Chapter 8 *Unobserved Heterogeneity in Auctions*

Empirical Industrial Organization

Andras Niedermayer[1]

[1]Department of Economics (THEMA), Cergy Paris Université

Winter 2024

Auctions without Heterogeneity

- Take the Guerre, Perrigne, and Vuong setting for auction econometrics with 2 bidders
- each bidder has valuation $V_i \sim F_i$
- bids are

$$B_1 = \beta_1(V_1)$$
$$B_2 = \beta_2(V_2)$$

Auctions with Observed Heterogeneity

- now assume that for each auction, there is a vector of observed covariates Z (e.g. for the sale of houses: the number of bedrooms, number of square meters, ZIP code)
- each bidders' valuation is $Z'\gamma + V_i$ where V_i is the idiosyncratic valuation and γ are the weights of the covariates, Z and V_i are independent
- it can be shown that the bids B_i are additive in $Z'\gamma$

$$B_1 = \beta_1(Z'\gamma + V_1) = Z'\gamma + \beta_1(V_1) = Z'\gamma + A_1$$
$$B_2 = \beta_2(Z'\gamma + V_2) = Z'\gamma + \beta_2(V_2) = Z'\gamma + A_2$$

- A_i is the "idiosyncratic bid"
- problem: B_1 and B_2 are correlated because of $Z'\gamma$, this violates Guerre, Perrigne, and Vuong's assumptions
- solution: run regression to estimate γ, use residuals $B_i - Z'\gamma$ as estimates of A_i

Auctions with Unobserved Heterogeneity

- assume that there is additionally unobserved heterogeneity Y, observed by market participants but not by the econometrician (e.g. how beautiful the view is from a house)
- Y, A_1, A_2 are independent
- again, Y is additive, observed bids B_i are

$$B_1 = Z'\gamma + Y + A_1$$
$$B_2 = Z'\gamma + Y + A_2$$

- problem: even after correcting for observables $B_1 - Z'\gamma$ and $B_2 - Z'\gamma$ are correlated because of Y
- solution: deconvolution method described by Krasnokutskaya (Identification and Estimation of Auction Models with Unobserved Heterogeneity, REStud, 2011)

Parametric Intuition

- before getting into the non-parametric estimation technique, we give an intuition based on parametric estimation
- assume we only want to know the variance of Y, A_1, and A_2 (e.g. because we assume that they are normally distributed, so that together with the mean, we would know everything about the distribution of Y, A_1, A_2)
- let us ignore $Z'\gamma$ (or assume we removed $Z'\gamma$ by taking residuals)

$$B_1 = Y + A_1$$
$$B_2 = Y + A_2$$

Parametric Intuition

- the covariance of B_1 and B_2 is

$$\begin{aligned}
&\mathrm{Cov}[B_1, B_2] \\
&= \mathrm{Cov}[Y + A_1, Y + A_2] \\
&= \mathrm{Cov}[Y, Y] + \cancel{\mathrm{Cov}[Y, A_1]} + \cancel{\mathrm{Cov}[Y, A_2]} + \cancel{\mathrm{Cov}[A_1, A_2]} \\
&= \mathrm{Var}[Y]
\end{aligned}$$

- the variance of A_1 and A_2 is then

$$\begin{aligned}
\mathrm{Var}[A_1] &= \mathrm{Var}[B_1] - \mathrm{Var}[Y] \\
\mathrm{Var}[A_2] &= \mathrm{Var}[B_2] - \mathrm{Var}[Y]
\end{aligned}$$

Parametric Intuition

Alternative Approach

$$B_1 = Y + A_1$$
$$B_2 = Y + A_2$$

- define a function Θ

$$\Theta(t_1, t_2) = \text{Var}[t_1 B_1 + t_2 B_2]$$

- $\Theta(t_1, t_2)$ can be written as

$$\Theta(t_1, t_2) = (t_1 + t_2)^2 \text{Var}[Y] + t_1^2 \text{Var}[A_1] + t_2^2 \text{Var}[A_2] \qquad (1)$$

- define $\Theta_1(t_1, t_2) = \partial\Theta(t_1, t_2)/\partial t_1$
- (1) can be transformed to

$$\Theta_1\left(0, \frac{1}{2}\right) = \text{Var}[Y]$$

- we get $\text{Var}[A_1]$ and $\text{Var}[A_2]$ as before

Parametric Intuition

$$B_1 = Y + A_1$$
$$B_2 = Y + A_2$$

- to get the mean of Y, A_1, A_2, take expectations

$$E[B_1] = E[Y] + E[A_1]$$
$$E[B_2] = E[Y] + E[A_2]$$

- $\Rightarrow$ 2 equations, 3 unknowns
- $\Rightarrow$ we know the means up to a normalization (increasing $E[Y]$ by a constant and decreasing $E[A_1]$ and $E[A_2]$ by the same constant is observationally equivalent)

