Estimating the Long-Term Effects of Novel Treatments

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Abstract

Policy makers typically face the problem of wanting to estimate the long-term effects of novel treatments, while only having historical data of older treatment options. We assume access to a long-term dataset where only past treatments were administered and a short-term dataset where novel treatments have been administered. We propose a surrogate based approach where we assume that the long-term effect is channeled through a multitude of available short-term proxies. Our work combines three major recent techniques in the causal machine learning literature: surrogate indices, dynamic treatment effect estimation and double machine learning, in a unified pipeline. We show that our method is consistent and provides root-n asymptotically normal estimates under a Markovian assumption on the data and the observational policy. We use a data-set from a major corporation that includes customer investments over a three year period to create a semi-synthetic data distribution where the major qualitative properties of the real dataset are preserved. We evaluate the performance of our method and discuss practical challenges of deploying our formal methodology and how to address them.

1. Introduction

Businesses frequently invent new ways of interacting with their customers. Marketing departments frequently devise new marketing campaigns. Pharmaceutical companies typically roll out trials of new drugs. In a multitude of domains, policy makers want to understand the long-term effects of novel treatments, recently deployed, while only having access to short-term data following their deployment. Such policy makers only have access to long-term historical data where other treatments had been deployed.

We propose an estimation methodology that leverages historical data to derive estimates of the long-term treatment effects of novel treatments. The seminal paper of Athey et al. (2020) proposes a surrogate index methodology as one solution to this problem. The fundamental assumption of the surrogate index is that there exist short-term proxies that are observed in the short-term dataset and that causal effects on long-term outcomes have to be primarily channeled through these short-term signals. In other words, the treatment has a long-term causal effect, if and only if it has an effect on a variety of short terms signals. This allows one to use the historical long-term data set to learn a mapping from short-term signals to a projected long-term reward — referred to as the surrogate index — and subsequently, estimate the causal effect of novel treatments on the surrogate index.

But often historical treatment policies are *dynamic*: treatments are assigned repeatedly, and their assignments depend on past treatments and short-term outcomes. This can easily break the assumptions needed for the surrogate index to work and introduce bias. For example, suppose that a firm offers multiple investments to a particular customer in the historical data and these investments are highly auto-correlated, i.e. if a customer receives an investment this month, then they will receive an investment with high probability in one of the subsequent months. These future investments can substantially increase the long-term outcome of interest, and this increase will be attributed to the short-term proxies. The surrogate index thus formed will tend to over-predict long-run outcomes, and so when it is used to measure the treatment effect of some new treatment in the short-run data, the estimated treatment effect will be bigger (in absolute magnitude) than the truth.

The main methodological innovation of this paper is to suggest using the dynamic treatment effect analysis of (Lewis & Syrgkanis, 2020) on the historical data in order to create an unbiased dynamically adjusted surrogate index. Our dynamically adjusted surrogate index takes the interpretation of the projected long-term reward in the absence of any future treatments.

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Applying this dynamically adjusted surrogate model to the short-term dataset leads to unbiased causal effect estimates of the long-term effects of the novel treatments.

A second contribution of the paper is to generalize the causal analysis step that uses the experimental sample to allow for multiple continuous treatments rather than a single binary treatment. We do this by proposing a new estimator for this expanded surrogate approach which allows for the construction of valid confidence intervals, even when using flexible machine learning models at both stages of the estimation to deal with high-dimensional data. In short, we show how one can combine three recently developed techniques, i.e. i) the surrogate index approach of (Athey et al., 2020), ii) the double machine learning approach of (Chernozhukov et al., 2018a) and iii) the dynamic treatment effect estimation approach of (Lewis & Syrgkanis, 2020), in a single data analysis pipeline to estimate treatment effects in the presence of dynamic treatment policies.

Our work lies in the broader field of estimating causal effects with machine learning and Neyman orthogonality (Neyman, 1979; Robinson, 1988; Ai & Chen, 2003; Chernozhukov et al., 2016; Chernozhukov et al., 2018b). Moreover, it relates to the work on machine learning estimation of treatment effects in the dynamic treatment regime (Nie et al., 2019; Thomas & Brunskill, 2016; Petersen et al., 2014; Kallus & Uehara, 2019b;a; Lewis & Syrgkanis, 2020; Bodory et al.; Singh et al., 2020) and on structural nested models in biostatistics (Robins, 1986; Robins et al., 1992; Robins, 1994; Robins & Ritov, 1997; Robins et al., 2000; Lok & DeGruttola, 2012; Vansteelandt et al., 2014; Vansteelandt & Sjolander, 2016). Finally, it relates to the surrogacy literature in causal inference (Prentice, 1989; Begg & Leung, 2000; Frangakis & Rubin, 2002; Freedman et al., 1992). Our work builds on insights in these works and proposes the first complete method that combines all three lines of work, so as to estimate the long-term effect of novel treatments from observational data that stem from a dynamic observational policy and in a manner that allows for high-dimensionality.

2. Problem and Methodology

Though our methodology applies in many domains, for concreteness we consider the running example of a firm making investments in its customers in order to increase subsequent purchases by those customers.

A firm has a number of distinct investments/treatments $T_1, T_2 \dots T_k$ it offers to its customers. At each period t (e.g. the period of a month), a vector of treatments $T_{i,t} = (T_{i,t,1}, \dots, T_{i,t,k}) \in \mathbb{R}^k$, is applied to each customer i. We also observe a vector of p characteristics $X_{i,t} = (X_{i,t,1}, \dots, X_{i,t,p}) \in \mathbb{R}^p$, some of which are constant within customer (e.g. industry) and some of which vary over time and customer (e.g. last month's revenue). We observe the outcome of interest $Y_{i,t} \in \mathbb{R}$ (e.g. monthly revenue).

We are interested in identifying the average effect of each treatment at some period t, on the cumulative outcome in the subsequent M periods, i.e. $\bar{Y}_{i,t} = \sum_{\kappa=0}^{M} Y_{i,t+\kappa}$. We assume that for most customers and periods we do not observe the subsequent M periods following a treatment. This makes direct inference from the historical data impractical. Instead, we assume that for every customer i and period t, we have access to a vector of d short-term proxies/surrogates, $S_{i,t} = (S_{i,t,1}, \dots, S_{i,t,d})$. In practice, this can for instance include the next few months of revenue and other measures that are indicative of a customer's trajectory.

Given these surrogate measures, Athey et al. (2020) propose the following estimation strategy:

1. Begin with a long-term *observational data set* that, for each period and customer, consists of customer characteristics $X_{i,t}$, customer surrogates for growth $S_{i,t+1}$ and realized M-period outcomes $\bar{Y}_{i,t}$. Notably, this data-set does not need to contain the treatments whose causal effect we want to measure and hence can use a longer span of historical data even if treatments are only measured recently. Based on this data-set we construct a model that tries to predict the target outcome $\bar{Y}_{i,t}$ from the surrogates $S_{i,t}$ and customer characteristics. Mathematically, this model $\hat{g}(S_{i,t}, X_{i,t})$ estimates the conditional expectation:

$$g_0(S_{i,t}, X_{i,t}) := \mathbb{E}[\bar{Y}_{i,t} \mid S_{i,t}, X_{i,t}]. \qquad \text{(surr. model)}$$

   and corresponds to a simple regression task. Any machine learning estimation algorithm can be used to construct $\hat{g}$.

2. Next, use a second short-term, *experimental data set* that includes customer features, $X_{i,t}$, surrogates, $S_{i,t}$, and treatments $T_{i,t}$.[1] This data set is restricted to the period where we can measure treatments, but only requires a few

[1] We note that the set of customers does not need to be the same in the two data-sets.

months of leading surrogates rather than M periods of leading outcomes. For any such sample we calculate the surrogate index:

$$I_{i,t} := \hat{g}(S_{i,t}, X_{i,t}), \qquad \text{(predicted surrogate index)}$$

which is the predicted long-term outcome from the observed short-term proxies. This predicted index becomes the outcome in the final causal model.

This standard approach will be consistent for the treatment effects under two assumptions. The first is that the relationship between the surrogate variables and the target outcome remains unchanged over our historical sample period and the periods into which we project future outcomes. We follow Athey et al. (2020) in maintaining this assumption. The model is able to account for steady growth in a natural way: as the surrogates grow, the predicted outcomes grow too.[2]

The second main assumption is that the only causal path from the treatment to the outcome goes through the surrogates. In other words, the treatment has an effect on M-period outcomes if and only if it affects these surrogates. This critical "sufficiency" assumption typically requires that a few periods of post-treatment data are available to build a relatively good understanding of the M-period path.

Serially correlated treatment policies will often violate this second assumption. A customer who receives a treatment in period t may be more likely to receive a second treatment over the next year, even controlling for observed features X_{it}. This positive correlation could be either because a customer that is treated subsequently becomes more salient to the firm, or because the treatment successfully drives surrogate metrics higher, thereby improving the perceived return on subsequent treatments of this customer (an example could be a firm that targets its fast-growing customers).

In either case, the standard estimated surrogate index model $\hat{g}$ will now be biased; we would wrongly estimate that even small increases in our proxy measures forecast strong growth in M-period outcomes on average. If we subsequently estimate the causal effect of novel treatments on the surrogate index, we will over-estimate the causal effect on M-period outcomes. Essentially, causal effect that should be attributed to the future treatment gets doubly attributed to the current treatment. To remove this bias we include a dynamic adjustment step based on (Lewis & Syrgkanis, 2020) that alters our observational data-set to remove the effect of future treatments from the target outcome before estimating our surrogate index model. To show this why this method is both necessary and sufficient for removing bias, we frame the problem and solution in approach in a two-period example before formally presenting the general case in section 3.

2.1. A two-period example

Assume that $\bar{Y}_{i,t} = Y_{i,t} + Y_{i,t+1}$ (i.e. the long-term outcome contains two periods). We assume a linear Markovian model of how the random variables evolve over time. We collapse $X_{i,t}$ and $S_{i,t-1}$ so that the surrogates at each period and the controls in the next period are all denoted by $S_{i,t-1}$. In other words, all customer observable characteristics in the next period are candidate surrogates and become controls for the period after next. This is without loss of generality. Finally, we focus on a single scalar investment.

With these simplifications, the structural model that describes how all the random variables evolve can be written in three equations:

$$\begin{aligned} S_{i,t} &= AT_{i,t} + BS_{i,t-1} + \epsilon_{i,t} && \text{(evolution of controls)} \\ Y_{i,t} &= \gamma' S_{i,t} + \zeta_{i,t} && \text{(outcome equation)} \\ T_{i,t+1} &= \kappa T_{i,t} + \lambda' S_{i,t} + \eta_{i,t} && \text{(treatment policy)} \end{aligned}$$

where A is an $p \times 1$ matrix, B is a $p \times p$ matrix, γ, λ are p-dimensional vectors and κ is a scalar. The terms $\epsilon_{i,t}, \zeta_{i,t}, \eta_{i,t}$ are exogenous mean-zero independent noise terms. For conciseness we drop the customer index i in the remaining equations.

Target effect estimand The effect of treatment T_t on the long-term outcome $\bar{Y}_t$ can be derived as follows:

$$\begin{aligned} Y_t &= \gamma' S_t + \text{noise} = \gamma' AT_t + \gamma' BS_{t-1} + \text{noise} \\ Y_{t+1} &= \gamma' S_{t+1} + \text{noise} = \gamma' AT_{t+1} + \gamma' BS_t + \text{noise} \\ &= \gamma' AT_{t+1} + \gamma' BAT_t + \gamma' B^2 S_{t-1} + \text{noise} \end{aligned}$$

[2] It is not robust to changes in the mapping between the surrogates and outcomes over time, and so we would recommend refitting the model periodically to account for this.

Thus we see that the effect of T_t on $Y_{t+1} + Y_t$, keeping all other variables fixed, is:

$$\theta_0 := \gamma'(B + I)A. \tag{1}$$

A one-unit increase in investment T_t, leads to a total of θ_0 units more revenue in the next two periods assuming future treatments are held constant.

