



*Geometry is one and eternal shining in the mind of God.*  
JOHANNES KEPLER.

God's plan for mathematics is beyond human reach but mathematics is the only light that can illuminate the mysteries of this world.

What we call the *science of physics* is filled with the radiance of mathematics; the fog of ignorance around what we now call LAWS OF NATURE receded in the brilliance of this physics.

But the beam of this light has barely touched the edges of the kingdom of LIFE and the face of PRINCESS THOUGHT remains hidden from us in the shroud of darkness.

And unless we know the ways of THOUGHT we can not understand what mathematics is.

The first part of the book – *Quotations and Ideas* is sprinkled with the ideas of those who saw sparks of light in the dark see of unknown. In the second part – *Memorandum Ergo* we reflect on what in mathematics could shed light on MYSTERY THOUGHT.

# Quotations and Ideas.

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## 1 Beautiful Elsewhere.

*Round us, near us, in depth and height,  
Soft as darkness and keen as light.*

ALGERNON SWINBURNE. LOCH TORRIDON.

"*Mathématiques, un dépaysement soudain*" was an exhibition organised by Fondation Cartier pour l'art contemporain in Paris. It featured, among other things, *The Library of Mysteries*:



THE MYSTERY OF PHYSICAL LAWS,  
THE MYSTERY OF LIFE,  
THE MYSTERY OF THE MIND  
THE MYSTERY OF MATHEMATICS.

These were presented as quotes from writings of great scientists in a film made by David Lynch – an artist’s visualizations of the ideas of Time, Space, Matter, Life, Mind, Knowledge, Mathematics.

Michel Cassé and Hervé Chandés have persuaded me to try to do *Naive Mathematician*’s version of what David Lynch has done – to project a vision of these ideas to the invisible screen in our mind, an image illuminated by the eternal shining of mathematics rather than by the reflection of the beauty of the goddess of arts.

I knew it would be impossible but I tried anyway. Much of what I wrote was done in discussions with Giancarlo Lucchini and some of my English was corrected by Bronwyn Mahoney. Below is a modified version of what came out of it.

## 2 Science.

*Nothing exists except atoms in the void;  
everything else is opinion.*

DEMOCRITUS OF ABDERA (?), 460 - 370 BCE.

*All men by nature desire knowledge. Thinking is the talking of the soul with itself. ... knowledge is the one motive attracting and supporting investigators... always flying before them... their sole torment and their sole happiness. To perceive is to suffer.*

*Common sense is the collection of prejudices acquired by age eighteen.  
...Science... [is] asking, not whether a thing is good or bad ... but of what kind it is?*

*Science is the belief in the ignorance of experts. What we know already ... often prevents us from learning. Our freedom to doubt was born out of a struggle against authority in the early days of science.*

*Science is no more a collection of facts than a heap of stones is a house. A fact is valuable only for the idea attached to it, or for the proof which it furnishes. He who does not know what he is looking for will not understand what he finds. The investigator should have a robust faith – and yet not believe.*

*Science increases our power in proportion as it lowers our pride. Ignorance more frequently begets confidence than does knowledge. Whoever undertakes to set himself up as a judge of Truth and Knowledge is shipwrecked by the laughter of the gods.*

*The gods are fond of a joke – the universe is not only queerer than we suppose, but queerer than we can suppose. And the most incomprehensible thing about the world is that it is comprehensible.*

*There is geometry in the humming of the strings, there is music in the spacing of the spheres. A hidden beauty is stronger than an obvious one. It is godlike ever to think on something beautiful and on something new.*

*The most beautiful thing we can experience is the mysterious. It is the source of ... art and ... science – branches of the same tree. He to whom this emotion is a stranger... is as good as dead: his eyes are closed.*

*When it comes to atoms, language can be used only as in poetry. Poetry is nearer to vital truth than history. Knowledge is limited. Imagination encircles the world.*

*But put off your imagination, as you put off your overcoat, when you enter the laboratory. Put it on again when you leave.*

*The objective reality of things will be hidden from us forever; we can only know relations. Everything we call real is made of things that cannot be regarded as real. The internal harmony of the world is the only true objective reality.*

*It is not nature which imposes time and space upon us, it is we who impose them upon nature because we find them convenient. ... the distinction between past, present, and future is a most stubbornly persistent illusion.*

*A human being is a part of a whole, called by us "universe" – a part ... restricting us to our personal desires and to affection for a few persons nearest to us. Our task must be to free ourselves from this prison by widening our circle of compassion to embrace all living creatures and the whole of nature in its beauty.*

PYTHAGORAS  
HERACLITUS  
PLATO  
ARISTOTLE

CHARLES DARWIN  
CLAUDE BERNARD  
JAMES CLERK MAXWELL  
HENRY POINCARE

NIELS BOHR  
ALBERT EINSTEIN  
JOHN HALDANE  
RICHARD FEYNMAN.

What these people think and how they write enlightens your mind and elevates your spirit but these thoughts have little life of their own. They do not grow, they do not transform, they do not shoot new green sprouts – they luminesce as crystals of fiery flowers frozen in eternity. They are not quite what mathematicians call *ideas*, they are halfway between *ideas* and *opinions*<sup>1</sup>.

Great scientific *ideas* are different – they are alive, they ignite your soul with delight, they invite you to fight and to contradict them. Unlock your spirit from the cage of mundane, let your imagination run free, start playing with such ideas as a little puppy with its toys – and you find yourself in the world of *Beautiful Elsewhere* – that is called *Mathematics*.

### 3 Numbers.

*All the mathematical sciences are founded on relations between physical laws and laws of numbers.*

JAMES CLERK MAXWELL (1856).

*The numbers named by me exceed the*

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<sup>1</sup>An OPINION about  $X$  is a function, say  $OP_X = OP_X(p)$ , that assigns YES (agree) or NO (disagree) to a person  $p$  who is dying to say what he/she thinks about  $X$ , e.g. about the existence (non-existence) of vacuum. Democritus did not care about specific YES/NO -values of  $OP_X(p)$ , except for  $p$  being his best friend, maybe. But the philosopher would love to see a correlation between  $OP_X(p)$  for  $X = \text{"vacuum exists"}$  and the distance from the house of  $p$  to Abdera.

*mass equal in magnitude to the universe.*  
ARCHIMEDES, THE SAND RECKONER, 250 BCE (?).



*Archimède... dont la vie est une époque dans l'histoire de l'homme,  
et dont l'existence paraît un des bienfaits de la nature.*

NICOLAS CONDORCET.

Archimedes estimated the diameter of the universe at about 2 light-years that is  $\approx 2 \cdot 10^{13} \text{ km}$  – twenty thousand billions kilometres, about the half, as we know to-day, of the distance to the nearest stars – the binary system Alpha Centauri A & B.

Then Archimedes invented an exponential representation of large numbers and evaluated the number of sand grains or rather of  $\approx 0.2$  millimetre poppy seeds needed to fill it at less than  $10^{63}$  in modern notation. (I took these numbers from the Wikipedia article. In fact the  $2 \cdot 10^{13}$  km cube has volume  $8 \cdot 10^{57} \text{ mm}^3$  that makes  $10^{60}$  cubes of  $0.2 \text{ mm}$ .)

If a philosopher would not be impressed and say that

*a good decision is based on knowledge and not on numbers,*

Archimedes might respond that decisions may be left to our mighty rulers but that are *numbers* which are guardians of our true knowledge.

Large numbers are everywhere. Even Socrates, Plato and Aristotle would admit that *knowledge of thyself* is incomplete if you are unaware of the roughly  $10^{14}$  (100,000 billion) bacteria in your guts – several bacteria per each cell of your own body.

(Bacteria are roughly  $1\mu\text{m}$  – one thousandth of a millimetre = one millionth of meter in size, a few thousand times smaller than your own cells volume-wise. If you had a bacterium inside every one of your cells you would hardly notice this – you will be safely dead by that time.)

A single bacterium, if there is enough nutrients, can divide every 20-30 minutes and, in 24 hours you may have a blob  $10^5 \mu\text{m} = 10\text{cm}$  in size with about  $2^{50} = (2^{10})^5 = 1024^5 \approx 1000^5 = 10^6 \times 10^9$  (million billions) bacteria in it.

A schoolboy now computes:

Day 2. The blob contains  $10^{15} \times 10^{15} = 10^{30}$  bacteria and has  $10^{10} \mu\text{m} = 10\text{km}$  in diameter, about 1 kg of bacteria per every square meter of the surface of the Earth.

Day 4. It increases to  $10^{20} \mu\text{m} = 10^{11} \text{km}$  and reaches the outermost regions of the Solar system. It will engulf the Sun ( $\approx 1.5 \times 10^8 \text{km}$  from Earth), all planets including Pluto ( $\approx 6 \times 10^9 \text{km}$ ) but not fully the orbit of Sedna at its farthest point from us ( $\approx 1.4 \times 10^{11} \text{km}$ ).

(Aristotle, who maintained that

*the shape of the heaven is of necessity spherical,*

would feel relieved if he knew that the bacteria are still contained by the *spherical heavenly shell* of the hypothetical *Oort cloud* of comets around the Sun about light-year away.)

Day 7. The blob contains  $10^{15 \times 7} = 10^{105}$  bacteria and has  $10^{5 \times 7} \mu\text{m} = 10^{26} \text{km} \approx 10^{13}$  light-years in diameter, hundred times the diameter of the observable universe ( $\approx 10^{11}$  light-years).<sup>2</sup>

Bacteria have been around for billions of days, but do numbers like  $10^{100000\dots}$  make any sense at all? The answer is *yes and no*. They can not be reached by counting 1,2,3,..., at least not in our space-time *continuum*, nor be represented by *collections of physical objects* of any kind. However, unrealistically large (and unrealistically small) numbers are instrumental in our treatment of NATURE's LAWS that are manifested in observable properties of *objects* in the Universe.

How does Nature, who, as Einstein says, *integrates empirically* manage to satisfy these laws?

Is it because she has something *much bigger* than space/time (kind of quantum fields?) at her disposal where *empirical integration* is possible?

Or is there a *secret logical something* built into Nature and she proceeds by *mathematical induction* as mathematicians do?

Or had she found a *simple logical bypass* for arriving at these laws but we can not reach it being bound to the mental routes available to our brains?

These questions, probably, make no sense; it is frustrating being unable to formulate a good one.

Yet, a mathematician may find a consolation in trying to estimate the number  $N_{can}$  of different logical arguments (brain routes), say in  $L$  words, that a sentient brain *can*, in principle, generate. Probably, if somebody told our mathematician what the words *can* and *in principle* signify, he/she would bound  $N_{can}$  by something like  $\sim L \log L$ , or even less than that – far from the number  $N_{all} \sim 2^L$  of *all* such arguments, well behind of what bacteria can do.

This might hurt his/her pride but then the mathematician will soon realize that numbers that linger behind his/her own logic/language beat the fastest replicating bacteria.

Indeed, *think* of Schrödinger's cat. The body of a cat is, roughly, composed of  $N \approx 10^{26}$  molecules, those of water and small molecular residues in macromolecules. Suppose each molecule can be in two states. Then there are  $S = 2^N \approx 10^{0.3N}$  states of a cat. Some of these states are judged being *alive* and some are classified as *dead*. The number, say *CAT*, of possible judgments/opinions is

$$2^S = 2^{2^{10^{26}}} > 10^{10^{10^{25}}}$$

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<sup>2</sup>Darwin computed the numbers of descendants of a couple of elephants and arrived at the number 15 000 000 after 500 years. This is much exaggerated, but in 5 000 years the Earth would be covered several times over by more than  $10^{15}$  elephants and the whole Universe would not contain  $10^{90}$  elephants after 30 000 years. (30 000 is yesterday on the Earth geological time scale, it is less than 0,02 of 1% of (about 200 000 000 years of) the evolution time of mammals on Earth.

How does one decide, how can one select a sound judgement from this super-duper-universe of possibilities? Mathematicians do not understand how it works, but a cat, if he/she is alive and oblivious of math., somehow manages, makes a right choice and ... stays alive.

Some courageous people play with unimaginably greater numbers, the descendants of Gödel's *incompleteness theorem*. If you meet such a number on your path of reasoning about "real world" your logic is as good as dead.<sup>3</sup> Fortunately, you do not meet them in "real life" unless you call these misshapen monsters by their names.

**THE MONSTER OF *STOP*.** If your computer has  $M$  bits of memory, say with  $M = 10^{10}$ , then whatever you "ask" the computer to do, it either *stops* after  $< 2^M$  steps or it goes into a cycle and runs forever. (You can use convenient *time units* instead of "steps"; the number  $2^{10^{10}}$  is so big, it makes little difference if you take nanoseconds or billions of years for these units.)

Here, what we call a *question* or a *program* that you "ask" your computer to perform, is a sequence of *letters* that are the names of keys on the keyboard you have to press in order to activate this *program*.

Let us leave the "real world" and allow your computer to have an *infinite* (unbounded) memory. As in the finite case, the computer may stop after finitely many steps or run for infinite time *depending on your question*, (and on the design of the hardware and the operating system in your computer) except that *infinite* does not have to be *cyclic* for infinite memory. For example, if asked to find a file named *cell-phan-nimber-Bull-Gytes* the computer either finds it and stops, or, if there is no such file, runs forever.

(A fundamental ability of your *slow brain*, not shared by the *fast Windows search system*, is an *almost instantaneous NO SUCH FILE* response to this kind of inquiry. A simple, yet structurally nontrivial, instance of this is *BITTER* on your tongue for almost anything chemically away from potential nutrients, with a few mistakes, e.g. *SWEET* for saccharin.

One may speculate together with Robert Hooke, Charles Babbage and Alan Turing on plausible brain memory architectures making this possible. There is no experimental means for checking non-trivial conjectures of this kind but there is a mathematically attractive model of such a memory suggested by Pentti Kanerva, called *sparse distributed memory*.)

Pick up a (moderately large) number  $L$  and take *all those programs* in  $L$  letters for which the computer *eventually stops*. Since the number of these programs is finite ( $< 100^L$  if there are  $< 100$  different letters at your disposal for writing programs) the longest (yet, finite!) time of executing such a program, say, the time measured in years, is also *finite*; call this time  $\text{STOP}(L)$ .

Although finite, this number of years, even for moderate  $L$ , say  $\text{STOP} = \text{STOP}(50\,000)$ , (a program in 50 000 letters takes a dozen pages to write down) is virtually indistinguishable from infinity in a certain precise logical sense.

Our Universe is pathetically small not only for containing anything of this size but even for holding a *writing* of an *explicit formula* or of an *explicit verbal description* of a number of such size, even if we use atoms for letters in such a writing. (The way we have just described this number does not count – distin-

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<sup>3</sup>These numbers lie at the heart of *Turing's halting theorem* and of Kolmogorov-Chaitin's complexity. A whiff of either of the two is poisonous for any *scientific theory*.



guishing stopping times from infinity without indicating a specific "experimental protocol" for this is not what we call *explicit*.)

By comparison,  $\mathcal{CAT}$  appears tiny – the corresponding exponential formula can be expressed with a few dozen binary symbols (and *physically* written down with a few thousand atoms manipulated by means of an *Atomic Force Microscope*).

To be fully honest we must admit there was something not quite right with our definition of  $\text{Stop}(L)$ .

Imagine, for example, that the memory of our computer does contain the string *cell-phan-nimber-Bull-Gytes* but it is positioned so far away that it can not be reached less than in  $T$  time units. Since you can choose  $T$  as large as you want, the logic of our definition inevitably makes  $\text{Stop}(L)$  equal infinity, even if you limit the length  $L$  of admissible programs to something quite small, say, less than one thousand letters.

Somewhat one has to prohibit such a possibility, by insisting that all "memory cells" in your computer that *can not* be reached in less than, say,  $10^{10}$  time units are empty, nothing is written in these "far away cells". Moreover, the computer is supposed to know when it crosses the boundary of "non-empty space" and will not spend any time in searching the empty one.

On the other hand, the computer is allowed, in the course of a computation, to write/erase in these "far away cells"; this is what may eventually generate an enormous volume of occupied memory cells and make its reading excruciatingly long.

With this provision, the definition of  $\text{Stop}(L)$  becomes correct, it does give you something *finite*, PROVIDED you have *precisely* defined what "far away cells" and "reaching something in the memory" mean.

But can one *explicitly* describe in *finitely many* words an *infinite* memory along with a description of a search program through this memory?

A commonly accepted solution articulated by Turing, is to assign the memory cells/units to *all* numbers  $1, 2, 3, 4, \dots, 1000$ . with labels "*empty*" / "*non-empty*" attached to them and with the symbols 1 and 0 written along with all "*non-empty*" marks. Then an individual step of a memory search is defined as moving from an  $i$ -cell to  $i+1$  or  $i-1$ , where each cell is labeled "*non-empty*" after having been visited.

If you are susceptible to the magic of the word "all" and believe this truly defines the infinity of numbers, then you will have a "mathematically precise" definition of  $\text{Stop}(L)$ .<sup>4</sup>

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<sup>4</sup>There are, probably, 5-10 mathematicians and logicians in the world who think that the monsters like  $\text{Stop}$  are indicators of fundamental flaws in our concepts: "number", finite", infinite".

All this being said, there is something not quite right with **STOP**: it is a byproduct of the machinery of formal logic – not a *true mathematical* number. But about thirty years ago, **pretty HUGE** numbers were discovered, e.g. the time Hercules needs to slay Hydra in a mathematical model of this regrettable parricide. But all such *GIGANT BEAUTIES* are incomparably smaller than the ugly looking **STOP**.

## 4 Laws.

Lex 1: *Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.*

ISAAC NEWTON, PHILOSOPHIE NATURALIS  
PRINCIPIA MATHEMATICA, 1687.<sup>5</sup>

*... les lois générales, connues ou ignorées,  
qui règlent les phénomènes de l'univers,  
sont nécessaires et constantes.*

NICOLAS CONDORCET.

*So much as we know of them [the laws of nature] has been developed by the successive energies of the highest intellects.*

MICHAEL FARADAY.

The most astounding single event in the history of science, a physicist argued, was the discovery of the *general relativity* in 1916 – this would have been postponed by 20-30 years if not for Einstein, twice as long as any other scientific discovery, according to our physicist's calculation on the back of an envelope.

Our physicist, a theoretician, could not confirm his calculation by an experiment, but the history has done it for him and... proved him wrong.

*The true logic of this world is the calculus of probabilities.*



JAMES CLERK MAXWELL.

Mendel's theory of genes – units of inheritance, was published in 1866, where Mendel derived their very existence and essential properties from

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<sup>5</sup>Historically, the *first recorded* LAW – the velocity/force formula for motion in viscous medium – was stated by Aristotle. (Later versions of this were suggested by Newton, 1687, and by Stokes, 1851.) A century after Aristotle, Archimedes discovered basic laws of statics of mechanical systems: the laws of the Lever and of equilibrium of solid bodies in liquids.

*the striking regularity with which the same hybrid forms always reappeared*  
in thousands of his experiments with pea plants.

Discovery of genes was the greatest event in biology since the discovery of cells by Robert Hooke in 1665 and of infusoria by Antonie van Leeuwenhoek in 1674.

Mendel's methodology of

combinatorial design of multistage interactive experiments  
+  
extracting specific structural information from statistics  
of observable data by mathematical means

was novel for all of science. This is why Mendel's paper was ignored by biologists for about thirty years.

When similar data were obtained and analyzed by de Vries, Correns, and Tschermak at the turn of the 20th century, biologists returned to Mendel's paper. Many, including Alfred Russel Wallace, were appalled by Mendel's ideas, while even the most sympathetic ones were confounded by "counterintuitive" and "biologically unfeasible" Mendel's algebra.

