Foreclosure Auctions

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Abstract

We develop a novel theory of real estate foreclosure auctions, which have the special feature that the lender acts as a seller for low and as a buyer for high prices. The theory yields several empirically testable predictions concerning the strategic behavior of the agents, both under symmetric and asymmetric information. Using novel data from Palm Beach County (FL, US), we find evidence of both strategic bidding and asymmetric information, with the lender being the informed party. Moreover, the data are consistent with mortgage securitization dissipating the original lender's information and leading to less aggressive bidding in the auction.

Keywords: foreclosure auctions, common value component, securitization

JEL Codes: to be added

1 Introduction

Foreclosure auctions of real estate have a substantial economic impact. In 2013, 609,00 residential sales in the U.S. were foreclosure related (foreclosure auctions or sales of real estate owned by a lender), amounting to 10.3% of total residential sales.[1] Foreclosures were one of the major concerns during the economic crisis, with many home owners losing their property because of the drop of real estate prices.

[1] See `http://www.realtytrac.com/Content/foreclosure-market-report/december-and-year-end-2013-us-residential-and-foreclosure-sales-report-7967`.

We develop a novel theory of foreclosure auctions, incorporating the institutional specificities of foreclosures. A foreclosure auction is run by a government agency (in our data set the Clerk and Comptroller's Office of Palm Beach County, FL) after a mortgagee stopped making payments to the lender. The lender, typically a bank, and third-party bidders, typically real estate brokers, participate in this auction. Payments up to the amount owed (the judgment amount) are paid to the lender, payments above the judgment value, if any, are paid to the owner of the property. The owner typically does not participate. In such an auction, the bank essentially acts as a seller below the judgment amount, and as a buyer above the judgment amount.

An important feature of foreclosure auctions is that the bank is likely the better informed party. There are several reasons for this. First, the bank assesses the value of the property when it grants the mortgage. Second, the value of the property is greatly affected by how much the owner invested in maintenance. The bank typically has more information concerning such maintenance investments: the bank can e.g. observe whether the owner was in financial distress for a longer period of time and hence neglected maintenance. Third, a major problem with foreclosed property is that the original owner may abandon the house, take with them everything they can get their hands on, and sometimes even actively damage the property. The bank's previous interactions with the owner may provide it with better information about the likelihood of this behavior. The law, however, prevents other bidders (e.g. real estate brokers) to enter and inspect the property. This further enhances the informational advantage of the bank.[2]

The broker's expected payoff depend both of the bank's information and on their own idiosyncratic components (signals). The latter may reflect their own value added to the property.[3] This value added may be largely independent of the baseline value of the house, so we assume that the broker and bank signals are independent. The relevance of the bank's information to

[2]This legal requirement is meant to protect the borrower, who may still live in the property during the foreclosure auction. An exception is that in some mortgage contracts there are provisions which allow the lender's representative to enter a vacant property to make repairs and to provide regular maintenance such as turning utilities on and off (see `http://www.nolo.com/legal-encyclopedia/deceptive-foreclosure-practices-when-banks-treat-occupied-homes-vacant.html`). For such mortgage contract, this gives an additional informational advantage to the lender.

[3]When visiting websites of several brokers participating in the foreclosure auctions in our dataset, we could see that they often specialize in renovating foreclosed properties.

the broker creates a common value component in the auction. However, unlike the standard common value model where buyers have relevant information, here is it assumed that only the bank is the informed party.

We derive participants' equilibrium bidding strategies in a foreclosure auction with a common value component, which, as we have argued, should be relevant in foreclosure auctions. However, it is also interesting to consider the symmetric information environment, where the bank's information is known to the brokers. This independent private value (IPV) setup is nested within our general model. Under IPV, brokers bid their own type as in a standard English auction. The bank bids its valuation plus a markup for low valuations, just as a seller would bid. The bank's bid takes into account the classical trade-off of a seller: a higher bid lowers the bank's probability of selling the house, but increases the bank's revenue in case of sale. However, one side of this trade-off disappears for banks that bid exactly the judgment amount (v_J): a higher bid still lowers the probability of sale, but does not lead to higher revenues for the bank, since the additional revenue goes to the original owner. This change of the trade-off causes a bunching of banks' bids at the v_J. If the bank's valuation is sufficiently high – higher than v_J- the bank does not want to sell the property, but would want to keep it itself. Hence, for high valuations, the bank bids its own type, just as a buyer would do in an English auction.

Under asymmetric information, the bank's bid also serves as a signal about the bank's information about the quality of the house. For values sufficiently below v_J, when the bank effectively acts as a seller, the bank's bid involves a markup over its value. But we also show that on top of this markup, it involves a *signalling premium* that makes the bid higher than what it would have been under symmetric information. The intuition for the signalling premium is that, under asymmetric information, the bank should have a lower probability of sale given any bid in order to make the bid incentive compatible. Under asymmetric information, the bank would have an additional incentive to deviate from equilibrium by bidding higher, as such a bid would signal a higher quality to the brokers and will, other things equal, lead to a higher probability of sale, which is beneficial to the bank. To compensate for this, the equilibrium price should be higher under asymmetric information.

The bunching result from the symmetric information setup carries over to the asymmetric information one. However, bunching has a surprising implication in the presence of a common value component: there cannot be an equilibrium where the bank's biding strategy is continuous around the bunch. The reason for this is that brokers who are supposed to drop slightly below v_J will instead prefer to wait for the price to rise above v_J, as they would then be certain to obtain a significantly higher house quality at a slightly higher price. Since brokers will not have an incentive to drop out at prices slightly below v_J, neither will have the bank: increasing the bid to v_J will lead to a higher revenue for the bank, while not changing the probability of selling. This logic implies that there has to be a discontinuity in the bank's bidding strategy below v_J. We show that the bank's equilibrium bidding strategy is strictly increasing and continuous for low valuations, exhibits a discontinuous jump to v_J and bunching at v_J, and is equal to the bidding strategy of a buyer for valuations above v_J.

Our theory has two striking empirically testable implications. First, there is bunching of banks' bids at the judgment amount v_J. Second, in the presence of both auctions with symmetric and asymmetric information, one should expect to observe the following. For values slightly below v_J, the probability of the bank selling the house to a third-party bidder first increases with the bank's bid and then drops down discontinuously at v_J. This is due to the following selection effect. For prices slightly below v_J, the symmetric information auctions are selected into the observable sample. This is because there is a *gap* in the bank's bids below v_J under asymmetric information. For prices at and slightly above v_J, both symmetric and asymmetric information auctions are selected. As the probability of sale ("demand") is lower under asymmetric information due to the signalling premium, the observable demand will exhibit a downward jump at v_J. Further, for bank's bids just below v_J, the probability of sale increases with the bank's bid. The reason is that the closer the bank's bid approaches v_J from below, the more one gets a selection into symmetric information auctions. This selection leads to a higher observed probability of sale, because of the aforementioned signalling premium in the asymmetric information auctions. This *demand discontinuity* and *non-monotonicity* due to the selection effect are testable predictions in the environment where both symmetric and asymmetric information auctions are present.

We have collected a novel data set with foreclosure auctions from Palm Beach County in Florida. We have data on 12,788 auctions from 2010 to 2013 with the total judgment amount

\$4.2bn and the total sale amount (the sum of the winning bids) \$0.5bn. The data reveals that bunching indeed occurs at the judgment amount. We also observe that the probability of sale increases with the bank's bid just below the judgment amount. Further, demand exhibits a downward discontinuity at the judgment amount. This points, as we have argued, to the presence of auctions with both symmetric and asymmetric information in our dataset.

Further empirical support for asymmetric information is provided by considering securitized and non-securitized mortgages. The securitization of mortgages was a major concern during the financial crisis. Originating banks securitized their mortgages through securitization agencies (mostly the Government Sponsored Enterprises Freddie Mac and Fannie Mae), the securitized assets were then sold on the capital market. There is a wide held suspicion that moral hazard played an important role during the financial crisis: the suspicion is that banks did not exert sufficient effort in gathering information before granting mortgages and hence excessively granted mortgages and securitized them, shifting the burden to the holders of securitized assets and Government Sponsored Entities (and ultimately to the taxpayer).

Different levels of information gathering for securitized and non-securitized mortgages have empirically testable implications for foreclosure auctions: a bank holding a non-securitized mortgage is expected to be better informed about the value of the property than a bank holding a securitized mortgage. This is for two reasons. First, the bank is more likely to have gathered information about the loan-to-value ratio of a property for non-securitized mortgages. Second, the bank's estimate of the probability of default of a borrower is informative about the value of the property at the time of the foreclosure (see **?**). A reason for the relation between the probability of default and the value of the property is that borrowers in financial distress are less likely to invest in the maintenance of their property.[4]

One would therefore expect the bank to have less private information about the quality of the house for securitized mortgages than for non-securitized mortgages. This implies that the non-monotonicity and the discontinuity of the probability of sale as a function of the bank's bid should be present for non-securitized mortgages, but these both features are less saleint,

[4]Causality can, of course, also go in the opposite direction: home owners with a higher loan-to-value ratio have more of an incentive to default. For our purposes, it is immaterial in which direction the causality goes. In either case, superior information about the probability of default also leads to superior information about the value of the property.

or not even statistically discernible for securitized mortgages. An analysis of the data reveals that this is indeed the case.

Next, we consider a version of our model where the broker's valuation depends on the bank's information *linearly*, with coefficient α measuring the degree of bank's informativeness. A higher α corresponds to more precise information concerning the common value component. We show that the bank will bid more aggresively if α is higher. This is *both* because of the direct effect on the broker values, and the indirect effect due to the higher incentive to signal the favourable information to the brokers. In order to be able to estimate the bank's bidding strategy, we collected additional data on the resale prices and the tax assessment values of foreclosed properties. Using the resale price and the assessment value as two independent noisy signals for banks' valuations, we constructed the distribution of banks' valuations for securitized and non-securitized mortgages. Matching the quantiles of banks' valuation distributions with banks' bid distributions gives us the (average) bidding function. Having an estimate of the bidding function for both securitized and non-securitized mortgages, we show that for a given valuation, a bank holding a non-securitized mortgage bids more than a bank holding a securitized mortgage. This is consistent with asymmetric information being more salient for non-securitized mortgages.

Related Literature. To the best of our knowledge, this is the first paper providing an economic analysis of the bidding behavior in foreclosure auctions. So far, the details of foreclosure auctions have been mainly analyzed in the law literature (see e.g. **?**).

For our theoretical analysis, we build on two strains of literature. The first analyzes bidding behavior of buyers with information about a common value component, starting with **?**. The second, more recent, literature analyzes auctions in which the seller has superior information, see **?**, **?**, **?**. For bids above the judgment amount, we can use results from the former; for bids sufficiently below the judgment amount, we can use results from the latter literature, in particular from **?**. Our theoretical contribution is to show that there are surprising predictions for the intermediate range of bids, which are between the informed seller and the informed buyer regions.

Evidence of strategic behavior and asymmetric information has long been a focus of the empirical auction literature, beginning with **?** and **?** for offshore oil auctions, and **?** and **?** for treasury auctions. **?** find evidence of dealers' informational advantage in Canadian Treasury auctions.

The application of our theory to securitization relates to the growing literature that has been sparked by the interest in the determinants of the financial crisis, such as **?**, **?**, **?**. Our analysis confirms the suspicion voiced in this literature that securitization led to moral hazard. Our analysis can be seen as complementary to the analysis in **?**, who provide evidence for asymmetric information with respect to the probability of default. Our analysis shows existence of asymmetric information with respect to the other determinant of the expected shortfall of a mortgage: the loss given default (which is determined by the value of the house). In a wider sense, our article relates to the growing literature that uses insights from auctions to analyze important questions in financial markets, such as **?**, **?**, **?**, **?**.

2 Foreclosure process

Property foreclosure is a remedy allowed by law to the lender if the borrower defaults on the mortgage. While it generally transfers the ownership of the property to the lender, the process may be a lengthy one and the details depend on the jurisdiction. In the US, roughy one third of the states adopt what is called a *judicial* foreclosure, while the rest is comprised of the *nonjudicial* foreclosure states. See Table **??**.

The main legal difference between judicial and nonjudicial foreclosure is that in the former, the process takes place in the court system, while in the latter, it takes place outside the courts. Judicial foreclosures always result in the property being sold at a public auction, while the nonjudicial foreclosure usually involves a sale of the property by the trustee under the power of sale clause. The trustee sale may or may not be conducted through auction, but still have to generally comply with various state laws, even though it is performed outside of the court system.[5]

In this paper, we focus on judicial foreclosures. In our empirical application, we study foreclosure auctions in a judicial state (Florida). The judicial foreclosure process generally

[5]A handful of states (Connecticut, Maine and Vermont) allow for a *strict foreclosure*, where the property title is transferred to the lender immediately after the default.

Judicial states	Nonjudicial states
Connecticut	Alabama
Delaware	Alaska
Florida	Arizona
Hawaii	Arkanzas
Illinois	California
Indiana	Colorado
Iowa	District of Columbia
Kansas	Georgia
Kentucky	Idaho
Louisiana	Maryland
Maine	Massachusetts
New Jersey	Michigan
New Mexico	Minnesota
New York	Mississippi
North Dakota	Missouri
Ohio	Montana
Oklahoma	Nebraska
Pennsylvania	Nevada
South Carolina	New Hampshire
South Dakota	New Mexico
Vermont	North Carolina
Wisconsin	Oklahoma
	Oregon
	Rhode Island
	South Dakota
	Tenessee
	Texas
	Utah
	Vermont
	Virginia
	Washington
	West Virginia
	Wyoming

Table 1: Judicial and nonjudicial foreclosure states. Some states use both types of foreclosure and are listed in both columns.

consists of the following steps.

1. The mortgage holder misses several payments on the mortgage. In some states, the lender is allowed to start the foreclosure process after the borrower has missed one payment. In practice, however, the process usually begins after three payments have been missed.

2. The lender informs the mortgage holder, usually through a letter of intent with a specified deadline, that it intends to begin the foreclosure process. This notice of intent, or "breach letter", allows the borrower to make up the missed payments before the deadline. The deadline is usually set at 30 days.

3. If the mortgage is still delinquent, the lender proceeds to file a lawsuit, usually in the county where the property is located. The complaint is served to the mortgagee, usually by the county sheriff. At this point, the mortgagee becomes a defendant in the foreclosure lawsuit.

4. The defendant is given a certain amount of time, usually 20 - 30 days, to respond. The defendant does not have to respond. The defendant may choose to respond if he or she is able to raise a substantive complaint, e.g. concerning the ownership of the mortgage or unfair lending practices.

5. If the complaint goes uncontested or the defendant does not raise a substantive complaint, the lender is granted a judgement of foreclosure. The judgement will specify the date of the foreclosure sale. The sale is conducted through a public auction. The bank is allowed to credit bid up to the debt amount (also called the judgement amount), while other participants (usually real estate brokers) are required to submit bids in cash or cash equivalent. The auction is usually conducted in an open format. Some counties have adopted Internet auctions.[6] The property title is awarded to the high bidder, and the auction price is equal to the highest bid. If the lender is *not* the high bidder, the lender receives the payment equal to the minimum of the auction price and the judgement value. If the sale price exceeds the judgement value, the surplus is used to satisfy the junior liens, if any. The remaining surplus is paid to the defendant.

