Results & Answers:

Answer 1) Shown in the code of Integrals.py.

Answer 2) The results for five different values for n have been given. We can conclude that as the value of n increases, both the trapezoidal and Simpson's approximations get closer to the true values of the integral. Simpson's rule provides a more accurate approximation than the trapezoidal rule for the given function and integral. The true error for Simpson's rule is consistently smaller than that for the trapezoidal rule. Finally, as n becomes larger, the differences between the true values and the approximations decrease. This aligns with the expected behavior of numerical integration methods as the number of subintervals increases.

Answer 3) We cannot find the exact answer; however, we can get a fairly accurate approximation after using the trapezoidal rule or Simpson's rule and reducing factors such as error tolerance in the numerical integration method. It has been attempted in the code of Integrals.py.

Answer 4 & 5)
The exact value of the function in point 4 is approximately 0.2118530.211853.

As the number of Intervals has not been defined, we will use n=3,5.

When n=3,
Trapezoid: 0.06803114898500677
Simpsons: 0.06372998730706916
True Error (Trapezoid): 0.03442867300643769
True Error (Simpsons): 0.0387298346843753
Estimated Error (Trapezoid): 0.011136953208404549
Estimated Error (Simpsons): 0.00248907680689897

When n=5,
Trapezoid: 0.0816744366947708
Simpsons: 0.07655217883058991
True Error (Trapezoid): 0.02078538529667366
True Error (Simpsons): 0.025907643160854546
Estimated Error (Trapezoid): 0.00659565015705918
Estimated Error (Simpsons): 0.0016526701141183727

For n=3, the results are significantly lower than the exact value. For n=5, the results are also lower than the exact value but closer than the results for n=3. As the number of intervals increases, we get closer to the exact value.

Answer 6)
Answer Using Trapezoidal: 9159.0

Answer Using Simpsons: 8970.75

The difference in results is expected, as Simpson's rule generally provides a more accurate approximation than the trapezoidal rule.