Homework 1 Answers.pdf

Homework 1.pdf

Homework 2 Answers.pdf

Homework 2.pdf

Homework 3 Answers.pdf

Homework 3.pdf

Homework 4 Answers.pdf

Homework 4.pdf

Economics 202
Winter 2017

Problem set 1: Answers

1. See US_data_ps1_answer.xlsx. Basic data is on sheet FRED_data, variables in logs and inflation rates are on sheet LOGS. See the tabs for the answers to parts (a)-(d).

2. See US_data_ps1_answer.xlsx, sheet Exercise 2. The volatility of the output gap fell roughly by half going from the earliest period to the 1985-2007 Great moderation period. Volatility has increased somewhat since.

3. See US_data_ps1_answer.xlsx, sheet Exercise 3. The volatility of the infaltio rate fell roughly by 60% going from the earliest period to the 1985-2007 Great moderation period. Volatility has increased since.

4. Romer, p. 46, Question 1.5.

   (a) The equation given in lecture to describe the evolution of the capital stock per unit of effective labor is

$$\begin{aligned}(1+g+n)(k_{t+1}-k_t) &= sy_t - (\delta+g+n))\,k_t \\ &= sk_t^{\alpha} - (\delta+g+n))\,k_t.\end{aligned}$$

   On the balanced growth path, k is constant so $k_{t+1} - k_t = 0$ and investment per unit of effective labor is equal to break-even investment per unit of effective labor. Let k^* denote the balanced growth (steady-state) value of k. Hence, k^* solves

$$sk^{\alpha} = (\delta+g+n))\,k \Rightarrow k^* = \left(\frac{s}{\delta+g+n}\right)^{\frac{1}{1-\alpha}}.$$

   The balanced-growth-path value of output per unit of effective labor is

$$y^* = (k^*)^{\alpha} = \left(\frac{s}{\delta+g+n}\right)^{\frac{\alpha}{1-\alpha}}$$

   while consumption is

$$c^* = (1-s)\,y^* = (1-s)\left(\frac{s}{\delta+g+n}\right)^{\frac{\alpha}{1-\alpha}}.$$

   (b) The golden-rule level of the capital stock is that level at which consumption per unit of effective labor is maximized. To derive this level of k, note that

$$c^* = y^* - sy^* = (k^*)^{\alpha} - (\delta+g+n)\,k^*$$

because $sy^* = (\delta + g + n)) k^*$, i.e., along the balanced growth path, investment equals break-even investment. Now maximize c^* with respect to k^*. The first-order condition is

$$\frac{\partial c^*}{\partial k^*} = \alpha (k^*)^{\alpha - 1} - (\delta + g + n) = 0$$

or the marginal product of capital $\alpha (k)^{\alpha-1}$ equals $\delta + g + n$ when c^* is maximzied. Solving for the golden rule k,

$$k_{GR}^* = \left(\frac{\alpha}{\delta + g + n}\right)^{\frac{1}{1-\alpha}}.$$

To determine the saving rate that would make k_{GR}^* the steady-state equilibrium, we need to find s_{GR} such that

$$s_{GR} (k_{GR}^*)^{\alpha} = (\delta + g + n)) k_{GR}^*$$

or

$$\begin{aligned} s_{GR} &= (\delta + g + n)) (k_{GR}^*)^{1-\alpha} \\ &= (\delta + g + n)) \left[\left(\frac{\alpha}{\delta + g + n}\right)^{\frac{1}{1-\alpha}}\right]^{1-\alpha} \\ &\alpha. \end{aligned}$$

5. Romer, p. 46, Question 1.8 (parts a and b only).

   (a) Because saving equals investment, this question is asking aout the effects of a rise in the saving rate from 0.15 to 0.18. The steady-state value of k is determined by the solution to

   $$sy = sk^{\alpha} = (g + n + \delta) k.$$

   Let s_1 be the initial saving rate, s_2 the new saving rate. Let k_1^* be the steady-state value of k for s_1 and let k_2^* be the steady-state value of k for s_2. hence,

   $$s_1 (k_1^*)^{\alpha} = (g + n + \delta) k_1^*.$$

   $$s_2 (k_2^*)^{\alpha} = (g + n + \delta) k_2^*.$$

   To solve for k_1^* or k_2^*, we would need to know $g + n + \delta$, but the problem only gives us α, s_1 and s_2. But we can find an expression for how k_2^* compares to k_1^* by dividing the equation ofr k_2^* by the one for k_1^*:

   $$\frac{s_2 (k_2^*)^{\alpha}}{s_1 (k_1^*)^{\alpha}} = \frac{(g + n + \delta) k_1^*}{(g + n + \delta) k_2^*} = \frac{k_2^*}{k_1^*}.$$

This equation implies

$$\left(\frac{k_2^*}{k_1^*}\right)^{\alpha-1} = \frac{s_1}{s_2} \Rightarrow \frac{k_2^*}{k_1^*} = \left(\frac{s_2}{s_1}\right)^{\frac{1}{1-\alpha}}.$$

From this solution for the ratio of k_2^* to k_1^*, we can obtain

$$\frac{y_2^*}{y_1^*} = \frac{(k_2^*)^\alpha}{(k_1^*)^\alpha} = \left(\frac{k_2^*}{k_1^*}\right)^\alpha = \left(\frac{s_2}{s_1}\right)^{\frac{\alpha}{1-\alpha}}.$$

Thus, for $\mathrm{s}_1 = 0.15$, $s_2 = 0.18$, and $\alpha = 1/3$,

$$\frac{y_2^*}{y_1^*} = \left(\frac{0.18}{0.15}\right)^{\frac{1/3}{1-(1/3)}} = 1.2^{\frac{1}{2}} = 1.0954.$$

The 20% rise in the saving rate from 15% to 18% increases income per effective unit of labor by 9.54%.

(b) The effect on c^* is obtained by noting that

$$\frac{c_2^*}{c_1^*} = \frac{(1-s_2)\,y_2^*}{(1-s_1)\,y_1^*} = \frac{(1-s_2)}{(1-s_1)}\left(\frac{s_2}{s_1}\right)^{\frac{\alpha}{1-\alpha}}.$$

Plugging in the numbers,

$$\frac{c_2^*}{c_1^*} = \frac{(1-0.18)}{(1-0.15)}1.0954 = 1.0568.$$

Consumption per effective unit of labor is 5.68% higher in the new steady state.

Economics 202
Winter 2017

Problem set 1: Due January 19, 2017

1. Using the FRED database of the Federal Reserve Bank of St. Louis (FRED: https://research.stlouisfed.org/fred2/) download quarterly time series starting in 1959:Q1 for the following macroeconomic variables: Real GDP (GDPC96), Real Consumption (PCECC96), Real Investment (GPDIC96), Real Potential GDP (GDPPOT), Output per hour in the nonfarm business sector (OPHNFB), the Consumer Price Index (CPIAUCSL), the PCE price Index (PCEPI), the GDP Deflator (GDPDEF), and the Civilian Unemployment Rate (UNRATE), all seasonally adjusted. (Note – if you use the "browse popular series" in the FRED add-in for excel, it will list investment as GDPIC96 but then give you a series error when you try to download the data. Change the name to GPDIC96 and it should work.)

    (a) Convert each series (except the unemployment rate) to natural logs. Plot the natural logs of GDP, potential GDP, and the output gap, defined as $100*(ln(GDP_t)-ln(PotentialGDP_t))$ and the unemployment rate. (Hint: put the output gap and the unemployment rate on a secondary axis.)

    (b) Plot inflation as measured by the CPI, the PCE price index, the core PCE, and the GDP deflator. For each price index, define the inflation rate (at annual rates) as $400 * (ln(P_t) - ln(P_{t-1}))$. Why is the log change multiplied by 400?

    (c) Redo part (b) defining inflation rates as year-over-year percent changes (i.e., as $100 * (ln(P_t) - ln(P_{t-4}))$).