In the next section, we present a stylized model of a foreclosure auction that captures the main institutional features described above.

[6]This is the case for Palm Beach county in Florida, the source of the data for the empirical application in our paper.

3 Model

Consider the owner, the bank (also called the seller, S) and n real estate brokers (or buyers, B) who participate in the foreclosure auction. The judgment amount (i.e. the balance of the mortgage) is denoted as v_J. The foreclosure auction is modelled as variant of the (button) English auction as in **?**. The auction website expedites bidding by allowing the participants to employ automatic bidding agents. The bidders provide their agents the maximum price they are willing to pay (their *dropout* price), and the agent then bids on their behalf. These proxy bids can be updated at any time.[7] The brokers are able to observe if the bank's maximum bid has been exceeded, while a broker's dropout price is unobservable to the bank. The broker may therefore change its maximum bid upon observing the bank's dropout price. As for the bank, we assume that it commits to its dropout price.

The key difference from a standard auction is that the proceeding of the foreclosure auction up to the judgment amount goes to the bank, anything above the judgment amount goes to the original owner. This effectively turns the bank into a seller for prices below and into a buyer for prices above the judgment amount.

The winner pays the auction price p. If the price exceeds the judgment amount, $p \geq v_J$, then the bank gets v_J and the owner pockets the difference $p - v_J$. If the price is below the judgment amount, $p < v_J$, then the banks gets p and the owner gets nothing. The property is transferred to a broker only if a broker wins; otherwise, the bank keeps the property. It follow that if the bank wins the auction, it effectively pays the auction price to itself, so in reality no money changes hands in this case. But if a broker wins, then there is an actual money transfer, from the broker to the bank and possibly the owner as well (if the auction price exceeds the judgment amount).

As is usual, we model the foreclosure sale (auction) as a game of incomplete information. As is explained in the empirical section of the paper, banks and brokers buy houses for different purposes: banks mostly sell the houses later on. Brokers typically renovate the property before reselling. Motivated by this, we make the following assumption concerning the information of the bank and the brokers. First, we assume that the ith broker's idiosyncratic signal, denoted

[7] Such proxy bidding makes the button model even more applicable here, as it alleviates the need to model difficult features such as e.g. jump bidding that may be present in the traditional open auctions.

as X_B^i, only concerns its renovation value added to the house. Second, we assume that the bank's signal X_S concerns the baseline resale value of the house. The signals X_B^i and X_S will be sometimes referred to as buyers' and seller's *types*. Their realizations will be denoted as x_B^i and x_S, respectively.

The bank is assumed to be the informed party. Its signal X_S is normalized to equal the expected value of the house in the market, so the bank's valuation is

$$u_S(x_S) = x_S.$$

The brokers do not observe X_S; they only privately observe their own signals X_B^i. Broker i's expected value of the house, given its own signal x_B^i and the bank's signal x_S, is denoted as $u_B(x_B^i, x_S)$.

We make the following assumptions concerning the expected valuations of the brokers.

Assumption 1 (broker valuations)**.** *A broker's expected valuation is differentiable and strictly increasing in own signal x_B, and nondecreasing in the bank's signal x_S,*

$$\frac{\partial u_B(x_B, x_S)}{\partial x_B} \geq \alpha, \quad \frac{\partial u_B(x_B, x_S)}{\partial x_S} \geq 0,$$

for some constant $\alpha > 0$. Moreover, the derivatives satisfy the single crossing *condition*

$$\frac{\partial u_B(x_B, x_S)}{\partial x_B} > \frac{\partial u_B(x_B, x_S)}{\partial x_S}$$

This assumption ensures that a broker's valuation of the house is increasing in its own signal x_B, and is non-decreasing in the bank's signal x_S. If u_B does not depend on x_S, we have a special case of *private values*. Otherwise, the valuations are interdependent.

For reasons that will be clear in the sequel, we normalize the broker signals so that the value conditional on winning the auction is equal to the signal,

$$u_B(x_B, x_B) = x_B. \tag{1}$$

This normalization is without loss of generality because Assumption 1 ensures that $u_B(x_B, x_B)$ is continuous and strictly increasing in x_B.

We make the following assumptions regarding the distribution of the signals.

Assumption 2 (Signals). *The bank's signal X_S is drawn from a distribution F_S supported on $\mathbb{R}_+$, with density f_S continuous and positive on the support. The broker signals X_B^i, $i = 1, ..., n$, are identically and independently distributed and drawn from the distribution F_B, supported on $\mathbb{R}_+$, with density f_B continuous and positive on the support.*

The independence assumption is made to simplify the analysis of the game, by eliminating the need to consider adjustments that brokers would otherwise make to their proxy bids following dropouts by other brokers. Under independence, we shall see that the information in the auction will be transmitted only from the bank to the brokers, following the bank's dropout from the auction. After that, the brokers would essentially have independent private values, and simply enter those values as their (updated) proxy bids, and there will be no updating from brokers' dropout prices.[8]

The *Myerson virtual value* is defined in the present setting as

$$J_B(x_B, x_S) = u_B(x_B, x_S) - \frac{\partial u_B(x_B, x_S)}{\partial x_B} \frac{1 - F_B(x_B)}{f_B(x_B)}. \tag{2}$$

We make the standard monotonicity assumption concerning $J_B(x_B, x_S)$.

Assumption 3 (Virtual value monotonicity). *The function $J_B(x_B, x_S)$ is strictly increasing in x_B.*

In the following, we will use $x_{(1)}$ and $x_{(2)}$ for the highest and second highest order statistics among n brokers' signals. Further, we will use $F_{(1)}$ and $F_{(2)}$ for the corresponding distributions.

We have to reasons to focus on a model in which the seller's superior information about the quality of the property being auctioned is at the center of interest. First, the bank is likely to have better information due to having gathered information about the property when granting the mortgage. Information about the mortgagee can also serve as an indication of how well the mortgagee maintains the house. Second, we will later consider securitized and non-securitized mortgages. As it will become clearer later on, there are reasons to believe that the informativeness of the bank's signal about the quality of the house is different for securitized and non-securitized mortgages.

[8]Independence is also assumed in **?**, while **?** and **?** allow the buyer signals to be correlated, but still independent of the seller's signal.

4 Symmetric information

It is useful to first start with the case of *symmetric information*, where the bank's information is known to the brokers. This independent private values (IPV) setup is particularly useful to highlight the role of the judgment amount, below which the bank acts as a seller and above which the bank acts as a buyer.

Standard arguments imply that it is a weakly dominant strategy for the broker to choose its valuation as the drop out price, so

$$p_B(x_B) = x_B.$$

The bank's bidding behavior can be best derived by first considering two hypothetical cases. First, nothing is owed to the bank ($v_J = 0$) and hence the bank always acts as a buyer. Second, an infinite amount is owed to the bank ($v_J = \infty$) and hence the bank always acts as a seller. After deriving these two hypothetical cases, we can put the two pieces together and additionally derive the bank's bidding behavior for the transitional region in which the bank turn from a seller to a buyer.

First, consider the case in which the bank always acts as a buyer ($v_J = 0$). By standard arguments for English auctions, the bank's optimal strategy is to bid its own type, i.e. $p_S(x_S) = x_S$.

Next, consider the case when $v_J = \infty$, i.e. the standard auction where the bank acts as the seller. The brokers will drop out at prices $u_B(x_B^i, x_S)$. If the bank decides to drop out at price p, its auction revenue will be equal to p if there is one, and only one broker that is active in the auction at that price. If there are multiple brokers active at p, the bank's revenue will be equal to the second-highest dropout broker price, i.e. $u_B(x_{(2)}, x_S)$. Denote as $\hat{x}_B$ the broker's type indifferent between staying in or dropping out at p, so that $p = u_B(\hat{x}_B, x_S)$. Then the bank's expected profit is equal to

$$\Pi_S(x_S, \hat{x}_B) = u_B(\hat{x}_B, x_S) n F_B(\hat{x}_B)^{n-1}(1 - F_B(\hat{x}_B)) + \int_{\hat{x}_B}^{\infty} u_B(y, x_S) f_{(1)}(y) dy + x_S F_B(\hat{x}_B)^n.$$

(If there is only one broker, $n = 1$, then the integral term should be excluded from the above formula.) The bank will choose the price, or, equivalently, the broker's marginal cutoff $\hat{x}_B$, optimally. The derivative of $\Pi(x_S, , \hat{x}_B)$ with respect to $\hat{x}_B$ is equal to

$$\frac{\partial \Pi(x_S, \hat{x}_B)}{\partial \hat{x}_B} = -n f_B(\hat{x}_B) F_B(\hat{x}_B)^{n-1} \Big(J_B(\hat{x}_B, x_S) - x_S \Big)$$

for any $n \geq 1$. Assumption 3 implies that $\Pi(\hat{x}_B, x_S)$ is *quasiconcave* in $\hat{x}_B$, and is uniquely maximized at $x_B^*(x_S)$ that satisfies the FOC

$$J_B(x_B^*(x_S), x_S) = x_S. \tag{3}$$

This is, of course, a well-known result in auction theory concerning the optimal reserve price, adapted to the setting where the buyer's valuations depend on the seller's information x_S, observable to the buyers.[9] In general, the bank's optimal strategy

$$p_S^0(x_S) = u_B(x_B^*(x_S), x_S) \tag{4}$$

may be non-monotone in x_S. Totally differentiating (22) with respect to x_S yields

$$\frac{dx_B^*(x_S)}{dx_S} = \frac{1 - \partial J_B / \partial x_S}{\partial J_B / \partial x_B}$$

So a sufficient condition for the monotonicity is

$$\frac{\partial J_B(x_B, x_S)}{\partial x_S} < 1 \tag{5}$$

It can be checked that (5) is implied by the stronger but more intuitive conditions

$$\frac{\partial u_B(x_B, x_S)}{\partial x_S} < 1, \qquad \frac{\partial^2 u_B(x_B, x_S)}{\partial x_B \partial x_S} \geq 0. \tag{6}$$

Now consider the case of our primary interest, $0 < v_J < \infty$. If the bank has valuation $x_S \leq v_J$, it will not drop out at the the price above v_J: the bank is not entitled to any revenue in excess of v_J, so staying in the auction beyond v_J will only reduce the probability of sale. So effectively, the bank's problem is to maximize its profit with the additional constraint $p \leq v_J$, or, equivalently, $u_B(\hat{x}_B, x_S) \leq v_J$. As we have seen, the bank's profit is quasiconcave in $\hat{x}_B$, so the optimal price is given by

$$p_S^0(x_S) = \begin{cases} u_B(x_B^*(x_S), x_S) & \text{if } x_S < \underline{x}_S, \\ v_J & \text{otherwise,} \end{cases}$$

[9]See **?**.

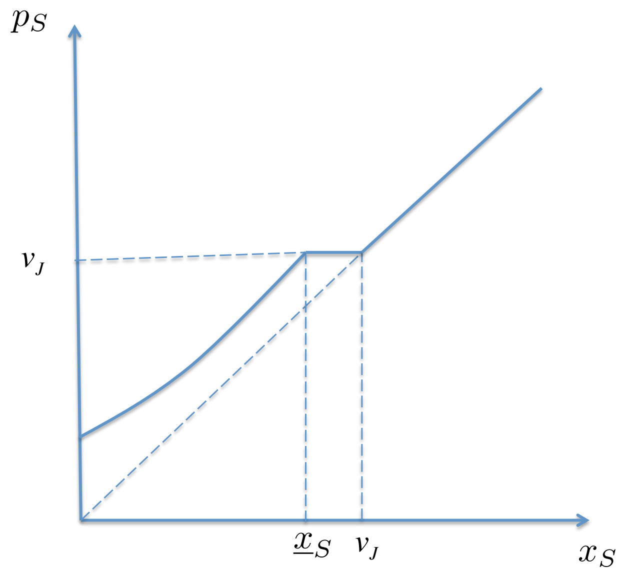


Figure 1: Equilibrium under IPV

where the border between the separating and the bunching region $\underline{x}_S$ is implicitly define by $u_B(x_B^*(\underline{x}_S), \underline{x}_S) = v_J$.

If, on the other hand, $x_S > v_J$, then the bank is not willing to sell the house at the price below x_S, but also will not benefit from a price above x_S as the excess $p - v_J$ will go to the owner. It follows that the bank's optimal strategy in this case is to drop out at x_S, $p_S(x_S) = x_S$.

We summarize these findings in the proposition below. Refer to Figure 1.

Proposition 1. *The bank's optimal strategy is given by*

$$p_S^0(x_S) = \begin{cases} u_B(x_B^*(x_S), x_S) & \text{if } x_S < \underline{x}_S, \\ v_J & \text{if } x_S \in [\underline{x}_S, v_J] \\ x_S, & \text{if } x_S > v_J \end{cases}$$

The optimal strategy is strictly increasing in x_S provided (5) *or* (6) *hold, in which case the bank's prices are pooled over an interval* $[\underline{x}_S, v_J]$.

Discussion The bank's equilibrium behavior is different depending on whether the bank's valuation is below or above the judgment amount. The bank acts in a seller role in the former case, and in a buyer role in the latter. The most interesting feature of the equilibrium is the

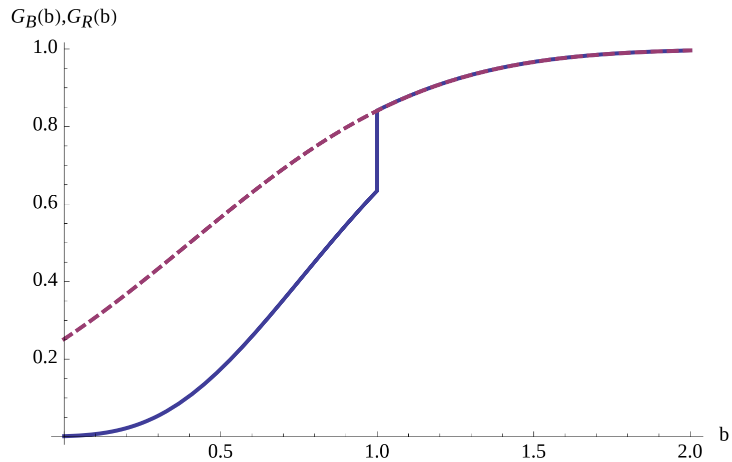


Figure 2: Computed equilibrium bid distributions for the bank ($G_S(\cdot)$; solid curve) and the broker ($G_B(\cdot)$; dashed curve). It is assumed $u_B(x_B, x_S) = x_B$, and the value distributions are specified as normal, with equal means $\mu_S = \mu_B = 0.4$ and standard deviation $\sigma_S = \sigma_R = 0.6$. The judgment amount is $v_J = 1$. Observe the mass point at the judgment amount in $G_S(\cdot)$.[AN: It's probably better to only plot the bank's bid distribution.]

bunching region. In this region, the bank's dropout prices are pooled at the judgment amount (see Figure 1).

