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Economics 202 (Fall 2015)

Final exam: Answers

Instructions: Do both part A and part B. Be sure to (a) put your name on each sheet and (b) label which questions you are answering.

Part A: Answer all three questions from this part. Question A3 is on back of this page.

A1 Consider a typical loan contract in which the loan amount is L, the borrower either repays $(1+r)L$ or defaults and loses collateral C. Briefly explain why, if the project return is risky, the borrower will prefer to invest in riskier projects while the lender will prefer that the borrower invest in safer projects. *If the project does well, the borrower gets the entire project return minus the fixed amount he/she has to repay. If the project does poorly and the borrower has to default, he/she loses the fixed amount of collateral. So the borrower likes projects that have a chance of doing really great and doesn't care how badly they do when the fail. risky projects are therefore attractive to the borrower. On the other hand, the lend gets fixed amount if the project does well and the collateral plus anything that's left and can be recovered from the project if it fails. So a lender prefers projects that still do okay when the borrower defaults and doesn't care at all about the return when the project does great since the lender only gets the repayment amount.*

A2 According to the expectational IS relationship that is central to the new Keynesian model, will aggregate demand rise or fall if the central bank cuts its policy interest rate from 3% to 2% and expected inflation falls from 2% to 0.5%, everything else held constant? Why? *The key here is that aggregate spending should depend on the real interest rate, i.e., the nominal rate minus the expected rate of inflation. So in this example, the real interest rate goes from* $0.03 - 0.02 = 0.01$ *to* $0.02 - 0.05 = 0.015$. *Since the real interest rate has risen (even though the nominal rate fell), we would* expected aggregate spending to fall.

A3 Using the basic new Keynesian model, briefly explain why a credible central bank will commit to future policies that do not ignore the past. Be specific. *This is easiest seen by looking at the new Keynesian Phillips Curve:*

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t.$$

If there is an inflation shock at time t (i.e., $e_t \neq 0$*), the central bank can do a better job of stabilizing* π_t *for a given output gap if it can commit to future policies and thereby affect expected future inflation. But it can only be credible if it fulfils the promises it made at time t that influence expectations of inflaiton at t+1, that is, at t+1 it must honor pledges it made in the past (i.e., at time t).*

Part B: Answer one question from this part.

B1 Consider the following new Keynesian model at a zero nominal interest rate:

$$x_t = \mathrm{E}_t x_{t+1} + \left(\frac{1}{\sigma}\right)\left(\mathrm{E}_t \pi_{t+1} + r^Z\right) \tag{1}$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t, \tag{2}$$

where $r^Z < 0$ is the negative demand shock that has pushed the economy to the zero lower bound. Denote the equilibrium at the zero lower bound by x^Z and π^Z. When the economy is not at the zero lower bound, assume $x = \pi = 0$. Assume the public expects that the economy will remain at the zero lower bound in the following period with probability q and will be out of the zero bound (so that $x = \pi = 0$) with probability $1 - q$.

(a) What do $\mathrm{E}_t\pi_{t+1}$ and $\mathrm{E}_t x_{t+1}$ equal? *$\mathrm{E}_t\pi_{t+1}$ equals π^Z with probability q and 0 with probability $1-q$, so $\mathrm{E}_t\pi_{t+1} = q\pi^Z$. Similarly, $\mathrm{E}_t x_{t+1} = qx^Z + (1-q)\times 0 = qx^Z$.*

(b) Use your results in (a) to eliminate expectations from (1) and (2) and solve for the equilibrium x^Z and π^Z. On a graph with the output gap on the horizontal axis and inflation in the vertical axis, plot the two equilibrium conditions. (Assume $\sigma(1-q)(1-\beta q) - q\kappa > 0$ so that the Phillips curve is flatter than the IS curve.) Indicate on your graph the point that represents the equilibrium values for x^Z and π^Z.

$$x^z = \mathrm{E}_t x_{t+1} + \left(\frac{1}{\sigma}\right)\left(\mathrm{E}_t\pi_{t+1} + r^Z\right) = qx^Z + \left(\frac{1}{\sigma}\right)\left(q\pi^Z + r^Z\right)$$

or

$$\sigma(1-q)x^Z = \left(q\pi^Z + r^Z\right)$$

or

$$\pi^Z = \frac{\sigma(1-q)x^Z - r^Z}{q} \tag{3}$$

and

$$\pi^Z = \beta \mathrm{E}_t\pi_{t+1} + \kappa x^Z = \beta q \pi^Z + \kappa x^Z$$

or

$$(1-\beta q)\pi^Z = \kappa x^Z$$

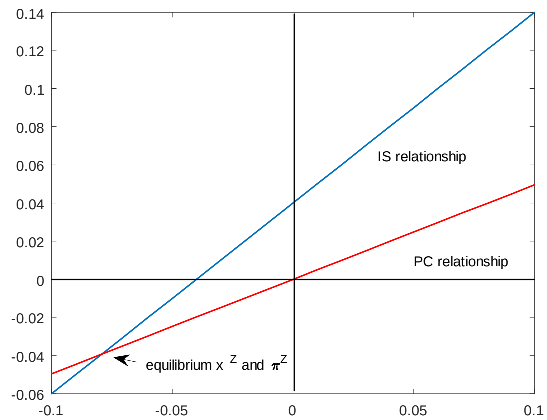
or

$$\pi^Z = \left(\frac{\kappa}{1-\beta q}\right)x^Z \tag{4}$$

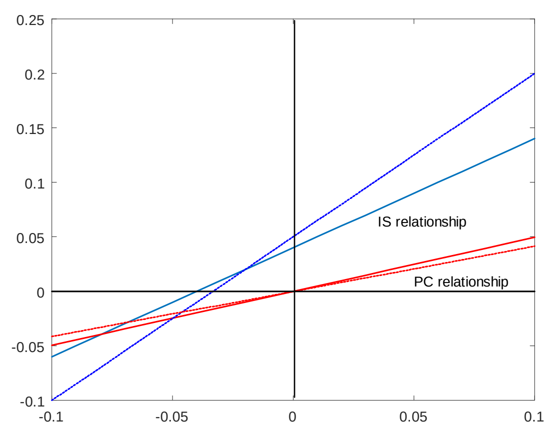
Plot (3) and (4). Both have positive slopes by given the assumption that $\sigma(1-q)(1-\beta q) - q\kappa > 0$, (3) is steeper than (4). See the figure:

(c) *Carefully explain* how the output gap and inflation are affected by a fall in q. Use your graph to illustrate the new equilibrium. *A fall in q increases the likelihood of exiting the ZLB, so expected future output rises as does expected future inflation. Both these movements in expectations act to increase current aggregate demand (the first directly since spending today depends on expected future income the second by lowering the real interest rate). The rise in expected inflation resulting from the fall in q also directly increases current inflation. See the figure – the new equilibrium with a smaller negative output gap and smaller negative inflation is where the dotted lines intersect.*

B2 Suppose the balance sheet of an individual bank consists of assets a_t, uninsured deposit liabilities d_t, loans from other banks b_t, and the bank's own capital n_t, so $a = d + b + n$. The bank can divert for its own use a fraction θ, $0 < \theta < 1$, of its "divertible" assets, defined as



0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.1
-0.05
0
0.05
0.1
IS relationship
PC relationship
equilibrium x ^{z} and π^z



0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.1
-0.05
0
0.05
0.1
IS relationship
PC relationship

$a - \omega_d d - \omega_b b$, where $0 \leq \omega_d,\, \omega_b \leq 1$ and $\omega_d < \omega_b$. Assume the bank wants to maximize the value of the bank $V \equiv r_a a - r_d d - r_b b$, where r_a is the return on assets, and r_i is the cost of borrowing for $i = d,\, b$.

(a) Under what conditions does the bank have an incentive to divert assets? *It has an incentive to divert whenever the value of the bank is less than the amount the bank can divert, i.e., whenever*

$$V = r_a a - r_d d - r_b b < \theta \left(a - \omega_d d - \omega_b b\right).$$

(b) If λ is the Lagrangian multiplier on the constraint that it not divert assets, show that the first order conditions for the individual's choice of a, d, and b implies

$$r_a - r_i = \left(\frac{\lambda}{1+\lambda}\right) \theta \left(1 - \omega_i\right) > 0$$

for $i = d,\, b$. *The bank's decision problem is*

$$\max_{a,b,d} r_a a - r_d d - r_b b + \lambda \left[r_a a - r_d d - r_b b - \theta \left(a - \omega_d d - \omega_b b\right)\right]$$

where $a = b + d + n$. Using the balance sheet, we can rewrite the problem as

$$\max_{a,b,d} \left(r_a - r_d\right) d + \left(r_a - r_b\right) b + r_a n + \lambda \begin{bmatrix} \left(r_a - r_d\right) d \left(r_a - r_b\right) b + r_a n \\ -\theta \left(\left(1 - \omega_d\right) d + \left(1 - \omega_b\right) b + n\right) \end{bmatrix}$$

and the first order conditions are

$$\left(r_a - r_d\right)\left(1 + \lambda\right) - \lambda\theta \left(1 - \omega_d\right) = 0$$
$$\left(r_a - r_b\right)\left(1 + \lambda\right) - \lambda\theta \left(1 - \omega_b\right) = 0$$

or

$$r_a - r_i = \left(\frac{\lambda}{1+\lambda}\right) \theta \left(1 - \omega_i\right) > 0$$

(c) If $\lambda > 0$ and $\theta > 0$, explain intuitively why the spread between the return on assets r_a and the cost of borrowing is larger on the bank's deposit liabilities than it is for the cost of borrowing from other banks. *In the absence of the moral hazard friction, arbitrage would ensure $r_a = r_b = r_d$. The presence of the friction ($\omega_i < 1$) leads to spreads as arbitrage is limited. The larger the friction (the smaller ω_i) the larger the spread will be. For every dollar raised via deposits, the bank can divert $1 - \omega_d$ of the assets it uses the funds to purchase, while from every dollar of bank loans, it can potentially divert $1 - \omega_b < 1 - \omega_d$. So te bank will be less limited in borrowing from other banks and it can arbitrage away more of the spread. Perhaps another way of seeing this is that the bank will borrower from each source until the marginal return on assets is equal to the marginal cost of borrowing from that source. The marginal cost of borrowing is r_i plus the premium arising because of concerns the bank will divert assets, so*

$$r_a = r_i + \left(\frac{\lambda}{1+\lambda}\right) \theta \left(1 - \omega_i\right).$$

The premium is smaller when borrowing from other banks as they do not have to worry as much about the moral hazard problem.

Economics 202
Fall 2015

Review questions (and some answers) for the final exam

1. What is the difference between the nominal rate of interest and the real rate of interest. Which one does economic theory suggest is the most relevant for aggregate spending (i.e., for consumption and investment)? If the nominal interest rate is 1.75% and expected inflation is 2%, what is the real rate of interest? *The real rate is* $i_t - E_t\pi_{t+1} = 1.75 - 2 = -0.25\%$.

2. Suppose

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t\pi_{t+1} - r_t^f\right)$$

$$\pi_t = \beta \mathrm{E}_t\pi_{t+1} + \kappa x_t,$$

and the central bank follows a policy rule given by

$$i_t = r_t^f + \phi \mathrm{E}_t\pi_{t+1}.$$

   (a) Explain why one equilibrium for this system is $x_{t+i} = \pi_{t+i} = 0$ for all $i \geq 0$. *Substitute the policy rule into the expectational IS relationship to obtain a two equation system of the form*

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(\phi - 1)\,\mathrm{E}_t\pi_{t+1}$$

$$\pi_t = \beta \mathrm{E}_t\pi_{t+1} + \kappa x_t.$$

   Now verify that $x_{t+i} = \pi_{t+i} = 0$ *for all* $i \geq 0$ *satisfies both these equations.*

   (b) Explain why this may not be the only equilibrium if $\phi < 1$. (Hint: Consider what would happen if, for some reason, private agents expected π_{t+1} will be greater than zero.) *Suppose for some extraneous reason, people expected* $E_t\pi_{t+1} > 0$*. Then since* $\phi - 1 < 0$*, this rise in expected inflation increases the output gap and the positive output gap pushes up actual inflation, thereby validating the original rise in expected inflation.*

3. Under the optimal time-consistent (discretionary) policy in the basic new Keynesian model with policy objectives given by a quadratic loss function in inflation and the output gap, $\kappa\pi_t + \lambda x_t = 0$, $\kappa\pi_{t+1} + \lambda x_{t+1} = 0$, $\kappa\pi_{t+2} + \lambda x_{t+2} = 0$, ... (using standard notation) while the optimal commitment policy ensures $\kappa\pi_t + \lambda x_t = 0$, $\kappa\pi_{t+1} + \lambda(x_{t+1} - x_t) = 0$, $\kappa\pi_{t+2} + \lambda(x_{t+2} - x_{t+1}) = 0$, ... Explain why the two policies differ? Why does commitment do better? Why does commitment introduce inertia (i.e., why does policy at $t+i$ involve the lagged output gap x_{t+i-1}?)

4. Consider the following new Keynesian model at a zero nominal interest rate:

$$x_t = \mathrm{E}_t x_{t+1} + \left(\frac{1}{\sigma}\right)\left(\mathrm{E}_t\pi_{t+1} + r^Z\right)$$

$$\pi_t = \beta \mathrm{E}_t\pi_{t+1} + \kappa x_t,$$

where $r^Z < 0$ is the negative demand shock that has pushed the economy to the zero lower bound. Denote the equilibrium at the zero lower bound by x^Z and π^Z. When the economy is not at the zero lower bound, assume $x = \pi = 0$. Assume the public expects that the economy will remain at the zero lower bound in the following period with probability q and will be out of the zero bound (so that $x = \pi = 0$) with probability $1 - q$.

(a) What are $\mathrm{E}_t \pi_{t+1}$ and $\mathrm{E}_t x_{t+1}$ equal to?

$$\mathrm{E}_t \pi_{t+1} = q\pi^Z + (1-q) \times 0 = q\pi^Z,$$

and

$$\mathrm{E}_t x_{t+1} = qx^Z + (1-q) \times 0 = qx^Z.$$

(b) Assume $\sigma(1-q)(1-\beta q) - q\kappa > 0$ and solve for the output gap and inflation at the zero lower bound (i.e., solve for x^Z and π^Z). *The two equation system for x^Z and π^Z is*

$$x^Z = qx^Z + \left(\frac{1}{\sigma}\right)\left(q\pi^Z + r^Z\right)$$

$$\pi^Z = \beta q \pi^Z + \kappa x^Z.$$

Rewrite this system as

$$\begin{bmatrix} \sigma(1-q) & -q \\ -\kappa & 1-\beta q \end{bmatrix} \begin{bmatrix} x^Z \\ \pi^Z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} r^Z$$

or

$$\begin{aligned} \begin{bmatrix} x^Z \\ \pi^Z \end{bmatrix} &= \begin{bmatrix} \sigma(1-q) & -q \\ -\kappa & 1-\beta q \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} r^Z \\ &= \left(\frac{1}{\sigma(1-q)(1-\beta q) - q\kappa}\right) \begin{bmatrix} (1-\beta q) \\ \kappa \end{bmatrix} r^Z \end{aligned}$$

Since $r^Z < 0$, both x^Z and π^Z are negative.

