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Macroeconomic Analysis
202 - Winter 2016

Alan Ledesma

UCSC

Sep. 2016
https://sites.google.com/a/ucsc.edu/alanledesma/teaching/2017q1

Information

Welcome to 202 sections!

1. The syllabus is your friend.
2. Check my website:
   `https://sites.google.com/a/ucsc.edu/alanledesma/teaching/2017q1`
   Search for: "`ucsc phd students economics`"
   Select: "`Ph.D. in Economics - Economics Department`"
   Scroll down to find: "`Alan Ledesma`"
   Additional material, slides and discussions will be posted there.
3. Sections:
   Thu. 5:20PM-6:25PM and 6:40AM-7:45AM at Kresge Clrm 325.
4. Office hours:
   Tue. 11:00AM-1:00PM at E2 building, Room 403G.
5. Although advisable, you are not required to attend sections ...
6. I love questions ...

Help with MS Excel for PS1: Add-in installation

- Download the add-in from https://research.stlouisfed.org/fred-addin/ (you may follow the instructions).
- Locate the xlam-file in a selected folder.
- Open the excel file and select: `File/Options` and click "`Add-Ins`" on the emerging window.
- Go to "`Manage: Excel Add-Ins / Go...`"
- On the emerging window select "`Browse`" locate the xlam-file and add it to the list of available add-ins. Verify that it is selected (✓).
- Add the xlam-file location to the excel "`trust center`": `File/ Options/ Trust Center/ Trust Center Settings.../ Trusted Locations/ Add New Location...`

The FRED Add-In is also available at Google spread sheet, look for it under the option: "`Add-ons/Get add-ons`" and search for "`FRED`".

Help with MS Excel for PS1: Add-in usage

Useful for Q1-3 in PS1!

1. Let's download Real GDP (GDPC96), Real Potential GDP (GDPPOT), and dates of recessions (US-REC)

- Type first the variable identifier, e.g., `GDPC96`.
- Right below introduce a transformation, e.g., `lin`. (see next page for all options available)
- Right below introduce the frequency, e.g., `q`.
- Right below introduce the starting period, e.g., 1905.

	A	B	C	D	E	F	G
1	GDPC96			GDPPOT			USREC
2	lin			lin			lin
3	q			q			q
4	01/01/1959			01/01/1959			01/01/1959

- Go to "`FRED/Get FRED Data`"

See the attached "`Help_PS1.xlsx`" for a more comprehensive example.
See https://docs.google.com/a/ucsc.edu/spreadsheets/d/1LHTAuS1iaQmDWqy0F
edit?usp=sharing for a similar application in Google Docs.

Help with MS Excel for PS1: Add-in usage

Useful for Q1-3 in PS1!

Transformations available:

- Level (`lin`): x_t.
- Change (`chg`): $x_t - x_{t-1}$.
- Change from Year Ago (`ch1`): $x_t - x_{t-n}$
- Percent Change (`pch`): $\left(\frac{x_t}{x_{t-1}} - 1\right) 100$
- Percent Change from Year Ago (`pc1`): $\left(\frac{x_t}{x_{t-n}} - 1\right) 100$
- Compound Annual Rate of Change (`pca`): $\left[\left(\frac{x_t}{x_{t-1}}\right)^n - 1\right] 100$
- Continuously Compounded Rate of Change (`cch`): $(\ln x_t - ln x_{t-1})100$
- Continuously Compounded Annual Rate of Change (`cca`): $(\ln x_t - \ln x_{t-1})100n$
- Natural Log (`log`): $\ln x_t$

Solow-Swan model: Basics

Useful for Q4-5 in PS1! Assumptions

A1. No government + closed economy: $y_t = c_t + i_t$ and $i_t = s_t$

A2. Constant marginal propensity to consume $c_t = (1 - s)y_t$ or $s_t = sy_t$.

A3. Neoclassical production function with Harrod neutral technology: $Y_t = F(K_t, A_t N_t)$ or $y_t = f(k_t)$ with $f(k_t) = F(k_t, 1)$.

A4. Exogenous growth of labor and technology: $N_t = (1 + n)N_{t-1}$ and $A_t = (1 + gA_{t-1})$

A5. Capital evolves according to
$K_{t+1} = (1 - \delta)K_t + I_t$ or $(1 + n + g)k_{t+1} = (1 - \delta)k_t + i_t$

From A1, A2 and A3, I get $s_t = i_t = sy_t = sf(k_t)$. Plugging the latter in A5 leads to the Solow-Swan's model fundamental equation

$$(1 + n + g)k_{t+1} = sf(k_t) + (1 - \delta)k_t \text{ or} \tag{1}$$

$$(1 + n + g)\Delta k_{t+1} = \underbrace{s \overbrace{f(k_t)}^{\substack{y_t \\ \text{output}}}}_{\substack{ic_t \\ \text{Investment} \\ \text{curve}}} - \underbrace{(n + g + \delta)k_t}_{\substack{dl_t \\ \text{Depreciation} \\ \text{line}}} \tag{2}$$

Solow-Swan model: Steady-state & golden rule

Useful for Q4-5 in PS1!

- The steady-state condition (replace k_t and k_{t+1} with $\overline{k}$) in the fundamental equation

$$\boxed{\frac{\overline{k}}{f\left(\overline{k}\right)} = \frac{s}{n+g+\delta}} \tag{3}$$

  Since $f(\cdot)$ has diminishing marginal productivity, the LHS is increasing in $\overline{k}$.

- Golden rule: Maximum attainable consumption at the steady-state. Consumption at the SS: in (3) $\overline{i} = sf\left(\overline{k}\right) = \overline{y} - \overline{c} = f\left(\overline{k}\right) - \overline{c}$; therefore

$$\overline{c} = f\left(\overline{k}\right) - (n+g+\delta)\overline{k}$$

$$\frac{\partial \overline{c}}{\partial \overline{k}} = f'\left(\overline{k}\right) - (n+g+\delta) \Rightarrow \boxed{\overline{k}^* : f'\left(\overline{k}^*\right) = n+g+\delta} \tag{4}$$

  That is, the steady-state capital should generate an slope of $f(\cdot)$ that coincides with the slope of the depreciation line.