The following intuition can be given for the bunching of banks' bids at the judgment amount. Consider comparative statics with respect to the banks valuation x_S. For low valuations x_S, the bank bids its valuation plus the monopoly markup. The optimal bid of the bank balances two effects in a trade-off: a higher bid increases the bank's expected profits conditional on selling, but also lowers the probability of selling the house to a third party. As we increase x_S, this trade-off changes in a way that makes higher bids more attractive to the bank. Hence, the banks bid increases with x_S. However, at the point where the bank's bid reaches v_J, one part of the trade-off disappears for price increase (but not decreases): a higher bid by the bank does not increase the bank's profit conditional on selling, since any additional revenues go to the original owner. Therefore, as x_S increases, the bank's optimal bid stays at v_J. Once x_S surpasses v_J, the bank actually prefers retaining the property, since it is worth more than the amount owed. Therefore, the bank's bid will again increase with x_S for x_S above v_J. This flat piece in the bank's bidding function corresponds to a mass point in banks' bid distributions. Figure 2 illustrates this in an example with normal distributions.

5 Asymmetric information

We now consider the general environment with common values, where the bank will be the informed party. As we shall see, there are some novel features in this setting compared to the symmetric information setup. The main new feature is that the equilibrium will involve a *gap* below the judgment amount. This should be intuitive since otherwise the bank would prefer to deviate from prices slightly below v_J to offering v_J, leading both to a higher average price and not changing the probability of sale.

As with symmetric information, the bank will act as a seller for lower types, and as a buyer for higher types. Again, we begin by considering the extreme cases $v_J = 0$, where the bank always acts as a buyer, and $v_J = \infty$, when the bank always acts as a seller.

5.1 Bank-buyer equilibrium ($v_J = 0$)

If $v_J = 0$, both the bank and the brokers act as buyers, bidding in a standard English auction.[10] The bank knows its valuation x_S, so will dropout at the price equal to x_S. In a symmetric equilibrium,[11] a broker's dropout strategy $p_B(x_B)$ is found through the standard indifference condition. The intuition is that, given the auction has reached price p, if a broker decided to drop out at price p instead to $p + \epsilon$, this decision will change the broker's expected payoff only if the bank (and the other brokers) dropped out at prices $p \in (p, p + \epsilon)$. If the bank drops out at $p = x_S$, then in the limit as $\epsilon \to 0$, the expected value of the house to the broker will be $u_B(x_B, p)$. In equilibrium, the broker will drop out at a price such that he is indifferent between dropping out and continuing, while single crossing implies that brokers with higher types will prefer to continue. Thus, a broker's type $X_B(p)$ dropping out at price p is (uniquely) found from the condition

$$u_B(X_B(p), p) = p.$$

Given our normalization $u_B(x, x) = x$, we see that $X_B(p) = p$, so the broker, even though uninformed, in equilibrium will also drop out at the price equal to its type. This equilibrium is described in the proposition below.

[10]We follow **?** and assume the *thermometer* model, where the price is risen continuously from 0 until there is only a single buyer left.

[11]By symmetric here we mean that the *brokers* adopt the same strategy.

Proposition 2 (Bank-buyer equilibrium)**.** *In a realtor-symmetric equilibrium, both the bank and the brokers will drop out at the prices equal to their signals,*

$$p_S(x_S) = x_S, \qquad p_B(x_B) = x_B.$$

5.2 Bank-seller equilibrium ($v_J = \infty$)

Auctions in which the seller has information about the common value component have been considered in **?**, **?** and **?**.[12] These papers consider the case of a public reserve price and characterize a separating equilibrium in strictly increasing, continuous and differentiable strategies. We begin by adapting the equilibrium characterization results in the aforementioned papers to our setting.

We restrict attention to equilibria where the bank adopts an increasing and continuous equilibrium dropout strategy $p_S^*(x_S)$, with a differentiable inverse $X_S^*(p)$. Given our assumption that broker signals are independent, only the bank's dropout price is relevant for information updating. Denote a broker's dropout strategy as $p_B^*(x_B)$, with the inverse denoted as $X_B^*(p)$. As in **?**, $X_B^*(p)$ is found by equating the object's expected value to the broker assuming the bank drops out at p, to the price p:

$$u_B(X_B^*(p), X_S^*(p)) = p \tag{7}$$

Following the bank's dropout at a price $\tilde{p}$, a broker's dropout strategy is simply $u_B(x_B, X_S(\tilde{p}))$ as the brokers will then have independent private values.

The following proposition describes the separating equilibrium in our model.

Proposition 3 (Bank-seller equilibrium)**.** *There is a unique equilibrium in monotone differentiable strategies. The bank's and the broker's inverse bidding strategies $X_S^*(p)$ and $X_B^*(p)$ are given by the (unique) solutions to the differential equations*

$$\frac{dX_S^*(p)}{dp} = \frac{(J_B(X_B^*, X_S^*) - X_S^*)]f_{(1)}(X_B^*)}{\frac{\partial u_B}{\partial x_S}(u_B(X_B^*, X_S^*) - X_S^*)f_{(1)}(X_B^*) + \frac{\partial u_B}{\partial x_B}\int_{X_B^*}^{\infty}\frac{\partial u_B}{\partial x_S}f_{(2)}(x)dx}, \tag{8}$$

$$\frac{dX_B^*(p)}{dp} = \frac{\frac{\partial u_B}{\partial x_S}(F_{(2)}(X_B^*) - F_{(1)}(X_B^*)) + \int_{X_B^*}^{\infty}\frac{\partial u_B}{\partial x_S}f_{(2)}(x)dx}{\frac{\partial u_B}{\partial x_S}(u_B(X_B^*, X_S^*) - X_S^*)f_{(1)}(X_B^*) + \frac{\partial u_B}{\partial x_B}\int_{X_B^*}^{\infty}\frac{\partial u_B}{\partial x_S}f_{(2)}(x)dx}, \tag{9}$$

[12] These papers consider the case of a publicly observable seller's reserve price in a second-price auction.

subject to the initial conditions $X_S^*(\underline{p}) = 0$ *and* $X_B^*(\underline{p}) = \underline{p}$. *The lowest price offered by the bank* $\underline{p}$ *is given by* $\underline{p} = p_S(0) = u_B(\underline{x}_B, 0)$, *where* $\underline{x}_B$ *is the lowest broker type that purchases with positive probability, given by the unique solution to* $J_B(\underline{x}_B, 0) = 0$. *For an out-of-equilibrium reserve price* $p < \underline{p}$, *brokers believe that the bank's type is the lowest possible.*

We provide the proof in the Appendix. The proof adapts **?** to our setting where the bank is an active bidder in the open auction. We also show that the full-information price is the only price that can be offered by the lowest-type bank in such a separating equilibrium. Thus the equilibrium outcome is unique (assuming that the strategies are differentiable and monotone). The out-of-equilibrium beliefs for prices lower than $\underline{p}$ are indeterminate, but must be sufficiently pessimistic so as to provide the bank with an incentive not to drop out at lower prices. The most pessimistic beliefs (i.e. believing that the bank's type is the lowest possible) are reasonable, and they indeed support this unique equilibrium outcome.

There is a noteworthy property of this separating equilibrium, in comparison with the symmetric information setup considered in Section 4, the *signalling premium*.

Corollary 1 (Signalling premium)**.** *Under asymmetric information, the bank bids higher,* $p_S^*(x_S) > p_S^0(x_S)$ *for* $x_S > 0$.

Proof. Our previous analysis in that section implies that the bank with valuation x_S will set the price so that the marginal broker type willing to purchase at this price, $X_B^0(p)$, is found from the "marginal revenue equals cost" equation $J_B(X_B^0(p), x_S) - x_S = 0$. The price strategy itself is given by $p_S^0(x_S) = u_B(X_B^0(p), x_S)$. How does this price compare to the one with asymmetric information, $p_S^*(x_S)$? Proposition 3 shows that there is no distortion at the bottom, so that the two price are equal: $p_S^*(0) = p_S^0(0)$.

For $x_S > 0$, we have $dX_S^*(p)/dp > 0$. Going back the differential equation (8), this means that for $p = p_S^*(x_S)$, we must have $J_B(X_B^*(p), x_S) > x_S$, which, by the monotonicity of $J_B(x_B, x_S)$ in x_B, implies $X_B^*(p) > X_B^0(p)$. Since $u_B(X_B^0(p), X_S^0(p)) = p$ and $u_B(X_B^*(p), X_S^*(p)) = p$, with $u_B(x_B, x_S)$ being monotone increasing in both arguments (under common values), we must have

$$X_S^*(p) < X_S^0(p) \implies p_S^*(x_S) > p_S^0(x_S)$$

for $x_S > 0$. □

The signalling premium leads to welfare losses under asymmetric information. Consider, for simplicity, the case of a single broker. Our normalization $u_B(x_B, x_B) = x_B$ implies that it is efficient to transfer the object from the seller with valuation x_S to the buyer with valuation x_B if and only if $x_B > x_S$. Denote as $x_B(x_S)$ the minimal buyer type that will, in equilibrium, trade with the seller with valuation x_S. It is found from $u_B(x_B(x_S), x_S) = p(x_S)$. Under asymmetric information, this type is determined from $u_B(x_B^*(x_S), x_S) = p_S^*(x_S)$, while under symmetric information, it is determined from $u_B(x_B^0(x_S), x_S) = p_S^0(x_S)$. Since $p_S^*(x_S) > p_S^0(x_S)$ due to the signalling premium, we must have

$$x_B^*(x_S) > x_B^0(x_S) > x_S.$$

Thus, the trading boundary, already distorted even under symmetric information due to the market power of the seller, is distorted even further under asymmetric information.

Motivated by our empirical application, we now investigate how the bank's strategy is affected as its information concerning the common value component becomes less precise, and hence less relevant to the brokers. Following **?**, we consider a linear specification in the form

$$u_B(x_B, x_S) = x_B + \alpha x_S, \qquad \alpha \in [0, 1).$$

Here, α reflects the relevance of the bank's information for the broker.[13] As α decreases, the bank's information becomes progressively less relevant to the broker. The case $\alpha = 0$ corresponds to independent private values. Note that this is different from the symmetric information case considered above since now the bank's information becomes irrelevant rather than being revealed to the brokers. We denote the bank's strategy $p_S(x_S; \alpha)$.

Proposition 4 (The effect of α)**.** *In the linear model, $p_S(x_S; \alpha)$ is increasing in α.*

Proof. See the Appendix □

5.3 General case: $v_J \in (0, \infty)$

In a foreclosure auction with $v_J \in (0, \infty)$, the equilibrium will combine the features of the bank-seller equilibrium for lower x_S, where the bank will drop out at $p_S^*(x_S)$ as described for

[13] This specification does not satisfy our normalization $u_B(x_B, x_B) = x_B$, but it will if we change the broker's signal to $\tilde{x}_B = \frac{x_B}{1-\alpha}$.

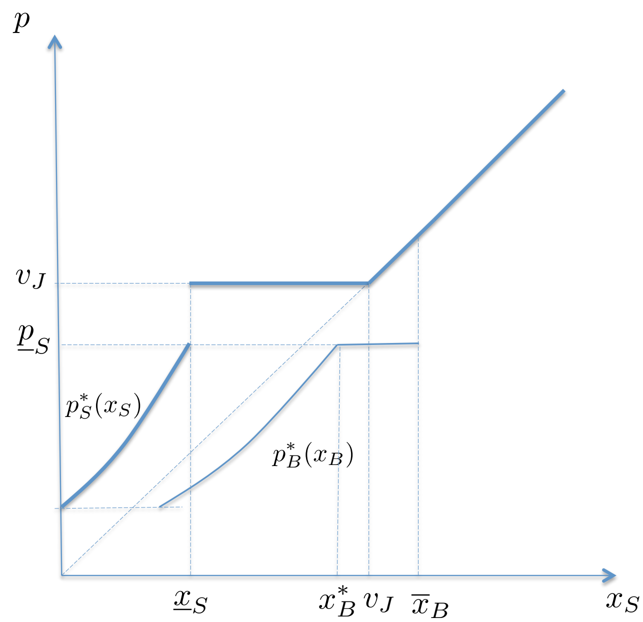


Figure 3: Equilibrium with common values

the $v_J = \infty$ case, and bank-buyer equilibrium for higher x_S, where the bank would drop out at $p_S^*(x_S) = x_S$). However, "stitching" these two equilibria under common values is by no means a simple matter. To see the difficulties that arise, suppose we have a bunching equilibrium with common values where the bank types over a certain interval bid v_J:

$$p_S(x_S) = v_J, \qquad x_S \in [\underline{x}_S, \overline{x}_S].$$

To begin, we observe that there does not exist an equilibrium in continuous strategies that involves bunching. If the bank's dropout strategy $p_S(x_S)$ were continuous and involved bunching, then the brokers' dropout strategy $p_B(x_B)$ would involve a gap below v_J, since otherwise the brokers who would contemplate bidding slightly below v_J, would prefer to wait and drop out at a price slightly above v_J, to take advantage of a dramatically higher quality of the house they would be able to get for banks in the bunching region. Such a gap in broker bids, however, creates an incentive for the bank types that bid somewhat below v_J to deviate to the bid v_J, since this deviation leads to a higher expected price, but does not change the probability of selling.

In order to prevent such deviations, an equilibrium with bunching must involve a *gap* below

v_J, with $\underline{x}_S$, the seller at the lower bound of support of the bunching region, being indifferent between bidding v_J and bidding $\underline{p}_S < v_J$. Sellers with valuations slightly below $\underline{x}_S$ will strictly prefer bidding slightly below $\underline{p}_S$ to bidding v_J. The bank's strategy in our (semi-)separating foreclosure equilibrium coincides with $p_S^*(x_S)$ derived previously for $v_J = \infty$, involves a jump at $\underline{x}_S$ to v_J, bunching at v_J, and truthful bidding for $x_S > v_J$.

The broker's dropout strategy will be a best response to the bank's strategy. It will coincide with $p_B^*(x_B)$ derived for the $v_J = \infty$ case for $x_B < x_B^*$, where x_B^* is implicitly defined by $p_B^*(x_B^*) = \underline{p}_S$. Then, the types $x_B \in (x_B^*, \overline{x}_B)$ will drop out as soon as the price has surpassed $\underline{p}_S$. These brokers realize that they cannot beat the bank profitably at any price. The types $x_B \geq \overline{x}_B$ will continue, and bid up to their values. We must have $\overline{x}_B > v_J$, since otherwise the broker bidding slightly above v_J would prefer a deviation to a bid slightly above $\underline{p}$ in order to avoid a loss due to a lower average quality over the bunch.

