Assignment 3B

Buildings as Energy and Indoor Climate Systems

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Question 1

The thermal nodes considered in the 4-node model are defined according to Table 1.

Table 1: Definition of the thermal nodes used in the 4-node model.

Node	Physical meaning
0	Outdoor air
1	Window surface
2	Indoor air
3	Interior walls surfaces
4	Opaque facade wall surface

Energy balance at the node 1 is:

$$\frac{\alpha_o U_g}{\alpha_o + U_g} A_1 (T_0 - T_1) + \alpha_i A_1 (T_2 - T_1) + \alpha_r F_{13} A_1 (T_3 - T_1) + \alpha_r F_{14} A_1 (T_4 - T_1) + a_1 A_1 q_z = 0. \tag{1}$$

The individual contributions in equation (1) are:

- Combined heat transfer between the outdoor air and the glass, accounting for external convection and heat conduction through the glazing.
- Convective heat exchange between the indoor air and the glass.
- Long-wave radiative heat exchange between the glass and the interior wall surfaces.
- Long-wave radiative heat exchange between the glass and the opaque facade wall.
- Fraction of the incident solar irradiance that is absorbed within the glazing.

$$\underbrace{\left[-\left(\frac{\alpha_o U_g}{\alpha_o + U_g} + \alpha_i + \alpha_r F_{13} + \alpha_r F_{14}\right) A_1\right]}_{M_{11}} T_1 + \underbrace{(\alpha_i A_1)}_{M_{12}} T_2 + \underbrace{(\alpha_r F_{13} A_1)}_{M_{13}} T_3 + \underbrace{(\alpha_r F_{14} A_1)}_{M_{14}} T_4 = \underbrace{-\frac{\alpha_o U_g}{\alpha_o + U_g} A_1 T_0 - a_1 A_1 q_z}_{B_1}. \tag{2}$$

Energy balance at the node 2 is:

$$\alpha_i A_1 (T_1 - T_2) + \alpha_i A_3 (T_3 - T_2) + \alpha_i A_4 (T_4 - T_2) + \dot{m}_{\text{vent}} c_p (T_0 - T_2) + Q_{\text{HVAC}} = 0. \tag{3}$$

The individual contributions in equation (3) are:

- Convective heat exchange between the indoor air and the glazing.
- Convective heat exchange between the indoor air and the interior wall surfaces.
- Convective heat exchange between the indoor air and the opaque facade wall.
- Heat exchange due to ventilation/infiltration with outdoor air.
- Heating (or cooling) power supplied to the room.

$$\underbrace{(\alpha_i A_1)}_{M_{21}} T_1 \;+\; \underbrace{\Big[-(\alpha_i A_1 + \alpha_i A_3 + \alpha_i A_4 + m_{\text{vent}} c_p)\Big]}_{M_{22}} T_2 \;+\; \underbrace{(\alpha_i A_3)}_{M_{23}} T_3 \;+\; \underbrace{(\alpha_i A_4)}_{M_{24}} T_4 \;=\; \underbrace{-\dot{m}_{\text{vent}} c_p T_0 \;-\; Q_{\text{heat}}}_{B_2}. \tag{4}$$

Energy balance at the node 3 is:

$$\alpha_r F_{31} A_3 (T_1 - T_3) + \alpha_r F_{34} A_3 (T_4 - T_3) + \alpha_i A_3 (T_2 - T_3) + d_1 A_1 q_z + -A_3 \Big[X_0 T_3 - \sum_{k=1}^{3} X_k\, T_3(t-k)\Big] = 0. \tag{5}$$

The individual contributions in equation (5) are:

- Long-wave radiative exchange between the interior wall surfaces and the glazing.
- Long-wave radiative exchange between the interior wall surfaces and the opaque facade wall.
- Convective heat exchange between the indoor air and the interior wall surfaces.
- Fraction of the solar irradiance transmitted through the glazing that reaches and is absorbed by the interior wall surfaces.
- Dynamic conductive heat flow into the construction associated with node 3, represented by the response factors X_0, X_1, X_2, X_3 and the past surface temperatures $T_3(t-k)$.

$$\underbrace{(\alpha_r F_{31} A_3)}_{M_{31}} T_1 + \underbrace{(\alpha_i A_3)}_{M_{32}} T_2 + \underbrace{\Big[-(\alpha_i + \alpha_r F_{31} + \alpha_r F_{34} + X_0) A_3\Big]}_{M_{33}} T_3 + \underbrace{(\alpha_r F_{34} A_3)}_{M_{34}} T_4 \;=\; \underbrace{-d_1 A_1 q_z + A_3 \sum_{k=1}^{3} X_k\, T_3(t-k)}_{B_3}. \tag{6}$$

Energy balance at the node 4 is:

$$\frac{\alpha_o U_f}{\alpha_o + U_f} A_4 \,(T_0 - T_4) + \alpha_i A_4 (T_2 - T_4) + \alpha_r F_{41} A_4 (T_1 - T_4) + \alpha_r F_{43} A_4 (T_3 - T_4) = 0. \tag{7}$$

