Econ 200 Fall 2015 Final Answers.pdf

Final Answers.pdf

Formula Sheet.pdf

Midterm Answer Key.pdf

Midterm Practice Problems.pdf

Practice Final Answers.pdf

PS1 Solutions.pdf

PS2 Solutions.pdf

PS3 Solutions.pdf

PS4 Solutions.pdf

Final Exam

Econ 200

1. Business is good at Acme Products. George, the owner, estimates that he can earn an additional 12 per year indefinitely starting next year if he invests in expansion now, George is risk-neutral: he is not averse to risk, but neither does he seek it. The investment will be sunk once George makes the decision.

(a) (4 points) If George can finance at rate $k = 0.10$, what is his maximum willingness to pay for that investment?

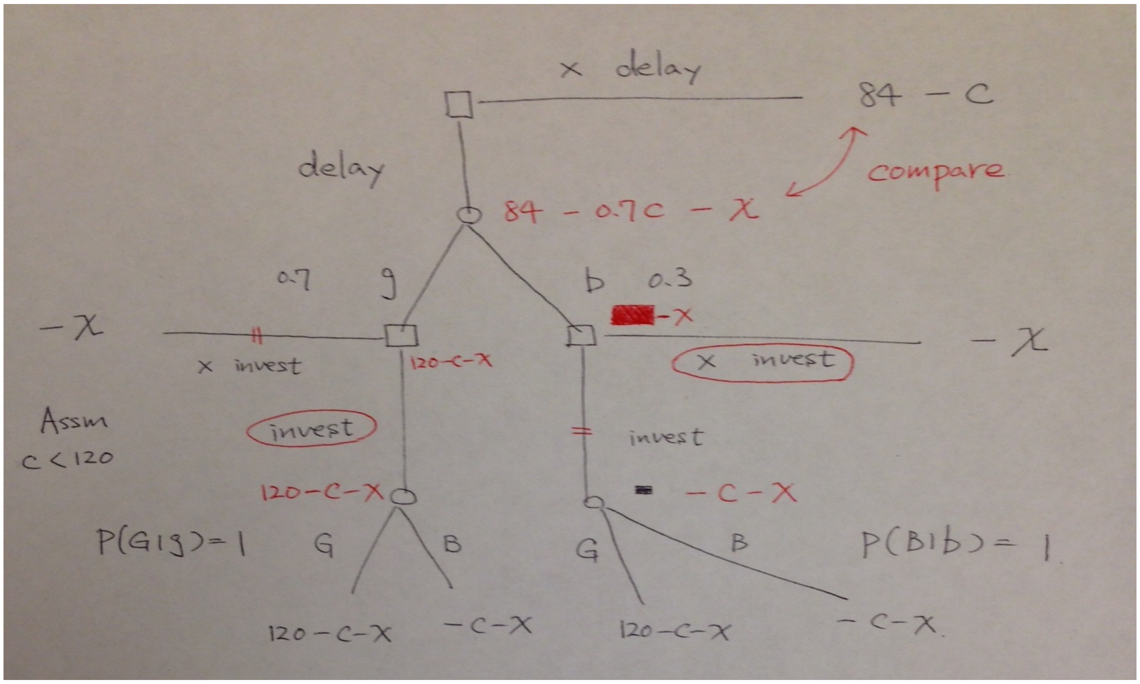
> **Solution:** Maximum willingness to pay for this investment is the present value of the expansion,
>
> $$\begin{aligned} \frac{12}{(1+0.1)} + \frac{12}{(1+0.1)^2} + \cdots &= \left(\frac{12}{1-\frac{1}{1.1}}\right)\left(\frac{1}{1+0.1}\right) \\ &= 120. \end{aligned} \tag{1}$$

(b) (6 points) Suddenly George realizes that things might go wrong during the next year, and if so the additional earnings will be zero. He estimate the probability if 0.3 that things will go wrong. Now how much is he willing to pay for that investment.

> **Solution:** Expected present value of the investment with possible failure is,
>
> $$0.7(120) + 0.3(0) = 84. \tag{2}$$

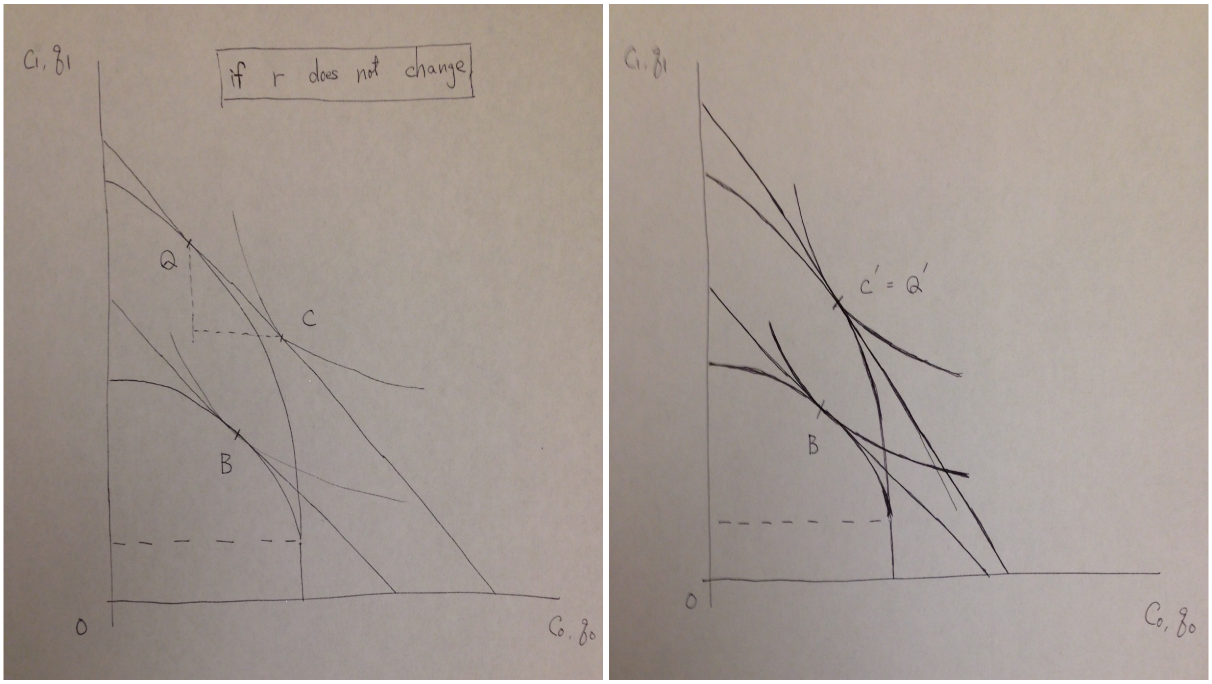
(c) (8 points) George then realizes that, at some cost, he can delay making the investment until he knows whether or not things will go wrong. How much cost of delay is he willing to incur?

> **Solution:** Let c be the investment cost and x be the cost to delay. The decision tree is as follows. ("Not delay" is marked "x delay" and similarly for Not invest.) Therefore, willingness to pay for delaying is $(84 - 0.7c) - (84 - c) = 0.3c$.



2. Developments in Silicon Valley increases productivity worldwide. Other things (i.e., thrift unaffected), what should be the impact on the real interest rate? Nominal interest rates? Current and future real consumption? Wealth? (20pts)

Solution: The increase in productivity shifts PPF upward as in the left panel, where the slope is steeper above each q_0. Therefore, at the old interest rate, the optimal q_0 decreases (this is the production effect) and the optimal c_0 increases. As shown in the left panel of the figure below, the upshot is that borrowing exceeds lending.
In equilibrium, therefore, the real interest rate must rise as in the right panel. The substitution effect decreases c_0 and increases c_1. Income effect (via the production effect in the left panel) increases both c_0 and c_1. Overall, c_1 increases but the effect on c_0 is ambiguous. Although real interest rate increases, q_1 also increases and thus effect on real wealth is ambiguous. In the graph, c_0 barely increases and real wealth also barely increases. If the inflation rate unaffected, then the nominal interest rate increases by the same amount as the real interest rate.



3. Suppose that Abe makes risky choices as if maximizing the expected value of the Bernoulli function $u(m) = \ln(m+1)$. He is faced with a situation in which he will receive either 0 or 24; the outcomes are equally likely.

(a) (4 points) What is the expected outcome? Variance of outcome?

Solution: Expected value and the variance of the situation is,

$$\begin{aligned} E[m] &= \frac{1}{2}(0) + \frac{1}{2}(24) = 12, \\ V[m] &= E[(m - E(m))^2] = \frac{1}{2}(0-12)^2 + \frac{1}{2}(24-12)^2 = 144. \end{aligned} \tag{3}$$

(b) (6 points) What is Abe's certainty equivalent of the risky outcome? What is the maximum amount he would be willing to pay an insurer to get the mean outcome for sure?

Solution: Certainty equivalence, m^{CE}, satisfies,

$$\begin{aligned} \ln(m^{CE} + 1) &= \frac{1}{2}\ln(0+1) + \frac{1}{2}\ln(24+1), \\ &= \ln(24+1)^{\frac{1}{2}} = \ln 5 = \ln(4+1). \end{aligned} \tag{4}$$

Thus, $m^{CE} = 4$. Max insurance payment = RP = $E[m] - m^{CE} = 12 - 4 = 8$.

(c) (4 points) What is Abe's coefficient of relative risk aversion at the mean outcome?

Solution: Since $u'(m) = (m+1)^{-1}$ and $u''(m) = -(m+1)^{-2}$, the coefficient of relative risk aversion is,

$$r(m) = -\frac{-(m+1)^{-2}}{(m+1)^{-1}}(m) = \frac{m}{m+1}. \tag{5}$$

Thus, $r(E[m]) = \frac{12}{13}$.

4. Ajax Inc and Bestco produce imperfectly substitutable products, each at per unit marginal costs of 1. Inverse demand for Ajax is $p_A = 6 - q_A - 0.5q_B$ and symmetrically Bestco's inverse demand is $p_B = 6 - q_B - 0.5q_A$, where q_A and q_B denote the quantities of the two firms. Find the Nash equilibria of the following games, where payoffs are profits.

(a) (6 points) Both firms choose quantity simultaneously and independently.

Solution: (Cournot competition) Firm A maximizes profit with respect to q_A,

$$\max_{q_A}(6 - q_A - \frac{1}{2}q_B)q_A - q_A. \tag{6}$$

First-order condition is,

$$5 - \frac{1}{2}q_B - 2q_A^* = 0. \tag{7}$$

Since firms are symmetric,

$$5 - \frac{1}{2}q_A - 2q_B^* = 0. \tag{8}$$

Thus, in Nash equilibrium, $q_A^{NE} = q_B^{NE} = 2$.

(b) (4 points) Ajax chooses quantity first, then Bestco observes it and then chooses its own quantity.

Solution: (Stackelberg) A moves first and B moves second. We solve this problem by backward induction. The best response function of firm B is the same as Cournot competition case,

$$\begin{aligned} &5 - \frac{1}{2}q_A - 2q_B^* = 0, \\ \Leftrightarrow \quad &BR_B(q_A) = \frac{5}{2} - \frac{1}{4}q_A. \end{aligned} \tag{9}$$

Firm A maximizes profit taking this best response function of firm B into consideration,

$$\max_{q_A} \left(6 - q_A - \frac{1}{2}\left(\frac{5}{2} - \frac{1}{4}q_A\right)\right) q_A - q_A. \tag{10}$$

First-order condition is,

$$\frac{15}{4} - \frac{7}{4}q_A^* = 0. \tag{11}$$

Thus, $q_A^{NE} = \frac{15}{7}$ and $q_B^{NE} = \frac{55}{28}$.

(c) (6 points) Both firms choose price simultaneously and independently.

Solution: (Bertrand) By rewriting the demand system, we obtain,

$$q_A = 4 - \frac{4}{3}p_A + \frac{2}{3}p_B, \tag{12}$$

$$q_B = 4 - \frac{4}{3}p_B + \frac{2}{3}p_B. \tag{13}$$

Firm A maximizes its profit with respect to p_A,

$$\max_{p_A} \quad p_A(4 - \frac{4}{3}p_A + \frac{2}{3}p_B) - (4 - \frac{4}{3}p_A + \frac{2}{3}p_B). \tag{14}$$

First-order condition is,

$$\frac{16}{3} - \frac{8}{3}p_A^* + \frac{2}{3}p_B = 0. \tag{15}$$

Since firms are symmetric,

$$\frac{16}{3} - \frac{8}{3}p_B^* + \frac{2}{3}p_A = 0. \tag{16}$$

Thus, $p_A^{NE} = p_B^{NE} = \frac{8}{3}$.

(d) (4 points) Ajax chooses price first, then Bestco observes it and then chooses its own price.

Solution: Again, we solve this problem by backward induction. The best response function of firm B is the same as Bertrand case,

$$
\begin{aligned}
&\frac{16}{3} - \frac{8}{3}p_B^* + \frac{2}{3}p_A = 0 \\
\Leftrightarrow \quad &BR_B(p_A) = 2 + \frac{1}{4}p_A.
\end{aligned}
\tag{17}
$$

Firm A maximizes profit taking this best response function of firm B into consideration,

$$
\max_{p_A} \quad (p_A - 1)\left(4 - \frac{4}{3}p_A + \frac{2}{3}\left(2 + \frac{1}{4}p_A\right)\right). \tag{18}
$$

First-order condition is,

$$
-\frac{7}{3}p_A^* + \frac{39}{6} = 0. \tag{19}
$$

Thus, $p_A^{NE} = \frac{39}{14}$ and $p_B^{NE} = \frac{151}{56}$.

5. Name at least three techniques of price discrimination commonly used by airlines. How does each technique overcome the main obstacles to price discrimination? Comment very briefly on the efficiency implications of each technique. (18pts)

Solution: Laws against one customer reselling an airline ticket to another customer virtually eliminate the arbitrage obstacle for any form of price discrimination. Self-selection, or incentive compatibility, helps cope with the unobservability of WTP obstacle, as noted below.

- Timing of purchase: Charge higher prices for those who buy tickets on closer dates to departure dates. Incentive compatible because last minute customers tend to have less price-elastic demand. Can be regarded as Third-degree PD.

- First-class vs Economy-class: Price way higher prices for consumers with high willingness to pay. Incentive compatible because of the difference in service and quality. Can be regarded as Second-degree PD.

- Mileage: Give bonuses to customers with low willingness to pay. Incentive compatible because customers with high willingness to pay does not have incentive to earn mileage because of opportunity cost. Similar to Second-degree/ quantity discount, and a little like a 2-part tariff.

- Payment history: This is similar to the example from one of the clip in class. Charges different prices to different customers based on histories of purchase, which reflect willingness to pay. Time will tell us whether this new way of price discrimination will work or not. Might approximate First-degree.

6. The price of crude oil decreased about 60% over the last 1.5 years. Suppose that a sector of the economy can be approximated reasonably well by a production function with crude oil as one input and labor and capital as the other inputs. If that production function is Leontieff, what impact would you expect to see on the sector's input demands, cost and output? How would you answer change if the production function were CES with substitution elasticity between 0 and 1? Which production function seems a more reasonable description in the short run of a month or two? (14pts)

Solution:

- When the relative price of input changes, ratios of input quantities do not change if production function is Leontieff. One way to see this is to draw isoquant. Isoquant has a kink and regardless of the input price ratio, cost is minimized at the kink. Still, the overall marginal cost is lower, and thus total cost. With lower marginal cost, the profit-maximizing output should increase.

- When the elasticity of substitution is not zero, there is a substitution toward the lowered priced goods. This additional effect even more lowers the total cost.

- In the short-run, it is hard to switch to different input ratios. As a result, production function with low elasticity of substitution is suitable.

ECON 200 - Final Exam - Fall 2016
Solutions

December 7, 2016

1 Risky Choice

Suppose that Anna makes risky choices as if maximizing the expected value of the Bernoulli function $u(m) = m^{2/3}$. She is faced with a situation in which she will receive either 0 (with probability 0.7) or 27 (with probability 0.3)

a) What is the expected (mean) outcome? Variance of outcome? (4 pts)
The mean is as follows:

$$\begin{aligned}\mu &= \sum_i p_i v_i \\ &= 0.7 \cdot 0 + 0.3 \cdot 27 = 8.1\end{aligned}$$

The variance is as follows:

$$\begin{aligned}\text{Var} &= \sum_i p_i (v_i - \mu)^2 \\ &= 0.7 \cdot (0 - 8.1)^2 + 0.3 \cdot (27 - 8.1)^2 \\ &= 45.927 + 107.163 = 153.090\end{aligned}$$

b) What is Anna's certainty equivalent for this situation? What is the maximum amount she would be willing to pay an insurer to get the mean outcome for sure? (6 pts)
We calculate this by setting equal the utility of the certainty equivalent c and the expected utility of the situation:

$$\begin{aligned}u(c) &= 0.7 \cdot u(0) + 0.3 \cdot u(27) \\ c^{2/3} &= 0 + 0.3 \cdot 27^{2/3} = 2.7 \\ c &= 2.7^{3/2} \approx 4.437\end{aligned}$$

Since she is receiving 8.1 with certainty, we could say the WTP for insurance (also known as the risk premium) is as follows:

$$\begin{aligned}u(8.1 - wtp) &= u(c) \\ 8.1 - wtp &= 4.437 \\ wtp &\approx 3.66\end{aligned}$$

c) What is Anna's coefficient of absolute risk aversion at the mean outcome? Her coefficient of relative risk aversion? (4 pts)

This is formulaic:

$$\begin{aligned}
\text{ARA}(x) &= -\frac{u''(x)}{u'(x)} \\
&= \frac{\frac{2}{9}x^{-4/3}}{\frac{2}{3}x^{-1/3}} = \frac{1}{3x} \\
\text{ARA}(8.1) &= \frac{1}{3\cdot 8.1} = \frac{1}{24.3} = 0.0412 \\
\text{RRA}(x) &= x\cdot \text{ARA}(x) = x\frac{1}{3x} = \frac{1}{3}.
\end{aligned}$$

2 Bayesian Updating

Exams are either tough or easy; overall 60% are tough. Looking at the first two problems on the exam gives a clue: these problems seem tough 80% of the time when the exam is tough, and the seem tough only 10% of the time when the exam is easy.

a) What is the updated probability that the exam is tough, after seeing that the first two problems are easy? (6 pts)
This is a form of Bayesian updating. We are looking for the posterior probability that the exam is tough given that the first two problems are easy:

$$\begin{aligned}
p(\text{Exam} = \text{tough}|\text{first two} = \text{easy}) &= \frac{p(\text{first two} = \text{easy}|\text{Exam} = \text{tough})\, p(\text{Exam} = \text{tough})}{p(\text{first two} = \text{easy})} \\
p(E = t|f2 = e) &= \frac{p(f2 = e|E = t)\, p(E = t)}{p(f2 = e|E = t)\, p(E = t) + p(f2 = e|E = e)\, p(E = e)} \\
p(E = t|f2 = e) &= \frac{0.2\cdot 0.6}{0.2\cdot 0.6 + 0.9\cdot 0.4} \\
&= \frac{0.12}{0.12 + 0.36} = \frac{1}{4}
\end{aligned}$$

The posterior probability is 0.25

b) Would you find it psychologically comforting to know the day before whether the first two problems seem easy? (1 pt) When would that information have economic value? (3 pts)
Any coherent answer gets the 1pt for psych comfort. You get the 3pts if you point out that the information is economically valuable to the extent that it would change your optimal plan. Otherwise put, the information would have economic value if and only if it enabled you to allocate your time more efficiently away from studying and toward something more productive, such as studying for another exam that would be relatively more difficult or towards a paying job. The key here is that the change in how likely the exam is to be difficult (in terms of your perception) must enable you to do something more productive; otherwise, there is no economic value. (That's the neoclassical answer. You could argue as a behavioral economics matter, there may be some willingness to pay to reduce anxiety).

