# Empirical Industrial Organization

CY Cergy Paris Université
Master in economic analysis and PhD, Economics track.
Problem Set 2
Due: February 28, 2024

## Problem 1

A seller auctions off an object to 3 buyers. Buyer valuations $\epsilon_\ell, \ell = 1, 2, 3$, are distributed i.i.d. with support $[0, 1]$ and distribution function $F(x) = x^2$. Each buyer $\ell$ knows the realization of $\epsilon_\ell$ but only knows the distribution of other buyer's valuations.

1. Determine the distribution function of the maximum valuation among 2 buyers.

The item is sold through a first price sealed bid auction. We look for a symmetric Bayes-Nash equilibrium where all bidders use the same bidding function $\beta$, where a bidder with value $\epsilon$ bids $b = \beta(\epsilon)$. The function $\beta$ is taken to be strictly increasing and differentiable.

2. Write bidder 1's problem and use the corresponding first order condition to determine the equilibrium bidding function $\beta$ (Instruction: you should write all the equations using the distribution function you derived in question 1. Do not use the general formulation that is in the slides.)

3. Assume now that a sealed bid second price auction with reserve price $r$ is used instead. Show that the optimal reserve price is $r = \frac{1}{\sqrt{3}}$.

## Problem 2

There are 2 firms with constant and identical marginal costs $c > 0$, that sell differentiated products for which consumers have unit demands. The utility of consumer $\ell$ from consuming product $i$ at price $p_i$ is $\varepsilon_{\ell i} - p_i + y_\ell$ where $y_\ell$ is income and the $\varepsilon_{\ell i}$ S are i.i.d. uniform on $[0, 1]$. The value of the outside good is assumed to be sufficiently low that all consumers choose to consume one of the products in equilibrium.

We look for a symmetric equilibrium where all firms charge price $p^*$.

1. Write Firm $i$ 's demand as a function of its own price, $p_i$, and the equilibrium price $p^*$ charged by the competitor (Hint: beware that the support of $\varepsilon_{\ell i}$ is from 0 to 1.)

2. Derive the symmetric equilibrium price. (Hint: you need to look for the optimal value of price $p_i$ for some firm $i$ given the other firm is charging $p^*$ and then use the symmetry to replace this optimal $p_i$ by $p^*$.)

3. Redo questions 1 and 2 assuming that the outside option yields utility $y_\ell$.

## Problem 3

Suppose that in the market for laptop computers, only 3 models are available. Model 1 is a high end computer with large memory, a high speed processor and top quality video whereas the other 2 have significantly more modest characteristics. Suppose the utility from buying product $i$ at price $p_i$ is

$$u_{\ell i} = x_i\beta - p_i + \epsilon_{\ell i} + y_\ell, \tag{1}$$

where $x_i$ is a vector of characteristics for product $i$ and $y_\ell$ is consumer $\ell$ 's income. Random terms $\epsilon_{\ell i}$ are i.i.d across products and consumers with distribution function $F(x) = e^{-e^{-x-\gamma}}$, where $\gamma$ is Euler's constant. Total consumer population is assumed to be $L = 1$. There is no outside option so consumers must buy one of the 3 product (market is covered).

1. Write the demand for product $i$. (Hint: do not derive the logit form; just write it out so it is consistent with the specification of utility and the distribution function $F$.)

2. Compute the derivatives of $D_1$ and $D_2$ with respect to $p_3$. Show that they can be written as $P_1P_3$ and $P_2P_3$ respectively (where $P_i$ is the probability that a consumer buys product $i$ ). What can you say about the impact of an increase in $p_3$ on the market shares of products 1 and 2 , assuming that initially, a consumer is equally as likely to buy products 2 and 3 and buys product 1 with probability .4. Do you find this is a reasonable prediction? Explain.

Suppose now that a consumer either has a high income (with probability $\alpha$ ) or a low income and that utility from buying product $i$ is modified as follows :

$$u_{\ell i} = x_i\beta - \rho_\ell p + \epsilon_{\ell i}, \tag{2}$$

where $\rho_\ell = \rho_A$ if $y_\ell$ is high and $\rho_\ell = \rho_B$ if $y_\ell$ is low, with $0 < \rho_A < \rho_B$, and the rest of the specification is unchanged.

3. Interpret the above specification in terms of income effects.

4. Write the probability that a consumer chooses product $i$ conditional on her income: $P_{iA}$ if income is high and $P_{iB}$ if income is low. Write the unconditional probability that a consumer chooses product $i$.

5. To simplify notation, assume now that $x_2 = x_3 = x$ and $p_2 = p_3 = p$ and denote $v_A = x\beta - \rho_A p$ and $v_b = x\beta - \rho_B p$. Show that if $p_1 > p$, then $P_{1A} > P_{1B}, P_{2A} < P_{2B}$ and $P_{3A} < P_{3B}$. Provide an intuitive interpretation for these results. (Hint: if you cannot derive the results you can still explain why they should be true intuitively.)

6. Explain why, provided that $P_{2B} > P_{1B}$, if $\alpha$ is not too large, then we can have $\frac{\partial D_1}{\partial p_3} < \frac{\partial D_2}{\partial p_3}$.