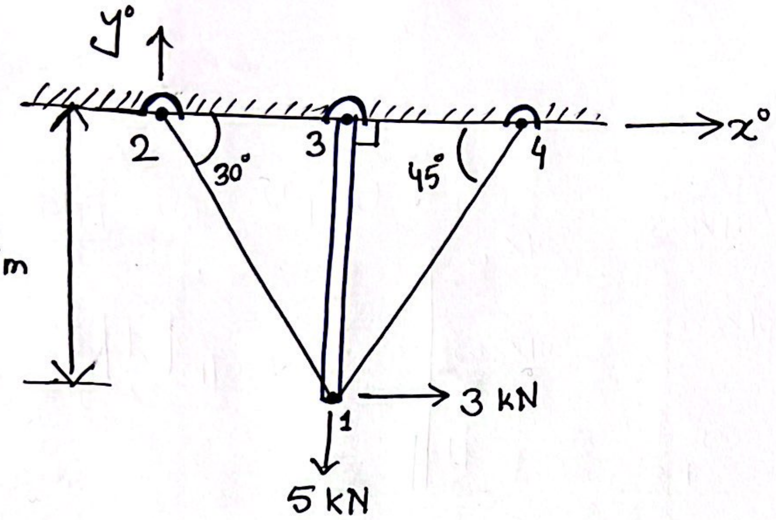
# #1



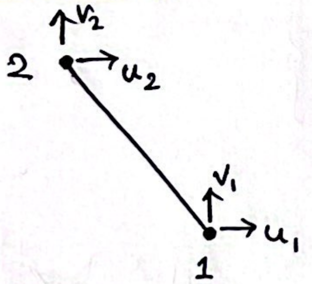

ME 441
FEM

Afnan Mostafa

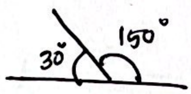
HW2

## Analysis :

### Bar 1-2:




$[K]^{1-2}$ for horizontal bar,




$$= \frac{EA}{l}\begin{bmatrix} \cos^2\alpha & \sin\alpha\cos\alpha & -\cos^2\alpha & -\sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha & \sin^2\alpha & -\sin\alpha\cos\alpha & -\sin^2\alpha \\ -\cos^2\alpha & -\sin\alpha\cos\alpha & \cos^2\alpha & \sin\alpha\cos\alpha \\ -\sin\alpha\cos\alpha & -\sin^2\alpha & \sin\alpha\cos\alpha & \sin^2\alpha \end{bmatrix}$$

$\alpha = 150°$

$$\begin{array}{cc} & \begin{array}{cccc} u_1 & v_1 & u_2 & v_2 \end{array} \\ [K_{12}] = \frac{EA}{l_{12}} & \begin{bmatrix} \frac{3}{4} & -\frac{\sqrt{3}}{4} & -\frac{3}{4} & \frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & \frac{1}{4} & \frac{\sqrt{3}}{4} & -\frac{1}{4} \\ -\frac{3}{4} & \frac{\sqrt{3}}{4} & \frac{3}{4} & -\frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & -\frac{1}{4} & -\frac{\sqrt{3}}{4} & \frac{1}{4} \end{bmatrix} \end{array}$$

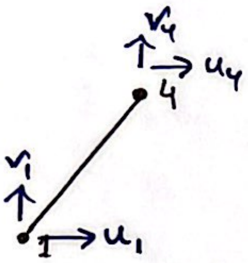
$$[K_{12}]_{global} = \begin{array}{cc} & \begin{array}{cccccc} u_1 & v_1 & u_2 & v_2 & u_4 & v_4 \end{array} \\ \begin{array}{c} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_4 \\ v_4 \end{array} & \begin{bmatrix} 3/4 & -\sqrt{3}/4 & -3/4 & \sqrt{3}/4 & 0 & 0 \\ -\sqrt{3}/4 & 1/4 & \sqrt{3}/4 & -1/4 & 0 & 0 \\ -3/4 & \sqrt{3}/4 & 3/4 & -\sqrt{3}/4 & 0 & 0 \\ \sqrt{3}/4 & -1/4 & -\sqrt{3}/4 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{6\times 6} \end{array} \times \frac{EA}{l_{12}}$$

$\sin 30^\circ = \frac{5}{l_{12}}$

$l_{12} = 5/\sin 30^\circ = 10$

$l_{14} = 5/\sin 45^\circ = 5\sqrt{2}$

**Bar 1-4:**



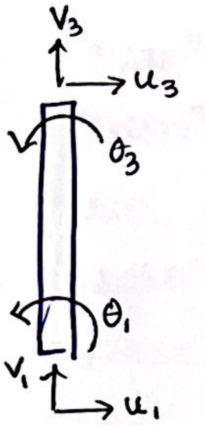

$$[K_{14}] = \frac{EA}{l_{14}} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$[K_{14}] = \frac{EA}{2 l_{14}} \begin{array}{c} \begin{array}{cccc} u_1 & v_1 & u_4 & v_4 \end{array} \\ \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix} \end{array}$$

$$[K_{14}]_{global} = \frac{EA}{2 l_{14}} \begin{array}{c|c} & \begin{array}{cccccc} u_1 & v_1 & u_2 & v_2 & u_4 & v_4 \end{array} \\ \begin{array}{c} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_4 \\ v_4 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 & -1 & -1 \\ 1 & 1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 1 \\ -1 & -1 & 0 & 0 & 1 & 1 \end{bmatrix}_{6\times 6} \end{array}$$

$$[K_{bar}]_{global} = \begin{bmatrix} 14.215 & 14.097 & -0.075 & 0.043 & -14.14 & -14.14 \\ 14.097 & 14.165 & 0.043 & -0.025 & -14.14 & -14.14 \\ -0.075 & 0.043 & 0.075 & -0.043 & 0 & 0 \\ 0.043 & -0.025 & -0.043 & 0.025 & 0 & 0 \\ -14.14 & -14.14 & 0 & 0 & 14.14 & 14.14 \\ -14.14 & -14.14 & 0 & 0 & 14.14 & 14.14 \end{bmatrix}$$

**Beam 1-3:**




$$[R] = \begin{bmatrix} c & s & 0 & 0 & 0 & 0 \\ -s & c & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c & s & 0 \\ 0 & 0 & 0 & -s & c & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$c = \cos 90^\circ$

$\sin 90^\circ = s$

$\therefore [K_{13}]_{local} = [R]^T [K_{13}] [R]$

$$[K_{13}] = \frac{EI}{l_{13}^3} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 12 & 6l_{13} & 0 & -12 & 6l_{13} \\ 0 & 6l_{13} & 4l_{13}^2 & 0 & -6l_{13} & 2l_{13}^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -12 & -6l_{13} & 0 & 12 & -6l_{13} \\ 0 & 6l_{13} & 2l_{13}^2 & 0 & -6l_{13} & 4l_{13}^2 \end{bmatrix}$$