- take a random variable $Y \sim G_Y$
- define the characteristic function

$$\Phi_Y(t) = E[\exp(itY)]$$

- property of Φ_Y: $\Phi_Y(0) = 1$, $\Phi'_Y(0) = iE[Y]$, $\Phi''_Y(0) = i^2 E[Y^2]$, $\Phi'''_Y(0) = i^3 E[Y^3]$, etc.
- assuming Y is continuously distributed, there is a one-to-one-mapping between the pdf g_Y and the characteristic function Φ_Y:

$$\Phi_Y(t) = \int_{\mathbb{R}} \exp(ity) g_Y(y) dy$$

$$g_Y(y) = \frac{1}{2\pi} \int_{\mathbb{R}} \exp(-ity) \Phi_Y(t) dt$$

- define the characteristic functions $\Phi_1(t) = E[\exp(itA_1)]$ for $A_1 \sim G_1$ and $\Phi_2(t) = E[\exp(itA_2)]$ for $A_2 \sim G_2$

Non-Parametric Identification

- we observe the joint distribution of B_1 and B_2
- alternatively, we observe the characteristic function of the joint distribution of B_1 and B_2

$$\begin{aligned}\Psi(t_1, t_2) &= E[\exp(it_1 B_1 + it_2 B_2)] \\ &= \Phi_1(t_1)\Phi_2(t_2)\Phi_Y(t_1 + t_2) \qquad (2)\end{aligned}$$

- define $\Psi_1(t_1, t_2) = \partial\Psi(t_1, t_2)/\partial t_1$
- we can solve (2) for Φ_Y:

$$\Phi_Y(t) = \exp\left(\int_0^t \frac{\Psi_1(0, x)}{\Psi(0, x)} dx - itE[A_1]\right)$$

- plugging Φ_Y into (2) we get Φ_1 by setting $t_2 = 0$ and Φ_2 by setting $t_1 = 0$

Non-Parametric Identification

- more explicitly,

$$\Psi(t_1, t_2) = \Phi_1(t_1)\Phi_2(t_2)\Phi_Y(t_1 + t_2)$$

- implies

$$\Phi_1(t) = \frac{\Psi(t, 0)}{\Phi_Y(t)}$$
$$\Phi_2(t) = \frac{\Psi(0, t)}{\Phi_Y(t)}$$

Non-Parametric Estimation

- we have observations $(B_{1j}, B_{2j})_{j=1}^{n}$ from n auctions
- we get the estimate $\hat{\Psi}$ using

$$\Psi(t_1, t_2) = E[\exp(it_1 B_1 + it_2 B_2)]$$
$$\Rightarrow \hat{\Psi}(t_1, t_2) = \frac{1}{n} \sum_{j=1}^{n} \exp(it_1 B_{1j} + it_2 B_{2j}) \qquad (3)$$

- we get the estimate $\hat{\Psi}_1$ using

$$\Psi_1(t_1, t_2) = E[iB_1 \exp(it_1 B_1 + it_2 B_2)]$$
$$\Rightarrow \hat{\Psi}_1(t_1, t_2) = \frac{1}{n} \sum_{j=1}^{n} iB_{1j} \exp(it_1 B_{1j} + it_2 B_{2j}) \qquad (4)$$

Non-Parametric Estimation

- using $\hat{\Psi}$ and $\hat{\Psi}_1$ and the equation from the identification of Φ_Y, Φ_1, Φ_2, we get estimates $\hat{\Phi}_Y$, $\hat{\Phi}_1$, $\hat{\Phi}_2$
- we get $\hat{g}_1$ using the one-to-one mapping between g_1 and Φ_1
- we get $\hat{g}_2$ analogously
- we get $\hat{G}_1$ and $\hat{G}_2$ by integrating $\hat{g}_1$ and $\hat{g}_2$
- using $\hat{g}_1$, $\hat{g}_2$, $\hat{G}_1$, $\hat{G}_2$ we can use the method from Guerre, Perrigne, Vuong to estimate the inverse bidding functions β_1^{-1} and β_2^{-1}
- β_1^{-1} and β_2^{-1} give us the pseudo-valuations for every possible idiosyncratic bid
- $\Rightarrow$ the distributions of the idiosyncratic bids $A_1 \sim G_1$ and $A_2 \sim G_2$ give us the distributions of the idiosyncratic valuations $V_1 \sim F_1$ and $V_2 \sim F_2$