Surrogate index without dynamic adjustment If we train a surrogate index by regressing $\bar{Y}_t = Y_t + Y_{t+1}$ on S_t, as is the method proposed in Athey et al. (2020), then this would result in the following surrogate index:

$$\begin{aligned} g_0(S_t) =\ & \mathbb{E}[\bar{Y}_t \mid S_t] = \mathbb{E}[Y_t \mid S_t] + \mathbb{E}[Y_{t+1} \mid S_t] \\ =\ & \gamma' S_t + \mathbb{E}[\gamma' S_{t+1} \mid S_t] \\ =\ & \gamma' S_t + \mathbb{E}[\gamma' A T_{t+1} + \gamma' B S_t \mid S_t] \\ =\ & \gamma'(I + B)S_t + \gamma' A \mathbb{E}[T_{t+1} \mid S_t] \end{aligned} \tag{2}$$

Subsequently, if we estimate the causal effect of T_t on $g_0(S_t)$, then this effect would be:

$$\begin{aligned} \theta_* =\ & \gamma'(I + B)A + \gamma' A \frac{\mathbb{E}[\mathbb{E}[T_{t+1} \mid S_t]\, T_t]}{\mathbb{E}[T_t^2]} \\ =\ & \theta_0 + \gamma' A \frac{\mathbb{E}[\mathbb{E}[T_{t+1} \mid S_t]\, T_t]}{\mathbb{E}[T_t^2]} \\ =\ & \theta_0 + \gamma' A \kappa \frac{\mathbb{E}[\mathbb{E}[T_t \mid S_t]\, T_t]}{\mathbb{E}[T_t^2]} + \gamma' A\, \lambda' A \end{aligned} \tag{3}$$

The standard estimate contains a bias stemming from the fact that investment today can lead to higher investment tomorrow, either directly (i.e. that $\kappa \geq 0$) or indirectly through the surrogates (i.e. that $\lambda' A \geq 0$). The standard surrogate approach is valid only when the investment policy is not adaptive, i.e. $\kappa = 0$ and $\lambda = 0$, since then T_{t+1} would be independent of S_t. We see here that the bias is larger if the investment policy is highly auto-correlated, which is typical in practice. The two possible channels for bias, through direct auto-correlation or surrogate-dependent treatment policies, can result in substantially biased estimates.

Dynamic adjustment The goal of the dynamic adjustment is to remove the effect of the next-period treatment from the long-term outcome $\bar{Y}_{i,t}$. We achieve this by estimating a separate causal effect of $T_{i,t+1}$ on $Y_{i,t+1}$, controlling for $X_{i,t+1}, S_{i,t}$. Observe that this conditional expectation is equal to:

$$\mathbb{E}[Y_{t+1} \mid T_{t+1}, S_t] = \gamma' A T_{t+1} + \gamma' B S_t. \tag{4}$$

We then subtract the causal effect, α_{t+1}, from $Y_{i,t+1}$ to get a *dynamically adjusted outcome*, $Y^{\text{adj}}_{i,t+1} := Y_{i,t+1} - \alpha'_{t+1} T_{t+1}$. Finally, we can create an *adjusted long-term outcome*, $\bar{Y}^{\text{adj}}_{i,t} := Y_{i,t} + Y^{\text{adj}}_{i,t+1}$.

We then build a new *dynamically adjusted surrogate index*, which is the projected adjusted long-term outcome, conditional on the observed surrogates:

$$\begin{aligned} g_0^{\text{adj}}(S_t) =\ & \mathbb{E}[\bar{Y}_t^{\text{adj}} \mid S_t] \\ =\ & \mathbb{E}[Y_t \mid S_t] + \mathbb{E}[Y_{t+1}^{\text{adj}} \mid S_t] \\ =\ & \gamma' S_t + \mathbb{E}[Y_{t+1} - \gamma' A T_{t+1} \mid S_t] \\ =\ & \gamma' S_t + \mathbb{E}[\gamma' B S_t \mid S_{t+1}] \\ =\ & \gamma'(I + B) S_t \end{aligned} \tag{5}$$

This new index captures the projected M-period outcomes as if the customer was offered no future treatments.

When we estimate the effect of the treatment based on this adjusted surrogate index we recover:

$$\begin{aligned} \mathbb{E}[g_0^{\text{adj}}(S_t) \mid T_t, S_{t-1}] =\ & \gamma'(I + B)\mathbb{E}[S_t \mid T_t, S_{t-1}] \\ =\ & \gamma'(I + B) A T_t + \gamma'(I + B) B S_{t-1} \end{aligned} \tag{6}$$

With the dynamic adjustment, the coefficient in front of T_t that we recover is the true causal effect $\theta_0 = \gamma'(I + B)A$. Lewis & Syrgkanis (2020) show how this adjustment approach can be extended to many periods, via a recursive peeling process, and also to high-dimensional surrogates and controls via a dynamic double machine learning approach.

Estimating causal effects of new treatments With this adjusted surrogate index in hand, we can also estimate the long-term effect of any other treatment that was introduced more recently. In this example, the stationarity assumption of both proposed surrogate approaches requires that B, which governs how the surrogate evolves, and γ, which governs how surrogates translate to per-period outcomes, do not change between the observational and experimental data-sets. These two parameters govern how surrogates today relate to future outcomes in the absence of any treatment.

Under such a condition, if we introduce a new treatment T_t^{new}, which has a different effect A^{new} on the surrogates (and hence on the long-term outcome), then the effect of this treatment on the long-term outcome is $\theta_0^{\text{new}} = \gamma'(I + B)A^{\text{new}}$. This θ_0^{new} is exactly the outcome of estimating the causal effect of T_t^{new} on $g_0^{\text{adj}}(S_{t+1})$ controlling for S_t.

3. Formal Guarantees

In this section we present our main results. There are a number of innovations beyond the basic strategy presented in the two period example above. First, we develop a generalization of the doubly robust estimation method in Athey et al. (2020) to the case of multiple continuous treatments, under a semi-parametric assumption and in the presence of a dynamic treatment policy in the observational data. Second, we make use of orthogonal machine learning techniques (Chernozhukov et al., 2018a) throughout, to allow for a rich set of potential confounders and valid analytic confidence intervals.

We assume that we have access to two sample populations: an experimental population, denoted as e, and an observational population, denoted as o. A sample from each population consists of a finite horizon time-series $(S_0, T_1, S_1, Y_1, T_2, S_2, Y_2, \ldots, T_M, S_M, Y_M)$. We observe the full M-period time series for each sample in the observational population, but we only observe (S_0, T_1, S_1) for each sample from the experimental population. As in the two period example, we simplify without loss of generality by merging the control variables in period t and the surrogates for period $t-1$. In other words, all next-period control variables serve as potential surrogates and vice versa. We assume that the data obey the Markovian assumptions depicted in the causal graph in Figure 1.

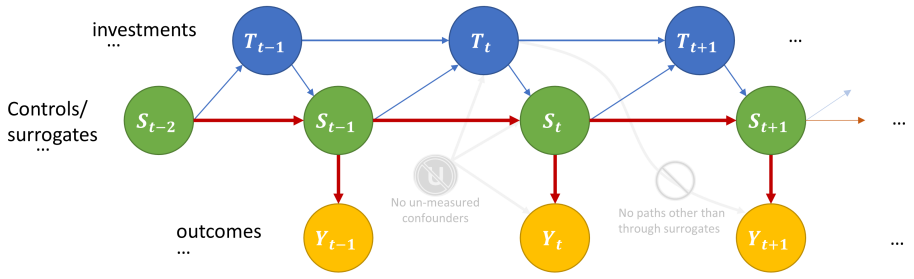

Figure 1. Causal graph representation of the main assumptions of our causal analysis, which are: i) there are no paths to future outcomes from past investments that do not go through surrogates, ii) the relationships designated with a red arrow have to remain un-changed between the long-term data-set and the short-term data-set, iii) there are no un-measured confounders at each period, i.e. variables that are not observed and which affect the treatment assignment and the outcome directly.

Our goal is to estimate the causal effect of treatment vector T_1 on the long term outcome $\bar{Y}_1 := \sum_{t=1}^{M} Y_t$ in the experimental/short-term sample, controlling for the effect of any future treatments. In other words, if we fix the future treatments that each sample receives and we change the treatment T_1 from some value t_0 to some other value t_1, then what is the change on the long term outcome $\bar{Y}_1$, i.e.:

$$\tau(t_1, t_0) := \mathbb{E}_e[\mathbb{E}[\bar{Y}_1 \mid do(T_1 = t_1), T_2, \ldots, T_M] - \mathbb{E}[\bar{Y}_1 \mid do(T_1 = t_0), T_2, \ldots, T_M]] \tag{7}$$

We present our theoretical results in two steps. In the first setting, we assume that treatments happen only at period 1. This

is the setting analyzed in Athey et al. (2020), albeit only for the case of a single binary treatment T_1.[3] We then show how this approach can be modified to incorporate a dynamic treatment policy in the observational and experimental sample.

Notation Throughout, we will denote with $\mathbb{E}_e[\cdot]$ the expectation conditional on the experimental population and $\mathbb{E}_o[\cdot]$ the expectation conditional on the observational population. Moreover, for any vector-valued function f that takes as input a random variable Z, we denote with:

$$\|f\|_{2,o} = \sqrt{\mathbb{E}_o\left[\|f(Z)\|_2^2\right]} \tag{8}$$

and analogously $\|f\|_{2,e}$. We denote with $\mathbb{E}_n[\cdot]$, the empirical expectation over all the samples, i.e. for any random variable Z, $\mathbb{E}_n[Z] := \frac{1}{n}\sum_i Z_i$, and with $\mathbb{E}_{e,n}$ and $\mathbb{E}_{o,n}$ the empirical expectation over the experimental and observational samples correspondingly, i.e. $\mathbb{E}_{e,n}[Z] = \frac{1}{n_e}\sum_{i\in e} Z_i$ and $\mathbb{E}_{o,n}[Z] = \frac{1}{n_o}\sum_{i\in o} Z_i$.

3.1. Double/Debiased Correction without Dynamic Adjustment

To begin, we assume that $T_2, \dots, T_M = 0$, i.e. treatment occurs only in period 1. Throughout, we assume a *partially linear relationship* between the treatment T_1 and the long-term outcome in the experimental sample:

$$\mathbb{E}_e[\bar{Y}_1 \mid T_1, S_0] = \theta_0^\top \phi(T_1, S_0) + b_0(S_0) \tag{PLR}$$

for some known feature map $\phi(\cdot,\cdot)$, but arbitrary function $b_0(\cdot)$.

Formally, the *invariance of the surrogate-outcome* relationship requires that that the mean-relationship between the surrogates S_1 and the long-term outcome does not change between the observational and the experimental sample:

$$g_0(S_1) := \mathbb{E}_o[\bar{Y}_1 \mid S_1] = \mathbb{E}_e[\bar{Y}_1 \mid S_1]. \tag{IR}$$

We denote with g_0 the surrogate index model and with $g_0(S_1)$ the surrogate index.

Under the PLR assumption and the causal graph governing our data, we have that:

$$\tau(t_1, t_0) = \theta_0^\top \mathbb{E}[\phi(t_1, S_0) - \phi(t_0, S_0)] \tag{9}$$

To estimate our treatment effect of interest, it suffices to find an estimate $\hat{\theta}$ of the parameter vector θ_0. Subsequently, we can also estimate:

$$\hat{\tau}(t_1, t_0) = \hat{\theta}^\top \mathbb{E}_{e,n}[\phi(t_1, S_0) - \phi(t_0, S_0)] \tag{10}$$

We establish three consistent estimators for θ_0 that follow the same intuition as Athey et al. (2020), adapted to consider linear effects of continuous treatments rather than a single binary treatment.

Theorem 3.1 (Identification)**.** *Denote the residual surrogate index and the residual treatment with:*

$$\tilde{g}_0(S_1) := g_0(S_1) - \mathbb{E}_e[g_0(S_1) \mid S_0] \tag{11}$$

$$\tilde{T}_1 := \phi(T_1, S_0) - \mathbb{E}_e[\phi(T_1, S_0) \mid S_0] \tag{12}$$

Then under assumptions (PLR) and (IR):

$$\theta_0 = \mathbb{E}_e[\tilde{T}_1 \tilde{T}_1^\top]^{-1} \mathbb{E}_e[\tilde{T}_1 \tilde{g}_0(S_1)] \qquad \text{(surrogate index rep.)}$$

$$\theta_0 = \mathbb{E}_e[\tilde{T}_1 \tilde{T}_1^\top]^{-1} \mathbb{E}_o\left[\frac{\Pr(e \mid S_1, S_0)}{\Pr(o \mid S_1, S_0)} \frac{\Pr(o)}{\Pr(e)} \mathbb{E}_e[\tilde{T}_1 \mid S_1, S_0]\, \bar{Y}_1\right] \qquad \text{(surrogate score rep.)}$$

$$\theta_0 = \mathbb{E}_e[\tilde{T}_1 \tilde{T}_1^\top]^{-1} \left(\mathbb{E}_e[\tilde{T}_1 \tilde{g}_0(S_1)] + \mathbb{E}_o\left[\frac{\Pr(e \mid S_1, S_0)}{\Pr(o \mid S_1, S_0)} \frac{\Pr(o)}{\Pr(e)} \mathbb{E}_e[\tilde{T}_1 \mid S_1, S_0](\bar{Y}_1 - g_0(S_1))\right]\right) \qquad \text{(orthogonal rep.)}$$

[3] We note that the work of Athey et al. (2020) also allowed for estimation of average treatment effects, even in the case when there is arbitrary treatment effect heterogeneity. In this work, we assume that treatment effects are constant. A generalization to the case of arbitrary treatment effect heterogeneity is feasible, but would require the estimation of conditional covariance matrices, which would make the estimation algorithm more brittle and the exposition much more complex.