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Dynamics in the Solow-Swan model

What happens with the steady-state growth, and with efficient units of capital, production and consumption right after the shock and in the long run in the following cases:

1. An earthquake destroys half of the phisical capital; however, the population size is unaffected.
    - No change in either α, n, g, δ or s; therefore no change in the SS.
    - Immediate changes: $k'_0 = \frac{K_0/2}{A_0(N_0)} = 0.5k_0$, $y'_0 = (0.5k_0)^\alpha = 0.5^\alpha y_0$ and $c'_0 = (1-s)(0.5k_0)^\alpha = 0.5^\alpha c_0$.
    - Long-run: no change.
2. A biological desaster reduces the economy population by half; however, the stock of capital is unaffected.
    - No change in either α, n, g, δ or s; therefore no change in the SS.
    - Immediate changes: $k'_0 = \frac{K_0}{A_0(N_0/2)} = 2k_0$, $y'_0 = (2k_0)^\alpha = 2^\alpha y_0$ and $c'_0 = (1-s)(2k_0)^\alpha = 2^\alpha c_0$.
    - Long-run: no change.

Dynamics in the Solow-Swan model

3. Capital depreciates faster.
    - δ increases $\Rightarrow k'_{ss} < k_{ss}$.
    - Immediate changes: $k'_0 = \frac{K_0}{A_0 N_0} = k_0 \Rightarrow$ no changes.
    - Long-run: $y_s s' < y_s s$, consumption depends on Golden rule and $\Delta\%Y' = n + g = \Delta\%Y$.
4. Population growht rate decreases.
    - n decreases $\Rightarrow k'_{ss} > k_{ss}$.
    - Immediate changes: $k'_0 = \frac{K_0}{A_0 N_0} = k_0 \Rightarrow$ no changes.
    - Long-run: $y_s s' > y_s s$, consumption depends on Golden rule and $\Delta\%Y' = n' + g < n + g = \Delta\%Y$.
5. Saving rate is increased.
    - s increases $\Rightarrow k'_{ss} > k_{ss}$.
    - Immediate changes: $k'_0 = k_0$ and $y'_0 = y_0$, but $c'_0 = (1 - s')y_0 < (1 - s)y_0 = c_0$.
    - Long-run: $y_s s' > y_s s$, consumption depends on Golden rule and $\Delta\%Y' = n + g = \Delta\%Y$.

Consumption decision: 2 periods

(This exercise is intuitive more than algebraical, please follow the graph in the section)

Assume that the household utility of consumption is $U(C_1, C_2)$ and that the sequence of income is $\{Y_1, Y_2\}$

1. Describe the consumption decision if there is no financial market and the consumption good is perishable. $C_1 = Y_1$ and $C_2 = Y_2$
2. Assume that there is a bond market, bonds can be purchased at period 1. A unit of consumption can purchase a real bond that repays at the next period $1+r$ per unit. Describe the budget constraint and the utility maximization process.

$$MRT = MRS \Rightarrow 1 + r = \frac{U_1(C_1, C_2)}{U_2(C_1, C_2)}$$

It comes from

$$\max_{C_1, C_2, B} U(C_1, C_2) \text{ s.t. } C_1 + B = Y_1 \text{ and } C_2 = Y_2 + (1 + r)B \text{ or}$$

$$\max_{C_1, C_2} U(C_1, C_2) \text{ s.t. } C_1 + \frac{C_2}{1 + r} = Y_1 + \frac{Y_2}{1 + r}$$

Consumption decision: 2 periods

3. Assume that $U(C_1, C_2) = u(C_1) + \beta u(c_2)$ with $\beta \in (0,1)$ as the impatience factor (i.e., $\beta = 1/(1+i)$). Re-state the optimality condition. Is that expresion familiar to us?

$$1 + r = \frac{u_c(C_1)}{\beta u_c(C_2)} \Rightarrow u_c(C_1) = \beta u_c(C_2)(1+r)$$

4. Further assume that $u(C) = \frac{C^{1-\sigma}-1}{1-\sigma}$ with $\sigma > 0$ as the risk aversion coefficient. Re-state the optimality condition.

$$C_1^{-\sigma} = \beta C_2^{-\sigma}(1+r)$$

Consumption decision: ∞ periods

Find the Euler condition in

$$\max_{C,B} \sum_{i=0}^{\infty} \beta^i L_{t+1} A_{t+i}^{1-\sigma} \frac{c_t^{1-\sigma}-1}{1-\sigma} \text{ s.t.}$$

$$k_t^{\alpha} + (1-\delta)k_t = c_t + (1+g+n)k_{t+1}$$

Notice that $u_c(c_t) = L_t A_t^{1-\sigma} c_t^{-\sigma}$; therefore

$$MRT = MRS \Rightarrow \frac{\alpha k_{t+1}^{\alpha-1} + 1 - \delta}{1+n+g} = \frac{L_t A_t^{1-\sigma} c_t^{-\sigma}}{\beta L_{t+1} A_{t+1}^{1-\sigma} c_{t+1}^{-\sigma}}$$

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Consumption-labor decision: 2 periods

Assume a 2-period consumer described by

$$\max_{C_t, C_{t+1}, N_t, N_{t+1}} u(C_t) - \chi v(N_t) + \beta \left[u(C_{t+1}) - \chi v(N_{t+1}) \right] \text{ s.t.}$$

$$C_t + \frac{C_{t+1}}{1+r_{t+1}} = w_t N_t + \frac{w_{t+1} N_{t+1}}{1+r_{t+1}} \text{ with } u', v' > 0 \ \& \ u'' < 0, v'' > 0.$$

