Supporting Joins and Numerical Computations over Encrypted Databases

Department of Information Engineering and
Computer Science
University of Trento

Author:

Alex Pellegrini

Supervisors:

Associate Prof. Dr. Bruno Crispo
Dr. Muhammad Rizwan Asghar

Background

- The system considers :
  - A cloud Server (SQL database)
  - Many Clients (users)
  - Key Management Authority

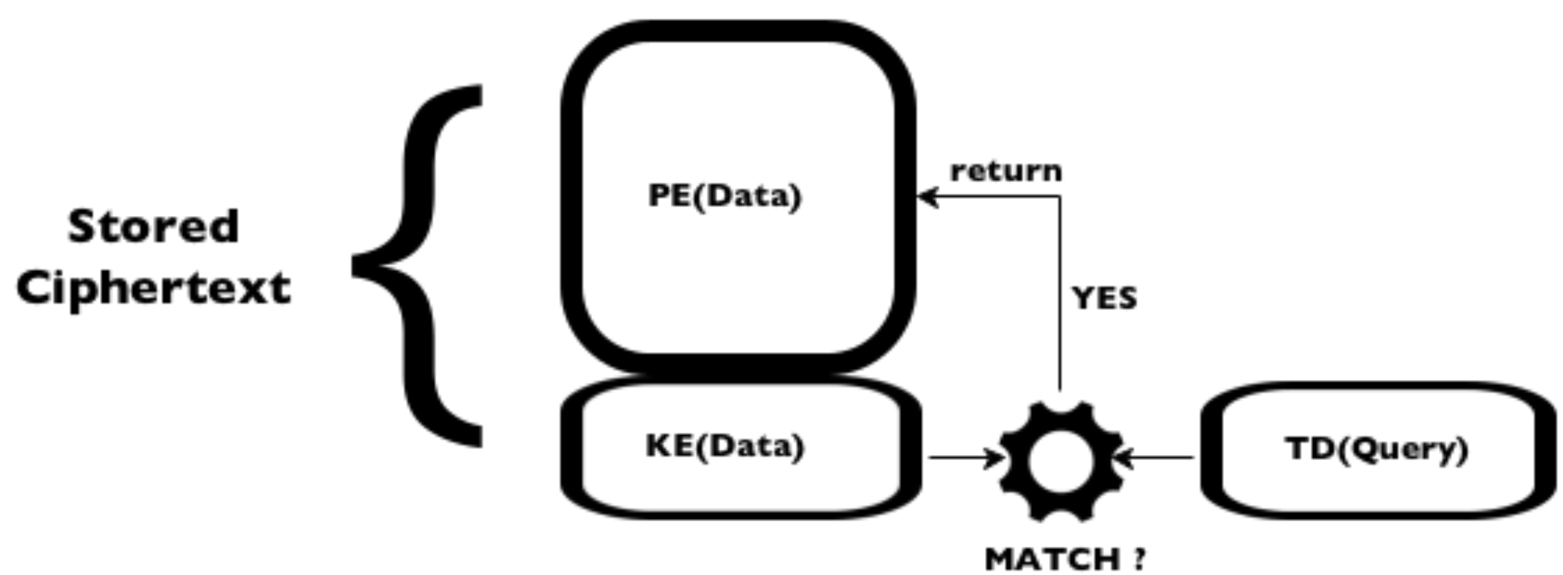
Encryption Schemes

- Discrete Logarithm Problem
- Data Retrieval
  - Proxy Encryption (PE)
- Data Search
  - Keyword Encryption (KE) :: Data searchable
  - Trapdoor Encryption (TD) :: Query (read) Encryption

Encrypted Match

```
function PE[] SEARCH(TD(Q), Data)
    PE[] matching ← new PE[Data.size];
    for all D in Data do:
        if MATCH( TD(Q), KE(D)):
            matching.append(PE(D));
        end if
    end for
    return matching;
end function
```

Match(KE(D), TD(Q)) → {0, 1}



Goals

- Support computations between numerical ciphertexts
- Evaluate range SQL encrypted queries on encrypted databases
- Combine encrypted records to join tables

Numerical Data

- Introduction of `Integer`(s) data type.
- Probabilistic Homomorphic Encryption (HE)
  - Adapted Paillier Cryptosystem
- PE is replaced with HE for numerical data