Using MATLAB,

$$\therefore [K_{13}]_{\substack{local \\ 6\times6}} = \begin{array}{c} \begin{array}{cccccc} u_1 & v_1 & \theta_1 & u_3 & v_3 & \theta_3 \end{array} \\ \begin{bmatrix} 50 & 0 & 125 & -50 & 0 & 125 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 125 & 0 & 416.67 & -125 & 0 & 208.33 \\ -50 & 0 & -125 & 50 & 0 & -125 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 125 & 0 & 208.3 & -125 & 0 & 416.67 \end{bmatrix}_{6\times6} \end{array}$$

$I = \frac{1}{12} bh^3$

$\therefore I = \frac{0.05^4}{12} m^4$

$\therefore l_{13} = 5m$

Now,

expanding $[K_{12}]_{\substack{global \\ 6\times6}}$, $[K_{14}]_{\substack{global \\ 6\times6}}$ & $[K_{13}]_{\substack{local \\ 6\times6}}$ to 10x10 Global matrix.

~~$[K_{12}]_{6\times6}$ includes → $u_1$ / $V_1$~~

All 3 matrices are 10x10 matrices that include 10 DOFs.

$u_1 \quad v_1 \quad \theta_1 \quad u_2 \quad V_2 \quad u_3 \quad V_3 \quad \theta_3 \quad u_4 \quad V_4$

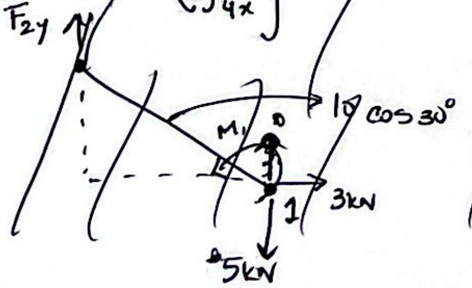
$$[K_r]_{\text{global}\ 10\times10} = \begin{bmatrix} 3.6\times10^5 & 6.8\times10^5 & 125 & -1.87\times10^5 & 1.08\times10^5 & -50 & 0 & 125 & -1.76\times10^5 & -1.76\times10^5 \\ 6.8\times10^5 & 2.3\times10^5 & 0 & 1.08\times10^5 & -62500 & 0 & 0 & 0 & -1.76\times10^5 & -1.76\times10^5 \\ 125 & 0 & 416.67 & 0 & 0 & -125 & 0 & 208.3 & 0 & 0 \\ -1.8\times10^5 & 1.08\times10^5 & 0 & 1.87\times10^5 & -1.08\times10^5 & 0 & 0 & 0 & 0 & 0 \\ 1.08\times10^5 & -62500 & 0 & 1.08\times10^5 & 62500 & 0 & 0 & 0 & 0 & 0 \\ -50 & 0 & -125 & 0 & 0 & 50 & 0 & -125 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 125 & 0 & 20833 & 0 & 0 & 125 & 0 & 416.67 & 0 & 0 \\ -1.76\times10^5 & -1.76\times10^5 & 0 & 0 & 0 & 0 & 0 & 0 & 1.76\times10^5 & 1.76\times10^5 \\ -1.76\times10^5 & -1.76\times10^5 & 0 & 0 & 0 & 0 & 0 & 0 & 1.76\times10^5 & 1.76\times10^5 \end{bmatrix}$$

$$\begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ \cancel{v_2} \to 0 \\ \cancel{u_3} \to 0 \\ \cancel{v_3} \to 0 \\ \cancel{\theta_3} \to 0 \\ u_4 \\ \cancel{v_4} \to 0 \end{Bmatrix}$$

$[K]_{\text{global}(5\times5)}$

$$= \begin{bmatrix} 3.6\times10^5 & 6.8\times10^5 & 125 & -1.87\times10^5 & -1.76\times10^5 \\ 6.8\times10^5 & 2.3\times10^5 & 0 & 1.08\times10^5 & -1.76\times10^5 \\ 125 & 0 & 416.67 & 0 & 0 \\ -1.8\times10^5 & 1.08\times10^5 & 0 & 1.87\times10^5 & 0 \\ -1.76\times10^5 & -1.76\times10^5 & 0 & 0 & 1.76\times10^5 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ \cancel{u_2} \to 0 \\ \cancel{u_4} \to 0 \end{Bmatrix}$$

$$\therefore [F] = \begin{Bmatrix} f_{1x} \\ f_{1y} \\ M_1 \\ f_{2x} \\ f_{4x} \end{Bmatrix} \qquad \therefore \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ \cancel{u_2} \to 0 \\ \cancel{u_4} \to 0 \end{Bmatrix} = [K]^{-1}_{\text{global}(5\times5)} \begin{Bmatrix} f_{1x} \\ f_{1y} \\ M_1 \\ f_{2x} \\ f_{4x} \end{Bmatrix}$$




$\therefore M_1 = F_{2y} \times \frac{10\times\sqrt{3}}{2}$

$\therefore M_1 = (-5000 \times 5\sqrt{3})$ N

$F_{2y} = -5$ kN

After solving $Ax = b$,

$$\begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} \\ \\ \\ \\ \end{Bmatrix}$$

$$\begin{Bmatrix} f_{x_1} \\ f_{y_1} \\ \cancel{M_1} \, 0 \end{Bmatrix} = \begin{bmatrix} 3.63\times10^5 & 6.85\times10^4 & 125 \\ 6.85\times10^4 & 2.39\times10^5 & 0 \\ 125 & 0 & 416.67 \end{bmatrix}^{-1} \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \end{Bmatrix}$$

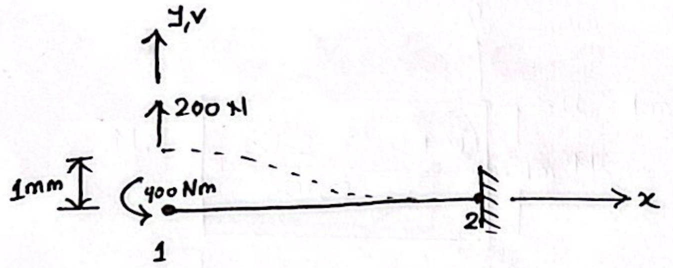
$$\{F\} = \begin{Bmatrix} 3000 \\ -5000 \\ 0 \end{Bmatrix}$$

Solving in MATLAB,

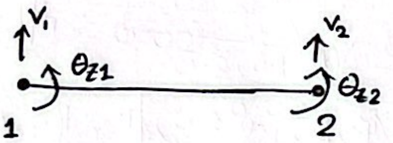
$$\begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \end{Bmatrix} = \begin{Bmatrix} 0.0129 \\ -0.0246 \\ -0.0039 \end{Bmatrix}$$

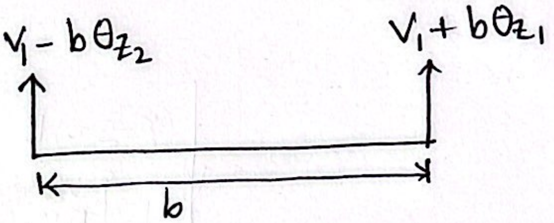
| DOF | Hw #2 | Abaqus |
|---|---|---|
| $u_1$ | 0.0129 | $9.11\times10^{-3}$ |
| $v_1$ | -0.0246 | $-5.6\times10^{-3}$ |
| $\theta_1$ | -0.0039 | $-3.095\times10^{-3}$ |

## #2.




**Analysis:**




There are 2 nodes and 4 DoF ($v_1, \theta_{z1}, v_2, \theta_{z2}$). Hence the stiffness matrix, [K], is a 4x4 matrix.