So the bank's dropout strategy $p_S(x_S)$ and the broker's dropout strategy when the bank hasn't dropped out, $p_B(x_B)$, are given respectively by

$$p_S(x_S) = \begin{cases} p_S^*(x_S), & x_S \in [0, \underline{x}_S], \\ v_J, & x_S \in [\underline{x}_S, v_J], \\ x_S, & x_S > v_J, \end{cases}$$

$$p_B(x_B) = \begin{cases} p_B^*(x_B), & x_B \in [\underline{x}_B, x_B^*), \\ \underline{p}_S, & x_B \in [x_B^*, \overline{x}_B], \\ x_B, & x_B > \overline{x}_B. \end{cases} \tag{10}$$

See Figure 3, where the bank's and broker's bidding strategies are shown respectively by the thick and the thin curves.[AN: maybe change lines to dashed and solid]

We now introduce the key two conditions that will uniquely pin down the cutoff types $\underline{x}_S$ and $\overline{x}_B$ and therefore pin down an equilibrium. First, the $\overline{x}_B$-type broker must be indifferent between dropping out at a price slightly above $\underline{p}$ or staying in up to $\overline{x}_B$. By deviating to a higher bid $\overline{x}_B$, the broker would gain the property value $u_B(\overline{x}_B, x_S)$ at a price equal to v_J if $x_S \in [\underline{x}_S, v_J]$, and x_S if $x_S \in [v_J, \overline{x}_B]$. So the broker's indifference condition takes the form

$$0 = \int_{\underline{x}_S}^{\overline{x}_B} \Big(u_B(\overline{x}_B, x_S) - \max\{v_J, x_S\} \Big) f_S^*(x_S) dx_S =: H(\underline{x}_S, \overline{x}_B), \tag{11}$$

where $f_S^*(x_S)$ is the density of the bank's types conditional on $x_S \in [\underline{x}_S, \overline{x}_B]$,

$$f_S^*(x_S) = \frac{f_S(x_S)}{F_S(\overline{x}_B) - F_S(\underline{x}_S)}.$$

The second condition specifies that the $\underline{x}_S$-type bank is either indifferent between bidding $\underline{p}_S = p_S(\underline{x}_S)$ or $p = v_J$ (if $\underline{x}_S > 0$), or weakly prefers the bid at v_J to $\underline{p}_S$ (if $\underline{x}_S = 0$):

$$\Pi_S(\underline{x}_S, \underline{p}_S) = \Pi_S(\underline{x}_S, v_J), \qquad (\underline{x}_S > 0) \tag{12}$$

$$\Pi_S(0, \underline{p}_S) \leq \Pi_S(0, v_J), \qquad (\underline{x}_S = 0) \tag{13}$$

where $\Pi_S(x_S, p)$ is the bank's expected equilibrium profit if its type is x_S and it bids p.[14]

Our first result is a technical lemma below that shows existence of cutoffs $\underline{x}_S, \overline{x}_B$ that solve the the indifference conditions (11) and (12). Note that it could happen that $\underline{x}_S = 0$, in which case the bunch extends all the way to the left.

Lemma 1 (Existence and uniqueness of the cutoffs)**.** *There exists a unique solution* $(\underline{x}_S, \overline{x}_B)$ *to the broker's and bank's indifference conditions* (11) *and* (12)*,* (13)*.*

Proof. See the Appendix. □

Given the existence of the cutoffs, we now establish existence (and uniqueness) of a semi-pooling equilibrium under asymmetric information of the kind described above.

Proposition 5 (Equilibrium existence)**.** *There exists a unique equilibrium in the class of strategies given by* (10)*.*

Proof of Proposition 5. We begin with the bank's equilibrium strategy $p_S(x_S)$. The arguments from the previous sections imply that $p_S(x_S)$ is an equilibrium best response for $x_S \leq \underline{x}_S$ and $x_S \geq \overline{x}_B$, so in this proof we only consider $x_S \in (\underline{x}_S, \overline{x}_B)$. For $x_S \in (v_J, \overline{x}_B)$, the bank acts as a buyer, and it is a best response for it to bid its value, x_S. So it only remains to consider $x_S \in (\underline{x}_S, v_J]$. These types will prefer to bid v_J over any bid in $(\underline{p}_S, v_J)$. The reason is that the brokers with values $x_B < \overline{x}_B$ drop out immediately once the price has gone over $\underline{p}_S$. The brokers who remain will bid up to their values $x_B \geq \overline{x}_S$. The bank will prefer to bid v_J over any $(\underline{p}_S, v_J)$ since doing so will not reduce the probability of selling to a broker, but will at least weakly increase the price conditional on sale.[15]

[14] See the Appendix for the explicit formula for $\Pi_S(x_S, p)$.

[15] The price conditional on sale will be strictly higher $v_J > \underline{p}$ if there is only one broker who is active, i.e. has $x_B \geq \overline{x}_S$. If there is more than one broker, the expected price conditional on sale will be unchanged.

Having shown equilibrium incentives for the bank, we now turn to the broker. The results in the previous section imply that, for $x_B \leq x_B^*$, $p_B(x_B)$ dominates any other bid $p \leq \underline{p}_S = p_B(x_B^*)$. Since bidding $p \in (\underline{p}_S, v_J)$ will not affect the broker's expected profit due to the fact that no other participant bids there, it remains to be shown that a broker with $x_B < \overline{x}_B$ will not have an incentive to deviate to a $p \geq v_J$. It is clear that such a broker will not have an incentive to deviate to a bid $p > \overline{x}_B$, since this would lead the broker to buy at prices that are too high and would result in a loss. Indeed, by single crossing, $u_B(x_B, p) < p$ for $p > x_B$, which implies $u_B(x_B, p) < p$ for $p > \overline{x}_B$.

The incremental expected profit from deviation to a price $p \in [v_J, \overline{x}_B]$ is

$$\begin{aligned}
\Delta\Pi_B &= \int_{\underline{x}_S}^{p} (u_B(x_B, x_S) - \max\{v_J, x_S\}) f_S(x_S) dx_S \\
&< \int_{\underline{x}_S}^{p} (u_B(\overline{x}_B, x_S) - \max\{v_J, x_S\}) f_S(x_S) dx_S \\
&\leq \int_{\underline{x}_S}^{\overline{x}_B} (u_B(\overline{x}_B, x_S) - \max\{v_J, x_S\}) f_S(x_S) dx_S = 0
\end{aligned}$$

where the first inequality follows from the fact that $u_B(x_B, x_S)$ is increasing in x_B, while the second inequality follows from the definition of $\overline{x}_B$ as the type that is indifferent This shows that the broker with $x_B < \overline{x}_B$ will not have an incentive to deviate to a $p \geq v_J$, and it also follows that the broker types $x_B \in [x_B^*, \overline{x}_B]$ will bunch at $\underline{p}_S$.

The equilibrium uniqueness follows because there are unique cutoff types $\underline{x}_S$ and $\overline{x}_S$ according to Lemma 1. □

6 Testable Hypotheses

Our theory predicts bunching of banks' bids at v_J for both symmetric and asymmetric information. Denoting the distribution of bank's bids as $G_S(\cdot)$, we therefore have the following testable hypothesis.

Hypothesis 1 (Bunching at v_J). *Under both symmetric and asymmetric information, the distribution $G_S(p)$ has an atom at $p = v_J$, formally,*

$$\lim_{p \uparrow v_J} G_S(p) < G_S(v_J).$$

In a hypothetical world, in which the econometrician could perfectly observe all forms of heterogeneity, it would be straightforward to test for asymmetric information: our theory predicts a gap in banks' bid distributions just below v_J for asymmetric information auctions, but not for symmetric information auctions. Hence, the presence or absence of a gap could serve as a test for asymmetric information.

However, one should not expect a gap if there is unobserved heterogeneity with respect to information asymmetry, as one certainly would expect in reality. If the sales of some of the houses are characterized by asymmetric information (ASI), whereas for others the auction is essentially under symmetric information (SI), then the SI auction will fill out the gap. Here, by SI we mean the houses where the bank's information is also available to the broker; in our model, this implies that x_S is observable to the broker.

But heterogeneity with respect to informational asymmetry does have empirically testable implications. The testable implication can be derived from Corollary 1 (the signaling premium), by observing that for ASI houses the probability of sale is lower than for SI houses for a given bid by the bank. The empirically testable implication stems from a selection due to the gap for ASI auctions as described below.

For SI houses, the bank and the brokers are symmetrically informed. In our framework, this means that the bank's information x_S is observable to the broker and that there is no gap below v_J. For ASI houses, x_S is not observable to the broker and there is a gap in banks' bids in the interval $(\underline{p}_S, v_J)$ for $\underline{p}_S = p_S(\underline{x}_S)$. For bank's bids below the gap for ASI houses ($p_S < \underline{p}_S$), we will observe a mixture of SI and of ASI houses. The probability of sale for such prices is the average of the high SI and the low ASI probability of sale. In the gap ($p_S \in (\underline{p}, v_J)$), we only observe high probability of sale SI houses. This selection effects leads to an increase of the probability of sale at $\underline{p}_S$. For $p_S = v_J$ and above, we again observe both SI and ASI houses, hence a downward jump in the probability of sale at $p_S = v_J$. This is illustrated in Figure 4.

In reality, one would expect that there are more than two types of properties, that there are different degrees of asymmetric information and hence different lower bounds $\underline{p}_S$ for the gaps $(\underline{p}_S, v_J)$. This will smooth out the discontinuity at $\underline{p}_S$ in Figure 4, but we should still expect that the probability of sale to increase in the bank's maximum bid in an interval below v_J, and jump downwards at v_J. Note that the upper bound of the gap v_J is the same irrespective of

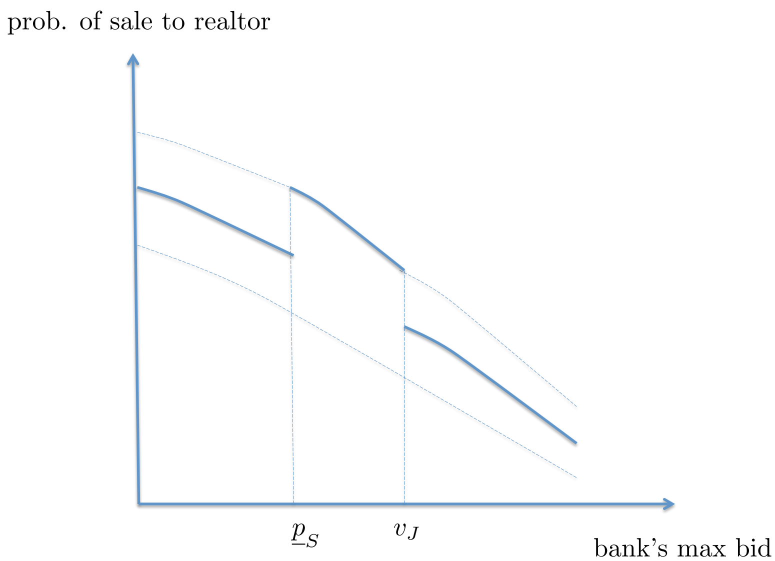


Figure 4: Probability of sale to the broker as a function of the bank's maximum bid for independent symmetric information houses (dotted, blue), adverse selection houses (red, dashed), and a mixture of both types of houses (black, solid).

the degree of asymmetric information, hence the discontinuity at v_J will not be smoothed out. We thus arrive at the following testable hypothesis concerning the probability of sale to the broker, denoted as $\rho(p_S)$. We assume that the prices are normalized by v_J, so that all houses have the same effective v_J.

Hypothesis 2 (Probability of sale)**.** *If all auctions in the data are under symmetric information, the probability of sale $\rho(p_S)$ is a continuous, decreasing function of p_S. If, on the other hand, the data exhibit a mixture of auctions with a varying degree of informational asymmetry, including SI auctions with a positive probability, then we should expect the probability of sale to the broker $\rho(p_S)$ to exhibit the following pattern. Initially, $\rho(p_S)$ decreases in p_S. Then, over a certain interval $p_S \in [\tilde{p}, v_J)$, where $\tilde{p} < v_J$, $\rho(p_S)$ increases in p_S. At $p_S = v_J$, $\rho(p_S)$ drops discontinuously to a lower value, $\rho(v_J) < \lim_{p_S \uparrow v_J} \rho(p_S)$, and from that point on, decreases in a continuous fashion.*

This pattern is illustrated in a simulated example in Figure 5. In this example, it is assumed that the broker's payoff is linear in the signals, $u_B(x_B, x_S) = (1-\beta)x_B + \beta x_S$, and the signals are lognormally distributed. The weight β put on the bank's information x_S is uniformly

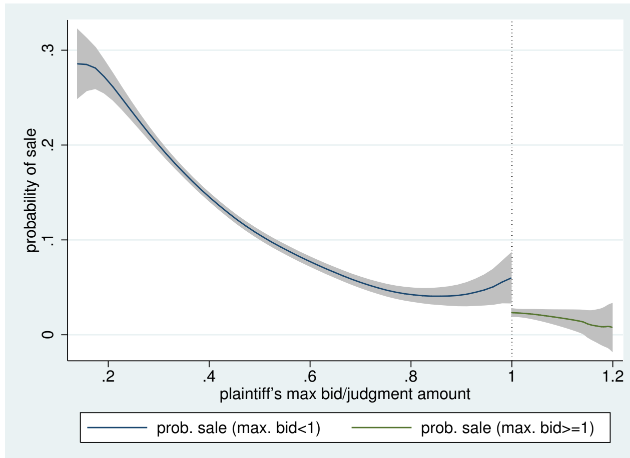


Figure 5: Simulated example for the probability of sale to the broker. Linear payoffs and lognormal distributions for x_B, x_S. 50,000 random draws in a Monte Carlo simulation. Probability of sale as a function of the bank's maximum bid. Locally linear kernel regression with the data split at $p/v_J = 1$ (confidence interval: 95%, Epanechnikov kernel and rule-of-thumb (ROT) bandwidth selection, see **?**).

distributed on $[0, 1/2]$, with $\beta = 0$ corresponding to symmetric information.

7 Data

The Clerk & Comptroller's auction website provides service for sales on the foreclosed properties in Palm Beach county, Florida, US. The website provides a platform for the banks (plaintiff, to whom property owners hold liability) and potential buyers (mosty brokers), to meet in this marketplace. The ClerkAuction online platform conducts foreclosure sales on all business days, which provides a large amount of data on these sales.

We collected data from the website for foreclosure sales between January 21, 2010 and November 27, 2013. Our data record all transaction details on these sales, including winning bid, winner identities, and judgment amounts.

Our dataset contains 12,788 auctions with a total judgment amount of \$4.2bn. The sum of winning bids is \$0.5bn. Table 2 reports the summary statistics for main variables. The variable *bank winning* indicates that 84% of auctions under study ended up having properties transferred to bank's ownership.

We now turn to empirical tests of our hypotheses.

Table 2: Summary statistics

Variable	Mean	Std. Dev.	Min	Max
bank wins	0.813	0.390	0	1
number of third-party bidders	1.065	1.362	0	14

Variable	Mean	Std. Dev.	1st Percentile	99th Percentile
Variables with original scales				
bank's bid	210,996	1,294,181	4,200	1,476,989
judgment amount	329,951	1,953,172	4,985	2,380,127
Variables normalized by judgment amount				
bank's bid	0.766	1.363	0.096	1.167

Hypothesis 1 can be easily checked by plotting the cumulative distribution function of banks' bids. Figure 6 shows the distribution of banks' maximum bids. Bunching shows up very clearly: there are roughly 4,000 observations bunched at the judgment amount.