HE : Encryption

- x : secret key from Z^*_q
  n : product of two large primes
  g : an nth-residue of $Z^*_{n^2}$
  D : numerical value
  r_D : random number
- Compute :
  $$C_{D_1} = \{ c'_1 = g^{r_{D_1}}, c''_1 = g^{x r_{D_1}}(1 + D_1 n) \}$$

HE : Sum

- $C_{D_2} = \{\ c'_2 = g^{r_{D_2}},\ c''_2 = g^{x r_{D_2}}(1 + D_2 n)\ \}$
- Sum (element-wise product):
  $C_D = C_{D_1} C_{D_2} = \{\ c' = c'_1 c'_2,\ c'' = c''_1 c''_2\ \} =$

$$\{$$
$$c' = g^{r_{D_1} + r_{D_2}},$$
$$c'' = g^{x(r_{D_1} + r_{D_2})}(1 + D_1 n + D_2 n + D_1 D_2 n^2)$$
$$\}$$

HE : Decryption

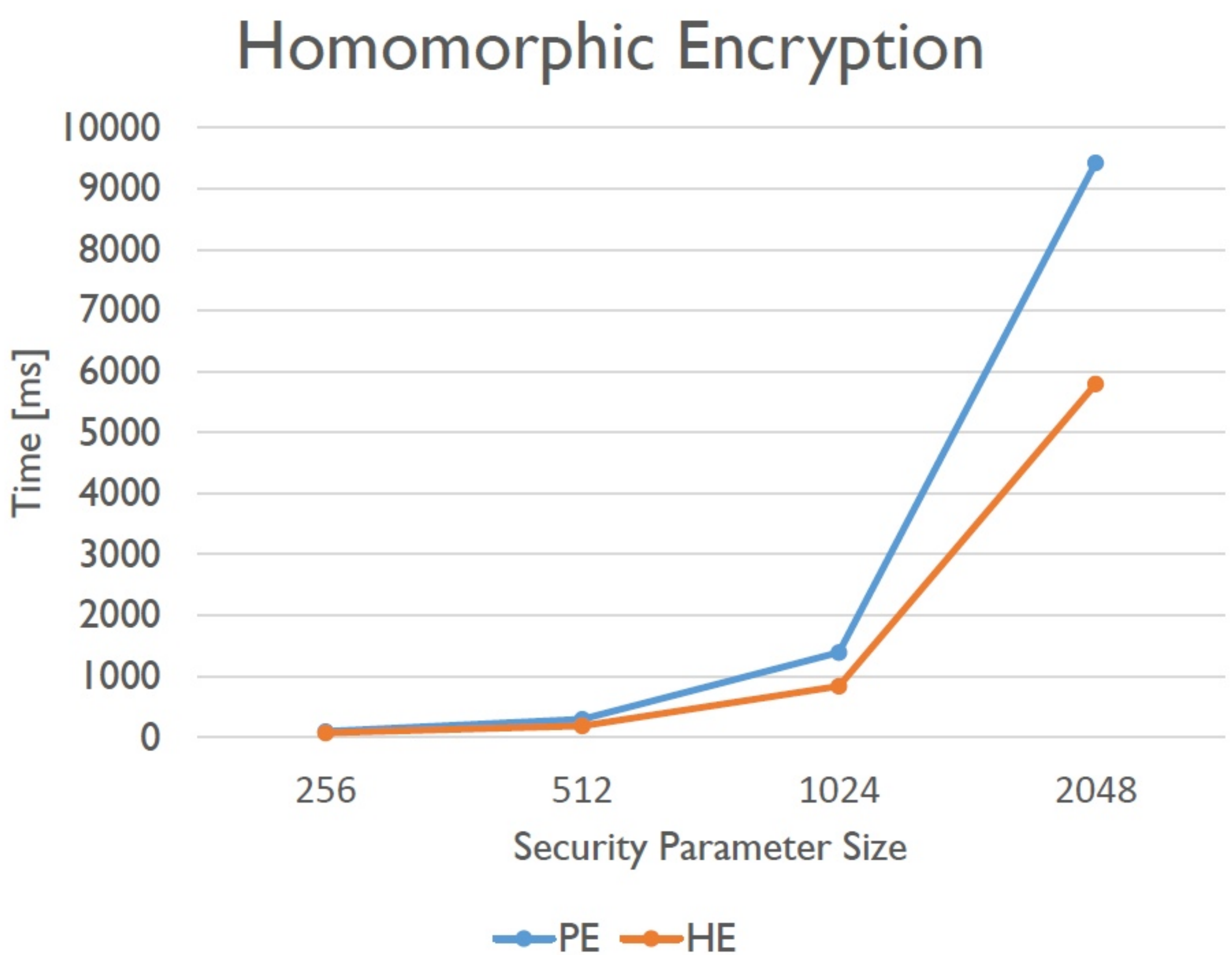
- Decrypt $C_D = \{ c', c'' \}$:

  $\lambda = c''(c')^{-x} = (1 + D_1 n + D_2 n + D_1 D_2 n^2)$

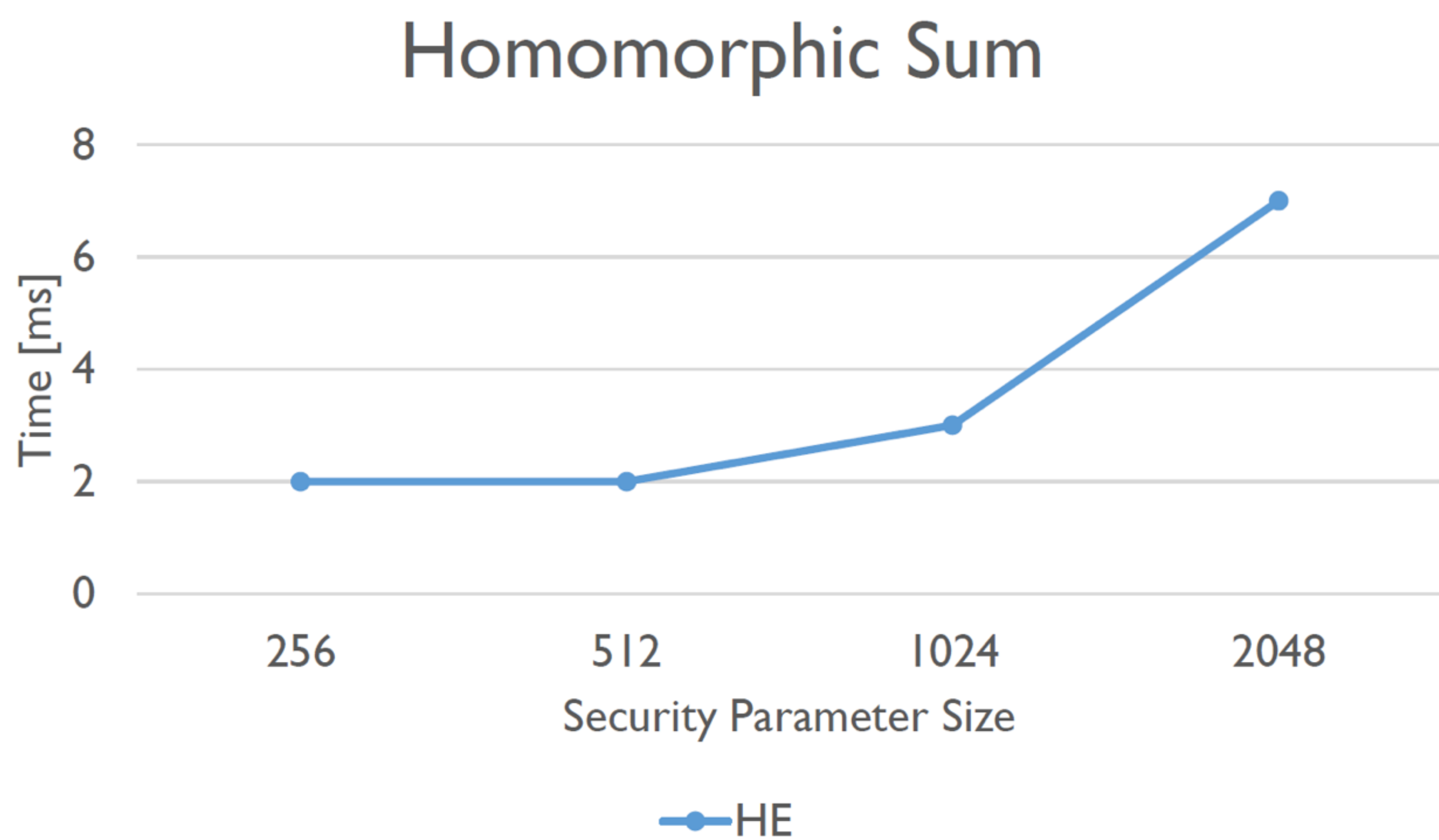
- Compute D :