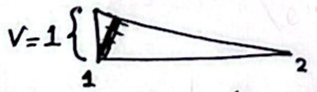
$$
\begin{array}{c|cccc}
 & v_1 & \theta_{z_1} & v_2 & \theta_{z_2} \\
\hline
v_1 & & & & \\
\theta_{z_1} & & & & \\
v_2 & & & & \\
\theta_{z_2} & & & &
\end{array}
\quad
[K] = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{bmatrix}
$$

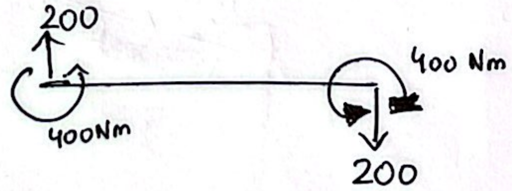
Since it is a symmetric matrix,

$K_{12} = K_{21}$, $K_{13} = K_{31}$ $K_{14} = K_{41}$

$K_{23} = K_{32}$ , $K_{34} = K_{43}$, $K_{24} = K_{42}$

let $v_1 = 1$, $\theta_{z1}, v_2, \theta_{z_2} = 0$




$F_1 = (200 \cancel{\ }) N$

$F_1 = 200 N$

$+\circlearrowleft M_1 = (400 \times 1000) Nmm$

$M_1 = 400{,}000 \ Nmm$

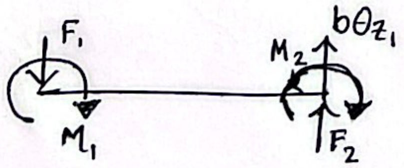
$\therefore F_2 = -F_1$ & $M_2 = +M_1$

$$\therefore [K] = \begin{bmatrix} 200 & ? & ? & ? \\ 400000 & ? & ? & ? \\ -200 & ? & ? & ? \\ +400000 & ? & ? & ? \end{bmatrix}$$

Using symmetry,

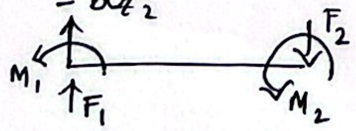
$$[K] = \begin{bmatrix} 200 & 400000 & -200 & +400000 \\ 400000 & ? & ? & ? \\ -200 & ? & ? & ? \\ +400000 & ? & ? & ? \end{bmatrix}$$

Now, let $\theta_{z_1} = 1$, $v_1 = v_2 = \theta_{z_2} = 0$



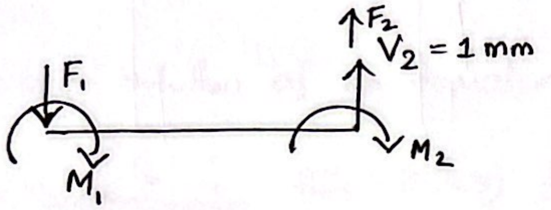

but, we don't know $F_1, M_1, F_2, M_2$ in this case when $v_1 = 0$

Similarly for $\theta_{z_2} = 1$,




$F_1, M_1, F_2, M_2 = ?$

But, for $V_2 = 1$, $V_1 = \theta_{z_1} = \theta_{z_2} = 0$.




We know $F_2 = 200\ N$ $\quad\quad$ $M_2 = M_1$

$= -400\ Nm$

$\therefore F_1 = -200\ N$

$= -400000\ Nmm$

$$\therefore [K] = \begin{bmatrix} 200 & 400000 & -200 & 400000 \\ 400000 & ? & -400000 & \\ -200 & & 200 & \\ 400000 & & -400000 & \end{bmatrix}$$

Again, using symmetry,

$$[K] = \begin{bmatrix} 200 & 4\times10^5 & -200 & 4\times10^5 \\ 4\times10^5 & ? & -4\times10^5 & ? \\ -200 & -4\times10^5 & 200 & -4\times10^5 \\ 4\times10^5 & ? & -4\times10^5 & ? \end{bmatrix}$$

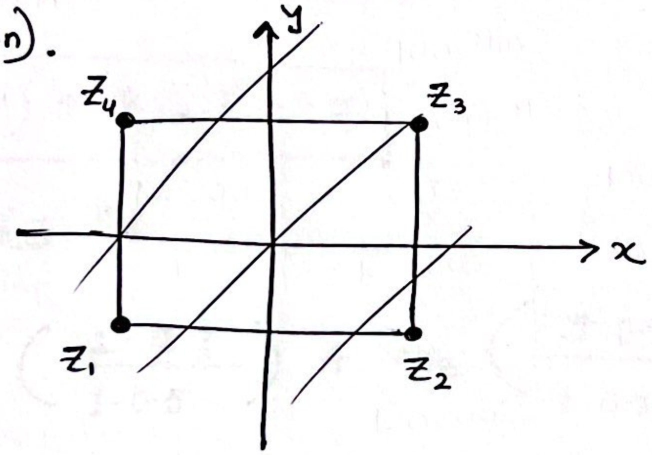
Not sure if there is a way to get $k_{22}$, $k_{24}$, $k_{42}$ & $k_{44}$.

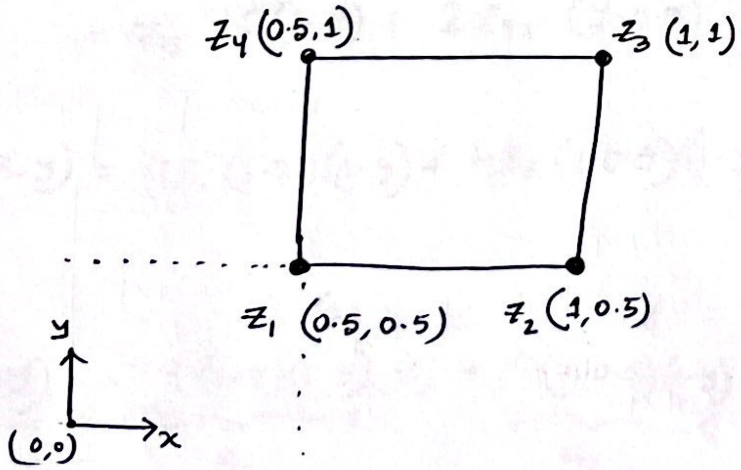
## #3.