In 1908 a leading English mathematician G. H. Hardy and independently a German physician Wilhelm Weinberg spelled out this *counterintuitive* as

$$\frac{[(p+q)^2 + (p+q)(q+r)]^2}{[(p+q)^2 + (p+q)(q+r)] \cdot [(p+q)(q+r) + (q+r)^2]} = \frac{(p+q)^2}{(p+q) \cdot (q+r)}$$

and MENDEL'S LAWS OF INHERITANCE were accepted by (almost) everybody.

(Hardy called this *a mathematics of the multiplication table type*. He overlooked the mathematical beauty of *Mendelian dynamics* of the *next generation maps*  $M$  in the spaces of *truncated polynomials* – the maps that represent transformations of *distributions of alleles* in populations under *random matings*. In a simple instance, such an  $M$  applies to matrices  $P = (p_{ij})$  by substituting each  $(i, j)$ -entry  $p_{ij}$  by the product of the sums of the entries in the  $i$ -row and the  $j$ -column in  $P$ ,

$$(p_{ij}) = P \xrightarrow{M} P^{next} = (p_{ij}^{next}) \text{ for } p_{ij}^{next} = \sum_i p_{ij} \cdot \sum_j p_{ij}.$$

It is amazing, albeit obvious, that  $M(M(P)) = const \cdot M(P)$  for  $const = \sum_{ij} p_{ij}$ , where this amounts to the above  $(p, q, r)$ -formula for symmetric  $2 \times 2$  matrices in Hardy's notation.)

*... are we justified in regarding them [genes] as material units; as chemical bodies of a higher order than molecules? ... It makes no difference in... genetics. Between the characters that are used by the geneticist and the genes his theory postulates lies... embryonic development.* THOMAS HUNT MORGAN, 1934.

In 1913, almost half a century after publication of Mendel's paper *Versuche über Pflanzen-Hybriden*, 21 years old Alfred Sturtevant made the next step along Mendelian lines of logic and determined *relative positions* of certain genes on one of the chromosomes of *Drosophila* by analyzing frequencies of specific morphologies in generations of suitably interbred flies.

Just think about it. You breed fruit flies, you count how many have particular *combinations* of certain features, e.g. you record the distribution of occurrences of the following eight ( $2 \times 2 \times 2$ )-possibilities:

[striped bodies]/[yellow bodies]  
[red eyes]/[white eyes]  
[normal wings]/[smallish wings],

and you distinctly see with your mathematically focused mind eye (even if you happened to be color blind as Sturtevant) that the corresponding genes – abstract entities of Mendel's theory that are associated with these features, are all lying in definite relative positions on an imaginary line, where, as for Mendel's genes, the existence of this line is derived from how *hybrid forms reappear*. In particular, you assign the "eyes gene" a position *between* the "body gene" and the "wing gene", since

[smallish wings] + [yellow bodies] **"implies"** [white eyes]  
in descendants of certain parents with an abnormally high probability.

Decades later, molecular biology and sequencing technology demystified "Sturtevant's line" by identifying it with a DNA string segmented by genes, but a mathematical unfolding of Sturtevant's idea still can be seen only in dreams.

ABOUT ROBERT HOOK, ANTONIE VAN LEEUWENHOEK,  
DROSOPHILA MELANOGASTER  
AND THE IDEA OF STURTEVANT.

*Hooke's name* is associated with *Hooke's elasticity law* but that was only one of many of his experimental discoveries, original concepts and practical inventions. For example, he recognized fossils as the remains of *extinct species*, he developed an almost modern *model of memory*, he proposed (1665?) the construction of *the spring balance watch* (Huygens' description of his own construction dates to 1675) and he suggested (1684) a detailed design of an *optical telegraph with semaphores*. (The first operational system with a network of  $\approx 500$  stations was built in 1792 in France.)

*Leeuwenhoek* found out how to obtain small glass balls for the lenses of his microscopes but he made others to believe he was grinding tiny lenses day and night by hand. Besides infusoria, he observed and described crescent-shaped bacteria (*large Selenomonads*) the vacuole of the cell and the spermatozoa. The secret of Leeuwenhoek's microscopes was rediscovered in 1957.



*Drosophila melanogaster* – the pomace fly ( $\approx 2.5$ mm long, normally with red-eyes and striped bellies) were introduced as a major model organism in genetics by Thomas Hunt Morgan. He and his students were counting the mutant characteristics of thousands of flies and studied their inheritance. Analysing these

data, Morgan demonstrated that genes are carried on chromosomes; also he introduced the concepts of *genetic linkage* and of *crossing over*.

*Sturtevant* mathematically "synthesised" his string of genes from the "substrate" of results and ideas taken from the work by Morgan almost as Kepler "crystallized" elliptical orbits from Tycho Brahe's astronomical tables. Sturtevant recalls the great moment as follows.

*I suddenly realised that the variations in strength of linkage, already attributed by Morgan to differences in the spatial separation of the genes, offered the possibility of determining sequences in the linear dimension of a chromosome. I went home and spent most of the night (to the neglect of my undergraduate homework) in producing the first chromosome map, which included the sex-linked genes  $y$ ,  $w$ ,  $v$ ,  $m$ , and  $r$ , in the order and approximately the relative spacing that they still appear on the standard maps.*

Sturtevant's idea of (re)construction of the (a posteriori linear) geometry of the genome is similar to Poincare's suggestion on how the brain (re)constructs the (a posteriori Euclidean) geometry of the external world from a set of samples of retinal images.

Grossly oversimplifying, an unknown geometric (or non-geometric) structure  $S$  from a given class  $\mathcal{S}$  on a set  $X$  under the study – be it the set of (types of) genes in the genomes (of organisms) of a given species or the set of photoreceptor cells in the retina – is represented by some probability measure on the set of subsets  $Y$  of  $X$ . What is essential, this measure is supported on  $\mathcal{S}$ -simple (special) subsets  $Y$  that admit *short descriptions in the language of  $\mathcal{S}$* ; this allows a reconstruction of  $S$  from relatively few samples.

This sampling is far from random. In genetics, such a  $Y = Y(O)$  is the subset of genes of particular allele versions in the genome of an individual organism  $O$ , where these  $O$  are obtained via a *controlled* breeding protocol that had been specifically designed by an experimentator.

In vision, such a  $Y = Y(t)$  is the set of excited photoreceptor cells in the retina of your eye at a given moment  $t$ , where more often than not the variation  $Y(t)$  with  $t$  is due to a motion of an object relative to your eye; the brain, that commands the muscles that move your eye, has an ability to design/control such variations.

The hardest step in finding  $S$  is guessing what  $\mathcal{S}$  is. After all, what is a structure?



Mendel's laws are no more than a Platonic shadow – a statistically averaged image of the workshop of Life on the flat screen of numbers. The molecular

edifice of the cell is crudely smashed on this screen and its exquisite structure can not be reconstructed from this image by pure thought. Hundreds (thousands?) ingenious experiments are needed to recover the enormity of information that had been lost.

*... I hold it true that pure thought can grasp reality,  
as the ancients dreamed.* ALBERT EINSTEIN.

Contrary to what we see in biology, the mathematical image of the basic machinery running the world of physics retains the finest details of this machinery. It even may seem, probably, only to our *Naive Mathematician*, that the less you know the better you understand how the Universe is run.

For example, forget about velocities, forces, accelerations. Imagine a world exclusively populated by wandering watches that have no perception of speed and force. But when two watches meet, they can recognize each other and compare their records of the intervals of times between consecutive meetings.

A watch-mathematician would sum up what he/she believes he/she "sees" in the watch-world as *self-obviuos axioms* and, after pondering for a few centuries on what they imply, he/she will figure out that there is a unique simplest most symmetric *watch space* of every given dimension. It is the *Lorentz-Minkowski time-space*, that is four dimensional in the Universe we happened to exist.

The mathematician will be delighted by this marvellous space-idea; yet perplexed, since his/her mental picture of the world does not explain

*why the watches that have no physical contact apart from their meeting points remain synchronous.*

(Here, on Earth, it is not this incredible synchronisation but its violation is regarded paradoxical.)

But then his friend physicist conceives the idea of speed and his colleague experimentalist designs fast traveling watches. The mathematician sighs with relief: the formulas of his/her theory (that is called *special relativity* on Earth), are perfectly right and desynchronisation is clearly seen for watches traveling with the mutual relative speeds close to 1. (On Earth, this 1, that is the speed of light, is elegantly expressed as 299 792 458 ... $\times$  another unit of speed the meaning of which no watch-mathematician has ever been able to grasp.)

*Lines of force convey a far better and purer idea...*  
MICHAEL FARADAY, 1833.<sup>6</sup>

*Thy reign, O force! is over. Now no more  
Heed we thine action;  
Repulsion leaves us where we were before,  
So does attraction.*

JAMES CLERK MAXWELL, 1876.

*The theory of relativity by Einstein...  
cannot but be regarded as a magnificent work of art.*  
ERNEST RUTHERFORD<sup>7</sup>.

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<sup>6</sup>*Faraday must have grasped with unerring instinct ... spatial states, today called fields...*  
ALBERT EINSTEIN, 1940.

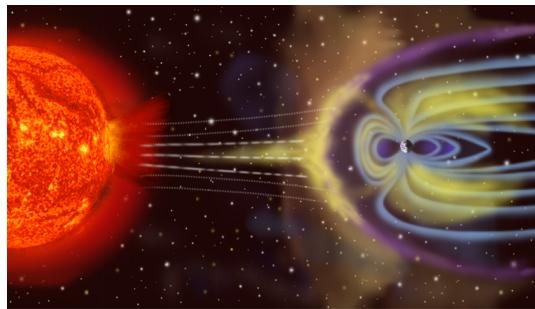
<sup>7</sup>Rutherford, named Faraday of nuclear physics, experimentally identified alpha, beta and gamma emissions, discovered atomic nuclei and proposed protons and neutrons for their constituents. "*All science is either physics or stamp collecting*" is a saying attributed to him.

Next our mathematician will look not for a most symmetric space but for the most symmetric *law* of "motion" applicable to all imaginable to him/her watch-spaces filled/strained by Faradayan-like *lines/fields of force*.

First he/she thinks that no such distinguished law is possible, but then some terms in his/her calculation miraculously cancel each other and a beautiful equation pops out. He/she, undoubtedly, will call it *Einstein vacuum equation*. (This was so derived by David Hilbert<sup>8</sup>.)

Then his friend physicist will bring in *energy/matter* and, to everybody's satisfaction, an experimentalist/cosmologist will show that the universe behaves as predicted by the resulting equation – the simplest mathematically imaginable description of [watch/space]-worlds – *general relativity theory*.

*The greatest change in ... our conception of the structure of reality - since Newton...was brought about by Faraday's and Maxwell's work on electromagnetic field phenomena.*



ALBERT EINSTEIN, 1931.

*... ten thousand years from now,  
the most significant event of the 19th century  
will be judged as Maxwell's discovery of the laws of electrodynamics.*

RICHARD FEYNMAN, LECTURES ON PHYSICS", 1964.<sup>9</sup>.

The watch world symmetry, that was acclaimed by the terrestrial physicists of the 20th century, originated in the work by Maxwell who, over 1855 – 1873, thought out (a system of twenty) differential (wave) equations that logically embrace and unify the *Ampere law* (1826) on magnetic effects of electric currents and *Faraday's law of induction* of electric currents by moving magnetic fields (1831).

Formally, these are *Lagrange-Hamilton equations* dictated by *Maupertuis' principle* for electromagnetic fields; they possess a remarkably high – what is now called *Lorentzian* – symmetry that, as Einstein wrote in 1953,

*transcended its connection with Maxwell's equations.*

---

He, apparently, regarded chemistry as a part of physics and he did not live to see the birth of molecular biology from the seed of the Mendelian-chromosome theory of heredity.

<sup>8</sup>Hilbert was the one who, along with Poincaré, guided the passage from the 19th to 20th century mathematics.

<sup>9</sup>The 120th century seems far off – *another age must be the judge* – Charles Babbage's writes in his *Ninth Bridgewater Treatise* (1837). His design of analytic engine (that implemented what is now called the *universal Turing machine*) will be the most likely number one on the **19TH CENTURY SIGNIFICANT EVENTS** list of a robot of the 22nd century.

(The historically first *wave equation* – the equation of a vibrating string, was written down and studied by d'Alembert in 1747; apparently, its *Lorentzian symmetry* went unnoticed until much later.)

The prima donna role for this symmetry in physics hinted at by FitzGerald in 1889 and suggested by Larmor in 1897 and by Lorentz in 1899, was consummated in the two 1905 papers:

*Sur la dynamique de l'électron*<sup>10</sup> by Poincare

and

*Zur Elektrodynamik bewegter Körper* by Einstein.

Two years later, Hermann Minkowski, who was usually preoccupied (besides convex bodies) with multidimensional geometry of *quadratic equations in many variables* (quadratic forms) and with their transformation groups suggested a 4-dimensional geometric realisation of the (Poincare)-Lorentz symmetry. The Minkowski 4-D geometry was extended by Einstein in 1916 to (*Einstein*)-*Lorentz spaces* that he used as a mathematical framework for *the general relativity theory*.

Now, imagine a technologically backward watch-civilization where no high energy experiment is available. Here a mathematician will have to invent some mystical *absolute time* that will synchronize non-interacting watches as windowless *monads* of Leibniz.

*Absolute, true and mathematical time, of itself, and from its own nature flows equally without regard to anything external, and by another name is called duration.* ISAAC NEWTON.

Then mathematician's friend physicist breaks somewhere a window from where he could see motions of other watches and the mathematician will realise that granting an *absolute time* to a Lorentz space necessarily implies an *absolute space* where motion is possible.

Eventually the two – the mathematician with his physicist friend – such a couple is called *Isaac Newton* on Earth – will arrive at the THREE LAWS and, when they come to Earth, they incorporate INVERSE SQUARE LAW for gravitation into their theory, where "square" is suggested (implied?) by the geometric

$\text{AREA} \sim R^2$ -LAW: the area of the surface of the  $R$ -sphere is proportional to  $R^2$  where the exponent 2 comes as  $2 = 3 - 1$  for 3 being the dimension of our physical space.

(Newton mathematically *proved* that *only* the gravitation  $\sim 1/R^2$ -LAW agrees with the astronomical observations<sup>11</sup>, but it is unclear who was the first to *conjecture*  $\sim 1/R^2$ . Robert Hooke claimed to be the one. Indeed, in his communication to the Royal Society of 1666 he states:

*... heavenly bodies... mutually attract each other within their spheres of action. ... so much the greater as the bodies are nearer.*

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<sup>10</sup>where the Lorentz group materialised in the words

*... the equations of the electromagnetic field are not altered by a certain transformation (which I will call by the name of Lorentz) of the form:....*

The principle of relative motion was discussed by Poincare in earlier papers but that was not quite what we call *special relativity theory* as formulated by Einstein.

<sup>11</sup>Planetary motions to which Newton's equations apply are *not* directly observable. It took the successive energies of the highest intellects of (not only) Copernicus, Tycho Brahe and Kepler to make the observable data amenable to mathematical analysis.

But others, including Galileo, Kepler and Newton himself, expressed similar ideas on gravitation.

Newton regarded the proof of  $\sim 1/R^2$  as by far more difficult than guessing it. Since he was the only person on Earth, if not in all astronomically observable<sup>12</sup> Universe, qualified to approach the problem and to evaluate its mathematical difficulty, we accept his judgement.)

But despite the remarkable agreement of the theory with the recorded planetary orbits on the short time intervals (thousands years) a disturbing question will nag our friends.

Are these laws – the laws of the classical mechanics + the inverse square law, *consistent* with the apparent stability of the solar system on the time scale of millions and hundreds millions years?

Newton himself believed the answer was NO and that planets would collide with the Sun if not for the divine intervention from time to time. But about 250 years after Newton an optimistic *maybe* was suggested by a counterintuitive mathematical theorem, usually referred to as *KAM*, that (very roughly) says that *quite a few* physically significant dynamical systems may behave "asymptotically rather periodically", contrary to what physicists (as well as mathematicians) had always believed. (Everybody's intuition suggested "asymptotically chaotic" behavior of *predominant majority* of mechanical systems with more than two degrees of freedom.)

This theorem depends on a hidden *symplectic symmetry* between the kinetic and potential energies as it is seen on yet another mathematical screen discovered by Hamilton  $\approx 100$  years after the death of Newton.

*Quantum electrodynamics describes Nature as absurd  
from the point of view of common sense.*

*And it fully agrees with experiment.*

RICHARD FEYNMAN. QED: THE STRANGE THEORY  
OF LIGHT AND MATTER, 1985.

*Nothing is too wonderful to be true,  
if it be consistent with the laws of nature;  
and in such things as these experiment  
is the best test of such consistency.*

MICHAEL FARADAY,  
LABRATORY JOURNAL ENTRY, 1849

Is the running of "watches" themselves governed by classical/relativistic mechanics?

Probably it is not hard to "prove" by an argument in the spirit of Zeno's paradoxes that no mechanical/electromagnetic model of Newton+Maxwell+Einstein can be compatible with the properties of *matter* we see everywhere around us. Apparently, our beautiful physical laws are "just" Mendelian kind of images of something else where this "else" is expected by physicists to live in the *quantum world*.

Mathematical fragments of *quantum* may be accessible to us but when we try to imagine it as a whole our mind revolts and if we insist it is getting giddy

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<sup>12</sup>"Observable" here, rather parochially, refers to an observer centred somewhere in the vicinity of *the Milky Way galaxy*.

in a tangle of paradoxes and ambiguities. And little consolation can be found in what physicists keep repeating after Niels Bohr<sup>13</sup>:

*If anybody says he can think about quantum physics without getting giddy, that only shows he has not understood the first thing about them.*

You hardly can see a clear picture of anything being giddy at the same time, but you will feel better if you could prove mathematically that, in principle, no common sense (including rigorous mathematics) model of *anything* resembling *physical world* is possible.

"Thinking quantum" is incompatible with our deeply ingrained intuition of "reality." As Wolfgang Pauli<sup>14</sup> says:

*The layman always means, when he says "reality" that he is speaking of something self-evidently known; whereas to me it seems the most important and exceedingly difficult task of our time is to work on the construction of a new idea of reality.*

He also writes:

*Just as in the theory of relativity a group of mathematical transformations connects all possible coordinate systems, so in quantum mechanics a group of mathematical transformations connects the possible experimental arrangements.*

This is only a hint on reality being a property of some transformation group, but a realistic idea of "physical reality" remains beyond our wildest dreams.

And quoting Pauli again,

*The best that most of us can hope to achieve in physics is simply to misunderstand at a deeper level.*

Yet, maybe, there remains a hope of seeing the image of the world in the mirror of mathematics.

*ἀγεωμέτρος μηδελταεὶς εἰστω.  
Let no one ignorant of geometry enter.*

This is associated with the name of Plato who did not thought much of those unfortunates who were

*ignorant that the diagonal of a square is incommensurate with its side.*

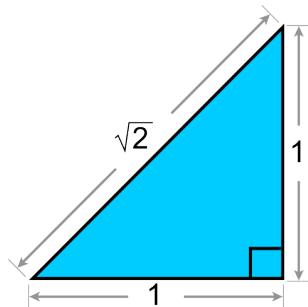
(Twenty four centuries later, certain Plato's disciples went further by suggesting that *the true human consciousness* is distinguished by the ability of its possessor to grasp validity, meaning and implications of "incommensurability" between certain *theories* of numbers (rather than between certain individual numbers) that follows from *Gödel's incompleteness theorem*.