The core estimation challenge that the surrogate approach resolves is that the treatments and outcome of interest are not observed in a single dataset. Intuitively, the first **surrogate index representation** approaches this challenge by using realized treatments from the experimental sample and, in place of realized outcomes, substitutes the expected outcome conditional on the surrogates, $g_0(S)$, which can be identified from the observational sample and then constructed in the experimental sample.

The second **surrogate score representation** reverses this substitution. The second term pairs an expectation of the treatment conditional on surrogates, $\mathbb{E}_e[\tilde{T}_1 \mid S_1, S_0]$, with the realized outcomes from the observational sample. This representation requires an added ratio of probabilities of appearing in each sample, $\Pr(e)$ and $\Pr(o)$, to adjust for sampling variation across the two datasets.

The third **orthogonal representation** blends the first two representations and satisfies *Neyman orthogonality*, which allows the construction of confidence intervals and double robustness.[4]

We note that the parameter identified in the equations in Theorem 3.1 is interpretable even if the partially linear assumption is violated. In this case, the equations are identifying the best linear projection of the variation in the long-term outcome that is not explained by the initial state, i.e. $\bar{Y}_1 - \mathbb{E}_e[\bar{Y}_1 \mid S_0]$, on the variation in the feature map $\phi(T_1, S_0)$, that is also un-explained by the initial state, i.e. $\tilde{T}_1$. That is the quantity:

$$\theta_0 = \mathbb{E}_e[\tilde{T}_1 \tilde{T}_1^\top]^{-1} \mathbb{E}_e[\tilde{T}_1 \ \left(\bar{Y}_1 - \mathbb{E}_e[\bar{Y}_1 \mid S_0]\right)] \tag{13}$$

We formulate the estimation of θ_0 based on the orthogonal representation as a Z-estimator based on a vector of moment equations that depends on a vector of nuisance functions f_0, i.e.:

$$m(\theta_0; f_0) := \mathbb{E}[\psi(Z; \theta_0, f_0))] = 0 \tag{14}$$

and such that it satisfies the Neyman orthogonality condition:

$$D[m(\theta_0; f_0), f - f_0] := \frac{\partial}{\partial t} m(\theta_0; f_0 + t\,(f - f_0))\Big|_{t=0} = 0$$

Subsequently, this will allow us to invoke the results in (Chernozhukov et al., 2018b), to derive an asymptotic normal estimator, even when high-dimensional, regularized approaches are used to estimate the nuisance functions f_0.

Theorem 3.2 (Orthogonal Moment Formulation)**.** *Denote with:*

$$\begin{aligned} q_0(S_1, S_0) &:= \frac{\Pr(e \mid S_1, S_0)}{\Pr(o \mid S_1, S_0)} \mathbb{E}_e[\tilde{T}_1 \mid S_1, S_0] \\ p_0(S_0) &:= \mathbb{E}_e[\phi(T_1, S_0) \mid S_0] \\ h_0(S_0) &:= \mathbb{E}_e[g(S_1) \mid S_0] \end{aligned}$$

and let $f = (g, q, p, h)$, denote the full set of nuisance functions. Then θ_0 is the solution to the moment equation:

$$\begin{aligned} m(\theta_0; f_0) &:= m_e(\theta_0; f_0) + m_o(\theta_0; f_0) = 0 \\ m_e(\theta; f) &:= \mathbb{E}\Big[1\{e\}\left(g(S_1) - h(S_0) - \theta^\top(\phi(T_1, S_0) - p(S_0))\right) \cdot (\phi(T_1, S_0) - p(S_0))\Big] \\ m_o(\theta_0; f) &:= \mathbb{E}\left[1\{o\}\, q(S_1, S_0)(\bar{Y}_1 - g(S_1))\right] \end{aligned}$$

Moreover, the moment m satisfies the Neyman orthogonality property with respect to f. Furthermore, it satisfies a stronger double robustness property with respect to g and q, i.e. if we denote with $\hat{\theta}$ the solution to $m(\theta; \hat{g}, \hat{q}, p_0, \hat{h}) = 0$, then:

$$\left\|\mathbb{E}_e[\tilde{T}_1 \tilde{T}_1^\top]\left(\hat{\theta} - \theta_0\right)\right\|_2 \leq \frac{\Pr(o)}{\Pr(e)} \|g_0 - \hat{g}\|_{2,o} \|q_0 - \hat{q}\|_{2,o}$$

[4] One practical difficulty with this third doubly robust approach is that it is less transparent and requires access to the raw historical dataset whenever estimating a new treatment option. In contrast, the surrogate index representation allows for segmentation: one can estimate the surrogate index in the observational data once and store only the g_0 model. Treatment effects can then be estimated using only these stored parameters and the experimental dataset, or multiple experimental datasets. This explicit construction of expected outcomes in the experimental data also makes the first approach particularly easy to interpret. However, one must then be careful to account for the additional uncertainty stemming from estimating outcomes in the first step, as standard confidence intervals in the second step will not account for this pre-estimated component.

Theorem 3.3 (Estimation and Asymptotic Normality)**.** *Consider the estimator based on the empirical version of the orthogonal moment, with plug-in nuisance estimates $\hat{f} = (\hat{g}, \hat{q}, \hat{p}, \hat{h})$ trained on a separate sample, i.e. $\hat{\theta}$ solves the empirical moment equation:*

$$\begin{aligned} m_n(\hat{\theta}; \hat{f}) :=&\ m_{e,n}(\hat{\theta}; \hat{f}) + m_{o,n}(\hat{\theta}; \hat{f}) = 0 \\ m_{e,n}(\theta; f) :=&\ \mathbb{E}_n\Big[1\{e\}\left(g(S_1) - h(S_0) - \theta^\top(\phi(T_1, S_0) - p(S_0))\right)(\phi(T_1, S_0) - p(S_0))\Big] \\ m_{e,n}(\theta; f) :=&\ \mathbb{E}_n\left[1\{o\}\, q(S_1, S_0)(\bar{Y}_1 - g(S_1))\right] \end{aligned}$$

If $\|\hat{h} - h_0\|_{2,e}, \|\hat{p} - p_0\|_{2,e} = o_p(n^{-1/4})$ and $\|g_0 - \hat{g}\|_{2,o} \cdot \|q_0 - \hat{q}\|_{2,o} = o_p(n^{-1/2})$, then:

$$\sqrt{n}\left(\hat{\theta} - \theta_0\right) \to_d N(0, J_e^{-1}\left(\Sigma_e + \Sigma_o\right) J_e^{-1})$$

where:

$$\begin{aligned} J_e :=&\ \mathbb{E}_e[\tilde{T}_1\, \tilde{T}_1^\top] \\ \Sigma_e :=&\ \frac{1}{\Pr(e)}\mathbb{E}_e\left[(\tilde{g}_0(S_1) - \theta_0^\top \tilde{T}_1)^2\, \tilde{T}_1 \tilde{T}_1^\top\right] \\ \Sigma_o :=&\ \frac{\Pr(o)}{\Pr(e)^2}\,\mathbb{E}_o\left[(\bar{Y}_1 - g_0(S_1))^2\, q_0(S_1, S_0)\, q_0(S_1, S_0)^\top\right] \end{aligned}$$

Asymptotically valid confidence intervals can be constructed using empirical analogues of the co-variance matrix.

3.2. Double/Debiased Correction with Dynamic Adjustment

In this section we deal with the case where the treatment policy in the observational and the experimental data is dynamic and we want to estimate only the effects of the treatment at period 1, conditioning on the future treatments, i.e. the part of the effect that does not go through future treatments but solely through the surrogates/control variables.

To achieve this we will make a further parametric assumption on the data generating process. In particular, as in the work of (Lewis & Syrgkanis, 2020) we will assume that all the relationships are linear, albeit potentially high-dimensional:

$$\begin{aligned} S_t =&\ AT_t + BS_{t-1} + \epsilon_t \\ Y_t =&\ CS_t + \eta_t \\ T_{t+1} =&\ DT_{t-1} + GS_{t-1} + \zeta_t \end{aligned} \tag{15}$$

where $\epsilon_t, \eta_t, \zeta_t$ are i.i.d. random shocks.

In this case, the main identification result in (Lewis & Syrgkanis, 2020), shows that:

$$\tau(t_1, t_0) = \sum_{t=1}^{M} \theta_{1,t}^\top (t_1 - t_0) = \theta_0^\top (t_1 - t_0) \tag{16}$$

where $\theta_0 := \sum_{t=1}^{M} \theta_{1,t}$ and $\theta_{1,t}$ are interpreted as the dynamic effect of treatment at period 1 on the outcome at period t, controlling for all future treatments. In this setting, this quantity is equivalent to the effect assuming that all future treatments are zero, since the effects of each period treatment under this linear model are linearly separable, i.e. we have that for all t_{-1}:

$$\begin{aligned} \tau(t_1, t_0) =& \mathbb{E}[\bar{Y}_1 \mid do(T_1 = t_1), T_{-1} = t_{-1}] \\ &- \mathbb{E}[\bar{Y}_1 \mid do(T_1 = t_0), T_{-1} = t_{-1}] \end{aligned}$$

This property simplifies both the estimation and the interpretation of the effect quantity as many causal quantities of potential interest collapse in this setting to the same object.

The identification results in (Lewis & Syrgkanis, 2020) also imply that we can identify the quantity $\theta_{1,t}$ by estimating the effect of T_1 on a dynamically adjusted target outcome:

$$Y_t^{\text{adj}} := Y_t - \sum_{\tau=2}^{t} \theta_{\tau,t}^\top T_t = \theta_{1,t}^\top T_1 + \beta_t S_0 + \mu_{1,t} \tag{17}$$

for some exogenous mean-zero random shock variable $\mu_{1,t}$. Thus in order to estimate $\tau(t_1, t_0)$ it suffices to estimate the effect of T_1 on the long-term dynamically adjusted outcome, which satisfies:

$$\bar{Y}_1^{\text{adj}} = \sum_{t=1}^{M} Y_t^{\text{adj}} = \theta_0^\top T_1 + \beta_0 S_0 + \mu_1 \tag{18}$$

for some exogenous mean-zero random shock variable μ_1. This is exactly the setup that we analyzed in the previous section, albeit with $\bar{Y}_1$ replaced by $\bar{Y}_1^{\text{adj}}$ and with $\phi(T_1, S_0) = T_1$.[5] Moreover, the PLR assumption with respect to this dynamically adjusted target immediately holds by the equation above.

Furthermore, we need to assume that the invariance of the surrogate-outcome assumption holds, albeit now for the dynamically adjusted long-term outcome:

$$g_0^{\text{adj}}(S_1) := \mathbb{E}_o[\bar{Y}_1^{\text{adj}} \mid S_1] = \mathbb{E}_e[\bar{Y}_1^{\text{adj}} \mid S_1] \tag{dynIR}$$

This assumption is much more permissive in practice than the standard invariance assumption as we no longer require that the dynamic treatment policy in the observational data be the same as in the experimental data, but simply that the adjusted outcomes retain the same relationship with the surrogates.

Thus we can apply the same analysis as in the previous section, now with this new target long-term outcome. One complication is that in this case, we also need to account for the variance of the $\hat{\theta}_t$ terms that are being estimated by the dynamic adjustment algorithm. This can be achieved with an expanded orthogonal vector moment equation that estimates simultaneously all the dynamic effects $\theta_{\tau,t}$ (using the orthogonal vector moment equation in Lewis & Syrgkanis (2020)) and appending the orthogonal moment vector θ_0 identified in the previous section via the surrogate approach. We formalize this in the following theorem.

