$$U = l^{\alpha} c^{\beta}$$

$$pc = wh + V$$

$$T = h + l$$

$$\frac{\partial U}{\partial l} = \alpha l^{(\alpha-1)} c^{\beta} = MU_l$$

$$\frac{\partial U}{\partial c} = \beta l^{\alpha} c^{(\beta-1)} = MU_c$$

$$\rightarrow \frac{MU_l}{MU_c} = \frac{\alpha c}{\beta l}$$

α **elasticity of utility w.r.t leisure;**

β **elasticity of utility w.r.t consumption;**

RW is the value of the slope of IC at the endowment point

$$\rightarrow RW = \left| \frac{MU_l}{MU_c} \right|_{l=T, c=\frac{V}{p}} = \frac{\alpha c}{\beta l}_{\, l=T, c=\frac{V}{p}} = \frac{\alpha V}{\beta p T}$$

$$\ln(RW) = \ln \alpha + \ln V - \ln \beta - \ln p - \ln T$$

$$\frac{\partial RW}{\partial \alpha} > 0; \frac{\partial RW}{\partial \beta} < 0; \frac{\partial RW}{\partial T} < 0; \frac{\partial RW}{\partial V} > 0; \frac{\partial RW}{\partial p} < 0$$

Decision Function/Participation Condition

$$If \;\; RW < w \;\; then \;\; work \;\; i.e. \;\; h^{*} > 0$$

$$\frac{\alpha V}{\beta p T} < w \rightarrow V < \frac{w \beta p T}{w}$$

Comparative Static # of Kids: n

$$\rightarrow RW = \frac{\alpha(n)V(n)}{\beta(n)\,pT(n)}$$

$$\frac{\partial RW}{\partial n} = ?$$

Before Kid (s)

$$If \;\; RW = w \; then \; indifferent \; to \; work \; versuss \; no \; wor$$

After Kid (s)

$$RW > w? \;\; OR \;\; RW < w$$

$$T = h + l \qquad U = l^{\alpha} c^{\beta} \qquad \frac{\partial U}{\partial l} = \alpha l^{(\alpha-1)} c^{\beta} = MU_l$$

$$pc = wh + V \qquad \frac{\partial U}{\partial c} = \beta l^{\alpha} c^{(\beta-1)} = MU_c$$

$$\Gamma = U + \lambda[w(T - l) + V - pc]$$

$$\left.\begin{aligned} \frac{\partial \Gamma}{\partial l} &= MU_l - \lambda w = 0 \ldots.(1) \rightarrow pMU_l = \lambda w \\ \frac{\partial \Gamma}{\partial c} &= MU_c - \lambda p = 0 \ldots.(2) \rightarrow MU_c = \lambda p \end{aligned}\right\} \rightarrow \frac{\alpha c^{*}}{\beta l^{*}} = \frac{w}{p} \ldots\ldots\ldots\ldots(4)$$

$$\frac{\partial \Gamma}{\partial \lambda} = 0 \Rightarrow w(T - l^{*}) + V - pc^{*} = 0 \ldots\ldots\ldots(3)$$

l* will give Leisure Demand Function. Using 3 and 4

$$\left.\begin{aligned} pc^{*} &= w(T - l^{*}) + V \\ c^{*} &= \frac{\beta w}{\alpha p} l^{*} \end{aligned}\right\} p \frac{\beta w}{\alpha p} l^{*} = w(T - l^{*}) + V = wT + V - wl^{*}$$

$$l^{*} = \frac{(wT + V)}{w\left(1 + \frac{\beta}{\alpha}\right)} = \frac{T}{\left(1 + \frac{\beta}{\alpha}\right)} + \frac{V}{w\left(1 + \frac{\beta}{\alpha}\right)}$$

$$l^* = \frac{(wT+V)}{w\left(1+\frac{\beta}{\alpha}\right)} = \frac{T}{\left(1+\frac{\beta}{\alpha}\right)} + \frac{V}{w\left(1+\frac{\beta}{\alpha}\right)}$$

$$c^* = \frac{\beta w}{\alpha p}\left[\left(\frac{T}{1+\frac{\beta}{\alpha}}\right) + \frac{V}{w\left(1+\frac{\beta}{\alpha}\right)}\right]$$

$$U^* = l^{*\alpha} c^{*\beta}$$

h* = T-l* will give Labor Supply Function.

$$h^* = T - l^* = T - \frac{(wT+V)}{w\left(1+\frac{\beta}{\alpha}\right)} = T - \frac{T}{\left(1+\frac{\beta}{\alpha}\right)} - \frac{V}{w\left(1+\frac{\beta}{\alpha}\right)}$$

$$\frac{\partial h^*}{\partial w} = \frac{V}{w^2\left(1+\frac{\beta}{\alpha}\right)} > 0$$

$$\varepsilon^S = \frac{w}{h^*}\frac{\partial h^*}{\partial w} = \frac{w}{h^*} \times \frac{V}{w^2\left(1+\frac{\beta}{\alpha}\right)} > 0$$