3 Yambits

Agil Corp recently launched a distinctive line of yambits. It finds that inverse demand for this product is $p = 80 - 2y$, while it can produce y units per month at cost $c(y) = 75 + 20y$.

a) What is the maximized profit, and corresponding price (p) and output (y)? (6 pts)
Take from the question that Agil has a distinctive product and therefore a monopoly pricing environment. We begin with profit maximization:

$$\max_y py - c(y)$$

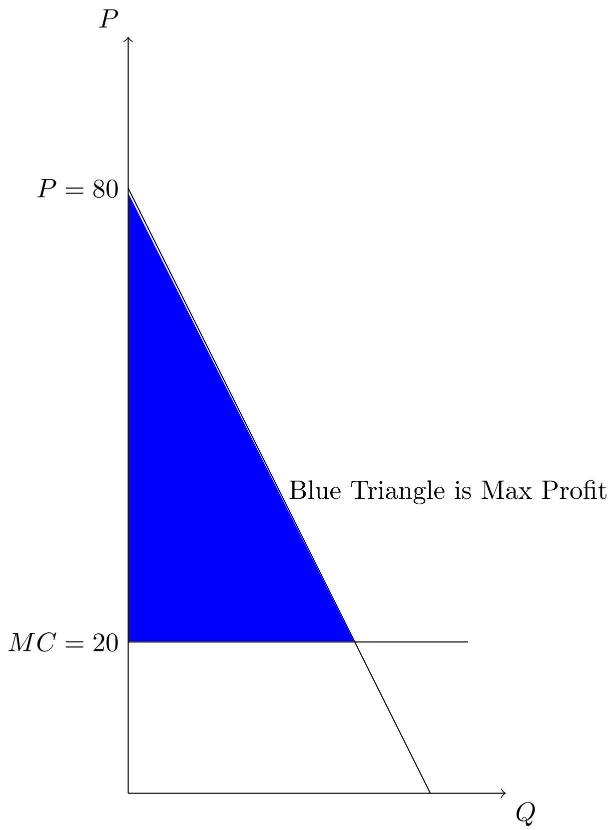


Figure 1: Perfect Price Discrimination

$$\max_y (80 - 2y)\, y - 75 - 20y = 60y - 2y^2 - 75$$

The first order condition is:

$$60 - 4y = 0$$
$$y^M = 15$$

We can check the second order condition:

$$-4 < 0$$

So this is a maximum. What is price?

$$p^M = 80 - 2y^M = 80 - 2 \cdot 15 = 50.$$

The corresponding profit is

$$\pi^M = p^M y^M - c\left(y^M\right) = 50 \cdot 15 - 75 - 20 \cdot 15 = 375.$$

b) Suppose Agil can charge different prices on different units sold. What is the largest profit that (absent obstacles) could then be obtained, and what is the corresponding output level? (3 pts)
It is instructive to draw a graph, but the point is essentially the following. If Agil Corp can charge every customer its willingness to pay, it the demand curve is the marginal revenue curve. As a result, the intersection of demand and marginal cost (20) is the optimal output:

$$p = 80 - 2y = 20 = MC$$
$$y = 30$$

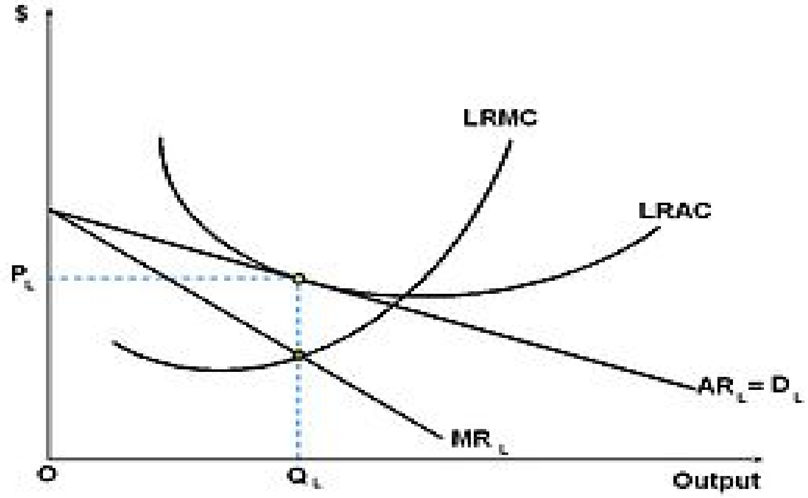


Figure 2: LR equilibrium for a Monopolistic Competitive firm. From Wikpedia article.

The price of each unit sold is different, so profit is the integral of the area under the marginal revenue (also demand) curve and above the marginal cost line:

$$\begin{aligned}
\pi^{max} &= \int_0^{30} (p(y) - mc)\, dy - FC \\
&= \int_0^{30} (80 - 2y - 20)\, dy - 75 \\
&= \int_0^{30} (60 - 2y)\, dy - 75 \\
&= 60y - y^2|_0^{30} - 75 \\
&= 60(30) - (30)^2 - 0 - 75 \\
&= (60 - 30) \cdot 30 - 75 = 30^2 - 75 = 825
\end{aligned}$$

c) What practical considerations limit Agil's ability to profitably charge different prices on different units? What are some possible ways to deal with these considerations? (4 pts)
Other than legal and moral (or customer attitude) considerations, (a) Agil likely has little information on each customer's willingness to pay. Furthermore, it is may be difficult to prevent arbitrage, which would allow all customers to buy at the lowest price. Some methods discussed in class to mitigate these problems include offering price/quantity menus, segmenting the market by observable characteristics correlated with WTP (or price elasticity), loyalty programs that collect data useful in determining willingness to pay, and in generated non-transferrable discounts.

d) Now assume that Agil must offer a unified price on all units sold each month. Suppose that there are no barriers to entry in the yambit business. What does standard (e.g., good undergrad level) economics predict regarding Agil's long run profit? Explain how the prediction works (e.g., in terms of shifts in the cost function or demand functions). A diagram may help you make your points. (6 pts)
The underlying intuition here is that economic profit is positive in the short run, which leads to entry. More firms will enter, especially those with relatively close substitutes (not exact substitutes, Agil's product remains distinctive). This will lower and flatten the demand curve until average cost is tangent to the demand curve, at which point further entry is unprofitable, as illustrated in Figure 2. Agil may earn economic profit during this transition, but long-run profit will be zero afterwards.

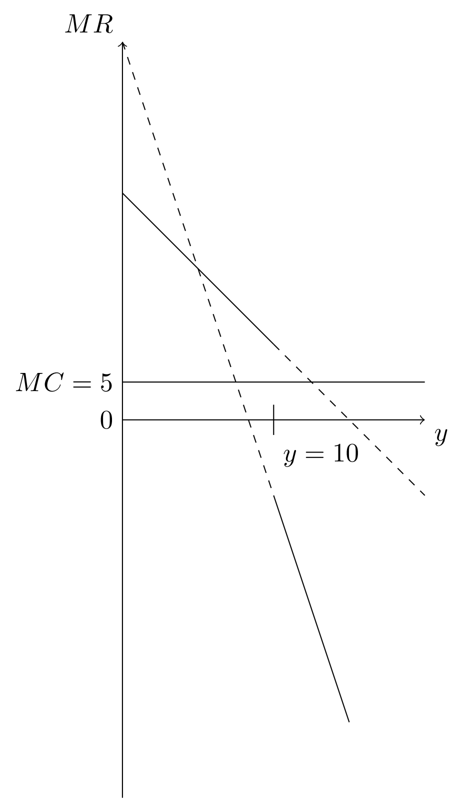


Figure 3: Discontinuous Marginal Revenue

4 Price Match Guarantee

Betamin Inc. currently sells $y = 10$ units per month of its main product at the prevailing price $p = 20$. When all firms in the industry shift their prices together, Betamin's inverse demand curve is well approximated by $p = 50 - 3y$. When rivals stay with the prevailing price, Betamin's inverse demand is approximately $p = 30 - y$. Its cost is $c(y) = 10 + 5y$.

a) What are Betamin's own price elasticities at the prevailing price when only it shifts price? When all firms shift in parallel? (2 pts)
When it is the only one, the own price elasticity is:

$$\varepsilon = \frac{\Delta y}{\Delta p} \cdot \frac{p}{y} = -1 \cdot \frac{20}{10} = -2$$

When all firms shift together, the own price elasticity is:

$$\varepsilon = \frac{\Delta y}{\Delta p} \cdot \frac{p}{y} = -\frac{1}{3} \cdot \frac{20}{10} = -\frac{2}{3}$$

b) Rivals offer a "not-undersold" policy in which they match the lowest price offered in the market if it is below $p = 20$, and otherwise stay at $p = 20$. Is it profit-maximizing for Betamin to continue to produce $y = 10$ units per month? Show your calculations. (5 pts)
Marginal cost is always 5. Marginal revenue is discontinuous at $y = 10$. For $y < 10$, $p > 20$, Betamin is the only one changing its price. Here, marginal revenue is given by:

$$\frac{\partial}{\partial y}(30 - y)\,y = 30 - 2y,$$

which always exceeds MC = 5. So Betamin would profitably increase output whenever it is below $y = 10$, i.e., would decrease price if above $p = 20$. On the other hand, for $y > 10$, $p < 20$, marginal revenue is given by:

$$\frac{\partial}{\partial y}(50 - 3y)\,y = 50 - 6y < 0,$$

so increasing output (and lowering price) would reduce revenue and increase cost, needlessly lowering profit. Clearly, the best situation is for Betamin to remain at the prevailing price with $y = 10$.

c) Betamin's COO is worried that its cost function might shift, and asks you what sort of economic factors could change it. Please list the standard items. (3 pts)
Standard things include shocks to input prices, changes in transportation costs (gasoline prices, etc), or changes in technology. Also economies of scope (if there are related product) and a learning curve (which tends to lower MC as experience accumulates).

d) For what range of marginal costs and fixed costs would Betamin find it profit-maximizing for Betamin to continue to charge the prevailing price? (6 pts)
From the diagram you can see the main condition is MC ≤ 10. If MC goes higher than that, Betamin should charge above 20 and reduce output below 10. The other condition is that Betamin covers its average cost (or in the SR, its avoidable cost). If the fixed cost is sunk (as the formula suggests) it is irrelevant in the SR. In the LR all costs are avoidable, and the revenue $10 \cdot 20 = 200$ must cover total cost $= y\cdot$ MC + FC. E.g., at MC=5, the firm will exit eventually if FC $> 200 - 10 \cdot 5 = 150$.

5 Ingots and Externalities

Demand in the ingot industry is $Y = 300 - 2p$. Supply is via 100 identical firms with production cost functions $c(y_i) = 50y_i + 0.5y_i^2$. People who live in towns that produce ingots suffer health and other costs increasing in total output approximated by $e(Y) = 0.5Y^2$.

a) What is the industry supply curve (also known as the private MC function)? (2 pts)
First we need to find individual private marginal cost:

$$mc(y_i) = \frac{\partial c(y_i)}{\partial y_i} = 50 + y_i$$

Each of 100 firms will supply according to this condition (noting min avc=0, supply at $p = mc$), so:

$$Y^S = 100y_i = 100 \cdot (p - 50) = 100p - 5000 \quad \forall p \geq 50$$

b) Find the competitive equilibrium output Y^c, and the corresponding total surplus $TS = PS + CS$. Assume that the costs $e(Y)$ are not included in producer surplus (PS) but are subtracted from consumer surplus. (5 pts)
Find competitive output by setting demand equal to supply:

$$\begin{aligned} 300 - 2p &= 100p - 5000 \\ 102p &= 5300 \\ p &= 51.96 \\ Y &= 300 - 2 \cdot \frac{5300}{102} \\ &= 196.08 \end{aligned}$$

Note that the producer surplus is above the supply curve and below the competitive price, from zero to the competitive output. This is

$$PS = \frac{1}{2}(51.96 - 50) \cdot 196.08 = 192.16$$

The consumer surplus before accounting for the externality is above the price and below the demand curve:

$$CS_{pre} = \frac{1}{2}(150 - 51.96) \cdot 196.08 = 9611.84$$

Calculate the externality

$$e(Y) = \frac{1}{2}(196.08)^2 = 19223.68$$

Total consumer surplus:

$$CS = 9611.84 - 19223.68 = -9611.84$$

Total surplus:

$$TS = CS + PS = 192.16 - 9611.84 = -9419.68$$

b2) What is the social MC function (which includes e as well as c costs)? (3 pts)
We can add the marginal external damages, $e'(Y) = Y$ directly to the aggregate industry supply function:

$$\begin{aligned} P\left(Y^S\right) &= 50 + \frac{1}{100}Y^S + e'\left(Y^S\right) \\ &= 50 + 1.01Y^S \\ Y^S &= \frac{100}{101}P - \frac{5000}{101} \quad \forall p \geq 50 \end{aligned}$$

c) Compute the output level Y^o that maximizes total surplus ($TS = PS + CS$). How much higher is TS here than at Y^c? (4 pts)
We know that the intersection of Y^S_{SMC} and demand maximizes TS:

$$\begin{aligned} \frac{100}{101}P - \frac{5000}{101} &= 300 - 2p \\ \frac{302}{101}p &= \frac{30300 - 5000}{101} \\ p &= \frac{25300}{302} \\ p &= 83.77 \\ Y^o &= 300 - 2 \cdot \frac{25300}{302} \\ &= 132.45 \end{aligned}$$

Note that producer surplus is still calculated as under the price line and above the supply curve. However, this is the triangle of surplus above social marginal cost and below the price line plus the external damages:

$$\begin{aligned} PS &= \frac{1}{2}(83.77 - 50) \cdot 132.45 + e(132.45) \\ &= 2236.42 + \frac{1}{2}(132.45)^2 \\ &= 2236.42 + 8771.50 = 11007.92 \end{aligned}$$

The pre-externality consumer surplus is still under the demand curve and above the price line

$$CS_{pre} = \frac{1}{2}(150 - 83.77) \cdot 132.45 = 4392.70$$

The externality is

$$e(Y) = \frac{1}{2}(192.31)^2 = 8771.50$$

Consequently,

$$CS = -4378.80$$

Note that while consumer surplus is negative, total surplus is positive and equal to:

$$\begin{aligned}TS &= \frac{1}{2}(83.77 - 50) \cdot 132.45 + \frac{1}{2}(150 - 83.77) \cdot 132.45 \\ &= 2236.42 + 4392.70 = 6692.12\end{aligned}$$

d) What are the main possible approaches to restore efficiency? Name a specific policy that seems most effective in this example. (3 pts)
The most effective method is to tax sales or production at Y, but because this needs to change with scale, it may need to be an approximation. Assuming perfect information, a tax of 31.81 would result in maximal total surplus.

6 Imperfect Substitutes

Anyway Inc. and Belton Co. produce imperfectly substitutable products, with inverse demand $p_A = 8 - q_A - 0.5q_B$ for Anyway and $p_B = 8 - q_B - 0.5q_A$ for Belton, where q_A and q_B denote their respective output quantities. They have cost functions $c(q_i) = 2 + 3q_i$, for $i = A, B$. Both firms choose quantity simultaneously and independently.

a) Find the best response functions for both firms. (4 pts)
This is a symmetric prolem,

$$\max_{q_i} \left(8 - q_i - \frac{1}{2}q_j\right) q_i - 2 - 3q_i$$

FOC

$$5 - 2q_i - \frac{1}{2}q_j = 0$$

Rearranging, for both A and B:

$$q_i = \frac{5}{2} - \frac{1}{4}q_j$$

b) Find Nash equilibrium outputs, prices, and payoffs (profits). (4 pts)
Note:

$$\begin{aligned}q_i &= \frac{5}{2} - \frac{1}{4}q_j \\ q_i^\star &= \frac{5}{2} - \frac{1}{4}\left(\frac{5}{2} - \frac{1}{4}q_i^\star\right) \\ &= \frac{15}{8} + \frac{1}{16}q_i^\star \\ q_i^\star &= 2 = q_A^\star = q_B^\star\end{aligned}$$

Note that

$$\begin{aligned}p_i &= 8 - q_i - \frac{1}{2}q_j \\ p_i^\star &= 8 - 2 - \frac{1}{2} \cdot 2 = 5\end{aligned}$$

for both A and B. The profits are therefore:

$$\pi_i = 5 \cdot 2 - 2 - 3 \cdot 2 = 2$$

for both A and B.

c) What is each firm's conjectural variation? What actual variation do the BR functions imply (e.g., $d\text{BR}_B/dq_A$)? (4 pts)
The conjectural variation is the assumption that firm i cannot affect firm j's level of output. In other words, i assumes that conjectural variation is zero. The actual variation is $-1/4$.

d) Now assume that both firms choose price (not quantity) simultaneously and independently. What now are their best response functions? NE prices, outputs and payoffs? Conjectural and implied actual variations? (6 pts)
Choosing price,

$$\begin{aligned} p_A &= 8 - q_A - .5q_B \\ p_B &= 8 - q_B - .5q_A \end{aligned}$$

$$\begin{aligned} q_i &= 8 - p_i - .5q_j \\ &= 8 - p_i - .5\left(8 - p_j - .5q_i\right) \\ &= 4 - p_i + .5p_j + .25q_i \\ q_i &= \frac{16}{3} - \frac{4}{3}p_i + \frac{2}{3}p_j \end{aligned}$$

$$\max_{p_i} p_i \cdot \left(\frac{16}{3} - \frac{4}{3}p_i + \frac{2}{3}p_j\right) - 2 - 3\left(\frac{16}{3} - \frac{4}{3}p_i + \frac{2}{3}p_j\right)$$

$$\max_{p_i} \frac{28}{3}p_i - \frac{4}{3}p_i^2 + \frac{2}{3}p_j p_i + constant$$

FOC

$$\begin{aligned} \frac{28}{3} - \frac{8}{3}p_i + \frac{2}{3}p_j &= 0 \\ p_i &= \frac{7}{2} + \frac{1}{4}p_j \\ p_i &= \frac{7}{2} + \frac{1}{4}\left(\frac{7}{2} + \frac{1}{4}p_i\right) \\ p_i^\star &= \frac{5}{4} \cdot \frac{7}{2} \cdot \frac{16}{15} = \frac{14}{3} \end{aligned}$$

for both A and B. Output is

$$q_i^\star = \frac{16}{3} - \frac{4}{3} \cdot \frac{14}{3} + \frac{2}{3} \cdot \frac{14}{3} = \frac{20}{9}$$

for both A and B. Conjectural variation is again zero, because firm i takes firm j's price as given. Yet, actual variation is $1/4$.

e) Now suppose that they are able to form a cartel. What is the maximized total profit for the two firms? What are the corresponding prices and outputs? What are the corresponding conjectural variations? (6 pts)
Maximize jointly

$$\max_{q_A, q_B} \left(8 - q_A - \frac{1}{2}q_B\right) q_A - 2 - 3q_A + \left(8 - q_B - \frac{1}{2}q_A\right) q_B - 2 - 3q_B$$

$$\max_{q_A, q_B} 5q_A + 5q_B - q_A^2 - q_B^2 - q_B q_A - 4$$

FOC

$$\begin{aligned} 5 - 2q_A - q_B &= 0 \\ 5 - 2q_B - q_A &= 0 \end{aligned}$$

Solving

$$\begin{aligned} q_A &= 5 - 2q_B \\ &= 5 - 2\left(5 - 2q_A\right) \\ &= -5 + 4q_A \\ q_A &= \frac{5}{3} = q_B \end{aligned}$$

Prices are as follows:

$$p = 8 - \frac{5}{3} - \frac{1}{2}\frac{5}{3} = 5.5$$

Profits are as follows:

$$\pi = 5.5 \cdot \frac{5}{3} - 2 - 3 \cdot \frac{5}{3} = \frac{13}{6} > 2$$

Collusion conjectural variation is that the other firm will maintain its share of output. Written properly, actual variation here is $-1/2$ for quantities.

