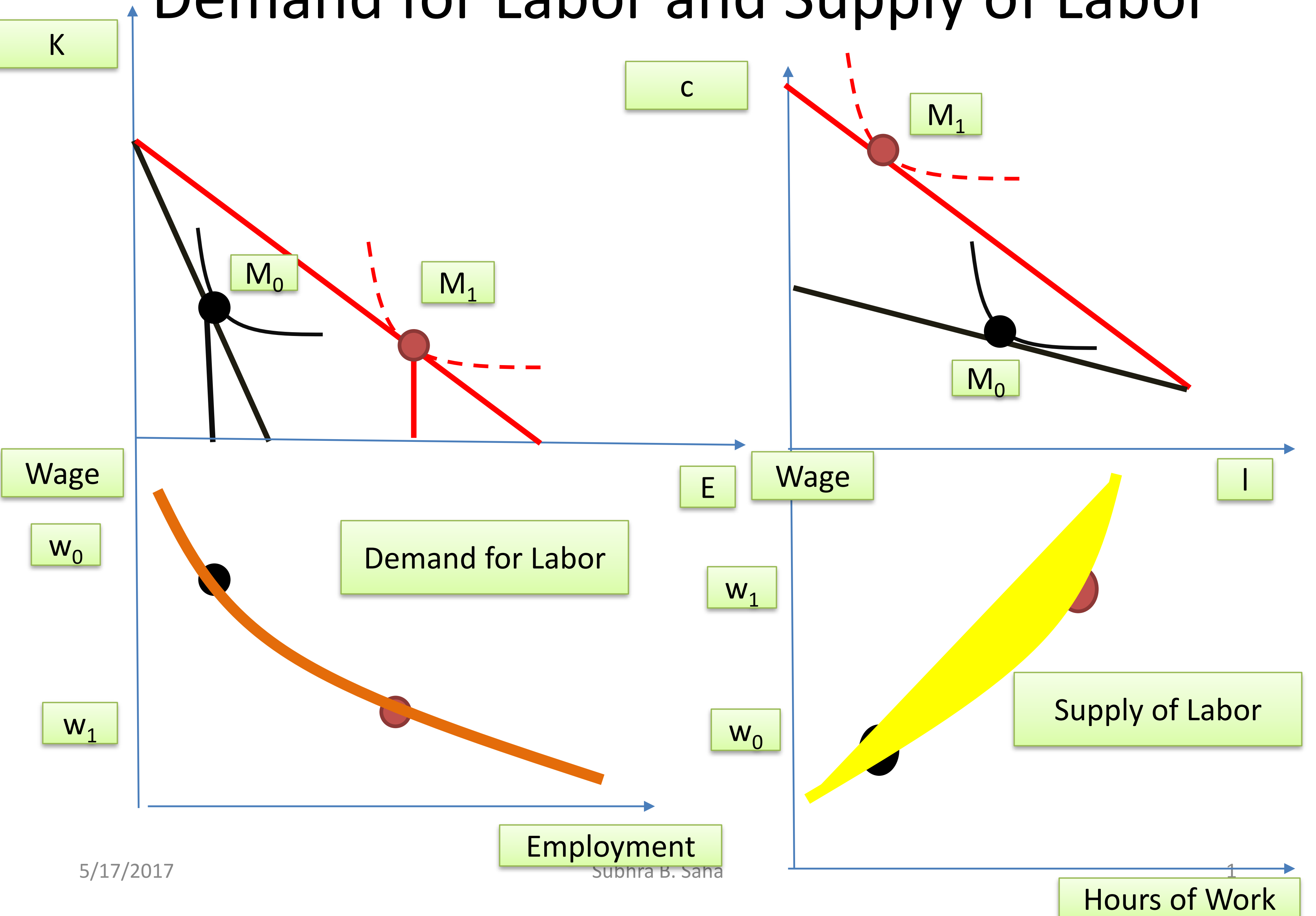
Demand for Labor and Supply of Labor



$$T = h + l$$
$$pc = wh + V$$

$$U = l^{\alpha} c^{\beta}$$

$$\frac{\partial U}{\partial l} = \alpha l^{(\alpha-1)} c^{\beta} = MU_l$$
$$\frac{\partial U}{\partial c} = \beta l^{\alpha} c^{(\beta-1)} = MU_c$$