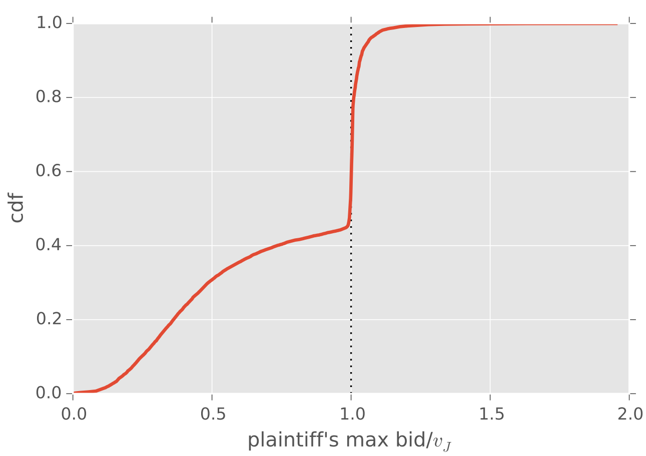


Figure 6: Distribution of the bank's maximum bids (normalized by the judgment amounts). Number of observations: 12,788.

Next, we plot the probability of sale as a function of the bank's maximum bid (Figure 7). Figure 7 shows the same kernel regression as the one shown for the Monte Carlo simulation in Figure 5: it is a locally linear kernel regression, the sample being split into observations below ($p_S < v_J$) and weakly above ($p_S \geq v_J$) the judgment amount. The most striking feature of the graph is that the probability of sale increases with the bank's bid just before the judgment

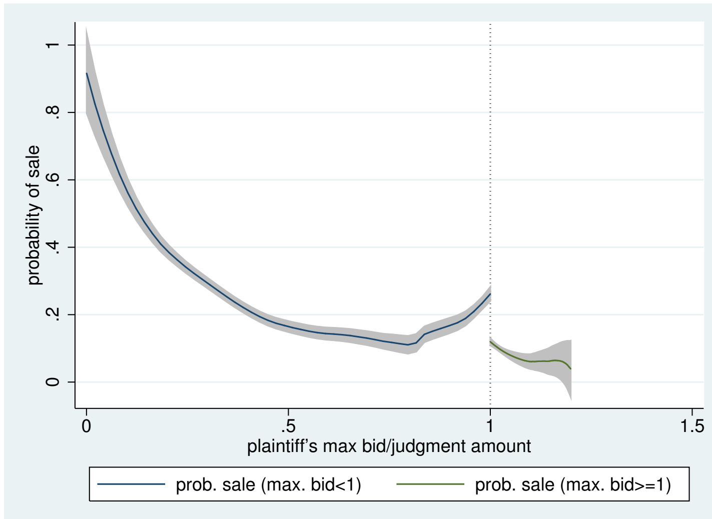


Figure 7: Probability of sale as a function of the bank's maximum bid. Locally linear kernel regression with the data split at $p/v_J = 1$ (confidence interval: 95%, Epanechnikov kernel and rule-of-thumb (ROT) bandwidth selection, see **?**).

amount and drops down discontinuously at v_J.

This is exactly what theory predicts in case that there is heterogeneity in terms of adverse selection (see Figures 4 and 5).

8 Applications

Our theory describes the effects of asymmetric information of the seller (the bank) on the outcome of the foreclosure auction, taking into account the judgment amount. Asymmetric information has been recognized to be highly relevant for the understanding of how markets works, on policy implications, and on welfare analyses at least since Akerlof's seminal contribution on the lemon's market (**?**). Dealing with all the implications of asymmetric information on the seller's side in (foreclosure) auctions is beyond the scope of this article. However, we will describe two examples in which the presence or absence of asymmetric information plays a crucial role: securitization of mortgages and the question of judicial versus non-judicial foreclosures.

8.1 Securitization

8.1.1 Overview

* Securitization played an important role during the financial crisis: mortgagees of a large number of securitized mortgages defaulted

—

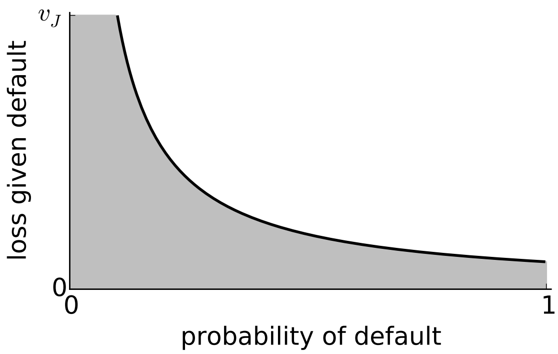
There is an ongoing controversy about the securitization of mortgages in the U.S. Often, the claim is made that securitization was one of the main causes of the financial crisis. According to this view, the combination of securitization and asymmetric information led to banks being too lax when granting mortgages, knowing that holders of securitized assets and the Government Sponsored Enterprises Freddie Mac and Fannie Mae would ultimately pay the bill. An opposing view is that a lack of securitization caused the crisis: during the crisis, the issuing of securitized assets was drastically reduced, leading to less liquidity for banks, which forced banks to cut back on lending and hence exacerbating the crisis.[16]

In the following, we will provide a short description of the securitization process, focusing on aspects that are relevant for our analysis. We will discuss some of these issues at the end of this section. For more details we refer the reader to **?**, **?**, **?**.

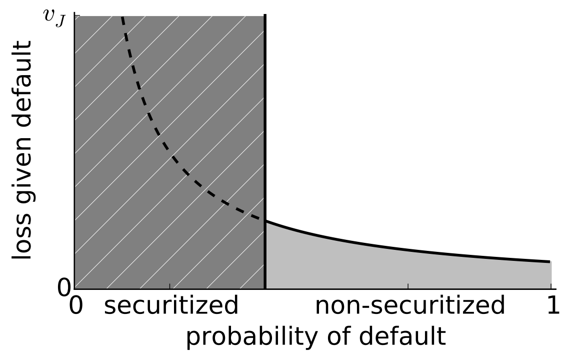
The securitization involves the following steps. First, the originating bank grants a mortgage to the home owner. Second, in order to get liquidity, the originating bank sells the cash flows from a pool of mortgages to a securitization agency, typically one of the Government Sponsored Enterprises Freddie Mac or Fannie Mae. The securitization agency splits the pool of assets into tranches and sells the tranches to investors on the capital market. Third, if a mortgage defaults, then the trustee of the pool of mortgages will apply for a foreclosure at the respective court. Subsequently, the trustee of the mortgage pool will act as the plaintiff in the foreclosure auction.

For a better understanding of the effects of securitization, first consider a bank's decision whether to grant a mortgage if there is no possibility to securitize mortgages. For a given interest rate, the bank is willing to grant a mortgage if the expected loss is below the spread between the mortgage interest rate and the bank's refinancing interest rate. The expected loss is the product of the probability of default (PD) and the loss given default (LGD). The loss

[16] The latter position is promoted e.g. by **?**. The former position is taken by all of the other papers we cite in the following.



(a) Securitization not possible



(b) Securitization possible

Figure 8: Choice of a bank to grant a mortgage based on probability of default and loss given default. The bank is willing to grant a mortgage in the shaded area in which the product of LGD and PD is below a threshold. If securitization is possible, the bank will also grant mortgages that fulfill formal requirements for securitization (hatched area), besides the non-securitized mortgages it keeps on its books (shaded area).

given default is the difference between the mortgage value and the proceedings from selling the property plus administrative costs. The combinations of PD and LGD that lead to the bank granting a mortgage are schematically shown in Figure 8a.

A bank's possibility to securitize mortgages can lead to moral hazard and adverse selection. We will restrict our discussion to moral hazard rather than adverse selection, since the existing literature suggests that moral hazard is the dominant effect (see **?**).

The moral hazard problem is due to the bank not having an incentive to put effort into screening applications for mortgages. It costs time and money to get reliable information about a borrower's probability of default and about the loss given default. Securitization agencies are aware of this moral hazard and choose two countermeasures. First, only mortgages can be securitized for which the probability of default is below a threshold, the probability of default being measured in terms of FICO scores.[17] Second, the bank has to choose a pool of mortgages and the agency randomly picks mortgages which would get securitized in order to reduce adverse selection.

[17]This is a somewhat simplified account, since securitization agencies chose two cutoffs for FICO scores: above a FICO score of 620, banks can securitize *low documentation mortgages*, below a score of 600, it is very difficult to securitize mortgages. Between 600 and 620, banks can securitize mortgages, but have to provide full documentation.

However, there is a reason to believe that these measure did not perfectly solve the problem of adverse selection due to asymmetric information. The literature so far has focused mainly on FICO scores not being a perfect measure for the probability of default (see **?**). However, even if the FICO score were a perfect measure of PD, uncertainty with respect to the LGD leads to moral hazard. As illustrated in Fig. 8b, the bank has an incentive not to exert sufficiently high effort in screening borrowers with respect to PD and LGD. In the stylized extreme case depicted in Fig. 8b, the bank exerts no effort in screening borrowers whose FICO score is above a threshold.

This has two effects. First, mortgages which would not have been granted without securitization because of a high LGD (relative to the PD), will be granted with securitization. Second, since the bank gathers less information about mortgagees, the bank has less private information about the LGD. The second effect is increased by the fact that the little soft information gathered by the bank may be lost when the mortgage is securitized and sold on the capital market.

For the foreclosure auction, this implies that for securitized mortgages, we should expect the bank to be less informated about the quality of the property than for non-securitized mortgages.

This leads us to the empirically testable implication that is an adaptation of Hypothesis 2. Given that holders of securitized mortgages are less informed about the quality of the house and holders of non-securitized mortgages are better informed, we get the following hypothesis:

Hypothesis 3 (Probability of sale for securitized and non-securitized mortgages)**.** *For securitized mortgages, the probability of sale $\rho(p_S)$ is a continuous, decreasing function of p_S. For non-securitized mortgges, we should expect the probability of sale to the broker $\rho(p_S)$ to exhibit the following pattern. Initially, $\rho(p_S)$ decreases in p_S. Then, over a certain interval $p_S \in [\tilde{p}, v_J)$, where $\tilde{p} < v_J$, $\rho(p_S)$ increases in p_S. At $p_S = v_J$, $\rho(p_S)$ drops discontinuously to a lower value, $\rho(v_J) < \lim_{p_S \uparrow v_J} \rho(p_S)$, and from that point on, decreases in a continuous fashion.*

In this description of securitization, we abstracted away from a number of issues which are orthogonal to our analysis, but matter for other purposes. Some of these issues are that information may not be lost, but merely reduced by securitization. The FICO score, which is used as a threshold for securitization, is an imperfect measure for the probability of default. Evidence for this is provided by **?**, who show that mortgages with a FICO score above the threshold were more likely to be securitized and also had higher probabilities of default than mortgages with FICO scores slightly below the threshold. Crossing the FICO score threshold from below increases the probability of default from 5% to 5.5%-6%.[18] This reinforces our point that the lender has better information about the loss-given-default for securitized than for non-securitized mortgages, since the probability of default is known to be correlated with the loss-given-default **?**.

Further, banks are required to keep part of the securitized mortgages on their balance sheets, so that banks have an incentive to exert less effort to collect information, not necessarily no effort at all. This will lead to a noisier, but not to completely uninformative signal for the lender in case of securitization. However, even with these additional issues we should expect the same prediction: that for a securitized mortgages the plaintiff is less informed than for non-securitized mortgages.

The hypothesis that the bank's information concerning the common value component is *less precise* for the securitized mortgages can be directly tested. We do so assuming the linear model

$$u_B(x_B, x_S) = x_B + \alpha x_S.$$

Proposition 4 shows that the bank's strategy $p_S(x_S; \alpha)$ is increasing in α. If some information is lost in the process of securitization, and the bank's information is therefore less precise for the securitized mortgages, then α is expected to be smaller. This leads to the following testable hypothesis.

Hypothesis 4. *For bids below v_J, we have*

$$p_S^{nonsec}(x_S) > p_S^{sec}(x_S), \qquad x_S > 0.$$

In the next section, we test these hypotheses with our data.

[18]To be more precise, there are two thresholds for FICO scores. The analysis of **?** mainly focuses on the threshold at a FICO score of 620 below which banks have to provide full documentation and below which banks can grant low documentation mortgages.

8.1.2 Data

Since we have the name of the plaintiff in each foreclosure auction, we can categorize mortgages as securitized vs non-securitized. We use a simple classification rule, we classify a mortgage as securitized if the name of the plaintiff contains at least one of the following keywords: "trust", "asset backed", "asset-backed", "certificate", "security", "securities", "holder".[19] This simple categorization does give false negatives (for some securitized mortgages none of the keywords shows up in the name of the plaintiff), but almost no false positives.

We have classified 3,249 mortgages as securitized and 9,539 as non-securitized. Figure 9 plots the distributions of banks' bids for securitized and non-securitized mortgages. The average difference of the bank's bid as a fraction of the judgment amount is much lower for securitized than for non-securitized mortgages, roughly 40 percentage points. This could be explained by two effects. First, a selection effect that leads to securitized mortgages being backed by houses of lower quality. Such a selection effect at the foreclosure auction stage can be due to moral hazard at a previous stage, when mortgages are granted. Second, the signaling premium leads to banks bidding higher for non-securitized than for securitized mortgages, since the common value component plays a larger role.

One cannot directly disentangle these two effects, hence it cannot be seen immediately whether there is more asymmetric information for non-securitized than for securitized mortgages. However, we can use our theory as a tool to uncover evidence of asymmetric information. We will take two approaches for this. The first relies on our theoretical results on the discontinuity of the probability of sale at the judgment amount. For the second approach, we use additional data that we hand collected for a subset of the data set.

8.1.3 Discontinuity

Our theoretical results on the judgment amount and the effect of the common value component help us gain more insight on asymmetric information. Recall that in the presence of asymmetric

[19]Two examples of plaintiff's names that are classified as securitized are "US BANK NATIONAL ASSOCIATION AS TRUSTEE ON BEHALF OF THE HOLDERS OF THE ASSET BACKED SECURITIES CORPORATION HOME EQUITY LOAN TRUST SERIES AEG 2006-HE1 ASSET BACKED PASS-THROUGH CERTIFICATES SERIES AEG 2006-HE1 HSBC MORTGAGE SERVCIES INC" and "DEUTSCHE BANK NATIONAL TRUST COMPANY AS TRUSTEE FOR ARGENT SECURITIES INC ASSET-BACKED PASS-THROUGH CERTIFICATES SERIES 2006-W2".