Formulas for Econ 200, Fall 2016, UCSC

Equations for Competitive Markets

Linear Demand: $q_d = a - bp$ **Linear Supply**: $q_s = x + yp$
Log-linear demand: $\ln(q_d) = \ln(a) + \varepsilon_d \ln p$ **Log-Linear Supply**: $\ln(q_s) = \ln(x) + \varepsilon_s \ln p$
Total Surplus=Consumer Surplus+Producer Surplus; **Revenue**=Producer Surplus + Variable Cost
Total Cost=Fixed Cost + Variable Cost; **Profit**=Revenue-Total Cost=Producer Surplus-Fixed Cost
Quantity Tax (tax per unit): $p_d = p_s + t$; **Value Tax** (tax on percentage spent): $p_d = (1+t)p_s$
Price Elasticity of Demand: $\varepsilon_d = \frac{\partial \ln(D)}{\partial \ln(p)} = \frac{\partial D}{\partial p}\frac{p}{D}$; If $|\varepsilon| > 1$ then curve is elastic.
Tax Incidence Formula: $p_s(t) = p^* - \frac{t|D'|}{S'+|D'|}$; $p_d = p^* + \frac{tS'}{S'+|D'|}$; If ε_d is constant: $\frac{\partial p_d}{\partial t} = \frac{\varepsilon_s}{|\varepsilon_d|+\varepsilon_s}$

Equations for Consumer Choice and Demand

Marginal Utility: $MU_i = \frac{\partial u}{\partial x_i}$; **Marginal Rate of Substitution**: $MRS_{ji} = \frac{MU_i}{MU_j}$ and at interior optimum $= \frac{p_i}{p_j}$
Perfect Substitutes: $u(x_1, x_2) = x_1 + cx_2$; **Cobb-Douglas**: $u(x_1, x_2) = \ln(x_1) + c\ln(x_2)$
CES Utility: $u(x_1, x_2) = \frac{1}{\rho}\ln(x_1^\rho + x_2^\rho)$; $\rho \in (-\infty, 1]$; **Quasilinear**: $u(x_0, x_1) = x_0 + g(x_1)$
Marshallian Demand: $\mathbf{x}^* = (x_1^*(\mathbf{p}, m), x_2^*(\mathbf{p}, m), ...)$ is the solution to $\max_{\mathbf{x}\geq 0} u(\mathbf{x})$ s.t. $m - \mathbf{p}\cdot\mathbf{x}$. The Lagrangian is $\mathcal{L} = u(\mathbf{x}) + \lambda(m - \mathbf{p}\cdot\mathbf{x})$. The FOCs can be written $MU_i = \lambda p_i$ or $MRS_{ji} = \frac{p_i}{p_j}$.
The solutions $x_i^*(\mathbf{p}, m)$ are homogeneous degree 0.
Hicksian Demand: $h_i^*(\mathbf{p}, u_0) : \min_x \mathbf{p}\cdot\mathbf{x}$ s.t. $u(\mathbf{x}) \geq u_0$
Roy's Identity: $x_i^*(\mathbf{p}, m) = -\frac{\partial v}{\partial p_i}/\frac{\partial v}{\partial m}$;
Slutsky Equation: $\frac{\partial x_i(\mathbf{p},m)}{\partial p_i} = \frac{\partial h_i(\mathbf{p},v(\mathbf{p},m))}{\partial p_i} - \frac{\partial x_i^*}{\partial m}x_i^*(\mathbf{p}, m)$; (**Elasticity Form**): $\varepsilon_i = \varepsilon_i^h - s_i\varepsilon_m$ for $s_i = \frac{p_i x_i}{m}$.
Demand Elasticity identity for product i: $\varepsilon_{im} + \varepsilon_{ii} + \sum_{j\neq i}\varepsilon_{ij} = 0$

Equations for Cost and Technology

Technical Rate of Substitution: $TRS_{ij} = -\frac{\frac{\partial f}{\partial x_j}}{\frac{\partial f}{\partial x_i}} = -\frac{mp_i}{mp_j} < 0$;
MC: $MC(y) = \frac{\partial c}{\partial y} = \frac{\partial c_v}{\partial y}$, and $\int MC = VC$.
Factor Prices: $\mathbf{w} = (w_1, w_2, ..., w_n)$; **Production Function**: $y = f(x_1, y_2)$
Cost Function with two factors: $c(\mathbf{w}, y) = w_1 x_1^*(w_1, w_2, y) + w_2 x_2^*(w_1, w_2, y)$
$= \min_{x_1, x_2 \geq 0} w_1 x_1 + w_2 x_2$ s.t. $y = f(x_1, x_2)$
Shepard's Lemma conditional factor demand: $x_i^*(\mathbf{w}, y) = \frac{\partial c(\mathbf{w},y)}{\partial w_i}$
Learning Curve: The typical specification is for $Y_t = \Sigma_{s\leq t} y_s$, AC falls proportionately, $\ln AC_t = AC_0 - b\ln Y_t$

Equations for Competitive Firms

SR Profit Maximization: $\max_{y, x_v \geq 0} \pi = \max_{y\geq 0}[\max_{x_v\geq 0} R(y) - w_v x_v - w_f x_f$ s.t. $y = f(x_v, \bar{x}_f)] = \max_{y\geq 0}[R(y) - c(y)]$
Revenue if firm is competitive: $R(y) = py = pf(x_v, \bar{x}_f)$ **FOC of *un*conditional factor demand**: $p\frac{\partial f(x_v, \bar{x}_f)}{\partial x_v} = w_v$
Hotelling's Lemma, Supply: $y^*(p, \mathbf{w}) = \frac{\partial \pi(p,\mathbf{w})}{\partial p}$; unconditional factor demands: $x_i(p, \mathbf{w}) = -\frac{\partial \pi(p,\mathbf{w})}{\partial w_i}$
Shutdown Condition (Competitive Firms): $-F > py - c_v(y) - F \implies AVC = \frac{c_v(y)}{y} > p$

Equations for Monopolies

FOC for a monopolist: $p(y) + p'(y)y = c'(y)$ which can be rewritten as $p = \frac{1}{1+\frac{1}{\varepsilon}} MC$; valid if $\varepsilon < -1$.

Passing Along Costs: $\frac{\partial p}{\partial c} = \frac{1}{2+yp''(y)/p'(y)}$.

Risky Choice. Given a lottery with monetary outcomes $m_1, ..., m_n$ and corresponding probabilities $p_1, ..., p_n$,
its **expected value** is $Em = \sum_i p_i m_i$ and its **variance** is Var $m = \sigma_m^2 = E(m - Em)^2 = \sum_i p_i(m_i - Em)^2$.
Given **Bernoulli function** $u(m)$ — so $u' > 0$ and, if the person is risk-averse, $u'' < 0$ —
the **certainty equivalent** m^{CE} to the lottery solves $u(m^{CE}) = Eu(m) = \sum_i p_i u(m_i)$.
The **coefficient of absolute risk aversion** is $a(m) = -u''(m)/u'(m)$ and
the **coefficient of relative risk aversion** is $r(m) = ma(m)$.
The **risk premium** is RP = $Em - m^{CE}$. It is also given by the second term of the Taylor expansion of u around Em.
Decision Theory. Probability Identities: $P(A|B) = \frac{P(A \cap B)}{P(B)}$; $P(A \cap B) = P(B \cap A)$; Given probability sets A,B,C,D:
$P(A) = P(A|B)P(B) + P(A|C)P(C) + P(A|D)P(D)$
Bayes Theorem: $p(s|m) = \frac{p(m|s)}{\sum_{t \in S} p(m|t)p(t)} p(s)$ or $\frac{p(m|s)}{p(t|m)} = \left[\frac{p(m|s)}{p(m|t)}\right] \left[\frac{p(s)}{p(t)}\right]$ or ln[posterior odds] $=$ ln[likelihood ratio]$+$ ln[prior odds]. Note: $s =$ state, $m =$ message
$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{P(B|A_i)P(A_i)}{\sum_j P(B|A_j)P(A_j)}$

Cournot. Given $D^{-1}(Y) = a - bY$, where $Y = y_i + Y_{-i} = \sum_{j=1}^{n} y_j$. Here $BR_i(Y_{-i}) = \underset{y_i}{\operatorname{argmax}}\ \pi_i = P(Y)y_i - c(y_i)$
$\implies P(Y) + P'(Y)y_i - MC_i(y_i) = 0$.
To solve for the Nash equilbrium, we want to find where the Best Response functions intersect.

$\Rightarrow NE_{Cournot} : Y^* = \frac{N}{(N+1)b}(a - c) \implies y_i^* = \frac{Y^*}{N} = \frac{a-c}{(N+1)b}$
Bertrand. For firms with homogeneous goods, $p = MC$ if equal MC, and $p =$ second lowest MC $-$ one tick if MC's differ.
Stackelberg. Leader solves $\max \pi_L(y_L, BR_F(y_L)) = D^{-1}(y_L + BR_F(y_L))y_L - c(y_L)$
Kinked Demand Curve & Sticky Prices. MR has discontinuous drop at $D(\bar{p})$ where $\bar{p}$ is the established price. Firms will match $p < \bar{p}$ and will not match $p > \bar{p}$. Profit maximization $\implies$ not changing quantity (or price) as long as $MC_{low} < MC < MC_{high}$ where MC_{low} is $MR_{match} \cap D(\bar{p})$, and MC_{high} is $MR_{no-match} \cap D(\bar{p})$.
Conjectural Variations. If $p(Y) = p(y_1 + y_{-1})$, then firm 1's **FOC** is $c'(y_1) = p(Y) + y_1 p'(Y)[1 + \nu]$. The conjectural variation $\nu = \frac{dy_{-1}}{dy_1}$ is 0 for Cournot, is -.5 for Stackelberg leader (in linear duopoly), is -1.0 in Bertrand, and $\nu = \frac{y_{-1}}{y_1}$ in collusion/Cartel.
Hotelling location models. *Duopoly case on* $[0, 1]$: Firm i's BR to location choice $z_j < .5$ is $z_i = z_j + \epsilon$, and to $z_j > .5$ is $z_i = z_j - \epsilon$. Unique NE will be back to back at $z = .5$. **Delivered Price** for firm j at location z is $p_j(z) = p + t|z - z_j|$.
Monopolistic Competition Solve standard monopoly problem $MR = MC$ and $p = D^{-1}(q^*)$ Determine whether economic profits are > 0 or < 0. In LR equilibrium $\pi = 0$ since $LRAC = LRAR$.
Public Goods & Externalities. Let $C(Y) = Y$. Assume agents $i = 1, \ldots, n$ have $u_i(m, Y) = m + g_i(Y)$. WTP for each agent is $g_i'(Y)$. Efficient quantity Y^o maximizes $B(Y) - C(Y) = \sum_{i=1}^n g_i(Y) - Y$. The FOC is $1 = B'(Y) = \sum_{i=1}^n g_i'(Y)$.
External cost $e(x)$ is subtracted from the usual TS in efficiency calculations. **Pigouvian tax** is $t = e'(x^o)$.

Midterm - ECON 200 - Fall 2016

October 26, 2016

1 Alkoids

Supply of alkoids is well approximated over the relevant range by the expression $p-3$, where p is price. Using the same units of measurement, demand is well approximated by $12-2p$.

1.1 What is the competitive equilibrium (CE) price $p^\star$ and quantity $Q^\star$?

Match supply and demand:

$$\begin{aligned}
Q_S^\star(p^\star) &= Q_D^\star(p^\star) \\
p^\star - 3 &= 12 - 2\cdot p^\star \\
3p^\star &= 15 \\
p^\star &= 5 \\
Q^\star(5) &= 5-3 = 2
\end{aligned}$$

1.2 What are the (own price) supply and demand elasticities at CE?

Own price elasticity is given by:

$$\begin{aligned}
\varepsilon_D &= \frac{\partial D}{\partial P}\cdot\frac{P}{D} = -2\cdot\frac{5}{2} = -5 \\
\varepsilon_S &= \frac{\partial S}{\partial P}\cdot\frac{P}{S} = 1\cdot\frac{5}{2} = 2.5
\end{aligned}$$

1.3 Suppose that a tax of $t=0.6$ euros per decigram is now imposed. What is the tax revenue? How much comes from consumers? From producers? What is the deadweight loss?

Note that

$$\begin{aligned}
Q_S^t(p_S) &= Q_D^t(p_D) = Q_D^t(p_S+t) \\
p_S^t - 3 &= 12 - 2\cdot\left(p_S^t + t\right) \\
3p_S^t &= 15 - 2\cdot 0.6 \\
p_S^t &= 5 - 0.4 = 4.6 \\
p_D^t &= 4.6 + 0.6 = 5.2 \\
Q^t &= p_S^t - 3 = 4.6 - 3 = 1.6\ kilotons
\end{aligned}$$

Tax revenue is

$$Q^t\cdot t = 1.6\cdot 0.6/kt = 0.96$$

Based on the slopes of the curves, we note that suppliers pay two-thirds of the tax in reduced price received, 0.64, while consumers pay one-third in increased price faced, 0.32. The deadweight loss is

$$\begin{aligned} DWL &= \frac{1}{2} \cdot \left(p_D^t - p_S^t\right) \cdot \left(Q^\star - Q^t\right) \\ &= \frac{1}{2} \cdot (5.2 - 4.6) \cdot (2 - 1.6) \\ &= 0.12 \end{aligned}$$

2 Cost Function

Suppose that you estimate the following equation:

$$\ln[c/y] = 2.1 + 0.6 \ln w_1 + 0.5 \ln w_2 \tag{1}$$

where ln is natural log, c=total cost, y=output quantity, and the w_i's are prices of key inputs.

2.1 What, if anything, can you say about the marginal cost function implied by eq. (1)?

Note that:

$$\begin{aligned} \ln c &= 2.1 + 0.6 \ln w_1 + 0.5 \ln w_2 + \ln y \\ c\left(w_1, w_2, y\right) &= e^{2.1} w_1^{0.6} w_2^{0.5} y \\ MC\left(w_1, w_2\right) &= e^{2.1} w_1^{0.6} w_2^{0.5} = constant \\ \therefore AVC\left(w_1, w_2\right) &= constant \\ &\therefore CRS\ production \end{aligned}$$

2.2 What can you say about returns to scale of the underlying production function?

As noted above, constant marginal cost implies constant average variable cost, which implies constant returns to production.

2.3 Is the cost function homogeneous of any degree? Should it be homogeneous of some degree k (and if so, what k)? Comment very briefly.

A cost function should be homogenous of degree 1 in w_1 and w_2. We can check:

$$\begin{aligned} &e^{2.1}\left(Tw_1\right)^{0.6}\left(Tw_2\right)^{0.5} y \\ &e^{2.1} T^{0.6} w_1^{0.6} T^{0.5} w_2^{0.5} y \\ &T^{1.1} e^{2.1} w_1^{0.6} w_2^{0.5} y = T^{1.1} c \end{aligned}$$

It is homogenous of degree 1.1. Close, but not quite a proper cost function.

2.4 (d) Write down the conditional demand function for input 1 implied by equation (1).

By Shephard's Lemma:

$$\begin{aligned} x_1^\star &= \frac{\partial c(\boldsymbol{w}, y)}{\partial w_1} \\ &= 0.6 \cdot e^{2.1} w_1^{-0.4} w_2^{0.5} y \\ &= \frac{0.6}{w_1} \cdot c(\boldsymbol{w}, y) \end{aligned}$$

3 Abe's preferences

Abe's preferences can be represented by $u(x_1, x_2) = \ln x_1 + 2 \ln x_2$.

3.1 Which of the following utility functions (if any) also represent his preferences?

1. $v(x_1, x_2) = x_1 + 2x_2$
   No, constant MRS, unlike u

2. $w(x_1, x_2) = x_1^{0.4} x_2^{0.8}$
   Yes, note:

$$\begin{aligned} \ln w &= 0.4 \ln x_1 + 0.8 \ln x_2 \\ 2.5 \ln w &= \ln x_x + 2 \ln x_2 \\ 2.5 \ln w &= u \end{aligned}$$

   w is just a positive monotonic transformation of u

3. $U(x_1, x_2) = x_1 + g(x_2)$
   No, $MU_1 = 1$, unlike u

3.2 Abe is currently consuming the bundle $(x_1, x_2) = (2, 3)$. How many extra microunits of good 2 would he require to just compensate him for the loss of a microunit of good 1?

This is just a question about the Marginal Rate of Substitution

$$\begin{aligned} MRS &= \frac{MU_1}{MU_2} = \frac{u_1}{u_2} \\ &= \frac{1/x_1}{2/x_2} \\ &= \frac{1}{2} \cdot \frac{x_2}{x_1} = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4} \end{aligned}$$

Abe would need 3/4 of a microunit of good 2 to compensate him.

3.3 What fraction of income m would you predict that Abe would spend on good 2 if, sometime later, prices were $(p_1, p_2) = (1, 4)$?

Solve for the optimal choice at this set of prices:

$$\begin{aligned} \frac{MU_1}{MU_2} &= \frac{p_1}{p_2} \\ \frac{1}{2} \cdot \frac{x_2}{x_1} &= \frac{1}{4} \\ x_2^\star &= \frac{1}{2} x_1^\star \end{aligned}$$

Plug this into the budget:

$$\begin{aligned} m &= p_1 x_1 + p_2 x_2 \\ m &= x_1^\star + 4 \cdot \frac{1}{2} \cdot x_1^\star \\ m &= 3x_1^\star \\ x_1^\star &= \frac{1}{3} m \end{aligned}$$

Abe will spend one-third of his income on good 1 and two-thirds on good 2.

4 Supply

Suppose that that a firm's cost function (for fixed input prices) is $c(y) = y^3 - 6y^2 + 10y + 8$.

4.1 What is its fixed cost? Variable cost? Average variable cost? Marginal cost?

- Fixed cost is 8.
- Variable cost is $y^3 - 6y^2 + 10y$
- Average variable cost is $\frac{c}{y} = y^2 - 6y + 10$
- Marginal cost is $\frac{\partial c}{\partial y} = 3y^2 - 12y + 10$

4.2 What is the firm's supply function y(p)? Its profit as a function of output price p?

First find minimum efficient scale:

$$\begin{aligned} MC &= AVC \\ 3y^2 - 12y + 10 &= y^2 - 6y + 10 \\ 2y^2 - 6y &= 0 \\ 2 \cdot y \cdot (y - 3) &= 0 \end{aligned}$$

The positive root is $y = 3$, so that is the minimum efficient scale, above which supply equals:

$$\begin{aligned} p &= MC \\ p &= 3y(p)^2 - 12y(p) + 10 \\ 0 &= 3y(p)^2 - 12y(p) + 10 - p \end{aligned}$$

Solving via the quadratic formula

$$\begin{aligned} y(p) &= \frac{1}{6} \cdot \left(12 + \sqrt{12^2 - 4 \cdot 3 \cdot (10 - p)}\right) \\ &= 2 + \frac{1}{6}\sqrt{144 - 120 + 12p} \\ &= 2 + \frac{1}{3}\sqrt{6 + 3p} \end{aligned}$$

It can only be the positive portion of the parabola, otherwise output falls with rising prices. This would not be optimizing. We need to find p at minimum efficient scale:

$$\begin{aligned} MC(3) &= 3(3)^2 - 12 \cdot 3 + 10 \\ &= 27 - 36 + 10 = 1 \end{aligned}$$

Therefore:

$$y(p) = \begin{cases} 2 + \frac{1}{3}\sqrt{6 + 3p} & if\ p \geq 1 \\ 0 & if\ p < 1 \end{cases}$$

Profit as a function of price:

$$\begin{aligned} \pi(p) &= p \cdot y(p) - y(p)^3 + 6 \cdot y(p)^2 - 10 \cdot y(p) - 8\ for\ p \geq 1 \\ &= (10 - p) \cdot \left(2 + \frac{1}{3}\sqrt{6 + 3p}\right) - \left(2 + \frac{1}{3}\sqrt{6 + 3p}\right)^3 + 6 \cdot \left(2 + \frac{1}{3}\sqrt{6 + 3p}\right)^2 - 8\ for\ p \geq 1 \end{aligned}$$

$$\pi(p) = \begin{cases} (10 - p) \cdot \left(2 + \frac{1}{3}\sqrt{6 + 3p}\right) - \left(2 + \frac{1}{3}\sqrt{6 + 3p}\right)^3 + 6 \cdot \left(2 + \frac{1}{3}\sqrt{6 + 3p}\right)^2 & for\ p \geq 1 \\ -8 & for\ p < 1 \end{cases}$$

5 HFCs

Hydrofluorocarbons (HFCs) are currently the main coolant used in air conditioners, but earlier this month a world-wide treaty was signed to ban HFCs starting in 2019. Suppose that current total output is $Q^o = 100$ at price $p^o = 200$ and that using a new sort of coolant instead of HFCs will raise marginal cost for air conditioners by about 20.

5.1 What other information would you need to estimate the impact on price and quantity sold?

Demand, or at least demand elasticity. Assuming income is not changing, we need the own price elasticity of demand and the cross-price elasticity of demand for substitutes, such as fans. We also need to know if supply is perfectly competitive and elastic or if it slopes upward what is the supply elasticity.

5.2 b

Write down a very rough numerical guess for each necessary piece of information (e.g., if income elasticity of demand for air conditioners were important, you might write down = 1.0) and compute the predicted impact on price and quantity.

Answer:

$$\begin{aligned} \varepsilon_D &= -3 \\ \varepsilon_S &= 1 \end{aligned}$$

Assume away cross-price effects. Supply contracts. Using the formula in the next part of the question:

$$\Delta P \approx 20 \cdot \frac{\varepsilon_S}{\varepsilon_S + |\varepsilon_D|} = 20 \cdot \frac{1}{4} = 5$$
$$\Delta Q \approx 20 \cdot \frac{|\varepsilon_D|\,\varepsilon_S}{\varepsilon_S + |\varepsilon_D|} \cdot \frac{Q_0}{P_0} = 20 \cdot \frac{3}{4} \cdot \frac{Q_0}{P_0} = 15 \cdot \frac{100}{200} = 7.5$$

5.3 Replace your numerical guesses by algebraic symbols (e.g., just use the symbol in the example above) and write your prediction as a function of the other information.

$$\Delta P \approx 20 \cdot \frac{\varepsilon_S}{\varepsilon_S + |\varepsilon_D|}$$
$$\Delta Q \approx 20 \cdot \frac{|\varepsilon_D|\,\varepsilon_S}{\varepsilon_S + |\varepsilon_D|} \cdot \frac{Q_0}{P_0} = 20 \cdot \frac{|\varepsilon_D|\,\varepsilon_S}{\varepsilon_S + |\varepsilon_D|} \cdot \frac{100}{200} = 10 \cdot \frac{|\varepsilon_D|\,\varepsilon_S}{\varepsilon_S + |\varepsilon_D|}$$

Midterm Practice Problems - ECON 200 - Fall 2016
ANSWER KEY

October 23, 2016

1 Grants

You manage a department whose mission involves two quantities, x_1 (say expenditures on safety) and x_2 (say expenditures on education). Preferences are strictly monotone and convex, and current consumption (expenditure levels in \$million) is $(x_1, x_2) = (12, 36)$.

1.1 a

The department is eligible for a $g = 10$ grant for x_1 from the Department of Homeland Security. Compare your optimal consumption here to that for an unresticted lump sum increase to your budget of 10. Does your answer depend on whether safety is a normal or inferior good? How would your answer change if the grant were 15 instead of 10? Graphs may help you make your point.

Answer: **Note**: This problem goes beyond the usual budget constraint, and so will be covered in more detail in lectures after the midterm.

Let $m > 0$ denote the original budget, so $p_1x_1 + p_2x_2 = m$. If we normalize the prices to 1 (we are purchasing 'expenditures'), then the pre-grant budget constraint becomes $x_1 + x_2 = 48$, graphed in Figure 1 as a black diagonal line.

Because the grant can only be spent on x_1, the budget shifts out by 10 but only at or below $x_2 = 48$, as shown in the blue line segments. (A lump sum would generate the orange budget line.) There is some grant size (around 16 or 17 for homothetic utility) for which this "kink" in the budget would constrain the choice. $g = 15$ is graphed in red.