According to this supposition, any conceivable *imitation* of human mind, be it by an electronic implementation of a Turing universal computer or by a bio-robot, would be instantaneously recognizable by the flagrant absence of this ability.

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<sup>13</sup>In 1913, Bohr invented the concept of *quantization of a physical system* in the context of the *planetary model of the atom* that was suggested by Rutherford in 1911.

<sup>14</sup>Pauli (1900 -1958) introduced spin into quantum mechanics formulated *Pauli exclusion principle* and he predicted the existence of *neutrino*.



However, no significant data on a presence/absence of this ability in human populations have been collected despite the renewed interests of neuropsychologists in the problem of consciousness.)

*Whoever despises the high wisdom of mathematics  
nourishes himself on delusion.*

LEONARDO DA VINCI.

*Every new body of discovery is mathematical in form,  
because there is no other guidance we can have.*

CHARLES DARWIN.

*To those who do not know mathematics  
it is difficult to get across a real feeling  
as to the beauty, the deepest beauty, of nature.*

RICHARD FEYNMAN.

When does a fragment of science qualify for a LAW OF NATURE? Why does finding these laws require successive energies of the highest intellects?

Isn't a LAW just a compression of information, a record of systematic correlations between facts where these correlations can be found by analyzing your observations?

"Yes" or "No" answer to this question depends on how you understand just, information, systematic, etc. where an essential (but not the only) difficulty facing a "law maker" is seen in the following example.

Imagine, that your facts or events are numbers, where the ones you happened to observe are

$$7, 19, 37, 56, 61, 91, 127, 189, 208, 296, 342, 386.$$

The only effective approach to finding "the law" behind these numbers is... to guess: THESE NUMBERS ARE DIFFERENCES OF CUBES<sup>15</sup>,

$$1 = 1^3, \quad 8 = 2^3, \quad 27 = 3^3, \quad 64 = 4^3, \quad 125 = 5^3, \quad 216 = 6^3, \quad 343 = 7^3, \dots$$

For instance,

$$91 = 216 - 125, \quad 127 = 343 - 216, \quad 189 = 216 - 27, \quad 386 = 9^3 - 7^3.$$

(This is a kind of how it happens in Bohr's rendition of *Balmer-Rydberg formula* for the wave length  $\lambda$  of the Hydrogen spectrum, e.g. observed in the stars. This formula, in suitable units, says that  $\frac{1}{\lambda} = \frac{1}{m^2} - \frac{1}{n^2}$ , with the rationale behind this provided by Bohr's quantized model of the Hydrogen atom.)

<sup>15</sup> A little mathematics of the multiplication table type tells you that the predominant majority of numbers are not differences of cubes. This makes this  $(m^3 - n^3)$ -law quite restrictive; hence, informative.

In general, a mathematician may say that a law  $\mathcal{L}$  is a "simple" map (function) from some "small and simple" *parameter set*  $P$ , (e.g. the set of pairs of cubes of integer numbers) *onto* a "complicated" subset  $Obs$  of observable events (differences of cubes of integers) within another "simple but possibly large" set  $Ima$  of all imaginable events (e.g. of all integers or, if you wish, of all real numbers that make a much larger, yet logically equally simple set).

The number of conceivable *simple laws*  $\mathcal{L}$ , e.g. of those describable in a few dozen words, is enormous – it grows exponentially with the number of words/symbols encoding an  $\mathcal{L}$ . There is no (?) general rule for guessing such an  $\mathcal{L}$ , from a realistic number of samples from  $Obs$  in each particular case<sup>16</sup>. And *he who does not know what he is looking for will not understand what he finds* – as Claude Bernard says.

(One may object by pointing out that an animal/human brain builds up a coherent mental picture of the external world by *systematically* finding correlations and cutting off redundancies in the flows of electric/chemical signals it receives from other parts of the body.

It is hard to argue since we do not know what is the *system* used by the brain, but, in any case, this system was designed by Nature *not at all* for discovering her inner workings.)

If you are a biochemist rather than a mathematician, you may visualize a *Law of Nature*. as an intricately shaped "molecule", a kind of "logical enzyme" for "catalytic synthesis of theories" out of "solution of empirical facts", where the role of solvent – the supporting matrix of the process, is played by mathematics.

The *architecture of logic* may vary from law to law and the corresponding *chemistries of facts* may have nothing in common (as statistical mechanics and classical genetics, for example) but the principles of *mathematical catalysis* remain the same for most (all?) *enzymatic laws* worth of the name **LAW OF NATURE**.

But beware: a shorthand expression of a law, e.g. of inertia – *Corpus omne perseverare...* – a string of a hundred characters, tells you as little of the structure and the function of a law, as a sequence of hundred amino acids says by itself about the physiological role of the corresponding protein. In order to read the message encoded by such a string you need to have a fair insight on the mathematical nature of the *solvent matrix* as much as on the *natural chemistry of facts*.

And if no mathematics in your head is available for this purpose, you shall not understand much of the corresponding law if at all.<sup>17</sup>

This was what happened to Mendel's ideas – apparently, mathematics and physics were not in the biology curricular of that time. If Mendel, instead of Nägeli had corresponded with somebody like Boltzmann, Guldberg and Waage or van 't Hoff, the timetable of genetics might have been shifted by a quarter of a century.

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<sup>16</sup> Making a correct guess is, in most cases, much harder than *verifying* one. There are several *conjectures* in mathematics, e.g.  $P \neq NP$ , that attempt to rigorously express this idea but, probably, we do not know yet how to *properly formulate* the problem.

<sup>17</sup> The laws of mechanics are special in this respect – they are built into the motor systems of the brain. *Instinctively*, we always try to find mechanical explanations (models) of everything around us. However, these "brain's laws" are, by necessity, non-Newtonian – true Newtonian mechanics is *instinctively rejected* by most (all?) people even if they formally understand it. (One wonders if this can be tested by experimental psychologists.)

NÄGELI, BOLTZMANN, GULDBERG, WAAGE, VAN 'T HOFF.

*Karl Wilhelm von Nägeli* (1817– 1891) – a leading botanist of the 19th century, introduced the concept of *meristem* – a group of plant cells capable of division; he also realized the role of sequences of cell divisions in plants morphology. But his fame rests on him being the first on the list of people who failed to understand Mendel's work.

*Ludwig Eduard Boltzmann* (1844 -1906) received his PhD on kinetic theory of gases in 1866 – the year of publication of Mendel's *Versuche über Pflanzen-Hybriden*.

*Cato Maximilian Guldberg* (mathematician) and *Peter Waage* (chemist) proposed in 1864 the *law of mass action* in chemical kinetics that was logically similar to Mendel's laws; their work went unnoticed until it was rediscovered by van 't Hoff in 1877.

*Jacobus Henricus van 't Hoff* (1852 -1911) was the first winner of the Nobel Prize in Chemistry for his discoveries in chemical kinetics, chemical equilibrium, osmotic pressure and stereochemistry.

The significance of Mendelian hybridization data, that are similar to the *simple volumes ratios property*, also called *the law of combining volumes*, of reacting gases discovered by Gay-Lussac and reported in his 1808 article

*Sur la combinaison des substances gazeuses, les unes avec les autres*  
would have been apparent to these people. (Gay-Lussac's law says, in Avogadro's words, that

*les combinaisons des gaz entre eux se font toujours selon des rapports très simples en volume, et que lorsque le résultat de la combinaison est gazeux, son volume est aussi en rapport très simple avec celui de ses composants.*)

Boltzmann especially, a proponent of atomic theory of matter and energy, would have been delighted by Mendel's idea of *atoms of inheritance* conceived with the logic similar to that involved in Avogadro's derivation of the Gay-Lussac law from his atomic conjecture:

*... the number of integral molecules in any gases is always the same for equal volumes, or always proportional to the volumes.*

This is called now-a-days the AVOGADRO LAW. It goes well along with the Gay-Lussac law if you assume that chemical substances are composed of distinct mutually identical units (atoms or molecules) but one can not a priori exclude these units from being *infinitesimally small* in the sense of Leibniz and, accordingly, the *number* of molecules in a finite volume of gas being *infinitely large*. (Such *infinitely large/small* numbers are describable now-a-days in the language of *non-standard analysis*. For instance, the number **STOP** that we met may be regarded as infinitely large and particles of mass **STOP<sup>-1</sup>** as infinitesimally small.)

But this *number* (defined now-a-days as the number of atoms in 12 grams of pure carbon  $^{12}\text{C}$ ) happened to be finite and not terribly large: the (*Avogadro*) number  $N_A$  of molecules in  $\approx 22.4$  liters of a gas at  $0^\circ\text{C}$  and 1 atm pressure is  $N_A \approx 6 \cdot 10^{23}$ . This also  $\approx$ equals the number of molecules in 18g of water  $\text{H}_2\text{O}$  of molecular weight  $\approx 18$ . If atoms were million times smaller with  $N_A = 6 \cdot 10^{41}$

instead of  $6 \cdot 10^{23}$ , the idea of atoms could forever remain only an idea.<sup>18</sup> (If atoms were so small, then the age of the Universe – a few billion years – would be too short for the  $1m\mu$  cell and, consequently, for our size organisms, to evolve. Then even the idea of atoms could not be possible – no brains no ideas... unless small atoms would allow brainy organisms of  $1 m\mu$  in size.)

Avogadro's ideas on atoms had no better luck than those of Mendel on genes – they were generally accepted only years afterwards. But if the problem with Mendel was because his contemporary biologists were not up-to his level of science and mathematics, the physicists of the 19th century were skeptical about atoms because of their (physicists') high standards for acceptance of new ideas. Faraday, for example, who understood the problem of atoms as much as Avogadro did, says:

*it is very easy to talk of atoms, it is very difficult to form a clear idea of their nature, especially when compounded bodies are under consideration.*

(The logical/philosophical problem of atoms had been also very much on the mind of Lavoisier; today, we know that the concept of atoms is self contradictory in the context of classical (non-quantum) physics. But in the natural science, unlike to how it is in mathematics, a contradiction does not necessarily invalidate an idea.)

Atoms and molecules were always on physicists' minds. For example, Maxwell writes:

*Assuming that the volume of the substance, when reduced to the liquid form, is not much greater than the combined volume of the molecules, we obtain from this proportion the diameter of a molecule. In this way Loschmidt, in 1865, made the first estimate of the diameter of a molecule. Independently of him and of each other, Mr. Stoney in 1868, and Sir W. Thomson, in 1870, published results of a similar kind, those of Thomson being deduced not only in this way, but from considerations derived from the thickness of soap-bubbles, and from the electric properties of metals.*

*According to the table, which I have calculated from Loschmidt's data, the size of the molecules of hydrogen is such that about two million of them in a row would occupy a millimetre, and a million million million million of them would weigh between four and five grammes!*

Maxwell might have been not 100% certain if there is any truth in the dynamical theory of gases... but eventually, atoms won under efforts by Boltzmann and when Einstein and Smoluchowski wrote down an equation for describing the stochastic, called *Brownian*<sup>19</sup>, motion of microscopic particles suspended in a fluid that allowed an evaluation of "the size of atoms".

The idea behind this equation is old and simple:

*... small compound bodies...  
are set in perpetual motion  
by the impact of invisible blows... .  
The movement mounts up from the atoms*

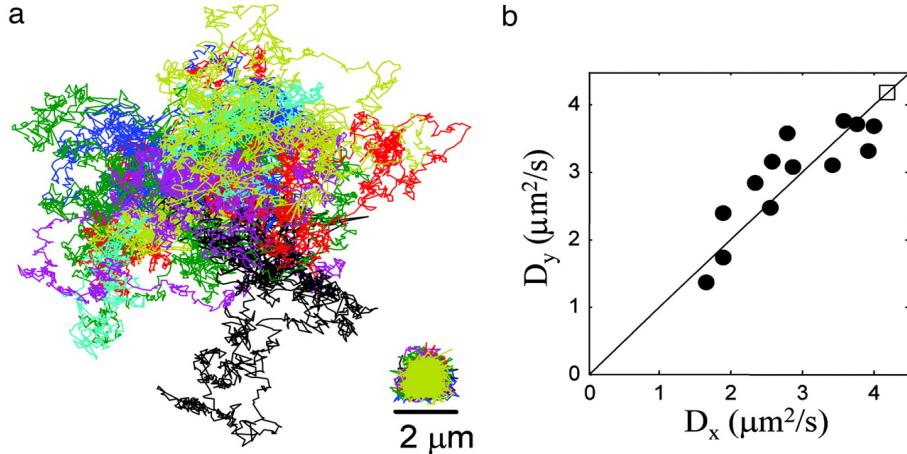
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<sup>18</sup>  $10^{-6}$  in liner size makes  $10^{-18}$  in volume and in mass for  $18 = 41 - 23$ .

<sup>19</sup> Robert Brown (1773 –1858) did not discover Brownian motion but he gave a detailed description of a cell nucleus and of other cellular structures. Brownian motion was systematically studied for the first(?) time by Jan Ingenhousz around 1785. (Ingenhousz also pinpointed the role of sunlight in photosynthesis.)

*and gradually emerges  
to the level of our senses.*

TITUS LUCRETIUS, 50 BCE(?)



But it took nearly two thousand years to turn poetry into science by translating these lines to mathematical language.

This "translation" was performed in different contexts independently(?) by:

*Thiele (1880), Bachelier (1900), Einstein (1905), Smoluchowski (1906), Wiener (1923)*, where the mathematics of this translation was not terribly far from *the multiplication table type*.

(The idea would have been apparent, to Pascal and to Buffon, while Euler and/or Laplace, would have had no problem performing the detailed computation, IF either of them had put his mind on it.)

Is it sheer luck or is it a consequence of the *weak anthropic principle* that Brownian displacement of particles is visible with an optical 1000× microscope?

Molecules are too small to be detectable by the human eye in the visible light spectrum, but their impacts are felt by objects of intermediate size.

Say, water molecules,  $\approx 0.3 \cdot 10^{-3} \mu\text{m}$ , are *only(!)* 1000 times smaller (twice on the logarithmic scale normalized by our eyesight), than the smallest optically distinguishable objects/displacements,  $\approx 0.2 \mu\text{m}$ . A  $1 \mu\text{m}$  bacterium, for example, scaled by the factor 1 000 000 to the human size ( $1\text{m} = 10^6 \mu\text{m}$ ) would "see" molecules of water as grains of sand.

And water molecules move fast: their (square) average speed is approximately  $650\text{m/s} \approx 1000^3 \cdot 0.5 \mu\text{m/s}$  at the room temperature. (Muzzle velocity of a bullet varies 200 -1200 m/s for most guns.)

What one observes in reality are displacements of particles suspended in water under *multiple* collisions with water molecules where disproportionately many of them move in a certain direction. The frequency and average size of these displacements is related to the Avogadro number (or, essentially equivalently, to the *Boltzmann constant*) via the Einstein – Smoluchowski formula (The reciprocal to the *Avogadro number* corresponds to "the volume size of an atom".)

Performing a measurement and an accurate evaluation of the Avogadro con-

stant was by no means simple – this was achieved by Jean Baptiste Perrin and his team around 1913-1914 with the resulting value for the Avogadro number:  $6.03 \cdot 10^{23}$ . (The accepted to-day value is  $6,0221\ldots \cdot 10^{23}$  that is obtained with X-ray crystallography of silicon single crystals.)

Do the laws of the classical (*non-molecular*) genetics, and/or the *classical* laws of physics and physical chemistry of the 19th century still have something new and interesting to offer to us?

A scientist would find it unlikely but a mathematician may think otherwise – we do not feel that anything is understood, unless it is expressed in the language of the 21st century mathematics. But even the 20th century mathematical rendition of something as innocuous as the 18th century *ideal gas laws*, (e.g. in terms of Riemannian and/or integral geometry) seems non-trivial. (The 20th century mathematics "dissolves" the law of inertia in the "matrix" of geodesic flows over spaces with affine connections; we expect something more interesting from such "matrix" in the 21st century.)

Mathematics also serves as a *cord* that ties laws to observations – the *empirical truth* of a law is not seen in passively observed *facts*. Mathematical ideas enter, often implicitly, into design and interpretation of experiments.

Nobody, for example, ever saw a *body* moving by itself with constant velocity along a straight line – the law of inertia stands in a stark contradiction with what you observe. Yet, you can not draw a mathematically consistent/elegant picture of mechanics without this law, e.g. you can not write down an esthetically acceptable (say, given by analytic functions) formula compatible with Galileo's rolling balls on inclined planes without *taking* the balls velocity being *constant* for the inclination angle *zero*.

## 5 Truth.

*La pensée ne doit jamais se soumettre,  
ni à un dogme, ni à un parti, ni à une passion,  
ni à un intérêt, ni à une idée préconçue.*

HENRI POINCARÉ .

When you point out to something naive or just plain silly in writings by a scientist of the past, one may say that you yourself are being foolish by judging an idea of yesterday from the standpoint of today.

Well... this would be convincing if not for people like Lavoisier, Claude Bernard, Faraday, Poincaré. They had fine ears for the ringing of scientific truth – they never (almost never?) said anything senseless; mistaken – maybe, but never trivial, vacuous or intrinsically inconsistent.

*Qu'un objet change d'état ou seulement de position,  
cela se traduit toujours pour nous de la même manière  
par une modification dans un ensemble d'impressions.*

HENRI POINCARÉ. LA SCIENCE ET L'HYPOTHÈSE, 1902.

Everything shined at the touch of Poincaré's mind, be it mathematics, physics or anything else. His analysis of the space perception in the ch. IV of *La science et l'hypothèse* remains unrivaled in its depth and clarity. His

conjecture on how the brain reconstructs the *a posteriori Euclidean* geometry of the external space with its rotational (orthogonal<sup>20</sup>) symmetry from the input of moving retinal images, that amazingly agrees with discoveries of the 20th century neurophysiology, stands as a challenge for experimental (mathematical?) psychologists of the 21st (22nd?) century.

*...science d'observation est une science passive.  
Dans les sciences d'expérimentation, l'homme ...  
provoque à son profit l'apparition de phénomènes,  
qui ... se passent toujours suivant les lois naturelles,  
mais dans des conditions que la nature  
n'avait souvent pas encore réalisées.  
... l'astronome fait des observations actives,  
c'est-à-dire des observations provoquées par  
une idée préconçue sur la cause de la perturbation.*

CLAUDE BERNARD, INTRODUCTION À L'ÉTUDE  
DE LA MÉDECINE EXPÉRIMENTALE, 1865.

If you quote a writing by Claude Bernard in an audience of mathematicians they will assume this must be Poincare, physicists would think of Einstein or Feynman and biologists of Darwin. But some may recall having seen these very words in a memorandum signed by several Nobel laureates in the last issue of Nature.

One hardly can add anything of substance to what Claude Bernard had already said about experimental science and what he says, applies to theories as well, since a design of a logical argument is similar to that of an experiment.

Claude Bernard insisted that an *active science* depends on experimentations and these must *control rather than confirm* our ideas. He could have continued by saying that an *active logic* must similarly control rather than confirm our ideas. We should not decide beforehand where the argument will bring us; yet, the rules of our logic must be set in advance. Then we go through an argument step by step and accept whatever comes out of it even if this contradicts our original ideas.

It is almost impossible to design such arguments without mathematics, where *going through an argument* being furnished by something like a proof of a theorem or "just" a computation; it is amazing how often the result of a simple computation may contradict your intuition.

An instance of that is the (Mendel)-Hardy-Weinberg formula we met before that expresses the (counterintuitive) idea that (naively understood) *evolution by natural selection* may stabilize on the second round of reproduction<sup>21</sup>.

