All symbols are consistent with the class discussion on this matter

$$\underset{\{0\le t\le 1\}}{Max}\, u^S = u^S(w(g(\bar{g},t)),1-t,e)$$

$$\underset{\{0\le t\le 1\}}{Max}\, u^S = u^S(w(g(\bar{g},t))) + (1-t) + e$$

$$where\;\; w(g(\bar{g},t) = \sqrt{\bar{g}} \times t$$

$$where\;\; u^S(x) = \log x$$

Q1) Following the class notes, find the student's reaction function.

Q2) Following the class notes, find the impact of grade inflation on student's effort

All symbols are consistent with the class discussion on this matter

$$\underset{\{0\le\bar{g},e\le 1\}}{Max}\ u^P = u^P(v((\bar{g},e)),1-e,t(\bar{g});\alpha,\beta)$$

$$\underset{\{0\le\bar{g},e\le 1\}}{Max}\ u^P = u^P(v((\bar{g},e)\times\alpha)+((1-e)\times t(\bar{g})\times\beta))$$

$$where\ v((\bar{g},e)=\bar{g}\times e \quad where\ u^P(x)=\log(x)$$

Q3) Following the class notes, assuming that the professor is a Stackelberg Leader, find the professor’s equilibrium grade inflation ($\bar{g}$) and teaching effort (e).

Q4) Following the class notes, find the impact of increasing teaching focus (α) by the university on professor’s choice of grade inflation

Answer 1 and 2: Differentiate the student's utility function with respect to "t"

$$\underset{\{0\le t\le 1\}}{Max}\, u^S = u^S(w(g(\bar{g},t))) + (1-t) + e$$

$$\underset{\{0\le t\le 1\}}{Max}\, u^S = \log(\sqrt{\bar{g}t}) + (1-t) + e$$

$$\frac{\partial u^S}{\partial t} = 0 \rightarrow \frac{1}{2t^*\sqrt{\bar{g}}} - 1 = 0 \rightarrow t^* = \frac{1}{2\sqrt{\bar{g}}} \text{....(1)}$$

$$\frac{\partial t^*}{\partial \bar{g}} = -\frac{\bar{g}^{-\frac{3}{2}}}{2} < 0$$

Grade inflation (increase in $\bar{g}$) leads to a decrease in student effort b/c students spend time in other activities

Answer 3 and 4: Differentiate the professor's utility function with respect to $\bar{g}$ and e

$$\underset{\{0\le \bar{g},e\le 1\}}{Max}\ u^P = \log(\bar{g}e\alpha) + \left(\left(1-e\right)\times t(\bar{g})\times\beta\right) \qquad t(\bar{g}) = t^* = \frac{1}{2\sqrt{\bar{g}}}$$

$$\underset{\{0\le \bar{g},e\le 1\}}{Max}\ u^P = \log(\bar{g}e\alpha) + \left(\left(1-e\right)\times\frac{1}{2\sqrt{\bar{g}}}\times\beta\right)$$

$$\frac{\partial u^P}{\partial \bar{g}} = 0 \rightarrow \frac{e\alpha}{\bar{g}e\alpha} - \frac{(1-e)\beta\bar{g}^{-\frac{3}{2}}}{4} = 0 \rightarrow \sqrt{\bar{g}} = \frac{(1-e)\beta}{4} \text{....(2)}$$

$$\frac{\partial u^P}{\partial e} = 0 \rightarrow \frac{\bar{g}\alpha}{\bar{g}e\alpha} - \frac{\beta}{2\sqrt{\bar{g}}} = 0 \rightarrow \sqrt{\bar{g}} = \frac{e\beta}{2} \text{....(3)}$$

Solve equation 2 and 3 to get the solutions for $\bar{g}$ and e

$$\frac{(1-e^*)\beta}{4} = \frac{e^*\beta}{2} \rightarrow \frac{3}{4}e^* = \frac{\beta}{4} \rightarrow e^* = \frac{\beta}{3} \text{...(4)} \qquad \sqrt{\bar{g}^*} = \frac{e^*\beta}{2} = \frac{\beta}{6} \rightarrow \bar{g}^* = \frac{\beta^2}{36} \text{...(5)}$$

Answer 3 and 4: Differentiate the professor's equilibrium teaching effort with respect to α

$$e^* = \frac{\beta}{3} \ldots(4) \qquad \bar{g}^* = \frac{\beta^2}{36} \ldots(5)$$

$$\frac{\partial e^*}{\partial \alpha} = 0 \ldots\ldots.(6)$$

This result indicates that if teaching effort or research effort is affected by institutional measures to boost teaching quality, depends on how the alpha parameter enter the professor's utility function.