The individual contributions in equation (7) are:

- Combined heat transfer between the outdoor air and the opaque façade surface, accounting for external convection and heat conduction through the façade, modelled as an equivalent heat-transfer coefficient.
- Convective heat exchange between the indoor air and the opaque facade surface.
- Long-wave radiative exchange between the opaque facade surface and the glazing.
- Long-wave radiative exchange between the opaque facade surface and the remaining interior wall surfaces.

$$\underbrace{(\alpha_r F_{41} A_4)}_{M_{41}} T_1 \;+\; \underbrace{(\alpha_i A_4)}_{M_{42}} T_2 \;+\; \underbrace{(\alpha_r F_{43} A_4)}_{M_{43}} T_3 \;+\; \underbrace{\Big[-\Big(\frac{\alpha_o U_f}{\alpha_o + U_f} + \alpha_i + \alpha_r F_{41} + \alpha_r F_{43}\Big) A_4\Big]}_{M_{44}} T_4 \;=\; \underbrace{-\frac{\alpha_o U_f}{\alpha_o + U_f} A_4 T_0}_{B_4}. \tag{8}$$

Collecting the terms in equations (2), (4), (6) and (8), the system can be written in matrix form as

$$\mathbf{M}\,\mathbf{T} = \mathbf{B}. \tag{9}$$

with

$$\mathbf{M} = \begin{bmatrix} -\Big(\frac{\alpha_o U_g}{\alpha_o + U_g} + \alpha_i + \alpha_r F_{13} + \alpha_r F_{14}\Big) A_1 & \alpha_i A_1 & \alpha_r F_{13} A_1 & \alpha_r F_{14} A_1 \\ \alpha_i A_1 & -(\alpha_i A_1 + \alpha_i A_3 + \alpha_i A_4 + \dot{m}_{\text{vent}} c_p) & \alpha_i A_3 & \alpha_i A_4 \\ \alpha_r F_{31} A_3 & \alpha_i A_3 & -(\alpha_i + \alpha_r F_{31} + \alpha_r F_{34} + X_0) A_3 & \alpha_r F_{34} A_3 \\ \alpha_r F_{41} A_4 & \alpha_i A_4 & \alpha_r F_{43} A_4 & -\Big(\frac{\alpha_o U_f}{\alpha_o + U_f} + \alpha_i + \alpha_r F_{41} + \alpha_r F_{43}\Big) A_4 \end{bmatrix} \tag{10}$$

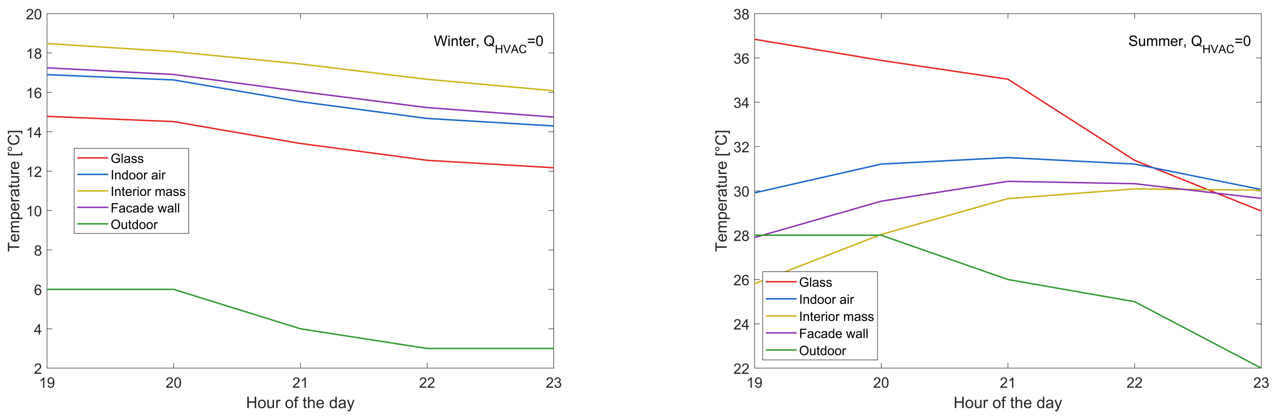
$$\mathbf{T} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} -\dfrac{\alpha_o U_g}{\alpha_o + U_g} A_1 T_0 - a_1 A_1 q_z \\ -\dot{m}_{\text{vent}} c_p T_0 - Q_{\text{HVAC}} \\ -d_1 A_1 q_z + A_3 \displaystyle\sum_{k=1}^{3} X_k T_3(t-k) \\ -\dfrac{\alpha_o U_f}{\alpha_o + U_f} A_4 T_0 \end{bmatrix} \tag{11}$$

Question 2

The MATLAB code implementation is available on GitHub at https://github.com/afonsofaraujo/BEICS_as3b, with all numerical parameters summarised in Table A.1 in Appendix A.