Theorem 3.4 (Surrogates with Dynamic Adjustment: Estimation and Asymptotic Normality)**.** *Let:*

$$\psi_0(Z; \theta, f_0) := 1\{e\} \underbrace{\left(\tilde{g}_0^{adj}(S_1) - \theta_0^\top \tilde{T}_1\right) \tilde{T}_1}_{\psi_{0,e}(Z;\theta,f_0)} + 1\{o\} \underbrace{\left(\bar{Y}_1^{adj} - g_0^{adj}(S_1)\right) q_0(S_1, S_0)}_{\psi_{0,o}(Z;\theta,f_0)}$$

$$\forall\, 2 \le \tau \le t \le M : \psi_{\tau,t}(Z; \theta, f_0) := 1\{o\} \left(\tilde{Y}_{t,\tau} - \sum_{\kappa=\tau+1}^{t} \theta_{\kappa,t}^\top \tilde{T}_{\kappa,\tau} - \theta_{\tau,t}^\top \tilde{T}_{\tau,\tau}\right) \tilde{T}_{\tau,\tau}$$

$$\forall\, 2 \le t \le M : \psi_t(Z; \theta_0, f_0) = \left[\psi_{t,t}(Z; \theta, f_0) \quad \dots \quad \psi_{t,2}(Z; \theta, f_0)\right]^\top$$

where we use the short-hand notation $\tilde{Y}_{t,\tau} = Y_t - \mathbb{E}[Y_t \mid S_{\tau-1}]$ *and* $\bar{Y}_1^{adj} = \sum_{t=1}^{M} \left(Y_t - \sum_{\tau=2}^{t} \theta_{\tau,t}^\top T_\tau\right)$ *and* $\tilde{T}_{t,\tau} = T_t - \mathbb{E}[T_t \mid S_{\tau-1}]$. *Let* $h_{t,\tau}(S_{\tau-1}) = \mathbb{E}[Y_t \mid S_{\tau-1}]$ *and* $p_{t,\tau}(S_{\tau-1}) = \mathbb{E}[T_t \mid S_{\tau-1}]$. *Let* $f = \{g, q, p, h, \{h_{\tau,t}, p_{\tau,t}\}_{2 \le \tau \le t \le M}\}$, *denote the set of all nuisance functions and let* f_0 *denote their true value. Consider the estimator based on the empirical version of the orthogonal moment, with plug-in nuisance estimates* $\hat{f}$ *trained on a separate sample, i.e.* $\hat{\theta}$ *solves the empirical moment equation:*

$$m_n(\hat{\theta}; \hat{f}) = 0 \qquad m_n(\theta; f) := \mathbb{E}_n[\psi(Z; \theta, f) := \begin{bmatrix} \mathbb{E}_n\left[\psi_0(Z; \theta, f)\right] \\ \mathbb{E}_n\left[\psi_2(Z; \theta, f)\right] \\ \dots \\ \mathbb{E}_n\left[\psi_M(Z; \theta, f)\right] \end{bmatrix} \tag{19}$$

If $\|\hat{r} - r_0\|_{2,o} = o_p(n^{-1/4})$, *for* $r \in \{\{h_{\tau,t}, p_{\tau,t}\}_{2 \le \tau \le t \le M}\}$ *and if* $\|\hat{h} - h_0\|_e = o_p(n^{-1/4})$, $\|\hat{p} - p_0\|_e = o_p(n^{-1/4})$ *and* $\|g_0 - \hat{g}\|_{2,o} \cdot \|q_0 - \hat{q}\|_{2,o} = o_p(n^{-1/2})$, *then:*

$$\sqrt{n}\left(\hat{\theta}_0 - \theta_0\right) \to_d N(0, J_e^{-1}\ (\Sigma_e + \Sigma_{o,1} + \Sigma_{o,2})\ J_e^{-1})$$

[5] We can also easily generalize the results in this section to known feature maps ϕ.

where J_e defined as in Theorem 3.3 and:

$$\Sigma_e := \frac{1}{\Pr(e)}\mathbb{E}_e\left[\Psi_{o,e}\,\Psi_{0,e}^\top\right]$$

$$\Sigma_{o,1} := \frac{\Pr(o)}{\Pr(e)^2}\mathbb{E}_o[\Psi_{0,o}\Psi_{0,o}^\top] \qquad \Sigma_{o,2} := \frac{\Pr(o)}{\Pr(e)^2}\sum_{t=2}^{M} B_t C_t^{-1}\cdot\left(\sum_{t'=2}^{M}\mathbb{E}_o[\Psi_t\Psi_{t'}^\top]C_{t'}^{-\top}B_{t'}^\top - \mathbb{E}_o[\Psi_t\Psi_{0,o}^\top]\right)$$

where we use the short-hand notation Ψ_t for the random variable $\psi_t(Z;\theta_0,f_0)$ and B_t, C_t are appropriately defined matrices.

We see that Theorem 3.4 is almost identical to that of Theorem 3.3, with the extra moments to estimate the dynamic effects of future treatments and with the extra variance term $\Sigma_{o,2}$, in the final co-variance formula for our target parameter θ_0, which represents the extra uncertainty that is introduced on the target parameter of interest by the fact that we are estimating the dynamic effects of future treatments in the observational/long-term sample.

Asymptotically valid confidence intervals can be constructed using empirical analogues of the co-variance matrix. Moreover, our proof of Theorem 3.4 essentially shows that the estimate $\hat{\theta}$ is asymptotically linear, as it can be framed as the solution to a vector of moment equations, with respect to a low-dimensional parameter vector, and depedenend on a vector of nuisance functions, satisfying Neyman orthogonality with respect to them. Thus our estimate is not only asymptotically normal but also asymptotically linear. Thus computationally convenient resampling methods, with better finite sample properties can be used to construct confidence sets. For instance, constructing intervals by running the Bootstrap on the final stage estimation (keeping the nuisance estimates fixed), will be asymptotically valid. Moreover, the computationally even more convenient multiplier Bootstrap can also be used (Chatterjee & Bose, 2005; Chernozhukov et al., 2013; 2014; Spokoiny & Zhilova, 2015; Zhilova, 2020), which can also be used for joint inference on multiple parameters.

4. Semi-Synthetic Experimental Evaluation

We evaluate the performance of our proposed estimation strategy on a semi-synthetic dataset. The semi-synthetic data retain qualitative characteristics of data on real-world incentive investments in customers at a major corporation, although all data series and relationships have been perturbed to retain confidentiality.

The semi-synthetic dataset, like the real-world dataset on which it is based, displays several patterns that are common across many potential applications. The treatments, in this case incentive investments, are lumpy: in most periods most customers get no investments. Proxies, which include single period values of the outcome of interest, are highly auto-correlated over time. Treatments are also auto-correlated, and correlated with past values of proxies. Finally, we include a set of time-invariant controls that affect both proxies/outcomes and treatments.

To build the semi-synthetic data we estimate a series of moments from a real-world dataset: a full covariance matrix of all proxies, treatments, and controls in one period and a series of linear prediction models (lassoCV) of each proxy and treatment on a set of 6 lags of each treatment, 6 lags of each proxy, and time-invariant controls. Using these values, we draw new parameters from distributions matching the key characteristics of each family of parameters. Finally, we use these new parameters to simulate proxies, treatments, and controls by drawing a set of initial values from the covariance matrix and forward simulating to match intertemporal relationships from the transformed prediction models. For further details on the data generation process, see Appendix Section C.

We now compare multiple possible approaches for estimating the effects of our three synthetic treatments on a long-term outcome. To construct this outcome we select one proxy to be the outcome of interest. We consider the effect of each treatment in period t on the cumulative sum of the outcome from period t to $t+3$, four periods, or t to $t+7$, eight periods. We can calculate the true treatment effects in the synthetic data as a function of parameters from the linear prediction models.

Because we construct a single, long synthetic dataset for this exercise it is possible to estimate the treatment effects on realized long-term outcomes directly, unlike the typical use case for a surrogate approach. Following a likely approach, we estimate the effect of each treatment at time t on outcomes over the next 4 or 8 periods using double machine learning and controlling for invariant customer characteristics and *contemporaneous and lagged* values of all proxies and other treatments. The blue, "total" bars in each panel of Figure 2 show the distribution of the estimation error[6] in the estimated treatment effects from this method across 100 simulated datasets. The top row plots the estimation error when estimating the effect on

[6]We use the ℓ_2 error $\|\hat{\theta}-\theta_0\|_2$.

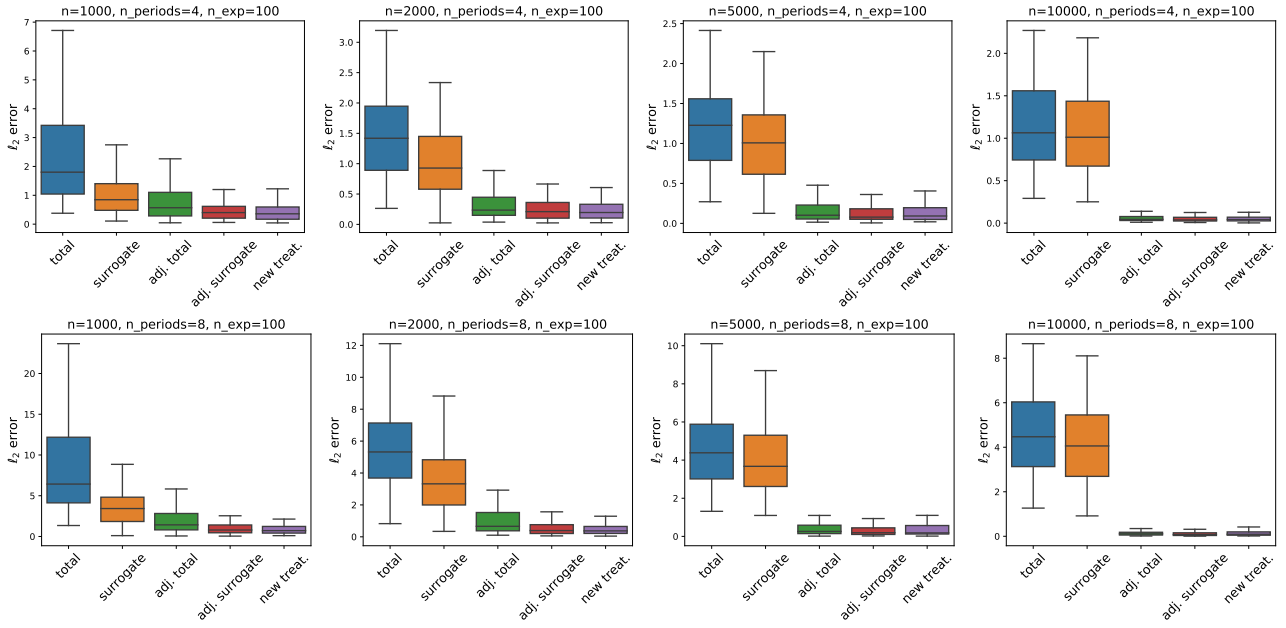

Figure 2. Experimental performance for $M = 4$ periods and $M = 8$ periods.

four periods of outcomes, increasing the sample size of each simulation from left to right, while the bottom row shows the same for the effect on eight periods of outcome. As predicted, the auto-correlation in treatments causes this method, which does not control for *future* treatments, to substantially overestimate treatment effects relative to their true values.

We then estimate the same set of treatment effects using the unadjusted surrogate approach described in Section 3.1. The distribution of estimation errors from this approach is represented in the orange "surrogate" bars in each panel of Figure 2. Since this approach still fails to control for future treatments when estimating the surrogate index, the estimated treatment effects are still substantially larger than the true effects on average. Note that the surrogate model exhibits slightly less bias than the direct "total" approach. Intuitively, because the surrogate approach is only capturing the relationship between treatment and outcome that passes through the surrogates it picks up less of the bias resulting from future correlated treatments than the direct approach.

The third set of green "adj. total" bars plot the distribution of estimation errors when estimating treatment effects on adjusted realized outcomes using the method of Lewis & Syrgkanis (2020). When a dataset containing both all treatments of interest and realized long-run outcomes is available, this should be the preferred approach. This third methodology, which removes the effects of future treatments from the long-run outcome in a first step, exhibits significantly less bias than the first two methods, particularly for reasonably large samples in the right two columns.