    (d) Plot the output gap and the inflation rate (measured as in part (b) by the PCE index) and add shading to indicate U.S. business cycle recession periods using the NBER business cycle dates. (The dates are available in at http://www.nber.org/cycles.html or in FRED as USREC. For help in adding shaded bars, see http://www.tvmcalcs.com/index.php/blog/comments/cha

2. Using Excel, calculate the standard deviation of the output gap (defined as in exercise 1) for the following periods: 1959:Q1-1984:Q4, 1985:Q1-2007:Q4, and 2008:Q1-2015:Q2. How has the volatility of the output gap changed over time?

3. Using Excel, calculate the standard deviation of CPI inflation at annual rates for the following periods: 1959:Q1-1984:Q4, 1985:Q1-2007:Q4, and 2008:Q1-2015:Q2. How has the volatility of inflation changed over time?

4. Romer, p. 46, Question 1.5.

5. Romer, p. 46, Question 1.8 (parts a and b only).

6. Optional:

   (a) Romer, p. 46-47, Question 1.9.

   (b) The business cycle models we will cover this quarter cannot be solved analytically, except in a few special cases. Instead, the common practice is to solve the models numerically. To do this, we will use Dynare (which runs in Matlab) to solve linear, rational expectations models of the business cycle, so it will be helpful to start learning the basics of Matlab and Dynare. A very basic introduction to matlab is Matlab_basics_2017.pdf available on the class web site. You will need the following matlab program files: plots_surfaces_examples.m and yearquarter.m. In addition, the data file Qvigstad.xls should be in the same directory. See also Slides_MatlabDynare_2017.pdf.

Economics 202
Winter 2017

Assignment 2: Answers

1. *Explain* why, in the basic Solow model, an increase in the saving rate affects the growth rate of output during the transition to a new steady state but does not affect the steady-state growth rate. *Explain* how capital per work and income per worker in the new steady state differ from their value in the old steady state. *Starting from a steady-state, an increase in the saving rate means that at the initial k, saving now exceeds the investment required to maintain k, that is, $sy > (\delta + g + n)\,k$. With higher investment, and more importantly with more investment than is needed to simply maintain k, k starts to grow. If k is growing, it means capital is growing faster than $g+n$, the growth rate of effective units of labor. Since y is a function of k, it too is growing faster during the tranistion. The economy converges to a new steady state with a higher k. In the new steady state, output per effective unit of labor and consumption per effective unit of labor are constant, so output grows at the same rate as effective units of labor and consumption grows at the rate of technology. Hence, neither the growth rate of total output nor the growth rate of consumption per worker are affected by the change in s. Total output grows at the rate $g+n$ and consumption per worker grows at the rate g, just as they did initially. However, the levels of k and y are high than initially; with higher saving, the economy can maintain a higher level of k and therefore y.*

2. In the basic Solow model, capital per effective unit of labor, k_t, evolves according to

$$(1+g+n)\,k_{t+1} = (1-\delta)\,k_t + sk_t^{\alpha},$$

where $g+n$ is the growth rate of effective units of labor (A_tL_t), δ is the depreciation rate, s is the saving rate, and k_t^{α} is output.

   (a) Solve for the steady-state value of k and given an expression for it in terms of the parameters of the model (i.e., in terms of g, n, δ, s, and α.

   (b) What is consumption per effective unit of labor (c_t) in the steady state?

   (c) Find an expression for the value of k that maximizes steady-state c_t.

   (d) What value of the saving rate would maximize steady-state c_t?

3. The Ramsey version of the Solow model developed in class (that is, the discrete time formulation used in lecture) consisted of the following four equations:

$$(1+g+n)\,k_{t+1} = (1-\delta)\,k_t + (k_t^{\alpha} - c_t)$$

$$c_t^{-\sigma} = (1 + r_{k,t+1} - r_k^*)\,c_{t+1}^{-\sigma}$$

$$r_{k,t} = \alpha k_t^{\alpha-1} - \delta$$

$$r_k^* = \rho + \sigma g$$

   (a) Give the steady-state versions of these three equations. The steady-state versions are

$$c^* = (k^*)^{\alpha} - (\delta + g + n)\,k^*$$

$$1 = (1 + r_k - r_k^*) \Rightarrow r_k = r_k^*$$

$$r_k = \alpha\,(k^*)^{\alpha-1} - \delta$$

$$r_k^* = \rho + \sigma g$$

(b) How is k in the steady state affected by σ? How is steady-state c affected? Explain. *Take the second part first. From Problem set 1, c^* is maximized when $k^* = k^*_{GR}$, with $\partial c^*/\partial k^* > 0$ for $k^* < k^*_{GR}$ and $\partial c^*/\partial k^* < 0$ for $k^* > k^*_{GR}$. I will focus on the $k^* < k^*_{GR}$ case. So c^* will change in the same direction os k^*. From the expression for $r^*_k = \rho + \sigma g$, a rise in σ increases the steady state return on capital. With diminshing marginal product of capital, a higher r^*_k implies the steady-state k^* must fall (to increase its marginal product in the new steady state). So a rise in σ decreases k^* and c^* when $k^* < k^*_{GR}$.*

(c) How is k in the steady state affected by δ? How is steady-state c affected? Explain. *Again focus on the $k^* < k^*_{GR}$ case (it should be straightforward to explain what happens if $k^* > k^*_{GR}$.). So c^* will change in the same direction os k^*. From the expression for $r^*_k = \rho + \sigma g$, r^*_k is unaffected by δ. Consequently, neither is r_k. But for this to be true, a rise in δ must be offset by a rise in $\alpha (k^*)^{\alpha-1}$, i.e., k^* must fall (recall $\alpha < 1$). Faster depreciation (a higher δ) increases the necessary investment level to maintain a given k^*. In a sense, it becomes more costly to maintain the original k^*, so steady-state k falls, and so does c^* if $k^* < k^*_{GR}$.*

4. For this question, use the dynare program *rbc.dyn* available on the class web site. Modify the program to add a new variable equal to investment to the model.

   (a) Find the standard deviation of investment using the baseline parameters. What is the ratio of the standard deviation of investment to the standard deviation of output? Is it greater than one? For later use, save your results as *baseline.mat* by using the the command `save baseline`.

   (b) Reduce the serial correlation coefficient of the productivity process (ρ) from 0.95 to 0.5. What happens to the ratio of the standard deviation of investment to the standard deviation of output? Explain (in words) why investment is more volatile relative to output when the productivity shock is more transitory (i.e., less persistent).

      i. Dynare saves the impulse response of variable x to shock v as a new variable named x_v. Save the impulse response of output to the productivity shock (y_e) using `save y_e_2 y_e (the` 2 to denote the solution with $\rho = 0.5$).

   (c) In matlab, enter the command `load baseline.` This loads the results from part (a). So now y_e will be the impulse response from the first simulation when $\rho = 0.95$. Plot y_e and y_e_2 in a single figure. Label the axis and add a legend and a title. What effect does ρ have on the response of output to a productivity shock?

5. Solve the model for $\eta = 0.5$, $\eta = 2$, and $\eta = 5$. Plot the responses of y for these three values of η in a single figure. Plot the responses of n for these three values of η in a single figure. Plot the responses of ω (the real wage – equal to the MPL of labor, or $y - n$ in terms of percent deviations around the steady state) for these three values of η in a single figure. (Keep all other parameters at the values used in part (a) of exercise 2.) How does η affect the response of output to the productivity shock? How does η affect the response of employment to the productivity shock? Explain (in words) why larger values of η cause a smaller response of employment. (Hint: what happens to the labor supply curve as η increases?) *See rbc_ps2Q5.dyn. As η increases, the labor supply curve becomes steeper – a positive productivity shock that shifts labor demand outward now leads to a larger rise in the real wage and a smaller increase in employment. As a result, output increases less.*

Economics 202
Winter 2017

Assignment 2: Due in class Tuesday, February 7, 2017

1. *Explain* why, in the basic Solow model, an increase in the saving rate affects the growth rate of output during the transition to a new steady state but does not affect the steady-state growth rate. *Explain* how capital per work and income per worker in the new steady state differ from their value in the old steady state.

