Auction Econometrics

Empirical Industrial Organization

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- Structural approach: we want to estimate primitives of the auction model, i.e. valuations
- From the observation of the bids $b_1, \cdots, b_n$ (and the auction rules) we want to recover the valuations $v_1, \cdots, v_n$ or equivalently the distribution $F(v)$.
- We can use observations of repeated auctions (assumption of the same bidders)
- When F is estimated, then the following questions can be addressed:
  - Market power of bidders: margin $v - p$
  - Optimal auction format (maximize revenue)
  - Optimal reserve price

Refresher Theory

- Assume there are I bidders
- bidder i has valuation U_i
- U_is are i.i.d. draws from a distribution F with density f
- in a second price auction, the equilibrium bid of bidder i is $b_i = U_i$
- in a first-price auction, the equilibrium bid of bidder i is given by

  $$\beta(U_i) = E\left[U_{-i} | U_{-i} < U_i\right]$$

  where $U_{-i} = \max_{j \neq i} U_j$ is the highest bid by a competitor
- the bid in a first-price auction can also be written as

$$\beta(U_1) = U_1 - \int_0^{U_1} \left(\frac{F(x)}{F(U_1)}\right)^{I-1} dx$$

Refresher Theory

- There is a revenue equivalence between first-price auctions and second-price auctions: both generate the same expected revenue
- Revenue equivalence also holds when there is a reserve price
- The optimal reserve price is given by

$$r - \frac{1 - F(r)}{f(r)} = c$$

where c are the (opportunity) costs of the seller

Estimation of First-Price auction

Distinct empirical approaches to infer distribution F:

1. Laffont, Ossard and Vuong (Econometrica 1995): "Econometrics of First-Price Auctions"
   - Use revenue equivalence theorem ("elegant")
2. Donald and Paarsch (1993):
   - "Brute-force" approach: computationally intensive
3. Guerre, Perrigne and Vuong (Econometrica 2000): Indirect inference approach
   - Very influential methodology, straightforward and relatively simple

Estimation of First-Price auction
Laffont, Ossard and Vuong (1995)

- Observe winning bids only
- Dutch auction (bidders with lower bids never have a chance to bid)
- Idea: revenue equivalence
- By revenue equivalence:

$$E[Winning\ Bid] = E[2nd\ Highest\ Valuation]$$

  Can infer directly the distribution of second highest valuation (second order statistic).
- Need parametric assumption on the distribution of valuation: $f(v|\theta)$, $F(.)$

Estimation of First-Price auction
Laffont, Ossard and Vuong (1995)

- Simulation estimator in practice: for a value of parameter θ and each auction l
  - Prepare S simulations $s = 1 : \cdots , S$
  - Draw $v_1^s, \cdots , v_N^s$, vector of simulated valuations for auction l
  - Sort the draws in ascending order
  - Set $b_l = v_{(2)}$ (2nd highest valuation)
  - Approximate $E(b_l; \theta) = \frac{1}{S} \sum b_l^s$
  - Estimate θ by simulated non linear least squares:

$$\min_{\theta} \frac{1}{L} \sum_{l} (b_l^w - E(b_l^w; \theta))^2$$

- Caveat:
  - Revenue equivalence assumes symmetric bidders (does not work for bidder heterogeneity)
  - Requires a parametric assumption on the distribution function

Estimation of First-Price auction
Direct inference approach

- Donald and Paarsch (1993), and others
- Idea: need to specify the density of observed data (which are bids) to write down likelihood.
- Find inverse bid function:

$$v = b^{-1}(b, \theta)$$

  where θ are parameters of density.
- Plug into distribution

$$F(b^{-1}(b, \theta), \theta)$$

  Distribution of bids:

$$H(b, \theta) = F(b^{-1}(b, \theta), \theta)$$

  with density

$$h(b, \theta) = f(b^{-1}(b, \theta), \theta) \cdot \frac{\partial b^{-1}(b, \theta)}{\partial b}$$

Estimation of First-Price auction
Direct inference approach

- Notice that:

$$\frac{\partial b^{-1}(b,\theta)}{\partial b} = \frac{1}{\frac{\partial b(v)}{\partial v}}$$

which has a simple analytic form for some distributions F.