Assuming homothetic preferences, we can identify the optimal choice, since any interior optimum would satisfy:

$$MRS(x_1, x_2) = \frac{MU_1}{MU_2} = \frac{p_1}{p_2} = 1$$
$$\text{implies } x_2 = 3x_1 \text{ based on observed consumption}$$

Such optima lie on the ray thru $(x_1, x_2) = (12, 36)$, on which $x_2 = 3x_1$, shown as a black dashed ray in Figure 1.

If preferences were not homothetic, and safety were an inferior good, then this relationship does not continue to hold as your income effectively rises with the grant. Instead, you would shift away from x_1 and toward even more x_2, implying that the kink in the budget line might constrain you to $(g, 48)$.

1.2 b

The grant changes to a 1:1 matching grant for x_1 only. What is your optimal consumption now? You may assume preferences are homothetic for simplicity.

Answer: Instead of a "kinked" budget, the budget line now acts as if the price for x_1 is half of what it was before, as shown in blue in Figure 2. An interior solution satisfies $\frac{MU_1}{MU_2} = \frac{p_1}{p_2} = \frac{1}{2}$. With homothetic preferences, each such point lies on a ray such as that graphed in a dashed line in Figure 2. Thus the solution is the intersection of that ray with the new budget line.

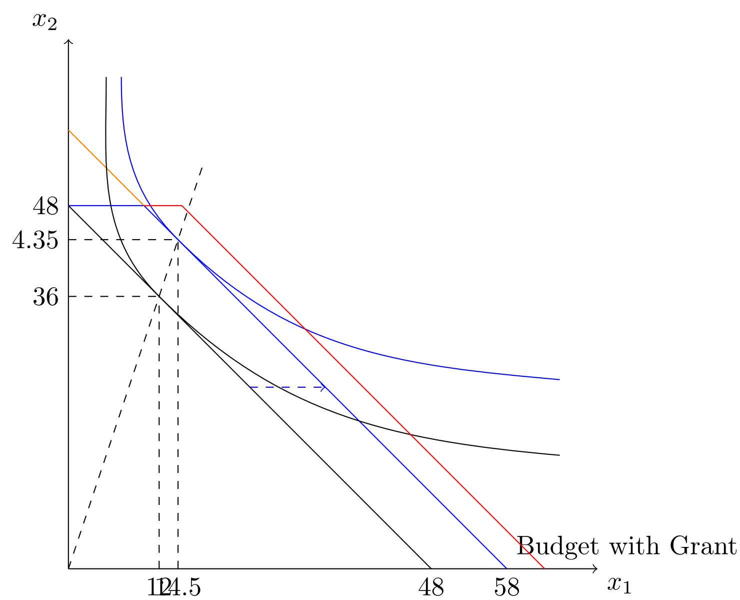


Figure 1: Grant v Lump Sum, Q1a

If preferences are Cobb-Douglas (a stronger assumption than homothetic), then we can say that expenditure shares of both goods remain constant as prices and income vary. Hence $\hat{x}_2$ remains at 36, while $\hat{x}_1 = \frac{12}{1/2} = 24$.

2 Estimating Demand

Your boss is considering a price increase for your firm's main product. He is not sure whether demand is best approximated as linear, or log linear. Your coworker tells him that either semilog or LES actually might be a better approximation. Before investing time and resources in determining the best approximation, your boss asks you to tell him the percentage change in demand he should expect for a 2-10% price increase in each case. He also wants to know which of these specifications have a choke price (above which demand is 0) or a saturation level (a maximum quantity demanded even at price 0). For each of the 4 specifications, write down the answer to his questions in terms of the relevant coefficient (which will be estimated from the data).

1. Linear:

$$D(P) = a - b \cdot P$$

   (a) There is a choke price: demand is zero when $P \geq \frac{a}{b}$

   (b) There is a maximum quantity demanded: a

   (c) The elasticity is $-b \cdot \frac{P}{Q}$, so the answer depends on current price and quantity. A 10% increase would lead to a decrease in quantity demanded of $10b\frac{P}{Q}$ percent

2. Log linear:

$$\log D(P) = a - b \cdot \log P$$

   (a) There is no choke price nor maximum quantity demanded.

   (b) The elasticity is $-b$, so a 10% increase in price would lead to a decrease in quantity demanded of $10b$ percent

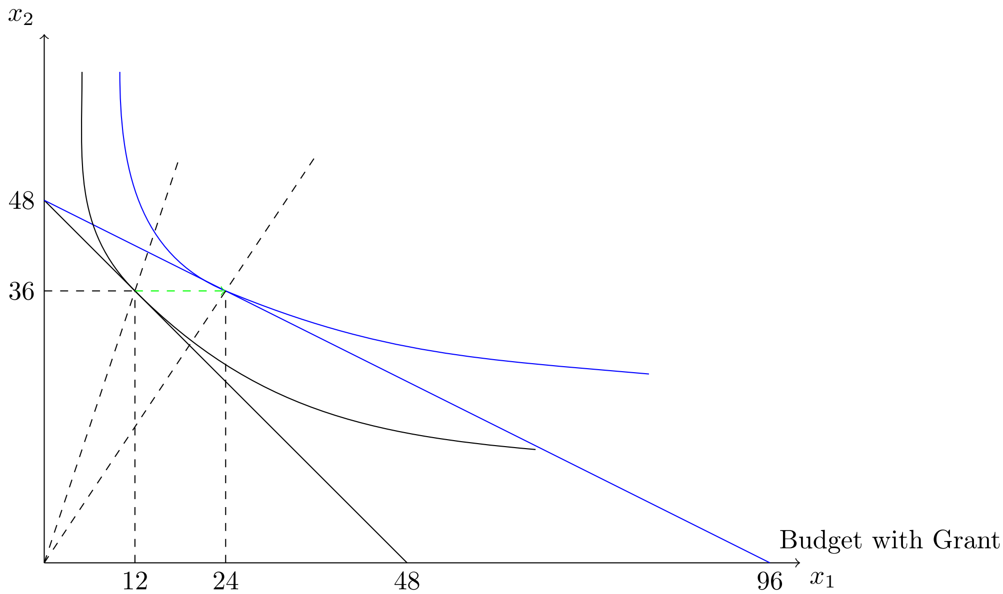


Figure 2: Matching Grant, Q1b

3. Semilog:

$$D(P) = a - b \cdot \log P$$

Note: Semi-log and LES have not been covered in class yet this quarter, so these parts should be considered very optional.

   (a) There is a choke price at $\log P \geq \frac{a}{b}$
   (b) There is no maximum quantity demanded
   (c) The elasticity is $-b/Q$, so the answer depends on current quantity. A 10% increase would lead to a decrease in quantity demanded of $10b/Q$ percent.

4. LES [Linear Expenditure System]

$$D(P) = b + \frac{a \cdot I}{P} - \sum_j \frac{P_j}{P} b_j$$

   (a) There is a choke price $P \geq \frac{1}{b} \cdot \left[\sum p_j b_j - a \cdot I\right]$ that depends on the prices of other goods, the marginal rates of substitution, and income
   (b) There is no maximum quantity demanded
   (c) The elasticity is $\frac{-aI + \sum p_j b_j}{PQ} < 0$, so the answer depends on current revenue, income, and the prices of other goods. A 10% increase in price would lead to a decrease in quantity demanded of $\frac{10 \sum p_j b_j - 10aI}{PQ}$ percent

3 Income Elasticity

Your firm sells scentroids to middle-income teenagers. Experience with these customers, who spend approximately 10% of their income on your product, indicates that a 1% increase in their income tends to increase your sales by 3%, while a 1% price increase tends to decrease the quantity they buy by about 0.8%. The opportunity soon will arise to sell scentroids to high-income teenagers. Assuming that they respond to changes in income and price the same way as current customers (except that they currently spend less than 1% of income on products like yours) estimate how they would respond to a 1% price decrease.

Answer: **Note:** Once again, we did not yet cover the Slutsky equation in class yet; that will be done in the first lecture after the midterm.

The Slutsky equation in elasticities (and expenditure share s_i) is:

$$\begin{aligned} \varepsilon &= \varepsilon^{Hicks} - s_i \cdot \varepsilon_m \\ -.8 &= \varepsilon^{Hicks} - .1 \cdot 3, \end{aligned}$$

So the pure substitution elasticity is $\varepsilon^{Hicks} = (-.8 + .3) = -0.5$. For the High Income (HI) kids we then have

$$\begin{aligned} \varepsilon_{HI} &= \varepsilon^{Hicks} - .01 \cdot 3 \\ \varepsilon_{HI} &= -0.5 - .01 \cdot 3 \\ &= -0.53 \end{aligned}$$

Thus the response would be an increase in quantity demanded of about 0.53 percent.

4 Preferences

4.1 Blutarsky

Blutarsky is planning a fraternity party. He cares only about alcohol content. Write down a utility function for him for x_1=bottles of beer and x_2= bottles of vodka, given that each bottle of vodka has the same amount of alcohol as three six packs of beer.

Answer: Alcohol content of a bottle of vodka is $3 * 6 = 18$ times that of a bottle of beer, so

$$u(x_1, x_2) = 18 \cdot x_1 + x_2.$$

4.2 Justine

Justine's preferences can be represented by $u(x_1, x_2) = \ln x_1 + \ln x_2$. Which of the following utility functions (if any) also represent her preferences? $v(x_1, x_2) = x_1 + x_2$, $w(x_1, x_2) = x_1^{0.4} x_2^{0.4}$, $U(x_1, x_2) = x_1 + g(x_2)$. Explain very briefly.

Answer:

Only $w(x_1, x_2)$, since it can be transformed as follows:

$$\begin{aligned} w(x_1, x_2) &= x_1^{0.4} x_2^{0.4} \\ \ln w &= 0.4 \cdot \ln x_1 + 0.4 \cdot \ln x_2 \\ 2.5 \ln w &= \ln x_1 + \ln x_2 = u(x_1, x_2) \end{aligned}$$

Thus, they represent the same preferences, since u is a positive monotonic transformation of w.

v and U have constant marginal utility of x_1, which is not a feature of u.

5 Tricounty Organic Strawberries

Your client, the Tricounty Organic Strawberry Association, provides data on their sales revenue, from which you estimate the demand function for their product.

5.1 Data

What other data will you need to estimate income elasticity η, own price elasticity ε, and cross price (for inorganic strawberries) elasticity ε_c?

Answer:

Prices for organic (P) and inorganic (P_C) strawberries, and mean income of consumers.

5.2 Estimation

Write out a convenient equation to estimate these elasticities from that data.

Answer:

We can impose a log-linear form:

$$\log D(P) = \alpha + \varepsilon \log P + \eta \log Y + \varepsilon_c \log P_C$$

5.3 Interpretation

Suppose that you estimate $\eta = 1.7$, $\varepsilon = -1.2$, and $\varepsilon_c = 0.4$. A former classmate comments that your estimates can't be right because the elasticities should sum to 0. How should you respond?

Answer:

It is unlikely that we have included all of the relevant other products' prices, and perhaps we missed some important substitutes (e.g., other fruit) and (more importantly, since the sum of elasticities is too high) complements (e.g., shortcake). However, if we believe that there aren't that many omitted complements, then we should be concerned. It likely implies that we have imposed a poor functional form, or we may be estimating from non-exogenous changes in prices and income. Or perhaps we made a simple mistake in entering the data, or something like that. We should see what happens when, after checking the data and estimation procedure carefully, we impose the constraint that the estimated elasticities sum to 0.

6 North Ifstan

North Ifstan (NI) has domestic suppliers of internet services whose monthly supply curve is well approximated by $S(p) = 10p$, while monthly demand is well approximated by $D(p) = 1000 - 10p$. International suppliers can provide any amount of access at $p = 25$.

6.1 a

Compute the current competitive equilibrium (CE) price, domestic producers surplus (PS), and consumer surplus (CS).

Answer:

$p_{CE} = 25$

$PS = \frac{1}{2} 25 \cdot 250 = 3,125$

$CS = \frac{1}{2} 75 \cdot 750 = 28,125$

6.2 b

The NI government is considering a rule that would eliminate foreign supply. How much (if at all) would domestic suppliers benefit? How much would consumers lose?

Answer:

Benefit to domestic suppliers:

$25 \cdot 250 + \frac{1}{2} \cdot 25 \cdot 250 = 9,325$

Loss to consumers:

$25 \cdot 500 + \frac{1}{2} \cdot 25 \cdot 250 = 15,625$

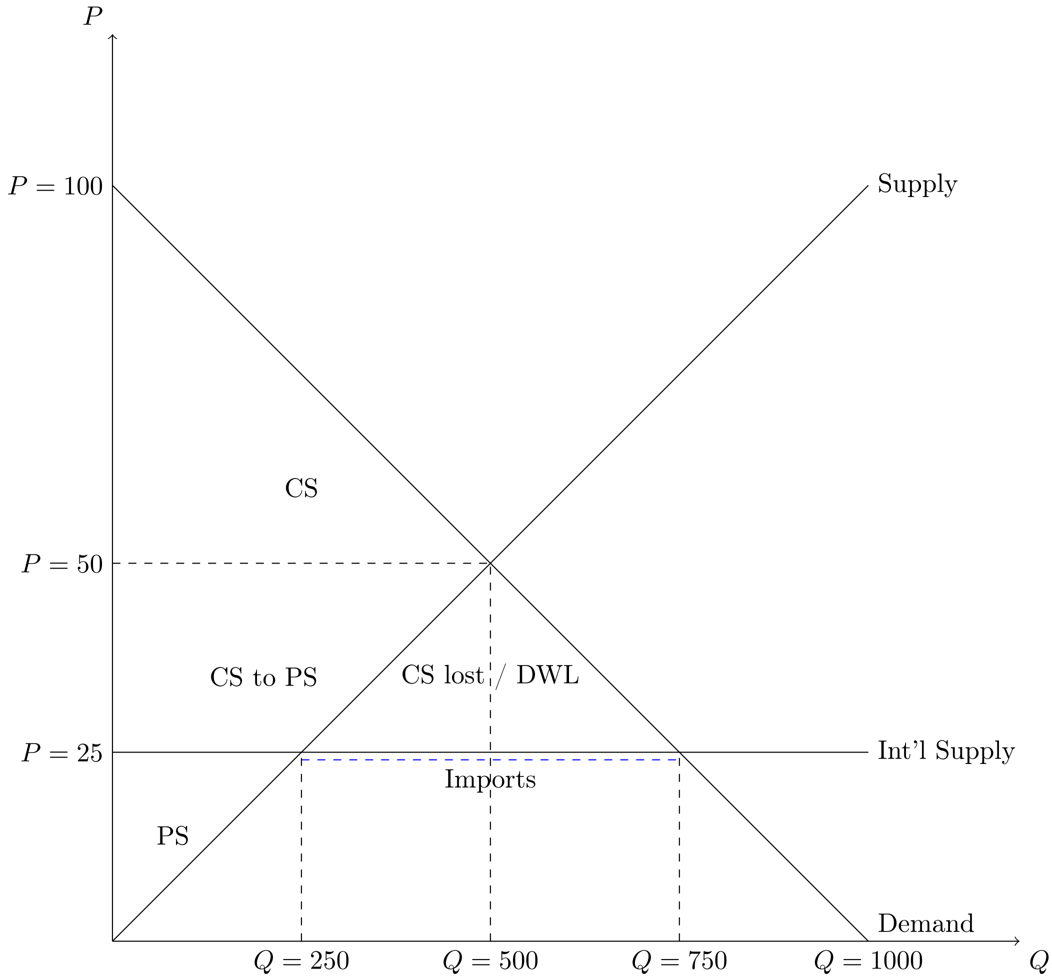


Figure 3: North Ifstan, Q6

Answers for Final practice problems

UCSC, Econ 200, Fall 2016

1. Business is good at Acme Products. George, the owner, estimates that he can earn an additional 12 per year indefinitely starting next year if he invests in expansion now, George is risk-neutral: he is not averse to risk, but neither does he seek it. The investment will be sunk once George makes the decision.

(a) (4 points) If George can finance at rate $k = 0.10$, what is his maximum willingness to pay for that investment?

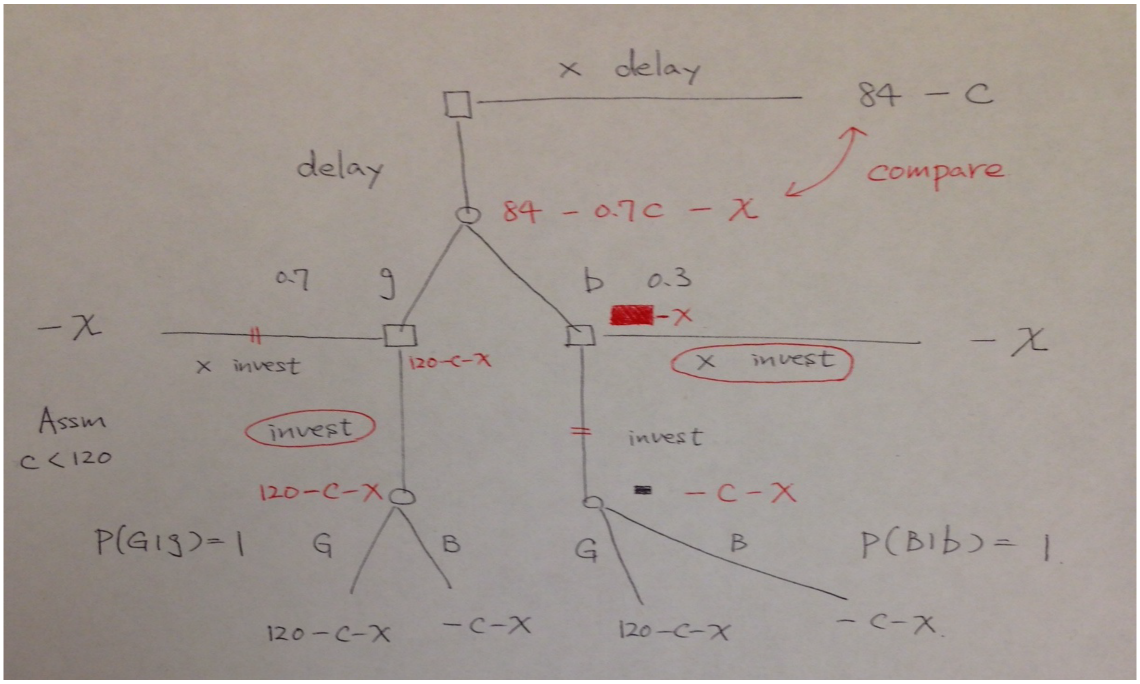
> **Solution:** Maximum willingness to pay for this investment is the present value of the expansion,
>
> $$\begin{aligned} \frac{12}{(1+0.1)} + \frac{12}{(1+0.1)^2} + \cdots &= \left(\frac{12}{1-\frac{1}{1.1}}\right)\left(\frac{1}{1+0.1}\right) \\ &= 120. \end{aligned} \tag{1}$$

(b) (6 points) Suddenly George realizes that things might go wrong during the next year, and if so the additional earnings will be zero. He estimate the probability if 0.3 that things will go wrong. Now how much is he willing to pay for that investment.

> **Solution:** Expected present value of the investment with possible failure is,
>
> $$0.7(120) + 0.3(0) = 84. \tag{2}$$

(c) (8 points) George then realizes that, at some cost, he can delay making the investment until he knows whether or not things will go wrong. How much cost of delay is he willing to incur?

> **Solution:** Let c be the investment cost and x be the cost to delay. The decision tree is as follows. ("Not delay" is marked "x delay" and similarly for Not invest.) Therefore, willingness to pay for delaying is $(84 - 0.7c) - (84 - c) = 0.3c$.



3. Suppose that Abe makes risky choices as if maximizing the expected value of the Bernoulli function $u(m) = \ln(m+1)$. He is faced with a situation in which he will receive either 0 or 24; the outcomes are equally likely.

(a) (4 points) What is the expected outcome? Variance of outcome?

Solution: Expected value and the variance of the situation is,

$$E[m] = \frac{1}{2}(0) + \frac{1}{2}(24) = 12,$$
$$V[m] = E[(m - E(m))^2] = \frac{1}{2}(0-12)^2 + \frac{1}{2}(24-12)^2 = 144. \tag{3}$$

(b) (6 points) What is Abe's certainty equivalent of the risky outcome? What is the maximum amount he would be willing to pay an insurer to get the mean outcome for sure?

Solution: Certainty equivalence, m^{CE}, satisfies,

$$\begin{aligned} \ln(m^{CE}+1) &= \frac{1}{2}\ln(0+1) + \frac{1}{2}\ln(24+1), \\ &= \ln(24+1)^{\frac{1}{2}} = \ln 5 = \ln(4+1). \end{aligned} \tag{4}$$

Thus, $m^{CE} = 4$. Max insurance payment = RP = $E[m] - m^{CE} = 12 - 4 = 8$.

(c) (4 points) What is Abe's coefficient of relative risk aversion at the mean outcome?

Solution: Since $u'(m) = (m+1)^{-1}$ and $u''(m) = -(m+1)^{-2}$, the coefficient of relative risk aversion is,

$$r(m) = -\frac{-(m+1)^{-2}}{(m+1)^{-1}}(m) = \frac{m}{m+1}. \tag{5}$$

Thus, $r(E[m]) = \frac{12}{13}$.