(c) Suppose the public becomes more pessimistic in the sense they think it is more likely the economy will remain at the zero lower bound (i.e., q increases). What happens to the output gap and inflation? Explain why. *From part (a),*

$$x^Z = \left(\frac{1-\beta q}{\sigma(1-q)(1-\beta q) - q\kappa}\right) r^Z$$

$$\pi^Z = \left(\frac{\kappa}{\sigma(1-q)(1-\beta q) - q\kappa}\right) r^Z$$

Let $\Delta \equiv \sigma(1-q)(1-\beta q) - q\kappa$. Then

$$\frac{\partial \Delta}{\partial q} = -\sigma(1-\beta q) - \sigma\beta(1-q) - \kappa < 0.$$

It follows a rise in q reduces both the numerator and the denominator of the expression for x^Z, so x^Z becomes more negative (remember r^Z is negative) while the denominator *in the expression for π^Z also falls so π^Z also becomes more negative. BOth the output gap and inflation depend on the expected future output gap and inflation. If the public thinks the economy is more likely to remain at x^Z and π^Z rather than return to $x = \pi = 0$, the fall in expected future x and π reduce current inflation and the output gap.*

5. Consider a simple new Keynesian model given by

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f\right)$$

$$\left(\pi_t - \pi^T\right) = \beta \mathrm{E}_t \left(\pi_{t+1} - \pi^T\right) + \kappa x_t + e_t$$

where x is the output gap, π is inflation, π^T is the central bank's inflation target, r^f and e are exogenous shocks. Monetary policy is designed to minimize

$$L_t = \frac{1}{2}\left[\left(\pi_t - \pi^T\right)^2 + \lambda x_t^2\right].$$

Under the optimal discretionary policy, show that the optimal targeting criterion takes the form

$$\kappa\left(\pi_t - \pi^T\right) + \lambda x_t = 0.$$

(a) What are the two equilibrium conditions that determine the output gap and inflation?

$$\left(\pi_t - \pi^T\right) = \beta \mathrm{E}_t \left(\pi_{t+1} - \pi^T\right) + \kappa x_t + e_t$$

$$\kappa\left(\pi_t - \pi^T\right) + \lambda x_t = 0$$

(b) Suppose the exogenous shocks are mean zero, serially uncorrelated processes. Plot the two equilibrium conditions for $e_t = 0$ (put the output gap on the horizontal axis and inflation gap on the vertical axis).

(c) Use your graph to illustrate *and explain* the effects on x_t and π_t of a negative inflation shock ($e_t < 0$).

(d) Use your graph to illustrate *and explain* the effects on x_t and π_t of a negative demand shock ($r_t^f < 0$).

6. Consider a simple new Keynesian model given by

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

where x is the output gap, π is inflation, i is the nominal interest rate, r^n and e are exogenous stochastic shocks, and σ, β, and κ are constants. Assume the exogenous shocks r^f and e are serially uncorrelated so that $\mathrm{E}_t r_{t+1}^f = \mathrm{E}_t e_{t+1} = 0$. In this case, $\mathrm{E}_t x_{t+1} = \mathrm{E}_t \pi_{t+1} = 0$. Suppose the central bank does not observe the demand shock r_t^f but instead has a forecast of it, denoted by r_t^{ff}. The policy rule is

$$i_t = r_t^{ff} + \phi \pi_t$$

where ϕ is a constant greater than 1.

(a) Solve for x_t and π_t. *Carefully explain* how inflation and the output gap are affected by the central bank's forecast error $err_t \equiv r_t^f - r_t^{ff}$. *With mean zero, serially uncorrelated shocks, the aggregate demand side of the model reduces to*

$$\begin{aligned} x_t &= -\left(\frac{1}{\sigma}\right)\left(i_t - r_t^f\right) \\ &= -\left(\frac{1}{\sigma}\right)\left(r_t^{ff} + \phi\pi_t - r_t^f\right) \\ &= -\left(\frac{\phi}{\sigma}\right)\pi_t + \left(\frac{1}{\sigma}\right)\left(r_t^f - r_t^{ff}\right) \\ &= -\left(\frac{\phi}{\sigma}\right)\pi_t + \left(\frac{1}{\sigma}\right) err_t \end{aligned}$$

so a forecast error acts like a positive shock to aggregate demand. If $err_t > 0$*, the central bank has underestimated* r_t^f *and so they have not raised the nominal interest rate sufficiently to offset the impact of the* r^f *shock on aggregate demand. Thus,* i_t *is too low relative to* r_t^f *which leads to an economic expansion. The Phillips curve is still given by*

$$\pi_t = \kappa x_t + e_t$$

So a positive err_t *increases* x_t *which pushes up inflation. As inflation rises, the central bank reacts by raising the nominal interest rate (this is the* $\phi\pi_t$ *term). In the equilibrium, some of the direct effect of* err_t *on* x_t *is offset by the rise in* i_t*, but there is still some rise in* x_t *and* π_t*.*

(b) *Explain* how the effects of the demand shock r_t^f and the inflation shock e_t on the output gap and inflation are affected by the central bank's choice of ϕ. *If the shock* r_t^f *is fully forecast by the central bank, there will be no effect on either the output gap or inflation as* i_t *is raised to offset the shock. So what will matter is* err_t*. Continuing with the explanation from part (a), if* ϕ *is large, than the rise in inflation induces a large rise in* i_t*. This acts to offset most of the rise in* x *and helps stabilize inflation. If* ϕ *is small, than both* x *and* π *will be affected more by the forecast error.*

7. Consider a firm that can invest in one of two projects. Project $i = 1, 2$ yields a gross rate of return of $R - x_i$ with probability 1/2 and $R + x_i$ with probability 1/2. Assume $x_2 > x_1 + 2[R - (1+r)L] > x_1$ so project 2 is a riskier project. The firm borrows L to undertake the project and has collateral C. The lender's opportunity cost of funds (the rate of return it can earn if it does not lend to the firm) is r. Both the firm and the lender are risk neutral. Assume the firm defaults when $R - x_i$ occurs in which case the lender receives $R - x_i + C$.

   (a) Suppose the lender can, without cost, monitor which project the firm chooses. What interest rate will the lender charge the firm if the firm picks project 1? What interest rate will it charge if the firm picks project 2? If the firm defaults on the loan (when the return is $R - x_i$), the lender gets $R - x_i + C < (1 + r^L)L$ if r^L is the interest rate on the loan. (Hint: for either project, the expected rate of return to the lender must equal r.) *On project* i*, the lender's expected return is*

$$\frac{1}{2}\left(1 + r_i^L\right)L + \frac{1}{2}\left[R - x_i + C\right]$$

and this must equal $(1+r)\,L$*. Hence,* r_i^L *must be such that*

$$\frac{1}{2}\left[\left(1+r_i^L\right)-(1+r)\right]L+\frac{1}{2}\left[R-x_i+C-(1+r)\,L\right]=0$$

$$\left(r_i^L-r\right)L=\left[(1+r)\,L-C-R+x_i\right]$$

$$\left(r_i^L-r\right)=r+\frac{\left[L+x_i-C-R\right]}{L}$$

so the lender charges a higher interest rate on the loan for the riskier project. (Note that $[(1+r)\,L-C-R+x_i]$ *is positive since by assuming, the firm has to default when* $R-x_i$ *occurs which means* $r_i^L-r>0$*.)*

(b) Using your answer from part (a), *explain* why the spread is lower if the borrower has more collateral.

(c) Now suppose the firm chooses which project to undertake after it receives the loan and the lender cannot observe which project is undertaken. What interest rate will the bank charge on loans? Can good, i.e., low risk, projects get funding? Explain. *The firm's expected return is higher on the high risk project 2. So the lender has to assume only type 2 projects will be undertaken. The loan rate will therefore be* $r_2^L>r_1^L$ and at this rate, the return to the firm with a safe project is

$$\begin{aligned}
\frac{1}{2}\left[R+x_1-\left(1+r_2^L\right)L\right]-\frac{1}{2}C &= \frac{1}{2}\left[R+x_1-C-2\,(1+r)\,L+R-x_2+C\right]\\
&= R-(1+r)\,L-\frac{1}{2}\,(x_2-x_1)\\
&= \frac{1}{2}\left\{x_2-x_1-2\left[R-(1+r)\,L\right]\right\}\\
&< 0
\end{aligned}$$

so firms with safe projects can borrow at r_2^L *but will choose not to borrow. (The inequality follows from the assumption made above about* x_2 *relative to* x_1*.*

8. Suppose the balance sheet of an individual consists of assets a_t, personal debts to friend d_t, bank loans of b_t, and a net worth of n_t, so

$$a=d+b+n.$$

Suppose the agent can default on its debts to friends and the bank and capture (or divert) a fraction θ, $0<\theta<1$, of its "divertible" assets. Divertible assets are defined as $a-\omega_d d-\omega_b b$, where $0\leq\omega_d,\,\omega_b\leq 1$ and $\omega_d<\omega_b$ (assume it is easier to divert assets brought with money borrowers from friends than with money borrowed from the bank since the bank will use the legal system to try to collect). Assume the individual wants to maximize

$$r_a a-r_d d-r_b b$$

where r_a is the return on assets, and r_i is the cost of borrowing for $i=d,b$.

(a) Explain why the individual has an incentive to default (divert assets) if

$$r_a a-r_d d-r_b b<\theta\left(a-\omega_d d-\omega_b b\right). \tag{1}$$

(b) If λ is the Lagrangian multiplier on the constraint, show that the first order conditions for the individual's choice of a, d, and b implies

$$r_b - r_d = \left(\frac{\lambda}{1+\lambda}\right)\theta\left(\omega_b - \omega_d\right) > 0$$

if $\lambda > 0$ and $\theta > 0$. Explain intuitive why the cost of borrowing from the bank is more expensive. *Using the balance sheet, the problem is*

$$\max_{d,b}\left\{\begin{array}{c} r_a\left(d+b+n\right) - r_d d - r_b b \\ +\lambda\left[r_a\left(d+b+n\right) - r_d d - r_b b - \theta\left(d+b+n-\omega_d d - \omega_b b\right)\right] \end{array}\right\}$$

$$\max_{d,b}\left\{\begin{array}{c} \left(1+\lambda\right)\left(r_a - r_d\right)d + \left(1+\lambda\right)\left(r_a - r_b\right)b + \left(1+\lambda\right)r_a n \\ -\lambda\theta\left[\left(1-\omega_d\right)d + \left(1-\omega_b\right)b + n\right] \end{array}\right\}$$

and the FOCs are

$$\left(1+\lambda\right)\left(r_a - r_d\right) - \lambda\theta\left(1-\omega_d\right) = 0$$

$$\left(1+\lambda\right)\left(r_a - r_b\right) - \lambda\theta\left(1-\omega_b\right) = 0$$

so

$$\left(1+\lambda\right)\left(r_a - r_d\right) - \lambda\theta\left(1-\omega_d\right) - \left(1+\lambda\right)\left(r_a - r_b\right) + \lambda\theta\left(1-\omega_b\right) = 0$$

$$\left(1+\lambda\right)\left(r_b - r_d\right) + \lambda\theta\omega_d - \lambda\theta\omega_b = 0$$

$$\left(1+\lambda\right)\left(r_b - r_d\right) - \lambda\theta\left(\omega_b - \omega_d\right) = 0$$

or

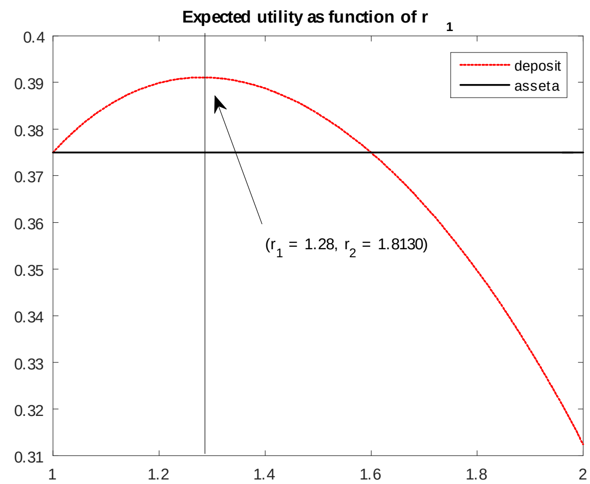
$$r_b - r_d = \left(\frac{\lambda}{1+\lambda}\right)\theta\left(\omega_b - \omega_d\right) > 0$$

implying, in this example, it is more expensive to borrow from the bank. The borrowing from the bank is more expensive because it is more likely it will have to repay.

9. Suppose a bank promises depositors that they can receive r_1 if they withdraw their funds at $t = 1$ and r_2 if they wait until $t = 2$. The bank excepts a fraction θ of depositors will want to take their money out at $t = 1$. The bank can invest in an asset, call it asset a, that pays off r if sold at $t = 1$ and $R > r$ if held until $t = 2$. The bank is risk neutral. Depositors utility is $1 - (1/c_1)$ for the fraction who need to consume at $t = 1$ and $1 - (1/c_2)$ for the $1 - \theta$ who do not need to consume until $t = 2$. Ex ante, depositors do not know when they will need to consume.

   (a) At $t = 0$, before depositors know when they will need to consume, what is the expected utility of investing in the bank deposit? What is the expected utility if the depositor decides instead to buy asset a? *The expected return to the deposit is* $\theta(1-(1/r_1)) + (1-\theta)(1-(1/r_2))$.

   (b) What is the expected return to the deposit? What is the expected return to asset a? $\theta r_1 + (1-\theta)r_2$ *and* $\theta r + (1-\theta)R$.



(c) What is the bank's profit if θ of the depositors withdraw at $t = 1$? Suppose $\hat{\theta} \geq \theta$ withdraw; what is the bank's profit? *At $t = 1$ the bank sells a fraction $\hat{\theta} r_1$ of its portfolio to meet withdrawals. That leaves it with $1 - \hat{\theta} r_1$ of its portfolio still invested in asset a and this then yields $(1 - \hat{\theta} r_1) R$ at $t = 2$. It has promised to pay out r_2 to each of its $1 - \hat{\theta}$ remaining depositors, so its profits are*

$$profit(\theta) = \left(1 - \hat{\theta} r_1\right) R - \left(1 - \hat{\theta}\right) r_2.$$

(The case when θ withdraw is obtained by setting $\hat{\theta} = \theta$.)

(d) Assume $r = 1$ and $R = 2$. Using your results in part (c), let r_1 go from 1 to 1.5 in increments of 0.02 and plot the value of r_2 for which bank profits are zero as a function of r_1. Add to your plot the expected utility of the deposit as a function of r_1 when r_2 is such that bank proficts are zero. Which combination of r_1 and r_2 maximizes depositors' utility? *BankRuns.m. Bank profits are zero when*

$$r_2 = \frac{2(1 - \theta r_1)}{1 - \theta}$$

so the expected utility of the deposit as a function of r_1 when bank profits are zero is

$$U(d) = \theta (1 - (1/r_1)) + (1 - \theta) \left[1 - \left(\frac{2(1 - \theta r_1)}{1 - \theta}\right)^{-1}\right]$$

The values of r_1 and r_2 that maximize expected utitlity are $r_1 = 1.28$, $r_2 = 1.813$. See the figure.