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<sup>20</sup>The orthogonal group  $O(3)$  of rotations of the 3-space is the basic *non-commutative atom* of our world geometry. Mathematicians' penetration into its amazing structure took two-and-a-half millenia – from *Pythagorus theorem* to the theory of *Lie groups* developed at the turn of the 20th century and the present day Euclidean (elliptic) gauge theory over 4-D spaces. Amazingly, the brains of all actively moving animals on Earth have developed (by learning?) fair models of this structure.

<sup>21</sup>The mathematics of this formula necessitates the passage from Darwinian idea of *variation* to more abstract (and more adequate) concept of *mutation* that was envisioned by Maupertuis in his picture of Life. (The term *mutation* for suddenly appearing variations was introduced by Hugo de Vries in his two volume *The Mutation Theory, 1900 – 1903*.)

Let us give another example that simultaneously illustrates Claude Bernard's remark that *averages confuse, while aiming to unify*.

Suppose that the average success of a certain brain surgery performed in two hospitals  $H_1$  and  $H_2$  depends on whether the father of a patient was right or left handed.

The *right* patients are sent to  $H_1$ , since

$H_1$  has **better average success** for *right*.

The *left* patients are also sent to  $H_1$ , since

$H_1$  has **better average success** for *left* as well.

But if you do not know whether you are in the *left* or *right* category you are sent to  $H_2$ , since

$H_2$  has **better average success** for *right + left* together.

Perplexed?<sup>22</sup> Then look at the following formula for the average success rate  $S_{r+l}$  for *right + left* together in terms of those for *right* and *left* separately, denoted  $S_r$ , and  $S_l$ , where  $N_r$  and  $N_l$  are the numbers of the *right* and *left* patients who have undergone this surgery,

$$S_{r+l} = \frac{N_r S_r + N_l S_l}{N_r + N_l}.$$

Now you see how this *can* happens in certain cases:  $S_{r+l}$  depends not only on  $S_r$  and  $S_l$  but also on  $N_r$  and  $N_l$ , where these numbers may be quite different in the two hospitals.

Rephrasing Einstein – *the laws of mathematics are certain even when they refer to reality; it is which reality they refer to is not certain*.

*...à ne chercher la vérité que dans l'enchaînement naturel  
des expériences et des observations, de la même manière que  
les mathématiciens parviennent la solution d'un problème ...*

ANTOINE LAVOISIER, DISCOURS PRÉLIMINAIRE  
AU TRAITÉ ÉLÉMENTAIRE DE CHIMIE, 1787.

*Je vous dénonce le coryphée des charlatans, le sieur Lavoisier.*  
JEAN-PAUL MARAT, L'AMI DU PEUPLE, JANVIER 1791.




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<sup>22</sup>All mathematicians I know refused to believe this could ever happen, but those of my friends who were not certain about their math pretended they could admit such possibility.

## *What unworthy motives ruled for the moment under high sounding phrases*

MICHAEL FARADAY.

Lavoisier was arrested in November 1793, accused of selling watered-down tobacco and guillotined on 8 May 1794 at the age of 50.

Lagrange<sup>23</sup> commented on Lavoisier's execution:

*Cela leur a pris seulement un instant pour lui couper la tête, mais la France pourrait ne pas en produire une autre pareille en un siècle.*

Claude Bernard was born in 1813 and Henry Poincaré in 1854.

*But who was Jean-Paul Marat?*. Marat performed numerous experiments and published work on fire, heat, electricity and light. This was not taken seriously by the contemporary French "academic establishment", in particular, by Lavoisier and Condorcet.

Some historians sympathetic to Marat argue that, according to certain evidence, he was a knowledgeable and dedicated scientist who did not get fair hearing at the French Royal Academy and that a comparison of Marat to somebody like Mesmer is unfair. (In 1784, Lavoisier and Benjamin Franklin, who were heading *la Commission royale d'enquête sur le magnétisme animal*, conducted the historically first controlled clinical trial and disproved Mesmer's claims.)

This might be true, but, apparently, Marat, however *knowledgeable*, was not appreciative of the extent of his *non-understanding*. Thus, he published a few hundred pages volume on his study of electricity. Lavoisier, on the other hand who believed that

*l'électricité ne jette pas seulement de la lumière sur les effets du tonnerre, elle sert encor à expliquer un grand nombre des operations de la nature,*

and who had been pondering on electricity most of his life, had published virtually nothing on it. Lavoisier thought at some point that

*le fluide électrique et le fluide magnétique sont eux-mêmes composés, comme l'eau, de deux fluides plus subtils encore, qui sont susceptibles de se réunir et de se séparer, et, en quelque façon, de se neutraliser l'un par l'autre,*

but he could not find a sufficient experimental evidence for his ideas and remained uncertain on the nature of electricity; but Marat believed he understood what electricity was. One tends to agree with Lavoisier's assessment of Marat's work as being insignificant.<sup>24</sup>

Marat refused to accept Lavoisier's judgement and hated Lavoisier, but he was not, however, directly involved in Lavoisier's condemnation: Marat was assassinated in July 1793.

## 6 Life.

*...strutture di ossa per uomini, cavalli o altri animali, che*

<sup>23</sup>Lagrange (1736 –1813), born Giuseppe Lodovico (Luigi) Lagrangia, was a mathematician. The imprints of his analytic rendition of the *least action principle* (more precise than that by Maupertuis and more general than that by Euler) are pronouncedly visible as *Lagrangians* (instead of Newtonian forces) of *dynamical systems* in many segments of mathematical physics along with their younger sisters *Hamiltonians*.

<sup>24</sup>Lavoisier could have said what he thought of Marat in Pauli's words: *I don't mind your thinking slowly; I mind your publishing faster than you think.*

*potessero sussistere e far proporzionalmente gli uffizi loro,  
mentre tali animali si dovesser argumentare ad altezze immense.*

GALILEO GALILEI, DISCORSI E DIMOSTRAZIONI  
MATEMATICHE INTORNO A DUE NUOVE SCIENZE, 1638.

The science of *allometry*, the birth of which you witness reading the above lines, still mystifies us by some of its finding. Why, for example, does the metabolic rate  $R$  of an animal of mass  $M$  closely follow

$$\text{Kleber's 3/4-law: } R \sim M^{3/4}?$$

*La Nature contient le fonds de toutes ces variétés,  
mais le hasard ou l'art les mettent en oeuvre.*

PIERRE-LOUIS MAUPERTUIS, VÉNUS PHYSIQUE, 1745.

Maupertuis pinpointed the role of natural selection in evolution, outlined a (not quite Mendelian) picture of heredity, sketched the theory of mutations and suggested a germ plasm mechanism close to that of Weismann. Since he was a mathematician, biologists are justified in having ignored his ideas.

*... que de machines... sont renfermés dans cette petite partie de matière  
qui compose le corps d'un animal! ... combinaisons... de principes,  
que nous ne connaissons que par des résultats si difficiles à comprendre,  
qu'ils n'ont cessé d'être des merveilles que par l'habitude que nous avons  
prise de n'y point réfléchir!<sup>25</sup>*

GEORGES-LOUIS BUFFON, HISTOIRE GÉNÉRALE DES ANIMAUX, 1749.

Buffon applied integral calculus to measure chances of geometric random events. For example, if one throws a needle of unit length to the plane divided into parallel strips of unit width, then the probability that the needle will cross a line between two strips equals  $2/\pi$  for  $\pi = 3.14\dots$  by Buffon's formula .

(This brought to life the fields of integral geometry and of geometric probability; also this inspired the currently accepted mathematical foundation of the probability theory laid out by Kolmogorov<sup>26</sup> in 1933.)

Buffon designed efficient lenses for lighthouses.

Buffon suggested a construction of concave mirrors that has been in use for two centuries afterwards.

Buffon offered the first scientific scenario of formation of planets, namely from a collision of a comet with the sun.

Buffon evaluated the age of Earth at several (hundred?) million years on the basis of sedimentation rates and argued that at least  $\approx 75,000$  years were needed for the Earth to cool down from the molten state, as he calculated by "rescaling" the results of heating and cooling iron balls.

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<sup>25</sup>Was Buffon able to see this because of his mathematical background? Was he the first to realize it? Was it possible at all to appreciate Life's complexity and our powerlessness in structural representation of live entities prior to the development of molecular biology in the last few decades?

<sup>26</sup>Beside modern probability, several other branches of the 20th century mathematics and mathematical physics grew out of the work by Kolmogorov including *KAM theory* in the classical (Hamiltonian) mechanics, a stochastic theory of *turbulence* and the *entropy* theory of dynamical systems.

(The idea of hot molten Earth was present in the writings by Descartes and Leibnitz.)

Buffon gave a definition of species:

*One should consider as being of the same species that which by means of copulation perpetuates itself and preserves the similarity of that species .... If the product of such mating is sterile, as is the mule, the parents are of different species. Any other criterion, particularly resemblance, is insufficient ....because the mule resembles the horse more than the water spaniel resembles the greyhound.*

This is rather different from how it was formulated by the natural philosopher John Ray who defined species by their

*... distinguishing features that perpetuate themselves in propagation from seed* (1686)

Apparently, Buffon, a mathematician in the depth of his heart, must have been mystified and fascinated by the remarkable fact that the obstruction to interbreeding shows in the *second* generation, defined an equivalence relation between groups of organisms that allowed an introduction of the concept of species.<sup>27</sup>

Buffon would be happy to see that his kind of mathematically active scientific thinking returned to taxonomy in biology at the end of the 20th century with the comparative study of genome sequences.

Buffon applied his definition to prove the unity of humanity:

*The Asian, European, and Negro all reproduce with equal ease with the American.*

Buffon writes:

*... one could equally say that man and ape have had a common origin like the horse and donkey that each family among the animals and vegetables have had but a single stem, and that all animals have emerged from but a single animal which, through the succession of time, has produced by improvement and degeneration all the races of animals.*

Buffon, who had already had enough trouble with ecclesiastical authorities, rejected this idea in writing, since it contradicted the biblical version of creation.

Buffon would find it ironic, that certain post-Darwinian evolutionary thinkers who have freely borrowed from his ideas, have, nevertheless, inherited a pre-Cartesian respect for authority and submission to ideology<sup>28</sup> from his ecclesiastical opponents. These thinkers have not admitted Buffon to membership in their club as a *non-believer* in Darwinism.

<sup>27</sup>In 1942, an influential evolutionary thinker Ernst Mayr reiterated Buffon's definition as  
*... interbreeding natural populations, which are reproductively isolated...*

What would Buffon make of the 20th century "evolutionary thinkers" who we unaware of mathematics of clusterization and who authoritatively stated that having a progeny needs mutual proximity of participating animals? (Never mind plants.)

<sup>28</sup>We value in ideas their *beauty* and their *novelty*; we appreciate in people the depth and originality of their *understanding* of non-trivial ideas. Carriers of an ideology, on the other hand, (often recognizable by *ist* attached to their tails) may be tolerant to those who are confused by what they (the carriers) proclaim – these are potential converts, but they are indignant against anyone who *clearly sees and rejects* their views.

Farady's

*Nothing [is] quite as frightening as someone who knows they are right*  
is about these people.

Buffon overhauled much of the knowledge of his time and developed a view on Nature and Life from a broad scientific perspective. But he wrote down only 36 volumes of his *Histoire naturelle, générale et particulière* out of planned 50 before he died in 1788.

(Buffon, like Einstein, complained that his natural laziness had precluded him from achieving more in science.)

Buffon's idea of evaluating the age of Earth was taken up by William (Kelvin) Thomson in 1862 who calculated the rate of thermal diffusion from the Earth crust of its initially molten state and, thus, estimated Earth being at most of order 100 million years old. (This was in contradiction with the sedimentary geological estimates by Maillet, Lyell and Darwin.)

Thomson is famous for the concept of the *absolute zero*,  $0 \text{ } ^\circ\text{K} \approx -273.15 \text{ } ^\circ\text{C}$  and for the idea of the *Heat Death of the Universe*:

*...a state of universal rest and death, if the universe were finite and left to obey existing laws. ... science points rather to an endless progress, ...involving the transformation of potential energy into palpable motion and hence into heat.... .*

As an engineer Thomson had greatly contributed to the transatlantic telegraph project.

But Thomson is also renown for his:

*heavier-than-air flying machines are impossible*

and the characteristically late Victorian:

*There is nothing new to be discovered in physics now. . .*

This contrasts with

*... we have no right to think thus of the unsearchable riches of creation....*

by Maxwell (who is often misquoted at this point).

Buffon direct impact on physics was, probably, rather limited but the flow of ideas emanating from his "Natural History" had been shaping the minds of generations of biologists and natural philosophers up to the middle of the 20th century.

*It is in relation to the statistical point of view that the structure of the vital parts of living organisms differs so entirely from that of any piece of matter that we physicists and chemists...*

*...living matter... is likely to involve "other laws of physics" hitherto unknown, which however, once they have been revealed, will form just as integral a part of science... .*

ERWIN SCHRÖDINGER, WHAT IS LIFE? 1944.

*The principle of continuity renders it probable that the principle of Life will hereafter be shown to be a part, or consequence of some general law.*

CHARLES DARWIN, LETTER TO GEORGE WALLICH, 1882.

Whereas Darwin dreamed of some *mathematical/philosophical PRINCIPLE* of LIFE, Schrödinger wanted to make sense of LIFE in the light of PHYSICAL LAWS.

But LIFE breaks physical symmetries/regularities and creates new ones<sup>29</sup>

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<sup>29</sup>It is not always clear what should be called "breaking" and what is "creating" a symmetry. Does, for example, selection of a particular chirality by biological systems break or create stochastic symmetry in ensembles of molecules?

of quite different nature, e.g.

*the striking regularity with which the same hybrid forms reappear.*

The laws of physics, obviously, restrict possible structures/behaviours of living systems as much as the rules of chess restrict the moves of pieces on a chess board.

But moves of a master in a chess party are no more accountable for by the rules of chess than mathematical theorems are representable by reflections of the laws of logic.

These rules, one might argue, well account for the statistics of the moves of somebody who plays chess for the first time in his/her life but not of a player with a couple of years experience. This is how it is with most *physical systems* – they start their games essentially from level zero, where the laws of physics apply.<sup>30</sup>

But the game of life on Earth had been played by Nature for a few billion years. The laws of physics, as Galileo pointed out, tell you that there are no flying elephants but they can not help you to draw the path that brought *walking* elephants to Earth. Compatibility with the laws of physics is a minuscule part of the reason for existence in biology.



It is not accidental that when Buffon and Schrödinger try to speak of Life in the language of physics they start speaking poetically.<sup>31</sup> The great painting of Life is not about the physics of the canvas and the chemistry of dyes.

But don't physical and biological theories share the hallmark feature of "true science": *falsifiability*?

*You don't need 100 famous intellectuals to disprove a theory.  
All you need is one simple fact.*

ALBERT EINSTEIN.

For instance, think of *Galileo's law of falling bodies*:

*if you let go down some objects ♦, ♠ and ♣ you hold in your hand, say of 300, 30 or 3 grams of weights, they all reach the floor in  $\approx \frac{1}{4}$  sec essentially independently of their weights.*

<sup>30</sup> The second law of thermodynamics is an exception: it applies, as Boltzmann pointed out, only to *prepared states* of physical systems, prepared by their *history*. This law appears to *Naive Mathematician* as a physical corollary to a general mathematical theorem, a theorem nobody has managed to formulate yet.

<sup>31</sup>This is reminiscent of Trevize in Asimov's *Foundation and Earth* who sees the Milky Way Galaxy as an unfolding of Cosmic Life.

Would the edifice of the classical mechanics collapse if, in some experiment on the see level of planet Earth, one of these objects, for instance  $\star$  of 30 grams of weight, had remained in the air for 10 seconds before reaching the ground?

The answer could be "Yes, it would" IF Earth were bare of life. And to get the correct answer, no 100 intellectuals are needed, a single *healthy adult sparrow* for  $\star$  will do.

But there is no way to say what "(bare of) *life*" signifies in the language of physics.

The main impediment to "reducing Life to Physics" is not that we lack some "Law of Nature", but incompatibility of the two contexts underlying the languages appropriate for description of physical and of biological phenomena. What we call *physics*, or rather *theoretical physics*, is not only a collection of mathematical models, but also a set of *tacit* rules of when, where and how these models should be related with results of experiments.

One can imagine a universe with physical laws very different from ours; yet, inhabited by somebody very much like ourselves, e.g. a virtual world of a sophisticated computer program. Probably, an outline of the ways of Life can be seen in how *mathematical order (mysterious)* "assembles" from simple "logical pieces", not in the pieces themselves.

But is there an abstract notion of *system* suitable for biology?

Can a cell be regarded as a *single* physical system?

What is a true *Grothendieck-style*<sup>32</sup> definition of "*single unit*" in biology?

A few facets of life can be described in the language of probabilities that is commonly used in physics, but these descriptions are dissimilar to the pictures that we see in physics and in chemistry as Schrödinger says. *Statistics of LIFE*, unlike *statistics of non-Life*, is characterized by

- *Improbably high multiplicity of occurrence of supposedly rare seemingly independent events,*

such as the presence of thousands *nearly identical* randomly looking *complex* protein molecules in a cell. (But hardly *two complex* snowflakes found in Antarctica are ever completely alike.)

On a different scale, and for a somewhat different reason(s), there may be trillions of *copies of thousands residues long* polynucleotide molecules (DNA and RNA) and/or of viral particles in a pond of water, not to mention seven billion enormous multi-molecular aggregates of almost indistinguishable compositions and shapes, that bipedally locomote themselves on the surface of Earth.

This *improbable multiplicity* phenomenon is partly due to

- *Amplification of rare and/or low energy events*, e.g. of advantages mutations in populations and of chemical signals in cells.  
(The latter would look similar to a chain reaction of combustion ignited by an accidental spark, if not for *key/lock mechanisms* ever-present in biological systems.)

This amplification is also responsible for

- *Gross discrepancy between "statistically averaged", "significant" and*

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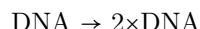
<sup>32</sup>Alexander Grothendieck (1928-2014) is the originator and crystallizer of a body of fundamental concepts of the 20th century mathematics.

"typical" events/behaviours.

Processes in Life often run as lotteries – no one's gain is close to the average – in stark contrast with comparably large *stochastically homogeneous* physical systems, such as the bulks of gases and liquids where *typical* tends to be close to average.

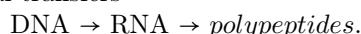
In many (but not all) cases the above features are associated to *information* that is *encoded* in a biological system and that can be *transmitted* from one biological (sub)system to another.

In fact, "information" is a commodity more valuable for Life than energy: the transfer of *sequential information* in the course of replication



is a defining factor of Life on Earth.

And the "*circulation of the information flow(s)*" that makes the cell tick starts with the sequential transfers



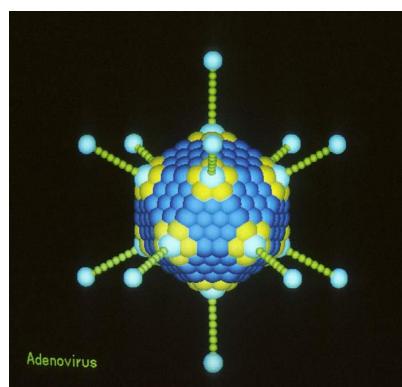
These transfers are "formal" and (essentially) expressible in terms of Shannon's style information theory. But there would be no Life if not for *macromolecular folding*. In a way, Life is created by



Albeit folding is an essentially physical process,<sup>33</sup> physics per se can not tell how and why it works, because, besides lacking a precise "physical/chemical formulae" for amino acid residues' interaction in the aqueous environment,<sup>34</sup> folding applies only to those *rare* polypeptide chains that were "selected by Life" for their specific roles in the Cell.