Question 3

Figure 1 shows the simulated temperature evolution of the indoor air and envelope nodes between 19:00 and 23:00 for winter and summer conditions, with no active heating or cooling applied ($Q_{\text{HVAC}} = 0$). Seasonal conditions are obtained by switching the outdoor temperature vector T_0 and the solar gains vector Q_{sol} according to the comments in the given code.



(a) Winter mode with $Q_{\text{HVAC}} = 0$.

(b) Summer mode with $Q_{\text{HVAC}} = 0$.

Figure 1: Temperature evolution of the room and envelope nodes between 19:00 and 23:00 for winter and summer conditions, with no active heating or cooling.

In winter mode, after the heating has stopped, the temperature of glass, indoor air and wall obviously decrease, following the trends in outdoor temperature. The wall is the warmest because of the accumulated solar heat, which is slowly released to the indoor air. The indoor air temperature is in-between the wall temperature and the glass temperature, which seems logical.

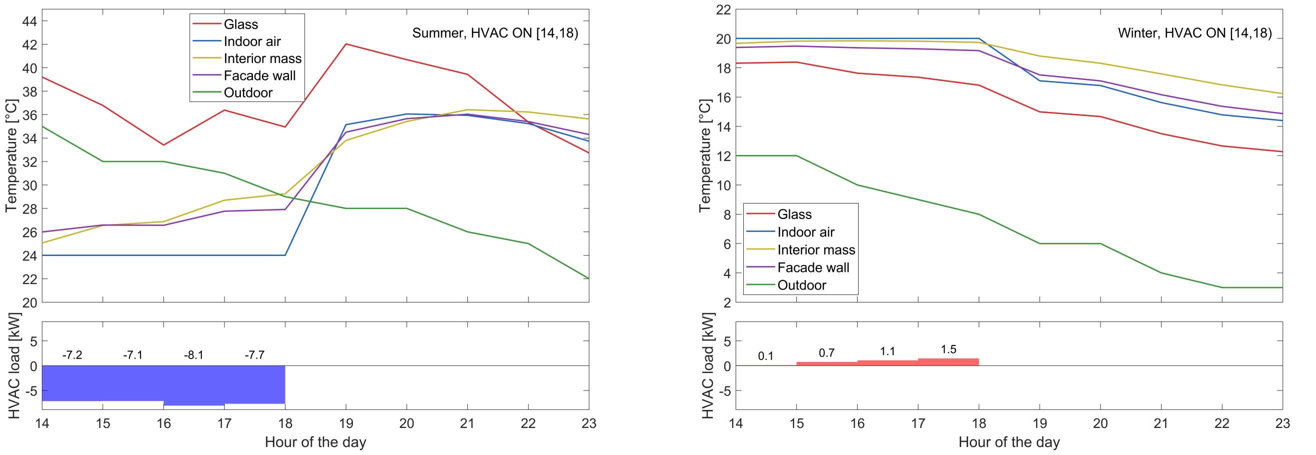
In summer mode, it looks completely different, after turning the cooling off, the temperature of the wall keeps increasing for a least 3 hours. His is because there is still incoming solar radiation (Q_{sol} vector in Matlab, it is not yet night), which is absorbed by the wall. As a consequence of this, the indoor air temperature increases as well, but more slowly, because it is also in contact with the glazing, which follows much more the temperature of the outdoor air.

In both cases, the façade wall temperature tracks the indoor air temperature much more closely than the glass temperature. Although the façade wall is also a node without thermal memory, its higher thermal resistance relative to the glass reduces its coupling with the outdoor environment, making it primarily governed by the indoor air temperature and, consequently, by the interior wall.

The past indoor air temperatures are not required in these dynamic calculations because the indoor air has a very low thermal storage capacity compared to the construction elements. Moreover, ventilation continuously replaces indoor air, preventing it from retaining long-term thermal memory. In contrast, the walls and other massive elements store heat and release it gradually, introducing time-lag effects that must be captured through their temperature history. Therefore, only the past wall temperatures are needed to correctly model the dynamic thermal behaviour of the room.

Question 4

The temperatures and heating and cooling loads are shown in Figure 2.



(a) Summer case with indoor air temperature set point of 24 °C. (b) Winter case with indoor air temperature set point of 20 °C.

Figure 2: Hourly evolution of indoor air, glazing, interior mass and facade wall temperatures, together with the corresponding HVAC heating or cooling load. The HVAC system is active between 14:00 and 18:00 to maintain the prescribed indoor air temperature set point, and switched off afterwards, resulting in free-floating conditions.

Note that temperatures represent end-of-hour states obtained from the energy balance over each time step, whereas the HVAC load corresponds to the power applied during the preceding interval. To reflect this distinction when plotting against real time, the HVAC load bars are shifted to the left and associated with the interval $[t-1, t)$, while temperatures are shown at hour t. This makes the physical interpretation consistent without altering the underlying numerical scheme.

Question 5

Model structure

The annual simulation is implemented as a modular Matlab workflow, where each function is responsible for a clearly defined task. The role of each function is summarised below.