1. Find FOCs and provide an interpretation

$$\overbrace{u'(C_t)}^{\text{MC of saving}} = \overbrace{\beta u'(C_{t+1})(1+r_{t+1})}^{\text{MR from saving}} \qquad \text{(Savings)}$$

$$\underbrace{v'(N_t)}_{\text{MC of working}} = \underbrace{w_t u'(C_t)}_{\text{MR of working}} \qquad \text{(Labor)}$$

2. Describe the effects of a change in prices (i.e., changes in r or w).

$$u'(Y_t - B_t) = \beta \overbrace{u'(Y_{t+1} + (1 + \mathbf{r_{t+1}}) B_t)}^{\text{Wealth effect}} \overbrace{(1 + \mathbf{r_{t+1}})}^{\text{Substitution effect}}$$

$$v'(N_t) = \underbrace{\mathbf{w_t}}_{\text{Substitution effect}} \underbrace{u'(\mathbf{w_t} N_t - B_t)}_{\text{Wealth effect}}$$

Substitution effect: Opportunity cost changes.
Wealth effect: Wealth/income changes.

Understanding the RBC model

Assume a 2-period, 1-household and 1-firm economy given by
Household:

$$\max_{C_t, C_{t+1}, N_t, I_t} \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} + \beta \frac{C_{t+1}^{1-\sigma}}{1-\sigma} \text{ s.t.}$$

$$C_t + I_t = w_t N_t \text{ and } (1 + r_{t+1}) I_t = C_{t+1}$$

Firm:

$$\max_{N_t} Y_t - w_t N_t \equiv \max_{N_t} A_t N_t^{\alpha} - w_t N_t, \text{ and}$$

$$\max_{I_t} Y_{t+1} - (1 + r_{t+1}) I_t \equiv \max_{I_t} A_{t+1} I_t^{\alpha} - (1 + r_{t+1}) I_t$$

Market clearing conditions:

$$Y_t = C_t + I_t \text{ and } Y_{t+1} = C_{t+1}$$

3. Find the RBC model equations ans present a graphical interpretation.

Understanding the RBC model

(*Graphical interpretation is more relevant than the algebra itself*)

Household FOCs:

$$r_{t+1} = \frac{C_t^{-\sigma}}{\beta C_{t+1}^{-\sigma}} - 1, \qquad \text{(Investment supply)}$$

$$w_t = \frac{N_t^{\eta}}{C_t^{-\sigma}}. \qquad \text{(Labor supply)}$$

Firm FOCs:

$$w_t = \alpha A_t N_t^{\alpha-1}, \qquad \text{(Labor demmand)}$$

$$r_{t+1} = \alpha A_{t+1} I_t^{\alpha-1} - 1. \qquad \text{(Investment demand)}$$

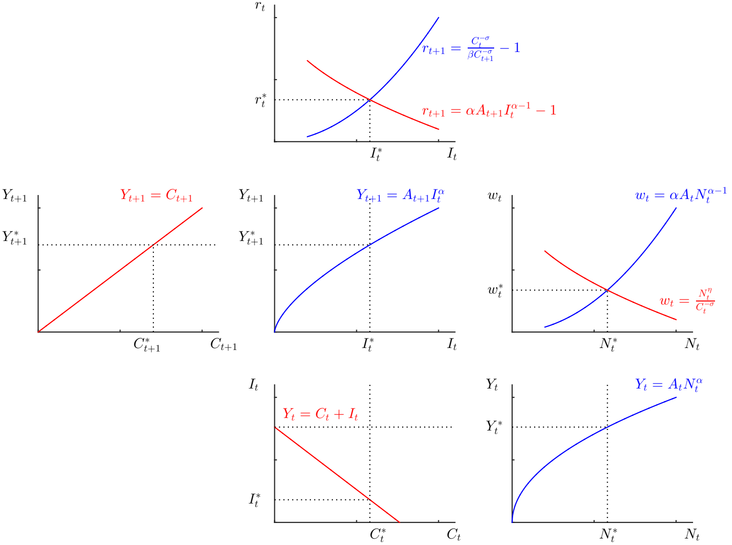
Market clearing conditions:

$$A_t N_t^{\alpha} = C_t + I_t, \qquad \text{(MC period 1)}$$

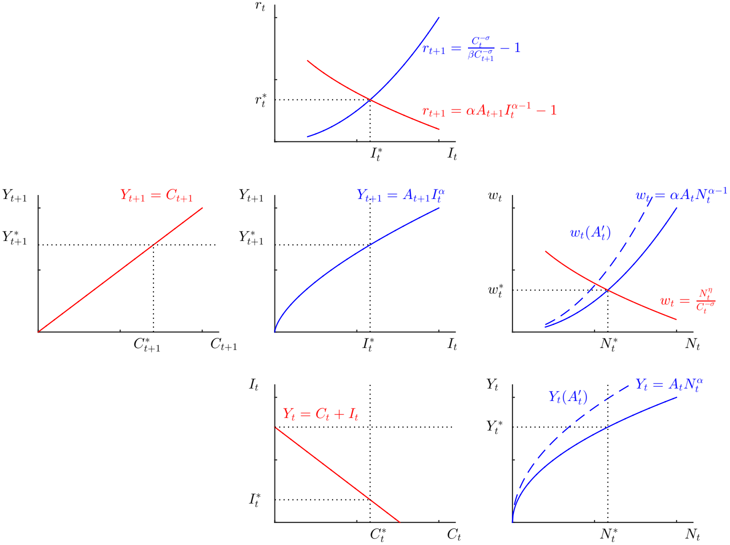
$$A_{t+1} I_t^{\alpha} = C_{t+1}. \qquad \text{(MC period 2)}$$

Endogenous variables: $\{C_t, C_{t+1}, N_t, I_t, r_{t+1}, w_t\}$
Exogenous variables: $\{A_t, A_{t+1}\}$
Coefficients: $\{\sigma, \beta, \eta, \alpha\}$