Neither physics nor present day mathematics can identify and formally describe polypeptide sequences that would fold to "*potentially live-cell proteins*".

Even worse conceptually, we possess no mathematical framework where we could formulate the idea of "*creative reduction of information by folding*".<sup>35</sup>



Similarly, there is no available mathematical formalism for the expression of

<sup>33</sup>Folding of some proteins needs "biological assistance" in the Cell.

<sup>34</sup>There is not even a satisfactory model of water itself on the nanoscale.

<sup>35</sup>Apparently, the bulk of the message carried by the amino acid sequence tells a protein  $P$  how it should fold rather than what it supposed to do upon folding. This "folding information" is, for the most part, invisible on the exposed surface of a folded  $P$  but the sequence can be recovered when  $P$  is unfolded. Thus, the term "reduction" is not quite appropriate.

the "economy of information principle" that dictates the icosahedral symmetry of viral particles (such as in the picture blow), where this information is what is needed to genetically encode an assembly of such a particle in the rotationally symmetric physical space.

And evolving the skyscrapers building ability by termites was, probably, grossly facilitated by the possibility of a *symmetric* implementation of the building program in *identical* genomes of the builders.

Is this kind of *information* it a precursor of a non-trivial mathematical concept or it will forever remain a poetic metaphor?

Physics has no means for addressing this kind of questions. But why – one asks with an insinuating smile – were there fundamental contribution to biology by physicists and not so many, if at all, by mathematicians?

I could give you an answer, but this would be ... opinion.

## 7 Evolution.

*Nothing in Biology Makes Sense Except in the Light of Evolution.*<sup>36</sup>

This is the title of a 1973 article by THEODOSIUS DOBZHANSKY'S.

*We now may be able to understand it [evolution] in biology.*

JACQUES MONOD, ON THE MOLECULAR THEORY OF EVOLUTION, 1975.

*We have an excellent idea of the core genetic makeup  
of the last common ancestor of all bacteria  
that probably lived more than 3.5 billion years ago.*

EUGENE KONONIN, THE LOGIC OF CHANCE, 2011.

The latter is not another excellently contrived JUST SO STORY but an outcome of statistical analysis (of *multiple alignments*) of genomes from data bases that have been accumulated on *petabyte* ( $10^{15}$  bytes) rate for the last ten years. From the petabyte hight, Dobzhansky's *Light* and Monod's *understand* appear as dim as the early 20th century picture of evolution, often called *modern evolutionary synthesis*, gauged by the molecular standards of the 1970's.

And the 18th and 19th centuries ideas serve for molecular biologists only as references to the names behind them. Yet, such concepts as *reproduction - inheritance - selection - competition* seem to carry still undeciphered messages that we would be able to read if we could translate *natural poetry* of olden days to our mathematical language, preferably, not exactly of *multiplication table type*.

*First forms minute, unseen by spheric glass,  
Move on the mud, or pierce the watery mass;  
These, as successive generations bloom,  
New powers acquire and larger limbs assume;  
Whence countless groups of vegetation spring,  
And breathing realms of fin and feet and wing.*

ERASMUS DARWIN, THE TEMPLE OF NATURE, 1802.

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<sup>36</sup>In mathematical terms, LIFE ON EARTH becomes a *connected*; thus, a topologically *non-trivially* structured entity *only* in the presence of the *time* coordinate. Is this connectivity or the *full topology* (geometry?) of LIFE ON EARTH what Dobzhansky calls *sense*?

*...difficult to believe ... that the more complex organs and instincts should have been perfected, by the accumulation of innumerable slight variations. Nevertheless,... all organs and instincts are, in ever so slight a degree, variable ... a struggle for existence ...*

CHARLES DARWIN, THE ORIGIN OF SPECIES, 1859.

Some pages later Darwin writes:

*... it is as easy to believe in the creation of a hundred million beings as of one; but Maupertuis' philosophical axiom "of least action" leads the mind more willingly to admit the smaller number... . Analogy would lead me one step further, namely to the belief that*

*all animals and plants have descended from one prototype.*

This sounds almost like a mathematical proposition! Pythagoras would have loved such a quintessential formulation of the idea of evolution. He might have learned this idea from Anaximander and he would have stated it almost as Darwin put it:

**Theorem 1.** *Every two organisms on Earth, be they plants, animals or humans, have a common ancestor.*

But only Pythagorus' corollary reached us across the chasm of time:

*As long as man continues to be the ruthless destroyer of lower living beings he will never know health or peace. For as long as men massacre animals, they will kill each other.*

Darwin, who hated slavery of any kind, adds:

*Animals, whom we have made our slaves, we do not like to consider our equal.*

Lamarck, Darwin and Wallace had scrutinised and systematised vast amount of material, in part collected by themselves, that led them to believe in transmutation of species and evolution in general. Darwin and his followers chose to present their ideas to general public in the language of political economy: *competition for resources, struggle for existence, etc.* That was well taken by their Victorian audience acquainted with *The Wealth of Nations* by Adam Smith and prepared to evolutionary ideas by *Vestiges of the Natural History of Creation* published anonymously in 1844 by Robert Chambers.

But **Theorem 1** was truly *proved* only a century later with the advent of molecular biology and sequencing techniques that uncovered *non-ambiguous* structural similarity between molecular architectures of all living cells.

Everyone who is able to *understand* the Pythagorus theorem will also *understand* that such degree of similarity could be neither accidental nor come from any kind of *convergent evolution*. You do not have to *convince* anybody anymore and can discard *struggle for existence, fittest survives* and *creative power of selection*. Granted the accumulated sequential and molecular structure data, all you need is a *little mathematics of the multiplication table type* in Hardy's terms.

(Darwinian *metaphor* of *struggle for existence* applies, with no possibility of being taken literally, to *numbers*  $R_1 > 1$  and  $R_2 > 1$ : the greatest of the two numbers survives in the formula  $\sqrt[T]{R_1^T + R_2^T}$  for time  $T$  being large. But when *struggle for existence* is applied to the animals the reproduction rates of which are represented by these numbers, it may give you wrong ideas.)

Yet, there are two problems with **Theorem 1**.

1. It logically (and obviously) implies that

there was a first *proto-cell* from which all other cellular organisms (viruses?) descended, where *proto* is a shorthand for *I have no idea what the Hell I am talking about.*<sup>37</sup>

2. The second question reads:

Where is the hidden beauty? What are physical/chemical/biological mechanisms besides heredity that drive the evolution?

The idea (opinion?<sup>38</sup>) of NATURAL SELECTION can be condensed to a single word:

NONE:<sup>39</sup>

*potentially exponential* rate of growth of populations allows *random* variations to cover *all* possibilities; all Nature/enviroment has to do is to *select* those she favors.

Darwin also insisted that consecutive steps of evolution are implemented by *small* variations<sup>40</sup>. These, he argued, are more likely to be non-deleterious; besides, this increases the chance of a (more or less) simultaneous occurrence of several, say two, variations that are advantageous only if they come together.

(Major transitions in evolution are, contrary to what Darwin would think, were associated not with accumulation of *point mutations* but with *abrupt* and significant genome rearrangements, including *duplications of genes and of the whole genomes*. For instance, the divergence of the lineages of humans and chimpanzees, that occurred about 5 million years ago, was, most likely, triggered by the *fusion of two chromosomes*: great apes have 24 pairs of chromosomes while humans have only 23 pairs.)

Besides, Darwin suggested several scenarios of how particular patterns of evolution e.g. that of the vertebrates eye, could have been gradually achieved by selection.

But Pythagorous would point out that *X is not impossible* does not imply *X is true*, e.g. for *X* being *selection and nothing else*<sup>41</sup> and that the true biological problem is concealed by dismissive besides attached to *heredity*.

In fact, until the development of the cellular/molecular biology that started at the end of 19th century, nobody was close to imagining the immensity and the beauty of the hidden structural complexity of Life; hence, of heredity. Even to-day, some may argue, we did not reach the point where *I know that I know*

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<sup>37</sup>The Earth is about 4.5 billion years old and *Great Oxygenation* of Earth's atmosphere happened  $\approx$ 2.5 billion years ago. There are  $\approx$ 2 billion years old fossils of multicellular organisms and traces of pre-oxygenic bacteria  $\approx$ 3.5 billion years old. But when and what was the age of proto-life?

<sup>38</sup>An assertive expression of an idea that has been around for a few decades classifies as an *opinion*.

<sup>39</sup>This answer brings to mind the famous apocryphal response of Laplace to Napoleon:

*Sire, je n'ai pas eu besoin de cette hypothèse.*

<sup>40</sup>The Western idea of gradualism can be traced back to Milo of Croton, an associate of Pythagoras, who, around 540 BCE, gradually developed his strength by having carried a calf growing to bull on his shoulders.

<sup>41</sup>Darwin himself wrote in the 1872 edition of *The Origin of Species* that he was ... *convinced that natural selection has been the main but not the exclusive means of modification.*

This, however, like prophecies by the oracle of Delphi, is open to many interpretations.

*nothing* coming from a biologist will be not just words. But as recently as at the beginning of the 19th century, some, e.g. Lamarck (but not Maupertuis) earlier, believed that spontaneous generation of worms from dirt was plausible.

There was a logical reason to this belief: a presence of rich flora of parasitic worms in the intestine of Adam would contradict the idea of Eden. But it became clear to (almost) everybody by the end of the 19th century that "spontaneous generation" of a single cell of a worm is no more probable, than spontaneous emergence of *Great Pyramid of Giza* – the first of the seven wonders of the world, out of stones, sand and dirt by some natural physical process some forty five centuries ago.

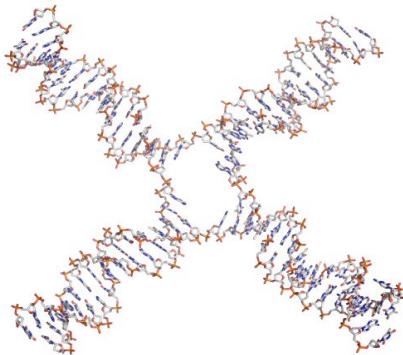
Eventually, accumulation of data on the *cell division*, in particular, the discovery of *meiosis* (cell division for production of *gametes*, e.g. sperm and egg cells in animals) by Oscar Hertwig in 1876, led to the *germ plasm* idea proposed by August Weismann around 1890 that turned the evolution theory toward *active science* in the sense of Claude Bernard.

In modern terms, the *Weismann principle* says:

*genomes vary but organisms are selected.*

(To confirm his idea and to disprove Lamarck, Weismann was chopping off the tails of several hundred mice for  $\approx 20$  generations and recorded no mouse born without a tail<sup>42</sup>.)

Soon thereafter, Mendel's ideas were rediscovered and an explosion of genetics followed by that of the molecular biology began. The focus of the evolution problem shifted from organisms to genomes.



We want to learn how genomes "fly", how their "engines" work. This, we believe, will bring us closer to understanding of how they fare in the adverse winds of competition and selection. (But doesn't this metaphor go too far when applied to natural selection? Granted, for instance, that the main factor for having a safe flight is a *selection* of an air company, can you seriously say that this selection is the main factor that keeps an airplane safely in the air? If not,

<sup>42</sup>Lamarck would point out that if Weismann had done it to hundred million mice for hundred million generations, then the tails would degenerate as do the eyes of fish living in dark caves. He also would conjecture that if Weismann had done it to a hundred million generations of lizards then, instead of regenerating their tails, they would start growing second heads with one not being enough to comprehend scientific significance of such experiment.

how can you take natural selection for the *main* factor in evolution as Darwin says?)

A rough quantitative picture of genome evolution can be seen with simple mathematics of *biased* (non-symmetric) *random walks* on *finite* (but very large) *graphs* with *absorption*, where the vertices of the graphs represent genomes and absorption corresponds to extinction/selection. But we are still far from formulating **Theorem 2**.

An amazing thing about old fashioned naturalists, especially Darwin, is that with *no support by an experiment* and/or with *no quantitative reasoning*,<sup>43</sup> they could built consistent pictures of large fragments of Nature and sometimes could do it (literarily) hundred times better than the most luminous physicists, astronomers and mathematicians of their time.

For example, Darwin and his friend a geologist Charles Lyell following ideas originated in the work by James Hutton estimated (somewhat differently) the age of Earth to be at least several hundred million years. (About 150 years earlier Benoît de Maillet evaluated the age of Earth by two billion years by estimating the rates of sedimentation and formation of earth crust but his argument is judged having been poorly founded.)

On the other hand, William Thomson (Kelvin), Hermann Helmholtz and Simon Newcomb came up with about thirty million years by evaluating the time needed for Earth to cool from the molten state and the Sun to be heated and sustained by the gravitational contraction. If physicists had taken Hutton, Lyell and Darwin seriously, they might have arrive at the matter-energy idea several decades before Becquerel's 1896 discovery of radioactivity and Einstein's  $E = mc^2$  of 1905.

And pondering over

*It is not the strongest of the species that survives, nor the most intelligent that survives. It is the one that is the most adaptable to change*

one starts wondering if these words may come true in a few hundred years from to-day: the Earth will be taken over by the most adaptable ones – protozoa, bacteria and viruses after Man will have done with multicellular life.

(The reader may be relieved: the above is a common *misquotation* of Darwin, he was neither that stupid nor *that* gloomy on the future of mankind.)

#### ABOUT HUTTON, LYELL, HELMHOLTZ AND NEWCOMB.

*James Hutton* (1726 – 1797) recognised the role of the subterranean heat in the creation of new rock followed by the gradual process of weathering and erosion on a very long geological time scale – *la route éternelle du temps* as Buffon says in *les Époques de la Nature* published in 1778. (Many of these ideas, including evolution, appear in *Protogaea* written by Gottfried Leibniz between 1691 and 1693, and published in 1749. For example, Leibniz writes: *The globe of the earth... has hardened from liquid, light or fire being the motive cause.*)

Hutton thought of Earth geo-dynamics as being cyclic with time stretched without limit into past and future. Concerning evolution of Life, he accepted

<sup>43</sup>Because of these shortcomings, the Darwinian scheme of evolution may appear exceedingly naive. For instance, biochemist Ernst Chain (1906 – 979), one of the main contributors to isolating and applying penicillin, writes: *...variants do arise in nature and ...their emergence can and does make some limited contribution towards the evolution of species. The open question is the quantitative extent and significance of this contribution.*

the dominant role of selection in what we now call *microevolution* but not, as Darwin and Wallace did, in *macroevolution*.

*Charles Lyell* (1797 – 1875) was a proponent of gradualism in geology whose uniformitarian ideas influenced Darwin.

*Hermann von Helmholtz* (1821 – 1894) invented *ophthalmoscope* for examining the inside of the eye and *Helmholtz resonator* to identify frequencies of sound waves.

Helmholtz measured the speed of propagation of nerve impulses and he developed mathematical and empirical theories on depth, color, sound and motion perceptions.

Helmholtz formulated the *law of the conservation of energy* in his mechanical foundation of thermodynamics, where he also introduced *Helmholtz free energy*.

Being a rare scientist whose discoveries had found immediate uses, he, nevertheless, states:

*Whoever in the pursuit of science, seeks after immediate practical utility may rest assured that he seeks in vain.*

In Maxwell's words:

*[Helmholtz] is ... who prosecutes physics and physiology, and acquires therein not only skill in developing any desideratum, but wisdom to know what are the desiderata.*

*Simon Newcomb* (1835 –1909) conducted a precise measurement of the speed of light. He discovered what is now known as *Benford's law*: more numbers, taken from "real life" data, will begin with 1, than with any other digit. Newcomb believed that the astronomy of his time was nearing the limit and, like Thomson, he was skeptical about *flying machines*.

*... those which depart most from the best adapted constitution, will be the most liable to perish, while,..., those organised bodies, which most approach to the best constitution for the present circumstances, will be ... multiplying the individuals of their race.*

JAMES HUTTON, AN INVESTIGATION OF THE PRINCIPLES  
OF KNOWLEDGE AND OF THE PROGRESS OF REASON,  
FROM SENSE TO SCIENCE AND PHILOSOPHY, 1794.

*And love Creation's final law  
Tho' Nature, red in tooth and claw*

ALFRED TENNYSON, IN MEMORIAM A.H.H., 1849.

*The powerful retractile talons of the falcon- and the cat-tribes...  
survived longest which had the greatest facilities for seizing their prey.*

ALFRED RUSSEL WALLACE. ON THE TENDENCY OF VARIETIES  
TO DEPART INDEFINITELY FROM THE ORIGINAL TYPE, 1858.

Teeth, claws, talons, nails – there are gentler designs of Mother Nature that are also more important for survival of her children.

The mortality rate for all animals is the highest before they reach maturity. If you are a bird or mammal, your survival depends 100% on the care of your



parents. Not enough mummy's milk – you are dead long before you learn what your talons are for.

But it is not easy to figure out how Nature managed *simultaneous* evolution of several *intrinsically uncorrelated*(?) functions, e.g. physiology + psychology of a kitten and then of the *same* animal in the role a mother cat where everything must function *in concert*. (Darwin's appeal to gradualism of evolution with no support by detailed/quantitative analysis of data is hardly acceptable for more than two "elementary functions"<sup>44</sup>)

But talons or no talons, Wallace was a great (the greatest?) 19th century naturalist. He collected more than 100,000 specimens in Malasia and Indonesia and discovered more than thousand new species, e.g. *Wallace's flying frog*.

As much as Darwin, he had been thinking on how and why species transform into new species and why the separation between different species is rather sharp<sup>45</sup>. He arrived at the same (but not quite) natural selection theory as Darwin, but, apparently, he was more ecologically minded than Darwin. A witness to that is Wallace's self-regulation principle in animal populations:

*The action of this principle is exactly like that of the centrifugal governor of the steam engine, which checks and corrects any irregularities almost before they become evident; and in like manner no unbalanced deficiency in the animal kingdom can ever reach any conspicuous magnitude, because it would make itself felt at the very first step, by rendering existence difficult and extinction almost sure soon to follow.*

I do not know if either he or Darwin realised that the *self-regulated* (negative feed-back) equilibrium can be oscillatory as in the *Lotka - Volterra equation*<sup>46</sup>. But Wallace gave a devastating analysis of how the ecology of island St Helena (famous for Napoleon who had *not* studied it) was interfered with by Europeans colonists and what happened to the equilibrium afterwards.

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<sup>44</sup>An outcome of any kind of such analysis would depend, in particular, on the comparative size of atoms and molecules versus cells, for example.

<sup>45</sup>This sharpness, that is *disjointness* of the corresponding *attractors in the dynamics of evolution*, probably, is determined by a presence of feedback loops in this dynamics. Negative feedbacks constrain the spread of attractors, that is the intraspecies variation, while the positive feedback loops make different attractors (representing different species) drift apart.

<sup>46</sup>This differential equation was used as a model of the predator-prey systems in the mid 1920es by the mathematical chemist Alfred Lotka and independently by the mathematician Vito Volterra. Prior to that, a similar equation was introduced in 1838 by the mathematician Pierre Verhulst for describing the number of individuals that an environment can support. Even earlier, in 1766, Daniel Bernoulli, stimulated by the inoculation controversy, solved an equation of this type in his study of smallpox epidemics.