2. In the basic Solow model, capital per effective unit of labor, k_t, evolves according to
$$(1+g+n)\,k_{t+1} = (1-\delta)\,k_t + sk_t^{\alpha},$$
where $g+n$ is the growth rate of effective units of labor (A_tL_t), δ is the depreciation rate, s is the saving rate, and k_t^{α} is output.
   (a) Solve for the steady-state value of k and given an expression for it in terms of the parameters of the model (i.e., in terms of g, n, δ, s, and α.
   (b) What is consumption per effective unit of labor (c_t) in the steady state?
   (c) Find an expression for the value of k that maximizes steady-state c_t.
   (d) What value of the saving rate would maximize steady-state c_t?

3. The Ramsey version of the Solow model developed in class (that is, the discrete time formulation used in lecture) consisted of the following four equations:
$$(1+g+n)\,k_{t+1} = (1-\delta)\,k_t + (k_t^{\alpha} - c_t)$$
$$c_t^{-\sigma} = (1 + r_{k,t+1} - r_k^*)\,c_{t+1}^{-\sigma}$$
$$r_{k,t} = \alpha k_t^{\alpha-1} - \delta$$
$$r_k^* = \rho + \sigma g$$
   (a) Give the steady-state versions of the first three of these four equations.
   (b) How is k in the steady state affected by σ? How is steady-state c affected? Explain.
   (c) How is k in the steady state affected by δ? How is steady-state c affected? Explain.

4. For this question, use the dynare program *rbc.dyn* available on the class web site. Modify the program to add a new variable equal to investment to the model.

(a) Find the standard deviation of investment using the baseline parameters. What is the ratio of the standard deviation of investment to the standard deviation of output? Is it greater than one? For later use, save your results as *baseline.mat* by using the the command `save baseline`.

(b) Reduce the serial correlation coefficient of the productivity process (ρ) from 0.95 to 0.5. What happens to the ratio of the standard deviation of investment to the standard deviation of output? Explain (in words) why investment is more volatile relative to output when the productivity shock is more transitory (i.e., less persistent).

  i. Dynare saves the impulse response of variable x to shock v as a new variable named x_v. Save the impulse response of output to the productivity shock (y_e) using `save y_e_2  y_e (the` 2 to denote the solution with $\rho = 0.5$).

(c) In matlab, enter the command `load baseline.` This loads the results from part (a). So now y_e will be the impulse response from the first simulation when $\rho = 0.95$. Plot y_e and y_e_2 in a single figure. Label the axis and add a legend and a title. What effect does ρ have on the response of output to a productivity shock?

5. Solve the model for $\eta = 0.5$, $\eta = 2$, and $\eta = 5$. Plot the responses of y for these three values of η in a single figure. Plot the responses of n for these three values of η in a single figure. Plot the responses of ω (the real wage – equal to the MPL of labor, or $y - n$ in terms of percent deviations around the steady state) for these three values of η in a single figure. (Keep all other parameters at the values used in part (a) of exercise 2.) How does η affect the response of output to the productivity shock? How does η affect the response of employment to the productivity shock? Explain (in words) why larger values of η cause a smaller response of employment. (Hint: what happens to the labor supply curve as η increases?)

Economics 202
Winter 2017

Assignment 3: Answers

1. Suppose the economy is characterized by

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f\right),$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t.$$

If $i_t = r_t^f$, show that $\pi_t = \pi_{t+1} = x_t = x_{t+1} = 0$ is an equilibrium. What problems might arise if the central bank decides to set its interest rate instrument according to the rule $i_t = r_t^f$? *If* $i_t = r_t^f$*, the equilibrium conditions become*

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f\right),$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t.$$

Substitute $\pi_t = \pi_{t+1} = x_t = x_{t+1} = 0$ *into these equations to verify this is a possible equilibrium. The proposed policy rule fails to satisfy the Taylor Principle, so there can exist multiple stationary rational expectations equilibra.*

2. Explain why inflation is costly in a new Keynesian model. *When not all firms can optimally adjust their price each period, inflation causes relative price movements. For example, if inflation becomes positive, firms that have recently had opportunities to adjust their price will have prices that are higher then those of firms that have not recently adjusted their price. This dispersion of relative prices is inefficient, since it means that households adjust the marginal rates of substitution among good to their relative prices while the marginal rate of transformation is always equal to one, given that the basic model assumes all firms share the same constant returns to scale production technology. Because of diminishing marginal utility, for a given level of total output, the increase in utility from consuming more of some goods is more than offset by the decrease in utility from consumption less of the goods with relatively high prices. If firms face increasing costs of producing output, then relative price dispersion also generates a production inefficiency as it leads firms with relative low prices to produce more and firms with relatively high prices to produce less. With convex costs of production, the cost saving from firms that produce less is more than offset by the cost increases of firms producing more.*

3. Suppose the economy is described by the following log-linearized system:

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f\right),$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \tag{1}$$

where r_t^f is a demand shock and e_t is a cost shock, both assumed to be white noise processes. The central bank sets the nominal interest rate i_t to minimize

$$\left(\frac{1}{2}\right)\mathrm{E}_t\left[\sum_{i=0}^{\infty}\beta^i\left(\pi_{t+i}^2 + \lambda x_{t+i}^2\right)\right].$$

(a) Derive the optimal time-consistent policy for the discretionary central banker. Write down the first-order conditions and the targeting rule that characterizes optimal discretionary policy linking x_t and π_t. *Since there are no endogenous dynamics in this problem (the exogenous disturbance terms are serially correlated but neither lagged inflation nor the lagged output gap appear), the central banker's problem under discretion reduces to a sequence of single period problems of the form*

$$\min \frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right)$$

subject to (1), where we can treat x_t as if it were the policy instrument. The first order condition is

$$\kappa\pi_t + \lambda x_t = 0.$$

(b) Assume private sector firms believe inflation follows an AR(1) process, so that they believe $\mathrm{E}_t\pi_{t+1} = \delta\pi_t$ for $0 < \delta < 1$. Using this in (*1*), show how the slope of the relationship between inflation and the output gap depends on δ. Is it steeper or flatter if δ increase (i.e., if firms believe the inflation process is more persistent)? Does this alteration in the Phillips curve affect the targeting rule for optimal discretionary policy you found in part (a). Explain why. T*he Phillips curve becomes*

$$\begin{aligned}\pi_t &= \beta\delta\pi_t + \kappa x_t + e_t \\ &= \left(\frac{\kappa}{1-\beta\delta}\right)x_t + \left(\frac{1}{1-\beta\delta}\right)e_t.\end{aligned}$$

The slope is now $\kappa/(1-\beta\delta)$, which is positive and increases with δ (i.e., the Phillips curve becomes steeper as δ inceases). Inflation is affected by three factors: expectations, the output gap, and the shock. An increase in the output gap increases marginal costs and inflation, but when $E_t\pi_{t+1} = \delta\pi_t$, the rise in inflation also increases expectd future inflation and this leads to a further increse in inflation as firms adjusting prices now raises prices more because they expected future inflation to be higher. Thus, the total increase in π_t when x_t increases is larger.

(c) Using your results in part (b), graph the Phillips curve and the first-order condition you obtained in part (a) in $(x_t\ \pi_t)$ space for $e_t = 0$. In this case, what are the equilibrium values of x_t and π_t? *The equilibrium values are $x = \pi = 0$ when $e = 0$.*

(d) Use your graph to illustrate how the economy responds to a positive inflation shock ($e_t > 0$). How does the response depend on δ? How does it depend on λ? In each case, explain your results.