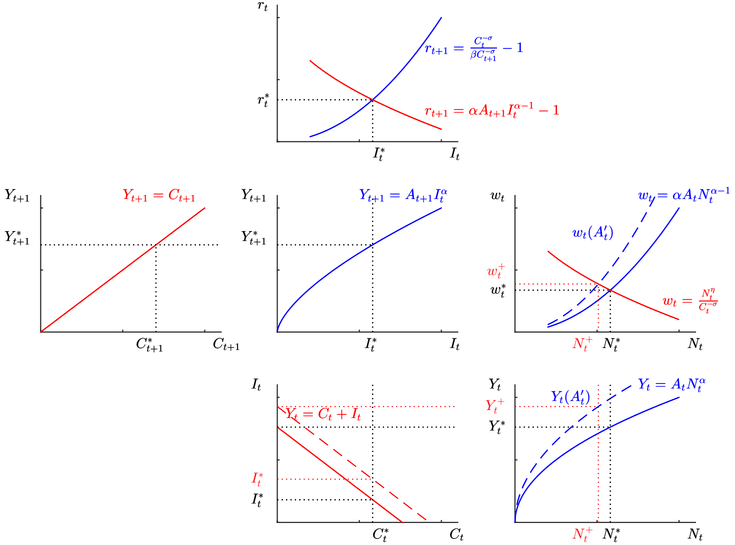
Understanding the RBC model



Understanding the RBC model: Shock to A_t (direct effect)



Understanding the RBC model: Shock to A_t (all effects)



Understanding the RBC model: Quick exercise

How would a shock to A_{t+1} affect this economy?

Dynare installation

- Download Dynare:
  http://www.dynare.org/download/dynare-stable
  Some instructions for Mac computers:
  http://www.dynare.org/DynareWiki/InstallOnMacOSX
- Double click the downloaded file, remember the installation directory. For example: "`C:\dynare\4.4.3`"
- Open Matlab
- Upload Dynare to your Matlab session, there are two ways:
  - Go to: "`Home\Set Path`" in the emerging window click the button "**`Add with Subfolders...`**", locate the installation directory ("`C:\dynare\4.4.3`"), select the folder "**`matlab`**" (only one click), and click the "**`Select Folder`**" button.
  - Type in the command window: "`addpath(genpath(C:\dynare\4.4.3\matlab))`"

Numerical simulation with Dynare

The RBC model to solve and simulate is

$$\widehat{y}_t = \alpha\widehat{k}_t + (1-\alpha)\widehat{n}_t + \widehat{z}_t \qquad \text{(Thecnology)}$$

$$\widehat{y}_t = C_y\widehat{c}_t + Y_k^{-1}\left(\widehat{k}_{t+1} - (1-\delta)\widehat{k}_t\right) \qquad \text{(Market clearing condition)}$$

$$\widehat{w}_t = \widehat{y}_t - \widehat{n}_t \qquad \text{(Labor demand)}$$

$$\widehat{r}_t = \alpha Y_k(\widehat{y}_t - \widehat{k}_t) \qquad \text{(Capital demand)}$$

$$\widehat{c}_t = E_t\widehat{c}_{t+1} - \frac{1}{\sigma}E_t\widehat{r}_{t+1} \qquad \text{(Euler equation)}$$

$$\widehat{w}_t = \eta\widehat{n}_t + \sigma\widehat{c}_t \qquad \text{(Labor supply)}$$

$$\widehat{z}_t = \rho\widehat{z}_{t-1} + \widehat{e}_t \qquad \text{(Persistent shock)}$$

In Dynare syntax

```
model(linear);
    y = alpha*k(-1) + (1-alpha)*n + z;
    y = C_Y*c + (1/Y_K)*(k-(1-delta)*k(-1));
    w = (y - n);
    r = alpha*Y_K*(y - k(-1));
    c = c(+1) - (1/sigma)*r(+1);
    w = eta*n + sigma*c;
    z = rho*z(-1)+e;
end;
```

Endogenous variables:
$\{\widehat{y}_t,\ \widehat{k}_t,\ \widehat{n}_t,\ \widehat{c}_t,\ \widehat{r}_t,\ \widehat{w}_t,\ \widehat{z}_t\}$
Exogenous variable:
$\{\widehat{e}_t\}$
Coefficients:
$\{\alpha,\ C_y,\ Y_k,\ \delta,\ \sigma,\ \eta,\ \rho,\ \sigma_e\}$

Numerical simulation with Dynare

In order to execute Dynare in Matlab, it is required to generate a text file (which extensions could be `*.mod` or `*.dyn`) with the following structure:

1. **Variables and parameter declaration**. `var`, `varexo` and `parameters` precede the list of endogenous variables, exogenous variables and model parameters, respectively:

```
var y c k n r w z;
varexo e;

parameters alpha delta rho sigma beta eta sde;
parameters R Y_K C_Y ;
```

2. **Model calibration**. The model calibration is introduced in Matlab format (separated by semicolons):

```
% Technology
alpha = 0.33; delta = 0.025; rho = 0.95; sde = 0.0033;
% Preferences
beta = 1/1.01; sigma = 1; eta = 2;
% Steady-state values
R   = 1/beta; Y_K = (1/alpha)*(R - 1 + delta);
C_Y = 1 - delta*(1/Y_K);
```

3. **The model equations**. As in the previous slide.