$$\Gamma = U + \lambda[w(T-l) + V - pc]$$

$$\left.\begin{aligned} \frac{\partial \Gamma}{\partial l} &= MU_l - \lambda w = 0 \ldots.(1) \rightarrow pMU_l = \lambda w \\ \frac{\partial \Gamma}{\partial c} &= MU_c - \lambda p = 0 \ldots.(2) \rightarrow MU_c = \lambda p \end{aligned}\right\} \rightarrow \frac{\alpha c^*}{\beta l^*} = \frac{w}{p} \;\ldots\ldots\ldots\ldots(4)$$

$$\frac{\partial \Gamma}{\partial \lambda} = 0 \Rightarrow w(T - l^*) + V - pc^* = 0 \ldots\ldots..(3)$$

l* will give Leisure Demand Function. Using 3 and 4

$$\left.\begin{aligned} pc^* &= w(T - l^*) + V \\ c^* &= \frac{\beta w}{\alpha p} l^* \end{aligned}\right\} p\frac{\beta w}{\alpha p} l^* = w(T - l^*) + V = wT + V - wl^*$$

$$l^* = \frac{(wT + V)}{w\left(1 + \frac{\beta}{\alpha}\right)} = \frac{T}{\left(1 + \frac{\beta}{\alpha}\right)} + \frac{V}{w\left(1 + \frac{\beta}{\alpha}\right)}$$

$$l^* = \frac{(wT+V)}{w\left(1+\frac{\beta}{\alpha}\right)} = \frac{T}{\left(1+\frac{\beta}{\alpha}\right)} + \frac{V}{w\left(1+\frac{\beta}{\alpha}\right)}$$

$$c^* = \frac{\beta w}{\alpha p}\left[\left(\frac{T}{1+\frac{\beta}{\alpha}}\right) + \frac{V}{w\left(1+\frac{\beta}{\alpha}\right)}\right]$$

$$U^* = l^{*\alpha} c^{*\beta}$$

h* = T-l* will give Labor Supply Function.

$$h^* = T - l^* = T - \frac{(wT+V)}{w\left(1+\frac{\beta}{\alpha}\right)} = T - \frac{T}{\left(1+\frac{\beta}{\alpha}\right)} - \frac{V}{w\left(1+\frac{\beta}{\alpha}\right)}$$

$$\frac{\partial h^*}{\partial w} = \frac{V}{w^2\left(1+\frac{\beta}{\alpha}\right)} > 0$$

$$\varepsilon^S = \frac{w}{h^*}\frac{\partial h^*}{\partial w} = \frac{w}{h^*} \times \frac{V}{w^2\left(1+\frac{\beta}{\alpha}\right)} > 0$$

$$q = E^{\alpha} K^{\beta} \quad \frac{\partial f(E,K)}{\partial E} = \alpha E^{\alpha-1} K^{\beta} = MP_E \quad \frac{\partial f(E,K)}{\partial K} = \beta E^{\alpha} K^{\beta-1} = MP_K$$

$$\Gamma = pE^{\alpha} K^{\beta} + \lambda[TC - wE - rK]$$

$$\left.\begin{array}{l} \dfrac{\partial \Gamma}{\partial E} = p\alpha E^{\alpha-1} K^{\beta} - \lambda w = 0 \ldots (1) \rightarrow p\alpha E^{\alpha-1} K^{\beta} = \lambda w \\ \dfrac{\partial \Gamma}{\partial K} = p\beta E^{\alpha} K^{\beta-1} - \lambda r = 0 \ldots (2) \rightarrow p\beta E^{\alpha} K^{\beta-1} = \lambda r \end{array}\right\} \rightarrow \frac{\alpha K^*}{\beta E^*} = \frac{w}{r} \ldots\ldots\ldots\ldots (4)$$

$$\frac{\partial \Gamma}{\partial \lambda} = 0 \Rightarrow TC - wE^* - rK^* = 0 \ldots\ldots\ldots (3)$$