(e) Show that the bank can meet its promise to depositors at $t = 2$ as long as the fraction who withdraw at time $t = 1$ is less than

$$\frac{R - r_2}{R r_1 - r_2}$$

If $\hat{\theta}$ depositors withdraw at $t = 1$, the bank can pay out

$$\frac{2\left(1-\hat{\theta}r_1\right)}{1-\hat{\theta}}$$

to teh remaining depositors at $t = 2$. This will equal or exceed r_2 iff

$$\frac{2\left(1-\hat{\theta}r_1\right)}{1-\hat{\theta}} \geq r_2$$

$$2 - 2\hat{\theta}r_1 \geq r_2 - \hat{\theta}r_2$$

$$\hat{\theta}\left(r_2 - 2r_1\right) \geq r_2 - 2$$

But $r_2 < 2$ so

$$\hat{\theta} \leq \frac{2-r_2}{2r_1-r_2} = \frac{R-r_2}{Rr_1-r_2}$$

(f) If you are depositor who does not need your funds until $t = 2$, what is your best strategy at $t = 1$ if you believe $\hat{\theta}$ depositors will withdraw funds at $t = 1$? *Your expected return at $T = 2$ if you do not withdraw will be*

$$\frac{2(1-\hat{\theta}r_1)}{1-\hat{\theta}}.$$

This has to be greater than r_1 – otherwise you would withdraw at $t = 1$. Let $\hat{\theta}^*$ be the critical tipping point. That is, $\hat{\theta}^*$ is the point such that

$$\frac{2(1-\hat{\theta}^* r_1)}{1-\hat{\theta}^*} = r_1 \Rightarrow \hat{\theta}^* = \frac{2-r_1}{r_1}.$$

10. Suppose the bank expected that it will need to audit a firm that it has lent to with probability q. The costs of auditing are c. Assume banks operate in a competitive environment. If there are no other frictions in the loan market and all borrowers are identical ex ante, explain why the spread between the interest rate on loans and the bank's opportunity cost of funds would be qc.

11. In Woodford's model with two interest rates, let ω denote the spread between the interest rate received by savers i^s and the interest rate charged to borrowers i^b.

    (a) If a negative shock to the financial intermediation sector reduces credit supply at each ω (the supply of credit curve shifts to the left), what happens to the spread? When happens to total credit? How should the central bank respond if it wants to stabilize inflation and the output gap?

    (b) If a positive shock to the demand for credit reduces credit demand at each ω (the demand for credit curve shifts to the right), what happens to the spread? When happens to total credit? How should the central bank respond if it wants to stabilize inflation and the output gap?

(c) From your results in parts (a) and (b), what do you conclude about how policy should react to an increase in interest rate spreads? Should the policy interest rate always be increased when spreads increase? Or should it be decreased? Or is the answer "It depends"? Explain.

12. In the basic flexible price model, why does a temporary rise in government purchases increase output? Explain.

13. In the basic flexible price model, what is the effect of a temporary rise in government purchases on consumption? Explain.

14. Suppose the economy is given by

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t,$$

and the central bank follows a policy rule given by

$$i_t = r_t^f + \phi \mathrm{E}_t \pi_{t+1}.$$

Assume that

$$r_t^f = r^f - a_1 z_t + a_2 g_t + \psi_t,$$

where r^f (without a time subscript) is the steady-state value of the flex-price real interest rate, z is a productivity shock, g is a government spending shock, and ψ represents other sources of demand shocks; a_1 and a_2 are positive constants.

(a) What is the effect of a fiscal spending shock on the output gap and inflation? Explain.

(b) Suppose ψ_t takes on a very large negative value, call it ψ^Z, that pushes i_t to 0. Suppose $\psi_{t+1} = \psi^Z$ with probability q and 0 with probability $1-q$. For simplicity, assume $z = 0$. In addition, assume g is constant at $\bar{g}$. Find the equilibrium values of x and π.

$$x_t = \mathrm{E}_t x_{t+1} + \left(\frac{1}{\sigma}\right)\left(\mathrm{E}_t \pi_{t+1} + r^f + a_2 g_t + \psi^Z\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t,$$

$$x_t^Z = q x_t^Z + \left(\frac{1}{\sigma}\right)\left(q \pi_t^Z + r^f + a_2 g_t + \psi^Z\right)$$

$$\pi_t^Z = \beta q \pi_t^Z + \kappa x_t^Z,$$

$$\sigma(1-q) x_t^Z = q \pi_t^Z + r^f + a_2 g_t + \psi^Z$$

$$(1-\beta q) \pi_t^Z = \kappa x_t^Z,$$

$$\sigma(1-q)(1-\beta q) x_t^Z = q(1-\beta q) \pi_t^Z + (1-\beta q)\left(r^f + a_2 g_t + \psi^Z\right)$$

$$\sigma(1-q)(1-\beta q) x_t^Z = q \kappa x_t^Z + (1-\beta q)\left(r^f + a_2 g_t + \phi^Z\right)$$

$$x_t^Z = \frac{(1-\beta q)\left(r^f + a_2\bar{g} + \psi^Z\right)}{[\sigma(1-q)(1-\beta q) - q\kappa]}$$

$$\pi_t^Z = \frac{\kappa}{1-\beta q} x_t^Z = \frac{\kappa\left(r^f + a_2\bar{g} + \psi^Z\right)}{[\sigma(1-q)(1-\beta q) - q\kappa]}$$

(c) Continuing with the example of part (b), draw the Phillips curve in a graph with the output gap on the horizontal axis and inflation on the vertical axis. Now assume $\bar{g} = 0$ and add the aggregate demand relationship to your graph.

(d) Using your graph from part (c), illustrate what the effects of a positive government spending shock $\bar{g} > 0$ would be on the equilibrium output gap and inflation. Explain.

15. What is a symmetric channel system for implementing monetary policy? Under such a system, explain why the central bank can affect the level of interest rates without altering its balance sheet.

Economics 202
Fall 2015

Review questions for the final exam

1. What is the difference between the nominal rate of interest and the real rate of interest. Which one does economic theory suggest is the most relevant for aggregate spending (i.e., for consumption and investment)? If the nominal interest rate is 1.75% and expected inflation is 2%, what is the real rate of interest?

2. Suppose
$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f\right)$$
$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t,$$
and the central bank follows a policy rule given by
$$i_t = r_t^f + \phi \mathrm{E}_t \pi_{t+1}.$$
   (a) Explain why one equilibrium for this system is $x_{t+i} = \pi_{t+i} = 0$ for all $i \geq 0$.
   (b) Explain why this may not be the only equilibrium if $\phi < 1$. (Hint: Consider what would happen if, for some reason, private agents expected π_{t+1} will be greater than zero.)

3. Under the optimal time-consistent (discretionary) policy in the basic new Keynesian model with policy objectives given by a quadratic loss function in inflation and the output gap, $\kappa\pi_t + \lambda x_t = 0$, $\kappa\pi_{t+1} + \lambda x_{t+1} = 0$, $\kappa\pi_{t+2} + \lambda x_{t+2} = 0$, ... (using standard notation) while the optimal commitment policy ensures $\kappa\pi_t + \lambda x_t = 0$, $\kappa\pi_{t+1} + \lambda\left(x_{t+1} - x_t\right) = 0$, $\kappa\pi_{t+2} + \lambda\left(x_{t+2} - x_{t+1}\right) = 0$, ... Explain why the two policies differ? Why does commitment do better? Why does commitment introduce inertia (i.e., why does policy at $t+i$ involve the lagged output gap x_{t+i-1}?)

4. Consider the following new Keynesian model at a zero nominal interest rate:
$$x_t = \mathrm{E}_t x_{t+1} + \left(\frac{1}{\sigma}\right)\left(\mathrm{E}_t \pi_{t+1} + r^Z\right)$$
$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t,$$
where $r^Z < 0$ is the negative demand shock that has pushed the economy to the zero lower bound. Denote the equilibrium at the zero lower bound by x^Z and π^Z. When the economy is not at the zero lower bound, assume $x = \pi = 0$. Assume the public expects that the economy will remain at the zero lower bound in the following period with probability q and will be out of the zero bound (so that $x = \pi = 0$) with probability $1 - q$.
   (a) What are $\mathrm{E}_t \pi_{t+1}$ and $\mathrm{E}_t x_{t+1}$ equal to?
   (b) Assume $\sigma(1-q)(1-\beta q) - q\kappa > 0$ and solve for the output gap and inflation at the zero lower bound (i.e., solve for x^Z and π^Z).

(c) Suppose the public becomes more pessimistic in the sense they think it is more likely the economy will remain at the zero lower bound (i.e., q increases). What happens to the output gap and inflation? Explain why.

5. Consider a simple new Keynesian model given by

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f\right)$$

$$\left(\pi_t - \pi^T\right) = \beta \mathrm{E}_t\left(\pi_{t+1} - \pi^T\right) + \kappa x_t + e_t$$

where x is the output gap, π is inflation, π^T is the central bank's inflation target, r^f and e are exogenous shocks. Monetary policy is designed to minimize

$$L_t = \frac{1}{2}\left[\left(\pi_t - \pi^T\right)^2 + \lambda x_t^2\right].$$

Under the optimal discretionary policy, show that the optimal targeting criterion takes the form

$$\kappa\left(\pi_t - \pi^T\right) + \lambda x_t = 0.$$

(a) What are the two equilibrium conditions that determine the output gap and inflation?

$$\left(\pi_t - \pi^T\right) = \beta \mathrm{E}_t\left(\pi_{t+1} - \pi^T\right) + \kappa x_t + e_t$$

$$\kappa\left(\pi_t - \pi^T\right) + \lambda x_t = 0$$

(b) Suppose the exogenous shocks are mean zero, serially uncorrelated processes. Plot the two equilibrium conditions for $e_t = 0$ (put the output gap on the horizontal axis and inflation gap on the vertical axis).

(c) Use your graph to illustrate *and explain* the effects on x_t and π_t of a negative inflation shock ($e_t < 0$).

(d) Use your graph to illustrate *and explain* the effects on x_t and π_t of a negative demand shock ($r_t^f < 0$).

6. Consider a simple new Keynesian model given by

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

where x is the output gap, π is inflation, i is the nominal interest rate, r^n and e are exogenous stochastic shocks, and σ, β, and κ are constants. Assume the exogenous shocks r^f and e are serially uncorrelated so that $\mathrm{E}_t r_{t+1}^f = \mathrm{E}_t e_{t+1} = 0$. In this case, $\mathrm{E}_t x_{t+1} = \mathrm{E}_t \pi_{t+1} = 0$. Suppose the central bank does not observe the demand shock r_t^f but instead has a forecast of it, denoted by r_t^{ff}. The policy rule is

$$i_t = r_t^{ff} + \phi \pi_t$$

where ϕ is a constant greater than 1.

(a) Solve for x_t and π_t. *Carefully explain* how inflation and the output gap are affected by the central bank's forecast error $err_t \equiv r_t^f - r_t^{ff}$.

(b) *Explain* how the effects of the demand shock r_t^f and the inflation shock e_t on the output gap and inflation are affected by the central bank's choice of ϕ.

7. Consider a firm that can invest in one of two projects. Project $i = 1, 2$ yields a gross rate of return of $R - x_i$ with probability 1/2 and $R + x_i$ with probability 1/2. Assume $x_2 > x_1 + 2\left[R - (1 + r)L\right] > x_1$ so project 2 is a riskier project. The firm borrows L to undertake the project and has collateral C. The lender's opportunity cost of funds (the rate of return it can earn if it does not lend to the firm) is r. Both the firm and the lender are risk neutral. Assume the firm defaults when $R - x_i$ occurs in which case the lender receives $R - x_i + C$.

   (a) Suppose the lender can, without cost, monitor which project the firm chooses. What interest rate will the lender charge the firm if the firm picks project 1? What interest rate will it charge if the firm picks project 2? If the firm defaults on the loan (when the return is $R - x_i$), the lender gets $R - x_i + C < (1 + r^L)L$ if r^L is the interest rate on the loan. (Hint: for either project, the expected rate of return to the lender must equal r.)

   (b) Using your answer from part (a), *explain* why the spread is lower if the borrower has more collateral.

   (c) Now suppose the firm chooses which project to undertake after it receives the loan and the lender cannot observe which project is undertaken. What interest rate will the bank charge on loans? Can good, i.e., low risk, projects get funding? Explain.

8. Suppose the balance sheet of an individual consists of assets a_t, personal debts to friend d_t, bank loans of b_t, and a net worth of n_t, so

$$a = d + b + n.$$

   Suppose the agent can default on its debts to friends and the bank and capture (or divert) a fraction θ, $0 < \theta < 1$, of its "divertible" assets. Divertible assets are defined as $a - \omega_d d - \omega_b b$, where $0 \leq \omega_d$, $\omega_b \leq 1$ and $\omega_d < \omega_b$ (assume it is easier to divert assets brought with money borrowers from friends than with money borrowed from the bank since the bank will use the legal system to try to collect). Assume the individual wants to maximize

$$r_a a - r_d d - r_b b$$

   where r_a is the return on assets, and r_i is the cost of borrowing for $i = d, b$.

   (a) Explain why the individual has an incentive to default (divert assets) if

$$r_a a - r_d d - r_b b < \theta\left(a - \omega_d d - \omega_b b\right). \tag{1}$$

   (b) If λ is the Lagrangian multiplier on the constraint, show that the first order conditions for the individual's choice of a, d, and b implies

$$r_b - r_d = \left(\frac{\lambda}{1 + \lambda}\right)\theta\left(\omega_b - \omega_d\right) > 0$$

   if $\lambda > 0$ and $\theta > 0$. Explain intuitive why the cost of borrowing from the bank is more expensive.

9. Suppose a bank promises depositors that they can receive r_1 if they withdraw their funds at $t = 1$ and r_2 if they wait until $t = 2$. The bank excepts a fraction θ of depositors will want to take their money out at $t = 1$. The bank can invest in an asset, call it asset a, that pays off r if sold at $t = 1$ and $R > r$ if held until $t = 2$. The bank is risk neutral. Depositors utility is $1 - (1/c_1)$ for the fraction who need to consume at $t = 1$ and $1 - (1/c_2)$ for the $1 - \theta$ who do not need to consume until $t = 2$. Ex ante, depositors do not know when they will need to consume.

   (a) At $t = 0$, before depositors know when they will need to consume, what is the expected utility of investing in the bank deposit? What is the expected utility if the depositor decides instead to buy asset a?

   (b) What is the expected return to the deposit? What is the expected return to asset a? $\theta r_1 + (1 - \theta) r_2$ *and* $\theta r + (1 - \theta) R$.

   (c) What is the bank's profit if θ of the depositors withdraw at $t = 1$? Suppose $\hat{\theta} \geq \theta$ withdraw; what is the bank's profit?

   (d) Assume $r = 1$ and $R = 2$. Using your results in part (c), let r_1 go from 1 to 1.5 in increments of 0.02 and plot the value of r_2 for which bank profits are zero as a function of r_1. Add to your plot the expected utility of the deposit as a function of r_1 when r_2 is such that bank proficts are zero. Which combination of r_1 and r_2 maximizes depositors' utility?

   (e) Show that the bank can meet its promise to depositors at $t = 2$ as long as the fraction who withdraw at time $t = 1$ is less than

$$\frac{R - r_2}{Rr_1 - r_2}$$

   (f) If you are depositor who does not need your funds until $t = 2$, what is your best strategy at $t = 1$ if you believe $\hat{\theta}$ depositors will withdraw funds at $t = 1$?