4. Suppose the economy's inflation rate is described by the following equation (all variables expressed as percentage deviations around a zero inflation steady state):

$$\pi_t = \beta\mathrm{E}_t\pi_{t+1} + \kappa x_t + e_t, \tag{2}$$

where x_t is the gap between output and the flexible-price equilibrium output level, and e_t is a white noise cost shock. The central bank sets the nominal interest rate i_t to minimize

$$\frac{1}{2}\mathrm{E}_t\left[\sum_{i=0}^{\infty}\beta^i\left(\pi_{t+i}^2 + \lambda x_{t+i}^2\right)\right].$$

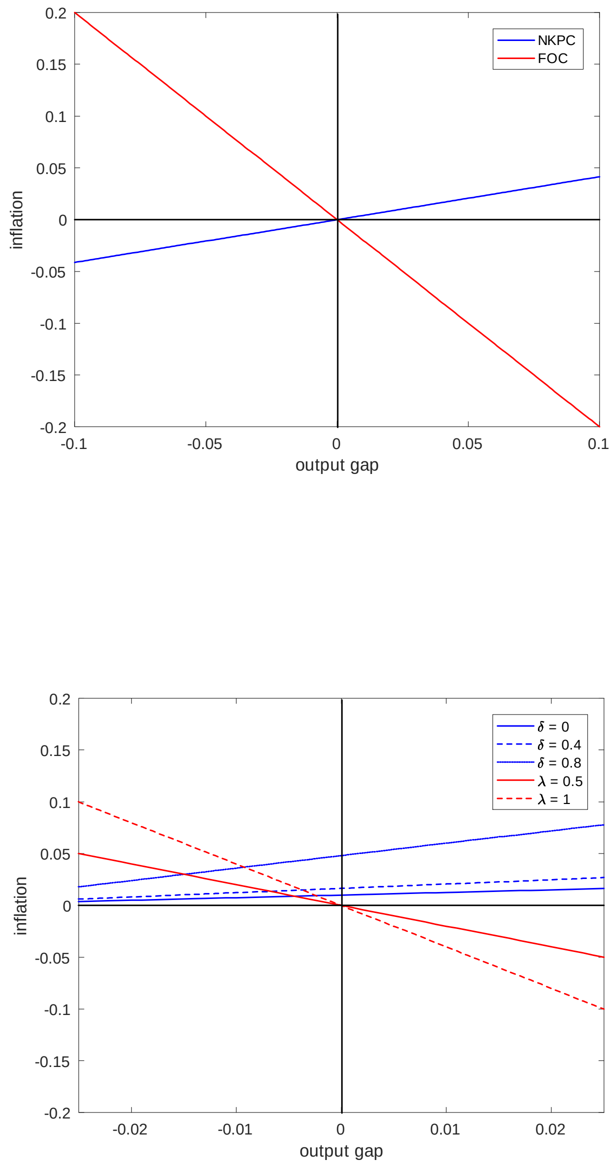


Figure 1: Effects of a positive cost shock: inflation raises, and it does so be a greater amount the more expected future inflation response to current inflation (the larger δ is) and the more the central bank focuses on stabilizing the output gap (the larger λ is).

(a) Derive the first-order conditions linking inflation and the output gap for the *fully* optimal commitment policy. *Under the fully optimal commitment policy, the central bank chooses current inflation and the output gap and planned sequences for future inflation and the output gap to minimize its loss function subject to the contraint imposed by the inflation adjustment equation. We can write the central bank's problem as*

$$\min \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\frac{1}{2} \left(\pi_{t+i}^2 + \lambda x_{t+i}^2 \right) + \psi_{t+i} \left(\pi_{t+i} - \beta \pi_{t+1+i} - \kappa x_{t+i} - e_{t+i} \right) \right]$$

where ψ_{t+i} is the Lagrangian multiplier on the time $t+i$ constraint. First order conditions are

$$\text{for } \pi_t\text{: } \pi_t + \psi_t = 0 \tag{3}$$

$$\text{for } \pi_{t+i}\text{: } \pi_{t+i} + \psi_t - \psi_{t+i-1} = 0 \text{ for } i > 0 \tag{4}$$

$$\text{for } x_{t+i}\text{: } \lambda x_t - \kappa \psi_t = 0 \text{ for } i \geq 0. \tag{5}$$

(b) Explain why the first-order conditions for time t differ from the first-order conditions for $t+i$ for $i > 0$. *The central bank's choice for future inflation in period $t+i$ also affects inflation in period $t+i-1$ because time $t+i-1$ inflation depends on expectations of π_{t+i}. When chosing π_{t+i} for $i > 0$, the central bank takes into account this affect on earlier inflation. However, this effect is absent when choosing π_t since π_{t-1} is already determined and can no longer be affected by π_t. Thus, the first order condition for the optimal choice of π_t depends only on ψ_t, the current marginal cost of inflation, while the choice of π_{t+1}, π_{t+2}, ... will also need to account for the effects on π_t, π_{t+1}, ...*

(c) Explain why, under commitment, the central bank promises a deflation in the period after a positive cost shock. *From the inflation adjustment equation, a central bank, faced with a cost shock, can better stabilize current inflation if it can adjust both the current output gap and the private sector's expectations about future inflation. Or, since the central bank cares about both output gap and inflation stabilization, we can express this by saying that if the cost shock is positive, a given rise ininflation can be achieved with a smaller decline in the output gap if expected future inflation is reduced. Thus, the optimal commitment policy will promise a deflation in period $t+1$ so that $\mathrm{E}_t \pi_{t+1} < 0$.*

5. The dynare program NKM_ps3.dyn is designed to solve the following model:

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma} \right) \left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f \right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + u_t$$

$$i_t = \phi_\pi \pi_t + \phi_x x_t + v_t.$$

$$r_t^f = \rho_r r_{t-1}^f + e_{r,t}$$

$$u_t = \rho_u u_{t-1} + e_{u,t}$$

$$v_t = \rho_v v_{t-1} + e_{v,t}$$

where $e_{z,t}$ is a mean zero, white noise process for $z = r, u, v$. Set $\sigma = 2$, $\beta = 0.99$, $\kappa = 0.5$, $\phi_x = 0$, $\rho_r = 0.9$, $\rho_u = 0.9$, and $\rho_v = 0.5$. Set the standard deviations to each of the $e_{z,t}$ processes equal to 0.01.

(a) Graph the responses of the output gap, inflation and the nominal interest rate to a 1 unit monetary policy shock (i.e., $e_{v,t} = 1$). Explain why x, π, and i behave the way they do. *See figure* **??** *below (see the version of NKM_ps3.dyn now posted). The positive shock to the interest rate is exactly like a negative aggregate demand shock. This can be seen by substituting the policy rule into the aggregate demand/IS relationship to yield*

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(\phi_\pi \pi_t + \phi_x x_t + v_t - \mathrm{E}_t \pi_{t+1} - r_t^n)$$

showing $v_t - r_t^n$ *is what matters. A positive policy shock pushes up the real interest rate* $i_t - \mathrm{E}_t \pi_{t+1}$ *and this reduces aggregate demand and so the output gap. With the fall in the fall in output, firms need fewer workers. This fall in the demand for labor reduces real wages and, hence, firms' marginal costs. Thus, those firms that do adjust prices reduce them in response to their lower real marginal costs. Therefore inflation falls. Notice that the fall in the output gap and inflation induces an interest rate cut according to the policy rule and this partially offsets the direct effect of the shock on the nominal interest rate.*

(b) Repeat part (b) for a shock to aggregate demand (i.e., $e_{r,t} = 1$). Explain why x, π, and i behave the way they do.*See figure* **??** *below. The shock to aggregate demand leads firms to increase production when prices are sticky. So the output gap rises. To product more, firms need to hire more workers and this increase in labor demand causes wages to rise. This, in turn, increases firms' marginal cost, so those firms that can adjust their price increase their price. This increases inflation. The rise in the output gap and inflation, the central bank's policy rule calls for an increase in the nominal interest rate in attempt to bring the output gap and inflation back to zero. Policy does so by ensuring the real interest rate rises to bring aggregage demand back down.*

(c) Now replace the policy rule with

$$i_t = r_t^f + \phi_\pi \pi_t + \phi_x x_t + v_t$$

and repeat part (c). Explain why your results differ. *Now, a demand shock has no effect on the output gap or inflation because the policy rate is adjusted to exactly neutralize the demand shock. This can be seen directly by substituting this new policy rule into the aggregate demand relationship to obtain*

$$\begin{aligned}
x_t &= \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f\right) \\
&= \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(r_t^n + \phi_\pi \pi_t + \phi_x x_t + v_t - \mathrm{E}_t \pi_{t+1} - r_t^f\right) \\
&= \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(\phi_\pi \pi_t + \phi_x x_t + v_t - \mathrm{E}_t \pi_{t+1})
\end{aligned}$$

while is independent of the demand shock r_t^f.