Disagreeing with Darwin, Wallace rejected the superficial similarity between artificial selection and natural selection in the wild:

*... varieties produced in a state of domesticity are more or less unstable, and often have a tendency, if left to themselves, to return to the normal form of the parent species; and this instability is considered to be a distinctive peculiarity of all varieties, even of those occurring among wild animals in a state of nature, and to constitute a provision for preserving unchanged the originally created distinct species.*

The current view (if I get it right) is that *instability of domesticity* and intraspecies variation in general are mainly due (besides ever-present Gaussian bell-shaped variations<sup>47</sup>) to genome *crossover recombination* during meiosis that, unlike mutation, is (quasi)reversible.

Wallace, who could not know anything of this, gave a 19th century explanation:

*If turned wild on the pampas, such [domesticated] animals would probably soon become extinct, or under favourable circumstances might each lose those extreme [artificially selected] qualities which would never be called into action, and in a few generations would revert to a common type.*

Isn't it amazing that the two explanations have hardly anything in common?

The first one was unthinkable hundred fifty years ago. It depends on cellular/molecular data that have been obtained by many technically involved experiments the outcomes of which were by no means predictable a priori.

On the other hand, Wallace's common sense argument could have come from Lamarck with *adaptation* instead of *selection* (implicit in the above quote) and even from Aristotle whose choice of words would be *openly teleological*.<sup>48</sup>

(Teleology is built into human language and it inadvertently pops up in our reasoning when we least expect it, especially in evolutionary biology and in psychology.<sup>49</sup> Even Darwin, who fought teleological thinking in biology for years, writes:

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<sup>47</sup>These variations, as was pointed out by Schrödinger and ≈150 years earlier by James Hutton, may be purely phenotypical; thus, *non-inheritable*, contrary to one of the main premises of the pre-Weismann Darwinism.

<sup>48</sup>Aristotle categorized "explanatory causes" into four classes:

*the material cause, the formal cause, the efficient/moving cause and the final cause*, where the latter corresponds to something like our *teleological*. It would be presumptuous however, even to raise it as a *question* what *exactly* Aristotle meant by these "causes" and what would be his vision on their respective roles in the performance of evolution; yet, this classification is inspiring anyway. For example, it brings the modern stochastic genome dynamics perspective on evolution to the same "material/efficient category" as Lamarckian ideas on means and causes of evolutionary modifications of organisms while "explanations by selection" go to the "formal category".

<sup>49</sup>Rephrasing von Brücke and/or Haldane,

*Teleology is like a mistress to an evolutionary biologist: he cannot live without her but he's apprehensive of being caught up in her company by a mathematical physicist.*

Ernst Wilhelm Ritter von Brücke (1819 – 1892), as much as Claude Bernard and Hermann Helmholtz, can be regarded as a father of modern physiology. Among many other things, he studied the change of colour in chameleons and the ways sounds of European and Oriental languages are produced.

John Burdon Sanderson Haldane (1892 – 1964) was one of the founders, along with Ronald Fisher and Sewall Wright, of *population genetics* – that is Mendelian dynamics in a *micro-evolutionary setting*. His idea (1941) of checking *semiconservative mode of DNA replication* with a use of <sup>15</sup>N – the heavy isotope of the nitrogen, was implemented in 1957 by Matthew Meselson and Franklin Stahl in one of the most logically beautiful experiments in biology.

*Natural selection cannot possibly produce any modification in a species exclusively for the good of another species.*

By logic, every explanatory non-teleological sentence about evolution with *for* in it must be either vacuous or self contradictory.)

Wallace was also skeptical about Darwin's powerful idea of *sexual selection*. He, apparently, found it too powerful. You can explain almost everything with it:

*a certain feature evolves because the opposite sex happened to like it.  
(This dispenses with the necessity of reflection – Poincare would say.)*

Thus, for example, Wallace identified the true role of what is now called *warning coloration in animals*, that Darwin originally attributed to sexual selection.

An essential point of divergence between Wallace and Darwin was in the dominant role of selection, in particular of sexual selection, in the *human* evolution that took merely 200 000 - 2 000 000 years depending on from where you count.

*Man was generated from all sorts of animals,  
since all the rest can quickly get food for themselves,  
but man alone requires careful feeding for a long time;  
such a being at the beginning could not have preserved his existence<sup>50</sup>.*

It is hard to disagree that sexual selection *is* the most probable cause of, say, sophisticated courtship of birds – just think of peacock burdened by his "useless" tale. Yet, only (adult?) males may be expendable: precarious childbirth and delayed maturity seems too high a price for the big brain in humans.

Apparently, the brain+language evolution went through a positive feed-back loop. Such loops, be they positive or negative, must be abundant in the organisms/environment systems since organisms modify/shape environment. (The interactions between opposite sexes of the same species is a basic instant of that, where positive loops are easier recognisable as they enhance sexual selection<sup>51</sup>.)

An obvious environment of an individual in the early human brain evolution, besides one's mates, was one's *tribe/clan identified by common language*:

*in a community of speakers and listeners,  
selection favors the best articulated ones.*

Besides, these tribes themselves became *units of selection*, where evolution speeds up by a significant factor for a population that is divided into  $N$  competing tightly knit groups. (*A little mathematics of the multiplication table type* suggests this factor may be almost as large as  $N$  but I am not certain about it.)

Wallace himself maintained that there must be *something transcendental* responsible for the emergence of higher cognition in humans. This does not look *scientific*, unless *transcendental* is read as

*a simple yet subtle abstract structure that underlies human cognition  
and that is evolutionary accessible.*

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<sup>50</sup> Attributed by (Pseudo?)-Plutarch (2nd century?) to Anaximander, 610 – 546 BCE.

<sup>51</sup> *Sexual preference for such developments in the female, must thus advance together, and so long as the process is unchecked by severe counterselection, will advance ... in geometric progression* – RONALD FISHER.

A remarkable instance of this kind of a structure –*imprinting in young animals*– was described in a 1872 paper by Douglas Spalding. This marked the birth of psychology as a *science* separated from neurophysiology and... Spalding’s results shared the faith of those by Mendel of being ignored for several decades.



An attractive feature of this structure for a theoretically inclined *Naïve Mathematician* as well as for practically minded Mother Nature with her unsophisticated strategy of evolution by selection, is the *universality* of imprinting: a baby animal (of a certain class of species) takes the first moving object, *whatever it is*, for its mother. (Does this very simplicity of imprinting disclosed by Spalding’s experiments that excites mathematicians holds it in low esteem among psychologists?)

Possibly, many (all?) fundamental patterns/units of human/animal psychology/behaviour are comparably mathematically simple/universal; thus, evolutionary accessible. But they may have no apparently straightforward manifestations and be hard to detect by direct experiments.<sup>52</sup>

*Les développements de la vie, la succession de ses formes,  
la détermination précise de celles qui ont paru les premières, la  
naissance simultanée de certaines espèces, leur destruction graduelle,  
nous instruirait peut-être autant sur l’essence de l’organisme.*

GEORGES CUVIER, RECHERCHES SUR LES OSSEMENS  
FOSSILES DES QUADRUPÈDES, 1812.

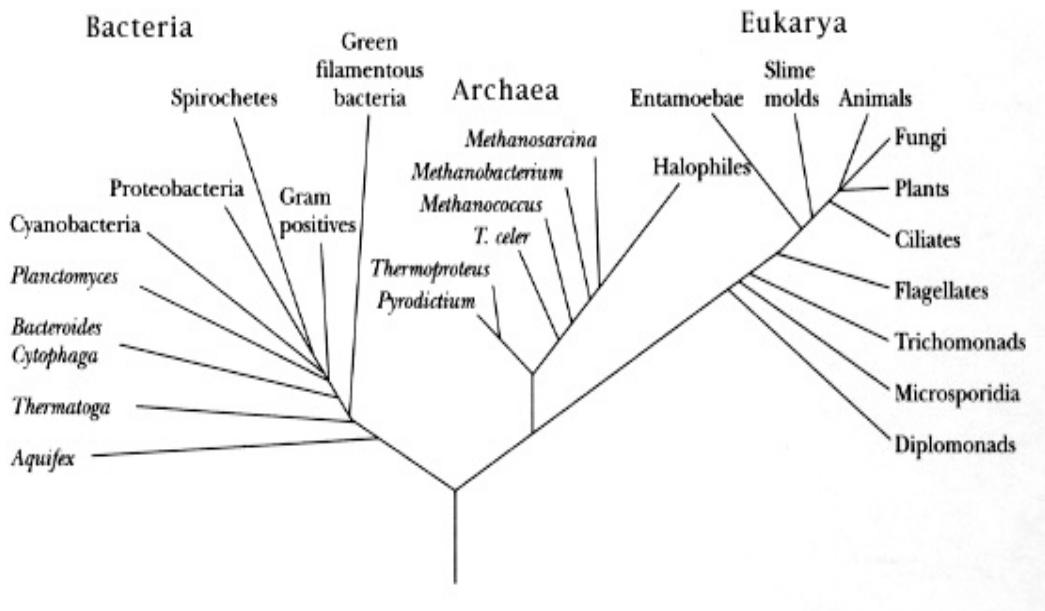
Cuvier was a magician of palaeontology and comparative anatomy:

...after inspecting a single bone, one [he modestly speaks of himself] can often determine the class, and sometimes even the genus of the animal to which it belonged, above all if that bone belonged to the head or the limbs. ... This is because the number, direction, and shape of the bones that compose each part of an animal’s body are always in a necessary relation to all the other parts, in such a way that - up to a point - one can infer the whole from any one of them and vice versa.

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<sup>52</sup>Another such pattern, *Hawk/Goose effect*, is a habituatory response of a baby animal who learns not to fear frequently observed shapes sliding overhead.

The structure of *imprinting* and of *Hawk/Goose* does not look simple if you go to the bottom of it. To see the problem, try to honestly describe these in the *brain language* – in terms of retinal images and/or properties of flows of signals received by the vision processing centres in the brain.



Cuvier's analysis of the fossil data was the primary source of the 19th century evolutionary theories, but Cuvier rejected the idea of gradual transmutation of species proposed by Lamarck, since fossil records indicated abrupt rather than gradual changes, and he scorned at the mechanisms of evolution suggested by Lamarck.

Would Cuvier accept natural selection as a scientifically valid solution to the problem of evolution? Would Huxley – *Darwin's bulldog* as he was called, stand a chance against arguments that Cuvier could master?<sup>53</sup>

(Thomas Henry Huxley, probably, the second great comparative anatomist after Cuvier, beat bishop Wilberforce in a public evolution debate. But that was rhetoric, not science.)

The discontinuity of fossil records that puzzled Cuvier remains a puzzle and his idea of the essential role of *catastrophes* in shaping evolution may be correct. Who knows, if not for the catastrophes, if the Earth were moving through time without hitting potholes and bumps on the way – nice, smooth and continuous ride, then Nature would be more than satisfied with Precambrian jelly fish – there would be no need for the luxury of "higher" plants, animals and people.

*Les végétaux sont des corps organisés vivants,  
jamais irritable dans leurs parties, ne digérant point,  
et ne se mouvant ni par volonté, ni par irritabilité excessive.*

JEAN-BAPTISTE LAMARCK, PHILOSOPHIE ZOOLOGIQUE, 1809.

<sup>53</sup>Cuvier would have no problem with the (pre-Darwinian) concept of *purifying selection*, that is similar to weathering that smoothes out the shapes of old mountains and hills, but creative power of selection in formation of species would sound to him as bizar as creative power of erosion in formation of mountains.

A definition worth its name – mathematicians learned this from Alexander Grothendieck is not a concise wording of what everyone knows but a pointer toward unknown. Many unexpected fruits have been harvested from what grew from the seeds of ideas in his definitions.

Lamarck had never read Grothendieck. Not only his definition misses moving carnivorous plants, such as *the Venus Flytrap* (undoubtedly known to Lamarck) but his criteria are inapplicable to most of the tree of life. The two minor branches of this tree – plants and animals are poor representatives of (mostly unicellular) life on Earth.

The tragedy of Lamarck was that he put wrong signs at the portals of the edifice of the evolution theory he had erected. Like Columbus who misread America for Indies on the coastline of the new land, Lamarck misspelled selection for adaptation.

Worse than that, unlike his evolutionary successors, Lamarck had proposed several biological mechanisms underlying evolution. His ideas had the "drawback" of having scientific (*material* and/or *moving* in Aristotelean terms) ingredients in them that made these ideas experimentally verifiable; they turned out to be (essentially but not fully) incorrect.<sup>54</sup>

If you take for granted the miracle of adaptation of organisms to environment during their lifetimes, you may equally accept the Lamarckian idea of adaptation on the evolutionary time scale via some non-teleological(!) mechanism that allows an environmentally channeled influence of the potential future on certain internal processes in an organism that could benefit descendants of this organism.

This idea creates no logical problem in many instances where selection and adaptation are interchangeable in "explaining" evolution; yet, as Darwin points out, no specific environmentally induced mechanism (e.g. what was suggested by Lamarck) seems plausible in *evolution of the mode of reproduction* – this most essential feature of any species. (An instance of such "non-Lamarckian" feature is the *stability of 1:1 sex ratio* for such species as elephant seals with harems of several dozen females.) Another "non-Lamarckian picture" is seen, as was also indicated by Darwin, in *evolution of social insects*. (There is no essential disagreement, however, between Lamarck and Darwin about the folk idea on inheritance of acquired traits.)

Geometrically speaking, the connectivity of LIFE ON EARTH along the time coordinate via the heredity threads between organisms is rather tenuous unlike the full-fledged spacial connectivity/unity of individual organisms.

But is it possible to directly disprove, Lamarck's idea of adaptation and show that beneficial mutations take place *prior* to the change in the environment where they turn out to be beneficial? You can not do this by staring at fossilized remnants of extinct animals.

Yet, in 1943, Salvador Luria (biologist) and Max Delbrück (physicist) thought of an experiment the logic and the beauty of which would delight Gregory Mendel. This goes, roughly, as follow.

Grow a colony with, say, about billion bacteria, starting from a single cell e.g. of *E. coli*, a bacterium adored by bacteriologists. (*E. coli*, colonizes your

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<sup>54</sup>Some Darwinists rejoice in finding inconsistencies in Lamarck as if this makes their own theories more substantial.

guts within several hours of birth, where it switches from aerobic to anaerobic life and it adheres to the mucus of your large intestine until you die.)

Suppose, that when we apply some factor  $X$  to such a colony, say a virus (e.g. Bacteriophage T1 - Escherichia coli's best friend) which is deadly for the cells, there is, nevertheless, a small probability, say  $p_1 = 0.02$ , that some cell in the colony survives. In fact, such a survivor is a result of a mutation that brings a certain new property  $\Pi$  to the mutated cell that is manifested in tolerance of the mutant to the  $X$  factor. Moreover, and this is essential,  $\Pi$  passes on to all descendants of our  $\Pi$ -cell.

If you apply this procedure, to, say, one thousand different colonies, then about twenty will have survivors in them. Discard dead colonies and look how many among remaining twenty have *two* (or more) surviving bacteria. There are two conjectural alternatives.

**1. Lamarckian Adaptation.** If property  $\Pi$  develops in response to the factor  $X$  after it has been introduced, then the probability of having two survivors will be  $p_2(\text{adapt}) = 0.0004 = p_1^2$  being the results of simultaneous occurrence of two independent  $p_1$ -events. In this case

**a presence of three colonies with two survivors  
is highly unlikely.**

**2. Sheer Luck.** If some bacteria mutate *prior* to introduction of  $X$ , then this could happen before the last round of division with *half* billion bacteria on the plate with probability  $0.01 = p_1/2$ . This makes  $p_2(\text{luck}) \geq 0.01$  and there must be something like

**from seven to thirteen colonies with two survivors in them.**

*This is what Claude Bernard calls active science where you make experiments not to confirm your ideas, but to control them.*

When you check what actually happened, you find out that, univocally, **2** is true and **1** is false: about half of not fully dead colonies have two or more active bacteria in them.

(To Lamarck this would be no more in a contradiction with his idea of evolution, than, say, a guillotined head could be a counterexample to how use/disuse of an organ implies an adaptive evolution of this organ in a *slowly* changing environment.)

Stop! How can you tell how many bacteria out of *billion* are alive?

*Elementary, my dear Watson.* Shake your cells in a liquid to randomly mix them and spread the culture on a nutrient plate. (Technically it is better first to spread the fully alive culture and then apply  $X$ .) Then each survivor – each  $\Pi$ -cell – starts dividing and in a short while you see as many colonies on the plate as you had  $\Pi$ -cells to start with.

(If we do not introduce  $X$  and keep alive colony at a limited size with a constant flow of nutrients, then, some time afterwards, only few cells will have surviving descendants and, eventually, only one cell. This follows from Anaximander's **Theorem 1** when it is empowered by an estimate of the *probability of extinction* that was calculated by Francis Galton and Henry William Watson in their 1874 paper. There is hardly one chance in million that the  $\Pi$ -cells from the original colony would have descendants in a few years.)

The *original* resistance to  $X$  is not preserved and none is visible in the colony; yet, the "dormant guardians" are there and they "fight back" when the

colony is attacked by  $X$ . Lamarck would say that colonies do adapt by "voting for survival" of their luckiest members and that female egg-cell, might be similarly selecting the fittest contender out of hundreds millions available spermatozoids; thus, ensuring adaptation of the progeny. Let Darwin and Weismann disprove this.)

Another idea of Lamarck is the presence of a *complexifying force* that *drives* organisms from simple to complex forms with the environmental "pressure" being superimposed on this force. You can not discard this by brandishing the banner of natural selection in the air, try something cleverer.

Probably, such a "force", if it exists, resides in the *logic of chance* in the evolutionary game (frame?) of LIFE, possibly, expressible in a language of a Grothendieck-like mathematics. But deciding one way or the other seems more difficult, than, for example, (quasi)*rigorously* "deriving" irreversibility in thermodynamics from the reversible laws of physics where the present day mathematical rendition of Boltzmann's arguments does not seem satisfactory.

In either case *mathematics of the multiplication table type* that suffices for the Luria-Delbrück experiment seems of little help to you. (But maybe the main problem is that we do not understand *multiplication table*.)

*... there perished many a stock, unable  
By propagation to forge a progeny.  
For whatsoever creatures thou beholdest  
Breathing the breath of life, the same have been  
Even from their earliest age preserved alive  
By cunning, or by valour, or at least  
By speed of foot or wing.*

TITUS LUCRETIUS, ON THE NATURE OF THINGS, 50 BCE(?).

*...et ces espèces que nous voyons aujourd'hui ne sont que  
la plus petite partie de ce qu'un destin aveugle avait produit.*

PIERRE-LOUIS MAUPERTUIS, ESSAI DE COSMOLOGIE, 1750.

*...just in proportion as this process of extermination has acted on  
an enormous scale, so must the number of intermediate varieties,  
which have formerly existed, be truly enormous.*

CHARLES DARWIN, ORIGIN OF SPECIES, 1859.

Dramatic cut-off of exponentially growing functions in a bounded space, called by Darwin *natural selection*, is an apparent logical necessity, not an intrinsically biological property of living systems.

The principle of natural selection no more "explains" evolution than differential equations "explain" mechanical motions, but this principle provides a conceptual framework and suggests a language for possible mathematical models of evolution.

This must have been obvious to Maupertuis and to anybody who had ever pondered on population growth and who could fathom the immensity of  $\exp T$ , e.g. to Buffon who along with Maupertuis was wondering

*What purposes then are served by this immense train of generations,  
this profusion of germs, many thousands of which are abortive for the  
one that is brought to life?*

to Benjamin Franklin, who wrote on some occasion in 1751:



*no bound to the prolific nature of plants or animals, but what is made by their crowding and interfering with each others means of subsistence,*

to Euler, who, in *Introductio in analysin infinitorum* (1748), illustrates the exponential function by examples from population dynamics and to Fibonacci who savores numerical subtleties of an idealized rabbit replication model in his *Liber Abaci* (1202).

