Econ 100A: Intermediate Microeconomics
Notes on Consumer Theory

Linh Bun*

Winter 2012 (UCSC)

1. Consumer Theory – Utility Functions

1.1. Types of Utility Functions

The following are some of the type of the utility functions that are important:
Perfect Complements
Perfect Substitutes
Cobb-Douglas
Quasilinear

1.2. Perfect Complements

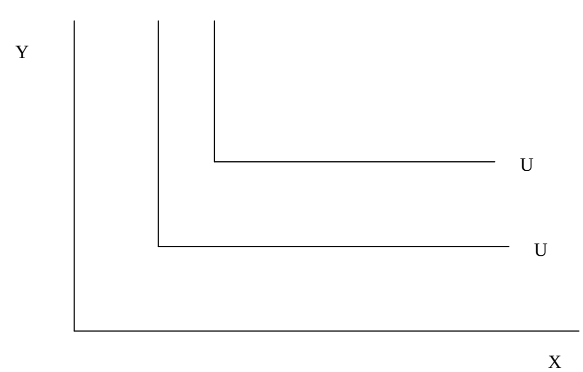
The Utility function:

$$U(x, y) = Min\{x, y\}$$

The Budget Constraint:

$$p_x x + p_y y = m$$

where m is income, and p_x and p_y is the price of good x and y respectively. The graph of the indifference curves for Perfect Completments is as follows:



Perfect complements: L-shaped indifference curves

*This notes is prepared with some help from Aadil Nakhoda

Example:

Right shoe and Left shoe: If we purchase one right shoe, we need to purchase one left shoe also.

The cosumer maximization problem for Perfect Complements:

$$Max\ U(x,y)$$

subject to

$$p_x x + p_y y = m$$

The optimal allocation, the consumption bundle that gives the highest utility (x^*, y^*), is when $x = y$
Replacing y with x into the budget constraint since $x = y$, we have the following:

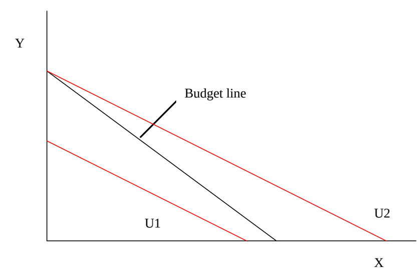
$$p_x x + p_y x = m$$

$$x(p_x + p_y) = m$$

Hence,

$$x^* = y^* = \frac{m}{p_x + p_y}$$

1.3. Perfect Substitutes



Consider the above graph:
The slope of the budget line is **Steeper** than the slope of the indifference curves.
The slope of the budge line in absolute value is $\left|\frac{p_x}{p_y}\right|$, where p_x and p_y is the price of good x and y respectively.
The slope of the indifference curves in absolute value is $|MRS|$, where MRS is the Marginal Rate of Substitutions

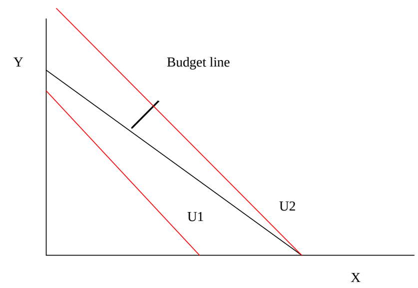
$$MRS = -\left[\frac{\text{Marginal Utility of Good } x}{\text{Marginal Utility of Good } y}\right] = -\left[\frac{MU_x}{MU_y}\right] = -\left[\frac{\frac{\partial U(x,y)}{\partial x}}{\frac{\partial U(x,y)}{\partial y}}\right]$$

The slope of the budget line is **Steeper** than the slope of the indifference curves. This is equivalent to having the following:

$$\left|\frac{p_x}{p_y}\right| > |MRS|$$

The **Optimal Allocation** (x^*, y^*) is $\left(0, \frac{m}{p_y}\right)$. Equivalently, the quantity of good x and y demanded is $\left(0, \frac{m}{p_y}\right)$.

Example:
Suppose we have two goods, Pepsi (x) and Coke (y). Which good would you prefer to purchase?
Spending all income m on Coke (y), i.e. purchasing only Coke (y), will put you on the highest indifference curve given the budget constraint.
The budget line is tangent to a higher indifference curve at the $y - Axis$, than it is at the $x - Axis$.



Consider the above graph:

The slope of the budget line is **Flatter** than the slope of the indifference curves.

The slope of the budge line in absolute value is $\left|\frac{p_x}{p_y}\right|$, where p_x and p_y is the price of good x and y respectively.