The final two bars in each panel of Figure 2 illustrate the success of the adjusted surrogate approach described in Section 3.2. We recommend this approach in the case when treatments are serially-correlated, as in the synthetic data, and it is not possible to collect a single dataset that contains both long-term outcomes and all the treatments of interest. As illustrated by the red "adj. surrogate" bars, this adjusted surrogate approach is highly accurate in predicting long-term effects with a performance comparable to that of having access to the raw long-term outcome itself. The final purple "new treat." bars show that the approach works equally well when considering the effect of a novel treatment that appears only in the experimental sample and was not part of the dynamic adjustment. Overall, this methodology overcomes a common data limitation when considering long-term effects of novel treatments and expands the surrogate approach to consider a common, and previously problematic, pattern of serially correlated treatments.

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A. Proofs of Theorems in Section 3

A.1. Proof of Theorem 3.1

Surrogate index representation. By the causal graph assumption and the IR assumption we have:

$$\begin{aligned}\mathbb{E}_e[\bar{Y}_1 \mid T_1, S_0] =\ & \mathbb{E}_e[\mathbb{E}_e[\bar{Y}_1 \mid S_1, T_1, S_0] \mid T_1, S_0] = \mathbb{E}_e[\mathbb{E}_e[\bar{Y}_1 \mid S_1] \mid T_1, S_0] = \mathbb{E}_e[\mathbb{E}_o[\bar{Y}_1 \mid S_1] \mid T_1, S_0] \\ =\ & \mathbb{E}_e[g_0(S_1) \mid T_1, S_0]\end{aligned}$$

Thus also we have that $\mathbb{E}_e[\bar{Y}_1 \mid S_0] = \mathbb{E}_e[g_0(S_1) \mid S_0]$ and that:

$$\mathbb{E}_e[\bar{Y}_1 \mid T_1, S_0] - \mathbb{E}_e[\bar{Y}_1 \mid S_0] = \mathbb{E}_e[g_0(S_1) \mid T_1, S_0] - \mathbb{E}_e[g_0(S_1) \mid S_0] = \mathbb{E}_e[\tilde{g}_0(S_1) \mid T_1, S_0]$$

By the PLR assumption and the definition of $\tilde{T}_1 := \phi(T_1, S_0) - \mathbb{E}[\phi(T_1, S_0) \mid S_0]$, we have that:

$$\mathbb{E}_e[\bar{Y}_1 \mid T_1, S_0] - \mathbb{E}_e[\bar{Y}_1 \mid S_0] = \theta_0^\top \tilde{T}_1 \implies \mathbb{E}[\tilde{g}_0(S_1) - \theta_0^\top \tilde{T}_1 \mid T_1, S_0] = 0 \implies \mathbb{E}\left[\left(\tilde{g}_0(S_1) - \theta_0^\top \tilde{T}_1\right)\tilde{T}_1\right] = 0$$

Solving for θ_0, yields the surrogate index representation:

$$\theta_0 := \mathbb{E}_e[\tilde{T}_1\, \tilde{T}_1^\top]^{-1}\mathbb{E}_e[\tilde{g}_0(S_1)\tilde{T}_1] \tag{20}$$

Surrogate score representation. We start by the definition of the surrogate score representation and use the causal graph assumptions to derive:

$$\begin{aligned}
I_{ss} :=\ & \mathbb{E}_e[\tilde{T}_1\tilde{T}_1^\top]^{-1}\mathbb{E}_o\left[\frac{\Pr(e \mid S_1, S_0)}{\Pr(o \mid S_1, S_0)}\frac{\Pr(o)}{\Pr(e)}\,\bar{Y}_1\,\mathbb{E}_e[\tilde{T}_1 \mid S_1, S_0]\right] \\
=\ & \mathbb{E}_e[\tilde{T}_1\tilde{T}_1^\top]^{-1}\mathbb{E}_o\left[\frac{\Pr(S_1, S_0 \mid e)}{\Pr(S_1, S_0 \mid o)}\,\bar{Y}_1\,\mathbb{E}_e[\tilde{T}_1 \mid S_1, S_0]\right] && \text{(Bayes-rule)} \\
=\ & \mathbb{E}_e[\tilde{T}_1\tilde{T}_1^\top]^{-1}\mathbb{E}_o\left[\frac{\Pr(S_1, S_0 \mid e)}{\Pr(S_1, S_0 \mid o)}\,\mathbb{E}_o[\bar{Y}_1 \mid S_1, S_0]\,\mathbb{E}_e[\tilde{T}_1 \mid S_1, S_0]\right] && \text{(tower-law)} \\
=\ & \mathbb{E}_e[\tilde{T}_1\tilde{T}_1^\top]^{-1}\,\mathbb{E}_e\left[\mathbb{E}_o[\bar{Y}_1 \mid S_1, S_0]\,\mathbb{E}_e[\tilde{T}_1 \mid S_1, S_0]\right] && \text{(change of measure)} \\
=\ & \mathbb{E}_e[\tilde{T}_1\tilde{T}_1^\top]^{-1}\mathbb{E}_e\left[\mathbb{E}_e[\bar{Y}_1 \mid S_1, S_0]\,\mathbb{E}_e[\tilde{T}_1 \mid S_1, S_0]\right] && \text{(IR assumption)} \\
=\ & \mathbb{E}_e[\tilde{T}_1\tilde{T}_1^\top]^{-1}\mathbb{E}_e\left[\mathbb{E}_e[\bar{Y}_1\,\tilde{T}_1 \mid S_1, S_0]\right] && \text{(conditional independence)} \\
=\ & \mathbb{E}_e[\tilde{T}_1\tilde{T}_1^\top]^{-1}\mathbb{E}_e\left[\bar{Y}_1\tilde{T}_1\right] && \text{(inverse tower-law)} \\
=\ & \mathbb{E}_e[\tilde{T}_1\tilde{T}_1^\top]^{-1}\left(\mathbb{E}_e\left[\tilde{T}_1\,\phi(T_1, S_0)^\top\theta_0\right] + \mathbb{E}_e\left[b_0(S_0)\,\tilde{T}_1\right]\right) && \text{(PLR assumption)} \\
=\ & \mathbb{E}_e[\tilde{T}_1\tilde{T}_1^\top]^{-1}\mathbb{E}_e\left[\tilde{T}_1\,\tilde{T}_1^\top\theta_0\right] && \text{(mean-zero } \mathbb{E}[\bar{T}_1 \mid S_0]) \\
=\ & \theta_0
\end{aligned}$$

Orthogonal representation. This follows easily by the fact that the second term in the orthogonal representation is mean zero and the first term is equal to θ_0 by the surrogate-index argument.

A.2. Proof of Theorem 3.2

Orthogonal moment formulation. First we see that we can re-write the orthogonal representation from Theorem 3.1 as follows:

$$\theta_0 = \mathbb{E}_e[\tilde{T}_1\,\tilde{T}_1^\top]^{-1}\left(\frac{\mathbb{E}[\tilde{g}_0(S_1)\tilde{T}_1 1\{e\}]}{\Pr(e)} + \frac{1}{\Pr(e)}\mathbb{E}\left[1\{o\}\frac{\Pr(e \mid S_1, S_0)}{\Pr(o \mid S_1, S_0)}(\bar{Y}_1 - g_0(S_1))\mathbb{E}_e[\tilde{T}_1 \mid S_1, S_0]\right]\right) \tag{21}$$

$$= \mathbb{E}[\tilde{T}_1\,\tilde{T}_1^\top 1\{e\}]^{-1}\left(\mathbb{E}[\tilde{g}_0(S_1)\tilde{T}_1 1\{e\}] + \mathbb{E}\left[1\{o\}\frac{\Pr(e \mid S_1, S_0)}{\Pr(o \mid S_1, S_0)}(\bar{Y}_1 - g_0(S_1))\mathbb{E}_e[\tilde{T}_1 \mid S_1, S_0]\right]\right) \tag{22}$$

Equivalently, the solution to the orthogonal moment equation:

$$\mathbb{E}\left[1\{e\}\left(\tilde{g}_0(S_1)-\theta_0^\top \tilde{T}_1\right)\tilde{T}_1+1\{o\}\,\frac{\Pr(e\mid S_1,S_0)}{\Pr(o\mid S_1,S_0)}\mathbb{E}_e[\tilde{T}_1\mid S_1,S_0](\bar{Y}_1-g_0(S_1))\right]=0 \tag{23}$$

By the definition of q_0, p_0 and h_0 the result follows that we can write θ_0 as the solution to $m(\theta_0; f_0) = 0$, where $f_0 = (q_0, p_0, h_0)$ and we remind that:

$$\begin{aligned}
m(\theta; f) :=&\ m_e(\theta; f) + m_o(\theta; f)\\
m_e(\theta; f) :=&\ \mathbb{E}\left[1\{e\}\left(g(S_1)-h(S_0)-\theta^\top(\phi(T_1,S_0)-p(S_0))\right)(\phi(T_1,S_0)-p(S_0))\right]\\
m_o(\theta_0; f) :=&\ \mathbb{E}\left[1\{o\}\,q(S_1,S_0)(\bar{Y}_1-g(S_1))\right]
\end{aligned}$$

Orthogonality. Orthogonality with respect to p, h, q, follows since:

$$\begin{aligned}
D_p[m(\theta_0; f_0), p-p_0] :=&\ \Pr(e)\left(\mathbb{E}_e[\tilde{T}_1\,(p(S_0)-p_0(S_0)] - \mathbb{E}_e\left[\left(\tilde{g}_0(S_1)-\theta_0^\top\tilde{T}_1\right)\,(p(S_0)-p_0(S_0)\right]\right)=0\\
D_h[m(\theta_0; f_0), h-h_0] :=&\ \Pr(e)\,\mathbb{E}_e[-\tilde{T}_1\,(h(S_0)-h_0(S_0)]=0\\
D_q[m(\theta_0; f_0), q-q_0] :=&\ \Pr(o)\,\mathbb{E}_o[(\bar{Y}_1-g_0(S_1))\,(q(S_1,S_0)-q_0(S_1,S_0))]\\
=&\ \Pr(o)\,\mathbb{E}_o[\mathbb{E}[\bar{Y}_1-g_0(S_1))\mid S_1]\,(q(S_1,S_0)-q_0(S_1,S_0)]=0
\end{aligned}$$

Orthogonality with respect to g is slightly more involved. First note that:

$$D_g[m(\theta_0; f_0), g-g_0] = \Pr(e)\,\mathbb{E}_e[\tilde{T}_1\,(g(S_1)-g_0(S_1)] - \Pr(o)\mathbb{E}_o[q_0(S_1,S_0)\,(g(S_1)-g_0(S_1)]$$

Now using a sequence of derivations almost identical to the ones we invoked in the surrogate score representation in Theorem 3.1, we can show that:

Lemma A.1. *For any scalar valued function $r(S_1, S_0)$,*

$$\mathbb{E}_o\left[q_0(S_1,S_0)\,r(S_1,S_0)\right] = \frac{\Pr(e)}{\Pr(o)}\mathbb{E}_e[\tilde{T}_1\,r(S_1,S_0)] \tag{24}$$

Proof.

$$\begin{aligned}
\frac{\Pr(e)}{\Pr(o)}\mathbb{E}_o\left[q_0(S_1,S_0)\,r(S_1,S_0)\right] :=&\ \mathbb{E}_o\left[\frac{\Pr(e\mid S_1,S_0)}{\Pr(o\mid S_1,S_0)}\frac{\Pr(o)}{\Pr(e)}\,\mathbb{E}_e[\tilde{T}_1\mid S_1,S_0]\,r(S_1,S_0)\right] && \text{(definition of } q_0\text{)}\\
=&\ \mathbb{E}_o\left[\frac{\Pr(S_1,S_0\mid e)}{\Pr(S_1,S_0\mid o)}\,\mathbb{E}_e[\tilde{T}_1\mid S_1,S_0]\,r(S_1,S_0)\right] && \text{(Bayes-rule)}\\
=&\ \mathbb{E}_e\left[\mathbb{E}_e[\tilde{T}_1\mid S_1,S_0]\,r(S_1,S_0)\right] && \text{(change of measure)}\\
=&\ \mathbb{E}_e\left[\mathbb{E}_e[\tilde{T}_1\,r(S_1,S_0)\mid S_1,S_0]\right]\\
=&\ \mathbb{E}_e\left[\tilde{T}_1\,r(S_1,S_0)\right] && \text{(inverse tower-law)}
\end{aligned}$$