An imbalance between the growth of the population and of resources also must have been known for centuries. The problem and an approach to it are discussed, for example, in Giovanni Botero's treatise *Delle cause della grandezza delle città* (1588).

A humanistic perspective on the solution of the overpopulation problem was proposed, in almost modern terms, by Nicolas de Condorcet in *Esquisse d'un tableau historique des progrès de l'esprit humain* that was published posthumously in 1795. But there is nothing humanistic in how Nature has been handling the (over)population problems for millions of years.

#### CONDORCET AND MALTHUS.

Condorcet is remembered by mathematicians for his 1785 essay  
*sur l'application de l'analyse à la probabilité des décisions rendues à la pluralité des voix*

where he proves his *jury theorem* on probability of a group of individuals to arrive at a correct decision and where he analyses the (*voting*) *paradox* of non-transitivity of the *order relation on decisions* defined by collective preferences.

His passionate manifesto

*Esquisse d'un tableau historique des progrès de l'esprit humain*

was written between October 1793 and March 1794 when Condorcet was in hiding having been condemned to death by Robespierre government for his humanistic political views.

(About a century afterwards, a monument was erected in Paris with the torso of Lavoisier, who was guillotined forty days after Condorcet was found

dead in his jail cell, and the head of Condorcet. This was not done on purpose.)

*Thomas Malthus*, in *An Essay on the Principle of Population* published in 1798, sides with Nature in her solution to the overpopulation problem and expresses his skepticism on the social practicability of the solution proposed by Condorcet.

Malthus' influence is due to the two facts.

1. In the 19th century, the number of readers who were willing to pay for a book with  $\exp T$  in it reached the critical mass needed to make the publication of such a book profitable.

2. Darwin and Wallace were among these readers.

But the idea that the technological progress can compensate for Malthusian  $\exp T$  remains as much in contradiction with the multiplication table to-day as it was at the time of Malthus. The rules of arithmetic do not change, at least not on a so short time scale.

#### ENTROPY VERSUS ENERGY BARRIERS IN THE DEVELOPMENT OF SCIENCE.

*The world is full of magical things patiently waiting for our wits to grow sharper.*

BERTRAND RUSSELL

There is a fundamental difference in the logical structure of principles of physics, such as *Newton's second law*, and those of evolutionary biology such as *adaptation by selection* that is reflected in our perception of these principles.

Learning and understanding physics is hard; one needs a firm mathematical background and a non-trivial intellectual effort to comprehend the meaning and the consequences of something like the second law, for example. Only a small percent of "educated people" even understand what it means to UNDERSTAND the second law.<sup>55</sup>

On the other hand the idea of *evolutionary adaptation by selection* strikes anybody, who is aware of the enormity of exponential growth, as obvious. (Re-alising insufficiency of this idea in its original Darwinian form needs a higher level of mathematical sophistication.<sup>56</sup>)

But why hadn't the full-fledged idea of evolution by selection sprouted from the brilliant mind of some mathematician such as Daniel Bernoulli or Leonhard Euler?

An idea in science, as a molecule in a chemical reaction, in order to reach the critical maturation point may need not to climb over a mountain of high energy, but rather to follow a narrow pathway in a labyrinth of tunnels through this mountain. Once such a pathway is found, everybody can follow effortlessly.

There may be yet other simple ideas in science waiting to be discovered that are as illuminating as the idea of the pivotal role of the exponential function in the evolutionary dynamics. If you think such ideas are "obvious", pinpoint a single one of them.

*... preservation of favourable variations, and the destruction of injurious*

---

<sup>55</sup>This is why the members of *flat Earth societies* do not argue against the second law.

<sup>56</sup>On the other hand, mathematically insensitive people may be driven against the very ideas of evolution and natural selection by some (Freudian?) psycho-sociological mechanism.

*variations, I call Natural Selection, or the Survival of the Fittest.*

CHARLES DARWIN, ORIGIN OF SPECIES 5TH EDN.1869.

It is not this survival rhetoric however that had been mainly occupying Darwin's mind but a persistent idea of turning the *cut-off effect on the exponential growth in biology* into a *principle of selection* (of continuity?), something like Maupertuis' *principle of least action*<sup>57</sup>.

Maupertuis, who had been working over his principle for two decades, believed that Nature always minimizes/optimizes whatever she does and he must have tried to figure out the formula for "evolutionary action" that is minimized by Nature in the course of selection. Part of this must be *time*, he would assume, since not so much *perfection* of your fitness makes you the winner, but how fast, let imperfect, your fitness can be evolutionary achieved. But he, probably, could not guess and write down other terms in this "action".

Besides, Maupertuis would observe, that mutability and the reproduction rate must be sufficiently high in order for *evolution by selection and nothing else* to be logically/mathematically feasible.

Very roughly,  $R^T$  for  $R$  being the reproduction rate, must "beat" something like  $2^P$  where  $P$  is the number of "variable parts" of an organism and where a presence of selection, that channels evolution by pruning off the majority of mathematically conceivable branches of development, makes the estimated time  $T$  needed for evolution longer rather than shorter.

On the other hand, the mutation rate can not be too high; otherwise, deleterious mutations, that are in a dominant majority, would result in extinction. The most dangerous are *dormantly deleterious mutations*, e.g. those that increase the mutation rate).

Being a student of Newtonian (non-Aristotelian) mechanics (with impulses as coordinates), Maupertuis would assume that the main *biological observable* (feature) that evolves by selection is "mutability", or rather its opposite – *fidelity of reproduction*. (All Nature knows is to keep mutation rate as low as she possibly can – evolution is a random dance on the razor's edge between stagnation and extinction<sup>58</sup>.)

Finally, Maupertuis, a mathematical physicist rather than a pure mathematician, would try to match rough numerical estimates of this kind with the schedule of the fossil data.

Well... Maupertuis, had done none of this and neither had Darwin who remained dissatisfied with the selection theory of evolution, judging by how insistently he was *convincing* himself that the natural selection explains evolution and by how eloquently he was declaring that he came to *believe* in the validity of the idea. It is for myths, not for science, to *explain* the world and life and it is up to preachers and politicians, not for scientists, to *convince* people of

<sup>57</sup>Maupertuis identified certain quantity, called *action*  $A = A(motion)$ , such that a *moving* physical system  $S$  minimizes this  $A$ , or rather, the *motion* of  $S$ , thought of as a spatial curve, satisfies the corresponding *Lagrange-Euler (differential) equations*. Lagrangian and Hamiltonian incarnations of this principle are present in all branches of mathematical physics.

<sup>58</sup>Nature faces the same problem as a public educational system – it should be *almost but not fully* perfect: if students 100% comply with the demands of their teachers, the society stagnates; but too little rigour results in *error catastrophe*, cancer of corruption and extinction.

What is amazing (for non-Lamarckians) is that besides sheer sloppiness, there are provisions by Nature for (quasi-random) "desirable mutations", e.g. in specified loci of genomes of parasites evading the immune systems of the hosts and in genomes of bacteria under stress.

anything.

Darwin wanted more of his selection theory than being merely a *convincing explanation* of the evolution of Life on Earth but mathematics of the 19th century had nothing to offer for substantiating his vision.

A cautious 21st century mathematician would not expect a single clear cut mathematical theorem/theory fulfilling Darwin's dream but he/she may still envisage a possibly vague, yet, *truly mathematical* setting embracing the idea of LIFE.

An instance of such a not quite precise but purely mathematical idea often used in physics is that of representing a state of a dissipative dynamical system, e.g. of a quasi-stationary flow of a viscous fluid, as an *ensemble of attractors* in the corresponding phase space.

But no such concept, nothing of what one expects to find in the 20th century dynamics, seems suitable for expressing much of the whole idea of LIFE.

(There may be dynamic-theoretic models of some LIFE's fragments. For instance, one possibly can describe the multi-scale time feature of the *evolution-by-selection*, with different classes of *units of selection*<sup>59</sup> on different time scales, in the language of "multistage dynamical systems" where attractors themselves are being subjects to dissipative-like *predominantly contractive* dynamics.)

It seems that a satisfactory understanding of genomes' stochastic dynamics and transforming the mathematical poetry of the Darwinian idea of evolution by selection to a hard-boiled scientific theory lies a long way off.

*.... our [19th] century will be called... the century of the mechanical view of nature, the century of Darwin.* LUDWIG BOLTZMANN.

The abstractly logical – *formal* in the Aristotelian sense – Darwinian SELECTION PRINCIPLE may be alluring to mathematicians and to mathematical physicists like Boltzmann; yet, this *formality* is disconcerting. How can some property of the logic/syntax/mathematics of the language that one uses stand for the *main reason* of something physically or biologically occurring?

Refined mathematical concepts may seem no more helpful for understanding the harsh truths of the real world than poetic metaphors for this purpose. Only *Naive Mathematician* may take, for instance, the summation rules of infinite series for "explanation" of Zeno's paradoxes or the general theory of (classical or/and quantum) Einstein-Lorentz spaces for a full physical model of SPACE-TIME. But *Naive Mathematician* may protest by pointing out, for example, that when Laplace writes about

*Une intelligence qui, pour un instant donné, connaît toutes les forces... embrasserait dans la même formule les mouvements des plus grands corps de l'univers et ceux du plus léger atome....*

he hardly thinks that the solution of the problem of determinism may be advanced by a study of the mind of this metaphoric *intelligence* by an experimental neuropsychologist (as some 21st centuries thinkers suggest), but he would accept that the (quasi)deterministic behaviour of planetary motions might be explicable in the light of *unique solvability of differential equations* and/or *KAM*-like theorems.

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<sup>59</sup>Mathematically speaking, units of selections are *particular* biological observables but it is unclear *particular* in what sense. For example, can *fidelity of reproduction* – one of the basic and evolutionary slowest observables – be taken for such a "unit"?

The skepticism of the hard core scientists should not divert us from a quest for mathematics that would bring "light of sense" to evolutionary biology.

We want to have maximally general, yet biologically sound mathematical concepts of

*gene, organism, population, unit of selection, environment, adaptation.*  
so that we can make sense of the following questions.

What are probability(?) spaces where selection works?

What defines (dis)connectivity of genomes, organisms, populations?

Why adaptations represented by (quasi)fixed points of (gradient?)  
evolutionary dynamics in *different* spaces, e.g. of genes in the  
environments of other genes and of organisms themselves in the "natural"  
environments, leads to *similarly looking* "coarsely grained adaptation" of  
populations, as we saw in in the example of domesticated species reverting  
to the wild type.

## 8 Brain.

*Tell me where is fancies bred,  
Or in the heart or in the head.*

SHAKESPEARE, THE MERCHANT OF VENICE.  
(Probably, written between 1596 and 1598.)

*... mental activities are entirely due to the behavior of  
nerve cells, glial cells, and the atoms, ions, and molecules  
that make them up and influence them.*

FRANCES CRICK, THE ASTONISHING HYPOTHESIS (1994)<sup>60</sup>

Shakespeare would not have been intimidated by the scientifically sounding "*mental activities*" instead of his "*fancies*"<sup>61</sup> and the Hypothesis would not have stricken him as especially astonishing.

After all, it has been known since about 2500 BCE that different types of head injury produce different symptoms, as it was recorded in *Edwin Smith Surgical Papyrus*, of about 1500 BCE where the idea of BRAIN appeared for the first time.<sup>62</sup>

<sup>60</sup>In early 1950s, Frances Crick and James Watson worked out, with the X-ray crystallography data contributed by Rosalind Franklin, a correct helical model of DNA. (Incorrect triple-stranded helix structure was earlier suggested by Linus Pauling). They concluded:

*it therefore seems likely hat that the precise sequence of the bases is the code that carries the genetical information.*

Crick also proposed

**THE CENTRAL DOGMA OF MOLECULAR BIOLOGY:** DNA→RNA→Proteins.

This ... deals with the detailed residue-by-residue transfer of sequential information.

*It states that such information cannot be transferred back from protein to either protein or nucleic acid.*

From age 60 until his death (2004) Crick worked on the Brain; in particular, he tried to identify specific neuronal processes responsible for consciousness.

<sup>61</sup>No matter how hard you try, you can not say much about the arrow [brain]↔[mind] without resorting to metaphors. (Poetic "*bred*" prompts in you a surge of ideas, while dry "*entirely due*" transmits zero positive information.)

<sup>62</sup>Surgical Papyrus is an incomplete copy of a text attributed to *Imhotep* (≈2650 – 2600 BCE), called "*inventor of healing*", who was high priest of the Old Kingdom of Egypt as well as chief builder and chief physician in his time. He is the first physician known by name.

Shakespeare could not have been acquainted with the *Surgical Papyrus* – this was discovered in the middle 1800s, but he might have been aware of the following.

*The seat of sensations is in the brain. ...  
All the senses are connected in some way with the brain...  
This power of the brain to synthesize sensations  
makes it also the seat of thought.*

[Attributed to] ALCMAEON OF CROTON (≈450 BCE).

*the source of our pleasure, merriment, laughter, and amusement,  
as of our grief, pain, anxiety, and tears, is none other than the brain.*

HIPPOCRATES(?), ON THE SACRED DISEASE, (≈425 BCE).

*... sense-perception in sanguineous animals is the region of the heart....  
ARISTOTLE, SLEEP AND SLEEPLESSNESS (≈350 BCE).*

It is hard to believe that clinical observations by Alcmaeon and Hippocrates did not convince Aristotle, who had not accepted "the Astonishing Hypothesis" because:

- (1) the heart, unlike the brain, connects with all the sense organs;<sup>63</sup>
- (2) the heart is more centrally placed;
- (3) the heart in embryos develops before the brain;
- (4) invertebrates, who have hearts but no brains, have sensations;
- (5) the heart but not the brain is affected by emotions;
- (6) the heart is warm but the brain is cold;
- (7) the heart but not the brain is essential for life.

Half a century later, Alexandria's anatomist Herophilus (335 - 280 BCE) discovered that that nerves spread from the brain throughout the body in agreement with the idea that the brain was the controlling organ in Man.

His younger colleague Erasistratus (304 - 250 BCE), who believed that the psychic pneuma was transmitted through motor nerves to muscles, appreciated the separate neural pathways for motor and sensory functions He also suggested that the degree of intelligence in animals is correlated with how much cerebral hemispheres are convoluted.

But the experimental evidence for the brain control over the body came four centuries later, when Galen of Pergamon (129 - 200?) had shown that cutting the recurrent laryngeal nerves which innervate the larynx makes a pig stop squealing but not struggling.<sup>64</sup>

Here is his reaction to those who were not convinced in the dominance of the brain after having attended this experiment.

*When I heard this, I left them and went off, saying only that I was mistaken in not realizing that I was coming to meet boorish skeptics; otherwise I should not have come.*

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<sup>63</sup>Nerves are less prominent than blood vessels.

<sup>64</sup>Besides founding experimental medicine including experimental neurophysiology, Galen greatly advanced anatomy, physiology, pathology and pharmacology of his time. For example, he showed that urine comes from the kidneys, he demonstrated that the larynx generates the voice and he recognize the essential difference between venous and arterial blood. He also developed several surgical techniques, including a one for correcting cataracts.

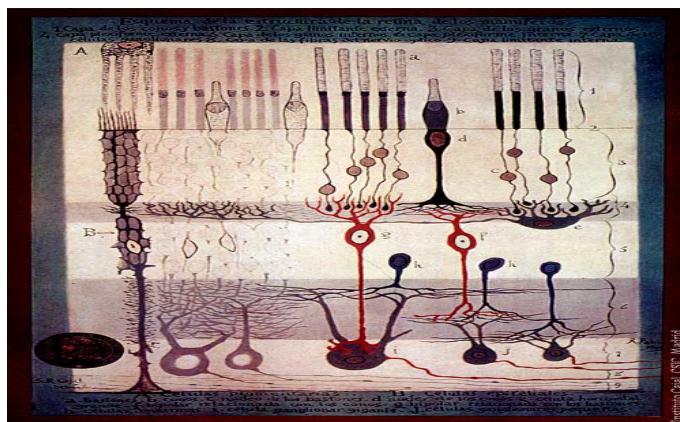
Ancients had no idea of a cell and could not fathom how the brain works. And when the cellular structure of the brain started unravelling itself in all its beauty, the inspired by it founders of the *neuronal theory* began to speak of the brain in poetic language:

*The brain is waking and with it the mind is returning.  
It is as if the Milky Way entered upon some cosmic dance.  
Swiftly the head mass becomes an enchanted loom  
where millions of flashing shuttles weave a dissolving pattern,  
always a meaningful pattern though never an abiding one;  
a shifting harmony of subpatterns.*

CHARLES S. SHERRINGTON, MAN ON HIS NATURE, 1942.

*To know the brain...is equivalent to ascertaining the material course  
of thought and will, to discovering the intimate history of life  
in its perpetual duel with external forces.*

SANTIAGO RAMON Y CAJAL, RECUERDOS DE MI VIDA 1917.



Cajal and Sherrington, apparently, speak of *the human brain* but the experimental neuroscience and, in part, the brain anatomy rely on the study of animal brains, starting from ox and pig brains dissected by Galen and continued with a microscopic study of insect brains since 18th century.

*... the brain of an ant is one of the most marvellous atoms of matter in the world, perhaps more so than the brain of a man.*

CHARLES DARWIN (1859)

For instance, the collective mind of ants is able to achieve shortest paths between locations in a rugged terrain:

*a busy ant highway between an anthill and a source of food  
usually implements a nearly shortest possibility.*

#### ANTS, CHEMISTRY AND LOGIC.

There are more than 10 000 different species of ants and variations within a species may be also high. On the average(?), there are about a quarter of

million of brain cells in an ant. But there are ants and ants with their weights ranging from 0.01mg to half a gram, where, in small ants, the brain makes more than 10% of the animal body weight – comparable to that in a newborn infant and by far more than 2% in adult humans.

*If my opinions are the result of the chemical processes going on in my brain, they are determined by the laws of chemistry, not those of logic.*

JOHN HALDANE, THE INEQUALITY OF MAN (1932).

Ants mark their trails with pheromones and themselves tend to choose the routes that have stronger pheromone odors. All things being equal,

*the number of ants that pass back and forth on some track, say during 1h, is inverse proportional to the length of this track;*

hence, the shortest track becomes the smelliest one, thus, eventually preferred by the ants.



What has made this algorithm evolutionary attainable is its simplicity and universality. Probably, basic programs running within our minds, just in order to exist at all, must be comparably simple and universal. But there is no any "law" in sight<sup>65</sup> that determines ants' opinions of where to go.

CAJAL AND SHERRINGTON.

Cajal studied nervous and brain tissues with a *silver staining technique* discovered by Camillo Golgi in 1873 and established that the *neurones* are basic units of nervous structure. This theory was completed with the introduction of the concept of *synapse* by Sherrington who writes in his 1906 book THE INTEGRATIVE ACTION OF THE NERVOUS SYSTEM:

*At the nexus between cells if there be not actual confluence, there must be a surface of separation.... In view, therefore, of the probable importance physiologically of this mode of nexus between neurone and neurone it is convenient to have a term for it. The term introduced has been synapse.*

## 9 Mind

*The energy of the mind is the essence of life.*

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<sup>65</sup>It is doubtful whether the expressions "laws of chemistry" and "laws of logic", unlike those of "laws of classical mechanics", make any sense at all.