10. Suppose the bank expected that it will need to audit a firm that it has lent to with probability q. The costs of auditing are c. Assume banks operate in a competitive environment. If there are no other frictions in the loan market and all borrowers are identical ex ante, explain why the spread between the interest rate on loans and the bank's opportunity cost of funds would be qc.

11. In Woodford's model with two interest rates, let ω denote the spread between the interest rate received by savers i^s and the interest rate charged to borrowers i^b.

   (a) If a negative shock to the financial intermediation sector reduces credit supply at each ω (the supply of credit curve shifts to the left), what happens to the spread? When happens to total credit? How should the central bank respond if it wants to stabilize inflation and the output gap?

   (b) If a positive shock to the demand for credit reduces credit demand at each ω (the demand for credit curve shifts to the right), what happens to the spread? When happens to total credit? How should the central bank respond if it wants to stabilize inflation and the output gap?

(c) From your results in parts (a) and (b), what do you conclude about how policy should react to an increase in interest rate spreads? Should the policy interest rate always be increased when spreads increase? Or should it be decreased? Or is the answer "It depends"? Explain.

12. In the basic flexible price model, why does a temporary rise in government purchases increase output? Explain.

13. In the basic flexible price model, what is the effect of a temporary rise in government purchases on consumption? Explain.

14. Suppose the economy is given by

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f\right)$$

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Assume that

$$r_t^f = r^f - a_1 z_t + a_2 g_t + \psi_t,$$

where r^f (without a time subscript) is the steady-state value of the flex-price real interest rate, z is a productivity shock, g is a government spending shock, and ψ represents other sources of demand shocks; a_1 and a_2 are positive constants.

(a) What is the effect of a fiscal spending shock on the output gap and inflation? Explain.

(b) Suppose ψ_t takes on a very large negative value, call it ψ^Z, that pushes i_t to 0. Suppose $\psi_{t+1} = \psi^Z$ with probability q and 0 with probability $1-q$. For simplicity, assume $z = 0$. In addition, assume g is constant at $\bar{g}$. Find the equilibrium values of x and π.

(c) Continuing with the example of part (b), draw the Phillips curve in a graph with the output gap on the horizontal axis and inflation on the vertical axis. Now assume $\bar{g} = 0$ and add the aggregate demand relationship to your graph.

(d) Using your graph from part (c), illustrate what the effects of a positive government spending shock $\bar{g} > 0$ would be on the equilibrium output gap and inflation. Explain.

15. What is a symmetric channel system for implementing monetary policy? Under such a system, explain why the central bank can affect the level of interest rates without altering its balance sheet.

Economics 202
Winter 2017

Review questions (and some answers) for the final exam

1. Using the basic Solow model, *carefully explain* how the steady-state value of income *per effective unit of labor* and the steady-state growth rates of total output and the growth rate of consumption *per worker* are affected by (a) a decrease in the saving rate; (b) a decrease in the population growth rate; (c) a decrease in the rate of growth of technology. Show graphically how the steady-state is affected by each of these changes. *(a) Starting from a steady-state, a decrease in the saving rate means that at the initial k, saving now is less than the investment required to maintain k, that is, $sy < (\delta + g + n)\,k$. With lower investment, and more importantly with less investment than is needed to simply maintain k, k starts to decline. The economy converges to a new steady state with a lower k. In the new steady state, output per effective unit of labor and consumption per effective unit of labor are constant, so output grows at the same rate as effective units of labor and consumption grows at the rate of technology. Hence, neither the growth rate of total output nor the growth rate of consumption per worker are affected by the change in s. Total output grows at the rate $g + n$ and consumption per worker grows at the rate g, just as they did initially. (b) and (c) Starting from a steady-state, declines in population growth or the growth rate of technology decrease the investment level required to maintain a constant capital per effective unit of labor as both increase the growth rate of effective units of labor. An decrease in either therefore makes the required investment line $(\delta + g + n)\,k$ flatter. At the initial k, sy is now more than $(\delta + g + n)\,k$, so with saving exceeding the level needed to maintain k, k begins to rise and the economy converges to a higher value of k. Since total output grows in the steady state at the rate $g + n$, it falls if either g or n decreases. Consumption per worker grows at the rate g so its growth rate decreases with a fall in g but is unaffected by a fall in n.*

2. Do the effects of a productivity shock on consumption and investment depend on how persistent the shock is? That is, suppose the productivity shock z_t follows $z_t = \rho z_{t-1} + e_t$ where e is white noise. Do the response of consumption and investment depend on whether $\rho = 0.95$, so that z is very persistent, or $\rho = 0.5$, so that z shocks die out more quickly. *The effects on consumption and investment do depend on the expected persistence of the shock. If a positive productivity shock dies out quickly, that is, temporary or transitory, households will not adjust their consumption very much. Thus, investment will respond more and consumption less. If the shock is long lasting, then households will increase their consumption more in response to what they expect to be a longer lasting rise in income. With consumption responding more, investment (saving) responds less.*

3. What is the difference between the nominal rate of interest and the real rate of interest. Which one does economic theory suggest is the most relevant for aggregate spending (i.e., for consumption and investment)? If the nominal interest rate is 1.75% and expected inflation is 2%, what is the real rate of interest? *The real rate is* $i_t - \mathrm{E}_t\pi_{t+1} = 1.75 - 2 = -0.25\%$.

4. A key component of both the real business cycle model and the new Keynesian model is the forward-looking behavior of consumption. The standard first-order condition for optimal

consumption in these models, when linearized, takes the form

$$c_t = \mathrm{E}_t c_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - \mathrm{E}_t \pi_{t+1}), \tag{1}$$

where the notation should be obvious. Does (1) means consumption is affected only by the *current* interest rate i_t? Explain.

5. Consider a typical loan contract in which the loan amount is L, the borrower either repays $(1+r)L$ or defaults and loses collateral C. Briefly explain why, if the project return is risky, the borrower will prefer to invest in riskier projects while the lender will prefer that the borrower invest in safer projects. *If the project does well, the borrower gets the entire project return minus the fixed amount he/she has to repay. If the project does poorly and the borrower has to default, he/she loses the fixed amount of collateral. So the borrower likes projects that have a chance of doing really great and doesn't care how badly they do when the fail. risky projects are therefore attractive to the borrower. On the other hand, the lend gets fixed amount if the project does well and the collateral plus anything that's left and can be recovered from the project if it fails. So a lender prefers projects that still do okay when the borrower defaults and doesn't care at all about the return when the project does great since the lender only gets the repayment amount.*

6. According to the expectational IS relationship that is central to the new Keynesian model, will aggregate demand rise or fall if the central bank cuts its policy interest rate from 3% to 2% and expected inflation falls from 2% to 0.5%, everything else held constant? Why? *The key here is that aggregate spending should depend on the real interest rate, i.e., the nominal rate minus the expected rate of inflation. So in this example, the real interest rate goes from* $0.03 - 0.02 = 0.01$ *to* $0.02 - 0.05 = 0.015$. *Since the real interest rate has risen (even though the nominal rate fell), we would* expected aggregate spending to fall.

7. Using the basic new Keynesian model, briefly explain why a credible central bank will commit to future policies that do not ignore the past. Be specific. *This is easiest seen by looking at the new Keynesian Phillips Curve:*

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t.$$

If there is an inflation shock at time t (i.e., $e_t \neq 0$*), the central bank can do a better job of stabilizing* π_t *for a given output gap if it can commit to future policies and thereby affect expected future inflation. But it can only be credible if it fulfils the promises it made at time t that influence expectations of inflaiton at t+1, that is, at t+1 it must honor pledges it made in the past (i.e., at time t).*

8. Suppose

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f\right)$$
$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t,$$

and the central bank follows a policy rule given by

$$i_t = r_t^f + \phi \mathrm{E}_t \pi_{t+1}.$$

(a) Explain why one equilibrium for this system is $x_{t+i} = \pi_{t+i} = 0$ for all $i \geq 0$. *Substitute the policy rule into the expectational IS relationship to obtain a two equation system of the form*

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(\phi - 1)\,\mathrm{E}_t \pi_{t+1}$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t.$$

Now verify that $x_{t+i} = \pi_{t+i} = 0$ *for all* $i \geq 0$ *satisfies both these equations.*

(b) Explain why this may not be the only equilibrium if $\phi < 1$. (Hint: Consider what would happen if, for some reason, private agents expected π_{t+1} will be greater than zero.) *Suppose for some extraneous reason, people expected* $E_t \pi_{t+1} > 0$. *Then since* $\phi - 1 < 0$, *this rise in expected inflation increases the output gap and the positive output gap pushes up actual inflation, thereby validating the original rise in expected inflation.*

9. Under the optimal time-consistent (discretionary) policy in the basic new Keynesian model with policy objectives given by a quadratic loss function in inflation and the output gap, $\kappa\pi_t + \lambda x_t = 0$, $\kappa\pi_{t+1} + \lambda x_{t+1} = 0$, $\kappa\pi_{t+2} + \lambda x_{t+2} = 0$, ... (using standard notation) while the optimal commitment policy ensures $\kappa\pi_t + \lambda x_t = 0$, $\kappa\pi_{t+1} + \lambda(x_{t+1} - x_t) = 0$, $\kappa\pi_{t+2} + \lambda(x_{t+2} - x_{t+1}) = 0$, ... Explain why the two policies differ? Why does commitment do better? Why does commitment introduce inertia (i.e., why does policy at $t+i$ involve the lagged output gap x_{t+i-1}?)

10. A credible central bank needs to be very forward looking and ignore the past because it's promises about future policy are its primary means of affecting the economy. Discuss. *Answer – see midterm.*

11. Consider a simple new Keynesian model given by

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

where x is the output gap, π is inflation, i is the nominal interest rate, r^f and e are exogenous stochastic shocks, and σ, β, and κ are all positive constants. Assume the exogenous shocks r^f and e are serially uncorrelated so that $\mathrm{E}_t r_{t+1}^f = \mathrm{E}_t e_{t+1} = 0$. In this case, $\mathrm{E}_t x_{t+1} = \mathrm{E}_t \pi_{t+1} = 0$. The central bank follows a policy rule given by

$$i_t = r_t^f + \phi \pi_t,\ \phi > 1.$$

Carefully explain how the volatility of inflation relative to the volatility of the output gap depends on the value of ϕ in the rule followed by the central bank.

12. Consider the following new Keynesian model at a zero nominal interest rate:

$$x_t = \mathrm{E}_t x_{t+1} + \left(\frac{1}{\sigma}\right)\left(\mathrm{E}_t \pi_{t+1} + r^Z\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t,$$

where $r^Z < 0$ is the negative demand shock that has pushed the economy to the zero lower bound. Denote the equilibrium at the zero lower bound by x^Z and π^Z. When the economy is not at the zero lower bound, assume $x = \pi = 0$. Assume the public expects that the economy will remain at the zero lower bound in the following period with probability q and will be out of the zero bound (so that $x = \pi = 0$) with probability $1 - q$.

(a) What are $\mathrm{E}_t \pi_{t+1}$ and $\mathrm{E}_t x_{t+1}$ equal to?

$$\mathrm{E}_t \pi_{t+1} = q\pi^Z + (1-q) \times 0 = q\pi^Z,$$

and

$$\mathrm{E}_t x_{t+1} = qx^Z + (1-q) \times 0 = qx^Z.$$

(b) Assume $\sigma(1-q)(1-\beta q) - q\kappa > 0$ and solve for the output gap and inflation at the zero lower bound (i.e., solve for x^Z and π^Z). *The two equation system for x^Z and π^Z is*

$$x^Z = qx^Z + \left(\frac{1}{\sigma}\right)\left(q\pi^Z + r^Z\right)$$

$$\pi^Z = \beta q \pi^Z + \kappa x^Z.$$

Rewrite this system as

$$\begin{bmatrix} \sigma(1-q) & -q \\ -\kappa & 1-\beta q \end{bmatrix} \begin{bmatrix} x^Z \\ \pi^Z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} r^Z$$

or

$$\begin{aligned} \begin{bmatrix} x^Z \\ \pi^Z \end{bmatrix} &= \begin{bmatrix} \sigma(1-q) & -q \\ -\kappa & 1-\beta q \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} r^Z \\ &= \left(\frac{1}{\sigma(1-q)(1-\beta q) - q\kappa}\right) \begin{bmatrix} (1-\beta q) \\ \kappa \end{bmatrix} r^Z \end{aligned}$$

Since $r^Z < 0$, both x^Z and π^Z are negative.

(c) Suppose the public becomes more pessimistic in the sense they think it is more likely the economy will remain at the zero lower bound (i.e., q increases). What happens to the output gap and inflation? Explain why. *From part (a),*

$$x^Z = \left(\frac{1-\beta q}{\sigma(1-q)(1-\beta q) - q\kappa}\right) r^Z$$

$$\pi^Z = \left(\frac{\kappa}{\sigma(1-q)(1-\beta q) - q\kappa}\right) r^Z$$

Let $\Delta \equiv \sigma(1-q)(1-\beta q) - q\kappa$. Then

$$\frac{\partial \Delta}{\partial q} = -\sigma(1-\beta q) - \sigma\beta(1-q) - \kappa < 0.$$

It follows a rise in q reduces both the numerator and the denominator of the expression for x^Z, so x^Z becomes more negative (remember r^Z is negative) while the denominator *in the expression for π^Z also falls so π^Z also becomes more negative. BOth the output gap and inflation depend on the expected future output gap and inflation. If the public thinks the economy is more likely to remain at x^Z and π^Z rather than return to $x = \pi = 0$, the fall in expected future x and π reduce current inflation and the output gap.*

13. Consider a simple new Keynesian model given by

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f\right)$$

$$\left(\pi_t - \pi^T\right) = \beta \mathrm{E}_t \left(\pi_{t+1} - \pi^T\right) + \kappa x_t + e_t$$

where x is the output gap, π is inflation, π^T is the central bank's inflation target, r^f and e are exogenous shocks. Monetary policy is designed to minimize

$$L_t = \frac{1}{2}\left[\left(\pi_t - \pi^T\right)^2 + \lambda x_t^2\right].$$

Under the optimal discretionary policy, show that the optimal targeting criterion takes the form

$$\kappa\left(\pi_t - \pi^T\right) + \lambda x_t = 0.$$

(a) What are the two equilibrium conditions that determine the output gap and inflation?

$$\left(\pi_t - \pi^T\right) = \beta \mathrm{E}_t \left(\pi_{t+1} - \pi^T\right) + \kappa x_t + e_t$$

$$\kappa\left(\pi_t - \pi^T\right) + \lambda x_t = 0$$

(b) Suppose the exogenous shocks are mean zero, serially uncorrelated processes. Plot the two equilibrium conditions for $e_t = 0$ (put the output gap on the horizontal axis and inflation gap on the vertical axis).

(c) Use your graph to illustrate *and explain* the effects on x_t and π_t of a negative inflation shock ($e_t < 0$).

(d) Use your graph to illustrate *and explain* the effects on x_t and π_t of a negative demand shock ($r_t^f < 0$).

14. Consider a firm that can invest in one of two projects. Project $i = 1, 2$ yields a gross rate of return of $R - x_i$ with probability 1/2 and $R + x_i$ with probability 1/2. Assume $x_2 > x_1 + 2\left[R - (1 + r) L\right] > x_1$ so project 2 is a riskier project. The firm borrows L to undertake the project and has collateral C. The lender's opportunity cost of funds (the rate of return it can earn if it does not lend to the firm) is r. Both the firm and the lender are risk neutral. Assume the firm defaults when $R - x_i$ occurs in which case the lender receives $R - x_i + C$.