Economics 202
Winter 2017

Assignment 3: Due in class February 28

1. Suppose the economy is characterized by

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t\pi_{t+1} - r_t^f\right),$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t.$$

If $i_t = r_t^f$, show that $\pi_t = \pi_{t+1} = x_t = x_{t+1} = 0$ is an equilibrium. What problems might arise if the central bank decides to set its interest rate instrument according to the rule $i_t = r_t^f$?

2. Explain why inflation is costly in a new Keynesian model.

3. Suppose the economy is described by the following log-linearized system:

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t\pi_{t+1} - r_t^f\right),$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \tag{1}$$

where r_t^f is a demand shock and e_t is a cost shock, both assumed to be white noise processes. The central bank sets the nominal interest rate i_t to minimize

$$\left(\frac{1}{2}\right)\mathrm{E}_t\left[\sum_{i=0}^{\infty}\beta^i\left(\pi_{t+i}^2 + \lambda x_{t+i}^2\right)\right].$$

(a) Derive the optimal time-consistent policy for the discretionary central banker. Write down the first-order conditions and the targeting rule that characterizes optimal discretionary policy linking x_t and π_t.

(b) Assume private sector firms believe inflation follows an AR(1) process, so that they believe $\mathrm{E}_t\pi_{t+1} = \delta\pi_t$ for $0 < \delta < 1$. Using this in (*1*), show how the slope of the Phillips Curve relationship between inflation and the output gap depends on δ. Is it steeper or flatter if δ increase (i.e., if firms believe the inflation process is more persistent)? Does this alteration in the Phillips curve affect the targeting rule for optimal discretionary policy you found in part (a). Explain why.

(c) Using your results in part (b), graph the Phillips curve and the first-order condition you obtained in part (a) in (x_t π_t) space for $e_t = 0$. In this case, what are the equilibrium values of x_t and π_t?

(d) Use your graph to illustrate how the economy responds to a positive inflation shock ($e_t > 0$). How does the response depend on δ? How does it depend on λ? In each case, explain your results.

4. Suppose the economy's inflation rate is described by the following equation (all variables expressed as percentage deviations around a zero inflation steady state):

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \tag{2}$$

where x_t is the gap between output and the flexible-price equilibrium output level, and e_t is a white noise cost shock. The central bank sets the nominal interest rate i_t to minimize

$$\frac{1}{2}\mathrm{E}_t\left[\sum_{i=0}^{\infty}\beta^i\left(\pi_{t+i}^2+\lambda x_{t+i}^2\right)\right].$$

(a) Derive the first-order conditions linking inflation and the output gap for the *fully* optimal commitment policy.

(b) Explain why the first-order conditions for time t differ from the first-order conditions for $t+1$.

(c) Explain why, under commitment, the central bank promises a deflation in the period after a positive cost shock..

5. The dynare program NKM_ps3.dyn is designed to solve the following model:

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t\pi_{t+1} - r_t^f\right)$$

$$\pi_t = \beta\mathrm{E}_t\pi_{t+1} + \kappa x_t + u_t$$

$$i_t = \phi_\pi\pi_t + \phi_x x_t + v_t.$$

$$r_t^f = \rho_r r_{t-1}^f + e_{r,t}$$

$$u_t = \rho_u u_{t-1} + e_{u,t}$$

$$v_t = \rho_v v_{t-1} + e_{v,t}$$

where $e_{z,t}$ is a mean zero, white noise process for $z = r, u, v$. Set $\sigma = 2$, $\beta = 0.99$, $\kappa = 0.5$, $\phi_\pi = 1.5$, $\phi_x = 0$, $\rho_r = 0.9$, $\rho_u = 0.9$, and $\rho_v = 0.5$. Set the standard deviations to each of the $e_{z,t}$ processes equal to 0.01.

(a) Graph the responses of the output gap, inflation and the nominal interest rate to a 1 unit monetary policy shock (i.e., $e_{v,t} = 1$). Explain why x, π, and i behave the way they do.

(b) Repeat part (b) for a shock to aggregate demand (i.e., $e_{r,t} = 1$). Explain why x, π, and i behave the way they do.

(c) Now replace the policy rule with

$$i_t = r_t^f + \phi_\pi\pi_t + \phi_x x_t + v_t$$

and repeat part (c). Explain why your results differ.

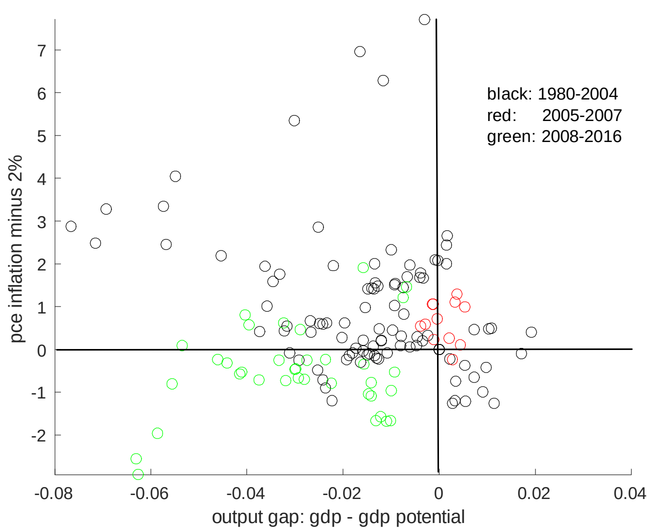
Economics 202
Winter 2017

Assignment 4: Due in class March 14 Answers

1. For this problem, you will need quarterly U.S. data from the first quarter of 1980 to the fourth quarter of 2016 on the following variables (with their FRED labels): real GDP (GDPC96), the Consumer Price Index (CPIAUCSL), Core CPI (CPILFESL), the Personal Consumption Chain Price Index (PCEPI), CORE PCE (PCEPILFE), the overall civilian unemployment rate (UNRATE), and the CBO estimates of potential GDP (GDPPOT) and the long-term (NROU) and short-term (NROUST) natural rate of unemployment. (The core price indexes remove food and energy prices.)

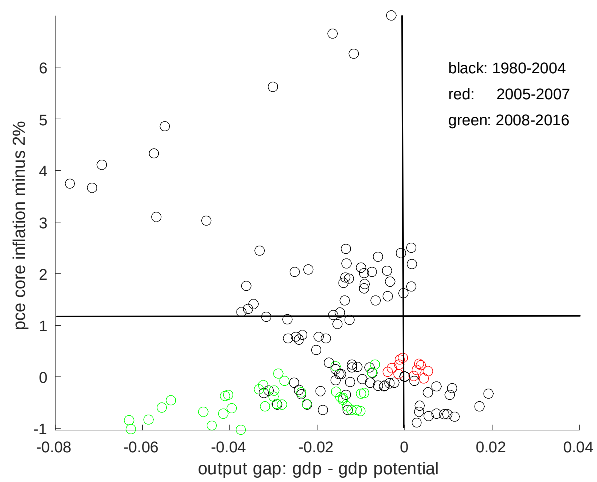
   (a) Take natural logs of all variables except the unemployment rates. Construct the output gap as the different between log GDP and log potential GDP. Calculate inflation rate gaps as the year over year change in the log price indexes, expressed at annual precentage rates (i.e., $\pi_t = 100 \times (p_t - p_{t-4})$ minus the Fed's 2% target. Construct unemployment gaps as the difference between the overall unemployment rate and each of the two measures of the natural rate.