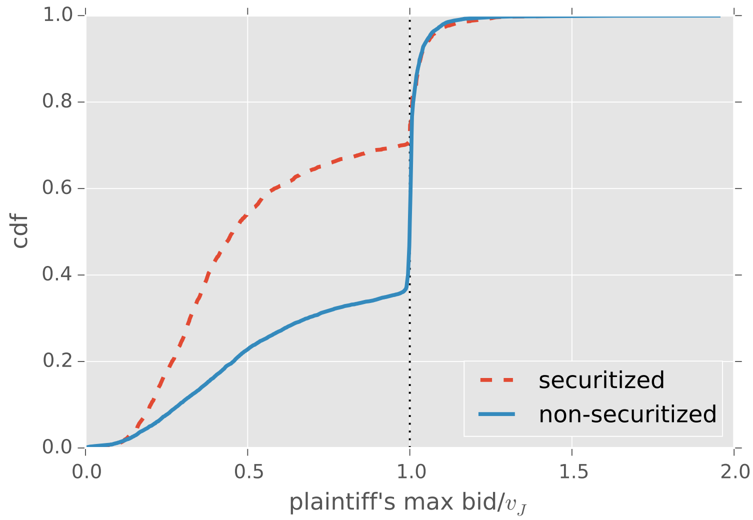


Figure 9: Distribution of the bank's maximum bid as a fraction of the judgment amount p_S/v_J for securitized (dashed line) and non-securitized (solid line) mortgages.

information, we should expect an increase and then a discontinuity in the probability of sale as a function of the bank's bid (Hypothesis 3).

Figures 10 and 11 show the probability of sale as a function of the bank's bid for securitized and non-securitized mortgages. For securitized mortgages, there is no increase in the probability of sale below the judgment amount. Further, the discontinuity at v_J is much less pronounced and indeed not statistically significant. For non-securitized mortgages, we still see the same pattern as in Figure 7. This is consistent with the theory that asymmetric information plays less of a role for securitized mortgages, since the trustee of the mortgage pool is less likely to have an informational advantage over brokers than a local bank.

8.1.4 Bidding strategy

Figures 10 and 11 provide indirect evidence for a common value component for non-securitized mortgages, while there is no such evidence for the securitized mortgages. However, for non-securitized mortgages there are relatively few observations above the judgement value, which might also explain the visual absence of the discontinuity. In this section, we develop and

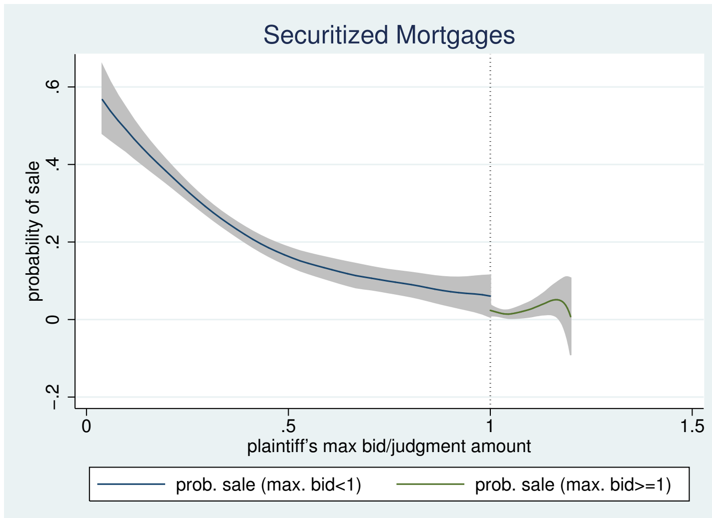


Figure 10: Probability of sale as a function of the bank's maximum bid for securitized mortgages. Locally linear kernel regression with the data split at $p/v_J = 1$ (confidence interval: 95%, Epanechnikov kernel and rule-of-thumb (ROT) bandwidth selection, see **?**).

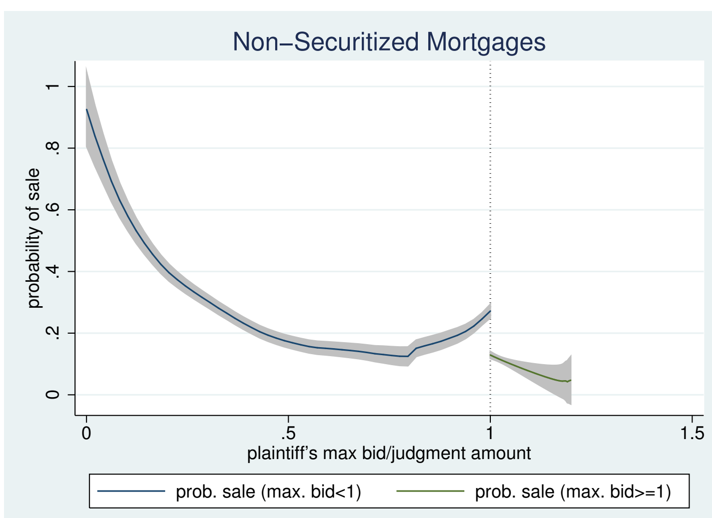


Figure 11: Probability of sale as a function of the bank's maximum bid for non-securitized mortgages. Locally linear kernel regression with the data split at $p/v_J = 1$ (confidence interval: 95%, Epanechnikov kernel and rule-of-thumb (ROT) bandwidth selection, see **?**)

implement a direct test of our Hypothesis 4,

$$p_S^{nonsec}(x_S) > p_S^{sec}(x_S).$$

While the theoretical prediction are clear, the empirical strategy is more involved. It is not sufficient to observe that banks' bids are higher for non-securitized than for securitized mortgages as depicted in Fig. 9. The reason for a different distribution of bids may be either the signaling premium (higher $p_S(x_S)$ for a given x_S), a selection effect (different distributions of x_S for securitized and non-securitized mortgages), or a combination of both. We have to control for x_S in order to identify the signaling premium.

In order to be able to control for x_S, we have hand-collected additional information for a subsample of our data set, namely for properties foreclosed in September 2011. In particular, we collected the information on property transactions after foreclosure sales from a different data source, a website powered by the Palm Beach County Property Appraiser (a government agent). The information regarding the property transactions are for ad valorem tax assessment purposes, as stated in the disclaimer of the website. This implies the appraiser exercises auditing procedures strictly to ensure the validity of any transaction received and posted by that office.

For each foreclosure case, we first traced back the original legal documents, from which we found the property address. We then used the address to search in the database for the detailed features regarding the property and its transaction history. The property appraiser database provides information regarding the type of property (single family, townhouse, zero lot line, etc.), its appraisal values for the most recent three years, the next sale date, the next sale value, and information on the next owner. Using this information, we were able to recover some of the next resale values of the properties at foreclosure.

A comparison of the data on foreclosed properties and the data from the Property Appraiser's database revealed that a perfect matching of observations in the two data sets is not possible, since the address listed in the legal documents is not always of the same format (or containing the same details) as the information recorded in the appraisal database. In order to err on the side of caution, we only used observations for which we can be certain about the identity of the buyer (being the property at foreclosure under study).[AN: Steven, I don't understand what "(being the property at foreclosure under study)" means. Could you please clarify.]

We identified 332 properties among 644 foreclosed cases in the month.[20] The next sale of the foreclosed properties happened mostly in the year of 2012, though a few of the properties were not resold until early 2013. This leaves us with 250 observations for which the resale price is available. Since our aim is to get an estimate of the bank's valuation distribution, we only used the data from the foreclosure auctions in which the bank won. This restricts our sample to 199 observations, with 77 of them in the category of securitized properties and 122 cases of non-securitized properties.

Table 3 provides descriptive statistics of the data collected.[AN: [POSSIBLY ADD COLUMN WITH ASSESSMENT VALUE/JUDGMENT AMOUNT.]] The table suggests that part of the difference in sales prices between securitized and non-securitized mortgages is explained by a selection effect: the bank's resale price in case it wins the auction is higher for non-securitized than for securitized mortgages. In the following, we will disentangle the selection and the signaling premium effect.

Table 3: Descriptive statistics for securitized and non-securitized mortgages.

	Resale Price/Judgment Amount Mean	#
All auctions	0.391	199
Securitized	0.349	77
Non-securitized:	0.417	122

The bank's bidding behavior is determined by its opportunity cost of selling x_S, which is the expected resale price. We identify the distribution of the unobservable x_S by specifying the following correlation structure between x_S, the observable resale price r and the (also observable) tax assessment value a.

$$\tilde{r} = \tilde{x}_S + \epsilon_r, \qquad \tilde{a} = \tilde{x}_S + \epsilon_a$$

where $\tilde{x}_S := \ln x_S - E[\ln x_S]$ is the de-meaned log-opportunity cost of the bank, and $\tilde{r}$, $\tilde{a}$ are the de-meaned log-resale price and tax assessment, respectively. In this specification, the noise terms ϵ_r and ϵ_S are mutually independent, and are also independent of $\tilde{x}_S$. Following **?** and **?**, the underlying distribution of x_S is non-parametrically identifiable under this assumption.

[20]There were 983 properties listed for foreclosure for September 2011, but 339 of them were canceled prior to the auction dates.

	μ_S	σ_S	mean	std
securitized	-1.17236	0.429416	0.33954	0.152791
non-securitized	-1.01577	0.588605	0.430615	0.277086

Table 4: Distribution of x_S for securitized and non-securitized mortgages. The estimate is based on a log-normal distribution $x_S/v_J \sim \ln N(\mu_S, \sigma_S)$.

One could use standard non-parametric deconvolution techniques based on Fourier transforms to obtain the distribution of x_S.

However, because of sample size issues, we take a semi-parametric approach and parametrically deconvolute the noise in the following way. We can identify the variance of $\tilde{x}_S$ by considering the variance of different combinations of r and a. A simple example for this is the following set of variances and the corresponding equations:

$$\mathrm{Var}[\tilde{r}] = \sigma_r^2 + \sigma_S^2$$

$$\mathrm{Var}[\tilde{a}] = \sigma_a^2 + \sigma_S^2$$

$$\mathrm{Var}\left[\frac{1}{2}\tilde{r} + \frac{1}{2}\tilde{a}\right] = \frac{1}{4}\sigma_r^2 + \frac{1}{4}\sigma_a^2 + \sigma_S^2$$

where σ_S^2, σ_r^2, and σ_a^2 are the variances of $\tilde{x}_S$, ϵ_r, and ϵ_a, respectively. The above is a system of three linear equations with three unknowns σ_i^2, $i \in \{S, r, a\}$, and has a unique solution.

The empirical variances of $\tilde{r}$, $\tilde{a}$, and $\tilde{r}/2 + \tilde{a}/2$ will thus lead to a consistent estimate of σ_S. Further, we use the empirical mean of r as an estimate for the mean of x_S, which is also consistent. We further make the parametric assumption that x_S, r, and a are log-normally distributed. Estimates of the distributions of x_S are reported in Table 4. The difference in means reveals that there is indeed a selection effect: securitized mortgages have a lower resale price to judgment amount ratio x_S/v_J than non-securitized mortgages. In the following, we will show that this selection effect cannot explain all the difference between securitized and non-securitized mortgages.

Next, observe that the distribution of banks' bids satisfies $G_S(p_S(x_S)) = F_S(x_S)$ if there is homogeneity with respect to the common value component. Since we only observe the distribution of the bank's resale prices if the bank wins the auction, it is useful to define $\tilde{F}_S(x_S)$

and $\tilde{G}_S(p_S)$ the distributions of x_S and p_S conditional on winning the auction. Because

$$\tilde{F}_S(x_S) = \tilde{G}_S(p_S(x_S)),$$

we can write

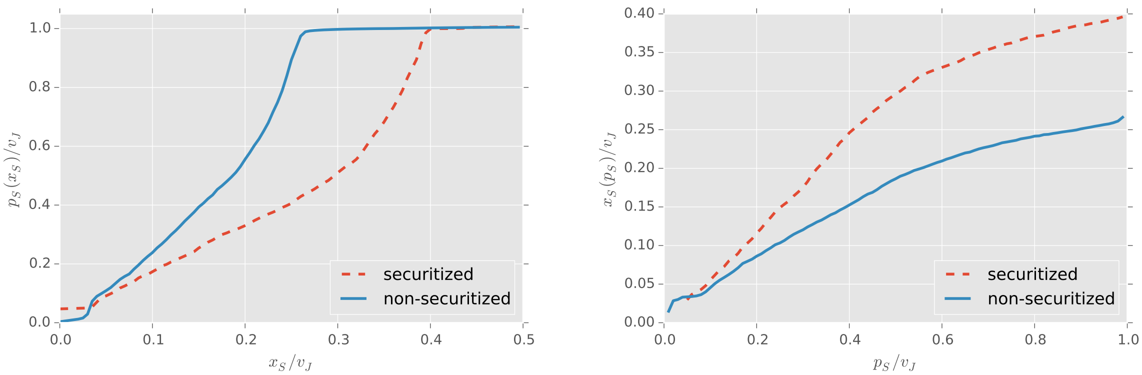
$$p_S(x_S) = \tilde{G}_S^{-1}(\tilde{F}_S(x_S))$$

Given that we have estimates of $\tilde{G}_S$ and $\tilde{F}_S$ both for securitized and for non-securitized mortgages, we can estimate p_S for both types of mortgages.

Figure 12a shows estimates of $p_S(x_S)$ for securitized and non-securitized mortgages. The estimates are consistent with the first alternative in our Hypothesis **??**, namely that the bank's bid is higher for non-securitized than for securitized mortgages, controlling for the bank's opportunity cost x_S.

Note that comparing the bidding functions for bids below v_J is difficult, since the bidding function below v_J has a different support for securitized and non-securitized mortgages. It is more practical to compare the inverse bidding function $x_S(p_S)$ for securitized and non-securitized mortgages, since the support is $[0, v_J]$ in both cases. The inverse bidding functions are shown if Figure 12b. Computing the 90% confidence interval of the difference of the inverse bidding functions for securitized and non-securitized mortgages reveals that the difference is significant for high values of p_S, see Figure 13.

The above analysis provides evidence for the existence of asymmetric information in mortgage markets. The analysis uncovers a difference between securitized and non-securitized mortgages and is consistent with the hypothesis that there is indeed information about the loss given default lost in the securitization process. This is consistent with the often held suspicion that there is moral hazard in the securitization process: if a mortgage is expected to be securitized, then the originating bank has less incentives to collect information about the quality of the mortgage and will be hence less informed than for non-securitized mortgages. While the literature so far has focused on moral hazard with respect to acquiring information about the probability of default, our analysis has uncovered another dimension along which there is moral hazard: The bank exerts insufficient effort to collect information about the loss given default. Our results suggest that the effect of securitization on the loss given default is similarly important for the expected loss as the effect on the probability of default. The bank's resale



(a) Bidding function

(b) Inverse bidding function

Figure 12: Bidding function $p_S(x_S)$ and inverse bidding function $x_S(p_S)$ relating the bank's opportunity cost of selling x_S and the bank's bid p_S for securitized (dashed line) and non-securitized (solid line) mortgages. The functions are constructed by matching quantiles of the estimated distribution of x_S with the quantiles of observed bids submitted by banks.