Practice Problems: Find l*,h*, U*, Elasticity of Labor supply for the following utility functions

1] $\underset{\{l,c\}}{Max}\ U = f(l,c) = l^{\alpha} + c$

2] $\underset{\{l,c\}}{Max}\ U = f(l-\bar{l}, c-\bar{c}) = \left(l-\bar{l}\right)^{\alpha} + \left(c-\bar{c}\right)$

3] $\underset{\{l,c\}}{Max}\ U = f(l,c) = \alpha l + \beta c$

4] $\underset{\{l,c\}}{Max}\ U = f(l,c) = Min\left\{\alpha l, \beta c\right\}$

The constraints are the same in each case of utility function

$$s.t\ \ pc = wh + V$$

$$s.t.\ T = l + h$$

$$T = h + l$$
$$pc = wh + V$$

$$U = (l)^{\alpha} + (c)$$

$$\frac{\partial U}{\partial l} = \alpha (l)^{(\alpha-1)} = MU_l$$

$$\frac{\partial U}{\partial c} = 1 = MU_c$$

$$\Gamma = U + \lambda[w(T-l) + V - pc]$$

$$\left.\begin{aligned} \frac{\partial \Gamma}{\partial l} &= MU_l - \lambda w = 0....(1) \;\rightarrow\; pMU_l = \lambda w \\ \frac{\partial \Gamma}{\partial c} &= MU_c - \lambda p = 0....(2) \;\rightarrow\; MU_c = \lambda p \end{aligned}\right\} \rightarrow \alpha\left(l^*\right)^{\alpha-1} = \frac{w}{p} \quad ....(4)$$

$$\frac{\partial \Gamma}{\partial \lambda} = 0 \Rightarrow w(T - l^*) + V - pc^* = 0........(3)$$

l* will give Leisure Demand Function. Use 4. Use 3 to find out c*

$$l^* = \left(\frac{w}{p\alpha}\right)^{\frac{1}{\alpha-1}}$$

$$h^* = T - \left(\frac{w}{p\alpha}\right)^{\frac{1}{\alpha-1}}$$

$$\frac{\partial h^*}{\partial w} = -\left(\frac{1}{\alpha-1}\right) w^{\frac{1}{\alpha-1}-1} \left(\frac{1}{p\alpha}\right)^{\frac{1}{\alpha-1}} > 0 \;\; if \;\; \alpha < 1$$

$$T = h + l$$
$$pc = wh + V$$

$$U = \left(l - \bar{l}\right)^{\alpha} + \left(c - \bar{c}\right)$$

$$\frac{\partial U}{\partial l} = \alpha\left(l - \bar{l}\right)^{(\alpha-1)} = MU_l$$

$$\frac{\partial U}{\partial c} = 1 = MU_c$$

$$\Gamma = U + \lambda[w(T - l) + V - pc]$$

$$\frac{\partial \Gamma}{\partial l} = MU_l - \lambda w = 0....(1) \rightarrow pMU_l = \lambda w$$

$$\frac{\partial \Gamma}{\partial c} = MU_c - \lambda p = 0....(2) \rightarrow MU_c = \lambda p$$

$$\rightarrow \alpha\left(l^* - \bar{l}\right)^{\alpha-1} = \frac{w}{p} \text{...........}(4)$$

$$\frac{\partial \Gamma}{\partial \lambda} = 0 \Rightarrow w(T - l^*) + V - pc^* = 0........(3)$$

l* will give Leisure Demand Function. Use 4. Use 3 to find out c*

$$l^* = \left(\frac{w}{p\alpha}\right)^{\frac{1}{\alpha-1}} + \bar{l}$$

$$h^* = T - \left(\frac{w}{p\alpha}\right)^{\frac{1}{\alpha-1}} - \bar{l}$$

$$\frac{\partial h^*}{\partial w} = -\left(\frac{1}{\alpha-1}\right) w^{\frac{1}{\alpha-1}-1}\left(\frac{1}{p\alpha}\right)^{\frac{1}{\alpha-1}} > 0 \;\; if \;\; \alpha < 1$$

$$T = h + l$$
$$pc = wh + V$$

$$U = f(l, c) = \alpha l + \beta c$$

$$\frac{\partial U}{\partial l} = \alpha = MU_l$$

$$\frac{\partial U}{\partial c} = \beta = MU_c$$

$$l^* = T \ \ if \ \ w < p$$
$$\rightarrow h^* = 0$$

$$l^* = 0 \ \ if \ \ w > p$$
$$\rightarrow h^* = T$$

$$T = h + l$$
$$pc = wh + V$$

$$U = f(l, c) = Min\{\alpha l, \beta c\}$$

$$\lambda l^* = c^*$$

$$\lambda l^* = wh + V = w(T - l^*) + V$$

$$\lambda l^* = -wl^* + (wT + V)$$

$$(\lambda + w)l^* = (wT + V)$$

$$\ln l^* = \ln(wT + V) - \ln(\lambda + w)$$

$$\frac{1}{l^*}\frac{\partial l^*}{\partial w} = \frac{T}{(wT + V)} - \frac{1}{(\lambda + w)}$$