- Example: Uniform distribution with N bidders: $v \sim U[0,\theta]$

$$\begin{aligned} b(v) &= \frac{n-1}{n} v \\ b'(v) &= \frac{n-1}{n} \end{aligned}$$

- Likelihood:

$$L(\theta) = \prod_{t=1}^{T} \prod_{i=1}^{N} f(b^{-1}(b_i^t,\theta),\theta) \cdot \frac{\partial b^{-1}(b_i^t,\theta)}{\partial b}$$

Estimation of First-Price auction

Direct inference approach

- Caveat: regularity condition of Maximum Likelihood is violated, support of bids depends on θ
- Donald and Paarsch (1993) derive asymptotic distribution of ML estimator.
- Asymmetric bidders, bidder heterogeneity $\Longrightarrow$ Need to numerically solve for the equilibrium (no analytic expression is known).
- Computationally very intensive.

- Guerre, Perrigne and Vuong (Econometrica, 2000)
- Idea: Use best response vis-a-vis the empirical distribution of opponents' bids.
- Observe bids, and thus observe density and distribution of bids.
- Bidder i's problem: win if $b_i \geq b_j$ for all $j \neq i$ Probability of winning is $H(b)^{N-1}$
- A risk neutral bidder will choose a bid that solves

$$\max_b [v - b] H(b)^{N-1}$$

Estimation of First-Price auction

Indirect Inference

- FOC:

$$-H(b)^{N-1} + [v - b](N-1)H(b)^{N-2}H'(b) = 0$$

  or

$$v = b + \frac{H(b)}{(N-1)H'(b)}$$

- We need first to estimate the distribution (and density) of bids $H(b)$

Estimation of First-Price auction
Indirect Inference

- Can estimate $\hat{H}$ consistently using the empirical distribution function

$$\hat{H}(b) = \frac{1}{TN} \sum_t \sum_i \mathbf{1}(b_i^t \leq b)$$

and Kernel estimator for $\hat{H}'(b)$

$$\hat{H}'(b) = \frac{1}{TN} \sum_t \sum_i \frac{1}{h_g} \kappa \left(\frac{b - b_i^t}{h_g} \right)$$

where $\kappa(\cdot)$ is a kernel function (e.g. normal pdf).

- h_g is bandwith parameter (goes to zero as T goes to infinity)
- We can find optimal bandwidth
- Rule of thumb: $h = std(bids) \times (\#observations)^{-1/5}$
- If h_g is zero we get empirical cdf
- Kernel:
  - Epanechnikov: $\kappa(u) = 0.01(1 - u^2)(|u| \leq 1)$
  - Normal:$\kappa(u) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{u^2}{2})$
  - Uniform: $\kappa(u) = \frac{1}{2}|u| \leq 1$
  - Triangular: $\kappa(u) = (1 - |u|)(|u| \leq 1)$

Estimation of First-Price auction
Indirect Inference

- To estimate distribution of v, generate *pseudo-values* $\hat{v}$:

$$\hat{v}_{it} = b_{it} + \frac{\hat{H}(b_{it})}{(N-1)\hat{H}'(b_{it})}$$

and then estimate distribution function

$$\hat{F}(v) = \frac{1}{TN}\sum_t \sum_i \mathbf{1}(\hat{v}_{it} \leq v)$$

and pdf

$$\hat{f}(v) = \frac{1}{TN}\sum_t \sum_i \frac{1}{h_f}\kappa\left(\frac{\hat{v}_{it} - v}{h_f}\right)$$

- With $\hat{F}$ in hand we can:
  - design optimal auction,
  - find optimal reserve price,
  - market design.
- Indirect inference approach extends to:
  - asymmetric bidders,
  - common values,
  - other auction rules.