E* will give Demand for Labor Function. Using 3 and 4

$$\left.\begin{array}{l} TC = wE^* + rK^* \\ K^* = \dfrac{\beta w}{\alpha r} E^* \end{array}\right\} \rightarrow TC = wE^* + r\frac{\beta}{\alpha r} wE^* \quad \boxed{\rightarrow E^* = \frac{TC}{w\left(1 + \dfrac{\beta}{\alpha}\right)}} \rightarrow \left|\eta^D E^*\right| = \left|\frac{w}{E^*} \frac{\partial E^*}{\partial w}\right| = 1$$

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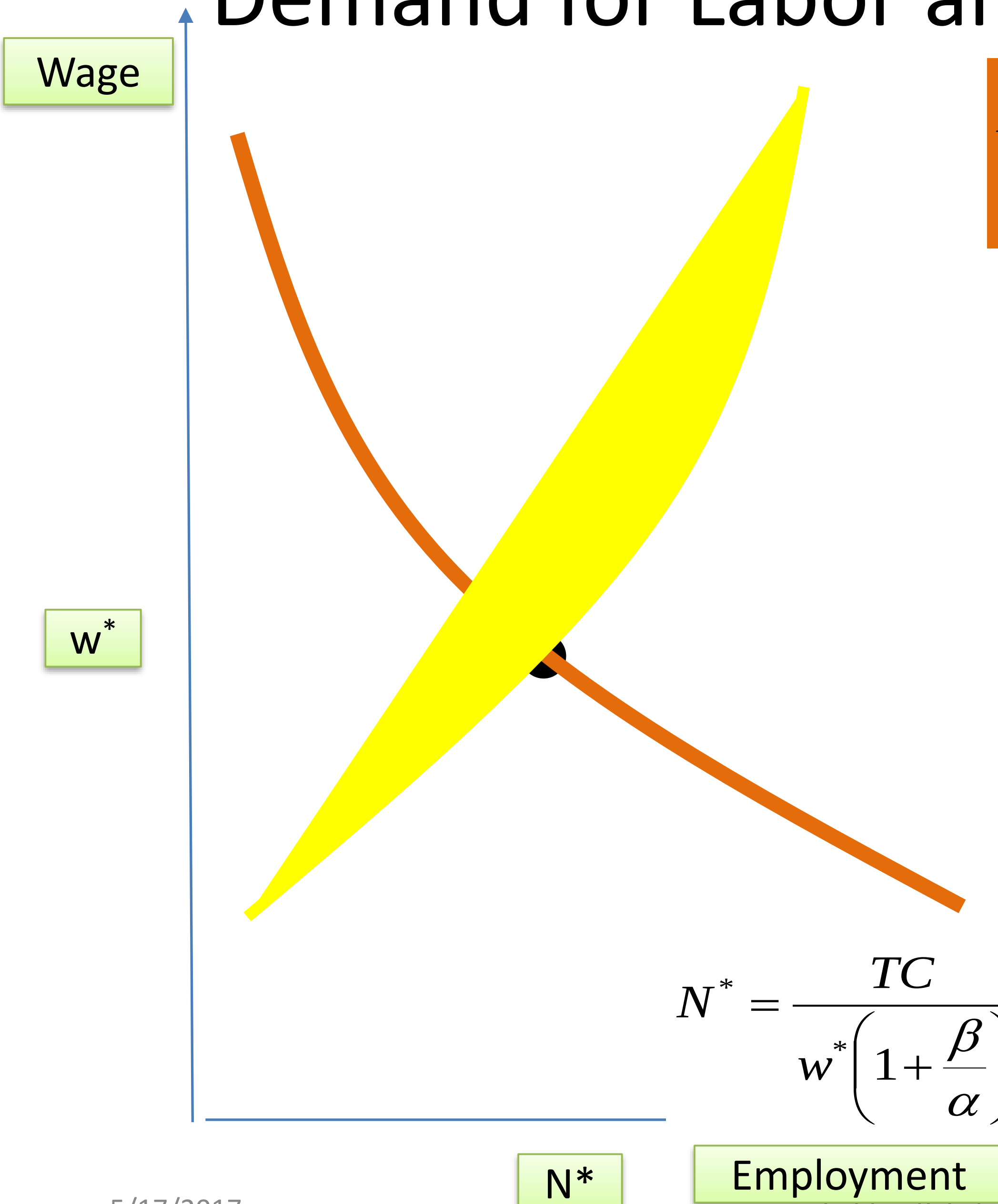