Given,

exact solution of an equation, $z(x,y) = \sin(x^2+y^2)$

We can approximate this $z(x,y)$ field using a 2D square element (bilinear interpolation).




$$z(x,y) = \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{Bmatrix}$$

$$[N] = \underbrace{\begin{bmatrix} 1 & x & y & xy \end{bmatrix}}_{[x]} \underbrace{\begin{bmatrix} 1 & x_1 & y_1 & x_1 y_1 \\ 1 & x_2 & y_2 & x_2 y_2 \\ 1 & x_3 & y_3 & x_3 y_3 \\ 1 & x_4 & y_4 & x_4 y_4 \end{bmatrix}^{-1}}_{[A]^{-1}}$$

$$z_{12} = z_1\left(\frac{1-x}{1-0.5}\right) + z_2\left(\frac{x-0.5}{1-0.5}\right)$$

$$\boxed{\therefore z_{12} = 2z_1(1-x) + 2z_2(x-0.5)}$$

$$z_{43} = z_4\left(\frac{1-x}{1-0.5}\right) + z_3\left(\frac{x-0.5}{1-0.5}\right)$$

$$\boxed{\therefore z_{43} = 2z_4(1-x) + 2z_3(x-0.5)}$$

Now, along the vertical line,

$$z(x,y) = z_{12}\left(\frac{1-\cancel{x}y}{1-0.5}\right) + z_{43}\left(\frac{y-0.5}{1-0.5}\right)$$

$$= z_{12} \times 2(1-y) + 2z_{43}(y-0.5)$$

$$\therefore z(x,y) = 4z_1(1-x)(1-y) + 4z_2(x-0.5)(1-y) + 4z_4(1-x)(y-0.5) + 4z_3(x-0.5)(y-0.5)$$

$$\therefore z(x,y) = \underbrace{4(1-x)(1-y)}_{N_1}z_1 + \underbrace{4(x-0.5)(1-y)}_{N_2}z_2 + \underbrace{4(1-x)(y-0.5)}_{N_4}z_4 + \underbrace{4(x-0.5)(y-0.5)}_{N_3}z_3$$

$$\Rightarrow z(x,y) = N_1z_1 + N_2z_2 + N_3z_3 + N_4z_4$$

$$= 4(1-y-x+xy)z_1 + 4(x-xy-0.5+0.5y)z_2 + 4z_4(y-0.5-xy+0.5x) + 4z_3(xy-0.5x-0.5y+0.25)$$

$$= z_1\underbrace{(4-4y-4x+4xy)}_{N_1} + z_2\underbrace{(4x-4xy-2+2y)}_{N_2} + z_4\underbrace{(4y-2-4xy+2x)}_{N_4} + z_3\underbrace{(4xy-2x-2y+1)}_{N_3}$$

$$= 4z_1 - 4yz_1 - 4xz_1 + 4xyz_1 + 4xz_2 - 4xyz_2 - 2z_2 + 2yz_2 + 4xyz_3 - 2xz_3$$
$$- 2yz_3 + z_3 + 4yz_4 - 2z_4 - 4xyz_4 + 2xz_4$$

$$= \underbrace{(4z_1 - 2z_2 + z_3 - 2z_4)}_{a_0} + x\underbrace{(-4z_1 + 4z_2 - 2z_3 + 2z_4)}_{a_1} + y\underbrace{(-4z_1 + 2z_2 - 2z_3 + 4z_4)}_{a_2}$$
$$+ xy\underbrace{(4z_1 - 4z_2 + 4z_3 - 4z_4)}_{a_3}$$