(a) Suppose the lender can, without cost, monitor which project the firm chooses. What interest rate will the lender charge the firm if the firm picks project 1? What interest rate will it charge if the firm picks project 2? If the firm defaults on the loan (when the return is $R - x_i$), the lender gets $R - x_i + C < (1 + r^L)L$ if r^L is the interest rate on the loan. (Hint: for either project, the expected rate of return to the lender must equal r.) *On project i, the lender's expected return is*

$$\frac{1}{2}\left(1+r_i^L\right)L+\frac{1}{2}\left[R-x_i+C\right]$$

and this must equal $(1+r)L$. Hence, r_i^L must be such that

$$\frac{1}{2}\left[\left(1+r_i^L\right)-(1+r)\right]L+\frac{1}{2}\left[R-x_i+C-(1+r)L\right]=0$$

$$\left(r_i^L-r\right)L=\left[(1+r)L-C-R+x_i\right]$$

$$\left(r_i^L-r\right)=r+\frac{\left[L+x_i-C-R\right]}{L}$$

so the lender charges a higher interest rate on the loan for the riskier project. (Note that $[(1+r)L-C-R+x_i]$ is positive since by assuming, the firm has to default when $R-x_i$ occurs which means $r_i^L-r>0$.)

(b) Using your answer from part (a), *explain* why the spread is lower if the borrower has more collateral.

(c) Now suppose the firm chooses which project to undertake after it receives the loan and the lender cannot observe which project is undertaken. What interest rate will the bank charge on loans? Can good, i.e., low risk, projects get funding? Explain. *The firm's expected return is higher on the high risk project 2. So the lender has to assume only type 2 projects will be undertaken. The loan rate will therefore be $r_2^L>r_1^L$* and at this rate, the return to the firm with a safe project is

$$\begin{aligned}\frac{1}{2}\left[R+x_1-\left(1+r_2^L\right)L\right]-\frac{1}{2}C &= \frac{1}{2}\left[R+x_1-C-2(1+r)L+R-x_2+C\right]\\ &= R-(1+r)L-\frac{1}{2}(x_2-x_1)\\ &= \frac{1}{2}\left\{x_2-x_1-2\left[R-(1+r)L\right]\right\}\\ &< 0\end{aligned}$$

so firms with safe projects can borrow at r_2^L but will choose not to borrow. (The inequality follows from the assumption made above about x_2 relative to x_1.

15. Suppose the balance sheet of an individual consists of assets a_t, personal debts to friend d_t, bank loans of b_t, and a net worth of n_t, so

$$a=d+b+n.$$

Suppose the agent can default on its debts to friends and the bank and capture (or divert) a fraction θ, $0<\theta<1$, of its "divertible" assets. Divertible assets are defined as $a-\omega_d d-\omega_b b$, where $0\leq\omega_d,\ \omega_b\leq 1$ and $\omega_d<\omega_b$ (assume it is easier to divert assets brought with money borrowers from friends than with money borrowed from the bank since the bank will use the legal system to try to collect). Assume the individual wants to maximize

$$r_a a-r_d d-r_b b$$

where r_a is the return on assets, and r_i is the cost of borrowing for $i=d,b$.

(a) Explain why the individual has an incentive to default (divert assets) if

$$r_a a - r_d d - r_b b < \theta \left(a - \omega_d d - \omega_b b\right). \tag{2}$$

(b) If λ is the Lagrangian multiplier on the constraint, show that the first order conditions for the individual's choice of a, d, and b implies

$$r_b - r_d = \left(\frac{\lambda}{1+\lambda}\right)\theta\left(\omega_b - \omega_d\right) > 0$$

if $\lambda > 0$ and $\theta > 0$. Explain intuitive why the cost of borrowing from the bank is more expensive. *Using the balance sheet, the problem is*

$$\max_{d,b}\left\{\begin{array}{c} r_a\left(d+b+n\right) - r_d d - r_b b \\ +\lambda\left[r_a\left(d+b+n\right) - r_d d - r_b b - \theta\left(d+b+n-\omega_d d - \omega_b b\right)\right] \end{array}\right\}$$

$$\max_{d,b}\left\{\begin{array}{c} \left(1+\lambda\right)\left(r_a - r_d\right)d + \left(1+\lambda\right)\left(r_a - r_b\right)b + \left(1+\lambda\right)r_a n \\ -\lambda\theta\left[\left(1-\omega_d\right)d + \left(1-\omega_b\right)b + n\right] \end{array}\right\}$$

and the FOCs are

$$\left(1+\lambda\right)\left(r_a - r_d\right) - \lambda\theta\left(1-\omega_d\right) = 0$$
$$\left(1+\lambda\right)\left(r_a - r_b\right) - \lambda\theta\left(1-\omega_b\right) = 0$$

so

$$\left(1+\lambda\right)\left(r_a - r_d\right) - \lambda\theta\left(1-\omega_d\right) - \left(1+\lambda\right)\left(r_a - r_b\right) + \lambda\theta\left(1-\omega_b\right) = 0$$
$$\left(1+\lambda\right)\left(r_b - r_d\right) + \lambda\theta\omega_d - \lambda\theta\omega_b = 0$$
$$\left(1+\lambda\right)\left(r_b - r_d\right) - \lambda\theta\left(\omega_b - \omega_d\right) = 0$$

or

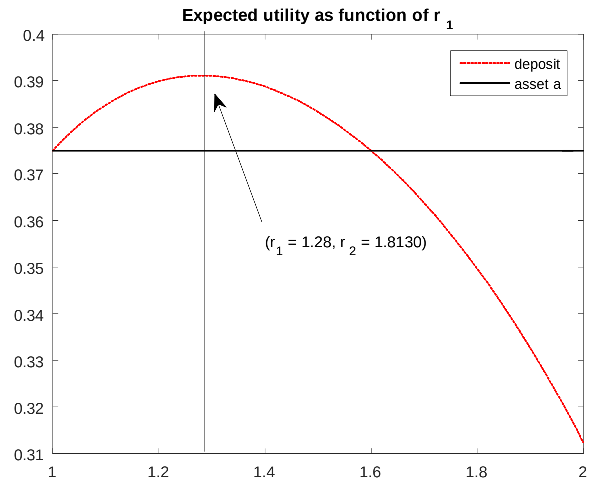
$$r_b - r_d = \left(\frac{\lambda}{1+\lambda}\right)\theta\left(\omega_b - \omega_d\right) > 0$$

implying, in this example, it is more expensive to borrow from the bank. The borrowing from the bank is more expensive because it is more likely it will have to repay.

16. Suppose a bank promises depositors that they can receive r_1 if they withdraw their funds at $t = 1$ and r_2 if they wait until $t = 2$. The bank excepts a fraction θ of depositors will want to take their money out at $t = 1$. The bank can invest in an asset, call it asset a, that pays off r if sold at $t = 1$ and $R > r$ if held until $t = 2$. The bank is risk neutral. Depositors utility is $1 - (1/c_1)$ for the fraction who need to consume at $t = 1$ and $1 - (1/c_2)$ for the $1 - \theta$ who do not need to consume until $t = 2$. Ex ante, depositors do not know when they will need to consume.

(a) At $t = 0$, before depositors know when they will need to consume, what is the expected utility of investing in the bank deposit? What is the expected utility if the depositor decides instead to buy asset a? *The expected return to the deposit is* $\theta(1 - (1/r_1)) + (1-\theta)(1-(1/r_2))$.

(b) What is the expected return to the deposit? What is the expected return to asset a? $\theta r_1 + (1-\theta) r_2$ *and* $\theta r + (1-\theta) R$.



(c) What is the bank's profit if θ of the depositors withdraw at $t = 1$? Suppose $\hat{\theta} \geq \theta$ withdraw; what is the bank's profit? *At $t = 1$ the bank sells a fraction $\hat{\theta} r_1$ of its portfolio to meet withdrawals. That leaves it with $1 - \hat{\theta} r_1$ of its portfolio still invested in asset a and this then yields $(1 - \hat{\theta} r_1) R$ at $t = 2$. It has promised to pay out r_2 to each of its $1 - \hat{\theta}$ remaining depositors, so its profits are*

$$profit(\theta) = \left(1 - \hat{\theta} r_1\right) R - \left(1 - \hat{\theta}\right) r_2.$$

(The case when θ withdrawn is obtained by setting $\hat{\theta} = \theta$.)

(d) Assume $r = 1$ and $R = 2$. Using your results in part (c), let r_1 go from 1 to 1.5 in increments of 0.02 and plot the value of r_2 for which bank profits are zero as a function of r_1. Add to your plot the expected utility of the deposit as a function of r_1 when r_2 is such that bank proficts are zero. Which combination of r_1 and r_2 maximizes depositors' utility? *BankRuns.m. Bank profits are zero when*

$$r_2 = \frac{2\left(1 - \theta r_1\right)}{1 - \theta}$$

so the expected utility of the deposit as a function of r_1 when bank profits are zero is

$$U(d) = \theta\left(1 - (1/r_1)\right) + (1 - \theta)\left[1 - \left(\frac{2\left(1 - \theta r_1\right)}{1 - \theta}\right)^{-1}\right]$$

The values of r_1 and r_2 that maximize expected utitlity are $r_1 = 1.28$, $r_2 = 1.813$. See the figure.

(e) Show that the bank can meet its promise to depositors at $t = 2$ as long as the fraction who withdraw at time $t = 1$ is less than

$$\frac{R - r_2}{R r_1 - r_2}$$

If $\hat{\theta}$ depositors withdraw at $t = 1$, the bank can pay out

$$\frac{2\left(1 - \hat{\theta}r_1\right)}{1 - \hat{\theta}}$$

to teh remaining depositors at $t = 2$. This will equal or exceed r_2 iff

$$\frac{2\left(1 - \hat{\theta}r_1\right)}{1 - \hat{\theta}} \geq r_2$$

$$2 - 2\hat{\theta}r_1 \geq r_2 - \hat{\theta}r_2$$

$$\hat{\theta}\left(r_2 - 2r_1\right) \geq r_2 - 2$$

But $r_2 < 2$ so

$$\hat{\theta} \leq \frac{2 - r_2}{2r_1 - r_2} = \frac{R - r_2}{Rr_1 - r_2}$$

(f) If you are depositor who does not need your funds until $t = 2$, what is your best strategy at $t = 1$ if you believe $\hat{\theta}$ depositors will withdraw funds at $t = 1$? *Your expected return at $T = 2$ if you do not withdraw will be*

$$\frac{2(1 - \hat{\theta}r_1)}{1 - \hat{\theta}}.$$

This has to be greater than r_1 – otherwise you would withdraw at $t = 1$. Let $\hat{\theta}^*$ be the critical tipping point. That is, $\hat{\theta}^*$ is the point such that

$$\frac{2(1 - \hat{\theta}^* r_1)}{1 - \hat{\theta}^*} = r_1 \Rightarrow \hat{\theta}^* = \frac{2 - r_1}{r_1}.$$

17. Suppose the bank expected that it will need to audit a firm that it has lent to with probability q. The costs of auditing are c. Assume banks operate in a competitive environment. If there are no other frictions in the loan market and all borrowers are identical ex ante, explain why the spread between the interest rate on loans and the bank's opportunity cost of funds would be qc.

18. Suppose the economy is given by

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t,$$

and the central bank follows a policy rule given by

$$i_t = r_t^f + \phi \mathrm{E}_t \pi_{t+1}.$$

Assume that

$$r_t^f = r^f - a_1 z_t + a_2 g_t + \psi_t,$$

where r^f (without a time subscript) is the steady-state value of the flex-price real interest rate, z is a productivity shock, g is a government spending shock, and ψ represents other sources of demand shocks; a_1 and a_2 are positive constants.

(a) What is the effect of a fiscal spending shock on the output gap and inflation? Explain.

(b) Suppose ψ_t takes on a very large negative value, call it ψ^Z, that pushes i_t to 0. Suppose $\psi_{t+1} = \psi^Z$ with probability q and 0 with probability $1-q$. For simplicity, assume $z = 0$. In addition, assume g is constant at $\bar{g}$. Find the equilibrium values of x and π.

$$x_t = \mathrm{E}_t x_{t+1} + \left(\frac{1}{\sigma}\right)\left(\mathrm{E}_t \pi_{t+1} + r^f + a_2 g_t + \psi^Z\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t,$$

$$x_t^Z = q x_t^Z + \left(\frac{1}{\sigma}\right)\left(q \pi_t^Z + r^f + a_2 g_t + \psi^Z\right)$$

$$\pi_t^Z = \beta q \pi_t^Z + \kappa x_t^Z,$$

$$\sigma(1-q)\, x_t^Z = q\pi_t^Z + r^f + a_2 g_t + \psi^Z$$

$$(1-\beta q)\, \pi_t^Z = \kappa x_t^Z,$$

$$\sigma(1-q)(1-\beta q)\, x_t^Z = q(1-\beta q)\, \pi_t^Z + (1-\beta q)\left(r^f + a_2 g_t + \psi^Z\right)$$

$$\sigma(1-q)(1-\beta q)\, x_t^Z = q\kappa x_t^Z + (1-\beta q)\left(r^f + a_2 g_t + \phi^Z\right)$$

$$x_t^Z = \frac{(1-\beta q)\left(r^f + a_2 \bar{g} + \psi^Z\right)}{[\sigma(1-q)(1-\beta q) - q\kappa]}$$

$$\pi_t^Z = \frac{\kappa}{1-\beta q} x_t^Z = \frac{\kappa\left(r^f + a_2 \bar{g} + \psi^Z\right)}{[\sigma(1-q)(1-\beta q) - q\kappa]}$$

(c) Continuing with the example of part (b), draw the Phillips curve in a graph with the output gap on the horizontal axis and inflation on the vertical axis. Now assume $\bar{g} = 0$ and add the aggregate demand relationship to your graph.

(d) Using your graph from part (c), illustrate what the effects of a positive government spending shock $\bar{g} > 0$ would be on the equilibrium output gap and inflation. Explain.

19. Suppose the balance sheet of an individual bank consists of assets a_t, uninsured deposit liabilities d_t, loans from other banks b_t, and the bank's own capital n_t, so $a = d + b + n$. The bank can divert for its own use a fraction θ, $0 < \theta < 1$, of its "divertible" assets, defined as $a - \omega_d d - \omega_b b$, where $0 \le \omega_d, \omega_b \le 1$ and $\omega_d < \omega_b$. Assume the bank wants to maximize the value of the bank $V \equiv r_a a - r_d d - r_b b$, where r_a is the return on assets, and r_i is the cost of borrowing for $i = d, b$.