  $(\lambda - 1) / n \bmod n = D_1 + D_2 = D$

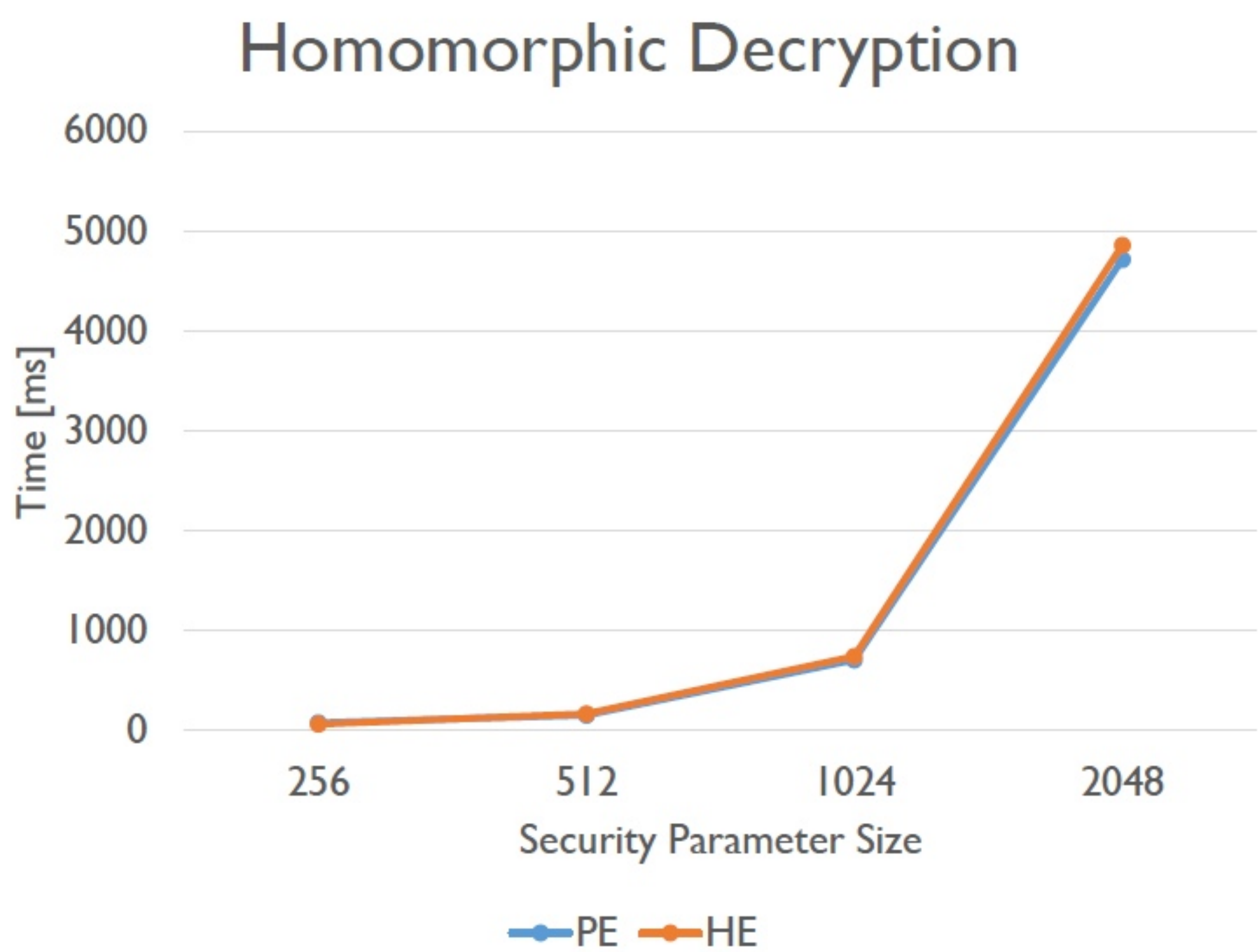
HE : Encryption Performances



HE Sum Performances



HE : Decryption Performances



Range Queries Evaluation

- Requirements :
  - Numerical columns' bit length known a priori
  - A supplementary table related to numerical columns
  - The Bag of Bits approach