   (b) Construct a scatter plot with the output gap on the horizontal axis and the PCE inflation gap on the vertical axis. John Taylor has argued that monetary policy was too expansionary during 2005-2007. If there evidence of this from your plot? Is there evidence that monetary policy was too contractionary during 2008-2016?

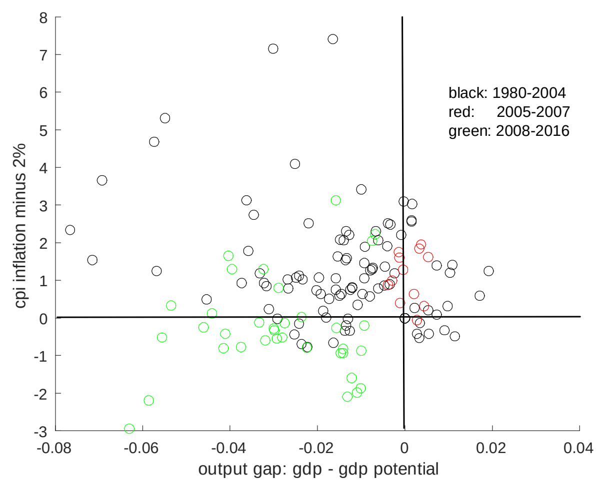


   There isn't much evidence from this figure that policy overly expansionary during 2005-2007 (the red circles): some points are in the 1st quadrant (consistent with policy being too expansionary) but as many are in quadrants 2 and 4. For 2008-2016, most (but not all points are in quadrant 3, suggestion policy was not expansionary enough.

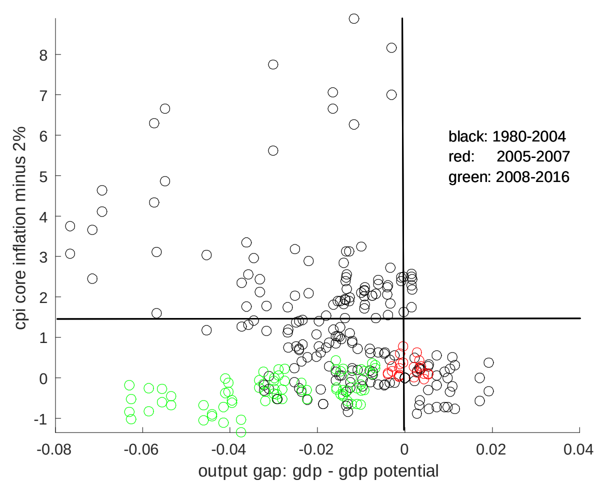
   (c) Redo part (b) using core PCE as the measure used to form an inflation gap. Are your conclusions from part b affected? What about if you use the CPI or core CPI measures of the inflation



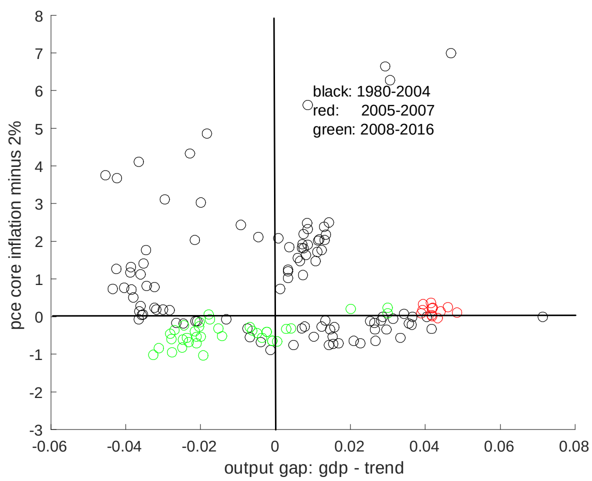
pce core inflation minus 2%
output gap: gdp - gdp potential
black: 1980-2004
red: 2005-2007
green: 2008-2016

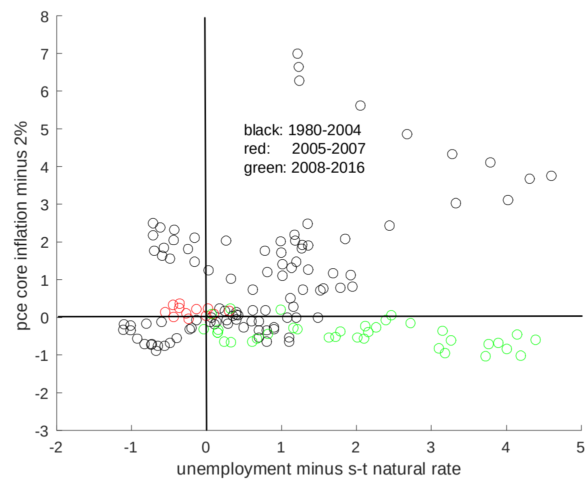


cpi inflation minus 2%
output gap: gdp - gdp potential
black: 1980-2004
red: 2005-2007
green: 2008-2016



cpi core inflation minus 2%
output gap: gdp - gdp potential
black: 1980-2004
red: 2005-2007
green: 2008-2016

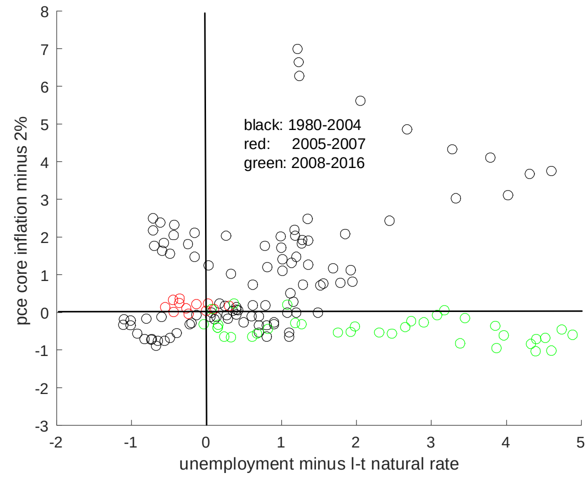




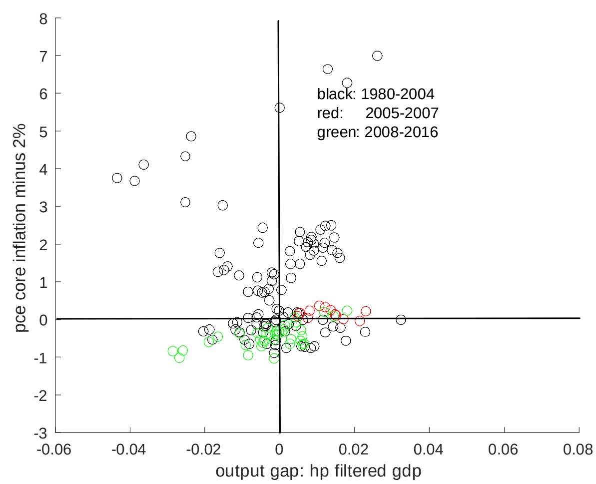
gap? *Using either core measure, it is clearer that policy should have been more expansionary after 2008 and none of the red cicles are in the 1st quadrant.*

(d) Redo part (b) using the following alternative measures of the output gap: (i) in place of the CBO measure of potential GDP, use the residuals of a least squares regression of log GDP on time and time squared (i.e., $\hat{u}_t$ from $y_t = a_0 + a_1 t + a_2 t^2 + \hat{u}_t$ where a_i are estimated using OLS); (ii) use unemployment minus the CBO estimate of the short-term natural rate of unemployment; (iii) use unemployment minus the CBO estimate of the long-term natural rate of unemployment. Are any of your conclusions affected? *For this questions, I will just report results using the OCE core measure of inflation. When the output gap is measured by detrending GDP using time and time squared, the evidence for Taylor's view is strongest. Recall that for the measures using unemployment, policy would be too expansionary (contractionary) if points show up in quadrant 2 (4).*