□

Applying Lemma A.1 for $r(S_1, S_0) = g(S_1) - g_0(S_1)$, we have:

$$D_g[m(\theta_0; f_0), g-g_0] = \Pr(e)\,\mathbb{E}_e[\tilde{T}_1\,(g(S_1)-g_0(S_1)] - \Pr(o)\frac{\Pr(e)}{\Pr(o)}\,\mathbb{E}_e[\tilde{T}_1\,(g(S_1)-g_0(S_1)]=0$$

Double robustness. Observe that by re-writing the orthogonal moment equation, and applying the definition of $g_0(S_1)$ and Lemma A.1 we have:

$$\begin{aligned}
\mathbb{E}_e[\tilde{T}_1\tilde{T}_1^\top]\hat{\theta} =&\ \mathbb{E}_e[\hat{g}(S_1)\,\tilde{T}_1] + \frac{\Pr(o)}{\Pr(e)}\mathbb{E}_o\left[(\bar{Y}_1 - \hat{g}(S_1))\hat{q}(S_1,S_0)\right] \\
=&\ \mathbb{E}_e[(\hat{g}(S_1) - g_0(S_1))\,\tilde{T}_1] + \frac{\Pr(o)}{\Pr(e)}\mathbb{E}_o\left[\mathbb{E}\left[\bar{Y}_1 - \hat{g}(S_1)\mid S_1\right]\,\hat{q}(S_1,S_0)\right] && \text{(tower-law and causal graph)} \\
=&\ \mathbb{E}_e[\hat{g}(S_1)\,\tilde{T}_1] + \frac{\Pr(o)}{\Pr(e)}\mathbb{E}_o\left[(g_0(S_1) - \hat{g}(S_1))\,\hat{q}(S_1,S_0)\right] && \text{(definition of } g_0\text{)} \\
=&\ \frac{\Pr(o)}{\Pr(e)}\mathbb{E}_o[\hat{g}(S_1)\,q_0(S_1,S_0)] + \frac{\Pr(o)}{\Pr(e)}\mathbb{E}_o\left[(g_0(S_1) - \hat{g}(S_1))\hat{q}(S_1,S_0)\right] && \text{(Lemma A.1)} \\
=&\ \frac{\Pr(o)}{\Pr(e)}\mathbb{E}_o[g_0(S_1)\,q_0(S_1,S_0)] + \frac{\Pr(o)}{\Pr(e)}\,\mathbb{E}_o\left[(g_0(S_1) - \hat{g}(S_1))(\hat{q}(S_1,S_0) - q_0(S_1,S_0))\right] \\
=&\ \mathbb{E}_e[\tilde{T}_1\,g_0(S_1)] + \frac{\Pr(o)}{\Pr(e)}\,\mathbb{E}_o\left[(g_0(S_1) - \hat{g}(S_1))(\hat{q}(S_1,S_0) - q_0(S_1,S_0))\right] && \text{(Lemma A.1)} \\
=&\ \mathbb{E}_e[\tilde{T}_1\tilde{T}_1^\top]\theta_0 + \mathbb{E}_o\left[(g_0(S_1) - \hat{g}(S_1))(\hat{q}(S_1,S_0) - q_0(S_1,S_0))\right]
\end{aligned}$$

Thus we get the desired property, i.e.:

$$\mathbb{E}_e[\tilde{T}_1\tilde{T}_1^\top]\left(\hat{\theta} - \theta_0\right) = \frac{\Pr(o)}{\Pr(e)}\mathbb{E}_o\left[(g_0(S_1) - \hat{g}(S_1))(\hat{q}(S_1,S_0) - q_0(S_1,S_0))\right] \tag{25}$$

which implies that:

$$\left\|\mathbb{E}_e[\tilde{T}_1\tilde{T}_1^\top]\left(\hat{\theta} - \theta_0\right)\right\|_2 \leq \frac{\Pr(o)}{\Pr(e)}\sqrt{\mathbb{E}_o\left[(g_0(S_1) - \hat{g}(S_1))^2\right]}\cdot\sqrt{\mathbb{E}_o\left[(\hat{q}(S_1,S_0) - q_0(S_1,S_0))^2\right]} \tag{26}$$

A.3. Proof of Theorem 3.3

Employing the general theorem in (Chernozhukov et al., 2018b) and since our estimator is the solution to the empirical analogue of a Neyman orthogonal moment equation:

$$m(\theta; f_0) = \mathbb{E}[\psi(Z;\theta,f_0)] = 0 \tag{27}$$

Thus as long as the nuisance functions f_0 are estimated on a separate sample and they satisfy an MSE rate of $o_p(n^{-1/4})$, then the final parameter estimate is asymptotically normal, i.e. $\sqrt{n}\,(\hat{\theta} - \theta_0) \to_d N(0, V)$ with $V = J^{-1}\Sigma J^{-\top}$, where $J = \nabla_\theta m(\theta_0; f_0)$ and $\Sigma = \mathbb{E}[\psi(Z;\theta_0,f_0)\,\psi(Z;\theta_0,f_0)^\top]$. We can also easily leverage the double robustness property in Theorem 3.2 to further relax the requirement on the nuisance functions g, q, so as to only require that the product of the two mean squared errors is of $o_p(n^{-1/2})$ as opposed to requiring the stronger property that each individual error is of $o_p(n^{-1/4})$.

Asymptotic variance characterization and estimation. We now analyze and simplify the variance term. First observe that in our setting:

$$J :=\ \mathbb{E}[\tilde{T}_1\,\tilde{T}_1^\top 1\{e\}] = \Pr(e)\mathbb{E}_e[\tilde{T}_1\,\tilde{T}_1^\top] =: \Pr(e)\,J_e \tag{28}$$

Moreover, if we let:

$$\begin{aligned}
\psi(Z;\theta,f) :=&\ 1\{e\}\,\psi_e(S_1,T_1,S_0;\theta,g,p,h) + 1\{o\}\,\psi_o(\bar{Y}_1,S_1,S_0;g,q) && (29)\\
:=&\ 1\{e\}\,\psi_e(Z_e;\theta,g,p,h) + 1\{o\}\,\psi_o(Z_o;\theta,g,q) && (30)\\
\psi_e(Z_e;\theta,g,p,h) :=&\ (g(S_1) - h(S_0) - \theta_0^\top\,(\phi(T_1,S_0) - p(S_0)))\,(\phi(T_1,S_0) - p(S_0)) && (31)\\
\psi_o(Z_o;g,q) :=&\ q(S_1,S_0)\,(\bar{Y}_1 - g(S_1)) && (32)
\end{aligned}$$

Then we also have that:

$$\begin{aligned}
\Sigma :=&\ \mathbb{E}[\psi(Z;\theta_0,f_0)\,\psi(Z;\theta_0,f_0)^\top] && (33)\\
=&\ \Pr(e)\,\mathbb{E}_e[\psi_e(Z_e;\theta_0,g_0,p_0,h_0)\,\psi_e(Z_e;\theta_0,g_0,p_0,h_0)^\top] + \Pr(o)\,\mathbb{E}_o[\psi_o(Z_o;g_0,q_0)\,\psi_o(Z_o;g_0,q_0)^\top] && (34)\\
=&\ \Pr(e)^2\,\Sigma_e + \Pr(e)^2\Sigma_o && \text{(definition of } \Sigma_e,\Sigma_o\text{)}
\end{aligned}$$

Thus the variance V can be re-written as desired:

$$V = J^{-1}\Sigma J^{-1} = \frac{1}{\Pr(e)^2} J_e^{-1}\Sigma J_e^{-1} = J_e^{-1}\left(\Sigma_e + \Sigma_o\right) J_e^{-1} \tag{35}$$

A.4. Proof of Theorem 3.4

Employing the general theorem in (Chernozhukov et al., 2018b) and since our estimator is the solution to the empirical analogue of a Neyman orthogonal moment equation:

$$m(\theta; f_0) = \mathbb{E}[\psi(Z; \theta, f_0)] = 0 \tag{36}$$

Neyman orthogonality of the moments that estimate the dynamic effects $\theta_{t,\tau}$, with respect to the nuisance functions $h_{t,\tau}, p_{t,\tau}$ is established in (Lewis & Syrgkanis, 2020), while orthogonality with respect to g, q, p, h can be established in a manner identical to the proof in Theorem 3.2.

Thus as long as the nuisance functions f_0 are estimated on a separate sample and they satisfy an MSE rate of $o_p(n^{-1/4})$, then the final parameter estimate is asymptotically normal, i.e. $\sqrt{n}\left(\hat{\theta} - \theta_0\right) \to_d N(0, V)$ with

$$V := \left[J^{-1}\Sigma J^{-\top}\right]_{1:k,1:k} \qquad J := \nabla_\theta m(\theta_*; f_0) \qquad \Sigma := \mathbb{E}[\psi(Z; \theta_*, f_0)\, \psi(Z; \theta_*, f_0)^\top] \tag{37}$$

where:

$$\psi(Z; \theta_*, f_0) := \left[\psi_0(Z; \theta_*, f_0);\quad \psi_2(Z; \theta_*, f_0);\quad \psi_3(Z; \theta_*, f_0);\quad \ldots;\quad \psi_M(Z; \theta_*, f_0)\right]^\top \tag{38}$$

and $[A]_{1:k,1:k}$ is the $k \times k$ top left diagonal block of matrix A and k is the number of treatments and with θ_* we denote the true value of the whole vector of parameters $\theta_0, \{\theta_{t,\tau}\}_{2 \le t \le \tau \le M}$. We can also easily leverage the double robustness property in Theorem 3.2 to further relax the requirement on the nuisance functions g, q, so as to only require that the product of the two mean squared errors is of $o_p(n^{-1/2})$ as opposed to requiring the stronger property that each error is of $o_p(n^{-1/4})$.

Asymptotic variance characterization and estimation. We further analyze and decompose the variance V above. In particular, observe that:

$$\begin{aligned}
J =& -\begin{bmatrix} A \Pr(e) & B \Pr(o) \\ 0 & C \Pr(o) \end{bmatrix} \\
A =& \mathbb{E}_e[\tilde{T}_1 \tilde{T}_1^\top] \\
B =& \begin{bmatrix} B_2 & \ldots & B_M \end{bmatrix} \\
B_t =& \begin{bmatrix} \mathbb{E}_o[q_0(S_1, S_0)\, T_t^\top] & \ldots & \mathbb{E}_o[q_0(S_1, S_0)\, T_2^\top] \end{bmatrix}
\end{aligned}$$

And:

$$C = \begin{bmatrix} C_2 & 0 & \ldots & 0 \\ 0 & C_3 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & C_M \end{bmatrix} \qquad C_t = \begin{bmatrix} \mathbb{E}_o[\tilde{T}_{t,t}\, \tilde{T}_{t,t}^\top] & 0 & \ldots & 0 \\ \mathbb{E}_o[\tilde{T}_{t,t-1}\, \tilde{T}_{t-1,t-1}^\top] & \mathbb{E}_o[\tilde{T}_{t-1,t-1}\, \tilde{T}_{t-1,t-1}^\top] & \ldots & 0 \\ \ldots & \ldots & \ldots & 0 \\ \mathbb{E}_o[\tilde{T}_{t,2}\, \tilde{T}_{2,2}^\top] & \mathbb{E}_o[\tilde{T}_{t-1,2}\, \tilde{T}_{2,2}^\top] & \ldots & \mathbb{E}_o[\tilde{T}_{2,2}\, \tilde{T}_{2,2}] \end{bmatrix}$$

Observe that we can write:

$$J = -\Pr(e)\begin{bmatrix} A & B\frac{\Pr(o)}{\Pr(e)} \\ 0 & C\frac{\Pr(o)}{\Pr(e)} \end{bmatrix} \qquad J^{-1} = -\frac{1}{\Pr(e)}\begin{bmatrix} A^{-1} & -A^{-1}BC^{-1} \\ 0 & C^{-1}\frac{\Pr(e)}{\Pr(o)} \end{bmatrix} \tag{39}$$

Moreover, we can write:

$$\Sigma = \Pr(e)\begin{bmatrix} \mathbb{E}_e[\psi_{0,e}(Z; \theta_*, f_0)\, \psi_{0,e}(Z; \theta_*, f_0)^\top] & 0 \\ 0 & 0 \end{bmatrix} + \Pr(o)\mathbb{E}_o[\psi(Z; \theta_*, f_0)\, \psi(Z; \theta_*, f_0)^\top] \tag{40}$$

Leading to:

$$V = V_e + V_o \tag{41}$$

where:

$$[V_e]_{1:k,1:k} = \frac{1}{\Pr(e)} A^{-1} \mathbb{E}_e[\psi_{0,e}(Z;\theta_*, f_0)\, \psi_{0,e}(Z;\theta_*, f_0)^\top] A^{-1} \tag{42}$$

and

$$V_o = \frac{\Pr(o)}{\Pr(e)^2} \begin{bmatrix} A & B\frac{\Pr(o)}{\Pr(e)} \\ 0 & C\frac{\Pr(o)}{\Pr(e)} \end{bmatrix}^{-1} \mathbb{E}_o[\psi(Z;\theta_*, f_0)\, \psi(Z;\theta_*, f_0)^\top] \begin{bmatrix} A & B\frac{\Pr(o)}{\Pr(e)} \\ 0 & C\frac{\Pr(o)}{\Pr(e)} \end{bmatrix}^{-\top} \tag{43}$$

$$= \frac{\Pr(o)}{\Pr(e)^2} \begin{bmatrix} A^{-1} & -A^{-1}BC_{-1} \\ 0 & C^{-1}\frac{\Pr(e)}{\Pr(o)} \end{bmatrix} \mathbb{E}_o\left[\psi(Z;\theta_*, f_0)\, \psi(Z;\theta_*, f_0)^\top\right] \begin{bmatrix} A^{-1} & 0 \\ -C^{-\top}B^\top A^{-\top} & C^{-1}\frac{\Pr(e)}{\Pr(o)} \end{bmatrix} \tag{44}$$

The V_e part of the variance is the variance if one ignores the uncertainty in the surrogate index model and simply estimates uncertainty as if the surrogate index was the target outcome. So this is the uncertainty in estimating the causal effect of the treatment on the surrogate index. The variance V_o is the influence of the uncertainty of estimating the surrogate index on the final treatment effect.