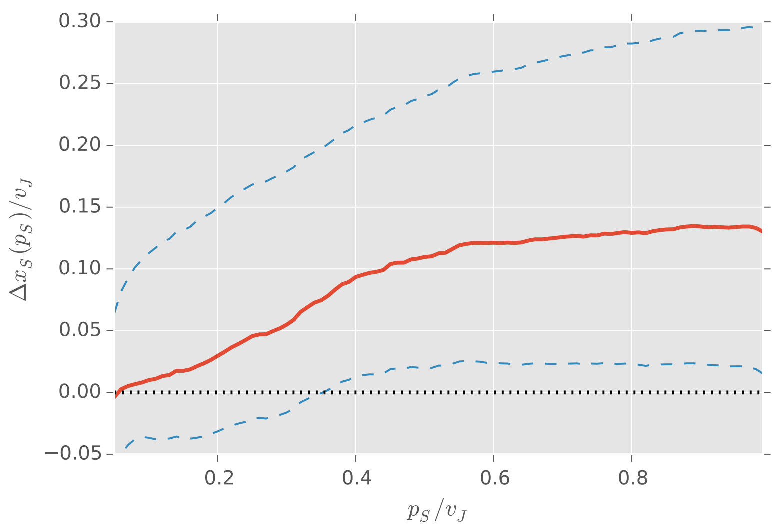


Figure 13: Difference between banks' inverse bidding functions (i.e. the bank's opportunity cost of selling x_S as a function of its bid p_S) for securitized and non-securitized mortgages. The difference (solid line) is constructed by matching quantiles of the estimated distribution of x_S with quantiles of observed reserve prices submitted by banks. The 5% and 95% percentiles of the difference estimate (dashed lines) are constructed using a bootstrap estimate.

price as a function of the judgment amount is 21% lower for securitized than for non-securitized mortgages (0.340 vs 0.431).[21] This is comparable to the 10% to 20% increase of the probability of default for securitized versus non-securitized mortgages (5.5% to 6% versus 5%) estimated by **?**.

9 Conclusions

We develop a novel theory of foreclosure auctions and test some of its predictions with data from Palm Beach county (Florida, US). We find evidence for strategic bidding and asymmetric information, with the bank being the informed party. First, the data reveal bunching in bids at the judgement amount as the theory predicts either under symmetric or asymmetric information. Second, there is a discontinuity in the probability of sale to the brokers, as the theory predicts under asymmetric information. Third, by looking separately at securitized and non-securitized mortgages, we find that banks bid lower for the securitized mortgages. Our theory predicts lower bids when the bank is less informed.

Recent literature, e.g. **?** has emphasized the role of soft information as the predictor of the probability of default. Only hard, verifiable information is priced when the mortgage is securitized. Knowing that the mortgage will be securitized, the lender may not have sufficient incentives to collect soft information. On the other hand, **?** present evidence that the probability of default and loss given default are positively correlated. Thus, soft financial information is relevant for the property resale values, and loss of this information for the securitized mortgages means also less information about the resale values, thereby leading to lower bids in the foreclosure auctions.

The implication of our findings in terms of the policy debate about securitization is that there is an additional reason to be cautious about securitization. This speaks in favor of some of the policies that have been proposed, such as requiring better documentation for securitized mortgages, reducing government support for securitization through securitization agencies, or holding the originating bank liable in case a mortgagee defaults (as is the case for European

[21] We provide results for the resale prices rather than losses given default, since there is more reliable data on this. A rough estimate of the difference in terms of the loss given default can be obtained by assuming that the loss given default is $v_J - x_S$. This would mean a 16% higher loss given default for securitized than for non-securitized mortgages (1-0.340 versus 1-0.431). The loss given default is typically higher than $v_J - x_S$, because of administrative costs.

covered bonds), see e.g. **?**, **?**, and **?**.

Our theory can also be used for a welfare comparison of judicial and non-judicial foreclosures. Roughly half of the states in the U.S. (including Florida) only allow judicial foreclosures, i.e. the foreclosure auction has to be run by a court with rules as the ones described in this article. The other half of the states allow for both judicial and non-judicial foreclosures. Mortgage contracts with a so called a "power of sale" clause allow the bank to choose a non-judicial foreclosure in case of a failure to repay, i.e. the bank can directly seize the property and sell it without going through a court.

The power of sale clause allows the bank to market the property, and to verifiably disclose, through inspections, the information regarding its condition. This eliminates the signalling premium, but introduces another distortion. The bank, acting as a de facto owner of the property, is no longer obligated to pay the owner back any auction proceed above the judgment amount. Thus the monopoly price distortion now extends to prices above the judgment amount. So the overall welfare effect of the power of sale is ambiguous. We plan to estimate this effect in future work.

Appendix

A Omitted Proofs

Proof of Proposition 3. For the exposition, we assume $n \geq 2$. The proof for $n = 1$ is parallel. It turns out convenient to restate the problem a bit differently. Consider the bank of type x_S that contemplates a dropout price p, and assume that the brokers hold the belief $\hat{x}_S$ concerning the bank's type following the bank's dropout. For now, this belief is not necessarily the equilibrium belief.

We restrict attention to equilibria where the broker's dropout strategy (if the bank has not dropped out) is a continuous, strictly increasing function, with a differentiable inverse $X_B^*(p)$. Then the bank's expected profit, as a function of own type x_S, the perceived type $\hat{x}_S$, and the price p is given by

$$\hat{\Pi}(x_S, \hat{x}_S, p) = p(F_{(2)}(X_B^*(p)) - F_{(1)}(X_B^*(p))) \\ + \int_{X_B^*(p)}^{\infty} u_B(x, \hat{x}_S) f_{(2)}(x)dx + x_S F_{(1)}(X_B^*(p)). \tag{14}$$

A direct calculation shows that

$$\frac{\partial \hat{\Pi}_S}{\partial p} = (p - u_B(X_B^*(p), \hat{x}_S)) f_{(2)}(X_B^*(p)) X_B^{*'}(p) \\ + (x_S - p) f_{(1)}(X_B^*(p)) X_B^{*'}(p) \\ + F_{(2)}(X_B^*(p)) - F_{(1)}(X_B^*(p)) \tag{15}$$

and

$$\frac{\partial \hat{\Pi}_S}{\partial \hat{x}_S} = \int_{\hat{X}_B(p)}^{\infty} \frac{\partial u_B(x, \hat{x}_S)}{\partial \hat{x}_S} f_{(2)}(x)dx. \tag{16}$$

The bank's expected profit following a deviation from the equilibrium price to some other price $\hat{p}$ is equal to $\hat{\Pi}_S(x_S, X_S(\hat{p}), \hat{p})$. In equilibrium, such a deviation should not be profitable,so that the following first-order condition (FOC) must hold for $p \in (\underline{p}, \infty)$:

$$\frac{\partial \hat{\Pi}_S(x_S, X_S(p), p)}{\partial p} = 0.$$

Substituting into this FOC the bank's type that bids p, $x_S = X_S^*(p)$, we obtain the following differential equation:

$$\frac{dX_S^*(p)}{dp} = -\frac{\partial \hat{\Pi}_S(X_S^*(p), X_S^*(p), p)/\partial p}{\partial \Pi_S(X_S^*(p), X_S^*(p), p)/\partial \hat{x}_S} \tag{17}$$

We now turn to the broker's equilibrium dropout strategy if the bank has not yet dropped out. We claim that the broker's strategy $X_B^*(p)$ defined as the solution (which will be shown unique later in the proof) to

$$u_B(X_B^*(p), X_S^*(p)) = p, \tag{18}$$

is a best response. Consider first the scenario when it is known to the broker that the bank will drop out at price p. Then it is optimal for the broker to drop out at the price $u_B(x_B, X_S^*(p))$, and the brokers with $x_B < X_B^*(p)$ will drop out at prices lower than p, while the brokers with

$x_B > X_B^*(p)$ will drop out at higher prices. If $X_B^*(p)$ defined by (18) is increasing, it follows that it is optimal to drop out at price p' for a broker of type $X_B^*(p')$. Since this best response does not depend on p, we see that $X_B^*(p')$ is a best response also when the broker does not know the bank's dropout price p.

Totally differentiating (18) yields another differential equation linking $X_S^*(p)$ and $X_B^*(p)$:

$$\frac{\partial u_B}{\partial x_B}\frac{dX_B^*(p)}{dp} + \frac{\partial u_B}{\partial x_S}\frac{dX_S^*(p)}{dp} = 1. \tag{19}$$

Equations (17) and (19) form a linear system for $\frac{dX_S^*(p)}{dp}$ and $\frac{dX_B^*(p)}{dp}$; solving this system yields (8) and (9) in Proposition 3:

$$\frac{dX_S^*(p)}{dp} = \frac{(J_B(X_B^*, X_S^*) - X_S^*)]f_{(1)}(X_B^*)}{\frac{\partial u_B}{\partial x_S}(u_B(X_B^*, X_S^*) - X_S^*)f_{(1)}(X_B^*) + \frac{\partial u_B}{\partial x_B}\int_{X_B^*}^{\infty}\frac{\partial u_B}{\partial x_S}f_{(2)}(x)dx}, \tag{20}$$

$$\frac{dX_B^*(p)}{dp} = \frac{F_{(2)}(X_B^*) - F_{(1)}(X_B^*) + \int_{X_B^*}^{\infty}\frac{\partial u_B}{\partial x_S}f_{(2)}(x)dx}{\frac{\partial u_B}{\partial x_S}(u_B(X_B^*, X_S^*) - X_S^*)f_{(1)}(X_B^*) + \frac{\partial u_B}{\partial x_B}\int_{X_B^*}^{\infty}\frac{\partial u_B}{\partial x_S}f_{(2)}(x)dx}, \tag{21}$$

subject to the initial conditions $X_S^*(\underline{p}) = 0$ and $X_B^*(\underline{p}) = \underline{x}$, where the cutoff $\underline{x}$ is uniquely determined from $u_B(\underline{x}, 0) = \underline{p}$.

With a given value for $\underline{x}$, the solution to the system (20) and (21) is unique by standard results in the theory of differential equations. We now claim that the only value of $\underline{x}$ compatible with equilibrium is the one given in the proposition, namely with $\underline{x} = \underline{x}_B$, uniquely determined from $J_B(\underline{x}_B, 0) = 0$. This value corresponds to the full information outcome.

Since $\hat{X}_B'(p) > 0$, it follows that the expected profit function $\hat{\Pi}(x_S, \hat{x}_S, p)$ satisfies the following *single-crossing* condition: the ratio of the slopes

$$\frac{\partial\hat{\Pi}_S(x_S, \hat{x}_S, p)/\partial p}{\partial\hat{\Pi}_S(x_S, \hat{x}_S, p)/\partial \hat{x}_S} \tag{22}$$

is increasing in x_S. This single-crossing condition implies that there are no profitable within-equilibrium deviations. Indeed, if $\hat{p} \geq \underline{p}$ is such a deviation, then differntial equation (17) implies

$$\frac{dX_S^*(\hat{p})}{dp} = -\frac{\partial\hat{\Pi}_S(X_S^*(\hat{p}), X_S^*(\hat{p}), \hat{p})/\partial p}{\partial\hat{\Pi}_S(X_S^*(\hat{p}), X_S^*(p), p)/\partial \hat{x}_S}$$

This, however, contradicts the single-crossing condition, according to which $X_S^*(\hat{p}) < x_S$ implies

$$\frac{\partial \Pi_S(X_S^*(\hat{p}), X_S^*(\hat{p}), \hat{p})/\partial p}{\partial \Pi_S(X_S^*(\hat{p}), X_S^*(\hat{p}), \hat{p})/\partial \hat{x}_S} < \frac{\partial \Pi_S(x_S, X_S^*(\hat{p}), \hat{p})/\partial p}{\partial \Pi_S(x_S, X_S^*(\hat{p}), \hat{p})/\partial \hat{x}_S}$$

Similarly, we can rule out an within-equilibrium deviation to a price $\hat{p} \in (p, \infty)$.

We now claim that only the full information cutoff $\underline{x} = \underline{x}_B$ is compatible with the separating equilibrium. First, observe that any value $\underline{x} < \underline{x}_B$ corresponds to the solution of the system that is not monotonically increasing and therefore cannot correspond to a separating equilibrium. Next, if $\underline{x} > \underline{x}_B$, then it is profitable for the type $x_S = 0$ to deviate to $p < \underline{p}$ even when the brokers' belief are the most pessimistic, $\hat{x}_S = 0$. The slope if the expected profit (15) is of the same sign as

$$-J_B(\underline{x}, 0) < J_B(\underline{x}, 0) = 0, \text{ if } \underline{x} > \underline{x}_B.$$

The cutoff $\underline{x} = \underline{x}_B$ does indeed yield the solution $(X_B^*(p), X_S^*(p))$ in which each function is monotonically increasing in p. First, note that equations (20) and (21) form an autonomous system of differential equations. Every solution curve that is entirely contained in the region

$$\mathcal{M} := \{(x_B, x_S) : x_S \geq 0, J_B(x_B, x_S) - x_S \geq 0$$

corresponds to a monotone increasing solution because the r.h.s. of (20) and (21) are non-negative in (positive in the interior of) $\mathcal{M}$. Next, observe that the solution curve that starts at the point $(x_B, x_S) = (\underline{x}_B, 0)$, i.e. in the south-west corner of $\mathcal{M}$ will never leave $\mathcal{M}$. The left boundary of $\mathcal{M}$ is given by the full information outcome, $\{(x_B, x_S) : J_B(x_B, x_S) - x_S = 0\}$. This boundary is an increasing locus because $J_B(x_B.x_S)$ is assumed increasing in x_B. The vector field of the system points inside $\mathcal{M}$, with $dX_S^*(p)/dp = 0$ and $dX_B^*(p)/dp \geq 0$. Therefore, any solution with the initial condition in $\mathcal{M}$ will stay in $\mathcal{M}$. Since the initial condition $(\underline{x}_B, 0) \in \mathcal{M}$, we conclude that $X_B^*(p)$ and $X_S^*(p)$ are increasing in p.

Finally, in order to ensure that the bank with $x_S = 0$ indeed prefers to choose $\underline{x}_B$, it is sufficient to assume that for any lower (out of equilibrium) price, the brokers believe that the bank's type that deviated is the lowest one, $\hat{x}_S = 0$. Then (15) implies that the slope of the expected profit is *positive* for $p < \underline{p}$.

□

Proof of Proposition 4. Instead of the bank's bid strategy directly, in this proof it turns out convenient to work with the function $s(x_B)$, which gives the the bank's type s as a function of the broker's type x_B that would drop out at the same price as the bank. Then the bank's bidding strategy is given by

$$p_S(x_S) = u_B(s^{-\mathbb{1}}(x_S), x_S) \tag{23}$$

$$= s^{-\mathbb{1}}(x_S) + \alpha x_S. \tag{24}$$

Dividing equation (8) by (9), we get the following differential equation for $s(x_B)$,

$$\frac{ds}{dx_B} = \frac{(\bar{J}(x_B) - (1-\alpha)s)f_{(1)}(x_B)}{\alpha \bar{F}_{(1)}(x_B)}$$

where

$$\bar{J}(t) = t - \frac{1 - F_B(t)}{f_B(t)}$$

and $\bar{F}_{(1)}(x_B) = 1 - F_{(1)}(x_B)$.