- `main.m`: Entry point of the model. Loads climate data, initialises the parameter set, runs the annual simulation, and saves the main outputs (nodal temperatures, HVAC loads, and annual energy demands).
- `defaults.m`: Defines all physical, geometrical, and operational parameters, including building properties, internal gains, ventilation settings, temperature setpoints, and HVAC capacity limits.
- `exceldataimport.m`: Imports and pre-processes the hourly climate data (outdoor temperature and solar radiation) from the provided Excel file and exports them to Matlab format.
- `run_simulation.m`: Core annual simulation routine. Executes an hourly time-stepping loop over the full year, applies the occupancy schedule, sets the operational states (ventilation and temperature control), and coordinates the calls to the HVAC controller and thermal model.
- `hvac_control.m`: Implements the HVAC control logic. Predicts the end-of-hour indoor air temperature under free-floating conditions and determines whether heating or cooling is required to enforce the active setpoint.
- `solve_Q_for_setpoint.m`: Computes the HVAC thermal power needed to meet the heating or cooling setpoint at the end of the current time step using a one-dimensional root-finding procedure.
- `HHS_model4.m`: Evaluates the reduced-order 4-node thermal model. Returns the updated nodal temperatures based on the current conditions, internal gains, ventilation heat exchange, solar inputs, HVAC load, and thermal memory effects.

The call sequence illustrated in Figure 3 starts in `main.m`, which loads the climate inputs and model parameters and then invokes `run_simulation.m`. As shown in the diagram, `run_simulation.m` executes an hourly loop over the full year, during which it updates the environmental inputs, applies the occupancy and control schedules, and coordinates the interaction between the HVAC control logic and the thermal model.

At each time step, `run_simulation.m` calls `hvac_control.m` to evaluate the indoor temperature evolution under free-floating conditions and to check compliance with the active heating or cooling setpoints. When temperature control is required, `hvac_control.m` calls `solve_Q_for_setpoint.m`, which computes the HVAC thermal load needed to enforce the setpoint at the end of the current hour.

The computed HVAC load, together with the current environmental inputs, internal gains, and ventilation state, is then passed to `HHS_model4.m`. As indicated by the feedback loop in Figure 3, `HHS_model4.m` updates the nodal temperatures and thermal memory states, which are returned to `run_simulation.m` and used as initial conditions for the next time step before the simulation advances.

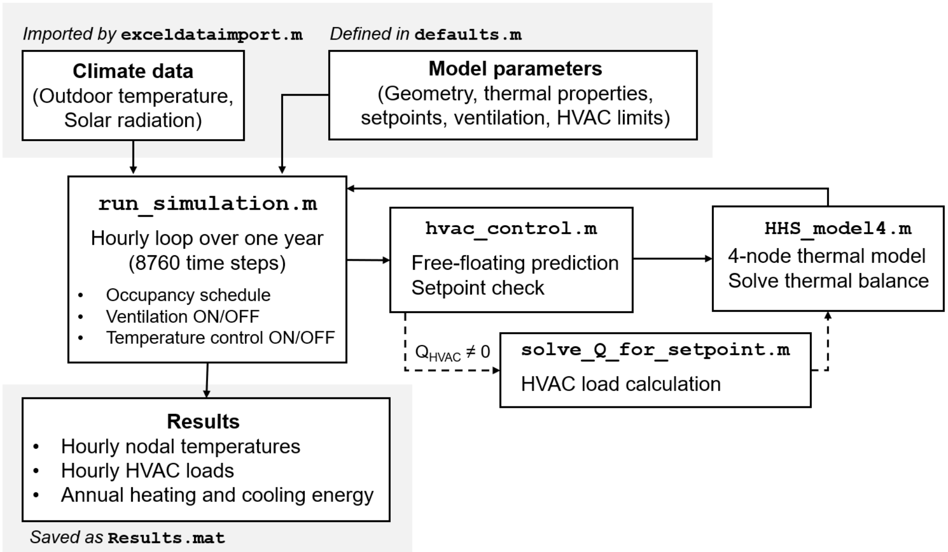


Figure 3: Structure and call sequence of the annual simulation model.

Assumptions

The following key assumptions were adopted to extend the 4-node model to annual hourly simulations:

- A fixed time step of 1 hour is used, and the simulation is advanced sequentially over the full year.
- The climate inputs (outdoor temperature and solar radiation) are taken directly from the provided hourly climate file and applied as piecewise-constant values over each hour. Solar radiation is computed as the sum of direct and diffuse components incident on the south-oriented facade.
- The occupancy schedule is fixed: the room is occupied from 8:00 to 18:00 and unoccupied from 18:00 to 8:00.
- During the unoccupied period, ventilation is switched off.
- During the unoccupied period, two alternative temperature control strategies are considered: (i) a free-floating strategy, in which heating and cooling are switched off and the indoor air temperature evolves freely and (ii) a setback strategy, in which the indoor air temperature is constrained within setback limits of 16 °C and 28 °C.
- During the occupied period, indoor temperature control is active using the specified heating and cooling setpoints of 20 ℃ and 24 ℃, respectively. A preheating strategy is applied, whereby the HVAC system may be activated during the hour preceding occupancy to ensure that the indoor temperature reaches the setpoint at 8:00 [1].
- Internal gains are applied according to the occupancy level (e.g., proportional to the number of occupants), and are set to zero during unoccupied periods.
- Ventilation heat exchange is modelled through a prescribed ventilation rate (dependent on occupancy) and is set to zero during unoccupied periods, in accordance with the assignment requirements.
- Initial conditions for the nodal temperatures and thermal-memory state variables are prescribed at the beginning of the year using the same values of assignment 3A.
- The HVAC system is represented as an ideal thermal power source/sink, able to deliver a constant load over the hour (within prescribed bounds) to enforce the end-of-step setpoint.

Results

Figure 4 shows the annual temperature evolution for the modelled nodes, illustrating both the seasonal variation driven by the climate boundary conditions and the daily modulation caused by the occupied/unoccupied control schedule. The corresponding daily-average HVAC thermal load is shown in Figure 5, highlighting the periods of dominant heating or cooling operation throughout the year.

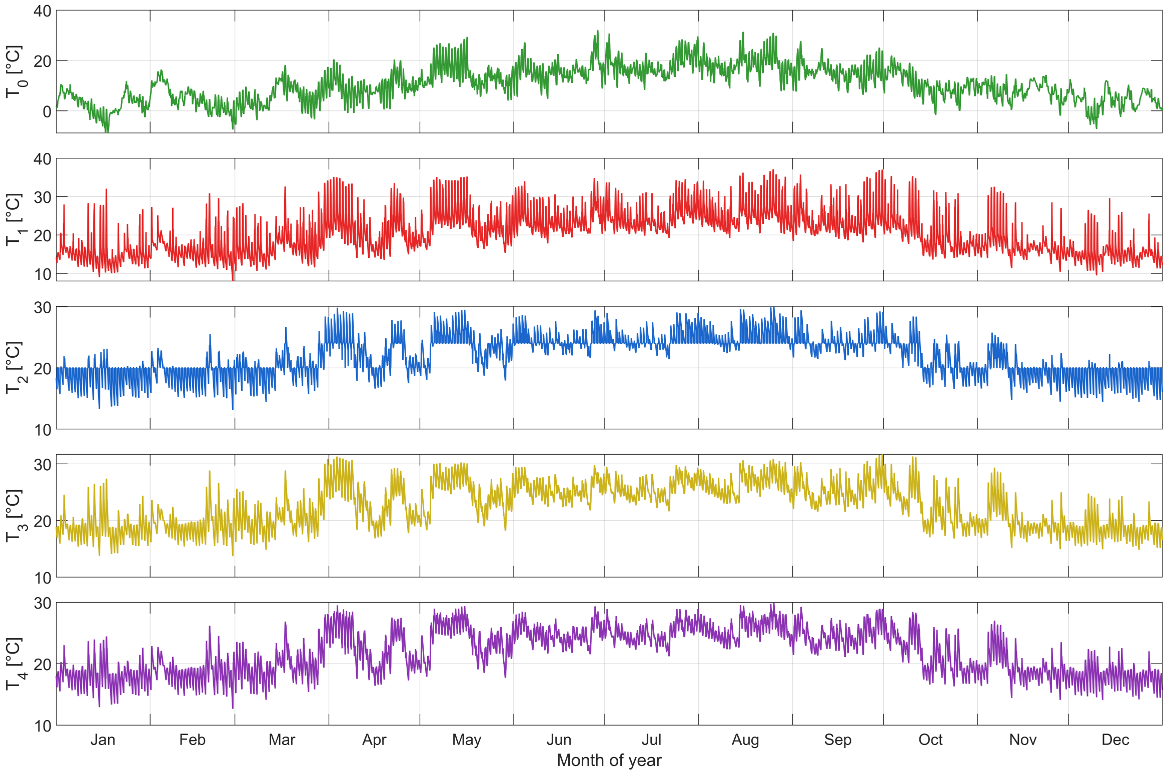


Figure 4: Annual temperature profiles for the modelled nodes.