4. Ajax Inc and Bestco produce imperfectly substitutable products, each at per unit marginal costs of 1. Inverse demand for Ajax is $p_A = 6 - q_A - 0.5q_B$ and symmetrically Bestco's inverse demand is $p_B = 6 - q_B - 0.5q_A$, where q_A and q_B denote the quantities of the two firms. Find the Nash equilibria of the following games, where payoffs are profits.

(a) (6 points) Both firms choose quantity simultaneously and independently.

Solution: (Cournot competition) Firm A maximizes profit with respect to q_A,

$$\max_{q_A}(6 - q_A - \frac{1}{2}q_B)q_A - q_A. \tag{6}$$

First-order condition is,

$$5 - \frac{1}{2}q_B - 2q_A^* = 0. \tag{7}$$

Since firms are symmetric,

$$5 - \frac{1}{2}q_A - 2q_B^* = 0. \tag{8}$$

Thus, in Nash equilibrium, $q_A^{NE} = q_B^{NE} = 2$.

(b) (4 points) Ajax chooses quantity first, then Bestco observes it and then chooses its own quantity.

Solution: (Stackelberg) A moves first and B moves second. We solve this problem by backward induction. The best response function of firm B is the same as Cournot competition case,

$$\begin{aligned} & 5 - \frac{1}{2}q_A - 2q_B^* = 0, \\ \Leftrightarrow \quad & BR_B(q_A) = \frac{5}{2} - \frac{1}{4}q_A. \end{aligned} \tag{9}$$

Firm A maximizes profit taking this best response function of firm B into consideration,

$$\max_{q_A} \quad \left(6 - q_A - \frac{1}{2}\left(\frac{5}{2} - \frac{1}{4}q_A\right)\right) q_A - q_A. \tag{10}$$

First-order condition is,

$$\frac{15}{4} - \frac{7}{4}q_A^* = 0. \tag{11}$$

Thus, $q_A^{NE} = \frac{15}{7}$ and $q_B^{NE} = \frac{55}{28}$.

(c) (6 points) Both firms choose price simultaneously and independently.

Solution: (Bertrand) By rewriting the demand system, we obtain,

$$q_A = 4 - \frac{4}{3}p_A + \frac{2}{3}p_B, \tag{12}$$

$$q_B = 4 - \frac{4}{3}p_B + \frac{2}{3}p_B. \tag{13}$$

Firm A maximizes its profit with respect to p_A,

$$\max_{p_A} \quad p_A(4 - \frac{4}{3}p_A + \frac{2}{3}p_B) - (4 - \frac{4}{3}p_A + \frac{2}{3}p_B). \tag{14}$$

First-order condition is,

$$\frac{16}{3} - \frac{8}{3}p_A^* + \frac{2}{3}p_B = 0. \tag{15}$$

Since firms are symmetric,

$$\frac{16}{3} - \frac{8}{3}p_B^* + \frac{2}{3}p_A = 0. \tag{16}$$

Thus, $p_A^{NE} = p_B^{NE} = \frac{8}{3}$.

(d) (4 points) Ajax chooses price first, then Bestco observes it and then chooses its own price.

Solution: Again, we solve this problem by backward induction. The best response function of firm B is the same as Bertrand case,

$$\begin{aligned} &\frac{16}{3} - \frac{8}{3}p_B^* + \frac{2}{3}p_A = 0 \\ \Leftrightarrow\ \ & BR_B(p_A) = 2 + \frac{1}{4}p_A. \end{aligned} \tag{17}$$

Firm A maximizes profit taking this best response function of firm B into consideration,

$$\max_{p_A} \ (p_A - 1)\left(4 - \frac{4}{3}p_A + \frac{2}{3}\left(2 + \frac{1}{4}p_A\right)\right). \tag{18}$$

First-order condition is,

$$-\frac{7}{3}p_A^* + \frac{39}{6} = 0. \tag{19}$$

Thus, $p_A^{NE} = \frac{39}{14}$ and $p_B^{NE} = \frac{151}{56}$.

5. Name at least three techniques of price discrimination commonly used by airlines. How does each technique overcome the main obstacles to price discrimination? Comment very briefly on the efficiency implications of each technique. (18pts)

Solution: Laws against one customer reselling an airline ticket to another customer virtually eliminate the arbitrage obstacle for any form of price discrimination. Self-selection, or incentive compatibility, helps cope with the unobservability of WTP obstacle, as noted below.

- Timing of purchase: Charge higher prices for those who buy tickets on closer dates to departure dates. Incentive compatible because last minute customers tend to have less price-elastic demand. Can be regarded as Third-degree PD.

- First-class vs Economy-class: Price way higher prices for consumers with high willingness to pay. Incentive compatible because of the difference in service and quality. Can be regarded as Second-degree PD.

- Mileage: Give bonuses to customers with low willingness to pay. Incentive compatible because customers with high willingness to pay does not have incentive to earn mileage because of opportunity cost. Similar to Second-degree/ quantity discount, and a little like a 2-part tariff.

- Payment history: This is similar to the example from one of the clip in class. Charges different prices to different customers based on histories of purchase, which reflect willingness to pay. Time will tell us whether this new way of price discrimination will work or not. Might approximate First-degree.

6. The price of crude oil decreased about 60% over the last 1.5 years. Suppose that a sector of the economy can be approximated reasonably well by a production function with crude oil as one input and labor and capital as the other inputs. If that production function is Leontieff, what impact would you expect to see on the sector's input demands, cost and output? How would you answer change if the production function were CES with substitution elasticity between 0 and 1? Which production function seems a more reasonable description in the short run of a month or two? (14pts)

Solution:

- When the relative price of input changes, ratios of input quantities do not change if production function is Leontieff. One way to see this is to draw isoquant. Isoquant has a kink and regardless of the input price ratio, cost is minimized at the kink. Still, the overall marginal cost is lower, and thus total cost. With lower marginal cost, the profit-maximizing output should increase.
- When the elasticity of substitution is not zero, there is a substitution toward the lowered priced goods. This additional effect even more lowers the total cost.
- In the short-run, it is hard to switch to different input ratios. As a result, production function with low elasticity of substitution is suitable.

1 Question 7

You have been hired as a consultant by a firm that produces a new, irreplaceable office gizmo (with no close substitutes) exclusively under a patent. The firm has two factories, one in Santa Cruz, the other in San Jose. The managers of the two factories have an ongoing dispute over what the company should do. The Santa Cruz manager argues that the San Jose plant should be closed because leasing and other fixed costs are much higher in San Jose and it makes no sense to produce at such an expensive facility. The SJ factory manager counters that the Santa Cruz plant should be closed because even though the factory lease is cheap, Santa Cruz workers are less productive. When the surf is big, they all call in sick and this reduces the plant's productivity. Your research shows that inverse demand is $P = 100 - Q$, where $Q = Q_{SC} + Q_{SJ}$. Costs at the San Jose factory are $C_{SJ} = 75 + 2 \cdot Q_{SJ}^2$, and costs are $C_{SC} = 25 + 3 \cdot Q_{SC}^2$ at Santa Cruz. Assume that you can't alter these cost functions.

a) With both plants operating, how should production be allocated between the two plants? What are the profits for the firm at this level of joint production?

The main point here is that you want to make sure you are producing at a point where the marginal costs of both plants are equalized. We can do that in a number of ways. Note from the profit maximization problem

$$\max_{Q_{SJ},Q_{SC}} P\left(Q_{SJ}+Q_{SC}\right)\cdot\left(Q_{SJ}+Q_{SC}\right)-C_{SJ}\left(Q_{SJ}\right)-C_{SC}\left(Q_{SC}\right)$$

we find

$$\frac{\partial P}{\partial Q_{SJ}} \cdot (Q_{SJ} + Q_{SC}) + P\left(Q_{SJ} + Q_{SC}\right) - C'_{SJ} = 0$$
$$\frac{\partial P}{\partial Q_{SC}} \cdot (Q_{SJ} + Q_{SC}) + P\left(Q_{SJ} + Q_{SC}\right) - C'_{SC} = 0$$

Plugging in the numbers,

$$\begin{aligned} 100 - 2Q_{SJ} - 2Q_{SC} &= 4Q_{SJ} \\ 100 - 2Q_{SJ} - 2Q_{SC} &= 6Q_{SC} \end{aligned}$$

Solving this,

$$\begin{aligned} Q_{SJ} &= \frac{1}{6}\left(100 - 2Q_{SC}\right) \\ Q_{SC} &= \frac{1}{8}\left(100 - 2Q_{SJ}\right) \\ Q_{SJ} &= \frac{50}{3} - \frac{1}{3}\left(\frac{25}{2} - \frac{1}{4}Q_{SJ}\right) \\ &= \frac{75}{6} + \frac{1}{12}Q_{SJ} \\ \frac{11}{12}Q_{SJ} &= \frac{75}{6} \\ Q_{SJ} &= \frac{150}{11} \\ Q_{SC} &= \frac{25}{2} - \frac{1}{4} \cdot \frac{150}{11} = \frac{22 \cdot 25 - 150}{44} = \frac{400}{44} = \frac{100}{11} \end{aligned}$$

Given this, profit is as follows:

$$\begin{aligned} \pi =& \left(100 - 2 \cdot \left(\frac{150}{11} + \frac{100}{11}\right)\right) \cdot \left(\frac{150}{11} + \frac{100}{11}\right) \\ & - 75 - 2 \cdot \left(\frac{150}{11}\right)^2 - 25 - 3 \cdot \left(\frac{100}{11}\right)^2 \\ \pi \approx& 519.83 \end{aligned}$$

Note that we should look for corner solutions. Suppose $Q_{SJ} = 0$, but we keep the San Jose plant.

$$\max_{Q_{SC}} P\left(Q_{SC}\right) Q_{SC} - 100 - 3Q_{SC}^2$$

$$\begin{aligned} P'Q_{SC} + P &= 6Q_{SC} \\ -Q_{SC} + 100 - Q_{SC} &= 6Q_{SC} \\ 8Q_{SC} &= 100 \\ Q_{SC} &= 12.5 \end{aligned}$$

Profit in this scenario is

$$\pi_{SC\ only} = (100 - 12.5) \cdot 12.5 - 100 - 3\,(12.5)^2 = 525$$

Suppose instead $Q_{SC} = 0$

$$\max_{Q_{SJ}} P\,(Q_{SJ})\,Q_{SJ} - 100 - 2Q_{SJ}^2$$

$$\begin{aligned} P'Q_{SJ} + P &= 4Q_{SJ} \\ -Q_{SJ} + 100 - Q_{SJ} &= 4Q_{SJ} \\ 6Q_{SJ} &= 100 \\ Q_{SJ} &= \frac{50}{3} \end{aligned}$$

Profit in this scenario is

$$\pi_{SJ\ only} = \left(100 - \frac{50}{3}\right) \cdot \frac{50}{3} - 100 - 2 \cdot \left(\frac{50}{3}\right)^2 = 733.33$$

b) Given the arguments of the two managers, what is your recommendation? Explain. What are the profits of the firm if they follow your recommendation?
As noted above, if the firm acts as a monopolist with only one or the other plant open, the profits are 600 closing the San Jose plant (add 75 to the SC only scenario) or 758.33 closing the Santa Cruz plant (add 25 to the SJ only scenario). Closing the Santa Cruz plant is optimal, essentially due to high marginal cost there. Of course, the answer could change if demand were to increase or decrease sufficiently.

2 Question 8

The incumbent firm and all potential entrants have marginal cost 4 in an industry with inverse demand $p = 48 - Q$ for a homogeneous product. All customers buy from the lowest price firm.

a. Before other firms can enter, what is the maximum profit for the incumbent?

$$\max_{p}\,(48 - q)\,q - 4q = 44q - q^2$$

$$\begin{aligned} 44 - 2q &= 0 \\ q^M &= 22 \\ \pi^M &= (48 - 22) \cdot 22 - 4 \cdot 22 = 484 \end{aligned}$$

b. After entry is possible, what is the maximum profit for the incumbent?

Entry enables unfettered competition, driving economic profit down to zero for all firms, including the incumbent, since marginal cost is flat and common across firms. With no barriers to entry and constant marginal cost, any number of firms $n > 2$ could split demand. At $p = MC = 4$, $Q = 48 - 4 = 44$, the firm's output is $q = 44/n$ and its profit is $\pi = (p - c)\, q = 0 \cdot q = 0$.

c. The incumbent has a unique option to lower marginal cost to 2 before entry is possible. How much would he be willing to pay to exercise this option?

If the incumbent has a lower marginal cost, he can charge $4 - \epsilon$ and take the whole market. Profit would then be

$$\pi = (4 - \epsilon)\, q - 2 \cdot q = (2 - \epsilon)\, q \approx 2 \cdot 44 = 88.$$

The incumbent is willing to pay up to approximately 88 per period to exercise the option. If the discount rate (adjusted for the risk of market disruption) is r and the cost advantage is permanent, then she is WTP $\approx 88/r$, e.g., about 900 if r is a bit under 10% per period.

Problem Set 1

Econ 200

I. Short Case Study Problems

1. At one time, the US domestic wheat supply (in millions of bushels per year) was fairly steady at approximately $S(p) = 1800 + 240p$, where p is the price in \$ per bushel. Demand (mainly for exports) fluctuated over this period, from an estimated $D(p) = 3550 - 266p$ in year 1 to $D(p) = 2580 - 194p$ in year 5.

(a) Compute the market clearing price in year 1 and in year 5.

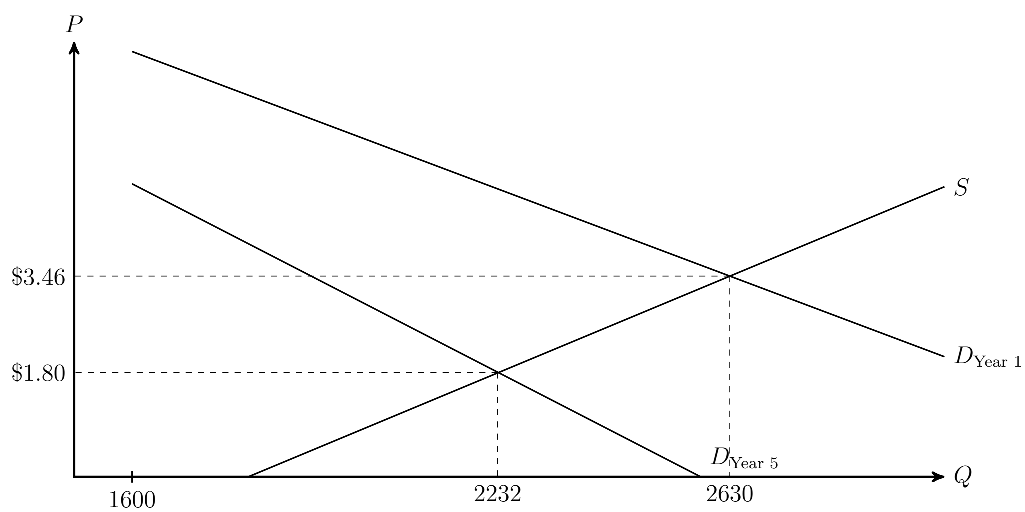
Solution: Set $S(p) = D_{\text{Year 1}}(p)$:

$$1800 + 240p = 3550 - 266p \Rightarrow p_{\text{Year 1}} = \frac{3550 - 1800}{240 + 266} = \$3.46$$

Likewise, $S(p) = D_{\text{Year 5}}(p)$:

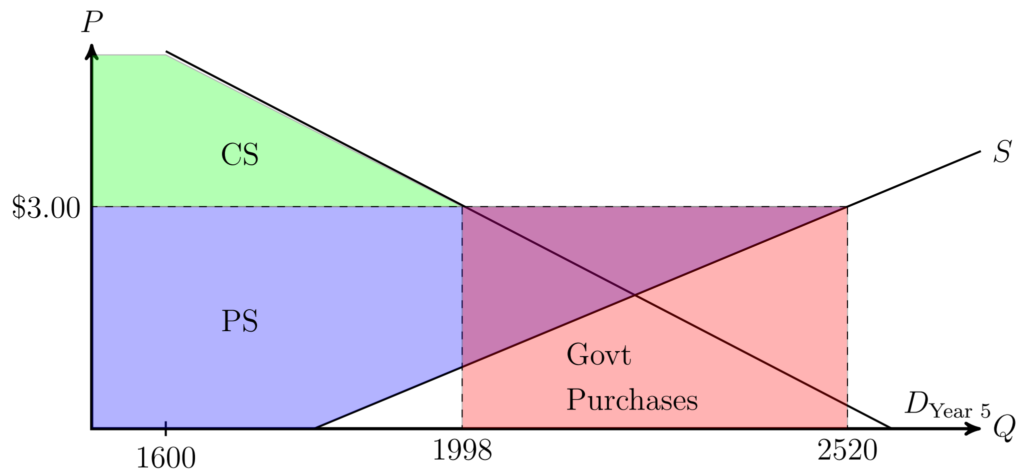
$$1800 + 240p = 2580 - 194p \Rightarrow p_{\text{Year 5}} = \frac{2580 - 1800}{240 + 194} = \$1.80$$

These prices give equilibrium quantities of $q_{\text{Year 1}} = 2630.4$ million bushels and $q_{\text{Year 5}} = 2232$ million bushels.



(b) Suppose the government supported a $p = \$3.00$ price throughout this period. (This is a rough approximation of actual policy.) Compute the prices and quantities seen by suppliers, demanders, and government price supporters (i.e. taxpayers) in year 1 and year 5. Also, relative to the competitive outcome in part (a), calculate the gains and losses in producer, consumer, and taxpayer surplus, expressed in $billions.

Solution: Note that the competitive price in year 1 is greater than the price floor of $p = \$3.00$. Thus, the government policy has no effect (i.e. the price floor is not binding) in year 1 and the price remains $p_{\text{Year 1}} = 3.46$ and the quantity remains $q_{\text{Year 1}} = 2630.4$ million bushels. There is no change in any of the surpluses.



In year 5, the price floor is binding so that the price becomes $p = \$3.00$. Consumers demand $D_{\text{Year 5}}(3) = 2580 - 194(3) = 1998$ million bushels while producers supply $S(3) = 1800 + 240(3) = 2520$ million bushels. In order to maintain the price floor, the government must buy the excess wheat at a price of $3.00. To calculate the surpluses, we need the inverse demand and supply functions:

$$p_d = \frac{2580}{194} - \frac{1}{194}q$$

$$p_s = \begin{cases} 0 & \text{if } q < 1800 \\ -\frac{1800}{240} + \frac{1}{240}q & \text{otherwise} \end{cases}$$

Thus, the change in consumer surplus is:

$$\begin{aligned}
CS' - CS &= \int_0^{1998} (p_d - 3)dq - \int_0^{2232} (p_d - 1.8)dq \\
&= \int_0^{1998} \left(\frac{1998}{194} - \frac{1}{194}q\right) dq - \int_0^{2232} \left(\frac{2230.8}{194} - \frac{1}{194}q\right) dq \\
&= 10289 - 12826 \\
&= -2537 \ \frac{\$\text{million}}{\text{year}}
\end{aligned}$$

or -2.537 billions of dollars per year. The change in producer surplus is:

$$\begin{aligned}
PS' - PS &= \int_0^{2520} (3 - p_s)dq - \int_0^{2232} (1.8 - p_s)dq \\
&= 3(1800) + \int_{1800}^{2520} (3 - p_s)dq - [1.8(1800) + \int_{1800}^{2232} (1.8 - p_s)dq] \\
&= 2160 + \int_{1800}^{2520} \left(\frac{2520}{240} - \frac{1}{240}q\right) dq - \int_{1800}^{2232} \left(\frac{2232}{240} - \frac{1}{240}q\right) dq \\
&= 2160 + (13230 - 12150) - (10379 - 9990) \\
&= 2851 \ \frac{\$\text{million}}{\text{year}}
\end{aligned}$$

or 2.851 billions of dollars per year. Taxpayer surplus without the price floor is zero. With the price floor, it is a negative amount equal to the total that is spent purchasing the excess wheat. Therefore the change in taxpayer surplus is:

$$TS' - TS = -3(2520 - 1998) - 0 = -1566 \ \frac{\$\text{million}}{\text{year}}$$

or -1.566 billions of dollars per year. Overall effect on Total Surplus is $-2.537 + 2.851 - 1.566 = -1.252$ billion dollars.

2. The US domestic sugar supply (in billions of pounds per year) is approximately $S(p) = -8.19 + 1.07p$, where p is the price in cents per pound, and domestic demand is approximately $D(p) = 23.86 - 0.25p$. The world price recently was 12 cents and world supply is extremely elastic. The US government has established a quota on imports of 3.0 billion pounds per year and assigns the import rights to specific firms.

(a) Relative to competitive equilibrium, estimate the impact of the quota on price, domestic competition of sugar, and surplus of domestic producers and of import right holders.