Numerical simulation with Dynare

4. **Initial values**. Values to initialize the simulation (commonly the steady-state):

```
initval;
  y = 0; k = 0; c = 0; r = 0; n = 0; z = 0; e = 0;
end;
```

5. **Shocks to simulate**. Declare the name and magnitude of shock to simulate:

```
shocks;
var e = sde;
end;
```

6. **Steady-state**. Verify the existence of the steady-state

```
steady;
```

7. **Simulate**. Simulate the model

```
%stoch_simul;
stoch_simul(dr_algo=0,periods=1000,irf=40); datatomfile('simudata',[]);
```

Reading errors (two types)

E1. **Syntax error**. Dynare file non-written properly. for instance: Undeclared variable or coefficient (Matlab is case-sensitive).

```
Starting Dynare (version 4.4.3).
Starting preprocessing of the model file ...
ERROR: rbc.dyn: line 44, col 37: Unknown symbol: z

Error using dynare (line 174)
DYNARE: preprocessing failed
```

Missing semicolon.

```
Starting Dynare (version 4.4.3).
Starting preprocessing of the model file ...
ERROR: rbc.dyn: line 25, cols 1-5: syntax error, unexpected NAME

Error using dynare (line 174)
DYNARE: preprocessing failed
```

Missing special instructions.

```
Starting Dynare (version 4.4.3).
Starting preprocessing of the model file ...
ERROR: rbc.dyn: line 67, col 7: syntax error, unexpected ';', expecting EQUAL

Error using dynare (line 174)
DYNARE: preprocessing failed
```

Reading errors (two types)

E2. **Model stability error**. Calibration leads to model instability.

```
Error using print_info (line 42)
Blanchard Kahn conditions are not satisfied: no stable equilibrium

Error in stoch_simul (line 98)
    print_info(info, options_.noprint, options_);

Error in rbc (line 167)
info = stoch_simul(var_list_);

Error in dynare (line 180)
evalin('base',fname) ;
```

Solutions

E1. (The easy one) Read the error message, locate the error in the file and correct the syntax.

E2. (Could be hard to solve) Change the calibration.

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Arrow-Pratt risk aversion

The Arrow-Pratt risk aversion or relative risk aversion (RRA) quantifies how much the household "dislike" uncertainty.

$$RRA(C) = -C\frac{u''(C)}{\sigma u'(C)}$$

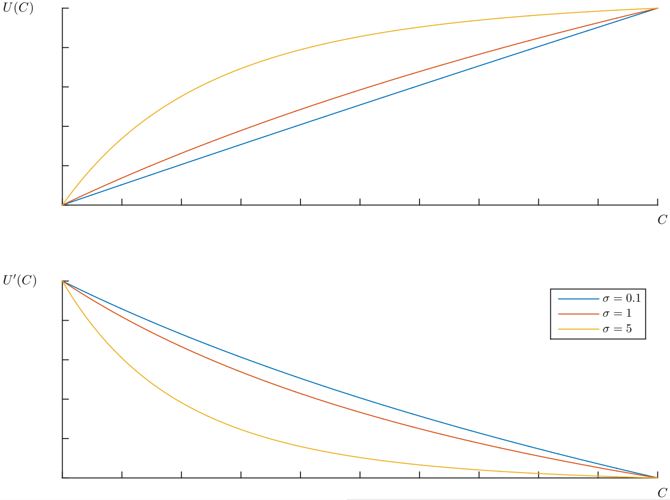
For instance, if $u(C) = \frac{C^{1-\sigma}}{1-\sigma}$ (i.e., the CES utility function); therefore

$$RRA(C) = \sigma$$

Large values of $RRA(C)$ imply "more convex" utility function, and "more convex" utility function imply more risk aversion.

Arrow-Pratt risk aversion

For instance, if $u(C) = \frac{C^{1-\sigma}}{1-\sigma}$; then



The role of concavity

Let's assume that a household have the oppotunity to purchase a lottery:

- The lottery cost is one unit of consumption and it has two equally-likely outcomes: win or lose.
- **Win**: household gets two units of consumption. **Lose**: nothing.

Preferences are $u(C) = \frac{C^{1-\sigma}}{1-\sigma}$, and there are three households with:

- Household 1: $\sigma > 0$, Household 2: $\sigma = 0$ and Household 3: $\sigma < 0$.

Can we predict which household would purchase (or not) the lottery?

The role of concavity

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- The lottery cost is one unit of consumption and it has two equally-likely outcomes: win or lose.
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Preferences are $u(C) = \frac{C^{1-\sigma}}{1-\sigma}$, and there are three households with:

- Household 1: $\sigma > 0$, Household 2: $\sigma = 0$ and Household 3: $\sigma < 0$.

Can we predict which household would purchase (or not) the lottery?
Notice that expected earnings are

$$E\{earnings\} = p(win)[2-1] + p(lose)[-1] = (1/2) \times 1 + (1/2) \times (-1) = 0.$$

However, **expected utility** is

$$E\{u(C)\} = p(win)u(C+2-1) + p(lose)u(C-1) = \frac{u(C+1) + u(C-1)}{2};$$

therefore, the household would purchase the lottery if

$$E\{u(C)\} > u(C) \equiv \frac{u(C+1) + u(C-1)}{2} > u(C).$$

The role of concavity

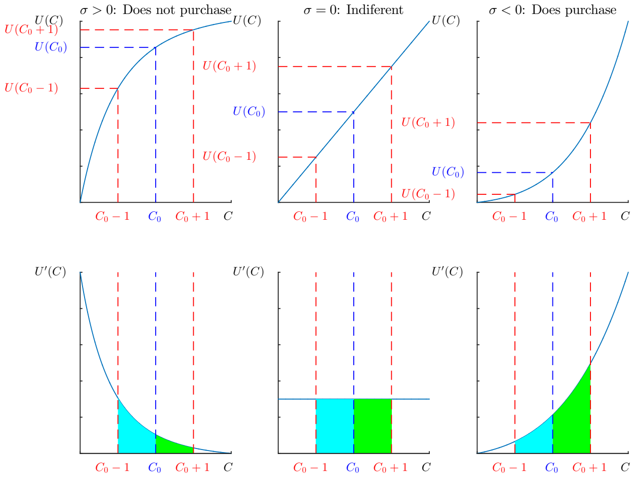
The last inequality could be written as

$$\frac{u(C+1)+u(C-1)}{2} > u(C) \Rightarrow [u(C+1)-u(C)] > [u(C)-u(C-1)]$$

The role of concavity

The last inequality could be written as

$$\frac{u(C+1)+u(C-1)}{2} > u(C) \Rightarrow [u(C+1)-u(C)] > [u(C)-u(C-1)]$$



Effects in the RBC model

Assume a 2-periods household that solves:

$$\max_{C_t, C_{t+1}, B_t} \frac{C_t^{1-\sigma}}{1-\sigma} + \beta E_t \frac{C_{t+1}^{1-\sigma}}{1-\sigma} \text{ s.t.}$$

$$C_t + B_t = Y_t \text{ and } C_{t+1} = Y_{t+1} + (1 + r_t) B_t$$

where $\beta = 0.8$, $\sigma = 1$ and $r_t = 0.25$

1. Consider $Y_t = 100$ and $Y_{t+1} = 100$ without uncertanty. How much would the household save at moment t.
2. Consider $Y_t = 100$ and $E_t Y_{t+1} = 100$ with uncertanty. Uncertanty: the exact Y_{t+1} is unckn own; however, it is known that with probability 0.5, outcome is $Y_{\ell,t+1} = 90$ and with probability 0.5, outcome is $Y_{h,t+1} = 110$ (i.e. $E_t Y_{t+1} = 0.5 Y_{\ell,t+1} + 0.5 Y_{h,t+1} = 100$). How much would this household save at moment t.

Effects in the RBC model

Assume a 2-periods household that solves:

$$\max_{C_t, C_{t+1}, B_t} \frac{C_t^{1-\sigma}}{1-\sigma} + \beta E_t \frac{C_{t+1}^{1-\sigma}}{1-\sigma} \text{ s.t.}$$

$$C_t + B_t = Y_t \text{ and } C_{t+1} = Y_{t+1} + (1+r_t)B_t$$

where $\beta = 0.8$, $\sigma = 1$ and $r_t = 0.25$

1. Consider $Y_t = 100$ and $Y_{t+1} = 100$ without uncertanty. How much would the household save at moment t.
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FOCs

$$\left.\begin{array}{l} C_t^{-\sigma} = \beta(1+r_t)E_t C_{t+1}^{-\sigma} \\ Y_t = C_t + B_t \\ Y_{t+1} = C_{t+1} - (1+r_t)B_t \end{array}\right\} [Y_t - B_t]^{-\sigma} = \beta(1+r_t)E_t\left[Y_{t+1} + (1+r_t)B_t\right]^{-\sigma}$$

Effects in the RBC model

1. Consider $Y_t = 100$ and $Y_{t+1} = 100$ without uncertanty. How much would the household save at moment t.

Effects in the RBC model

1. Consider $Y_t = 100$ and $Y_{t+1} = 100$ without uncertanty. How much would the household save at moment t.

$$[100 - B_t]^{-1} = 0.8(1 + 0.25)\,[100 + (1 + 0.25)B_t]^{-1} \Rightarrow B_t = 0$$

2. Consider $Y_t = 100$ and $E_t Y_{t+1} = 0.5Y_{\ell,t+1} + 0.5Y_{h,t+1} = 100$. How much would this household save at moment t.

Effects in the RBC model

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$$[Y_t - B_t]^{-\sigma} = \beta(1+r)\left(0.5\,[Y_{\ell,t+1} + (1+r)B_t]^{-\sigma} + 0.5\,[Y_{h,t+1} + (1+r)B_t]^{-\sigma}\right)$$
$$[100 - B_t]^{-1} = 0.8(1+0.25)\left(0.5\,[90 + (1+0.25)B_t]^{-1} + 0.5\,[110 + (1+0.25)B_t]^{-1}\right.$$
$$\Rightarrow B_t = 0.4420;$$

3. What if $Y_{\ell,t+1} = 50$ and $Y_{h,t+1} = 150$.

Effects in the RBC model

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$$[Y_t - B_t]^{-\sigma} = \beta(1+r)\left(0.5\left[Y_{\ell,t+1} + (1+r)B_t\right]^{-\sigma} + 0.5\left[Y_{h,t+1} + (1+r)B_t\right]^{-\sigma}\right)$$
$$[100 - B_t]^{-1} = 0.8(1+0.25)\left(0.5\left[90 + (1+0.25)B_t\right]^{-1} + 0.5\left[110 + (1+0.25)B_t\right]^{-1}\right.$$
$$\Rightarrow B_t = 0.4420;$$

3. What if $Y_{\ell,t+1} = 50$ and $Y_{h,t+1} = 150$.

$$\Rightarrow B_t = 9.89$$

Example with social-planner RBC

Consider the social-planner RBC model:

$$\begin{aligned}
\widehat{y}_t =& \alpha \widehat{k}_t + (1-\alpha)\widehat{n}_t + \widehat{z}_t && \text{(Thecnology)} \\
\widehat{y}_t =& C_y \widehat{c}_t + Y_k^{-1}\left(\widehat{k}_{t+1} - (1-\delta)\widehat{k}_t\right) && \text{(Aggregation)} \\
\widehat{c}_t =& E_t \widehat{c}_{t+1} - \frac{\alpha}{\sigma} Y_k (E_t \widehat{y}_{t+1} - \widehat{k}_{t+1}) && \text{(Euler)} \\
\widehat{y}_t =& (1+\eta)\widehat{n}_t + \sigma \widehat{c}_t && \text{(Labor)} \\
\widehat{z}_t =& \rho \widehat{z}_{t-1} + \widehat{e}_t && \text{(Persistent shock)}
\end{aligned}$$

Endogenous variables:
$\{\widehat{y}_t,\ \widehat{k}_t,\ \widehat{n}_t,\ \widehat{c}_t,\ \widehat{z}_t\}$
Exogenous variable:
$\{\widehat{e}_t\}$
Coefficients:
$\{\alpha,\ C_y,\ Y_k,\ \delta,\ \sigma,\ \eta,\ \rho,\ \sigma_e\}$