ARISTOTLE

*As followers of natural science we know nothing of any relation between thoughts and the brain, except as a gross correlation in time and space.*

CHARLES SHERRINGTON.

#### WHAT IS THE MIND? WHAT ARE THOUGHTS.

How much have we to know about the brain to understand the mind?

Imagine, by the middle of 21st century, we will have learned as much about the brain as we know to-day about the fundamental quantum mechanical laws of matter and energy or about the nanoscale dynamics of the cell. Would it help?<sup>66</sup>

Or, we will only clearer realize our helplessness and, following Pauli, will say that "mind", like "reality" is

*something self-evidently known... [but it is] an exceedingly difficult task ... to work on the construction of a new idea of mind.<sup>67</sup>*

But is it possible at all to translate the following poetic mage of brain/mind to the language of science?

*The eye sends ... into the cell-and-fibre forest of the brain throughout the waking day continual rhythmic streams of tiny, individually evanescent, electrical potentials.*

*This throbbing streaming crowd of electrified shifting points in the sponge-work of the brain bears no obvious semblance in space pattern, and even in temporal relation resembles but a little remotely the tiny two-dimensional upside-down picture of the outside world which the eyeball paints on the beginnings of its nerve fibers to the brain.*

But that little picture sets up an electrical storm. And that electrical storm so set up is one which effects a whole population of brain cells. Electrical charges having in themselves not the faintest elements of the visual - having, for instance, nothing of "distance," "right-side-upness," nor "vertical," nor "horizontal," nor "color," nor "brightness," nor "shadow," nor "roundness," nor "squareness," nor "contour," nor "transparency," nor "opacity," nor "near," nor "far," nor visual anything - yet conjure up all these.

*A shower of little electrical leaks conjures up for me, when I look, the landscape; the castle on the height, or, when I look at him approaching, my friend's face, and how distant he is from me they tell me.*

*Taking their word for it, I go forward and my other senses confirm that he is there.*

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<sup>66</sup>The molecular machinery of a living cell transcribes + translates each codon – triple of four basic nuclear acids in DNA (except for three stop codons) to one out of twenty standard amino acids (with one stop codon sometimes coding for Selenocysteine). The particular coding/translation rule used by Nature, that is *the same for* (almost) *all* organisms on Earth and that is called *genetic code*, is, for an experimentalist, the most fundamental law of biology. On the other hand, nothing(?) in life would visibly change if the code was somewhat different. What is essential in biology from a mathematician's perspective is the *principle* of coding, rather than *specificity* of the code being used.

The brain may harbor an immensity of such "coding arbitrariness"; an accumulating avalanche of the detailed knowledge of this "code" may block out rather than promote our understanding of the arrow [brain] ~ [mind].

<sup>67</sup>The idea of "reality" is inseparable from "mind": *Cogito ergo sum.*

SHERRINGTON, MAN ON HIS NATURE.

What does it mean to understand mind? What are the right questions to ask?

It seems there is an invisible interface between the brain and what we perceive as our mind – an interface that is comparable in its structural complexity to the *machinery of embryological development* that, as Thomas Hunt Morgan says, effectuates the transformation of what is "written" in the genes into [phenotypic] characters that are used by the [classical] geneticist.

*What I cannot create, I do not understand.*  
RICHARD FEYNMAN

Here we speak of creating *abstract models* not necessarily implemented by physical devices.<sup>68</sup> and some of them may not need a knowledge of the brain, at least for modelling "parts" of the mind, such as *memory*, for instance.



*Memory ... an Organ, as the Eye, Ear, or Nose,  
where the Nerves from the other Senses concur and meet. ...  
... I conceive to be nothing else but a Repository  
of Ideas formed partly by the Senses, but chiefly...  
... receiving, and being excited by such Impressions,  
they do again renew their former Impression... .*

ROBERT HOOK, ON HYPOTHETICAL EXPLICATION  
OF MEMORY, LECTURE TO THE ROYAL SOCIETY, 1682.

This looks almost as a verbal description of a *mathematical model* of memory, but some do not like such idea.

*Intellectual snobbery makes them* [mathematicians?] *feel they can produce*  
*results that are mathematically both deep and powerful and also apply*  
*to the brain.*

FRANCES CRICK, WHAT MAD PURSUIT (1988).

However, new ideas on mind/brain came from mathematical side along with the concepts of *information flows* and of *Turing machines* that allowed formulations of new kind of questions, such as:

*Can machines do what we (as thinking entities) can do?*

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<sup>68</sup>Ability to create does not imply understanding. Animals (as well as plants and bacteria) can create their approximate copies but even the smartest of the human stock have only the vaguest images in their minds of the ways of embryological development of their progeny.

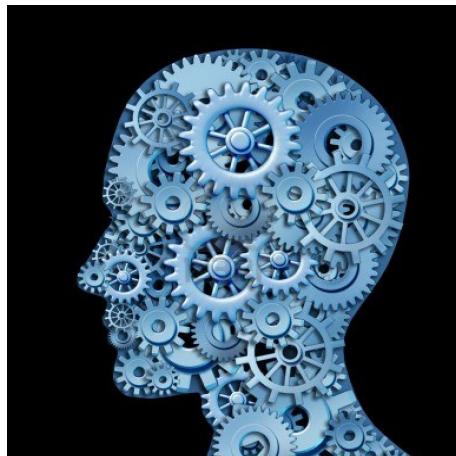
ALAN TURING, COMPUTING MACHINERY AND INTELLIGENCE (1950)

Attempts to disprove Turing's reasoning that a digital computer can imitate human intelligence are in the line with the argument that people could not have come from the same stock as monkeys, since monkeys lack in moral virtues and do not abide by traffic laws.

And it is not only apes and monkeys – also bacteria in our guts are our remote cousins as far as biology is concerned. The unity of life on Earth lies deeper than similarity in anatomy and physiology. Something of this kind must be true about "intelligence" except we do not what it is.

The difficulty in understanding the human mind is that it is *unique of its kind*; hence, there is no language to speak about it. The brain, on the contrary, is shared by almost all animals on this planet. This is why the neuroscience advances but the scientific theory of mind has been staying still for several millennia.

And how, on Earth, can one disprove Turing? – We *are* biological machines as much as, say, ants are, albeit, individually, we have more neurons in our brains:



*Man is so complicated a machine that it is impossible to get a clear idea of the machine beforehand, and hence impossible to define it.*

JULIEN OFFRAY DE LA METTRIE, MAN A MACHINE. 1748.

But maybe, what happens in our minds with wheels within wheels within wheels in them is too complicated to be representable by mere math? This is what Alain Edgar Poe thought in connection with chess.

*Arithmetical or algebraical calculations are, from their very nature, fixed and determinate.... [But] no one move in chess necessarily follows upon any one other.*

EDGAR ALLAN POE, MAELZEL'S CHESS-PLAYER, APRIL 1836.

Of course, Poe could not have known of Turing, but he was aware of the calculating machine of Babbage and argued that playing chess *can not* be modelled

on such a machine. And although what he was saying was formally incorrect, a fundamental difficulty<sup>69</sup> he pinpointed in such a modelling remained unresolved up to the present day.

Another problem with understanding human mind is that our intuitive concept of *intelligence* shaped by the existential ego of Cartesian *cogito ergo sum*, is buried in the multilayered cocoon of *teleology – purpose, function, usefulness, survival*. Turing purposefully avoids the question – *What is intelligence?* – since you can not answer this question unless you develop an appropriate language. (It is the same as with the question – *What is LIFE?* that can not be answered in the language of *LIFE ON EARTH*.)

However, Turing suggests that a "human-like intelligence", can be built in a machine by subjecting it to education. Indeed, *learning* is a more intelligent concept than *intelligence*<sup>70</sup>, but one will not go far with it if one assumes that the child brain is *something like ....lots of blank sheets*.

*Instead of trying to produce a programme to simulate the adult mind,  
why not rather try to produce one which simulates the child's?  
If this then subjected to an appropriate course of education  
one would obtain the adult brain.*

*Presumably the child brain is something like a notebook as one buys  
it from the stationer's. Rather little mechanism, and lots of blank sheets.*

ALAN TURING, COMPUTING MACHINERY AND INTELLIGENCE.

An unbridled delight in the brilliance of our "intelligence" blinds us to seeing the essence of the mind but modelling the learning process by a child may bring the light into the picture.

*Man is most nearly himself when he achieves  
the seriousness of a child at play.* HERACLITUS.

All a child knows is play and this is how he/she learns. The mathematics of this *playful learning process*, that is a transformation of the flow of electric/chemical signals the brain receives into a coherent picture of *external world* in the first two years of human life, is as intricate and mysterious as that of emergence and evolution of *live structures*.

And it is, so to speak, *free learning*: if you subject a human (or animal) infant to "education" you only impair the learning, since you have no idea of what happens in the brain/mind of a child. (Not that you know better how your own mind works.) Applying your "adult intelligence" to aid infant's development is like a use of forceps in helping an ameba to divide. No surprise, learning programs that have been developed until today are far cry from Turing's dream.

*In order to progress*, instead of inventing more and more "intelligent" definitions of *intelligence*, *we must recognize our ignorance* as Feynman says, forget/rectify<sup>71</sup> how we "naturally" see ourselves and start with potentially an-

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<sup>69</sup>As one tries to analyze a position many moves ahead, one is getting overwhelmed by the exponentially growing numbers of branches of possible game strategies.

<sup>70</sup>*Learning* brings along a relatively slow *time coordinate* in the description of *mind* similarly to how *evolution* brings such a coordinate in the picture of *Life on Earth* where *evolutionary learning* is described in the language of *natural selection*.

<sup>71</sup>*L'esprit humain se plie à une manière de voir* – Lavoisier says in his paper on *phlogiston* – the common sense idea of *warmth* disguised as a scientific concept in pre-Lavoisier chemistry.

swerable questions.

What are *logical (molecular-like) units*<sup>72</sup> abstract intelligence is composed from?

An absence (presence?) of what structure(s) in the mind of a baby ape makes it ability and willingness to learn after the age  $\approx 1.5$  years to fall below that of a human child but still keeps its learning gradient above that of an adult human?

What is the mechanism(s) responsible for rarely occurring structurally elaborate patterns within "human intelligence" such as exceptional musical and mathematical abilities? Are there counterparts for these in animals?

Do these patterns, that are harmful rather than helpful for *survival in the wild*, come by the way of natural selection and if so what is the pool of possibilities/variations of these patterns?

What is a non-teleological description of the role played by *predictable future* in functioning of an "intelligent system"?

What is a simple/short list of questions (or, rather an interactive algorithm generating questions depending on the history of a conversation) that would be easily answerable by a human of any culture, e.g. by a Cro-Magnon child (IF the language problem is somehow taken care of) but would cause an insurmountable difficulty for currently available "imitation programs".<sup>73</sup>

We need to design arguments and/or experiments for finding the answers that would serve to control rather than to confirm our ideas, being most satisfied with an outcome if it does not resemble anything we had imagined. Thus we shall be able to trace and to seal the route by which the self-gratifying illusion of our "intelligence" came from and to start a walk on a road toward understanding what the mathematical essence of mind/learning/ intelligence is.

Quite likely, a full instruction for making a human like learning system can be written down on a couple hundred pages.

But even if you are on the right track for doing so, it may be a long way to go. It took Nature half billion years and quadrillions of tries to arrive at the present design of our nervous system and we may need quadrillions of man/computer hours for designing a comparable system ourselves.<sup>74</sup> And the detailed knowledge of the wiring of the human brain would not necessarily help because of the indecipherable stochastic redundancy in its architecture.

## 10   Mysteries Remain.

*Ignoramus et ignorabimus.*

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If you follow Lavoisier's path of reasoning, you will find his words applicable in connection with "intelligence" and "consciousness" as well. These notions are helpful in the practical psychology but no common sense definition of *intelligence* and/or *consciousness* can be taken for anything *scientific* in Lavoisier's understanding of the idea of science.

<sup>72</sup>These units must be functions of *abstract context/purpose free* "variables", like *imprinting*, that applies to [first/second...] (moving object) and of *Hawk/Goose* that depends on [frequently/rarely] (occurring event).

<sup>73</sup>Such questions must refer to (and/or recycle) phrases that had been already used in the course of a dialog. Probably, the length  $L$  of the (suitably defined) *shortest naive* imitation program that is undetectable by a clever algorithm after an exchange by  $n$  sentences must grow at least exponentially,  $L \sim 2^n$ .

<sup>74</sup>One may accept this as a possibility if one believes that  $NP \neq P$ .

EMIL DU BOIS-REYMOND (1872)<sup>75</sup>

*In mathematics there is no ignorabimus.*

DAVID HILBERT (1900)

*Wir müssen wissen — wir werden wissen!*

DAVID HILBERT (1930)



Du Bois-Reymond suggested<sup>76</sup> that, possibly, humans shall *never* understand the following:

1. *Nature of matter and force.*
2. *Origin of motion.*
3. *Origin of Life.*
4. *Emergence of seemingly purposeful organisation in Nature.*
5. *Generation of conscious sensations by unconscious nerves.*
6. *Source of intelligent thought and Language.*
7. *Non-determinism of free will in deterministic Universe.*

Do you agree with what Du Bois-Reymond thinks of these issues?

To answer this, you need to find mathematical counterparts to the words there that are used (implicitly) metaphorically.

For instance what is DETERMINISTIC Universe"?

Most likely, Du Bois-Reymond carries on the back of his mind Laplace's idea of *une intelligence qui, pour un instant donné, connaît trait toutes les forces...* that models the universe by a system of ordinary differential equations.

This *model* is deterministic due to the unique solvability of such systems under due regularity conditions.

Of course, this is absurd, nobody would literally think that the state(?) of Universe is given by a set  $u(t) = (u_1(t), \dots, u_k(t)) \in U = \{u_1, \dots, u_k\}$  at a moment(?)  $t$  by EXACT values of some real parameters  $u_i$  and that determinism amounts to some technical uniqueness theorem.

However, one can make sense of it by formulating the question in the following terms.

*Selective Instability of Dynamical Systems.* Let  $F = F_1$  by the "future" map from  $U$  to itself, for  $F : u(t = 0) \mapsto u(t = 1)$ . Given a point  $u$  in  $U$  define its *instability profile*  $IP(u)$  as the set of those  $u'$  in the "vicinity" of  $u$  for

<sup>75</sup> Du Bois-Reymond discovered *nerve action potential* and proposed chemical nature of synaptic transmission.

<sup>76</sup> Über die Grenzen des Naturerkennens, 1872 and Über die Grenzen des Naturerkennens: Die sieben Welträtsel, 1891.

which "essential parameters" of  $F(u)$  in  $U$  undergo "improbably large" (nearly discontinuous) variations under "small variations" of  $u$  in  $IP(u)$ .

*Selective Instability* of a state  $u$  signifies in this terms some particular mathematical property of the set  $IP(u)$  that corresponds to the vague<sup>77</sup> idea of being "structurally interesting".

States  $u$  of the Universe that depict *physical* systems  $P = P(u)$  are (tacitly) assumed NOT to be *selectively unstable*.<sup>78</sup>

But if there is something *alive* about  $u$ , e.g. a protein molecule  $M(u)$ , a cell  $C = C(u)$  or, even better, an "intelligent" humanoid ape  $A = A(u)$ , then  $IP(u)$  does look very interesting. And it may (or may not) be possible to correlate particular features of  $IP(u)$  with what  $A(u)$  calls "my free will" thus refuting what Du Bois-Reymond says about it.

In order to make the above "rigours" one needs to specify various terms used: "small variations", "improbably large", etc. which is done by "adjusting logical parameters" with a use of a representative set of examples. Below is one.

*Unlockig The Door.* Let you universe is reduced to a room with door and a lock in it, with a key  $K$  that may be in any position in this room and a mechanical contraption/apparatus  $A$  with say 100 degrees of freedom and such that, according to your record, it is equally likely to occupy all its own states,

If  $A$  is "non-intelligent" and, at  $t = 0$ , the key in a state  $u$  is positioned, say in the middle of the room, then the probability of the door being unlocked at the moment  $t = 1$ , if it was locked at  $t = 0$ , is virtually zero.

But if you observe that this probability greatly increases whenever some appendage of  $A$  holds  $K$ , then, you bet,  $A$  is intelligent.<sup>79</sup>

"Nature of mathematics" is absent from Du Bois-Reymond's list. Probably, he did not realise there was "ignorabimus problem" in mathematics, while Hilbert who had thought about it emphatically stated several times that all mathematical problems will be *eventually* solved, and he even suggested a program for *mathematically* proving this assertion.

#### WHERE DO WE STAND TO-DAY?

The twentieth century taught us humility. We are not even certain any more what exactly "eventually" means for human civilization. Our time may be short.

And the fascinating depth of the present day *non-understanding* of the physical world – matter, force, energy, motion – was inconceivable to people of the 19th century, who were oblivious of the ideas of relativity and quantum fields and who possessed no knowledge of the large scale dynamics of the observable Universe.

Also, contrary to what Hilbert believed, his naively "optimistic" conjectures on the logical organization of mathematics were mathematically *disproved*.<sup>80</sup>

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<sup>77</sup>If you are student of ergo-logic, there is nothing vague for you in the word "interesting".

<sup>78</sup>Yet, some physical systems exhibit some "selectively unstable features", such as localisation/concentration of energy to few (selective) degrees of freedom.

<sup>79</sup>All this suggests the existence of a rigorous, yet remaining interesting, theory of *selective instability*; possibly, the other of the above seven question (with possible exception of 1 and 2), if you concentrate hard, may also direct you toward non-trivial mathematics.

<sup>80</sup>Gödel has shown (1931) that there are unquestionably *true but unprovable statements* in Mathematics. More dramatically, Paul Cohen proved (1963) that there are several parallel worlds of Mathematics, such that some statements that are *true in one of the worlds may be false in another one*.

Mathematics is not a "logical bore" as Hilbert thought, but a miraculous structure that has no right to exist neither intrinsically and nor in how it is intertwined with the fundamental physics and with human minds.

(Unexplicably, despite the failure of Hilbert's program, the questions we ask in mathematics are getting answered almost instantaneously – often in the course of significantly less than a million of man-hours of people seriously thinking on these problems. But the answers are, sometimes, far from the expected ones.

Probably, it is not hard to design a random device generating mathematical problems that admit simple solutions but these will be not only unfindable but *unfathomable* in realistic time, say in a billion years of intense research by a prosperous Galactic civilisation with a quadrillion of active mathematicians in it. But the human brain is, apparently, unable to formulate such problems at will.<sup>81)</sup>

And we believe that the fundamental questions about Nature can not be formulated but in mathematical terms. The deepest nature of the two arrows

$$\text{SPACE/TIME/MATTER/ENERGY} \xrightarrow[?_1]{} \text{LIFE/BRAIN}$$

and

$$\text{LIFE/BRAIN} \xrightarrow[?_2]{} \text{MIND/THOUGHT}$$

can not be comprehended but in the ambience of MATHEMATICS.



*The Great Circle of Mysteries* is closed with

$$\text{BRAIN/MIND/THOUGHT} \xleftrightarrow[?_3]{} \text{MATHEMATICS}$$

and

$$\text{MATHEMATICS} \xleftrightarrow[?_4]{} \text{SPACE/TIME/MATTER/ENERGY}.$$

We know something about ?<sub>4</sub> – mathematical physicists tell us story after story about it. But no mathematics available to us elucidates the mysteries of ?<sub>1</sub>, ?<sub>2</sub> and ?<sub>3</sub>. The time of this mathematics is yet to come.

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<sup>81</sup>Maybe, the P≠NP problem is of this kind.