(a) Under what conditions does the bank have an incentive to divert assets? *It has an incentive to divert whenever the value of the bank is less than the amount the bank can divert, i.e., whenever*

$$V = r_a a - r_d d - r_b b < \theta(a - \omega_d d - \omega_b b).$$

(b) If λ is the Lagrangian multiplier on the constraint that it not divert assets, show that the first order conditions for the individual's choice of a, d, and b implies

$$r_a - r_i = \left(\frac{\lambda}{1+\lambda}\right)\theta\left(1-\omega_i\right) > 0$$

for $i = d$, b. *The bank's decision problem is*

$$\max_{a,b,d} r_a a - r_d d - r_b b + \lambda\left[r_a a - r_d d - r_b b - \theta\left(a - \omega_d d - \omega_b b\right)\right]$$

where $a = b + d + n$. *Using the balance sheet, we can rewrite the problem as*

$$\max_{a,b,d}\left(r_a - r_d\right)d + \left(r_a - r_b\right)b + r_a n + \lambda\left[\begin{array}{c}\left(r_a - r_d\right)d\left(r_a - r_b\right)b + r_a n \\ -\theta\left(\left(1-\omega_d\right)d + \left(1-\omega_b\right)b + n\right)\end{array}\right]$$

and the first order conditions are

$$\left(r_a - r_d\right)\left(1+\lambda\right) - \lambda\theta\left(1-\omega_d\right) = 0$$

$$\left(r_a - r_b\right)\left(1+\lambda\right) - \lambda\theta\left(1-\omega_b\right) = 0$$

or

$$r_a - r_i = \left(\frac{\lambda}{1+\lambda}\right)\theta\left(1-\omega_i\right) > 0$$

(c) If $\lambda > 0$ and $\theta > 0$, explain intuitively why the spread between the return on assets r_a and the cost of borrowing is larger on the bank's deposit liabilities than it is for the cost of borrowing from other banks. *In the absence of the moral hazard friction, arbitrage would ensure* $r_a = r_b = r_d$. *The presence of the friction* ($\omega_i < 1$) *leads to spreads as arbitrage is limited. The larger the friction (the smaller* ω_i*) the larger the spread will be. For every dollar raised via deposits, the bank can divert* $1-\omega_d$ *of the assets it uses the funds to purchase, while from every dollar of bank loans, it can potentially divert* $1-\omega_b < 1-\omega_d$. *So te bank will be less limited in borrowing from other banks and it can arbitrage away more of the spread. Perhaps another way of seeing this is that the bank will borrower from each source until the marginal return on assets is equal to the marginal cost of borrowing from that source. The marginal cost of borrowing is* r_i *plus the premium arising because of concerns the bank will divert assets, so*

$$r_a = r_i + \left(\frac{\lambda}{1+\lambda}\right)\theta\left(1-\omega_i\right).$$

The premium is smaller when borrowing from other banks as they do not have to worry as much about the moral hazard problem.

20. In the basic flexible price model, why does a temporary rise in government purchases financed by taxes increase output? Explain. *the rise in taxes reduces the income of households. They respond by reducing consumption and leisure (i.e., they increase labor supply). The increase in labor supply (the labor supply curve shifts to the right at each real wage) leads to a fall in real wages and a rise in employment. Total income in a flexible price environment rises but by less than the rise ingovernment purchases.*

21. In the basic flexible price model, what is the effect of a temporary rise in government purchases on consumption that is financied by an increase in taxes? Explain. *A temporary rise in government spending has the effects explained in question 20. Current consumption falls as government purchases rise more than the rise in incmome, so* $C = Y - G$ *falls. Since the fall is temporary (by assumption), the real interest rate has to move to make households willing to have lower consumption today relative to the future. This requires a rise in the real interest rate to induce household to willingly postpone consumption.*

22. According to the new Keynesian model, why does inflation volatility reduce welfare?

23. Suppose both prices and wages are sticky. Will monetary policy be able to ensure actual output fluctuates in line with movements in the efficient level of output and so ensure the output gap remains at zero? Carefully explain.

24. Suppose thre is a temporary need for government purcases to rise. Explain why it can be optimal for the government to borrow to finance this temporary rise in G_t.

Economics 202
Winter 2017

Review questions (and some answers) for the final exam

1. Using the basic Solow model, *carefully explain* how the steady-state value of income *per effective unit of labor* and the steady-state growth rates of total output and the growth rate of consumption *per worker* are affected by (a) a decrease in the saving rate; (b) a decrease in the population growth rate; (c) a decrease in the rate of growth of technology. Show graphically how the steady-state is affected by each of these changes.

2. Do the effects of a productivity shock on consumption and investment depend on how persistent the shock is? That is, suppose the productivity shock z_t follows $z_t = \rho z_{t-1} + e_t$ where e is white noise. Do the response of consumption and investment depend on whether $\rho = 0.95$, so that z is very persistent, or $\rho = 0.5$, so that z shocks die out more quickly.

3. What is the difference between the nominal rate of interest and the real rate of interest. Which one does economic theory suggest is the most relevant for aggregate spending (i.e., for consumption and investment)? If the nominal interest rate is 1.75% and expected inflation is 2%, what is the real rate of interest?

4. A key component of both the real business cycle model and the new Keynesian model is the forward-looking behavior of consumption. The standard first-order condition for optimal consumption in these models, when linearized, takes the form

$$c_t = \mathrm{E}_t c_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - \mathrm{E}_t \pi_{t+1}), \tag{1}$$

where the notation should be obvious. Does (1) means consumption is affected only by the *current* interest rate i_t? Explain.

5. Consider a typical loan contract in which the loan amount is L, the borrower either repays $(1+r)L$ or defaults and loses collateral C. Briefly explain why, if the project return is risky, the borrower will prefer to invest in riskier projects while the lender will prefer that the borrower invest in safer projects.

6. According to the expectational IS relationship that is central to the new Keynesian model, will aggregate demand rise or fall if the central bank cuts its policy interest rate from 3% to 2% and expected inflation falls from 2% to 0.5%, everything else held constant? Why?

7. Using the basic new Keynesian model, briefly explain why a credible central bank will commit to future policies that do not ignore the past. Be specific.

8. Suppose

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f\right)$$
$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t,$$

and the central bank follows a policy rule given by

$$i_t = r_t^f + \phi \mathrm{E}_t \pi_{t+1}.$$

(a) Explain why one equilibrium for this system is $x_{t+i} = \pi_{t+i} = 0$ for all $i \geq 0$.

(b) Explain why this may not be the only equilibrium if $\phi < 1$. (Hint: Consider what would happen if, for some reason, private agents expected π_{t+1} will be greater than zero.)

9. Under the optimal time-consistent (discretionary) policy in the basic new Keynesian model with policy objectives given by a quadratic loss function in inflation and the output gap, $\kappa\pi_t + \lambda x_t = 0$, $\kappa\pi_{t+1} + \lambda x_{t+1} = 0$, $\kappa\pi_{t+2} + \lambda x_{t+2} = 0$, ... (using standard notation) while the optimal commitment policy ensures $\kappa\pi_t + \lambda x_t = 0$, $\kappa\pi_{t+1} + \lambda (x_{t+1} - x_t) = 0$, $\kappa\pi_{t+2} + \lambda (x_{t+2} - x_{t+1}) = 0$, ... Explain why the two policies differ? Why does commitment do better? Why does commitment introduce inertia (i.e., why does policy at $t+i$ involve the lagged output gap x_{t+i-1}?)

10. A credible central bank needs to be very forward looking and ignore the past because it's promises about future policy are its primary means of affecting the economy. Discuss.

11. Consider a simple new Keynesian model given by

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

where x is the output gap, π is inflation, i is the nominal interest rate, r^f and e are exogenous stochastic shocks, and σ, β, and κ are all positive constants. Assume the exogenous shocks r^f and e are serially uncorrelated so that $\mathrm{E}_t r_{t+1}^f = \mathrm{E}_t e_{t+1} = 0$. In this case, $\mathrm{E}_t x_{t+1} = \mathrm{E}_t \pi_{t+1} = 0$. The central bank follows a policy rule given by

$$i_t = r_t^f + \phi \pi_t,\ \phi > 1.$$

Carefully explain how the volatility of inflation relative to the volatility of the output gap depends on the value of ϕ in the rule followed by the central bank.

12. Consider the following new Keynesian model at a zero nominal interest rate:

$$x_t = \mathrm{E}_t x_{t+1} + \left(\frac{1}{\sigma}\right)\left(\mathrm{E}_t \pi_{t+1} + r^Z\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t,$$

where $r^Z < 0$ is the negative demand shock that has pushed the economy to the zero lower bound. Denote the equilibrium at the zero lower bound by x^Z and π^Z. When the economy is not at the zero lower bound, assume $x = \pi = 0$. Assume the public expects that the economy will remain at the zero lower bound in the following period with probability q and will be out of the zero bound (so that $x = \pi = 0$) with probability $1 - q$.

(a) What are $\mathrm{E}_t \pi_{t+1}$ and $\mathrm{E}_t x_{t+1}$ equal to?

(b) Assume $\sigma(1-q)(1-\beta q) - q\kappa > 0$ and solve for the output gap and inflation at the zero lower bound (i.e., solve for x^Z and π^Z).

(c) Suppose the public becomes more pessimistic in the sense they think it is more likely the economy will remain at the zero lower bound (i.e., q increases). What happens to the output gap and inflation? Explain why.

13. Consider a simple new Keynesian model given by

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f\right)$$

$$\left(\pi_t - \pi^T\right) = \beta \mathrm{E}_t \left(\pi_{t+1} - \pi^T\right) + \kappa x_t + e_t$$

where x is the output gap, π is inflation, π^T is the central bank's inflation target, r^f and e are exogenous shocks. Monetary policy is designed to minimize

$$L_t = \frac{1}{2}\left[\left(\pi_t - \pi^T\right)^2 + \lambda x_t^2\right].$$

Under the optimal discretionary policy, show that the optimal targeting criterion takes the form

$$\kappa\left(\pi_t - \pi^T\right) + \lambda x_t = 0.$$

(a) What are the two equilibrium conditions that determine the output gap and inflation?

$$\left(\pi_t - \pi^T\right) = \beta \mathrm{E}_t \left(\pi_{t+1} - \pi^T\right) + \kappa x_t + e_t$$

$$\kappa\left(\pi_t - \pi^T\right) + \lambda x_t = 0$$

(b) Suppose the exogenous shocks are mean zero, serially uncorrelated processes. Plot the two equilibrium conditions for $e_t = 0$ (put the output gap on the horizontal axis and inflation gap on the vertical axis).

(c) Use your graph to illustrate *and explain* the effects on x_t and π_t of a negative inflation shock ($e_t < 0$).

(d) Use your graph to illustrate *and explain* the effects on x_t and π_t of a negative demand shock ($r_t^f < 0$).

14. Consider a firm that can invest in one of two projects. Project $i = 1, 2$ yields a gross rate of return of $R - x_i$ with probability 1/2 and $R + x_i$ with probability 1/2. Assume $x_2 > x_1 + 2\left[R - (1 + r)L\right] > x_1$ so project 2 is a riskier project. The firm borrows L to undertake the project and has collateral C. The lender's opportunity cost of funds (the rate of return it can earn if it does not lend to the firm) is r. Both the firm and the lender are risk neutral. Assume the firm defaults when $R - x_i$ occurs in which case the lender receives $R - x_i + C$.

(a) Suppose the lender can, without cost, monitor which project the firm chooses. What interest rate will the lender charge the firm if the firm picks project 1? What interest rate will it charge if the firm picks project 2? If the firm defaults on the loan (when the return is $R - x_i$), the lender gets $R - x_i + C < (1 + r^L)L$ if r^L is the interest rate on the loan. (Hint: for either project, the expected rate of return to the lender must equal r.)

(b) Using your answer from part (a), *explain* why the spread is lower if the borrower has more collateral.

(c) Now suppose the firm chooses which project to undertake after it receives the loan and the lender cannot observe which project is undertaken. What interest rate will the bank charge on loans? Can good, i.e., low risk, projects get funding? Explain.

15. Suppose the balance sheet of an individual consists of assets a_t, personal debts to friend d_t, bank loans of b_t, and a net worth of n_t, so

$$a = d + b + n.$$

Suppose the agent can default on its debts to friends and the bank and capture (or divert) a fraction θ, $0 < \theta < 1$, of its "divertible" assets. Divertible assets are defined as $a - \omega_d d - \omega_b b$, where $0 \leq \omega_d,\ \omega_b \leq 1$ and $\omega_d < \omega_b$ (assume it is easier to divert assets brought with money borrowers from friends than with money borrowed from the bank since the bank will use the legal system to try to collect). Assume the individual wants to maximize

$$r_a a - r_d d - r_b b$$

where r_a is the return on assets, and r_i is the cost of borrowing for $i = d, b$.

(a) Explain why the individual has an incentive to default (divert assets) if

$$r_a a - r_d d - r_b b < \theta \left(a - \omega_d d - \omega_b b\right). \tag{2}$$

(b) If λ is the Lagrangian multiplier on the constraint, show that the first order conditions for the individual's choice of a, d, and b implies

$$r_b - r_d = \left(\frac{\lambda}{1+\lambda}\right) \theta \left(\omega_b - \omega_d\right) > 0$$

if $\lambda > 0$ and $\theta > 0$. Explain intuitive why the cost of borrowing from the bank is more expensive.

16. Suppose a bank promises depositors that they can receive r_1 if they withdraw their funds at $t = 1$ and r_2 if they wait until $t = 2$. The bank excepts a fraction θ of depositors will want to take their money out at $t = 1$. The bank can invest in an asset, call it asset a, that pays off r if sold at $t = 1$ and $R > r$ if held until $t = 2$. The bank is risk neutral. Depositors utility is $1 - (1/c_1)$ for the fraction who need to consume at $t = 1$ and $1 - (1/c_2)$ for the $1 - \theta$ who do not need to consume until $t = 2$. Ex ante, depositors do not know when they will need to consume.

(a) At $t = 0$, before depositors know when they will need to consume, what is the expected utility of investing in the bank deposit? What is the expected utility if the depositor decides instead to buy asset a? T

(b) What is the expected return to the deposit? What is the expected return to asset a?

(c) What is the bank's profit if θ of the depositors withdraw at $t = 1$? Suppose $\hat{\theta} \geq \theta$ withdraw; what is the bank's profit?

(d) Assume $r = 1$ and $R = 2$. Using your results in part (c), let r_1 go from 1 to 1.5 in increments of 0.02 and plot the value of r_2 for which bank profits are zero as a function of r_1. Add to your plot the expected utility of the deposit as a function of r_1 when r_2 is such that bank proficts are zero. Which combination of r_1 and r_2 maximizes depositors' utility?

(e) Show that the bank can meet its promise to depositors at $t = 2$ as long as the fraction who withdraw at time $t = 1$ is less than

$$\frac{R - r_2}{Rr_1 - r_2}$$

(f) If you are depositor who does not need your funds until $t = 2$, what is your best strategy at $t = 1$ if you believe $\hat{\theta}$ depositors will withdraw funds at $t = 1$? Y

17. Suppose the bank expected that it will need to audit a firm that it has lent to with probability q. The costs of auditing are c. Assume banks operate in a competitive environment. If there are no other frictions in the loan market and all borrowers are identical ex ante, explain why the spread between the interest rate on loans and the bank's opportunity cost of funds would be qc.

18. Suppose the economy is given by

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t,$$

and the central bank follows a policy rule given by

$$i_t = r_t^f + \phi \mathrm{E}_t \pi_{t+1}.$$

Assume that

$$r_t^f = r^f - a_1 z_t + a_2 g_t + \psi_t,$$

where r^f (without a time subscript) is the steady-state value of the flex-price real interest rate, z is a productivity shock, g is a government spending shock, and ψ represents other sources of demand shocks; a_1 and a_2 are positive constants.