Example with social-planner RBC

1. Simulate the above model in Dynare. See: rbc_sp.dyn
2. (Similar to Q4) Modify the program to add two a new variable equal to real wage. See: rbc_sp_mod.dyn
3. (Similar to Q4a) Find the std of the real wage and and compare it against the std of output. See: rbc_sp_mod.dyn - Section: Q3
4. (Similar to Q4b) Reduce the shock persistence to 0.95 to 0.5 and repeat the question above. See: rbc_sp_mod.dyn - Section: Q4
5. (Similar to Q4c) Plot the output IRF for the first and the second simulation in a single graph. See: Single_IRF_ForQ5slides.m
6. (Similar to Q5) Set the persistence in 0.95 again and change the household risk aversion from 1.5 to 0.5, 1 and 10. Plot the three IRFs of output, consumption and wages (one graph per variable). See: rbc_sp_mod.dyn - Section: Q6 + file: Single_IRF_ForQ6slides.m

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Go in details over some Midterm review questions.

- RBC: Question 7.
- NK: Question 9.
- NK: Question 10.
- (If time allows) RBC: Question 5.
- (If time allows) RBC: Question 6.
- (If time allows) NK: Question 8.
- (If time allows) As many other questions as possible.

Consider the linearized equilibrium conditions:

$$
\begin{aligned}
y_t &= (1-\alpha)n_t + z_t && \text{Technology (1)} \\
y_t - n_t &= w_t && \text{Labor demand (2)} \\
\eta n_t + \sigma c_t &= w_t && \text{Labor supply (3)} \\
y_t &= c_t && \text{Market clearing (4)}
\end{aligned}
$$

(a) Explain what each of these equilibrium conditions represents.

(b) Find the equilibrium value of y_t as function of z_t.

(c) Find the equilibrium values of n_t and c_t as function of z_t.

(d) Explain how y_t, n_t an w_t are affected by a negative productivity shock if $\sigma < 1$.

(e) Consider the special case in which $\alpha = 0$. How would your answers to part (d) be affected if $\sigma > 1$? Explain.

(a) Check equation tags in previous slide.

(b) (4) in (3): $\eta n_t + \sigma y_t = w_t$ (5)
(5) in (2): $n_t = \frac{1-\sigma}{1+\eta} y_t$ (6)
(6) in (1): $y_t = \frac{1+\eta}{\eta+\sigma+\alpha(1-\sigma)} z_t$ (7)

(c) (7) in (6): $n_t = \frac{1-\sigma}{\eta+\sigma+\alpha(1-\sigma)} z_t$ (8)
(7) and (8) in (2): $w_t = \frac{\eta+\sigma}{\eta+\sigma+\alpha(1-\sigma)} z_t$ (9)

(d) If $0 < \sigma < 1$ then all coefficients multiplying z_t in (7), (8) and (9) are positive $\Rightarrow$ a negative z_t reduces y_t, n_t and w_t.

(e) If $\sigma > 1$ and
(7) with $\alpha = 0$: $y_t = \frac{1+\eta}{\eta+\sigma} z_t \Rightarrow \frac{\partial y_t}{\partial z_t} > 0$ (10)
(8) with $\alpha = 0$: $n_t = \frac{1-\sigma}{\eta+\sigma} z_t \Rightarrow \frac{\partial n_t}{\partial z_t} < 0$ (11)
(9) with $\alpha = 0$: $w_t = \frac{\eta+\sigma}{\eta+\sigma} z_t \Rightarrow \frac{\partial w_t}{\partial z_t} > 0$ (12)
$\therefore$ a negative z_t reduces y_t and w_t but increases n_t.

Assume that $E_t e_{t+j} = E_t r^n_{t+j} = 0$ and consider:

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r^n_t) \qquad \text{Dynamic IS (1)}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t \qquad \text{Phillips curve (2)}$$

$$i_t = r^n_t + \phi \pi_t \qquad \text{Policy rule (3)}$$

(a) Solve for x_t and π_t.

(b) Why r^n_t does not affect x_t or π_t.

(c) How the effects of e_t on x_t and π_t are affected by the CB's choice of ϕ.

(d) What do (1) and (2) represent?

(a) (3) in (1): $x_t = -\frac{\phi}{\sigma}\pi_t$ (4)

(4) in (2): $\boxed{\pi_t = \frac{\sigma}{\sigma+\kappa\phi} e_t}$ (5)

(5) in (4): $\boxed{x_t = -\frac{\phi}{\sigma+\kappa\phi} e_t}$ (6)

(b) The CB can perfectly predict r_t^n; therefore, it can offset any variation in x_t (as a consequence of a shock in r_t^n) by changing the interest rate accordingly.

(c) In (5): $|\frac{\partial \pi_t}{\partial e_t}|$ is reduced $\Rightarrow$ the effects of the shock are diminished when ϕ rises.
In (6): $|\frac{\partial x_t}{\partial e_t}|$ is increased $\Rightarrow$ the effects of the shock are magnified when ϕ rises.

(d) (1): Household decision $\rightarrow$ Aggregated demand, Euler equation or dynamic IS.
(2): Firm decision $\rightarrow$ Aggregated supply, or dynamic Phillips curve.

Assume that $E_t e_{t+j} = E_t r^n_{t+j} = 0$ and and consider:

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r^f_t) \qquad \text{Dynamic IS (1)}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t \qquad \text{Phillips curve (2)}$$

$$i_t = r^{ff}_t + \phi \pi_t \qquad \text{Policy rule (3)}$$

where r^{ff}_t is a CB's imperfect prediction of r^f_t.

(a) Solve for x_t and π_t.

(b) How r^f_t affects x_t or π_t if it is not well predicted.

(c) How the effects of e_t and r^f_t on x_t and π_t are affected by the CB's choice of ϕ.