$$\equiv a_0 + xa_1 + ya_2 + xya_3$$

$z_1 = \sin(0.5^2 + 0.5^2) = .4794$

$z_2 = \sin(1^2 + 0.5^2) = .949$

$z_3 = \sin(1^2 + 1^2) = .909$

$z_4 = \sin(0.5^2 + 1^2) = .949$

~~$a_0 = 0$~~

$a_0 = -0.9694$

$a_1 = 1.9584$

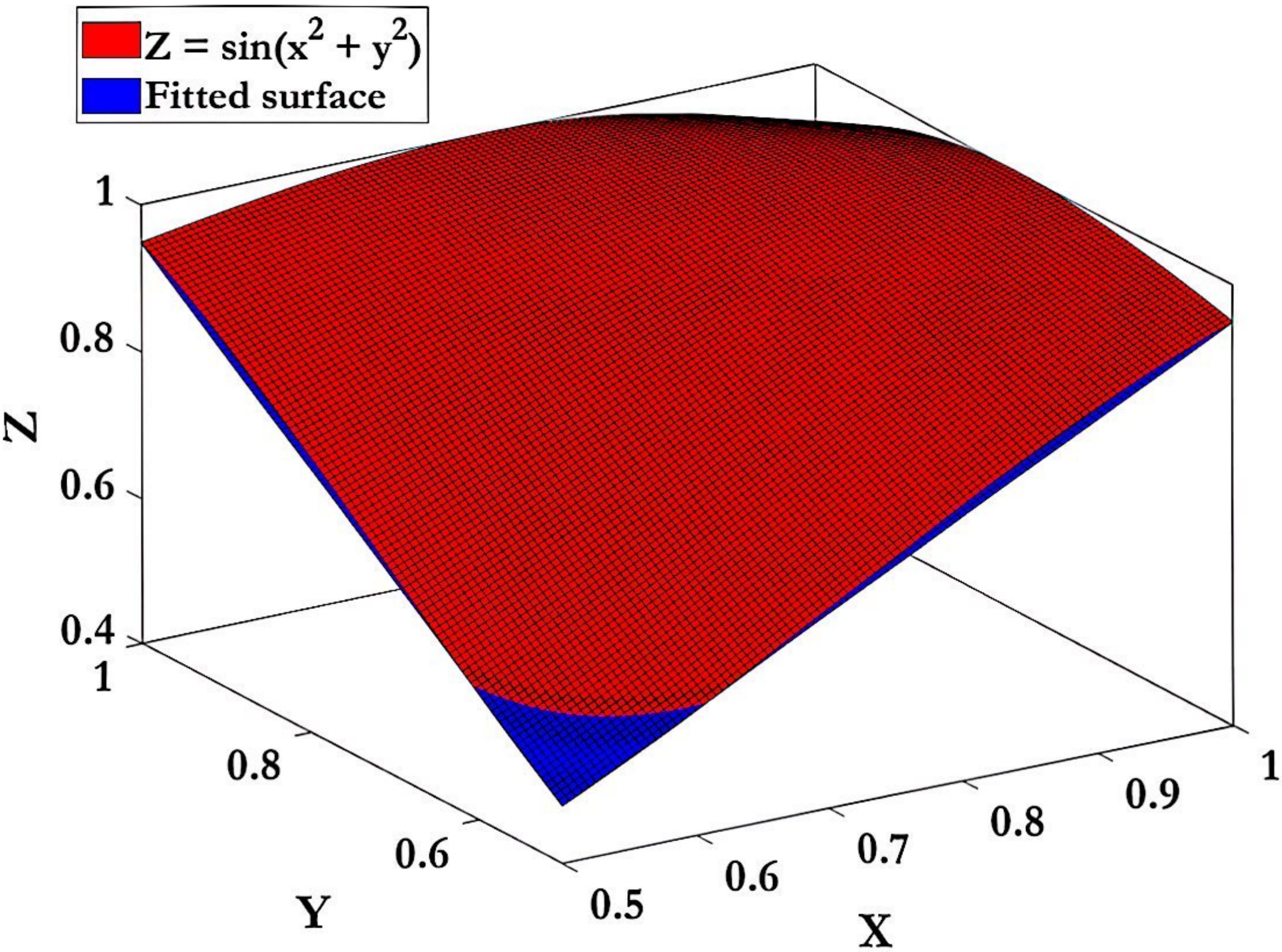