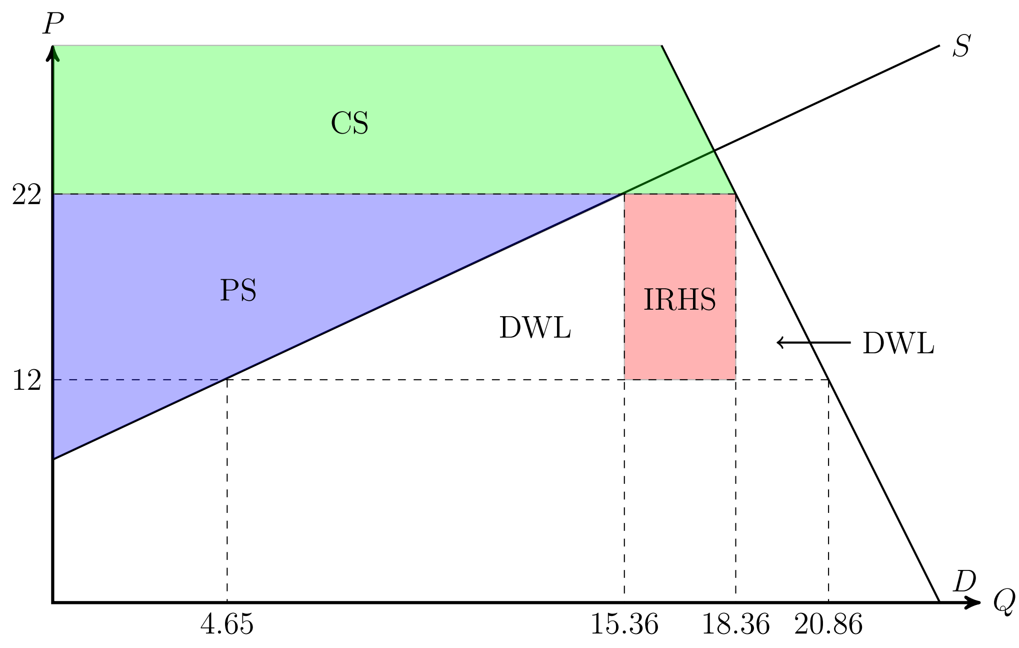
Solution: For this problem, we assume that world supply being "extremely elastic" means that it is perfectly elastic. The first step is to find the equilibrium in the absence of the quota. The price of sugar in this case is just the world price, $p^* = 12$. Then domestic supply is $S^* = 4.65$ and domestic consumption is $D^* = 20.86$.

Since imports would be $20.86 - 4.65 = 16.21$ billion pounds without the quota, the quota is effective in limiting the quantity of imports. With the quota, we know that $S_{\text{Quota}} + 3 = D_{\text{Quota}}$. Therefore,

$$-8.19 + 1.07p + 3 = 23.86 - 0.25p \Rightarrow p_{\text{Quota}} = 22.01 \text{ cents/pound}$$

which gives $S_{\text{Quota}} = 15.36$ and $D_{\text{Quota}} = 18.36$. Therefore, the quota has the following effect on price and domestic consumption:

$$\Delta p = p_{\text{Quota}} - p^* = 10.01 \text{ cents/pound}$$
$$\Delta D = D_{\text{Quota}} - D^* = -2.5 \text{ billion pounds/year}$$



The inverse supply function is

$$p_s = 7.654 + 0.935q$$

Therefore the change in domestic producer surplus is:

$$\begin{aligned} PS' - PS &= \int_0^{15.36} (22.01 - p_s)dq - \int_0^{4.65} (12 - p_s)dq \\ &= 100\ (\frac{\text{billion pounds}}{\text{year}})(\frac{\text{cents}}{\text{pound}}) \\ &= \$1 \text{ billion/year} \end{aligned}$$

The change in the surplus of import right holders is:

$$\begin{aligned} 3(22.01 - 12) - 16.21(12 - 12) &= 30.0\ (\frac{\text{billion pounds}}{\text{year}})(\frac{\text{cents}}{\text{pound}}) \\ &= \$300 \text{ million/year} \end{aligned}$$

(b) What is the maximum willingness to pay (in \$millions to be spent on campaign contributions, lobbying, etc) of the last two groups to maintain the existing quota system? What is the total efficiency loss?

Solution: The total efficiency loss is the loss that consumers experience minus the benefits that producers and import rights holders receive:

$$\begin{aligned} \Delta CS + \Delta PS + 30.0 &= -66.3\ (\frac{\text{billion pounds}}{\text{year}})(\frac{\text{cents}}{\text{pound}}) \\ &= \$663 \text{ million/year} \end{aligned}$$

Together, domestic producers and import rights holders are willing to pay up to their combined benefit from the quota, \$1.3 billion per year, in order to maintain the current system. If the import regime is determined every x years, then domestic producers and import rights holders would be willing to contribute the net present value (NPV) of their total expected benefit over the x-year term. (Issues to think about: what discount rate should be applied? how to adjust for risk?)

3. Suppose that demand for gasoline has elasticity -0.3 in the short run and -0.8 in the long run, that supply elasticity is 0.6 in both SR and LR, that the current price is \$3.50/gal and that current quantity is 0.5 billion gallons/day with essentially no

sales tax. A \$0.35/gal sales tax is proposed.

(a) Predict the impact of the proposed tax on price, quantity, tax revenue, and producer and consumer surplus.

Solution: In the short run, $\frac{\partial p_d}{\partial t} = \frac{0.6}{0.6+|-0.3|} = \frac{2}{3}$. The impact on p_d is

$$\Delta p_d = \frac{\partial p_d}{\partial t} \cdot \Delta t = \frac{2}{3}(0.35) = \frac{0.7}{3}$$

In the short run the tax raises the price consumers pay from \$3.50/gal to \$3.73/gal (of which producers receive \$3.38/gal) and the quantity sold falls by

$$\Delta D = \frac{\partial D}{\partial p_d}\Delta p_d = \epsilon_d \frac{D}{p_d}\Delta p_d = -0.3\left(\frac{0.5}{3.5}\right)\frac{0.7}{3} = -0.01$$

from 0.5 to 0.49 billion gallons per day. The tax revenue raised is

$$(0.35\ \frac{\$}{\text{gallon}})(0.49\ \frac{\text{billion gallons}}{\text{day}}) = \$172 \text{ million/day}$$

The easiest approach to find the change in producer and consumer surplus is to assume that supply and demand are linear. If this is the case, then the loss in consumer surplus is given by the area of the rectangle, $0.49(3.73 - 3.5)$, plus the area of the triangle, $0.5(0.5 - 0.49)(3.73 - 3.5)$. This sum is equal to 0.114, or \$114 million/day. Likewise, the loss in producer surplus is

$$0.49(3.5 - 3.38) + 0.5(0.5 - 0.49)(3.5 - 3.38) = 0.0594$$

or \$59.4 million/day.

In the long run, $\frac{\partial p_d}{\partial t} = \frac{0.6}{0.6+|-0.8|} = \frac{3}{7}$. Compared to the short run where the majority of the tax was born by consumers, in the long run consumers bear less than half the tax. The price for consumers rises from \$3.50/gal to \$3.65/gal (of which producers receive \$3.30/gal) and the quantity sold falls from 0.5 to 0.483 billion gallons/day. The tax revenue raised is $0.35(0.483) = 0.169$, or \$169 million/day. The loss in consumer surplus is now

$$0.483(3.65 - 3.5) + 0.5(0.5 - 0.483)(3.65 - 3.5) = 0.0737$$

or \$73.7 million/day and the loss in producer surplus is now

$$0.483(3.5 - 3.30) + 0.5(0.5 - 0.483)(3.5 - 3.30) = 0.0983$$

or \$98.3 million/day.

(b) What is the deadweight loss as conventionally computed? What important additions and subtractions do you think are warranted in a sensible public interest cost-benefit analysis of the tax?

Solution: The conventional deadweight loss in the short-run is

$$0.5(0.35)(0.5 - 0.49) = 0.00175 = \$1.75 \text{ million/day}$$

while in the long-run it is $2.98 million/day. If one believes there are social costs to gasoline consumption (pollution, traffic) that are not captured in the market price of gasoline, then a reduction in gasoline consumption also reduces these social costs, offsetting some of the conventional deadweight loss.

4. Abel, Bob and Chris each are interested in purchasing an hour long amateur Chiropractic adjustment (ACA). Abel values an ACA at $30, Bob at $40 and Chris at $60. Ryan, Sam and Trudy are amateur Chiropractors. In their day jobs, Ryan earns $10 per hour, Sam earns $20 per hour and Trudy earns $50 per hours; each would have to miss work to offer an appointment. Assume each supplier can only supply one ACA and each demander can only consume one ACA.

(a) Use the competitive equilibrium model to predict price and quantity. Who gets ACAs and who supplies them? What is the consumer, producer and total surplus?

Solution: We can construct the following supply and demand schedules:

Quantity	Demand Price	Supply Price	Joint Surplus
1	60	10	50
2	40	20	20
3	30	50	-20

In the competitive equilibrium, two ACAs are sold by Ryan and Sam. The market price cannot be lower than $20, otherwise Sam would not be willing to supply the second ACA. Similarly, the market price cannot be higher than $40 or else only Chris would be willing to buy an ACA. If the price were less than $30, then both Abel or Bob would be willing to buy the second ACA.

In equilibrium, however, Bob should be the one to purchase the second ACA since he is able to pay more than Abel and still benefit from the transaction. Therefore the market price must be $p^* \in [30, 40]$.

Consumer surplus is $(60 - p^*) + (40 - p^*) = 100 - 2p^*$. Producer surplus is $(p^* - 10) + (p^* - 20) = 2p^* - 30$. Total surplus is $(100 - 2p^*) + (2p^* - 30) = 70$.

(b) Imagine that a phone app does the matching, using an algorithm that maximizes the number of ACA transactions. Suppose for the moment that consumers truthfully report their valuations to the app, and suppliers report their true costs. What matching would you predict? What is the corresponding consumer, producer and total surplus?

Solution: The app can match consumers and producers such that three transactions are made. One such match would be: Chris-Trudy, Bob-Sam, and Abel-Ryan. The total surplus for this match is $(60 - 50) + (40 - 20) + (30 - 10) = 50$. The split between CS and PS depends on how the app assigns price to each transaction. In any case, the total surplus of 50 is less than in CE; the app is not efficient, although it may be profitable for its creators.

(c) Briefly compare the feasibility of implementing the models/algorithms suggested in (a) and (b) above. Which algorithm would you recommend for the app? NB: you don't have to restrict yourself to (a) or (b).

Solution: The algorithm in part (b) depends on truthful reporting of valuations and costs. As both buyers and sellers can benefit by lying, this throws the feasibility of the algorithm in doubt. The competitive equilibrium, on the other hand, requires no knowledge about any of the market participants.

Part (b) may still be preferable if the app's revenues depend on the number of transactions conducted. In the long run, however, the app may attract more users if it maximizes total surplus by charging the competitive price.

II. Short Essays

1. Print shop

Solution: Explain using non-technical language: (1) the fact that marginal revenue is negative at certain points and (2) why that is a bad pricing scheme. Any alternative with sufficient economic logic is acceptable.

2. Water conservation

Solution: Again using non-technical language, explain that (per Dr. Friedman) consumers optimize at the margin so increasing the unit price for the last unit of water is what promotes conservation by selfish rational consumers. However, evidence on responses to marginal or average prices is currently mixed, so increasing tier structures may increase consumption in a revenue-neutral setting.

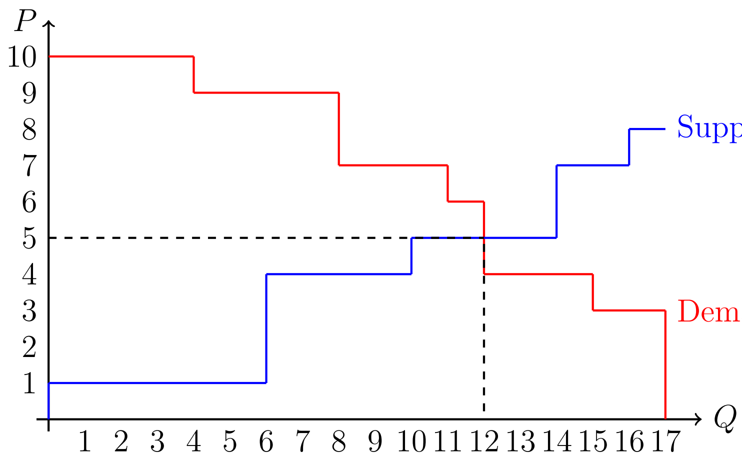
3. Personal data market

Solution: A few relevant concerns might be: (1) potentially very few buyers (who purchase individual users' data, bundle it and sell to advertisers, etc.), resulting in monopoly power; (2) legal hurdles present barriers to entry — in particular, users' property rights to their personal data do not have solid legal backing; (3) personal data is not homogenous, so prices could be hard to establish; (4) externalities, risk, adverse selection effects may also be present. In Dan's opinion, with good laws and market design, a viable market is possible, but it is not yet clear that one will ever emerge.

Supplement to Problem Set 1

Econ 200

1) Demand curve and supply curve given for the experiment of double auction in the first class are as follows.



3) Competitive Equilibrium
For in-class data, competitive equilibrium is $p^* = 5, q^* = 12$.

4) Comparison with the experiment of double auction in class
For period 1: mean price was 4.84 and median price was 5.7. A handful of prices were far from the equilibrium. The number of transactions was 12, exactly as predicted.

For period 2: mean price was 5.48 and median price was 5.5; all prices were between 5 and 6. The number of transactions was 13.
Someone paid more than their valuation, since there were not 13 buyers with valuations of 5 or higher.

Given that the setting of double auction was quite different from the assumed setting of perfectly competitive market, the theoretical outcomes of competitive equilibrium is surprisingly close to the experimental outcomes. This suggests that even if some of the textbook assumptions on perfectly competitive markets are not satisfied, the theoretical outcome of competitive equilibrium may serve as an very useful benchmark.

Problem Set 2 - ECON 200 - Fall 2016
Answer Key

October 18, 2016

Part I
Short Case Study Problems

1 Supply in the Short and Long Run

A firm has production function $\ln y = \frac{1}{3}\ln x_1 + \frac{1}{2}\ln x_2$. Input 2 is unchangeable for now at the level $x_2 = 8$. Prices are \$8 and \$5 respectively for inputs 1 and 2.

1.1 a. What is the firm's variable cost? fixed cost (if any)? total cost? marginal cost?

$$\begin{aligned} y &= x_1^{1/3} x_2^{1/2} \\ y &= \sqrt{8} \cdot x_1^{1/3} \\ x_1 &= \frac{y^3}{8^{3/2}} \end{aligned}$$

$$\begin{aligned} \text{Cost} &= 8 \cdot x_1 + 5 \cdot 8 \\ C(y) &= 8 \cdot \frac{y^3}{8^{3/2}} + 40 \\ &= \frac{y^3}{8^{1/2}} + 40 \end{aligned}$$

The variable component is $\frac{y^3}{8^{1/2}}$, and the fixed component is 40. The equation above is total cost. Marginal cost:

$$\frac{dC(y)}{dy} = \frac{3}{8^{1/2}} y^2 \approx 1.06 \cdot y^2$$

1.2 b. What is the firm's supply function?

Above the minimum efficient scale, $s_i(P)$ satisfies $P = MC$

To find minimum efficient scale:

$$\begin{aligned} AVC &= MC \\ \frac{y^2}{8^{1/2}} &= \frac{3}{8^{1/2}} y^2 \end{aligned}$$

This is only satisfied for $y = 0$, after which $MC > AVC$ for all y. Thus:

$$P = \frac{3}{8^{1/2}} \cdot s_i^2$$
$$s_i = \left(\frac{8^{1/2}}{3} \cdot P\right)^{1/2} = \frac{2^{1/4}}{3^{1/2}} \cdot P^{1/2} \approx .6866 \cdot \sqrt{P}$$

1.3 c. Now assume that both inputs can be adjusted freely. What are the firm's conditional input demands? What is its average cost? Marginal cost? Supply function?

$$y = x_1^{1/3} x_2^{1/2}$$
$$x_1 = \frac{y^3}{x_2^{3/2}}$$

$$\frac{\mathrm{MP}_1}{\mathrm{MP}_2} = \frac{w_1}{w_2}$$
$$\frac{\frac{1}{3}\frac{x_2^{1/2}}{x_1^{2/3}}}{\frac{1}{2}\frac{x_1^{1/3}}{x_2^{1/2}}} = \frac{w_1}{w_2}$$
$$\frac{2}{3}\frac{x_2}{x_1} = \frac{w_1}{w_2}$$
$$x_2 = \frac{3}{2}\frac{w_1}{w_2} x_1 = \frac{3}{2}\frac{w_1}{w_2}\frac{y^3}{x_2^{3/2}}$$
$$x_2^{5/2} = \frac{3}{2}\frac{w_1}{w_2} y^3$$
$$x_2^{\star}(y; w_1, w_2) = \left(\frac{3}{2}\frac{w_1}{w_2} y^3\right)^{2/5} = \left(\frac{3}{2}\frac{w_1}{w_2}\right)^{2/5} y^{6/5}$$
$$x_1^{\star}(y; w_1, w_2) = \frac{2}{3}\frac{w_2}{w_1}\left(\frac{3}{2}\frac{w_1}{w_2} y^3\right)^{2/5} = \left(\frac{2}{3}\frac{w_2}{w_1}\right)^{3/5} y^{6/5}$$

The above are the conditional input demands.

$$C(y, w_1, w_2) = w_1 x_1^{\star}(y, w_1, w_2) + w_2 x_2^{\star}(y, w_1, w_2)$$
$$C(y; w_1, w_2) = w_1 \cdot \left(\frac{2}{3}\frac{w_2}{w_1}\right)^{3/5} y^{6/5} + w_2 \cdot \left(\frac{3}{2}\frac{w_1}{w_2}\right)^{2/5} y^{6/5}$$
$$= \left(\left(\frac{2}{3}\right)^{\frac{3}{5}} + \left(\frac{3}{2}\right)^{\frac{2}{5}}\right) \cdot w_1^{2/5} \cdot w_2^{3/5} \cdot y^{6/5}$$
$$= \left(\frac{2}{3}\right)^{\frac{3}{5}} \cdot \left(1 + \frac{3}{2}\right) \cdot w_1^{2/5} \cdot w_2^{3/5} \cdot y^{\frac{6}{5}}$$
$$= \left(\frac{2}{3}\right)^{\frac{3}{5}} \cdot \frac{5}{2} \cdot 8^{2/5} \cdot 5^{3/5} \cdot y^{\frac{6}{5}} \approx 5.15 \cdot y^{6/5}$$

$$AC = \left(\frac{2}{3}\right)^{\frac{3}{5}} \cdot \frac{5}{2} \cdot 8^{2/5} \cdot 5^{3/5} \cdot y^{\frac{1}{5}} \approx 5.15 \cdot y^{1/5}$$
$$MC = \left(\frac{2}{3}\right)^{\frac{3}{5}} \cdot 3 \cdot 8^{2/5} \cdot 5^{3/5} \cdot y^{\frac{1}{5}} \approx 15.59 \cdot y^{1/5}$$

Minimum efficient scale is zero, and the supply function satisfies $P = MC$

$$P = \left(\frac{2}{3}\right)^{\frac{3}{5}} \cdot 3 \cdot 8^{2/5} \cdot 5^{3/5} \cdot s_i^{\frac{1}{5}}$$

$$s_i(P) = \frac{1}{2^3 \cdot 3^2 \cdot 8^2 \cdot 5^3} \cdot P^5 = \frac{P^5}{72 \cdot w_1^2 \cdot w_2^3} = \frac{P^5}{576,000}$$

2 Long-Run Competitive Equilibrium and the Number of Firms

Suppose that each firm in an industry has long run total cost function $c(y_i) = y_i^3 - 9y_i^2 + 36y_i$, and they face industry demand curve $y_T = 200 - 10p$.

2.1 a. What is each firm's marginal cost function? Average cost function? Fixed cost? Supply curve?

$$\begin{aligned} MC_i &= 3y_i^2 - 18y_i + 36 \\ AC_i &= y_i^2 - 9y_i + 36 \\ FC_i &= 0 \end{aligned}$$

Individual firm supply satisfies $P = MC$ above minimum efficient scale

$$\begin{aligned} MC_i(y_{min}) &= AC_i(y_{min}) \\ 3y_{min}^2 - 18y_{min} + 36 &= y_{min}^2 - 9y_{min} + 36 \\ 2y_m^2 - 9y_m &= 0 \end{aligned}$$

The roots of the above are $y_m = 0$ and $y_m = 4.5$, so minimum efficient scale is 4.5, above which:

$$P = s_i^{-1}(y_i) = 3y_i^2 - 18y_i + 36$$

This means the minimum price is $4.5^2 - 9 \cdot 4.5 + 36 = 63/4 = 15.75$

To obtain the supply function, solve that equation for y_i in terms of p in the relevant range. Using the quadratic formula and simplifying, you get

$$s_i^2 - 6s_i + 12 - \frac{1}{3}P = 0$$

$$\begin{aligned} s_i &= \frac{1}{2} \cdot \left(6 + \sqrt{6^2 - 4 \cdot \left(12 - \frac{1}{3}P\right)}\right) \\ s_i(P) &= 3 + \frac{1}{2}\left(\frac{4}{3}P - 12\right)^{1/2} \\ s_i(P) &= 3 + \left(\frac{1}{3}P - 3\right)^{1/2} \end{aligned}$$

$$s_i(P) = \begin{cases} 3 + \left(\frac{1}{3}P - 3\right)^{1/2} & \text{if } p \geq \frac{63}{4}, \\ 0 & \text{if } p < \frac{63}{4}. \end{cases}$$

The total supply curve is zero below $p = 63/4$ and infinite (due to unlimited entry) above $p = 63/4$.