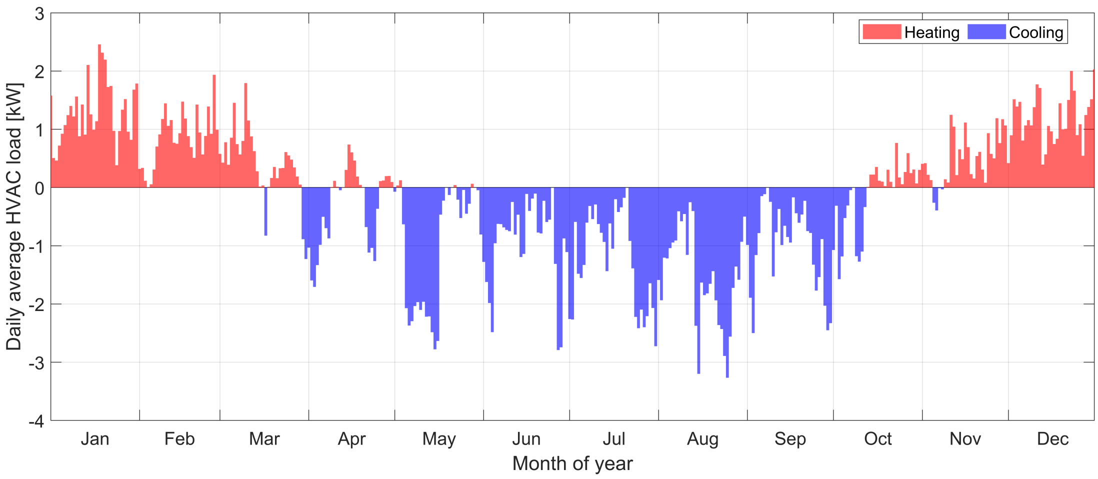


Figure 5: Daily average HVAC load over the year (positive for heating, negative for cooling).

To provide a physically interpretable view of the hourly dynamics, Figures 6 and 7 present representative winter, spring, summer, and autumn days. These plots show the coupled evolution of indoor and envelope temperatures together with the HVAC load and internal gains, illustrating how temperature control is only active during the occupied period and how the envelope dynamics contribute to delayed thermal responses.

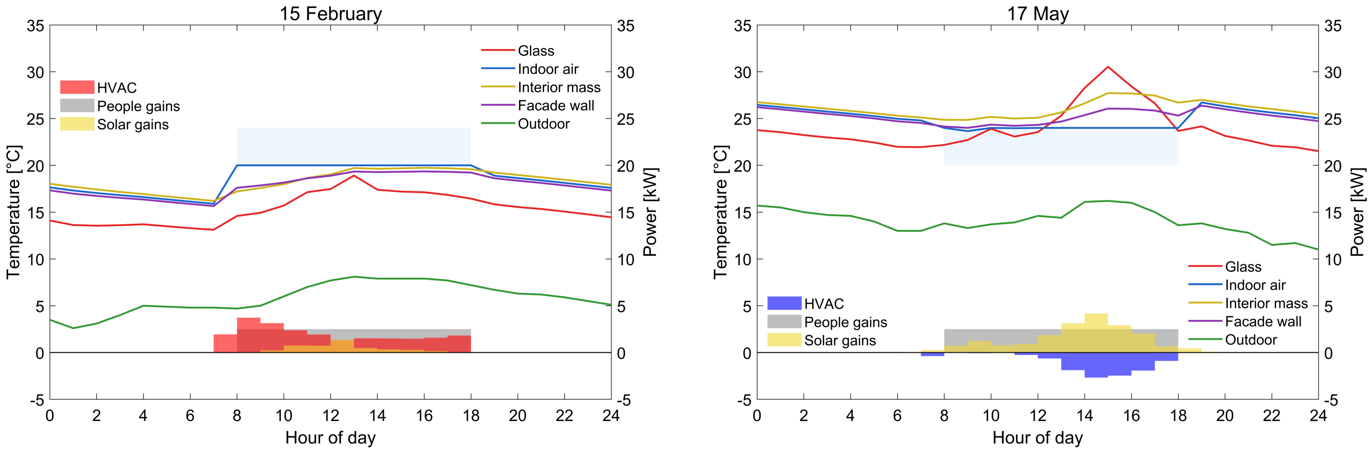
Table 2 shows annual energy consumption and the peak power for both heating and cooling.

Validation and consistency checks

To verify that the implementation yields logical results, a set of consistency checks was performed by varying key input parameters and confirming the expected qualitative trends:

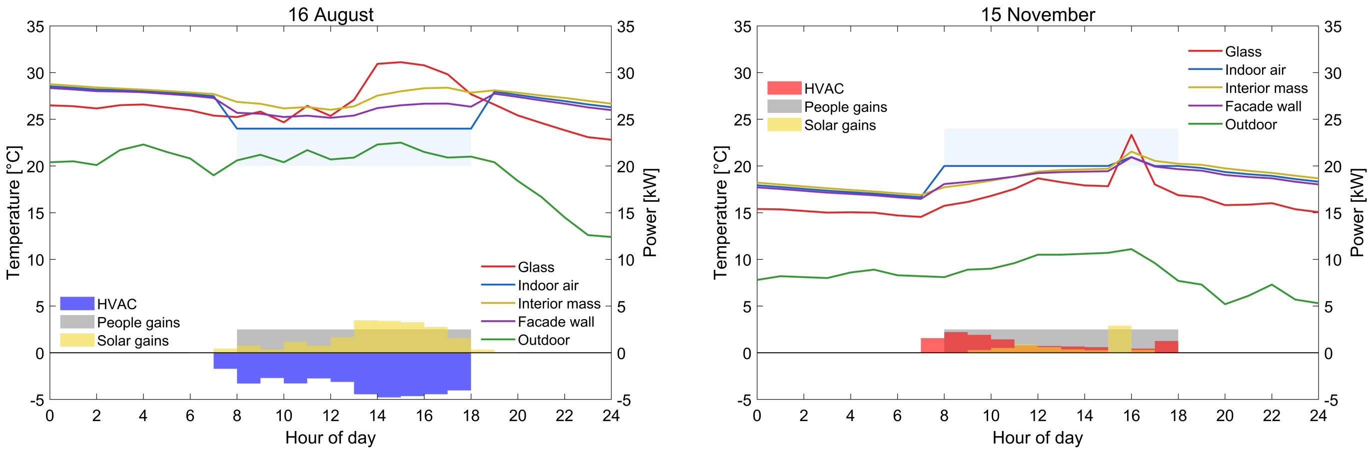
- Ventilation sensitivity: increasing ventilation during occupied hours increases the heating demand in winter and the cooling demand in summer, due to the stronger heat exchange with outdoor air.
- Setpoint sensitivity: increasing the heating setpoint results in higher annual heating energy use and higher controlled indoor temperatures during occupied periods. Decreasing the cooling setpoint increases cooling energy use in warm periods.

[1]This preheating strategy contradicts the previous point since the heating or cooling is work in unoccupied period but it was considered nontheless because otherwise comfort temperature would not be reached within the 8:00 to 9:00 interval.



(a) Day 46. (b) Day 137.

Figure 6: Hourly evolution of indoor and envelope temperatures together with HVAC load and internal gains for representative winter and spring days.



(a) Day 228. (b) Day 319.

Figure 7: Hourly evolution of indoor and envelope temperatures together with HVAC load and internal gains for representative summer and autumn days.

- Solar gains sensitivity: reducing solar inputs decreases cooling demand in warm seasons and increases heating demand in cold seasons, consistent with the role of solar radiation as an external gain.

Overall, the annual results and the sensitivity trends are consistent with the expected building-physics behaviour.

Discussion

An aspect that may initially appear counterintuitive is observed in Figure 6a, where the HVAC load between 7:00 and 8:00 is lower than in the subsequent hour, despite being responsible for a comparatively large increase in indoor temperature and occurring in the absence of solar or occupancy gains. This behaviour is a direct consequence of the ventilation constraints imposed by the assignment.

During the unoccupied period, ventilation is disabled, effectively rendering the room airtight. As a result, the heating energy supplied between 7:00 and 8:00 is used almost entirely to increase the indoor air and envelope temperatures, leading to a pronounced temperature rise with a relatively modest HVAC load. After 8:00, when occupancy begins and ventilation is activated, a significant fraction of the supplied heating energy is lost through ventilation heat exchange with the outdoor air. Consequently, a higher HVAC load is required to maintain the indoor temperature at the setpoint, even when internal gains from occupants are present.

To further assess the influence of temperature setbacks, the model allows an alternative operating mode by setting `p.unocc_mode = "SETBACK"` in `defaults.m`. In this mode, setback temperatures of 16 ℃ for heating and 28 ℃ for cooling are applied during unoccupied periods. The results show a slight reduction in the maximum HVAC power demand in both heating and cooling scenarios, at the expense of a small increase in the annual energy consumption. This peak reduction remains limited, as the load that the setback strategy aims to mitigate is already partially alleviated by the ventilation effect discussed above.

Table 2: Annual HVAC energy consumption and peak power for different unoccupied control strategies.

Strategy	Heating [kWh]	Cooling [kWh]	Total [kWh]	Peak Heat [kW]	Peak Cool [kW]
Free-floating	3 349.6	4 834.1	8 183.7	7.94	9.24
Setback (16–28°C)	3 357.9	4 845.0	8 202.9	7.93	9.19

Question 6

Figure 8 presents the Mean Radiant Temperature (MRT) as a function of the indoor air temperature during occupied periods. The MRT was computed using the physical T^4 formulation, where radiative view factors from the room centre were approximated by surface area ratios, $w_i = A_i/A_{\text{tot}}$.

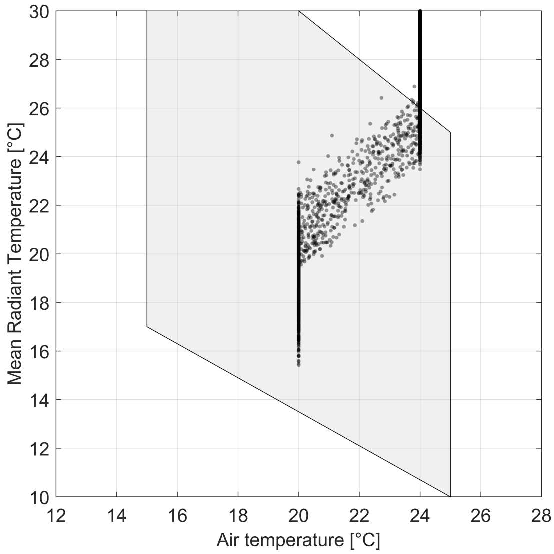


Figure 8: Mean Radiant Temperature (MRT) versus indoor air temperature during occupied hours. The shaded region indicates the thermal comfort domain.