When is the distribution of private information identified?

- Distribution is identified if for any F_1, F_2 consistent with data it must be that $F_1 = F_2$.
- Problem illustration: Suppose we have many data points (bids) Question: When is true distribution F uniquely determined from the data?
- Note that non-identification may arise in pooling equilibria $\Longrightarrow$ need a separating equilibrium (single crossing).
- Ideally: Examine identification problem prior to estimation.

- From earlier result we know that the inverse bid function

$$v = b + \frac{H(b)}{(N-1)H'(b)}$$

  is strictly monotone.
- Guerre, Perrigne and Vuong (Econometrica, 2000):
  Proposition: Distribution F is identifed if and only if $\frac{H(b)}{H'(b)}$ *is strictly monotone.*
- Does identification result extend to bidder asymmetry in first-price auctions?
- Yes, because the bid function remains strict monotone.
- FOC characterizes v as a residual vis-a-vis the distribution of opponents' bids.

Estimation of First-Price auction

Second-price auction

- Consider the dominant strategy equilibrium $b(v) = v$
- Distribution of bids is identical to the distribution of private values.
- Hence: Distribution F is identified.

Estimation of a Second-Price Auction

- Assumption: Data, $\left((b_i^t)_{i=1}^N \right)_{t=1}^T$ on a cross section of auctions, $t = 1, ..., T$, is available, each auction with
  1. an identical object
  2. fixed number of bidders N
  3. independent observations
- Assumption: bids are generated from the dominant strategy equilibrium in which

$$b_i(v_i) = v_i \quad \text{for all } i$$

- Estimate distribution function F using frequency estimator

$$\hat{F}(v) = \frac{1}{TN} \sum_t \sum_i \mathbf{1}(b_i^t \leq v)$$

- What if bids are not generated from dominant strategy equilibrium?

Estimation of a Second-Price Auction

- When strategic equivalence applies then the earlier results from second-price auction extend.
- Note: English auctions used in practice may not share this strategic equivalence.
- English auction may feature:
  1. Discrete price increases sometimes step-size in the increment is chosen by bidder
  2. Open access: bidders may re-enter later-on, the number of remaining bidders may not be known
  3. Bidding costs bid preparation costs, costs to participating in the auction, etc.
- Ebay: late bidding (e.g. Bajari and Hortacsu, Rand (2003)).
- Haile and Tamer (2003) Incomplete model of an English auction.

Collusion

- Auction is effective price discovery mechanism **under competition**
- Auction very sensitive to collusion
  - Small number of bidders, easily identifiable
  - Often repeated auctions
  - Collusion among bidders can benefit participants at the expense of the seller
- Collusion here is explicit, involves direct communication and side payments

Collusion
Collusive mechanisms

- Second price auction with IPV: ex post efficient collusion by any subset of bidders is possible
  - Implementation with a pre-sale knockout auction
- First price auction with IPV: efficient collusion possible if the ring includes all the bidders
  - E.g. territorial division of bidding privileges
  - Submits many identical bids at the reserve price

- Main difficulty: bidders can submit artificially low bids to hide collusion
- Test for collusion (Porter & Zona 1993):
  - New-York state highway paving jobs
  - They know which bidders were part of the ring
  - Compare the distribution of bids within the two groups
  - Specifically the order of the bids (not the value/magnitude)
  - Exploit the theoretical relation between the order of the bid and cost measures
  - One should expect high cost bidders to place higher bids
  - Cost variables: firm capacity, distance, utilization rate
  - Bids from the cartel do not pass the test while bids outside the cartel pass the test

- Test for collusion (Porter & Zona 1999):
  - School milk procurement process
  - They know which bidders were part of the ring
  - Compare the magnitude of bids near the firm's plant and beyond their local territories.
  - Find that bids further away were not higher than bids for local territory
  - Suggests that bids in the local territory not competitive
  - Consistent with territory allocation
  - Key element here: distance as a cost shifter