2.2 b. What is long run competitive equilibrium price? Output per firm? Number of firms?

In the long run, $P = MC = AVC$

$$P^{\star}_{LR} = \frac{63}{4}$$

Output per firm is a function of this price:

$$\begin{aligned} s_i^{\star}\left(P^{\star}_{LR}\right) &= 3 + \left(\frac{1}{3}\frac{63}{4} - 3\right)^{1/2} \\ &= 3 + \left(\frac{21}{4} - \frac{12}{4}\right)^{1/2} \\ &= 3 + \sqrt{\frac{9}{4}} = 3 + \frac{3}{2} = 4.5 \end{aligned}$$

As implied above by the minimum efficient scale. The number of firms matchesthis to total demand:

$$\begin{aligned} n_{LR} \cdot s_i^{\star}\left(P^{\star}_{LR}\right) &= y_T\left(P^{\star}_{LR}\right) \\ n_{LR} &= \frac{1}{4.5} \cdot \left(200 - 10 \cdot \frac{63}{4}\right) = \frac{1}{4.5}\left(200 - 157.5\right) \approx 9.44 \end{aligned}$$

There will be very small rents to firms from the slight excess demand that is too little to warrant a tenth firm. Moreover, firms will produce only at or above 4.5 units, no firm will enter and produce less.

2.3 c. Compute the producer surplus (PS), consumer surplus (CS) and total surplus (TS) at long run competitive equilibrium.

Ignoring the slight imperfection, we will calculate surplus assuming $P^{\star} = 15.75$ and $q^{\star} = 42.5$ exactly. Note that inverse demand is $20 - y$

$$CS = \frac{1}{2}\left(20 - 15.75\right) \cdot 42.5 = \frac{1445}{16} \approx 90.3125$$

$PS = 0$ note that the supply curve is horizontal and infinite at $P = 63/4$

$$TS = CS + PS = \frac{1445}{16} \approx 90.3125$$

3 Translog Cost and Profit Functions

You estimated a 3-factor translog cost function for a client. Write down a possible numerical function of this sort (about half of its coefficients can be zero).

3.1 Check that the coefficients satisfy the main required conditions (homogeneity, etc)

Demonstrate that you know the four conditions and how to verify that they are satisfied, at least locally for a reasonable set of prices.

1. Non-decreasing in input prices and output
2. Homogeneous of degree 1 in input prices
3. Concave in input prices (negative semi-definite Hessian)
4. Continuous and differentiable (any combination of logarithmic functions will satisfy this unless you put additional terms inside the logarithmic functions)

An example is

$$
\begin{aligned}
\ln \text{Cost} =& .5 \cdot \ln(w_1) - .2 \cdot [\ln(w_1)]^2 \\
&+ .25 \cdot \ln(w_2) + .1 \cdot [\ln(w_2)]^2 \\
&+ .25 \cdot \ln(w_3) + .1 \cdot [\ln(w_3)]^2
\end{aligned}
$$

3.2 Write down the implied factor demand equations.

Use Shepherd's Lemma

$$\frac{\partial c(w_1, w_2, w_3, y)}{\partial w_i} = x_i^{\star}(w_1, w_2, w_3, y)$$

3.3 Suppose instead that you estimated a translog profit function. How would you modify your answers to parts a and b?

Demonstrate that you know the four conditions and how to verify that they are satisfied, at least locally for a reasonable set of prices.

1. Non-decreasing in output price and non-increasing in input prices
2. Homogeneous of degree 1 in output price and input prices
3. Convex in output price (can only be satisfied locally)
4. Continuous and differentiable (any combination of logarithmic functions will satisfy this unless you put additional terms inside the logarithmic functions)

An example is

$$
\begin{aligned}
\ln \text{Cost} = & \ln(P) + \ln(P) \cdot \ln(w_1) - \ln(P) \cdot \ln(w_2) \\
& .5 \cdot \ln(w_1) - .2 \cdot [\ln(w_1)]^2 \\
& + .25 \cdot \ln(w_2) + .1 \cdot [\ln(w_2)]^2 \\
& + .25 \cdot \ln(w_3) + .1 \cdot [\ln(w_3)]^2
\end{aligned}
$$

Use Shepherd's Lemma for profit functions

$$\frac{\partial \pi(w_1, w_2, w_3, p)}{\partial w_i} = x_i^{\star}(w_1, w_2, w_3, p)$$

4 Varian questions

4.1 Question 4.6

Two-plant problem.

$$c(y) = \min\{4\sqrt{y_1} + 2\sqrt{y_2}; y_1 + y_2 \geq y\}$$

Since the cost is concave, rather than convex, the optimal solution will always occur at a boundary. If you derive optimality conditions without checked the second derivative, you will maximize cost (instead of minimizing). The solution is to produce entirely at the cheaper plant, 2. $c(y) = 2\sqrt{y_2}$.

4.2 Question 5.14

Use time series data to estimate marginal cost in each period.

Take a total derivative of the cost function to get the following:

$$
\begin{aligned}
dc &= \sum_{i=1}^{n} \frac{\partial c}{\partial w_i} dw_i + \frac{\partial c}{\partial y} dy \\
\frac{\partial c}{\partial y} dy &= dc - \sum_{i=1}^{n} \frac{\partial c}{\partial w_i} dw_i \\
\frac{\partial c}{\partial y} &= \frac{1}{dy} \cdot \left[dc - \sum_{i=1}^{n} \frac{\partial c}{\partial w_i} dw_i \right] \\
MC &= \frac{1}{\Delta y} \cdot \left[\Delta c - \sum_{i=1}^{n} x_i^{\star} \Delta w_i \right]
\end{aligned}
$$

Using Shepherd's Lemma to find marginal cost as a function of your known variables.

Part II
Short Essay

5 Short Essay 1

1. Your client, a specialty materials supplier, wants to plan how to respond to changes in key input prices (electricity, gadolinium ore, labor) to achieve given levels of output. What sort of data should you gather, and how might you analyze it? Write an overview memo of no more than 250 words that is intelligible to the client's non-technical managers as well as to its data scientists; please print it on a separate page.

This question is about estimating conditional factor demands, which probably is best done via estimating a cost function and using Shepard's lemma. Credit will be given for sensible ideas on how to do it, and for writing clearly.

6 Short Essay 2

2. What are the differences between (a) increasing returns to scale, (b) learning curve, and (c) decreasing (average) cost? What are the different implications of (a-c) for competitive equilibrium price in the long run? Write about 100 words intelligible to your TA.

- Increasing returns to scale is a production function where doubling inputs results in a more than doubling of output
- A learning curve relates to increasing efficiency with experience
- Decreasing average cost is related to increasing returns to scale; it implies that marginal cost is below average cost (potentially indefinitely, such as in an industry where fixed costs are the primary costs). This generally occurs because of increasing returns to scale with constant factor prices.

All these things cause problems for CE in the LR; a more likely industry structure in each case is oligopoly or even monopoly.

ECON 200 - Fall 2016 - Problem Set 3
Answer Key

November 6, 2016

Part I
Short Case Study Problems

1 Lottery Ticket

Your Bernoulli function is $u(m) = \sqrt{m}$, and currently your wealth is $m = 4$. You have a lottery ticket that will pay 12 with probability 0.1 and otherwise will pay 0.

a. What is the mean and variance of the lottery ticket value?
The mean is the possible values times their probabilities:

$$\begin{aligned}\mathbb{E}\left[\text{ticket}\right] &= p_1\text{outcome}_1 + p_2\text{outcome}_2 \\ &= 0.1 \cdot 12 + 0.9 \cdot 0 = 1.2\end{aligned}$$

The variance formula is as follows:

$$\begin{aligned}\text{Var}\left[\text{ticket}\right] &= \sum p_i \cdot \left(\text{outcome}_i - \mathbb{E}\left[\text{ticket}\right]\right)^2 \\ &= 0.1 \cdot (12 - 1.2)^2 + 0.9 \cdot (0 - 1.2)^2 \\ &= 12.96\end{aligned}$$

b. What is your expected utility with the ticket? Without the ticket?
Expected utility with the ticket is the probability-weighted sum of possible outcomes:

$$\begin{aligned}\mathbb{E}\left[U\left(\text{ticket}\right)\right] &= p_1 \cdot u\left(m + \text{outcome}_1\right) + p_2 \cdot u\left(m + \text{outcome}_2\right) \\ &= 0.1 \cdot \sqrt{4 + 12} + 0.9 \cdot \sqrt{4 + 0} \\ &= 0.1 \cdot \sqrt{16} + 0.9 \cdot \sqrt{4} \\ &= 0.1 \cdot 4 + 0.9 \cdot 2 = 2.2\end{aligned}$$

Without the ticket, expected utility is your utility of the only possible outcome:

$$\mathbb{E}\left[U\left(m\right)\right] = u\left(m\right) = \sqrt{4} = 2$$

c. What is the maximum price that you rationally would turn down if someone offered to buy your ticket?
You would rationally turn down any price that makes you worse off. To find out what range of prices that is, suppose you are given a price that makes you equally well off. That is:

$$\begin{aligned}u\left(m + p'\right) &= \mathbb{E}\left[U\left(\text{ticket}\right)\right] \\ \sqrt{4 + p'} &= 2.2 \\ 4 + p' &= 4.84 \\ p' &= 0.84\end{aligned}$$

For any price greater than $p' = 0.84$, that is $p > 0.84$, you would be happier with the money than the ticket. For any price lower than 0.84, you would be happier with the ticket, so you would turn down that offer. The maximum price is 0.84.

d. What is your risk premium for the lottery ticket?

The risk premium is the amount you are willing to pay for certainty. Here, the lottery ticket has a higher expected payoff than its value to us (or the value of that payoff in utility terms). The difference between the valuation we discovered in (c) and the expected value of the lottery is our risk premium:

$$\begin{aligned} \text{Risk Premium} &= \mathbb{E}\left[\text{ticket}\right] - p' \\ &= 1.2 - 0.84 = 0.36 \end{aligned}$$

2 Mean-Variance and Bernoulli EU

2. Finance textbooks often assume that utility takes the form $U(L) = \mathbb{E}[L] - c \cdot \text{Var}[L]$, for some $c > 0$ parametrizing the degree of risk aversion. Consider two investors, A and B. Investor A maximizes $U[L]$ for $c = 0.01$, while B maximizes the expected value of the Bernoulli function $u(m) = m^{0.9}$. Consider two lotteries: S pays \$1 for sure, while R pays \$1 with probability 0.999 and pays \$1000 with probability 0.001.

a. Compute the mean and variance of each lottery.

As before.

$$\begin{aligned} \mathbb{E}[S] &= 1 \\ \mathbb{E}[R] &= 0.999 \cdot 1 + 0.001 \cdot 1000 \\ &= 0.999 + 1 = 1.999 \end{aligned}$$

$$\begin{aligned} \text{Var}[S] &= 0 \\ \text{Var}[R] &= 0.999 \cdot (1 - 1.999)^2 + 0.001 \cdot (1000 - 1.999)^2 \\ &= 0.997 + 996.006 = 997.003 \end{aligned}$$

b. Compute U (for investor A) for each lottery.

We are given

$$U_A(L) = \mathbb{E}[L] - 0.01 \cdot \text{Var}[L]$$

$$\begin{aligned} U_A(S) &= \mathbb{E}[S] - 0.01 \cdot \text{Var}[S] \\ &= 1 - 0.01 \cdot 0 = 1 \end{aligned}$$

$$\begin{aligned} U_A(R) &= \mathbb{E}[R] - 0.01 \cdot \text{Var}[R] \\ &= 1.999 - 0.01 \cdot 997.003 = -7.971 \end{aligned}$$

c. Compute Eu (for investor B) for each lottery.

We are given

$$u(m) = m^{0.9}$$

Expected utility is then:

$$\mathbb{E}[U_B(L)] = \sum_i p_i u(m(\lambda_i))$$

Which is just saying that the expected utility is a probability-weighted sum of the utility of each possible outcome.

$$\begin{aligned} \mathbb{E}[U_B(S)] &= \sum_i p_i u(m(s_i)) \\ &= 1 \cdot 1^{0.9} = 1 \end{aligned}$$

$$
\begin{aligned}
\mathbb{E}\left[U_B\left(R\right)\right] &= \sum_i p_i u\left(m\left(r_i\right)\right) \\
&= 0.999 \cdot (1)^{0.9} + 0.001 \cdot (1000)^{0.9} \\
&= 0.999 + 0.501 = 1.5
\end{aligned}
$$

d. If faced with the choice between R and S, which would each investor choose?
Investor A would choose S, while investor B would choose R.

e. Now do Short Essay 1 below.
To be shown below, the point is that R stochastically dominates S, or in simpler terms, R is weakly better than S for any probabilities and preferences. The mean-variance approximation falters under certain circumstances, such as this one.

3 Bayesian Updating

A patient has either disease A or disease B. Diagnostic test 1 is positive with probability 0.7 (and negative with probability 0.3) with disease A, and is positive with probability 0.4 with disease B. Similarly, diagnostic test 2 is positive with probability 0.2 when the disease is A, and with probability 0.5 when the disease is B. Overall, the relative incidence of the two diseases is 60% A and 40% B.

a. Compute the joint probabilities $p(d, t_1, t_2)$ for each disease$d =$ A, B and each test result $t_1 =$ pos, neg and $t_2 =$ pos, neg.
The joint probabilities are

$$
\begin{aligned}
p\left(A, t_1^+, t_2^+\right) &= p\left(t_1^+|A\right) \times p\left(t_2^+|A\right) \times p\left(A\right) = 0.7 \cdot 0.2 \cdot 0.6 = 0.084 \\
p\left(A, t_1^+, t_2^-\right) &= p\left(t_1^+|A\right) \times p\left(t_2^-|A\right) \times p\left(A\right) = 0.7 \cdot 0.8 \cdot 0.6 = 0.336 \\
p\left(A, t_1^-, t_2^+\right) &= p\left(t_1^-|A\right) \times p\left(t_2^+|A\right) \times p\left(A\right) = 0.3 \cdot 0.2 \cdot 0.6 = 0.036 \\
p\left(A, t_1^-, t_2^-\right) &= p\left(t_1^-|A\right) \times p\left(t_2^-|A\right) \times p\left(A\right) = 0.3 \cdot 0.8 \cdot 0.6 = 0.144 \\
p\left(B, t_1^+, t_2^+\right) &= p\left(t_1^+|B\right) \times p\left(t_2^+|B\right) \times p\left(B\right) = 0.4 \cdot 0.5 \cdot 0.4 = 0.08 \\
p\left(B, t_1^+, t_2^-\right) &= p\left(t_1^+|B\right) \times p\left(t_2^-|B\right) \times p\left(B\right) = 0.4 \cdot 0.5 \cdot 0.4 = 0.08 \\
p\left(B, t_1^-, t_2^+\right) &= p\left(t_1^-|B\right) \times p\left(t_2^+|B\right) \times p\left(B\right) = 0.6 \cdot 0.5 \cdot 0.4 = 0.12 \\
p\left(B, t_1^-, t_2^-\right) &= p\left(t_1^-|B\right) \times p\left(t_2^-|B\right) \times p\left(B\right) = 0.6 \cdot 0.5 \cdot 0.4 = 0.12
\end{aligned}
$$

b. Compute the prior probabilities of each test result and each disease.
First of all, this should say the message probabilities of each test result and the prior probabilities of each disease. The priors are given, .6 for A and .4 for B. The message probabilities are just the marginals

$$
\begin{aligned}
p\left(m = t_1^+\right) &= p\left(t_1^+|\text{has A}\right) \cdot p\left(\text{has A}\right) + p\left(t_1^+|\text{has B}\right) \cdot p\left(\text{has B}\right) \\
&= 0.7 \cdot 0.6 + 0.4 \cdot 0.4 = 0.42 + 0.16 = 0.58 \\
p\left(m = t_1^-\right) &= p\left(t_1^-|\text{has A}\right) \cdot p\left(\text{has A}\right) + p\left(t_1^-|\text{has B}\right) \cdot p\left(\text{has B}\right) \\
&= 0.3 \cdot 0.6 + 0.6 \cdot 0.4 = 0.18 + 0.24 = 0.42 \\
p\left(m = t_2^+\right) &= p\left(t_2^+|\text{has A}\right) \cdot p\left(\text{has A}\right) + p\left(t_2^+|\text{has B}\right) \cdot p\left(\text{has B}\right) \\
&= 0.2 \cdot 0.6 + 0.5 \cdot 0.4 = 0.12 + 0.2 = 0.32 \\
p\left(m = t_2^-\right) &= p\left(t_2^-|\text{has A}\right) \cdot p\left(\text{has A}\right) + p\left(t_2^-|\text{has B}\right) \cdot p\left(\text{has B}\right) \\
&= 0.8 \cdot 0.6 + 0.5 \cdot 0.4 = 0.48 + 0.2 = 0.68
\end{aligned}
$$

Note that the probability of receiving a message here is unconditional of the message for the other test. Thus, the probability of receiving a positive result plus the probability of receiving a negative result should equal 1.

Table 1: Posterior Probabilities

	t_2^+	t_2^-
t_1^+	0.512	0.808
t_1^-	0.231	0.545

c. Calculate the posterior probability of disease A when t_2 is not performed and t_1 is positive (and also t_1 negative). Similarly, when t_1 is not performed and t_2 is positive (and also t_2 negative).
The formula for calculating the posterior is as follows:

$$p\left(A|t_1^+\right) = \frac{p\left(t_1^+|A\right)p\left(A\right)}{p\left(t_1^+\right)} = \frac{0.7 \cdot 0.6}{0.58} = 0.724$$

$$p\left(A|t_1^-\right) = \frac{p\left(t_1^-|A\right)p\left(A\right)}{p\left(t_1^-\right)} = \frac{0.3 \cdot 0.6}{0.42} = 0.429$$

$$p\left(A|t_2^+\right) = \frac{p\left(t_2^+|A\right)p\left(A\right)}{p\left(t_2^+\right)} = \frac{0.2 \cdot 0.6}{0.32} = 0.375$$

$$p\left(A|t_2^-\right) = \frac{p\left(t_2^-|A\right)p\left(A\right)}{p\left(t_2^-\right)} = \frac{0.8 \cdot 0.6}{0.68} = 0.706$$

d. Calculate and put into a table the posterior probability of disease A for each possible combination of the test results (t_1, t_2) when both tests are performed.
These can be calculated as follows:

$$\begin{aligned} p\left(A|t_1^+, t_2^+\right) &= \frac{p\left(t_1^+, t_2^+|A\right)p\left(A\right)}{p\left(t_1^+, t_2^+\right)} \\ &= \frac{p\left(t_1^+, t_2^+|A\right)p\left(A\right)}{p\left(t_1^+, t_2^+|A\right)p\left(A\right) + p\left(t_1^+, t_2^+|B\right)p\left(B\right)} \\ &= \frac{0.14 \cdot 0.6}{0.164} = 0.512 \end{aligned}$$

See Table 1 for all for posteriors.

Part II
Short Essay

4 Commentary on problem 2

In problem 2 above, which investor's choice (if either) makes sense to you? How can you generalize beyond the specific values (e.g., c = 0.01 and the probs and payoffs of R and S) in that problem to make a general point? Write about 100 words intelligible to your TA.

4.1 Answer Guidance

The purpose here is to identify the flaws of the mean-variance formula as an approximation for expected utility theory. While the mean-variance formulation works for gambles with small variance and utility functions with constant relative or absolute risk aversion, it breaks down with other cases. The lottery R stochastically dominates R for all probabilities and utility functions, since the expected gain can only be greater (or equal for $p_2 = 0$). The choice of investor A makes no sense in this context.

ECON 200 - PS 4 - Fall 16
ANSWER KEY

November 23, 2016

Part I

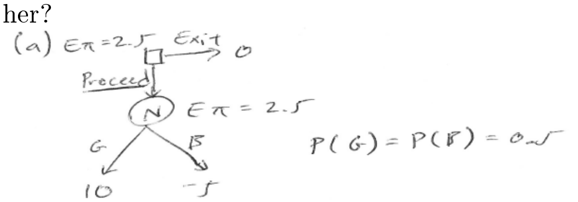
Short Case Study Problems

1 Consulting Fees and Cost of Information

If the market for your new product turns out well (G) you will gain incremental profit of 10 (millions of $), but otherwise (B) you will lose 5 if you bring out the product. You can assure an incremental profit of 0 if you cancel now. You believe the probability of G is 0.5.

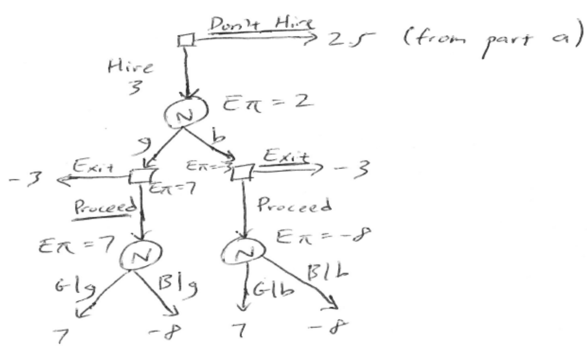
a. Draw the decision tree and solve it; state whether or not you should you bring out the product.

b. Suppose a consultant can tell you now whether to expect G or B. Her fee is 3. Is it worthwhile to hire her?