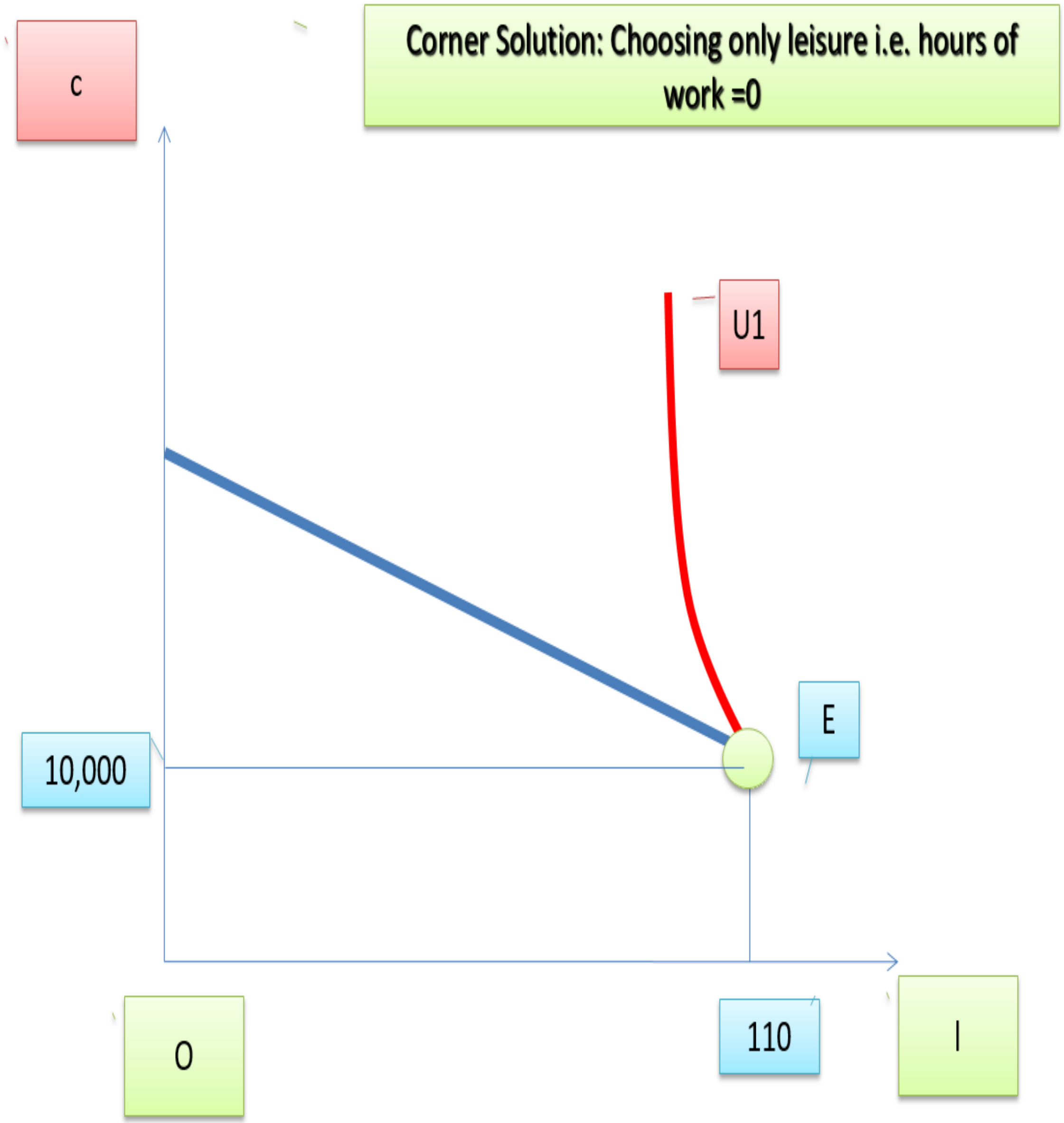
$$l^* = \frac{(wT + V)}{(\lambda + w)}$$

$$\frac{1}{l^*}\frac{\partial l^*}{\partial w} = \frac{\lambda T + w - wT - V}{(wT + V)(\lambda + w)}$$

$$h^* = T - \frac{(wT + V)}{(\lambda + w)}$$

$$\frac{\partial l^*}{\partial w} < 0 \rightarrow \quad \frac{\partial h^*}{\partial w} > 0 \ \ if \ \ w > \frac{\lambda T - V}{(T - 1)}$$

Graphical and Parametric : Corner Solution: Little More Detail



$$w \leq RW$$

OR

$$|MRS| > |\text{Re}\,al \quad Wage|$$

$$Or, \frac{MU_l}{w} > \frac{MU_c}{p}$$

All Play No Work

$$l^* = T ....(3)$$

$$h^* = T - l^* = 0..(4)$$

$$c^* = V ....(5)$$

$$U^* = (l^*)^\alpha \times (c^*)^\beta$$

$$U^* = T^\alpha V^\beta ....(6)$$