Question 7

The most energy efficient way possible to make the room comfortable with the restrictions given in the assignment is the unoccupied strategy of free-floating which led to the results in Table 2.

Appendix

Table A.1: Complete set of geometrical, physical, and operational parameters used in the 4-node building model. [a] Although the statement reports 0.3 $\mathrm{m^2\,K\,W^{-1}}$ as a "U-value", this unit corresponds to a thermal resistance. In this work, it was interpreted as $U_f = 0.3$ $\mathrm{W\,m^{-2}\,K^{-1}}$, which is consistent with an insulated façade.

Category	Parameter	Symbol	Value	Unit / Description
Geometry	Room length	L	10.8	m
	Room width	W	7.2	m
	Room height	H	2.8	m
Air properties	Air density	ρ_{air}	1.2	$\mathrm{kg\,m^{-3}}$
	Air heat capacity	$c_{p,\mathrm{air}}$	1005	$\mathrm{J\,kg^{-1}\,K^{-1}}$
Envelope and solar	Glazing U-value	U_g	2.0	$\mathrm{W\,m^{-2}\,K^{-1}}$
	Opaque facade U-value	U_f	0.3[a]	$\mathrm{W\,m^{-2}\,K^{-1}}$
	Glazing fraction of facade	p_{glass}	0.80	-
	Solar absorptance (glass)	a_1	0.12	-
	Solar absorptance (interior)	d_1	0.30	-
Internal gains and ventilation	Sensible heat per person	Q_{pp}	100	$\mathrm{W\,person^{-1}}$
	Base ventilation rate	$\dot{V}_{\mathrm{base}}$	0.0	$\mathrm{m^3\,s^{-1}}$
	Ventilation per person	$\dot{V}_{\mathrm{pp}}$	0.010	$\mathrm{m^3\,s^{-1}\,person^{-1}}$
	Ventilation mass flow	$\dot{m}_{\mathrm{vent}}$	$\rho_{\mathrm{air}}\dot{V}$	$\mathrm{kg\,s^{-1}}$
Thermal transfer coefficients	Indoor convection	α_i	7.8	$\mathrm{W\,m^{-2}\,K^{-1}}$
	Outdoor convection	α_o	2.5	$\mathrm{W\,m^{-2}\,K^{-1}}$
	Linearised LW radiation	α_r	5.0	$\mathrm{W\,m^{-2}\,K^{-1}}$
	Time step	Δt	3600	s
	Max HVAC power	Q_{max}	50000	W
Areas	Facade area	A_{fac}	LH	$\mathrm{m^2}$
	Glazing area	A_1	$p_{\mathrm{glass}}A_{\mathrm{fac}}$	$\mathrm{m^2}$
	Opaque facade area	A_4	$(1-p_{\mathrm{glass}})A_{\mathrm{fac}}$	$\mathrm{m^2}$
	Floor area	A_{floor}	LW	$\mathrm{m^2}$
	Ceiling area	A_{ceiling}	LW	$\mathrm{m^2}$
	Back wall area	A_{back}	LH	$\mathrm{m^2}$
	Side wall area	A_{side}	WH	$\mathrm{m^2}$
View factors	Glass $\rightarrow$ interior mass	F_{13}	1	-
	Interior mass $\rightarrow$ glass	F_{31}	$(A_1/A_3)F_{13}$	-
	Glass $\rightarrow$ facade	F_{14}	0	-
	Facade $\rightarrow$ glass	F_{41}	0	-
	Facade $\rightarrow$ interior mass	F_{43}	1	-
	Interior mass $\rightarrow$ facade	F_{34}	$(A_4/A_3)F_{43}$	-
Construction interior walls	Thermal conductivity	λ	0.2	$\mathrm{W\,m^{-1}\,K^{-1}}$
	Density	ρ_{con}	720	$\mathrm{kg\,m^{-3}}$
	Heat capacity	$c_{p,\mathrm{con}}$	840	$\mathrm{J\,kg^{-1}\,K^{-1}}$
	Wall thickness	e	0.10	m
Control and operation	Heating setpoint (occ.)	$T_{\mathrm{heat,occ}}$	20	°C
	Cooling setpoint (occ.)	$T_{\mathrm{cool,occ}}$	24	°C
	Heating setpoint (unocc.)	$T_{\mathrm{heat,unocc}}$	16	°C
	Cooling setpoint (unocc.)	$T_{\mathrm{cool,unocc}}$	28	°C
	Preheating horizon	n_{preheat}	1	h
	Unoccupied mode	-	SETBACK / FREEFLOAT	-
Initial conditions	Initial past temperatures	$\mathbf{T}_{\mathrm{past,init}}$	[20, 19, 18, 19]	°C