(a) What is the effect of a fiscal spending shock on the output gap and inflation? Explain.

(b) Suppose ψ_t takes on a very large negative value, call it ψ^Z, that pushes i_t to 0. Suppose $\psi_{t+1} = \psi^Z$ with probability q and 0 with probability $1-q$. For simplicity, assume $z = 0$. In addition, assume g is constant at $\bar{g}$. Find the equilibrium values of x and π.

(c) Continuing with the example of part (b), draw the Phillips curve in a graph with the output gap on the horizontal axis and inflation on the vertical axis. Now assume $\bar{g} = 0$ and add the aggregate demand relationship to your graph.

(d) Using your graph from part (c), illustrate what the effects of a positive government spending shock $\bar{g} > 0$ would be on the equilibrium output gap and inflation. Explain.

19. Suppose the balance sheet of an individual bank consists of assets a_t, uninsured deposit liabilities d_t, loans from other banks b_t, and the bank's own capital n_t, so $a = d + b + n$. The bank can divert for its own use a fraction θ, $0 < \theta < 1$, of its "divertible" assets, defined as $a - \omega_d d - \omega_b b$, where $0 \leq \omega_d$, $\omega_b \leq 1$ and $\omega_d < \omega_b$. Assume the bank wants to maximize the value of the bank $V \equiv r_a a - r_d d - r_b b$, where r_a is the return on assets, and r_i is the cost of borrowing for $i = d$, b.

    (a) Under what conditions does the bank have an incentive to divert assets?

    (b) If λ is the Lagrangian multiplier on the constraint that it not divert assets, show that the first order conditions for the individual's choice of a, d, and b implies

    $$r_a - r_i = \left(\frac{\lambda}{1+\lambda}\right)\theta\left(1 - \omega_i\right) > 0$$

    for $i = d$, b.

    (c) If $\lambda > 0$ and $\theta > 0$, explain intuitively why the spread between the return on assets r_a and the cost of borrowing is larger on the bank's deposit liabilities than it is for the cost of borrowing from other banks.

20. In the basic flexible price model, why does a temporary rise in government purchases financed by taxes increase output? Explain. *the rise in taxes reduces the income of households.*

21. In the basic flexible price model, what is the effect of a temporary rise in government purchases on consumption that is financied by an increase in taxes? Explain.

22. According to the new Keynesian model, why does inflation volatility reduce welfare?

23. Suppose both prices and wages are sticky. Will monetary policy be able to ensure actual output fluctuates in line with movements in the efficient level of output and so ensure the output gap remains at zero? Carefully explain.

24. Suppose thre is a temporary need for government purcases to rise. Explain why it can be optimal for the government to borrow to finance this temporary rise in G_t.

Glossary

Alan Ledesma

Some coefficients' interpretation for Econ-202. notice that some of them may have different meanings depending on the context.

	Source	Description
δ	$K_{t+1} = (1-\delta)K_t + I_t$	Rate of capital depreciation.
α	$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$	Cobb-Douglas Coef: Capital share in production.
α	$Y_t = e^{z_t} N_t^{1-\alpha}$	Cobb-Douglas Coef: $\alpha \in (0,1)$ implies decreasing return to scale while $\alpha = 0$ implies constant return to scale.
n	$L_t = (1+n)^t L_0$	Exogenous growth rate of the working force.
g	$A_t = (1+g)^t A_0$	Exogenous growth rate of technology.
s	$S_t = sY_t$	Saving rate.
β	$Y_t = K_t^{\alpha}(A_t L_t)^{\beta}$	Cobb-Douglas Coef: $\beta/(\alpha+\beta)$ $\equiv$Labor share in production.
β	$\sum_{i=1}^{\infty} \beta L_{t+1} U\left(\frac{C_{t+i}}{L_{t+i}}\right)$	Household's impatience discount factor. That is, $\beta = \frac{1}{1+\rho}$ where ρ is the household's impatience rate.
ρ	$\rho \equiv \frac{1-\beta}{\beta}$ or $\beta \equiv \frac{1}{1+\rho}$	Household's impatience rate.
ρ	$z_t = \rho z_{t-1} + e_t$	Productivity shock persistence.
σ_e	$z_t = \rho z_{t-1} + e_t$	Standard deviation of e_t.
σ	$U(X) = \frac{X^{1-\sigma}}{1-\sigma}$	There are two interpretations: $i)$ $1/\sigma$ is the household's inter-temporal consumption elasticity of substitution, and $ii)$ σ is also the household's relative risk aversion.
r_k^*	$r_k^* = \rho + \sigma g$	Steady-state rent of capital.
η	$v(L_t) = \chi \frac{L_t^{1+\eta}}{1+\eta}$	Household's labor supply to wage elasticity (also known as Frisch elasticity).
χ	$v(L_t) = \chi \frac{L_t^{1+\eta}}{1+\eta}$	Weight of labor dis-utility in total utility.
κ	$\pi_t = \beta E_t \pi_{t+1} + \kappa(\eta+\sigma)x_e + \epsilon_t^p$	Inflation to marginal cost elasticity. $\kappa(\eta+\sigma)$ if the inflation to output-gap elasticity. The coefficient comes from the model log-familiarization, it takes the form $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}$ where ω is the Calvo coefficient.
ω	Sticky price	Calvo coefficient: Firm's probability of not updating prices optimally.
θ	$C_t = \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$	Dixit-Stiglitz aggregator: Good to price elasticity.
μ	$\mu = \frac{\theta}{\theta-1}$	Firm's real markup at the steady-state.
a_π, a_x	$i_t = a_\pi \pi_t + a_x x_t$	Policy coefficients: central bank reaction to changes in π_t or x_t

Economics 202
Winter 2017

Midterm: Answers

1. Using the basic Solow model, *carefully explain* how the steady-state value of income *per effective unit of labor* and the *steady-state growth rate* of total output and the *growth rate* of consumption *per worker* are affected by (a) a decrease in the saving rate; (b) a decrease in the population growth rate. Show graphically how the steady-state is affected by each of these changes. Show graphically how the steady-state is affected by each of these changes. *(a) Starting from a steady-state, a decrease in the saving rate means that at the initial k, saving now is less than the investment required to maintain k, that is, $sy < (\delta + g + n)\,k$. With lower investment, and more importantly with less investment than is needed to simply maintain k, k starts to decline. The economy converges to a new steady state with a lower k. In the new steady state, output per effective unit of labor and consumption per effective unit of labor are constant, so output grows at the same rate as effective units of labor and consumption grows at the rate of technology. Hence, neither the growth rate of total output nor the growth rate of consumption per worker are affected by the change in s. Total output grows at the rate $g + n$ and consumption per worker grows at the rate g, just as they did initially. (b) Starting from a steady-state, declines in population growth decrease the investment level required to maintain a constant capital per effective unit of labor as both increase the growth rate of effective units of labor. An decrease in n therefore makes the required investment line $(\delta + g + n)\,k$ flatter. At the initial k, sy is now more than $(\delta + g + n)\,k$, so with saving exceeding the level needed to maintain k, k begins to rise and the economy converges to a higher value of k. Since total output grows in the steady state at the rate $g + n$, it falls if n decreases. Consumption per worker grows at the rate g so its growth rate is unaffected by a fall in n.*

2. Consider a simple real business cycle (RBC) model characterized by perfectly competitive markets and flexible prices and wages. The only source of fluctuations is a stochastic productivity shock. Suppose this economy experiences a negative productivity shock. Carefully explain how the RBC model predicts consumption, investment, output, and employment will move in response to this shock. Do the effects of a productivity shock on consumption and investment depend on whether a shock to z_t is very persistent or dies out quickly? *The negative productivity shock reduces output and the marginal products of both labor and capital. As a consequence, output is directly reduced by the negative productivity shock, but in addition, real wages fall, reducing the incentive for households to supply labor. Thus, output is also reduced by the decline in employment as the labor market reaches a new equilibrium with lower wages and employment. With households wishing to smooth consumption over time, they cut consumption less than the decline in output (income) by primarily reducing saving in the form of capital, something reinforced by the fall in the marginal product off capital that reduces the incentive to invest. As a consequence, investment declines and next period's capital stock is lower than it would have been if the negative shock to productivity had not occurred. Thus. with lower capital, output in the period after the shock will remain below its steady-state level even if the negative productivity shock has gone away. Thus, persistent effects on output are generated. The effects on consumption and investment do depend on the expected persistence of the shock. If a negative productivity shock dies out quickly, that is, it is temporary or transitory, households that want to smooth their consumption will not adjust their consumption very*

much. Thus, saving (investment) will respond more and consumption less when the shock is temporary. If the shock is longer lasting, then households decrease their consumption more in response to what they expect to be a longer lasting fall in income. With consumption responding more, saving (investment) responds less.

3. Consider a simple new Keynesian model given by

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

where x is the output gap, π is inflation, i is the nominal interest rate, r^f and e are exogenous stochastic shocks, and σ, β, and κ are all positive constants. Assume the exogenous shocks r^f and e are serially uncorrelated with mean zero and variances $\sigma_{r^f}^2$ and σ_e^2 respectively. In this case, $\mathrm{E}_t x_{t+1} = \mathrm{E}_t \pi_{t+1} = 0$. The central bank follows a policy rule given by

$$i_t = r_t^f + \phi \pi_t, \ \phi > 1.$$

(a) Solve for the equilibrium values of x_t and π_t. Are inflation and the output gap affected by the shock to the flexible price real interest rate r_t^f? *Carefully explain your answer. With mean zero, serially uncorrelated shocks, the aggregate demand side of the model reduces to*

$$\begin{aligned} x_t &= -\left(\frac{1}{\sigma}\right)\left(i_t - r_t^f\right) \\ &= -\left(\frac{1}{\sigma}\right)\left(r_t^f + \phi\pi_t - r_t^f\right) \\ &= -\left(\frac{\phi}{\sigma}\right)\pi_t. \end{aligned}$$

Using this expression to eliminate x_t from the inflation adjustment equation, inflation given by

$$\pi_t = \kappa x_t + e_t = -\kappa\left(\frac{\phi}{\sigma}\right)\pi_t + e_t \Rightarrow \pi_t = \left(\frac{\sigma}{\sigma + \phi\kappa}\right)e_t.$$

The output gap is then equal to

$$x_t = -\left(\frac{\phi}{\sigma}\right)\pi_t = -\left(\frac{\phi}{\sigma + \phi\kappa}\right)e_t.$$

The output gap and inflation are unaffected by r_t^f. The central bank completely neutralizes the impact of r_t^f by moving i_t one-or-one with r_t^f so that neither x_t nor π_t are affected. That is, a positive realization of r_t^f acts to increase the output gap (by causing consumption to rise) but the central bank offsets this by raising the nominal rate (since expected inflation is zero, the rise in i is equivalent to a rise in the actual real rate of interest). With the output gap unaffected, inflation is also unaffected.

(b) Suppose the central bank decides to respond more strongly to inflation (i.e., it increases ϕ). *Explain* why this change will lead to an increase in the volatility of the output gap relative to the volatility of inflation. *By responding more strongly to inflation, an inflation shock leads to a larger change in the nominal interest rate (and the real interest rate since expected inflation does not change). The larger movement in the real interest rate causes a larger movement in the output gap. This output gap change acts to partially offset the effect of the shock on inflation. Thus, a large ϕ leads to more stable inflation and a more volatile output gap. More formally (but not necessary to answer the problem): Define $A \equiv \sigma / (\sigma + \phi\kappa)$ and $B \equiv \phi / (\sigma + \phi\kappa)$. From part (a), the variance of the output gap is $\sigma_x^2 = B^2 \sigma_e^2$ and that of inflation is $\sigma_\pi^2 = A^2 \sigma_e^2$, where σ_e^2 is the variance of the shock e_t. Then*

$$\frac{\partial \left(\sigma_x^2 / \sigma_\pi^2\right)}{\partial \phi} = \frac{\partial \left(B/A\right)^2}{\partial \phi} = \frac{\partial \left(\phi/\sigma\right)^2}{\partial \phi} = 2\left(\frac{\phi}{\sigma^2}\right) > 0.$$

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Winter 2017

Midterm review questions

1. Using the basic Solow model, *carefully explain* how the steady-state value of income per effective unit of labor and the steady-state growth rate of total output are affected by (a) an increase in the saving rate; (2) an increase in the population growth rate; (c) an increase in the rate of growth of technology. Show graphically how the steady-state is affected by each of these changes. *(a) Starting from a steady-state, an increase in the saving rate means that at the initial k, saving now exceeds the investment required to maintain k, that is, $sy > (\delta + g + n)\,k$. With higher investment, and more importantly with more investment than is needed to simply maintain k, k starts to grow. The economy converges to a new steady state with a higher k. In the new steady state, output per effective unit of labor and consumption per effective unit of labor are constant, so output grows at the same rate as effective units of labor and consumption grows at the rate of technology. Hence, neither the growth rate of total output nor the growth rate of consumption per worker are affected by the change in s. Total output grows at the rate $g + n$ and consumption per worker grows at the rate g, just as they did initially. (b) and (c) Starting from a steady-state, changes in population growth or the growth rate of technology increase the investment level required to maintain a constant capital per effective unit of labor as both increase the growth rate of effective units of labor. An increase in either therefore makes the required investment line $(\delta + g + n)\,k$ steeper. At the initial k, sy is now less than $(\delta + g + n)\,k$, so with saving not enough to maintain k, k begins to fall and the economy converges to a lower value of k. Since total output grows in the steady state at the rate $g + n$, it rises if either g or n increases. Consumption per worker grows at the rate g so it increases with a rise in g but is unaffected by a rise in n.*

2. According to the basic Solow model discussed in lecture, the evolution of the capital stock per unit of effective labor is

$$(1 + g + n)\,(k_{t+1} - k_t) = sy_t - (\delta + g + n))\,k_t$$

and output per effective unit of labor was $y_t = k_t^{\alpha}$. What is the marginal product of capital in the steady state? *In the steady state, $k = k^*$ is constant, so the above equation implies*

$$0 = sy - (\delta + g + n)\,k^* \Rightarrow sk^{*\alpha} = (\delta + g + n)\,k^*.$$

Solving for k^*,

$$k^* = \left(\frac{s}{\delta + g + n}\right)^{\frac{1}{1-\alpha}}. \tag{1}$$

The marginal product of capital is

$$\frac{\partial y}{\partial k} = \alpha k^{\alpha - 1},$$

so evaluated at k^*, the marginal product of capital is

$$\alpha\left[\left(\frac{s}{\delta + g + n}\right)^{\frac{1}{1-\alpha}}\right]^{\alpha - 1} = \alpha\left(\frac{\delta + g + n}{s}\right).$$

3. Define $\hat{k}^*$ as the value of k in the Solow model that would maximize consumption in the steady state. Show that the marginal product of capital at $\hat{k}^*$ is $\delta + g + n$. Illustrate this result in a graph with k on the horizontal axis and c on the vertical axis. What saving rate would make $\hat{k}^*$ the steady-state value of k? *In the steady state, investment is equal to $(\delta + g + n)\,k^*$ since this is the investment requires to maintain a constant k. Consumption in the steady state is then given by*

$$c = y - (\delta + g + n)\,k = k^{*\alpha} - (\delta + g + n)\,k^*.$$

The value of k^* that maximizes this is the solution to

$$\alpha\left(\hat{k}^*\right)^{\alpha-1} = \delta + g + n \Rightarrow \hat{k}^* = \left(\frac{\alpha}{\delta + g + n}\right)^{\frac{1}{1-\alpha}} \tag{2}$$

Comparing (1) and (2) shows that $k^* = \hat{k}^*$ if and only if $s = \alpha$.