We can further expand and simplify the variance V_o in particular, we can write the top left $k \times k$ diagonal block in the following simplified form:

$$[V_o]_{1:k,1:k} = \frac{\Pr(o)}{\Pr(e)^2} A^{-1} \left(\mathbb{E}_o[\Psi_{0,o}\Psi_{0,o}^\top] - \sum_{t=2}^{M} B_t C_t^{-1} \left(\mathbb{E}_o[\Psi_t \Psi_{0,o}^\top] - \sum_{t'=2}^{M} \mathbb{E}_o[\Psi_t \Psi_{t'}^\top] C_{t'}^{-\top} B_{t'}^\top \right) \right) A^{-1} \tag{45}$$

which nicely also decomposes into the part that we had without the dynamic effect estimation and the extra part that stems from the estimation of the dynamic effects.

B. A Step-by-Step Algorithmic Description

In this section we describe in a step-by-step algorithmic manner the exact data analysis pipeline we describe in the main text.

We assume access to two data-sets:

1. a long-term historical data set that contains n_o samples of tuples $(X_{i,t}, T_{i,t}, S_{i,t}, Y_{i,t})$, that for each sample i consists of M periods
2. a short-term historical data set that contains samples n_e samples of triplets $(S_{i,0}, X_{i,1}, T_{i,1}^{\text{new}}, S_{i,1})$.

The treatments $T_{i,t}$ in the long-term dataset are any potential historical investments not necessarily overlapping with new treatments.

We then perform the following steps:

1. On the long-term data-set:
   (a) Estimate dynamic effects, $\theta_{t,\tau}$ for each $2 \le t \le \tau \le M$ of the dynamic effect of treatment at time t on outcome at time τ, via double/debiased dynamic treatment effect estimation method outlined in (Lewis & Syrgkanis, 2020), applied to each target period separately.
   (b) For each sample i and for each $t = \{1, \dots, M\}$, calculate dynamically adjusted cumulative outcome:

$$Y_{i,t}^{\text{adj}} = Y_{i,t} - \sum_{\tau=2}^{t} \theta_{\tau,t}^\top T_{i,\tau} \tag{46}$$

   (c) For each sample i, calculate dynamically adjusted cumulative long-term outcome:

$$\bar{Y}_{i,1}^{\text{adj}} := \sum_{t=1}^{M} Y_{i,t}^{\text{adj}} \tag{47}$$

(d) Using data $(X_{i,1}, S_{i,1}, \bar{Y}_{i,1}^{\text{adj}})$, for each sample i: estimate the model of the dynamically adjusted surrogate index $\hat{g}^{\text{adj}}$, by training a model to predict $\bar{Y}_{i,1}^{\text{adj}}$ from $S_{i,1}, X_{i,1}$. Equivalently, learn a model of

$$g_0^{\text{adj}}(S_{i,1}, X_{i,1}) := \mathbb{E}[\bar{Y}_{i,1}^{\text{adj}} \mid S_{i,1}, X_{i,1}]. \tag{48}$$

In our experiments, we estimated this model via Lasso regression. We used k-fold cross-validation to choose the weight on the Lasso penalty.

2. On the short-term data-set:

(a) For each sample i, impute a surrogate index:

$$I_i = \hat{g}^{\text{adj}}(S_{i,1}, X_{i,1}) \tag{49}$$

(b) Estimate the causal effect θ^{new} of $T_{i,t}^{\text{new}}) \in \mathbb{R}^{k'}$ of the baseline investments on $I_{i,t}$, controlling for $X_{i,t}, S_{i,t-1}$, via Double Machine Learning (Chernozhukov et al., 2018a):

i. Split the samples in half.

ii. On the first half, estimate a model $\hat{p}$ that predicts $T_{i,t}^{\text{new}}$ from $X_{i,t}, S_{i,t-1}$, i.e. learns the conditional expectation $p_0(X_{i,t}, S_{i,t-1}) := \mathbb{E}[T_{i,t}^{\text{new}} \mid X_{i,t}, S_{i,t-1}]$. In our analysis we used Lasso regression for this prediction task, with a cross-validated penalty weight.

iii. On the first half, estimate a model $\hat{q}$ that predicts $I_{i,t}$ from $X_{i,t}, S_{i,t-1}$, i.e. learns the conditional expectation $h_0(X_{i,t}, S_{i,t-1}) := \mathbb{E}[I_{i,t} \mid X_{i,t}, S_{i,t-1}]$. In our analysis we used Lasso regression for this prediction task, with a cross-validated penalty weight.

iv. For every period and sample on the other half of customers, calculate surprise in treatment and surprise in surrogate index:

$$T_{i,t}^{\text{new,res}} = T_{i,t}^{\text{new}} - \hat{p}(X_{i,t}, S_{i,t-1}) \tag{50}$$

$$I_{i,t}^{\text{res}} = I_{i,t} - \hat{h}(X_{i,t}, S_{i,t-1}) \tag{51}$$

v. Reverse the role of the two samples and repeat the last three steps, to calculate residuals for the other half of the samples.

vi. Using the treatment and surrogate index residuals from all samples, run a Linear Regression, regressing $I_{i,t}^{\text{res}}$ on $T_{i,t}^{\text{new,res}}$, to learn a vector of coefficients $\hat{\theta}^{\text{new}}$.

vii. Estimate the co-variance matrix $V \in \mathbb{R}^{k'} \times \mathbb{R}^{k'}$ of the estimate $\hat{\theta}^{\text{new}}$ using heteroskedasticity robust co-variance estimation for an ordinary least squares regression.

(c) Construct 99% confidence intervals using asymptotic normal approximation of the distribution of estimate $\hat{\theta}_j^{\text{new}}$, with standard error $\sigma_j^2 := V_{j,j}/n_e$, i.e.:

$$\theta_{0,j} \in \hat{\theta}_j^{\text{new}} \pm 2.577 \cdot \sigma_j \tag{52}$$

Debiasing correction. To account for the uncertainty in the surrogate index and achieve asymptotically valid intervals on the true dynamic effect, we need to add a few further steps. In particular:

1. Split all samples in half (both in long-term and short-term dataset)

2. Train a classification model $\hat{t}$ that predicts the population $\{o, e\}$, based on S_1, X_1, S_0, i.e.:

$$t_0(S_1, X_1, S_0) = \Pr(e \mid S_1, X_1, S_0) \tag{53}$$

In our experiments we used a penalized logistic regression with a cross-validated penalty weight.

3. Train a regression model $\hat{r}$ that predicts $\tilde{T}_{i,t}^{\text{new,res}}$ from S_1, X_1, S_0, i.e.:

$$r_0(S_1, X_1, S_0) = \mathbb{E}[\tilde{T}_{i,t}^{\text{new,res}} \mid S_1, X_1, S_0] \tag{54}$$

In our experiments we used a lasso regression with a cross-validated penalty weight.

4. Evaluate these models on the other half to construct a surrogate score:

$$Q_{i,t} = \frac{\hat{t}(S_{i,1}, X_{i,1}, S_{i,0})}{1 - \hat{t}(S_{i,1}, X_{i,1}, S_{i,0})} \hat{r}(S_{i,1}, X_{i,1}, S_{i,0}) \tag{55}$$

5. Reverse the roles of the split-samples and repeat last three steps.

6. Add a de-biasing correction to the estimate $\hat{\theta}^{\text{new}}$:

$$\hat{\theta}^{\text{new},deb} = \hat{\theta}^{\text{new}} + \frac{1}{n_o} \sum_{i \in o} \left(\bar{Y}_{i,1}^{\text{adj}} - \hat{g}^{\text{adj}}(S_{i,1}, X_{i,1}) \right) Q_{i,t} \tag{56}$$

7. Add variance correction to the computed variance V:

$$V^{deb} = \frac{n}{n_e} V + J_{e,n}^{-1} \Sigma_{o,n} J_{e,n}^{-1} \tag{57}$$

where $J_{e,n}$ and $\Sigma_{o,n}$ are empirical analogues of the quantities J_e and Σ_o of Theorem 3.4.

8. Construct 99% confidence intervals using asymptotic normal approximation of the distribution of estimate $\hat{\theta}_j^{\text{new}}$, with variance $\sigma_{j,deb}^2 := V_{j,j}^{deb}/n = V/n_e + J_{e,n}^{-1}\Sigma_{o,n}J_{e,n}^{-1}/n = \sigma_j^2 + J_{e,n}^{-1}\Sigma_{o,n}J_{e,n}^{-1}/n$, i.e.:

$$\theta_{0,j} \in \hat{\theta}_j^{\text{new},deb} \pm 2.577 \cdot \sigma_{j,deb} \tag{58}$$

C. Description of Data Generating Processes

C.1. Synthetic Data

For fully synthetic data we simply generate data based on the linear data generating process presented in Equation (15), with Gaussian exogenous shocks and randomly initialized parameter matrices.

C.2. Semi-Synthetic Data

As mentioned in Section 4, we generate the semi-synthetic data by leveraging the correlation matrix and some pre-trained models from a real world dataset. In this section, we describe in details how we simulate the dataset step by step.

The real world dataset contains approximately 10k customers. For each customer we collect a time series of their monthly investments, proxies, and revenue trajectory, along with a set of fixed customer characteristics. We extract meaningful information from the real data and then simulate a new synthetic data as follows:

Generate data for initial period. From the real world dataset we filter one month data on some period t and derive the normalized covariance matrix. In order to not expose the real correlation among variables, we decompose this matrix and recreate eigenvalues and eigenvectors ourselves. For eigenvalues, we keep the top 4 eigenvalues and fit a discontinuous linear regression on the true eigenvalues curve. For eigenvectors, we keep the corresponding 4 vectors and impute random remaining vectors. We then combine these new eigenvalues and eigenvectors to create a new covariance matrix. From this perturbed covariance matrix we draw a sample on a multi-variate gaussian distribution, which we use as the data for the initial period t_0 for each customer.

Learn intertemporal auto and cross-correlations. We first train linear models (e.g. LassoCV) on each proxy and investment to predict each of these outcomes using 6 lagged periods of each investment, 6 lagged periods of each proxy, and a set of time-invariant demographics. For both proxies and investments, we find a large amounts of auto-correlation and small cross-correlation effects, along with some effects from customer characteristics including customer size and level of engagement. Based on these insights, we then draw new coefficients for each model, with decaying trend on lagged variables in different scale, and randomly draw coefficients for demographics.

Simulate the residual distribution corresponds to each model. From the pre-trained models, we see how unobserved heterogeneity behaves on each investment and proxy. we find that majority of our residuals follows as a normal distribution, however, we have some unpredictable large outliers on both sides. In order to capture those surprising behaviours, we fit a mixture of different models. For a residual distribution from a single model, we fit mixture gaussian model (n_component=2) on the 5% and 95% band of the residuals and for the two tails, we fit log normal distributions.