Bag of Bits

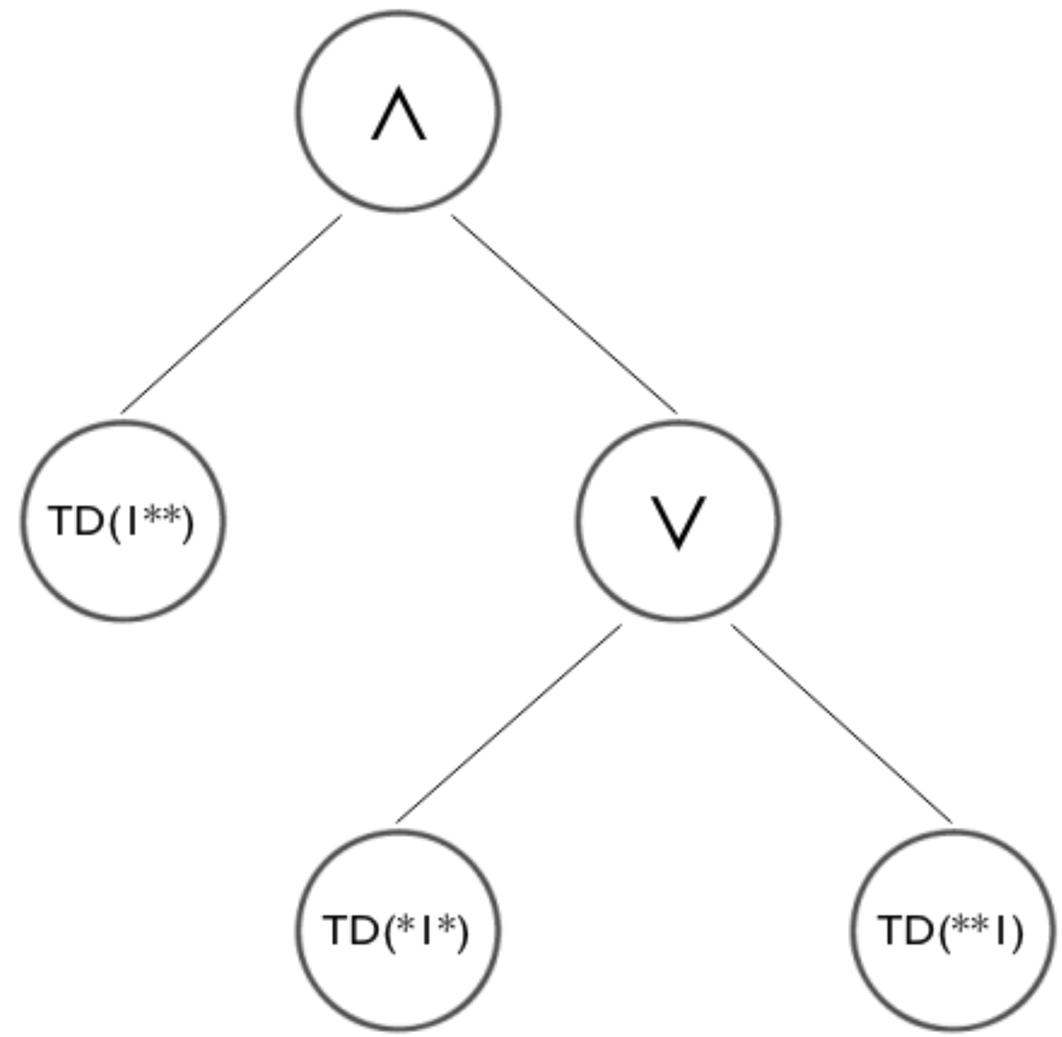
- A set (for each value) made up by strings composed by every bit of the value and filled up with $*$
- $n = 5$, $n_2 = 101$
  $k = 3$
- Bag of Bits for n is then made up by :

$$\{ KE(1{*}{*}), KE({*}0{*}), KE({*}{*}1) \}$$

Policy Tree

- `n > 4;`
- `5 = 101,`
  `6 = 110,`
  `7 = 111`

  `1**` AND (`*1*` OR `**1`)



Join

- Data Stored as PE/HE and KE (probabilistic)
- No way to compare stored data.