4. According to the basic Solow/Ramsey model discussed in lecture, the evolution of the capital stock per unit of effective labor is

$$(1 + g + n)(k_{t+1} - k_t) = y_t - c_t - (\delta + g + n) k_t,$$

output per effective unit of labor was $y_t = k_t^\alpha$, and consumption per effective unit of labor satisfies

$$c_t^{-\sigma} = (1 + r_{k,t} - r_k^*) c_{t+1}^{-\sigma},$$

where $0 < \beta < 1$, $\sigma > 0$ and $r_{k,t} = \alpha k_t^{\alpha-1} - \delta$. In this notation, r_k^* is the steady-state value of $r_{k,t}$ and $r_k^* = \rho + \sigma g$ where ρ is the discount factor (future utility is discounted by $1/(1+\rho)$).

(a) Explain (in words) why the steady-state marginal product of capital in this model depends on ρ. *An economy on which households discount the future more (i.e., have a larger ρ) prefers current utility relative to future utility more strongly than do households in an economy with a smaller ρ. With a larger ρ, households are less willing to save and accumulate capital in order to have higher future income and consumption – they prefer consumption now. By saving less, the economy accumulates less capital. With less capital, its marginal product is higher. Hence, the steady-state marginal product of capital is higher in the economy with the larger ρ.*

(b) How does the steady-state value of k depend on the depreciation rate. Does an increase in δ raise or lower the steady-state level of capital per effective unit of labor? Explain (in words). Does an increase in δ raise or lower the steady-state level of capital per effective unit of labor? Explain (in words). *With a higher rate of depreciation, a higher level of investment is required to maintain any given steady-state level of k. But starting from the initial steady state (i.e., before* δ increases). *the net return (the marginal product of capital net of depreciation) falls (k hasn't changed yet but more disappears through depreciation). This reduces the attractiveness of saving and investing, so households reduce investment. As a result, k declines. As k falls, the marginal product of capital increases, so in the new steady state with a smaller k, the marginal product r_k^* is higher.*

5. A characteristic of business cycles is persistence – expansions last for several quarters as do recessions. Does the effects of a productivity shock on investment and the evolution of the capital stock provide a means for generating persistent effects of temporary productivity shocks? Explain. Does this channel seem quantitatively important? If not, what aspect of a basic real business model's calibration accounts for persistence. Explain. *The effects of productivity shocks, combined with the assumption that households desire to smooth consumption, can generate some persistence by influencing the evolution of the capital stock. The basic idea can be illustrated by considering the impact of a negative productivity shock. This reduces output and the marginal products of both labor and capital. As a consequence, output is directly reduced by the negative productivity shock, but in addition, real wages fall, reducing the incentive for households to supply labor. Thus, output is also reduced by the decline in employment as the labor market reaches a new equilibrium with lower wages and employment. With households wishing to smooth consumption over time, the cut consumption less than the decline in output (income) by primarily reducing saving in the form of capital, something reinforced by the fall in the marginal product off capital that reduces the incentive to invest. As a consequence, next period's capital stock is lower than it would have been if the negative shock to productivity had not occurred. Thus. with lower capital, output in the period after the shock will remain below its steady-state level even if the negative productivity shock has gone away. Thus, persistent effects on output are generated. However, quantitatively, this effect is very small as even large*

fluctuations in investment have small effects on the aggregate capital stock when we are thinking of a period being something like a quarter (3 months). In fact, essentially all of the persistence in calibrated real business cycle models comes from assuming the exogenous productivity shock is very persistent.

6. A characteristic of business cycles in the U.S. is that employment varies a lot while real wages do not. The basic real business cycle model developed in lecture in which productivity shocks are the source of fluctuations can match one of these characteristics but not both. Suppose the wage elasticity of labor supply is given by γ (this was $1/\eta$ in the lecture notes). Which characteristic is matched if γ is large? Which is matched if γ is small? Illustrate your answers using a simple diagram of labor demand and labor supply. *Explain. The basic real business model discussed in class cannot explain the lack of real wage variation combined with large variations in employment if labor supply is inelastic. The reason is that equilibrium in the labor market occurs where labor demand and labor supply curves intersect and as productivity shocks shift labor demand around, the equilibrium will basically trace out the labor supply curve. With inelastic labor supply, a positive productivity shock that increases the marginal product of labor shifts the labor demand curve to the right. With firms wishing to hire more labor, the real wage must rise to induce households to supply more labor (a movement along the labor supply curve). If households do not respond much to the real wage, it takes a large rise in the real wage to bring labor demand and labor supply back into balance. The large rise in the real wage acts to dampen labor demand (a movement along the new labor demand curve). As a result, the real wage moves a lot, while employment varies much less. (There is a wealth effect that also is at work but it also acts to cause wages to rise more and employment less in the face of a positive productivity shock.)*

7. Consider a simple RBC model without capital. If the aggregate production function is $Y_t = e^{z_t} N_t^{1-\alpha}$, the linearized equilibrium conditions for the model are

$$y_t = (1-\alpha)\, n_t + z_t$$

$$y_t - n_t = \omega_t$$

$$\eta n_t + \sigma c_t = \omega_t$$

$$y_t = c_t$$

where y is output, c is consumption, n is employment, and ω is the real wage, all expressed as percent deviations around the steady state.

   (a) Explain what each of these equilibrium conditions represents (i.e., where does it come from?).*The first equation is the linearize production function expressed as percent deviations around the steady state. Output is produced using labor and it also depends on productivity. Because of decreasing returns to scale in the specified production function ($0 < \alpha < 1$), a one percent increase in n increases y by less than one percent. The second equation comes from the firm's first order condition for its hiring of labor. It sets the marginal product of labor equal to the real wage. Since the marginal product of labor is $(1-\alpha)\, Y_t/N_t$, it is $y_t - n_t$ in terms of percent deviations around the steady state. The third equation is basically the labor supply equation (it can be written as $n_t = (1/\eta)\, \omega_t - (\sigma/\eta)\, c_t$). It comes from the household's first order condition balancing the utility cost of an additional hour of work (ηN_t^{η}) with the utility value of the extra income earned ($\omega_t C_t^{-\sigma}$). When linearized around the steady state, this becomes $\eta n_t = \omega_t - \sigma c_t$. The final equation is the condition for the goods market to clear : $Y_t = C_t$*

   (b) Show that the equilibrium value of y_t is given by

$$y_t = \left(\frac{1+\eta}{\sigma + \eta + \alpha\,(1-\sigma)} \right) z_t.$$

   Set labor demand equal to labor supply to eliminate the real wage. Use goods clearing to replace consumption with output. Use the production function to eliminate employment. Rearrange to obtain the desired result.

(c) Find the equilibrium values of n and ω as function of z.

$$\begin{aligned}
\omega_t &= y_t - n_t = y_t - \left(\frac{y_t - z_t}{1-\alpha}\right) \\
&= -\left(\frac{\alpha}{1-\alpha}\right) y_t + \frac{1}{1-\alpha} z_t \\
&= -\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1+\eta}{\sigma+\eta+\alpha(1-\sigma)}\right) z_t + \left(\frac{1}{1-\alpha}\right) z_t \\
&= \left(\frac{\sigma+\eta}{\sigma+\eta+\alpha(1-\sigma)}\right) z_t
\end{aligned}$$

$$\begin{aligned}
n_t &= y_t - \omega_t \\
&= \left(\frac{1-\sigma}{\sigma+\eta+\alpha(1-\sigma)}\right) z_t
\end{aligned}$$

(d) Explain how output, employment an the real wage are affected by a negative productivity shock if $\sigma < 1$. *A negative productivity shock ($z_t > 0$) decreases the marginal product of labor, decreasing labor demand at the initial real wage. This shifts the labor demand curve to the left and leads, in equilibrium, to a rise in the real wage and employment.*

(e) Consider the special case in which $\alpha = 0$. How would your answers to part (d) be affected if $\sigma > 1$? Explain. (Hint: use a diagram of the labor market to explain what is going on.) *If $\alpha = 0$, the last equation of part (c) becomes*

$$n_t = \left(\frac{1-\sigma}{\sigma+\eta}\right) z_t.$$

A negative z_t now increases employment if $1 - \sigma < 0$. A large σ corresponds to a large wealth effect on labor supply. The negative productive shock and fall in wages and income reduces consumption. Households respond by supplying more labor and employment rises. Output still falls ($y_t = n_t + z_t = [(1+\eta)/(\sigma+\eta)] z_t < z_t$ but by less than the direct impact of the negative z_t.

8. Consider a simple new Keynesian model given by

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - \mathrm{E}_t \pi_{t+1} - r_t^n)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

$$i_t = r_t^n + \phi \pi_t,$$

where x is the output gap, π is inflation, i is the nominal interest rate, r^n and e are exogenous stochastic shocks, and σ, β, κ, and ϕ are constants.

(a) Describe (in words) what each of these three equations represents? Whose decisions to they represent? *The first equation comes from households' first-order condition for optimal consumption in which the marginal utility of consumption today is balanced against the expected real return and the expected marginal utility of consumption tomorrow. In this basic NK model, equilibrium requires that output equals consumption, so this first equation can be expressed in terms of the output gap (output relative to what would occur with flexible prices) rather than a consumption gap (consumption relative to what would occur with flexible prices). The second equation comes from the optimal price setting decisions of firms when firms adjust prices infrequently. When prices are changed, they adjust to real marginal cost, which is a function of the output gap. Because a firm may not adjust its price for several periods, if must be forward looking when it does change its price, so that pricing decisions are based on expectations of future inflation. The third equation represents a policy rule that says the central bank adjusts the nominal interest rate in response to the shock r_t^n and to movements in inflation.*

(b) What problems might arise in this model if $\phi < 1$? *Multiple stationary equilibria can occur if* $\phi < 1$.

9. Using the model of question 8, assume the exogenous shocks r^n and e are serially uncorrelated so that $\mathrm{E}_t r^n_{t+1} = \mathrm{E}_t e_{t+1} = 0$. In this case, $\mathrm{E}_t x_{t+1} = \mathrm{E}_t \pi_{t+1} = 0$.

   (a) Solve for x_t and π_t. *The model become*

$$x_t = -\left(\frac{1}{\sigma}\right)(i_t - r^n_t)$$

$$\pi_t = \kappa x_t + e_t$$

$$i_t = r^n_t + \phi \pi_t,$$

   which, when i is eliminated, becomes

$$x_t = -\left(\frac{1}{\sigma}\right)(r^n_t + \phi\pi_t - r^n_t) = -\left(\frac{\phi}{\sigma}\right)\pi_t$$

$$\pi_t = \kappa x_t + e_t.$$

   Solving,

$$\sigma x_t = -\phi\kappa x_t - \phi e_t \Rightarrow x_t = -\left(\frac{\phi}{\sigma + \phi\kappa}\right) e_t$$

   and

$$\pi_t = \kappa x_t + e_t = -\kappa\left(\frac{\phi}{\sigma + \phi\kappa}\right) e_t + e_t = \left(\frac{\sigma}{\sigma + \phi\kappa}\right) e_t$$

   (b) From your results in part (a), explain why neither the output gap nor inflation are affected by the demand shock r^n_t. *The central bank moves i_t one-for-one with r^n_t. This prevents r^n_t from affecting aggregate spending x_t. With x_t unaffected, marginal costs and therefore inflation are unaffected.*

   (c) From your results in part (a), explain how the effects of the inflation shock on the output gap and inflation are affected by the central bank's choice of ϕ. *From (a), we have that $x_t = -Ae_t$ and $\pi_t = Be_t$ where $A = \phi/(\sigma + \phi\kappa) > 0$ and $B = \sigma/(\sigma + \phi\kappa) > 0$. A is increasing in ϕ and B is decreasing in ϕ. If the central bank responses more strongly to inflation (ϕ increases), then in the face of a positive inflation shock $e_t > 0$ the central bank raises the nominal interest rate more. This reduces the output gap more, reducing marginal costs and offsetting some of the increase in inflation caused by the inflation shock. Hence, the larger is ϕ, the less π is affected by the shock and the more the output gap is.*

10. Consider a simple new Keynesian model given by

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t\pi_{t+1} - r^f_t\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

where x is the output gap, π is inflation, i is the nominal interest rate, r^n and e are exogenous stochastic shocks, and σ, β, κ, and ϕ are constants. Assume the exogenous shocks r^f and e are serially uncorrelated so that $\mathrm{E}_t r^f_{t+1} = \mathrm{E}_t e_{t+1} = 0$. In this case, $\mathrm{E}_t x_{t+1} = \mathrm{E}_t \pi_{t+1} = 0$. Suppose the central bank does not observe the demand shock r^f_t but instead has a forecast of it, denoted by r^{ff}_t. The policy rule is

$$i_t = r^{ff}_t + \phi\pi_t.$$

(a) Solve for x_t and π_t. *Carefully explain* how inflation and the output gap are affected by the central bank's forecast error $err_t \equiv r_t^f - r_t^{ff}$. *With mean zero, serially uncorrelated shocks, the aggregate demand side of the model reduces to*

$$\begin{aligned} x_t &= -\left(\frac{1}{\sigma}\right)\left(i_t - r_t^f\right) \\ &= -\left(\frac{1}{\sigma}\right)\left(r_t^{ff} + \phi\pi_t - r_t^f\right) \\ &= -\left(\frac{\phi}{\sigma}\right)\pi_t + \left(\frac{1}{\sigma}\right)\left(r_t^f - r_t^{ff}\right) \\ &= -\left(\frac{\phi}{\sigma}\right)\pi_t + \left(\frac{1}{\sigma}\right) err_t \end{aligned}$$

so a forecast error acts like a positive shock to aggregate demand. If $err_t > 0$, the central bank has underestimated r_t^f and so they have not raised the nominal interest rate sufficiently to offset the impact of the r^f shock on aggregate demand. Thus, i_t is too low relative to r_t^f which leads to an economic expansion. The Phillips curve is still given by

$$\pi_t = \kappa x_t + e_t$$

So a positive err_t increases x_t which pushes up inflation. As inflation rises, the central bank reacts by raising the nominal interest rate (this is the $\phi\pi_t$ term). In the equilibrium, some of the direct effect of err_t on x_t is offset by the rise in i_t, but there is still some rise in x_t and π_t.

(b) Does the effect of a fall in r^f on x and π depend on whether the central bank was able to forecast the decline accurately? *Yes. If the central bank made no forecast errors, inflation and the output gap would not be affected by r_t^f.*

(c) *Explain* how the effects of the demand shock r_t^f and the inflation shock e_t on the output gap and inflation are affected by the central bank's choice of ϕ. *If the shock r_t^f is fully forecast by the central bank, there will be no effect on either the output gap or inflation as i_t is raised to offset the shock. So what will matter is err_t. Continuing with the explanation from part (a), if ϕ is large, than the rise in inflation induces a large rise in i_t. This acts to offset most of the rise in x and helps stabilize inflation. If ϕ is small, than both x and π will be affected more by the forecast error.*