The slope of the indifference curves in absolute value is $|MRS|$, where MRS is the Marginal Rate of Substitutions

$$MRS = \frac{\text{Marginal Utility of Good } x}{\text{Marginal Utility of Good } y} = \frac{MU_x}{MU_y} = \frac{\frac{\partial U(x,y)}{\partial x}}{\frac{\partial U(x,y)}{\partial y}}$$

The slope of the budget line is **Flatter** than the slope of the indifference curves. This is equivalent to having the following:

$$\left|\frac{p_x}{p_y}\right| < |MRS|$$

The **Optimal Allocation** (x^*, y^*) is $\left(\frac{m}{p_x}, 0\right)$. Equivalently, the quantity of good x and y demanded is $\left(\frac{m}{p_x}, 0\right)$.

Example:

Suppose we have two goods, Pepsi (x) and Coke (y). Which good would you purchase?

Spending all income m on Pepsi (x), i.e. purchasing only Pepsi (x), will put you on the highest indifference curve given the budget constraint.

The budget line is tangent to a higher indifference curve at the $x - axis$, than it is at the $y - axis$.

Perfect Substitutes:

Rules to follow:

If the slope of the budget constraint is **Steeper** than the slope of the indifference curve, we consume the good on the $y - axis$.

In particular,

$$\left|\frac{p_x}{p_y}\right| > |MRS|$$

Where p_x and p_y is the price of good x and y respectively.

If the slope of the budget constraint is **Flatter** than the slope of the indifference curve, we consume the good on the $x - axis$.

In particular,

$$\left|\frac{p_x}{p_y}\right| < |MRS|$$

Where p_x and p_y is the price of good x and y respectively.

If we were to consume the good on the $x - axis$, we represent it as:

Example:

Suppose we have the utility function:

$$U(x, y) = 4x + 5y$$

and the budget constraint is as follows:

$$2x + 3y = 10$$

The Marginal Rate of Substitution is as follows:

$$MRS = -\frac{4}{5}$$

The MRS in absolute value is:

$$|MRS| = \left|-\frac{4}{5}\right| = \frac{4}{5}$$

The slope of the budget line is as follows:

$$Slope\,(BC) = -\frac{2}{3}$$

The slope of the budget line in absolute value

$$|Slope\,(BC)| = \left|-\frac{2}{3}\right| = \frac{2}{3}$$

Hence,

$$|MRS| > |Slope\,(BC)|$$

As a result, agents consume **only** goods x. The quantity of good x and y demanded, i.e. the Optimal Allocation, is $(x^*, y^*) = \left(\frac{10}{2}, 0\right)$

Note that when $y^* = 0$ we have $2x^* = 10$ from the budget constraint.

Hence,

$$x^* = 5, y^* = 0$$

The highest level of utility is

$$U(5, 0) = (4)(5) + (5)(0) = 20$$

1.4. Cobb-Douglas Utility Function

The Cobb-Douglas utility function:

$$U(x, y) = x^a y^b, \text{ where } a > 0 \text{ and } b > 0$$

Alternatively, using monotonic transformation, Cobb-Douglas utility function could also be represented as follows:

$$U(x, y) = a \log(x) + b \log(y), \text{ where } a > 0 \text{ and } b > 0$$

.

This indifference curve will have a negative slope, which will incorporate the individual's willingness to make tradeoffs between good x and y.

How to solve for an **Optimal Bundle or Optimal Allocation** given a Cobb-Douglas function:

$$MRS = -\left(\frac{p_x}{p_y}\right)$$

$$MRS = -\left(\frac{ax^{a-1}y^b}{bx^a y^{b-1}}\right) = -\left(\frac{a}{b}\right)\left(\frac{y}{x}\right)$$

$$\left(\frac{a}{b}\right)\left(\frac{y}{x}\right) = \frac{p_x}{p_y}$$

To solve for y^*, we can rewrite the above as:

$$\left(\frac{a}{b}\right) p_y y = p_x x$$

From the budget constraint, we have the following:

$$\left(\frac{a}{b}\right) p_y y + p_y y = m$$

$$\left(\frac{a}{b} + 1\right) p_y y = m$$

$$\left(\frac{a+b}{b}\right) p_y y = m$$

$$y^* = \left(\frac{b}{a+b}\right)\left(\frac{m}{p_y}\right)$$

And, now to solve for x^*
Substitute y^* above into the budget constraint:

$$p_x x + p_y \left[\left(\frac{b}{a+b}\right)\left(\frac{m}{p_y}\right)\right] = m$$

Cancelling p_y in above equation, we have the following:

$$p_x x + \left(\frac{b}{a+b}\right) m = m$$

$$p_x x = m - \left(\frac{b}{a+b}\right) m$$

$$p_x x = \left(1 - \frac{b}{a+b}\right) m$$

$$p_x x = \left(\frac{a+b-b}{a+b}\right) m$$

$$x^* = \left(\frac{a}{a+b}\right) m$$

The **Optimal Bundle** is:

$$(x^*, y^*) = (\left(\frac{a}{a+b}\right)\left(\frac{m}{p_x}\right), \left(\frac{b}{a+b}\right)\left(\frac{m}{p_y}\right))$$

This should be similar to the case presented in the textbook.