$a_2 = 1.9584$

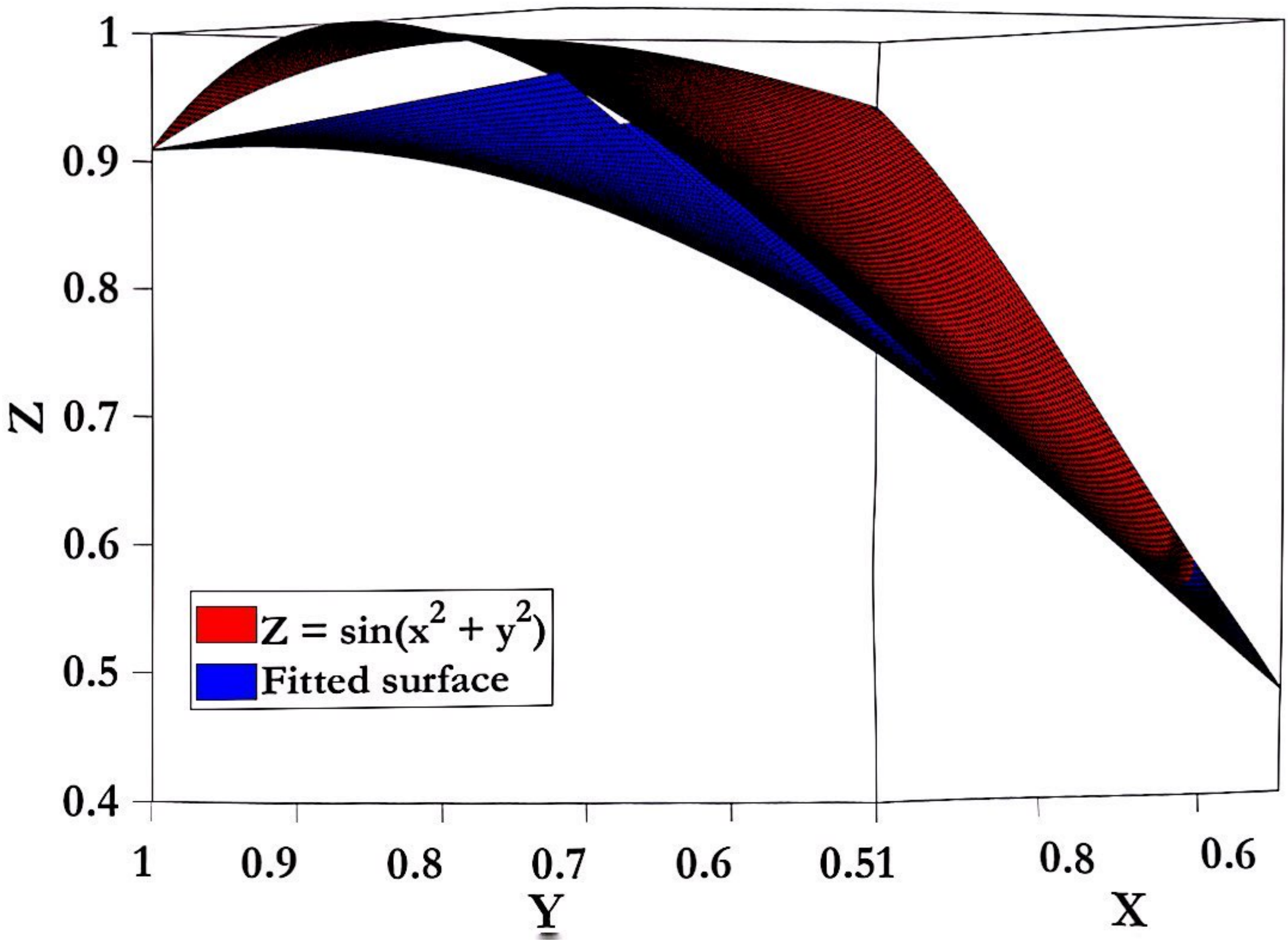
$a_3 = -2.0384$

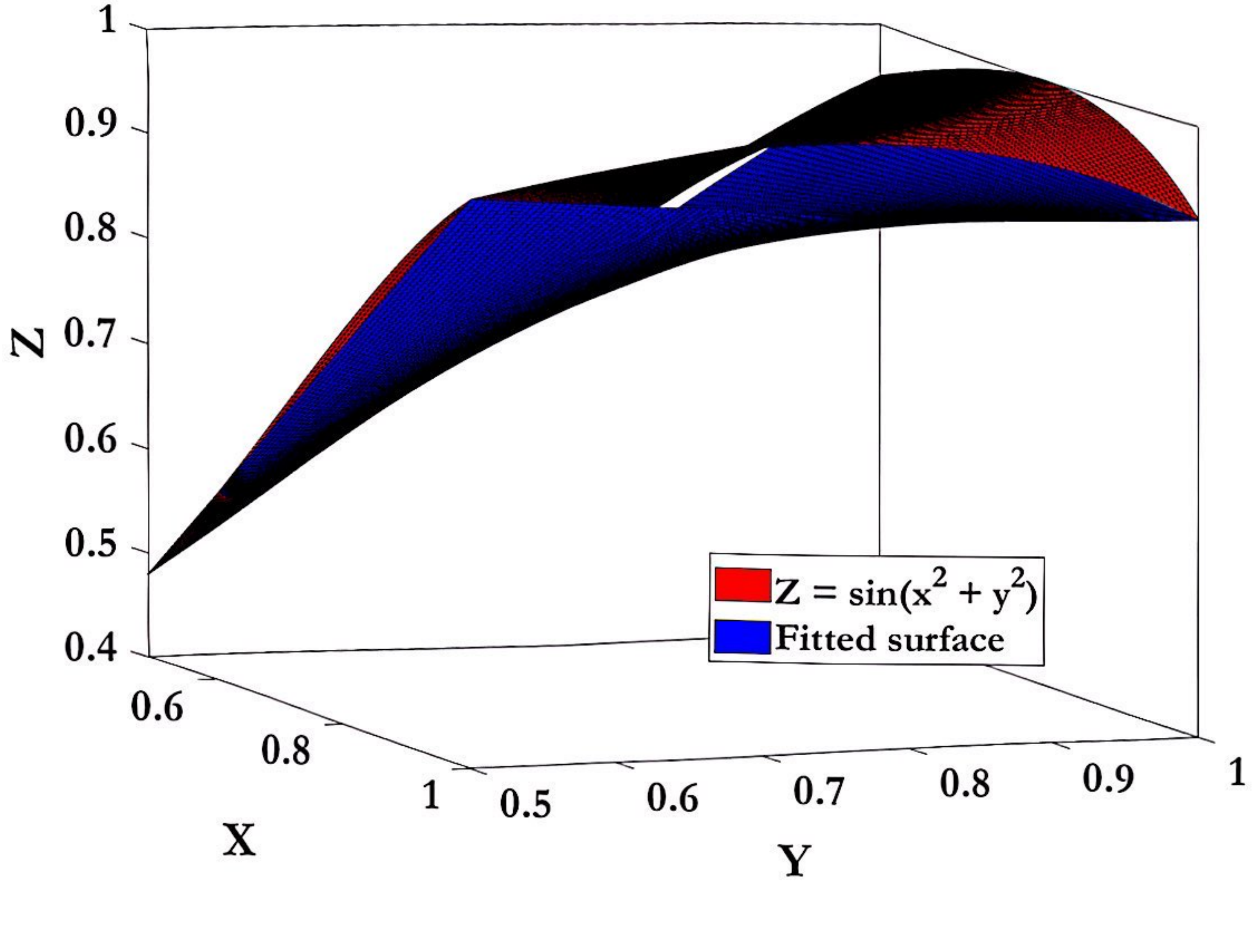
Now,

Plot $\rightarrow z = a_0 + a_1x + a_2y + a_3xy$

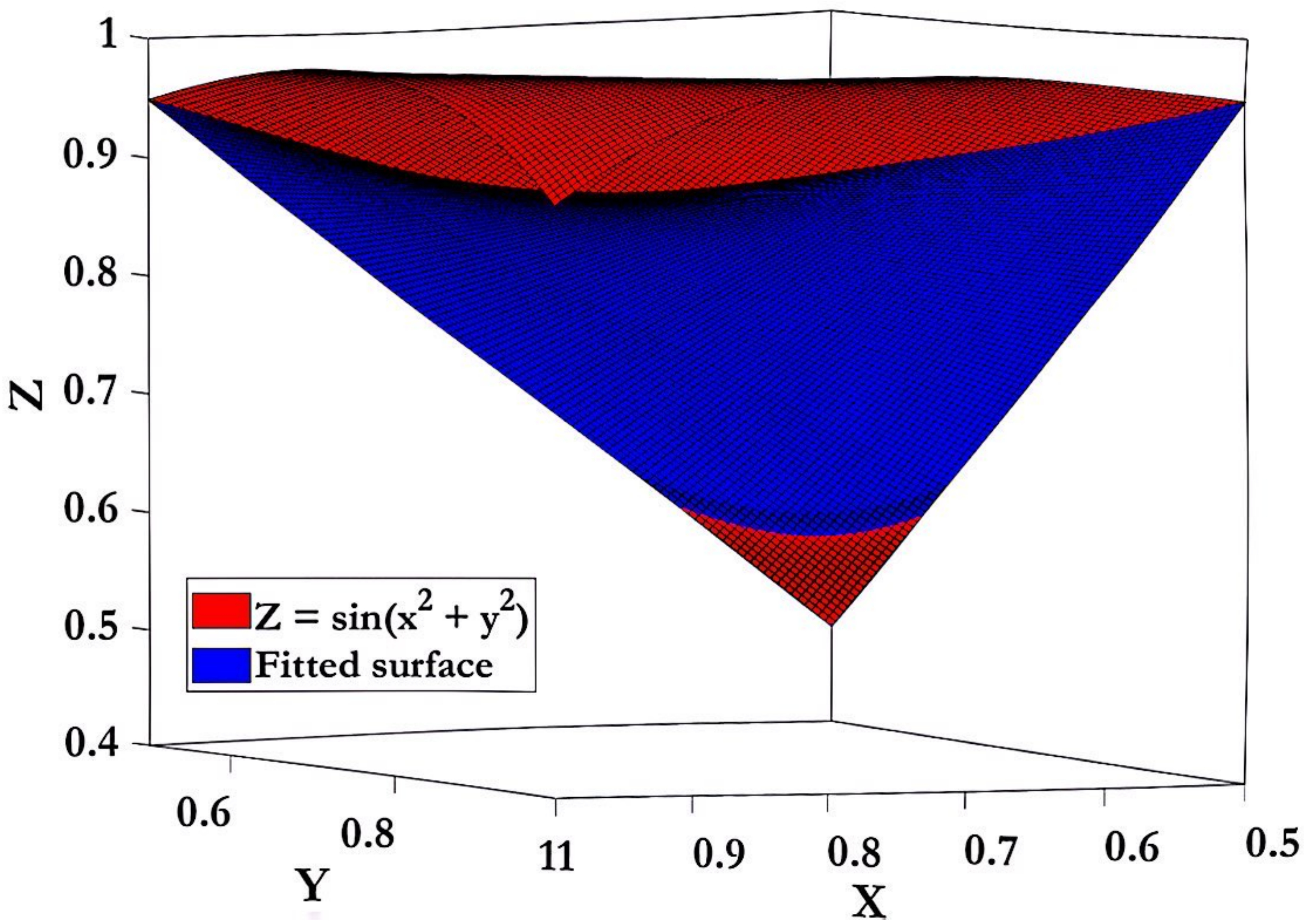
& compare with $z = \sin(x^2 + y^2)$; $x, y \in [0.5, 1]$