(e) Optional: Measure the output gap by HP filtering GDPC96 (use a weight of 1400) and redo part (b). *Here is the figure using the core pce to measure inflation:*



black: 1980-2004
red: 2005-2007
green: 2008-2016
pce core inflation minus 2%
unemployment minus l-t natural rate



black: 1980-2004
red: 2005-2007
green: 2008-2016
pce core inflation minus 2%
output gap: hp filtered gdp

2. Consider the following new Keynesian model at a zero nominal interest rate:

$$x_t = \mathrm{E}_t x_{t+1} + \left(\frac{1}{\sigma}\right)\left(\mathrm{E}_t \pi_{t+1} + r^Z\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t,$$

where $r^Z < 0$ is the negative demand shock that has pushed the economy to the zero lower bound. If the economy is not at the zero lower bound, assume $x = \pi = 0$. Denote the equilibrium at the zero lower bound by x^Z and π^Z. If the public expects that the economy will remain at the zero lower bound in the following period with probability q, then

$$\mathrm{E}_t \pi_{t+1} = q\pi^Z + (1-q) \times 0 = q\pi^Z,$$

and

$$\mathrm{E}_t x_{t+1} = qx^Z + (1-q) \times 0 = qx^Z.$$

(a) Solve for the output gap and inflation at the zero lower bound, i.e., solve for x^Z and π^Z (For this question, assume $\sigma(1-q)(1-\beta q) - q\kappa > 0$.) *The two equation system for x^Z and π^Z is*

$$x^Z = qx^Z + \left(\frac{1}{\sigma}\right)\left(q\pi^Z + r^Z\right)$$

$$\pi^Z = \beta q \pi^Z + \kappa x^Z.$$

Rewrite this system as

$$\begin{bmatrix} \sigma(1-q) & -q \\ -\kappa & 1-\beta q \end{bmatrix} \begin{bmatrix} x^Z \\ \pi^Z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} r^Z$$

or

$$\begin{aligned} \begin{bmatrix} x^Z \\ \pi^Z \end{bmatrix} &= \begin{bmatrix} \sigma(1-q) & -q \\ -\kappa & 1-\beta q \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} r^Z \\ &= \left[\frac{1}{\sigma(1-q)(1-\beta q) - q\kappa}\right] \begin{bmatrix} (1-\beta q) \\ \kappa \end{bmatrix} r^Z \end{aligned}$$

Since $r^Z < 0$, both x^Z and π^Z are negative.

(b) Suppose the public becomes more pessimistic in the sense they think it is more likely the economy will remain at the zero lower bound (i.e, q increases). What happens to the output gap and inflation? Explain why. *From the part (a),*

$$x^Z = \left(\frac{1-\beta q}{\sigma(1-q)(1-\beta q) - q\kappa}\right) r^Z$$

$$\pi^Z = \left(\frac{\kappa}{\sigma(1-q)(1-\beta q) - q\kappa}\right) r^Z$$

Let $\Delta \equiv \sigma(1-q)(1-\beta q) - q\kappa$. Then

$$\frac{\partial \Delta}{\partial q} = -\sigma(1-\beta q) - \sigma\beta(1-q) - \kappa < 0.$$

It follows a rise in q reduces both the numerator and the denominator of the expression for x^Z, so x^Z becomes more negative (remember r^Z is negative) while the denominator in the expression for π^Z also falls so π^Z also becomes more negative. Both the output gap and inflation depend on the expected future output gap and inflation. If the public thinks the economy is more likely to remain at x^Z and π^Z rather than return to $x = \pi = 0$, the fall in expected future x and π reduce current inflation and the output gap.

3. Suppose the model for the output gap and inflation is given by

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(r_t^L - r_t^n\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + u_t$$

$$r_t^n = \rho_r r_{t-1}^n + e_{r,t}$$

$$u_t = \rho_u u_{t-1} + e_{u,t}$$

where r_t^L is the real long-term interest rate. For simplicity, assume the long rate is a three period rate and r_t^L is related to the short term real interest rate r_t by the expectations model of the term structure:

$$r_t^L = \left(\frac{1}{3}\right)\left(r_t + \mathrm{E}_t r_{t+1} + \mathrm{E}_t r_{t+2}\right) + \varphi_t,$$

where φ_t is a shock to the long rate. Assume

$$\varphi_t = \rho_\varphi \varphi_{t-1} + e_{\varphi,t}$$

The short-term real rate is given by

$$r_t = i_t - \mathrm{E}_t \pi_{t+1},$$

and the nominal interest rate is set by the central bank according to

$$i_t = r_t^n + \rho_i i_{t-1} + (1 - \rho_i)\left(\phi_\pi \pi_t + \phi_x x_t\right).$$

For the exogenous shocks and parameters, $e_{z,t}$ is a mean zero, white noise process for $z = r, u, v, \varphi$. Set $\sigma = 2$, $\beta = 0.99$, $\kappa = 0.15$, $\rho_i = 0.5$, $\phi_\pi = 1.5$, $\phi_x = 0$, $\rho_r = 0.9$, $\rho_u = 0.9$, and $\rho_\varphi = 0.9$. Set the standard deviations of each $e_{z,t}$ to 0.01.

(a) Use dynare to plot the impulse responses of the output gap, inflation, the long-term real interest rate, the one-period real interest rate, and the nominal interest rate to a one standard deviation shock to φ_t. Explain why the different variables behave the way they do. *By substituting the term structure equation for r_t^L into the expectational IS relationship, we obtain*

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left[\left(\frac{1}{3}\right)\left(r_t + \mathrm{E}_t r_{t+1} + \mathrm{E}_t r_{t+2}\right) + \varphi_t - r_t^n\right]$$

which illustrates that the shock affects output and inflation in the same way (but with opposite sign) as a demand shock r_t^n. See NKM_ps4_3.dyn.

(b) Repeat part (a) but assume $\rho_\varphi = 0$. How do the results differ from those you obtained in part (a)? Explain why. *With no persistence in the term structure shock, expectations of future interests rates are affected very little when $\rho_\varphi = 0$ so the effects on the output gap and inflation are smaller and less persistent than those obtained in part (a).*

4. Consider a firm that can invest in one of two projects. Project $i = 1, 2$ yields a gross rate of return of $R - x_i$ with probability $1/2$ and $R + x_i$ with probability $1/2$. Assume $x_2 > x_1$ so project 2 is a riskier project. The firm borrows L to undertake the project and has collateral C. The lender's opportunity cost of funds (the rate of return it can earn if it not lend to the firm) is r. Both the firm and the lender are risk neutral. Assume the firm defaults when $R - x_i$ occurs; if the firm defaults, the lender gets $R - x_i + C < (1 + r^l)L$ if r^L is the interest rate on the loan.

   (a) Suppose the lender can, without cost, monitor which project the firm chooses. What interest rate will the lender charge the firm if the firm picks project 1? What interest rate will it charge if the firm picks project 2? (Hint: for either project, the expected rate of return to the lender must equal r.) *See the closely related question 7 in the review sheet for the final.*

   (b) Now suppose the firm chooses which project to undertake after it receives the loan and the lender cannot observe which project is undertaken. What interest rate will the bank charge on loans? Can good, i.e., low risk, projects get funding? Explain. *See the closely related question 7 in the review sheet for the final.*

5. Suppose the balance sheet of an agent (the agent could be a household or a firm) consists of assets a_t, debts of d_t, and a net worth of n_t, so

$$a_t = d_t + n_t.$$

Define the leverage ratio of this agent as $l_t \equiv a_t/n_t \geq 1$. Suppose the agent can default on its debts and capture (or divert) a fraction θ of its assets.

   (a) Explain why the agent has an incentive to default if

$$\theta a_t > n_t,$$

   where $0 < \theta < 1$.