$$E^* = \frac{TC}{w\left(1+\frac{\beta}{\alpha}\right)}$$

$$h^* = T - l^* = T - \frac{(wT+V)}{w\left(1+\frac{\beta}{\alpha}\right)}$$

$$\exists\ w = w^* \ni E^* = h^* = N^*$$

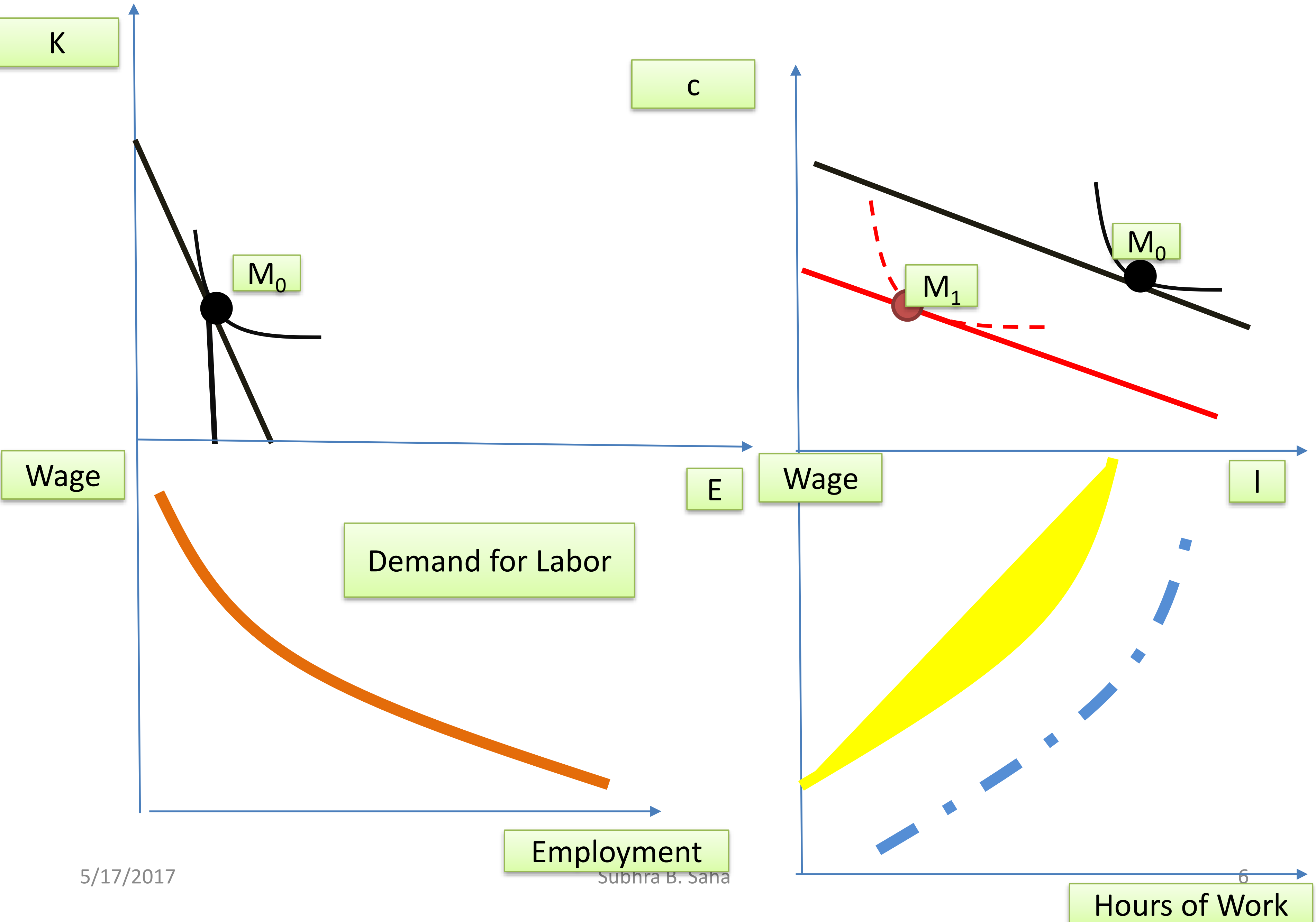
$$T - \frac{(wT+V)}{w\left(1+\frac{\beta}{\alpha}\right)} = \frac{TC}{w\left(1+\frac{\beta}{\alpha}\right)}$$