Build the panel dataset in a feed-forward manner. With the initial data set, proxy and investment parametric model coefficients and the residuals, we simulate the data following the equations below:

$$T_{i,t} = \sum_{j=1}^{6} \kappa_j T_{i,t-j} + \sum_{j=1}^{6} \alpha_j S_{i,t-j} + \lambda D_{i,t} + \eta_{i,t} \tag{59}$$

$$S_{i,t} = \theta T_{i,t} + \sum_{j=1}^{6} \gamma_j S_{i,t-j} + \beta D_{i,t} + \epsilon_{i,t} \tag{60}$$

where T represents treatment, S represents surrogates(proxies), and D represents demographics. First of all, we use the investment model to predict the current period investment, and add random residuals drawing from our fitted residual mixture distribution. Then we use the proxy model to predict current period proxy from all the controls, and add residuals as well. Other than that, we also add the effect of current period investment to the proxy. Moving to the next period, all the predictive outcomes and controls in the current period will become controls for next period, we could then repeat the process mentioned above again to get the next period prediction. Figure 1 also shows how this forward-feeding iteration works. After repeating this process m times, we have a semi-synthetic panel data ready to run experiment.

D. Coverage Results of Asymptotic Normal Based Intervals

D.1. Synthetic Data

	$n_t =$	2000	5000	10000
$n_t =$	2	96	93.5	96
	4	79	76	86
	8	53.3	57.5	62

(a) Dynamic Adjusted Surrogate

	$n_t =$	2000	5000	10000
$n_t =$	2	96.5	93	97
	4	84.5	83.5	87
	8	75	71.5	81

(b) De-biased Dynamic Adjusted Surrogate

Figure 3. Synthetic data, high-dimensional, lasso models. Coverage levels averaged across $n_{exp} = 100$ experiments and across the $k = 2$ treatments. n is number of samples and n_t number of periods of long-term outcome. Target coverage level is 99%.

	$n_t =$	2000	5000	10000
$n_t =$	2	98.5	99	98.5
	4	89	91	89.5
	8	80	75	80

(a) Dynamic Adjusted Surrogate

	$n_t =$	2000	5000	10000
$n_t =$	2	98.5	99	98.5
	4	95.5	94	93
	8	90.5	90.5	91

(b) De-biased Dynamic Adjusted Surrogate

Figure 4. Synthetic data, low-dimensional, linear regression models. Coverage levels averaged across $n_{exp} = 100$ experiments and across the $k = 2$ treatments. n is number of samples and n_t number of periods of long-term outcome. Target coverage level is 99%.

D.2. Semi-Synthetic Data

	$n_t =$	2000	5000	10000
$n_t =$	2	53	42	34
	4	49	42	36
	8	42	26	

(a) Dynamic Adjusted Surrogate

	$n_t =$	2000	5000	10000
$n_t =$	2	65	60	52
	4	76	71	70
	8	87	84	

(b) De-biased Dynamic Adjusted Surrogate

Figure 5. Semi-Synthetic data, lasso models. Coverage levels averaged across $n_{exp} = 100$ experiments and across the $k = 3$ treatments. n is number of samples and n_t number of periods of long-term outcome. Target coverage level is 99%.

E. MSE Results on Further Synthetic and Semi-Synthetic Data

We present results on the ℓ_2 error of the recovered coefficients, for synthetic and semi-synthetic data. The algorithms that whose performance we present is as follows:

1. total: estimating the effect with hypothetical access to the long-term outcome and no dynamic adjustment

2. total: estimating the effect with hypothetical access to the long-term outcome and no dynamic adjustment, but first projecting to the surrogates and then estimating the effect on the surrogate index.

3. adj. total: estimating the effect with hypothetical access to the long-term outcome, applying dynamic adjustment

4. adj. surrogate: estimating the effect with hypothetical access to the long-term outcome, applying dynamic adjustment, after first projected the adjusted outcome on the surrogates and then estimating the effect on the dynamically adjusted surrogate index

5. new treat: our dynamically adjusted surrogate index algorithm that uses a separate long-term dataset to estimate the surrogate index and then applying it to the short term dataset.

6. deb new treat: the debiased (fully orthogonal) version of our dynamically adjusted surrogate index algorithm, based on orthogonal score.

E.1. Synthetic Data

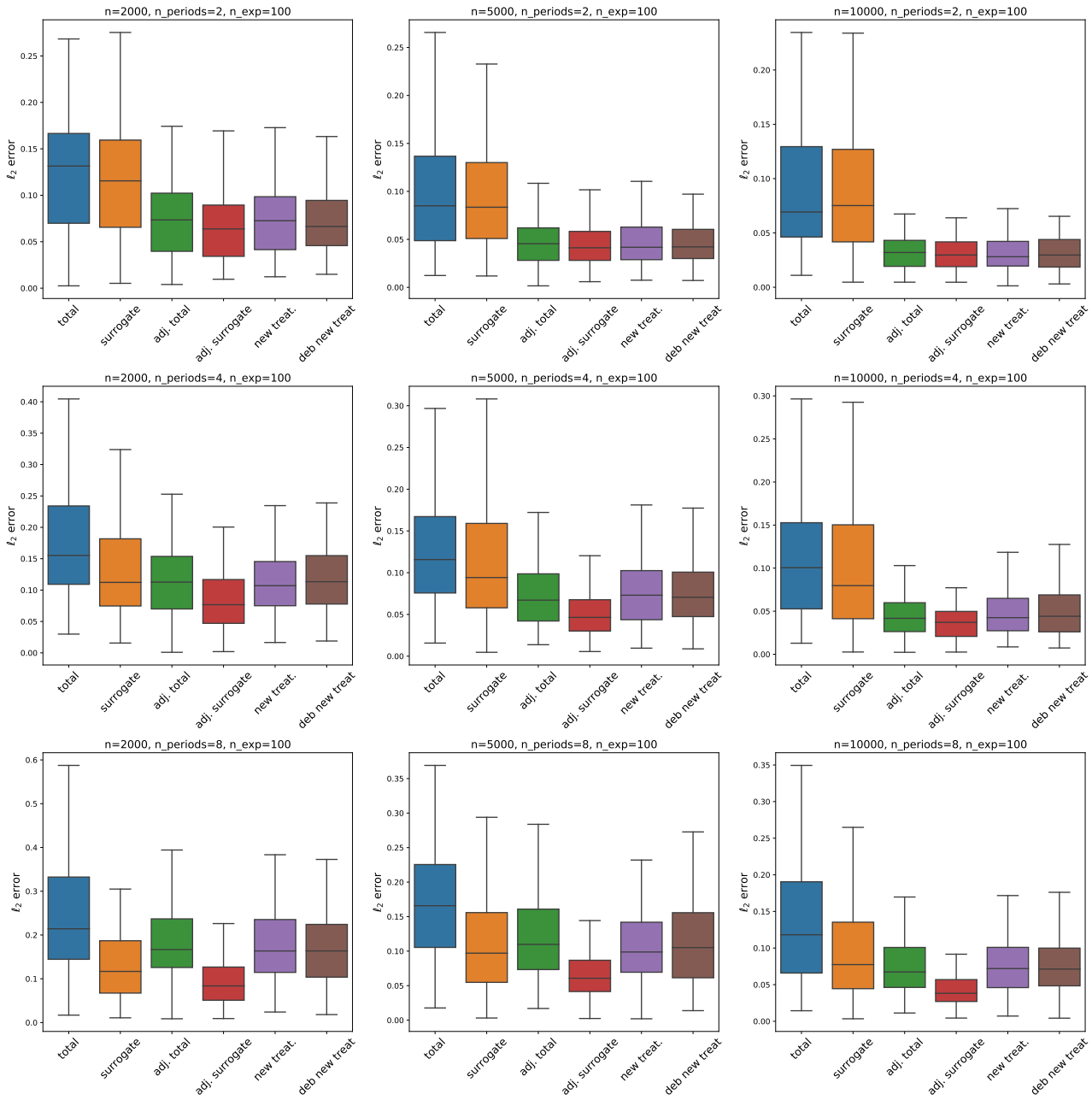

Figure 6. Synthetic data, high-dimensional, lasso models. n is number of samples and $n_{periods}$ number of periods of long-term outcome. ℓ_2 error of recovered coefficients.

E.2. Semi-Synthetic Data

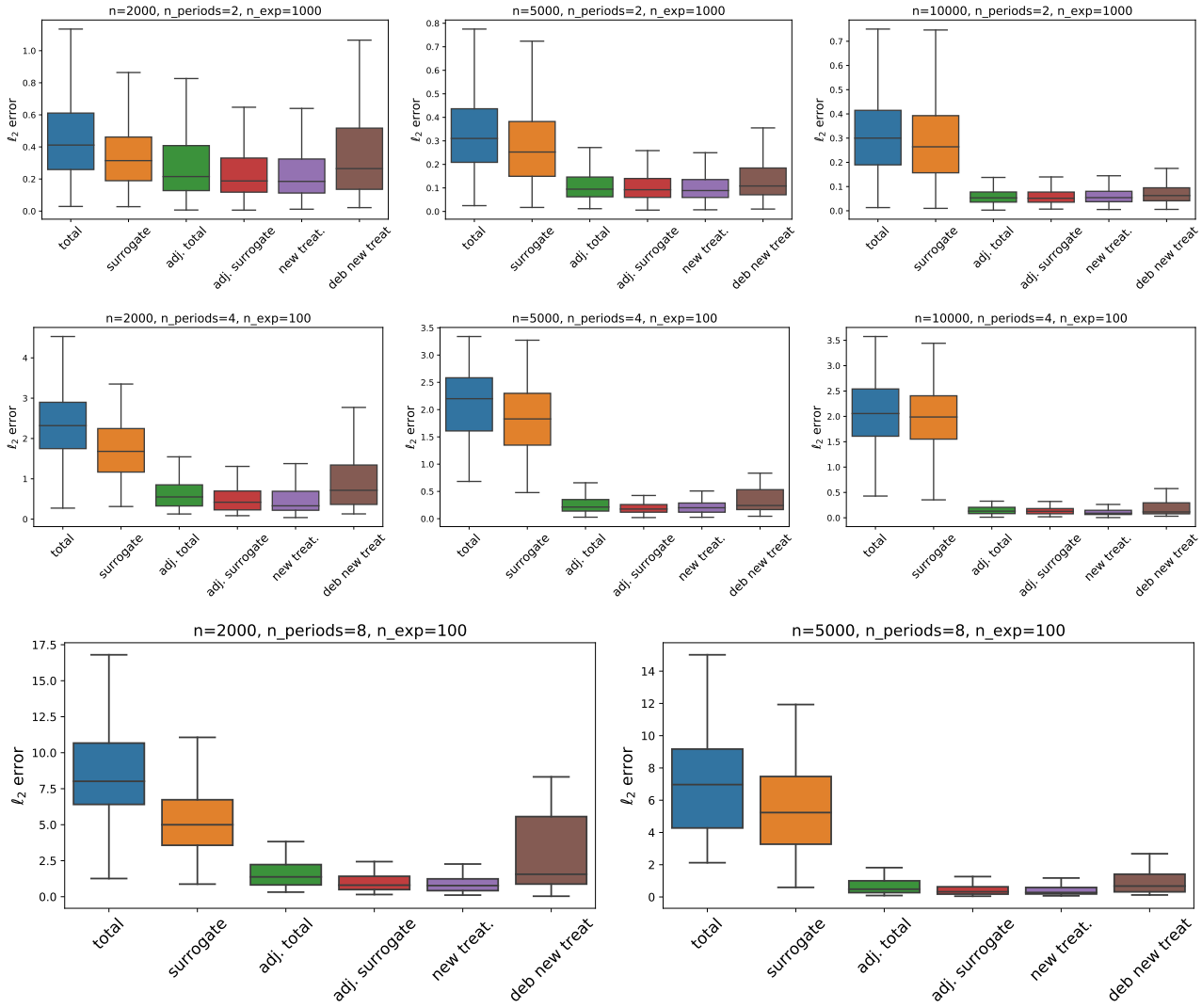

Figure 7. Semi-Synthetic data, high-dimensional, lasso models. n is number of samples and $n_{periods}$ number of periods of long-term outcome. ℓ_2 error of recovered coefficients.