This equation can be integrated explicitly[22]

$$s(x_B) = \gamma \bar{F}_{(1)}(x_B)^{\gamma-1} \int_{\underline{x}_B}^{x_B} \bar{F}_{(1)}(t)^{-\gamma} f_{(1)}(t) \bar{J}(t) dt$$

where

$$\gamma = \frac{1}{\alpha}$$

and $\underline{x}_B$ is the lowest broker participating type, the same as under symmetric information,

$$\bar{J}(\underline{x}_B) = 0.$$

The slope of $s(x_B)$ is given by

$$s'(x_B) = \gamma(\gamma-1) f_{(1)}(x_B) \bar{F}_{(1)}(x_B)^{\gamma-2} \int_{\underline{x}_B}^{x_B} \bar{F}_{(1)}(t)^{-\gamma} f_{(1)}(t) \bar{J}(t) dt + \gamma \bar{F}_{(1)}(x_B)^{-1} f_{(1)}(x_B) \bar{J}(x_B) \tag{25}$$

With a change of variable

$$y = \log \frac{\bar{F}_{(1)}(x_B)}{\bar{F}_{(1)}(t)},$$

[22] The integration method is essentially the same as in the proof of Theorem 3 in **?**. However, our linear specification is different and is not a special case of the linear specification in **?**.

we have

$$s'(x_B) = f_{(1)}(x_B) \int_{\underline{y}}^{0} \gamma(\gamma-1)e^{(\gamma-1)y}\tilde{J}(y)dy + \gamma\bar{F}_{(1)}(x_B)^{-1}f_{(1)}(x_B)\bar{J}(x_B)$$

where $\tilde{J}(y) = \bar{J}(t(y))$,

$$\underline{y} < 0, \qquad \tilde{J}(\underline{y}) = 0.$$

Taking the derivative of the slope $s'(x_B)$ with respect to γ, and using the estimate

$$\begin{aligned} \frac{d}{d\gamma}\Big(\gamma(1-\gamma)e^{(\gamma-1)y}\Big) &= \Big(2\gamma - 1 + \gamma(\gamma-1)\Big)e^{(\gamma-1)y} \\ &\geq \Big(1 + (\gamma-1)y\Big)e^{(\gamma-1)y} \end{aligned}$$

we obtain the estimate

$$\frac{ds'(x_B)}{d\gamma} \geq f_{(1)}(x_B) \int_{\underline{y}}^{0} \Big(1 + (\gamma-1)y\Big)e^{(\gamma-1)y}\tilde{J}(y)dy + \bar{F}_{(1)}(x_B)^{-1}f_{(1)}(x_B)\bar{J}(x_B).$$

The second term above is positive. As far as the first term, the extended mean-value theorem for integrals implies for some $a \in [\underline{y}, 0]$[23]

$$\begin{aligned} \int_{\underline{y}}^{0} \Big(1 + (\gamma-1)y\Big)e^{(\gamma-1)y}\tilde{J}(y)dy &= \tilde{J}(0) \int_{\underline{y}}^{0} \Big(1 + (\gamma-1)y\Big)e^{(\gamma-1)y}dy \\ &= J(x_B) \int_{\underline{y}}^{0} d\Big(ye^{(\gamma-1)y}\Big) \\ &= -J(x_B)ae^{(\gamma-1)\underline{a}} \geq 0 \end{aligned}$$

where the last line follows from $a < 0$. So we conclude

$$\frac{ds'(x_B)}{d\gamma} > 0.$$

Since $\gamma = 1/\alpha$, this implies $\frac{ds'(x_B)}{d\alpha} < 0$, which in turn implies that the slope of the inverse $s^{-\mathbb{1}}(x_S)$ is increasing in α. Since $p_S(x_S;\alpha) = s^{-\mathbb{1}}(x_S) + \alpha x_S$, we conclude that the slope of $p_S(x_S;\alpha)$ is increasing in α. Since $p_S(0) = \underline{x}_B$ is independent of α, this implies $p_S(x_S;\alpha)$ increases in α.

$\square$

[23]The second mean-value theorem for integrals states that $\int_a^b f(t)g(t)dt = g(a)\int_a^c f(t)dt + g(b)\int_c^b f(t)dt$ whenever f, g are continuous functions on $[a, b]$. See Theorem 2.12.17 on p.150 in **?**. Here, this theorem is applied with $g(t) = \tilde{J}(t)$, taking into account $\tilde{J}(\underline{y}) = 0$.

Proof of Lemma 1. Indifference condition (12) can be equivalently stated as

$$(\hat{p}(\underline{p}_S) - \underline{x}_S)(1 - F_{(1)}(X_B^*(p_S^*(\underline{x}_S)))) = (\hat{p}(v_J) - \underline{x}_S)(1 - F_{(1)}(\overline{x}_B)), \tag{26}$$

where $\hat{p}(p)$ denotes the equilibrium price received by the bank *conditional* on winning the auction with a reserve p. The bank's indifference condition (26) defines $\overline{x}_B$ as an implicit function of $\underline{x}_S$. This function is denoted as $y_B(\cdot)$. The indifference condition (26) can be re-written as

$$F_{(1)}(\overline{x}_B) = 1 - \frac{\Pi_S(\underline{x}_S)}{\hat{p}(v_J) - \underline{x}_S}$$

where we denoted the broker's type that corresponds to $\underline{x}_S$ as $\tilde{x}_B(\underline{x}_S) = X_B(p_S^*(\underline{x}_S)))$, and

$$\Pi_S(\underline{x}_S) = (\hat{p}(p_S^*(\underline{x}_S)) - x_S)(1 - F_{(1)}(\tilde{x}_B(\underline{x}_S)).$$

Next, we show that $\Pi_S(\underline{x}_S)/(\hat{p}(v_J) - \underline{x}_S)$ is increasing in $\underline{x}_S$, which implies that $\overline{x}_B$ is decreasing in $\underline{x}_S$. The derivative of this function is

$$\frac{d}{d\underline{x}_S}\frac{\Pi_S(\underline{x}_S)}{\hat{p}(v_J) - \underline{x}_S} = \frac{\Pi_S'(\underline{x}_S)(\hat{p}(v_J) - \underline{x}_S) + \Pi_S(\underline{x}_S)}{(\hat{p}(v_J) - \underline{x}_S)^2}$$

The envelope theorem implies $\Pi_S'(\underline{x}_S) = -(1 - F_{(1)}(\tilde{x}_B(\underline{x}_S)))$. So the numerator is equal to

$$\begin{aligned}
&\Pi_S(\underline{x}_S) - (\hat{p}(v_J) - \underline{x}_S)(1 - F_{(1)}(\tilde{x}_B(\underline{x}_S)))\\
&= \Pi_S(\underline{x}_S) - (\hat{p}(p_1) - \underline{x}_S)(1 - F_{(1)}(\tilde{x}_B(\underline{x}_S))) - (\hat{p}(v_J) - \hat{p}(p_1))(1 - F_B(\tilde{x}_B(\underline{x}_S)))\\
&= -(\hat{p}(v_J) - p_1)(1 - F_{(1)}(\tilde{x}_B(\underline{x}_S))) < 0
\end{aligned}$$

where the inequality follows since $p_1 < v_J$ and $\hat{p}(\cdot)$ is a an increasing function. Hence, $y_B(\underline{x}_S)$ is a *decreasing* function.

The broker's indifference condition (11), $H(\underline{x}_S, \overline{x}_B) = v_J$, defines $\overline{x}_B$ as an implicit function of $\underline{x}_S$,

$$\overline{x}_B = z(\underline{x}_S).$$

Indeed, we have $H(\underline{x}_S, v_J) < 0$ and, for $\overline{x}_B \geq v_J$,

$$\begin{aligned}
\frac{\partial H(\underline{x}_S, \overline{x}_B)}{\partial \overline{x}_B} &= \Big(u_B(\overline{x}_B, \overline{x}_B) - \overline{x}_B\Big) f_S(\overline{x}_B) + \int_{\underline{x}_S}^{\overline{x}_B} \frac{\partial u_B(\overline{x}_B, x_S)}{\partial \overline{x}_B} f_S(x_S) dx_S\\
&= \int_{\underline{x}_S}^{\overline{x}_B} \frac{\partial u_B(\overline{x}_B, x_S)}{\partial \overline{x}_B} f_S(x_S) dx_S\\
&\geq \alpha(F_S(\overline{x}_B) - F_S(v_J)),
\end{aligned}$$

where we have used the assumption that $\partial u_B/\partial x_B \geq \alpha > 0$. By integration, it then follows that for $\overline{x}_B \geq v_J$,

$$H(\underline{x}_S, \overline{x}_B) \geq H(\underline{x}_S, v_J) + \int_{v_J}^{\overline{x}_B} (F_S(y) - F_S(v_J))dy \to \infty$$

as $\overline{x}_B \to \infty$. So for a given $\underline{x}_S < v_J$, $H(\underline{x}_S, \overline{x}_B)$ is an increasing function of $\overline{x}_B$, tending to ∞, with $H(\underline{x}_S, v_J) < 0$. This implies that the equation $H(\underline{x}_S, \overline{x}_B) = 0$ defines $\overline{x}_B$ as an implicit function of $\underline{x}_S$. This function will be denoted as $z(\underline{x}_S)$,

$$H(\underline{x}_S, z_B(\underline{x}_S)) = 0.$$

We now show that $z_B(\cdot)$ is an *increasing* function. This will follow from the fact that $H(\underline{x}_S, \overline{x}_B)$ defined in (11) is an increasing function in first two arguments. We have already shown that $H(\underline{x}_S, \overline{x}_B)$ is increasing in $\overline{x}_B$ for $\overline{x}_B \geq v_J$. Now

$$\frac{\partial H(\underline{x}_S, \overline{x}_B)}{\partial \underline{x}_S} = (v_J - u_B(\overline{x}_B, \underline{x}_S)) f_S(\underline{x}_S).$$

We claim that $u_B(\overline{x}_B, \underline{x}_S) < v_J$, so that $H(\underline{x}_S, \overline{x}_B)$ is indeed increasing in $\underline{x}_S$. We argue by contradiction. If not, we would have

$$H(\underline{x}_S, \overline{x}_B) \geq \int_{v_J}^{\overline{x}_B} \Big(u_B(\overline{x}_B, x_S) - x_S\Big) f_S(x_S) dx_S, \tag{27}$$

Given our assumption $\partial u_B/\partial x_S < 1$, we have for $x_S < \overline{x}_B$,

$$\begin{aligned} u_B(\overline{x}_B, x_S) &= u_B(\overline{x}_B, \overline{x}_B) - \int_{x_S}^{\overline{x}_B} \frac{\partial u_B(\overline{x}_B, x_S)}{\partial x_S} dx_S \\ &> \overline{x}_B - \int x_S \overline{x}_B dx_S = x_S. \end{aligned}$$

Substituting this bound into (27), we get

$$\begin{aligned} H(\underline{x}_S, \overline{x}_B) &> \int_{v_J}^{\overline{x}_B} \Big(u_B(\overline{x}_B, x_S) - x_S\Big) f_S(x_S) dx_S \\ &> 0, \end{aligned}$$

a contradiction to $H(\underline{x}_S, \overline{x}_B) = 0$. So we conclude

$$\frac{\partial H(\underline{x}_S, \overline{x}_B)}{\partial \underline{x}_S} > 0.$$

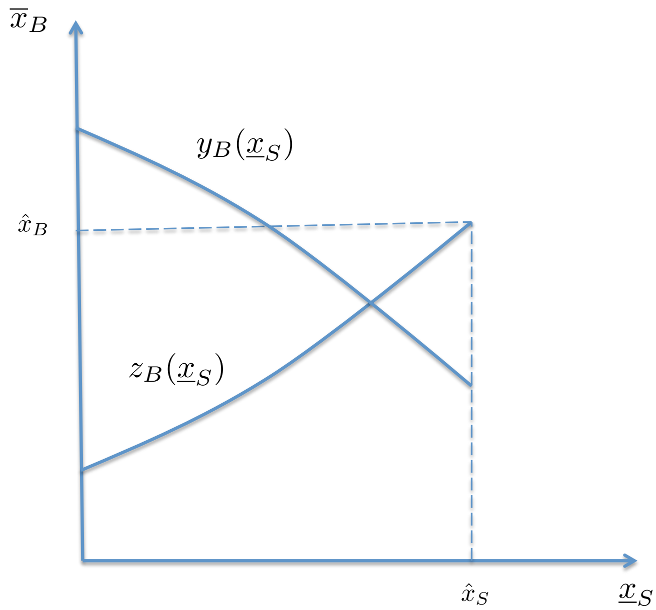


Figure 14: Functions $y_B(\cdot)$ and $z_B(\cdot)$.

The Implicit Function Theorem now implies that $z_B(\cdot)$ is increasing:

$$z_B'(\underline{x}_S) = -\frac{\partial H/\partial \underline{x}_S}{\partial H/\partial \overline{x}_B} > 0.$$

Thus $y_B(\underline{x}_S)$ and $z_B(\underline{x}_S)$ are both continuous, and are, respectively, decreasing and increasing functions. Continuing with the proof, let $\hat{x}_S = X_S^*(v_J)$ and $\hat{x}_B = X_B^*(v_J)$ be the bank's and broker's types in the bank-seller equilibrium ($v_J = 0$). The lower cutoff values are restricted between the lowest possible value 0, and $\hat{x}_S$. Both mapping $y_B(\cdot)$ and $z_B(\cdot)$ are defined on the domain $[0, \hat{x}_S]$.

Since the $\hat{x}_B$ type breaks even if the bank drops out at v_J in the bank-seller equilibrium ($v_J = 0$), we must have $H(\hat{x}_S, \hat{x}_B) > 0$. The monotonicity of $H(\underline{x}_S, \overline{x}_B)$ in $\overline{x}_B$ implies $z_B(\hat{x}_S) < \hat{x}_B$. At the same time, the definition of $y_B(\cdot)$ implies $\hat{x}_B = y_B(\hat{x}_S)$, and it follows that $z_B(\hat{x}_S) < y_B(\hat{x}_S)$. Refer to Figure 14. In view of the monotonicity of $y_B(\cdot)$ and $z_B(\cdot)$, there are two possibilities. First, if $z_B(0) \geq y_B(0)$, then type-0 bank prefers to bid $\underline{p}$ rather than v_J. In this case, the curves $y_B(\underline{x}_S)$ and $z_B(\underline{x}_S)$ have a unique intersection given by

$$\overline{x}_B = y_B(\underline{x}_S) = z_B(\underline{x}_S), \tag{28}$$

with $\overline{x}_S \in (0, \hat{x}_S)$. If, on the other hand, $z_B(0) < y_B(0)$, then type-0 bank prefers to bid v_J, so that the equilibrium involves $\overline{x}_B = 0$ and the bunching extends all the way to $x_S = 0$.

$\square$