$$T = \frac{TC}{w\left(1+\frac{\beta}{\alpha}\right)} + \frac{(wT+V)}{w\left(1+\frac{\beta}{\alpha}\right)}$$

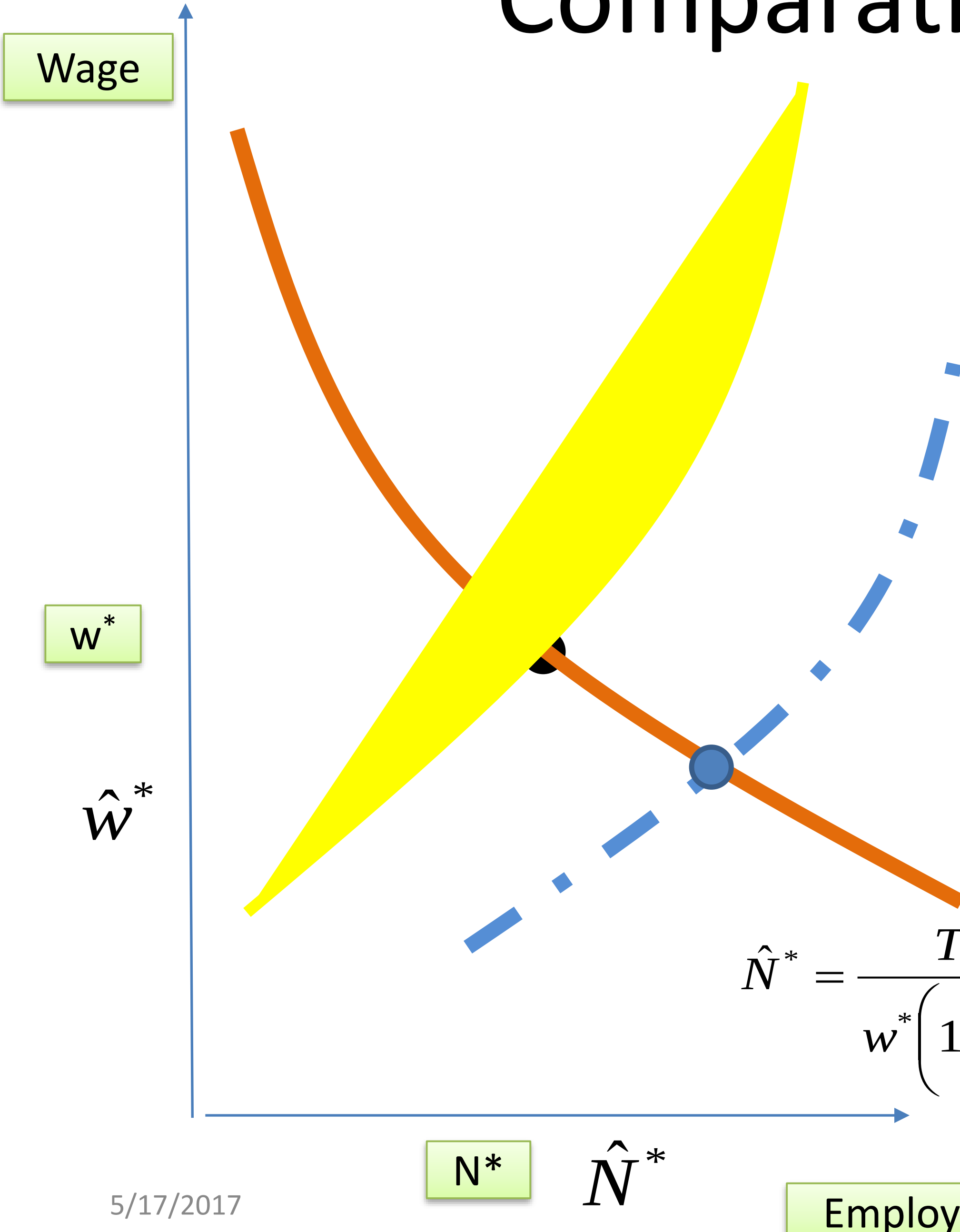
$$w^* = \frac{Z_1}{T};\ \ where\ Z_1 = \left[\frac{TC}{\left(1+\frac{\beta}{\alpha}\right)} + \frac{(wT+V)}{\left(1+\frac{\beta}{\alpha}\right)}\right]$$