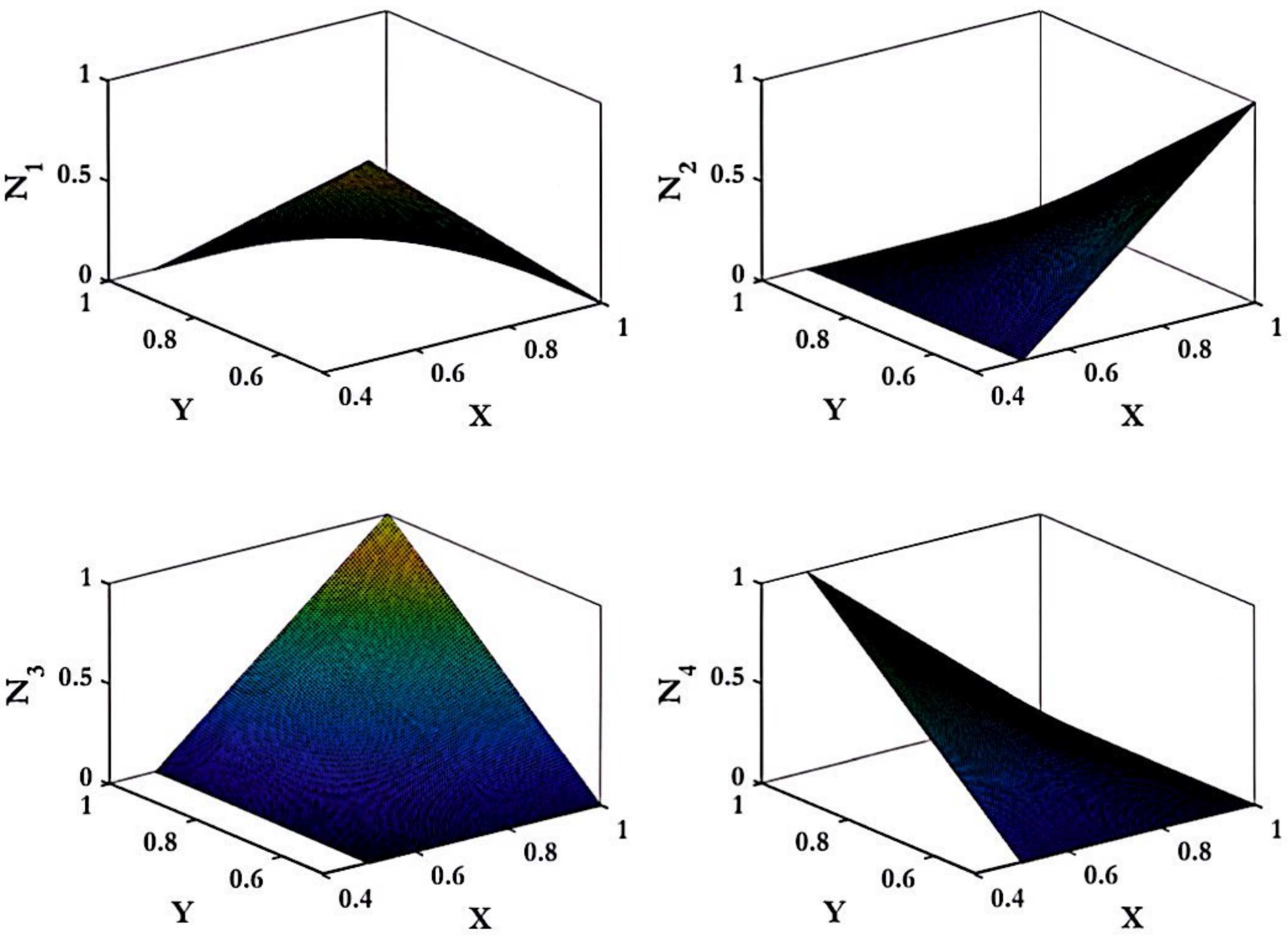

Z = sin(x^2 + y^2)
Fitted surface
1
0.8
Z
0.6
0.4
1
0.8
0.6
Y
0.5
0.6
0.7
0.8
0.9
1
X


1
0.9
0.8
0.7
0.6
0.5
0.4
Z
$Z = \sin(x^2 + y^2)$
Fitted surface
1
0.9
0.8
0.7
0.6
0.51
0.8
0.6
Y
X



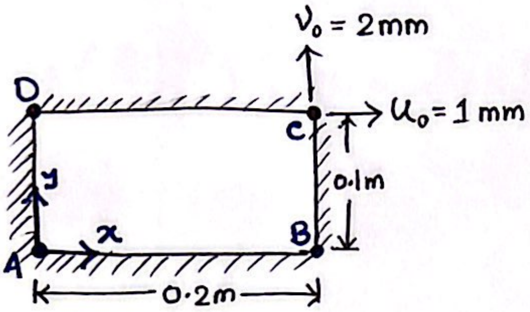
Z
0.4
0.5
0.6
0.7
0.8
0.9
1
X
0.6
0.8
1
Y
0.5
0.6
0.7
0.8
0.9
1
Z = sin(x^2 + y^2)
Fitted surface


$Z = \sin(x^2 + y^2)$
Fitted surface
Z
1
0.9
0.8
0.7
0.6
0.5
0.4
Y
0.6
0.8
1
X
1
0.9
0.8
0.7
0.6
0.5



1
$N_1$ 0.5
0
1
0.8
0.6
Y
0.4
0.6
0.8
1
X
1
$N_2$ 0.5
0
1
0.8
0.6
Y
0.4
0.6
0.8
1
X
1
$N_3$ 0.5
0
1
0.8
0.6
Y
0.4
0.6
0.8
1
X
1
$N_4$ 0.5
0
1
0.8
0.6
Y
0.4
0.6
0.8
1
X

## #4.




Given,

$E = 10^{11}$ Pa (Young's Modulus)

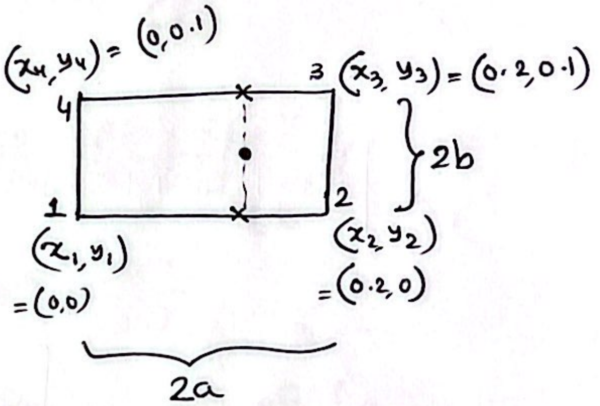
$\nu = 0.3$ (Poisson's Ratio)

Analysis:

A(0,0), B(0.2,0), C(0.2,0.1), D(0,0.1)

$AB = 2a = 0.2$ m $\quad$ $BC = 2b = 0.1$ m

Shape Functions:




$\cancel{N_A} =$

$$\phi_{12} = \phi_1\left(\frac{x_2 - x}{2a}\right) + \phi_2\left(\frac{x - x_1}{2a}\right)$$

$$= \phi_1\left(\frac{0.2 - x}{0.2}\right) + \phi_2\left(\frac{x - 0}{0.2}\right)$$

$$\phi_{12} = \phi_1\left(\frac{0.2 - x}{0.2}\right) + \phi_2 \times \frac{x}{0.2}$$

$$\phi_{43} = \phi_4\left(\frac{x_3 - x}{2a}\right) + \phi_3\left(\frac{x - x_4}{2a}\right)$$

$$= \phi_4\left(\frac{0.2 - x}{0.2}\right) + \phi_3\left(\frac{x - 0}{0.2}\right)$$

$$\phi_{43} = \phi_4\left(\frac{0.2 - x}{0.2}\right) + \phi_3 \times \frac{x}{0.2}$$

$$\therefore \phi(x,y) = \phi_{12}\left(\frac{0.1 - y}{2b}\right) + \phi_{43}\left(\frac{y - 0}{2b}\right)$$

$$\phi(x,y) = \phi_{12}\left(\frac{0.1 - y}{0.1}\right) + \phi_{43} \times \frac{y}{0.1}$$

$$\phi(x,y) = \phi_1 \underbrace{\left(\frac{0.2 - x}{0.2}\right)\left(\frac{0.1 - y}{0.1}\right)}_{N_1} + \phi_2 \underbrace{\left(\frac{x}{0.2}\right)\left(\frac{0.1 - y}{0.1}\right)}_{N_2} + \phi_4 \underbrace{\left(\frac{0.2 - x}{0.2}\right)\frac{y}{0.1}}_{N_4} + \phi_3 \underbrace{\left(\frac{x}{0.2} \times \frac{y}{0.1}\right)}$$

$$N_1 = \left(\frac{0.2-x}{0.2}\right)\times\left(\frac{0.1-y}{0.1}\right), \quad N_2 = \frac{x}{0.2}\times\left(\frac{0.1-y}{0.1}\right), \quad N_3 = \frac{xy}{0.2\times0.1}, \quad N_4 = \frac{(0.2-x)\,y}{0.2\times0.1}$$

$$N_1 = \frac{(0.2-x)(0.1-y)}{0.02}\ ; \quad N_2 = \frac{x(0.1-y)}{0.02}\ ; \quad N_3 = \frac{xy}{0.02}\ ; \quad N_4 = \frac{y(0.2-x)}{0.02}$$

$$\therefore N = \left[\frac{(0.2-x)(0.1-y)}{0.02} \quad \frac{x(0.1-y)}{0.02} \quad \frac{xy}{0.02} \quad \frac{y(0.2-x)}{0.02}\right]$$

$$\therefore N = \frac{1}{0.02}\left[(0.2-x)(0.1-y) \quad x(0.1-y) \quad xy \quad y(0.2-x)\right]$$

Now, $$u(x,y) = [N]\{u_i\} = \left[\frac{(0.2-x)(0.1-y)}{0.02} \quad \frac{x(0.1-y)}{0.02} \quad \frac{xy}{0.02} \quad \frac{y(0.2-x)}{0.02}\right]\begin{Bmatrix}0\\0\\u_3\\0\end{Bmatrix}$$

$$= \frac{xy}{0.02}\times u_3 = \frac{xy}{0.02}\,1\times10^{-3} = \frac{xy}{20}$$

$$v(x,y) = [N]\{v_i\} = \left[\frac{(0.2-x)(0.1-y)}{0.02} \quad \frac{x(0.1-y)}{0.02} \quad \frac{xy}{0.02} \quad \frac{y(0.2-x)}{0.02}\right]\begin{Bmatrix}0\\0\\v_3\\0\end{Bmatrix}$$

$$= \frac{xy}{0.02}\times v_3 = \frac{xy}{0.02}\,2\times10^{-3} = \frac{xy}{10}$$

Now $$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{xy}\end{Bmatrix}_{3\times1} = \begin{bmatrix}\frac{\partial}{\partial x} & 0\\ 0 & \frac{\partial}{\partial y}\\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x}\end{bmatrix}_{3\times2}\begin{Bmatrix}\frac{xy}{20}\\ \frac{xy}{10}\end{Bmatrix}_{2\times1} = \begin{Bmatrix}\frac{y}{20}\\ \frac{x}{10}\\ \frac{x}{20}+\frac{y}{10}\end{Bmatrix}_{3\times1}$$

Now, $$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}_{3\times1} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & 0\\ \nu & 1-\nu & 0\\ 0 & 0 & 1-2\nu\end{bmatrix}_{3\times3}\begin{Bmatrix}\frac{y}{20}\\ \frac{x}{10}\\ \frac{x}{20}+\frac{y}{10}\end{Bmatrix}_{3\times1} = \begin{Bmatrix}\frac{(1-\nu)y}{20}+\frac{x\nu}{10}\\ \frac{y\nu}{20}+\frac{(1-\nu)x}{10}\\ \frac{(1-2\nu)x}{20}+\frac{(1-2\nu)y}{10}\end{Bmatrix}_{3\times1}\times\frac{E}{(1+\nu)(1-2\nu)}$$

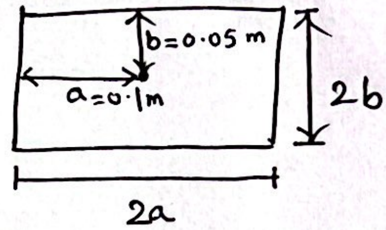
$$= \left\{\cancel{\frac{(1-\nu)y}{20}+\frac{x\nu}{10}} \quad \cancel{\frac{y\nu}{20}}\right.$$

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}_{\text{centroid}} = \begin{Bmatrix} \dfrac{1-0.3}{20}\times 0.05 + \dfrac{0.1\times 0.3}{10} \\[2ex] \dfrac{0.05\times 0.3}{20} + \dfrac{(1-0.3)\,0.1}{10} \\[2ex] \dfrac{(1-2\times0.3)\times0.1}{20} + \dfrac{(1-2\times0.3)\times0.05}{10} \end{Bmatrix} \times \frac{10^{11}}{(1+0.3)(1-0.6)}$$

centroid

$(x=0.1, y=0.05)$

$$= \begin{Bmatrix} \dfrac{19}{4000} \\[2ex] \dfrac{31}{4000} \\[2ex] \dfrac{1}{250} \end{Bmatrix} \times \begin{matrix}1.92\times10^{11} \\ \cancel{3.157\times10^{11}}\end{matrix}$$




$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix} = \cancel{\begin{Bmatrix}1.606\\ 2.767\\ 1.428\end{Bmatrix}\text{GPa}} \quad \begin{Bmatrix}0.912\\ 1.488\\ 0.768\end{Bmatrix}\text{GPa}$$

Ans