   (b) Suppose lenders will not lend if the borrower hass an incentive to default (i.e., if $\theta a_t > n_t$). Show that this implies the agent will face a borrowing constraint of the form

$$d_t \leq \left(\frac{1-\theta}{\theta}\right) n_t. \tag{1}$$

   The no default requirement is

$$\theta a_t = \theta\left(d_t + n_t\right) \leq n_t$$

   which can be rewritten as

$$\theta d_t \leq (1-\theta)\, n_t$$

$$d_t \leq \left(\frac{1-\theta}{\theta}\right) n_t$$

   (c) Rewrite (1) in a form that shows the agent will face a constraint on the maximum leverage a_t/n_t it can take on. How is the limit on leverage affected by an increase in θ? Explain.

$$\theta\left(d_t + n_t\right) = \theta a_t \leq n_t$$

$$l_t \equiv \frac{a_t}{n_t} \leq \frac{1}{\theta}.$$

An increase in the ability of the agent to divert assets (a rise in θ) reduces the maximum leverage the agent can take on. That is, it tightens the borrowing constraint the agent faces. If borrower can potentially run off with more of what you have lend, the less you will be willing to lend.

Economics 202
Winter 2017

Assignment 4: Due in class March 14.

1. For this problem, you will need quarterly U.S. data from the first quarter of 1980 to the fourth quarter of 2016 on the following variables (with their FRED labels): real GDP (GDPC96), the Consumer Price Index (CPIAUCSL), Core CPI (CPILFESL), the Personal Consumption Chain Price Index (PCEPI), CORE PCE (PCEPILFE), the overall civilian unemployment rate (UNRATE), and the CBO estimates of potential GDP (GDPPOT) and the long-term (NROU) and short-term (NROUST) natural rate of unemployment. (The core price indexes remove food and energy prices.)

   (a) Take natural logs of all variables except the unemployment rates. Construct the output gap as the different between log GDP and log potential GDP. Calculate inflation rate gaps as the year over year change in the log price indexes, expressed at annual percentage rates (i.e., $\pi_t = 100 \times (p_t - p_{t-4})$ minus the Fed's 2% target. Construct unemployment gaps as the difference between the overall unemployment rate and each of the two measures of the natural rate.

   (b) Construct a scatter plot with the output gap on the horizontal axis and the PCE inflation gap on the vertical axis. John Taylor has argued that monetary policy was too expansionary during 2005-2007. If there evidence of this from your plot? Is there evidence that monetary policy was too contractionary during 2008-2016?

   (c) Redo part (b) using core PCE as the measure used to form an inflation gap. Are your conclusions from part b affected? What about if you use the CPI or core CPI measures of the inflation gap?

   (d) Redo part (b) using the following alternative measures of the output gap: (i) in place of the CBO measure of potential GDP, use the residuals of a least squares regression of log GDP on time and time squared (i.e., $\hat{u}_t$ from $y_t = a_0 + a_1 t + a_2 t^2 + \hat{u}_t$ where a_i are estimated using OLS); (ii) use unemployment minus the CBO estimate of the short-term natural rate of unemployment; (iii) use unemployment minus the CBO estimate of the long-term natural rate of unemployment. Are any of your conclusions affected?

   (e) Optional: Measure the output gap by HP filtering GDPC96 (use a weight of 1400) and redo part (b).

2. Consider the following new Keynesian model at a zero nominal interest rate:

$$x_t = \mathrm{E}_t x_{t+1} + \left(\frac{1}{\sigma}\right)\left(\mathrm{E}_t \pi_{t+1} + r^Z\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t,$$

where $r^Z < 0$ is the negative demand shock that has pushed the economy to the zero lower bound. If the economy is not at the zero lower bound, assume $x = \pi = 0$. Denote the equilibrium at the zero lower bound by x^Z and π^Z. If the public expects that the economy will remain at the zero lower bound in the following period with probability q, then

$$\mathrm{E}_t \pi_{t+1} = q\pi^Z + (1-q) \times 0 = q\pi^Z,$$

and

$$\mathrm{E}_t x_{t+1} = qx^Z + (1-q) \times 0 = qx^Z.$$

(a) Solve for the output gap and inflation at the zero lower bound, i.e., solve for x^Z and π^Z (For this question, assume $\sigma(1-q)(1-\beta q) - q\kappa > 0$.)

(b) Suppose the public becomes more pessimistic in the sense they think it is more likely the economy will remain at the zero lower bound (i.e., q increases). What happens to the output gap and inflation? Explain why.

3. Suppose the model for the output gap and inflation is given by

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(r_t^L - r_t^n\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + u_t$$

$$r_t^n = \rho_r r_{t-1}^n + e_{r,t}$$

$$u_t = \rho_u u_{t-1} + e_{u,t}$$

where r_t^L is the real long-term interest rate. For simplicity, assume the long rate is a three period rate and r_t^L is related to the short term real interest rate r_t by the expectations model of the term structure:

$$r_t^L = \left(\frac{1}{3}\right)\left(r_t + \mathrm{E}_t r_{t+1} + \mathrm{E}_t r_{t+2}\right) + \varphi_t,$$

where φ_t is a shock to the long rate. Assume

$$\varphi_t = \rho_\varphi \varphi_{t-1} + e_{\varphi,t}$$

The short-term real rate is given by

$$r_t = i_t - \mathrm{E}_t \pi_{t+1},$$

and the nominal interest rate is set by the central bank according to

$$i_t = r_t^n + \rho_i i_{t-1} + (1-\rho_i)\left(\phi_\pi \pi_t + \phi_x x_t\right).$$

For the exogenous shocks and parameters, $e_{z,t}$ is a mean zero, white noise process for $z = r, u, v, \varphi$. Set $\sigma = 2$, $\beta = 0.99$, $\kappa = 0.15$, $\rho_i = 0.5$, $\phi_\pi = 1.5$, $\phi_x = 0$, $\rho_r = 0.9$, $\rho_u = 0.9$, and $\rho_\varphi = 0.9$. Set the standard deviations of each $e_{z,t}$ to 0.01.

(a) Use dynare to plot the impulse responses of the output gap, inflation, the long-term real interest rate, the one-period real interest rate, and the nominal interest rate to a one standard deviation shock to φ_t. Explain why the different variables behave the way they do.

(b) Repeat part (a) but assume $\rho_\varphi = 0$. How do the results differ from those you obtained in part (a)? Explain why.

4. Consider a firm that can invest in one of two projects. Project $i = 1, 2$ yields a gross rate of return of $R - x_i$ with probability 1/2 and $R + x_i$ with probability 1/2. Assume $x_2 > x_1$ so project 2 is a riskier project. The firm borrows L to undertake the project and has collateral C. The lender's opportunity cost of funds (the rate of return it can earn if it not lend to the firm) is r. Both the firm and the lender are risk neutral. Assume the firm defaults when $R - x_i$ occurs; if the firm defaults, the lender gets $R - x_i + C < (1 + r^l)L$ if r^L is the interest rate on the loan.

(a) Suppose the lender can, without cost, monitor which project the firm chooses. What interest rate will the lender charge the firm if the firm picks project 1? What interest rate will it charge if the firm picks project 2? (Hint: for either project, the expected rate of return to the lender must equal r.)

(b) Now suppose the firm chooses which project to undertake after it receives the loan and the lender cannot observe which project is undertaken. What interest rate will the bank charge on loans? Can good, i.e., low risk, projects get funding? Explain.

5. Suppose the balance sheet of an agent (the agent could be a household or a firm) consists of assets a_t, debts of d_t, and a net worth of n_t, so

$$a_t = d_t + n_t.$$

Define the leverage ratio of this agent as $l_t \equiv a_t/n_t \geq 1$. Suppose the agent can default on its debts and capture (or divert) a fraction θ of its assets.

(a) Explain why the agent has an incentive to default if

$$\theta a_t > n_t,$$

where $0 < \theta < 1$.

(b) Suppose lenders will not lend if the borrower has an incentive to default (i.e., if $\theta a_t > n_t$). Show that this implies the agent will face a borrowing constraint of the form

$$d_t \leq \left(\frac{1-\theta}{\theta}\right) n_t. \tag{1}$$

(c) Rewrite (1) in a form that shows the agent will face a constraint on the maximum leverage a_t/n_t it can take on. How is the limit on leverage affected by an increase in θ? Explain.