$$N^* = \frac{TC}{w^*\left(1+\frac{\beta}{\alpha}\right)} = \frac{Z_2}{w^*} = \frac{Z_2}{Z_1}T;\ \ where\ Z_2 = \frac{TC}{\left(1+\frac{\beta}{\alpha}\right)}$$

New Look at Comparative Statics: Lumpsum tax on households: PS 2



Comparative Statics



$$E^* = \frac{TC}{w\left(1+\frac{\beta}{\alpha}\right)}$$

$$h^* = T - l^* = T - \frac{\left(wT+\hat{V}\right)}{w\left(1+\frac{\beta}{\alpha}\right)}$$

$$\exists\ w = \hat{w}^* \ni E^* = h^* = \hat{N}^*$$

$$T - \frac{\left(wT+\hat{V}\right)}{w\left(1+\frac{\beta}{\alpha}\right)} = \frac{TC}{w\left(1+\frac{\beta}{\alpha}\right)}$$

$$T = \frac{TC}{w\left(1+\frac{\beta}{\alpha}\right)} + \frac{\left(wT+\hat{V}\right)}{w\left(1+\frac{\beta}{\alpha}\right)}$$

$$\hat{w}^* = \frac{\hat{Z}_1}{T};\ \ where\ \ Z_1 = \left[\frac{TC}{\left(1+\frac{\beta}{\alpha}\right)} + \frac{\left(wT+\hat{V}\right)}{\left(1+\frac{\beta}{\alpha}\right)}\right]$$

$$\hat{N}^* = \frac{TC}{w^*\left(1+\frac{\beta}{\alpha}\right)} = \frac{Z_2}{w^*} = \frac{Z_2}{\hat{Z}_1}T;\ \ where\ \ Z_2 = \frac{TC}{\left(1+\frac{\beta}{\alpha}\right)}$$