed{\hat{z}\,dz}$

$\therefore \bar{A}\cdot d\bar{l} = (x^2-y^2)\,dx+(x^2-xy)\,dy$

~~For C_1: $y=0$, and~~ $dy=0$

$\therefore \bar{A}\cdot d\bar{l} = (x^2-0)\,dx+(x^2-0)\cdot 0$

$= x^2\,dx$

$\therefore \int_{C_1} = A\cdot d\bar{l} = \int_0^2 x^2\,dx$

$= \left[\frac{x^3}{3}\right]_0^2$

$= \frac{8}{3}$

For C_2: $x=2,\ dx=0$

$$\bar{A}\cdot d\bar{l} = (2^2 - 2y)\,dy$$
$$= (4-2y)\,dy$$

$$\therefore \int_{C_2} \bar{A}\cdot d\bar{l} = \int_0^2 (4-2y)\,dy$$
$$= \left[4y - y^2\right]_0^2$$
$$= 8-4 = 4$$

For C_3: $y=2,\ dy=0$

$$A\cdot d\bar{l} = (x^2-4)\,dx$$

$$\therefore \int_{C_3} \bar{A}\cdot d\bar{l} = \int_2^0 (x^2-4)\,dx$$
$$= \left[\frac{x^3}{3} - 4x\right]_2^0$$
$$= 0 - \left(\frac{8}{3} - 8\right)$$
$$= \frac{16}{3}$$

For C_4: $x=0,\ dx=0$

$$\bar{A}\cdot dl = 0$$

$$\therefore \int_{C_4} \bar{A}\cdot d\bar{l} = 0$$

$$\int_C \bar{A}\cdot dl = \frac{8}{3} + 4 + \frac{16}{3} = \frac{36}{3} = 12$$