1.5. Quasilinear Utility Functions

A Quasilinear utility function is as follows:

$$U(x,y) = \ln(x) + y$$

where $\ln(x)$ is the natural logarithm.
The function $U(x,y)$ is linear in y.
Let us solve for the above function:
At the **Optimal Bundle** (x^*, y^*), we have the following equations

$$MRS = -\left(\frac{p_x}{p_y}\right)$$

i.e. the slope of the indifference curve at the **Optimal Bundle** is equal to the slope of the budget line.
The Marginal Rate of Substitutions MRS is as follows:

$$MRS = -\left[\frac{\text{Marginal Utility of Good } x}{\text{Marginal Utility of Good } y}\right] = -\left[\frac{MU_x}{MU_y}\right] = -\left[\frac{\frac{\partial U(x,y)}{\partial x}}{\frac{\partial U(x,y)}{\partial y}}\right]$$

$$\frac{\partial U(x,y)}{\partial x} = \frac{\partial\left[\ln(x) + y\right]}{\partial x} = \frac{\partial \ln(x)}{\partial x} = \frac{1}{x}$$

Recalling the rule for differentiating a natural logarithm function $\ln(x)$:

$$\frac{d\left[\ln(x)\right]}{dx} = \frac{1}{x}$$

$$\frac{\partial U(x,y)}{\partial y} = \frac{\partial\left[\ln(x) + y\right]}{\partial y} = 1$$

$$MRS = -\left(\frac{\frac{1}{x}}{1}\right) = -\left(\frac{1}{x}\right)$$

$$MRS = -\left(\frac{p_x}{p_y}\right)$$

Hence,

$$\frac{1}{x^*} = \frac{p_x}{p_y}$$

$$x^* = \frac{p_y}{p_x}$$

Substitute the above into the following budget constraint:

$$p_x x + p_y y = m$$

$$p_x\left(\frac{p_y}{p_x}\right) + p_y y = m$$

$$y^* = \frac{m - p_y}{p_y} = \frac{m}{p_y} - 1$$

So our optimal bundle is:

$$(x^*, y^*) = \left(\frac{p_y}{p_x}, \frac{m}{p_y} - 1\right)$$

2. Practice Problems:

2.1. Complements:

Utility function is

$$U(x_1, x_2) = Min(x_1, 2x_2)$$

Suppose an accountant needs 1 eraser for every 2 pencils he/she uses. Any more pencils will not be useful as the accountant will not be able to erase the caluclations. Any more erasers will also not serve his purpose also. Therefore, x_1 is pencil and x_2 is eraser.

To solve for maximization problem:

$$Max\ U(x_1, x_2)$$

where

$$U(x_1, x_2) = Min(x_1, 2x_2)$$

The budget constraint:

$$p_1 x_2 + p_2 x_2 = m$$

where p_1 and p_2 is the price of good x_1 and x_2 respectively.
With $x_1^* = 2x_2^* \rightarrow$

$$m = 2p_1 x_2^* + p_2 x_1^*$$

$$x_2^* = \frac{m}{2p_1 + p_2}$$

And for x_1^* :

$$m = p_1 x_1^* + \frac{1}{2} p_2 x_1^*$$

$$x_1^* = \frac{m}{p_1 + \frac{1}{2} p_2}$$

2.2. Substitutes:

$$U = 3\,(Coke) + 6\,(Pepsi)$$

The price of Pepsi is \$2, and the price of Coke is \$0.8. What is the **Optimal Consumption Bundle** for the individual?

First assume, Coke to be on $y - axis$

$$MRS = -\left(\frac{3}{6}\right) = -\left(\frac{1}{2}\right)$$

$$slope\ of\ budget\ line = -\left(\frac{2}{\frac{4}{5}}\right) = -\left(\frac{10}{4}\right) = -2.5$$

We know that the slope of the budget line is greater than the MRS, the slope of the indifference curve. So what good will the individual consume and why?

2.3. Cobb-Douglas:

Find the **Optimal Consumption Bundles** of x and y for the following utility functions:

$$U(x, y) = x^{\frac{2}{3}} y^{\frac{4}{5}}$$

$$U(x, y) = x^2 + y$$

Where the price of good x is \$2 and the price of good y is \$1, and income is \$10.