Bring out the product since the expected payoff of doing so (2.5) is greater than exiting (0).

(b) If the consultant is always correct, $P(g|G) = P(b|B) = 1$:



It is not worthwhile to hire the consultant.

Note that, even if the consultant is completely accurate, the informed tree (worth 2) is worth less than the uninformed tree (worth 2.5)

2 Startup and Timing of Costs

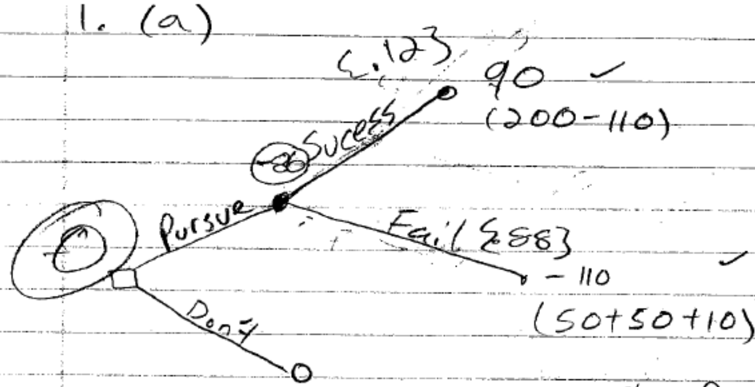
As a financial analyst for a biotech startup company, you are to advise on whether to pursue a line of research that will cost \$10 ("R&D") now and with probability 0.3 will lead to a potential product. It costs \$50 to test such a potential product, and the test leads to FDA approval ("validation") with probability 0.4. At that point the product is marketable, and at a further cost of \$50 ("production") it can be sold, bringing on average \$200 in revenue (net of other costs not mentioned). Assume that all dollar figures are present values in \$millions. Revenues come only from marketable products.

a. Draw and solve the decision tree, assuming that you must sink all mentioned costs (R&D, validation and production) immediately.

b. Redraw and solve the decision tree assuming that you can wait to see the R&D outcome before sinking the validation cost, and can wait for FDA approval before sinking the production cost.

c. Now assume that the situation is as in b. above except that with probability 0.2 a marketable product is a "blockbuster." At an additional cost of \$100 (expansion) a blockbuster generates net revenue of 1000 instead of 200. Once more, draw and solve the decision tree. Also note the probability that the line of research will ultimately produce a blockbuster.

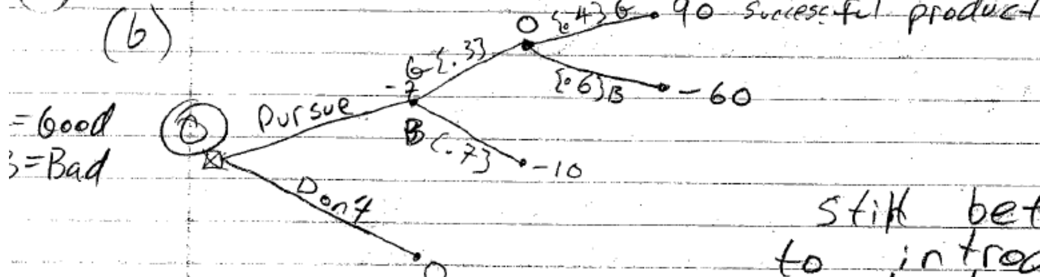
1. (a)



3/3

Don't Pursue $-86 < 0$

(b)

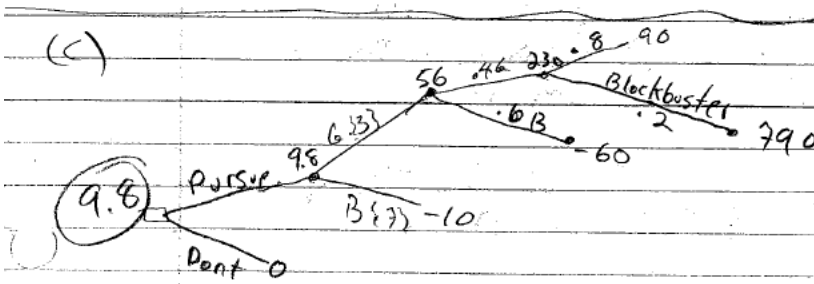


4/4

still better not to introduce the product

$-7 < 0$

(c)



you should pursue the product since expected payoff is now 9.8

chance of blockbuster $= .3 \cdot .4 \cdot .2$
$= .024 = 2.4\%$

Note: it is possible to interpret that you know a product will be a blockbuster or not. The above assumes that you can choose to expand only if the product is a blockbuster. Always state your assumptions.

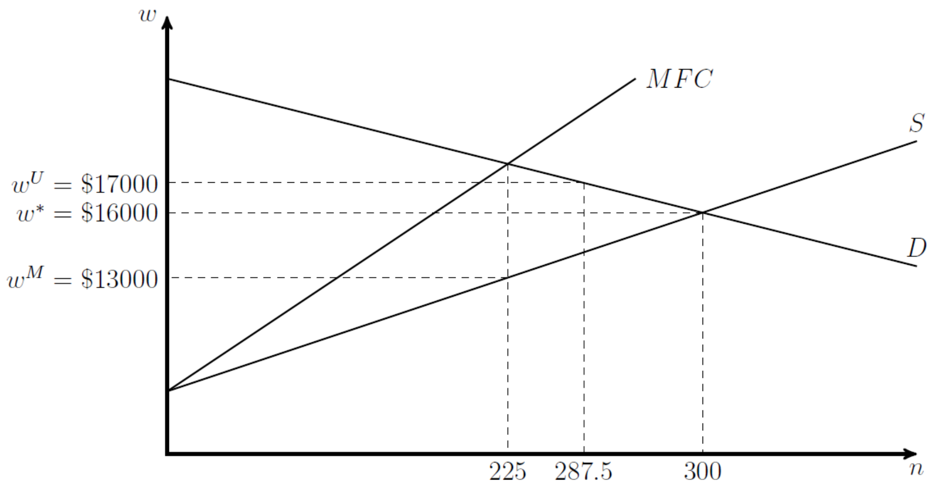
3 Monopsony

The demand for TAs at the University of Chico is approximated by the inverse demand function $w = 40000 - 80n$, where w is the annualized wage and n is the number of TAs hired. The supply is approximated by $w = 4000 + 40n$. If U Chico uses its market power fully, what wage will it pay and how many TAs will it hire? If TAs unionize and enforce $w = \$17,000$, how many TAs will be hired? Comment briefly on the efficiency ($TS = CS + PS$) implications of the union wage.

3.1 Instructor Solution

The university chooses the number of TAs to maximizes its objective function, which is the benet from TAs minus cost to hire. First-order condition is marginal benefit equals marginal factor cost. Total cost is given by $(4000 + 40n)\,n$, so marginal factor cost is $w = 4000 + 80n$. Setting marginal factor cost equal to inverse demand function, the university hires $n^M = 225$ TAs and sets wages equal to $w^M = \$13,000$ when it acts as a monopsonist. When students unionize and demand wages of $w^U = \$17,000$, the university will hire $n^U = 287.5$ TAs.

When the TAs unionize, the effect is that there is a price floor on wages. When we looked at price floors earlier in the quarter, we saw that they resulted in deadweight loss. But in this problem, the price floor actually reduces deadweight loss since it brings us closer to competitive equilibrium. This is because the monopsonist is already inefficient, so introducing a second distortion such as a price floor can actually reduce inefficiency. If the union compromises and sets the wage \$16,000, this achieves the most efficient allocation.



4 Lost Lake and Regulated Monopolies

Lost Lake CA is an isolated community with 20 households, each of whose demand for electricity is well approximated by p = 60 - q. The total cost of electricity generation and distribution there is c(Q) = 900 + Q. The local city council regulates electricity suppliers.

a. If the city council wants to avoid deadweight loss, what price ceiling will they choose for electricity? What is the corresponding CS and PS? What profit does the electrical power supplier earn?
To avoid deadweight loss, the council will have to set the price to what it would be in competitive equilibrium, $p^\star$. In this problem, since marginal cost is constant and equal to 1, the supply curve is just a flat line given by $p = 1$. Therefore, in competitive equilibrium the price is $p^\star = 1$.
Producer surplus is equal to zero because the supply curve is flat. To find consumer surplus, we first aggregate household demand q into demand for the entire town: $Q = 20q = 1200 - 20p$. At a price of $p^\star = 1$, $Q^\star = 1180$. Consumer surplus is then $\frac{1}{2}(1180)(60-1) = 34810$. The power company's profit is $p^\star Q^\star - c(Q) = \$900$, so it actually is losing \$900.

b. The supplier threatens to shut down. What price can the city council set to ensure nonnegative economic profit?
In general, the supplier's profit $p(Q)Q - c(Q)$ is a function of Q. Therefore, we can set this equation equal to zero and find its roots in order to find which values of Q generate zero profits. The equation is

$$\frac{1}{20}Q^2 - 59Q + 900 = 0$$

Applying the Quadratic Formula, we find that the roots are $Q = \{15.5; 1164.5\}$. The profit is nonnegative when $1.77 \leq p \leq 59.2$.

c. An imaginative city council member suggests charging each household a fixed annual fee plus a per-unit price for electricity consumption. Is there a fee+price combination that would maximize efficiency, and also induce participation by the supplier and all households? If so, compute it; if not, explain why none exists.

The only way to maximize efficiency (and avoid deadweight loss) is to set the per-unit price $p = p^{\star} = 1$. However, we know that the supplier is making a loss at that price so we will need to charge households a fixed fee sucient to make up for the supplier's loss in order to guarantee its participation. Since the supplier's loss is \$900, we would need to charge a fixed fee of at least \$900/20 = \$45 per household. However, if we charge too high of a fee then households may not wish to participate. In general, households will choose to participate if their surplus is nonnegative. In part (a), we calculated the surplus of all households when p = 1 and there is no fixed fee. Therefore, a household's surplus will be nonnegative as long as the fixed fee does not exceed \$34810/20 = \$1740.5.

To summarize, the per-unit price must be equal to $p^{\star} = 1$ in order to achieve maximum efficiency. When $p = p^{\star}$, the fixed fee for each household must be between \$45 and \$1740.5. If the fee is lower than \$45, then the supplier would rather shut down than continue to make a loss. If the fee is higher than \$1740.5, then households would rather avoid using electicity altogether. This way of pricing is called a two-part tariff.

5 Two-Market Monopolist

Baytech sells gizmos in the home market where it faces the demand function $q_H = 100 - 3p_H$. It also sells the same product in a foreign market where it faces the demand function $q_F = 200 - 7p_F$. Its cost function is$c(q_T) = 30 + 12q_T + q_T^2$, where $q_T = q_H + q_F$.

a. What output and price choices maximize Baytech's profit?

Write profit as

$$\mathrm{TR}_H(q_H) + \mathrm{TR}_F(q_F) - c(q_H + q_F)$$

where TR_H and TR_F are total revenues in the home and foreign markets, respectively. By taking derivatives, we see that Baytech's profit-maximizing conditions are

$$\max_{q_H, q_F} \mathrm{TR}_H(q_H) + \mathrm{TR}_F(q_F) - c(q_H + q_F)$$

$$\frac{\partial \pi}{\partial q_H} = 0 \implies \mathrm{MR}_H(q_H) - c'(q_H + q_F) = 0$$
$$\frac{\partial \pi}{\partial q_F} = 0 \implies \mathrm{MR}_F(q_F) - c'(q_H + q_F) = 0$$

Therefore,

$$\mathrm{MR}_H(q_H) = c'(q_H + q_F) = \mathrm{MR}_F(q_F)$$

This says that marginal revenue at home equals marginal revenue abroad equals the marginal cost of producing for the combined market. (By the way, $\frac{dc(q_T)}{dq_H} = \frac{dc(q_T)}{dq_T} \cdot \frac{dq_T}{dq_H} = c'(q_H + q_F)$, since $\frac{dq_T}{dq_H} = 1$. Likewise, $\frac{dc(q_T)}{dq_F} = c'(q_H + q_F)$.)

Substituting in the marginal revenues (obtained by taking the derivative of $\mathrm{TR}_H(q_H) = p_H(q_H) \cdot q_H$ and $\mathrm{TR}_F(q_F) = p_F \cdot (q_F) \cdot q_F$) and marginal cost,

$$\mathrm{MR}_H = \frac{100}{3} - \frac{2}{3}q_H = 12 + 2(q_H + q_F) = c'$$
$$\mathrm{MR}_F = \frac{200}{7} - \frac{2}{7}q_F = 12 + 2(q_H + q_F) = c'$$

Solve the system of two equations in two unknowns to find

$$q_H^{opt} = 7.45$$
$$q_F^{opt} = 0.73$$

Which imply

$$p_H = 30.85$$
$$p_F = 28.47$$

b. Antidumping regulations force Baytech to unify its prices. What choice of $p = p_H = p_F$ now maximizes profit? What is the maximum Baytech would rationally spend to eliminate the antidumping regulation? With $p = p_H = p_F$, we now have

$$q_T(p) = q_H(p) + q_F(p) = 200 + 100 - 3p - 7p = 300 - 10p$$

Inverse demand is therefore $p(q_T) = 30 - \frac{1}{10}q_T$. Baytech's optimization problem is now

$$\max_{q_T \geq 0} \left(30 - \frac{1}{10}q_T\right) q_T - \left(30 + 12q_T + q_T^2\right)$$
$$s.t.\ q_H \geq 0,\ q_F \geq 0,\ q_H + q_F = q_T$$

The first-order condition gives $q_T = 8.18$ and $p = 29.18$. Remember, though, that an interior solution must also satisfy $q_H \geq 0$ and $q_F \geq 0$. It turns out that $q_F(29.18)$ is negative. Therefore, the rst-order condition does not gives us an interior solution so we have to check corner solutions.
First, find the solution when $q_F = 0$. The problem is

$$\max_{q_H \geq 0} \left(\frac{100}{3} - \frac{1}{3}q_H\right) q_H - \left(30 + 12q_H + q_H^2\right)$$

which gives $q_H = 8$, $p = 30.67$, and profits of 55.33. The other possible corner solution is when $q_H = 0$. In this case, the problem is

$$\max_{q_F \geq 0} \left(\frac{200}{7} - \frac{1}{7}q_F\right) q_F - \left(30 + 12q_F + q_F^2\right)$$

which gives $q_F = 7.25$, $p = 27.535$, and profits of 30.07.
Therefore, Baytech will choose to produce its gizmos only in the home market and make a profit of 55.33. Baytech's unregulated profit in part (a) is 55.54, implying that Baytech should be willing to spend up to $55.54 - 55.33 = 0.21$, \$0.21 per period to overturn the regulation.

$$7.45 \cdot 30.85 + 0.73 \cdot 28.47 - \left(30 + 12 \cdot (7.45 + 0.73) + (7.45 + 0.73)^2\right) = 55.54$$

6 Price Discrimination in Airfare

Econometric analysis of air travel demand yields the following elasticity estimates for (first class, unrestricted coach, discount) demand: income = (1.8, 1.2, 1.1), and own-price = (-0.9, -1.2, -3.7). Predict the fares that a profit-oriented airline would pick for a route with no serious competition and marginal cost \$150 per passenger.

6.1 Instructor Solution

This is an example of third-degree price discrimination. Applying the markup formula

$$p_i = \frac{|\epsilon_i|}{|\epsilon_i - 1|}c$$

from the notes, we get

$$p_F = \frac{0.9}{0.9 - 1}(100) = -\$1350$$
$$p_U = \frac{1.2}{1.2 - 1}(100) = \$900$$
$$p_D = \frac{2.7}{2.7 - 1}(100) = \$205.56$$

Plainly, the formula does not give a valid price for first-class tickets, p_F. This is because demand for these tickets is inelastic ($|-0.9| < 1$). Faced with inelastic demand for first-class tickets, the profit-maximizing firm will raise p_F to an arbtrarily high level until demand is no longer inelastic. (As long as demand is inelastic, when a firm raises its price, total revenue will increase and total costs will fall.)

7 Comparing Market Structures

Direct demand is $Y = 86 - p$, where Y is the sum of output across all firms, and each firm has a cost function $c(y) = 14y$. a. Firms operate in the market by setting quantities. What are outputs, prices, profits and deadweight losses for monopoly, duopoly, triopoly and perfect competition markets? Show all work, but then collect your answers into a table, with columns for market structure and rows for performance measures. Which market structure is most efficient and WHY?
With perfect competition, the prot maximization condition p = MC implies that $86 - Y = 14$, or $Y = 72$, $p = 14$, and $= 0$. Of course, with perfect competition there is no deadweight loss.
Rather than solving the cases with 1 firm, 2 firms, and 3 firms separately, it's easier to just solve for N firms. Then, rm i maximizes

$$\max_{y_i} [86 - (y_1 + ... + y_N)] y_i - 14y_i$$

which has the first-order condition

$$72 - (y_1 + ... + y_{i-1} + 2y_i + y_{i+1} + ... + y_N) = 0$$

which can be re-arranged as

$$y_i = 72 - Y$$

Note that this condition holds for each of the N identical firms. If we add all these conditions together, we get

$$\sum_{i=1}^{N} y_i = N(72 - Y)$$

Of course, the sum on the left-hand side is just equal to Y , so we can solve the above for Y to get $Y = 72\frac{N}{N+1}$.
Note that when we let N go to infinity, we get $\lim_{N\to\infty} Y = Y^\star$.
For the monopolist case where $N = 1$, we get $Y^M = 36$, $p^M = 50$, $\pi^M = 1296$, and deadweight loss of $\frac{1}{2}(72 - 36)(50 - 14) = 648$. Note that to find the deadweight loss, we first had to find industry supply. Since the firm has inelastic supply at $p = 14$, inverse industry supply is also the horizontal line $p = 14$.
With a duopoly ($N = 2$), we get $Y^D = 48$, $p^D = 38$, $\sum_{i=1}^N \pi_i^D = 1152$, and deadweight loss of $\frac{1}{2}(72 - 48)(38 - 14) = 288$. With a tiopoly ($N = 3$), we get $Y^T = 54$, $p^T = 32$, $\sum_{i=1}^N \pi_i^T = 972$, and deadweight loss of $\frac{1}{2}(72 - 48)(38 - 14) = 162$.
To summarize:

	Y	p	$\sum \pi_i$	DWL
Monopoly	36	50	1296	648
Duopoly	48	38	1152	288
Triopoly	54	32	972	162
Perfect Comp.	72	14	0	0

Not surprisingly, perfect competition comes out as the most efficient market structure. With the other market structures, the firm(s) restrict(s) output in order to raise prices and ear profits. This behavior, however, generates deadweight loss.

b. Suppose firms set price instead of quantity. Again prepare a table of the same size and shape, and compare it to that in part a. How does your answer on efficiency change?
When firms compete by choosing prices, then in (short-run) Bertrand equilibrium, the firm with lowest cost can capture the whole market by choosing a price just below the marginal cost of the firm with the next-lowest cost. If all firms are identical, then they lower prices until they reach marginal cost; they divide up the market and all earn zero profits.
To summarize:

	Y	p	$\sum \pi_i$	DWL
Monopoly	36	50	1296	648
Duopoly	72	14	0	0
Triopoly	72	14	0	0
Perfect Comp.	72	14	0	0

Compared to part (a), the monopolist's behavior is still the same but now, as soon as we introduce any competition, we go immediately to the efficient (zero-profit) outcome.

Part II

Short Essay

8 Natural and Unnatural Monoploies and Public Policy

Which of these firms are natural or unnatural monopolists? What are reasonable public policies towards them? Google, Facebook, AirBnB, Amazon, Uber, Golden State Warriors.

8.1 Instructor Guidance

Natural monopolies are ones which arise from insufficient demand or high (natural) barriers to entry, among other things. Facebook might be considered a natural monopoly, because it would be extremely difficult to lure new customers away from Facebook to competitors until you have a large group using the competitor (network effects). A reasonable public policy toward a natural monopoly is to enforce anti-trust law to ensure that these companies do not unfairly overcharge customers, undersupply services, or otherwise take advantage of their market power.

Unnatural monopolies arise from policies or interventions that restrict market entry. An example might be the NBA allowing only one team to serve the San Francisco Bay Area, though it could be natural if there is only enough demand for one team. The best policies here are those which remove barriers to entry or otherwise encourage competition.

Suggestion: Google, Facebook, AirBnB, and Uber are natural monopolists due to network externalities. Amazon is a conglomerate, trying to use monopoly power in one area to undercut incumbents in related services. The Warriors are an unnatural monopoly generated by an anti-trust exemption for the NBA. The NBA regulates entry, hiring, draft picks, etc.

9 Imperfect Competition Market Structures in Practice

Pick an industry you know something about; if none, pick bookstores. Use one or two models of imperfect competition to organize a discussion of output and pricing decisions and profitability in the chosen industry. For example, for bookstores, you might consider Bertrand oligopoly (covered in class), or monopolistic competition, or dominant firm/competitive fringe (look them up if you are interested). Max 250 words; please print it on a separate page.

9.1 Instructor Guidance

Any relevant, well-argued response is appropriate. Friedman's postion: retail bookstores, before Borders and Amazon, were monopolistic competitors, or faced a kinked demand curve. Now Amazon is a monopolist in most local markets (not Santa Cruz), but is keeping prices low to prevent entry.