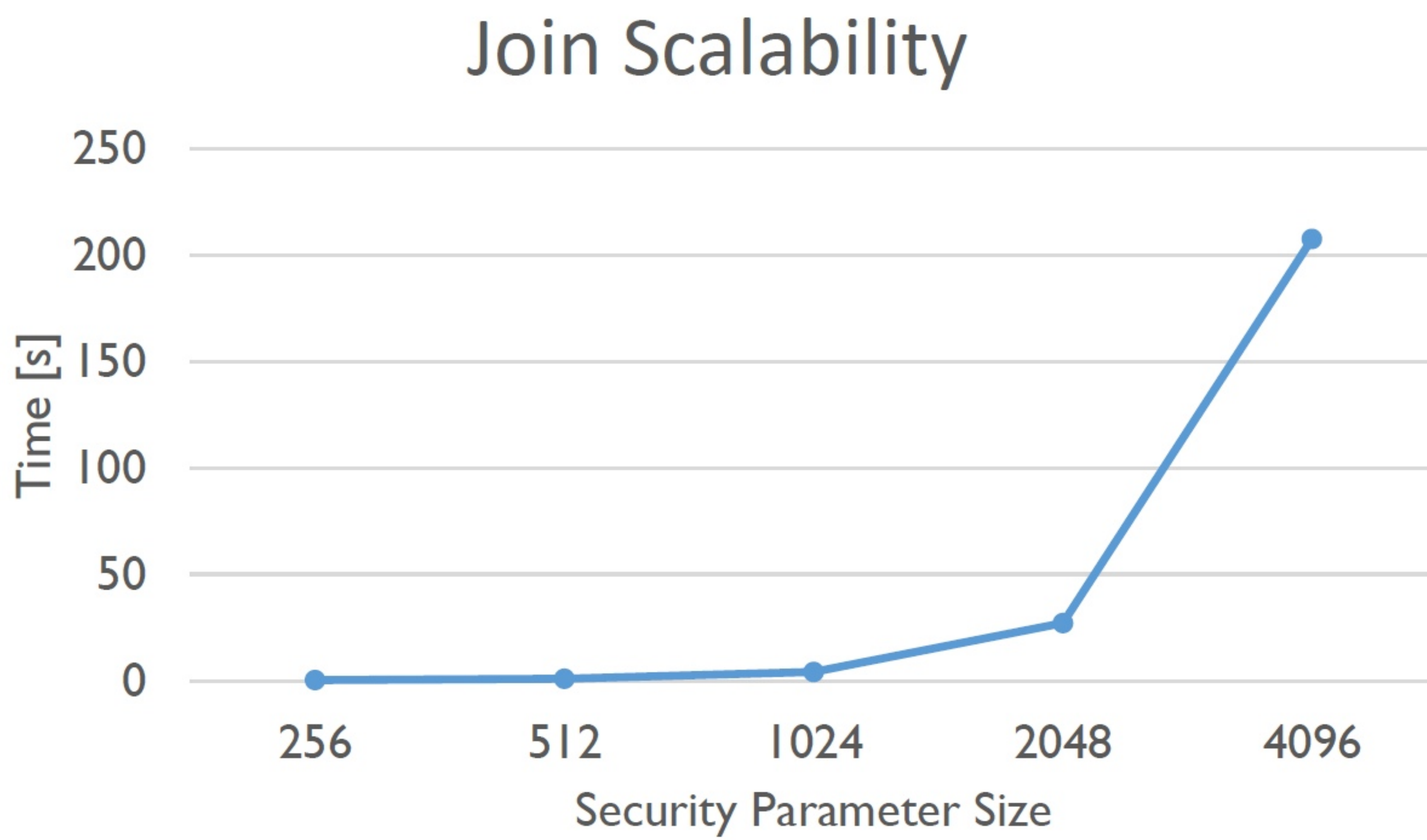
Cross Join

- No constraints on field values
- Combine records of a table with records of other tables (Cartesian Product)
- The DBMS can do the work.

ON Policy

- Joinable columns known a priori
- Store values in joinable columns as PE and TD ciphertexts
    - No need of Match function evaluation
    - String equality comparison

Join Performances



Conclusions

- Relatively fast Paillier Cryptosystem adaption
- Bag of Bits approach for numerical range queries
- Performant Join solution

That’s it !

Thank you for paying attention.

Alex