(a) Define $err_t^f \equiv r_t^{ff} - r_t^f$, then

(3) in (1): $x_t = -\frac{1}{\sigma}(err_t^f + \phi\pi_t)$ (4)

(4) in (2): $\boxed{\pi_t = -\frac{\kappa}{\sigma+\kappa\phi} err_t^f + \frac{\sigma}{\sigma+\kappa\phi} e_t}$ (5)

(5) in (4): $\boxed{x_t = -\frac{1}{\sigma+\kappa\phi} err_t^f - \frac{\phi}{\sigma+\kappa\phi} e_t}$ (6)

(b) In (5): $\frac{\partial \pi_t}{\partial err_t^f} < 0$

In (6): $\frac{\partial x_t}{\partial err_t^f} < 0$

Both are reduced if $err_t^f > 0$.

(c) In (5): $|\frac{\partial \pi_t}{\partial e_t}|$ is reduced $\Rightarrow$ the effects of the shock are diminished when ϕ rises.

In (6): $|\frac{\partial x_t}{\partial e_t}|$ is increased $\Rightarrow$ the effects of the shock are magnified when ϕ rises.

In (5): $|\frac{\partial \pi_t}{\partial err_t}|$ is reduced $\Rightarrow$ the effects of the shock are diminished when ϕ rises.

In (6): $|\frac{\partial x_t}{\partial err_t}|$ is reduced $\Rightarrow$ the effects of the shock are diminished when ϕ rises.

A characteristic of business cycles is persistence. Does the effects of a productivity shock on investment and the evolution of the capital stock provide a means for generating persistent effects of temporary productivity shocks?

Yes, There is capital inertial because capital accumulation takes time.

Does this channel seem quantitatively important? If not, what aspect of a basic real business model's calibration accounts for persistence.

No.

δ for endogenous persistence (which is small) and ρ for exogenous persistence.

A characteristic of business cycles in the U.S. is that employment varies a lot while real wages do not. The basic RBC model developed in lecture in which productivity shocks are the source of fluctuations can match one of these characteristics but not both. Suppose the wage elasticity of labor supply is given by γ ($1/\eta$ in lecture notes). Which characteristic is matched if it is large? Which is matched if small?

Large γ causes labor supply to be flatter; therefore, a shift up of labor demand (productivity shock) leads to a new equilibrium with much more employment and not much more wages.

Over the NK model in question 9

(a) Describe what each of these three equations represents? Whose decisions to they represent?

(b) What problems might arise in this model if $\phi < 1$?

Solution

(a) See equation tags and solution to part (d) of problem 9.

(b) The model becomes dynamically unstable: number of forward looking variables is larger than the number of explosive roots.

Random between 1 and 4

https://www.random.org/integers/?num=1&min=1&max=4&col=1&base=10&format=html&rnd=new

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Comments on the Midterm

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PS3: Topics

Q1. Model stability.

Q2. Cost of inflation.

Q3. Optimal policy under discretion.

Q4. Optimal policy under commitment.

Q5. Dynare: IRF in NK model with different policy rules.

Questions?

Any burning question on the assignment/lectures?

Today:

1. (In details) Similar exercise to Q3.
2. (In details) Similar exercise to Q4.
3. (Some details) Similar exercise to Q5.
4. (If time allows it) Help on Q2.

Similar to Q3

Consider:

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r_t^f) \qquad \text{Dynamic IS (3.1)}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t \qquad \text{Phillips curve (3.2)}$$

and a CB that minimices

$$\frac{1}{2} E_t \sum_{i=1}^{\infty} \beta^i \left[\pi_{t+i}^2 + \lambda x_{t+i}^2\right] \qquad \text{CB loss function (3.3)}$$

(a) Find FOCs for a discretionary CB.

(b) Assume $E_t \pi_{t+1} = \gamma x_t$ and repeat (a) (Please, notice that this assumption is **different from the problem set**).

(c) Explain using graphs.

Similar to Q3

(a) Lagrangian: Max (3.3) subject to (3.1) and (3.2)

$$\mathcal{L} = E_t \sum_{i=1}^{\infty} \beta^i \begin{bmatrix} \frac{1}{2}\left(\pi_{t+i}^2 + \lambda x_{t+i}^2\right) + \\ \ell_{x,t+i}\left(x_{t+i+1} - \frac{1}{\sigma}(i_{t+i} - \pi_{t+i+1} - r_{t+i}^f) - x_{t+i}\right) + \\ \ell_{\pi,t+i}\left(\beta\pi_{t+i+1} + \kappa x_{t+i} + e_{t+i} - \pi_{t+i}\right) \end{bmatrix} \tag{3.3}$$

- Discretionary CB: 'future' taken as given

$$\mathcal{L} = E_t \begin{bmatrix} \frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right) + \\ \ell_{x,t}\left(-\frac{1}{\sigma}(i_t - r_t^f) - x_t\right) + \\ \ell_{\pi,t}\left(\kappa x_t + e_t - \pi_t\right) \end{bmatrix} + E_t Future \tag{3.4}$$

- $\pi_t: \ \pi_t - \ell_{\pi,t} = 0 \Rightarrow \pi_t = \ell_{\pi,t}$ (3.5)
- $x_t: \ \lambda x_t - \ell_{x,t} + \kappa \ell_{\pi,t} = 0 \Rightarrow x_t + \kappa \ell_{\pi,t} = \ell_{x,t}$ (3.6)
- $i_t: \ -\frac{1}{\sigma}\ell_{x,t} = 0 \Rightarrow \ell_{x,t} = 0$ (i.e. not binding) (3.7)
- (3.7) and (3.5) in (3.6): $\boxed{\lambda x_t + \kappa \pi_t = 0}$ (3.8)

Similar to Q3

(b) Since restriction (3.1) is not binding (i.e. $\ell_{x,t} = 0$), it can be taken out of the optimization. For a discretionary CB, the Lagrangian in (3.3) can be reduced to:

$$\mathcal{L} = E_t \begin{bmatrix} \frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right) \\ \ell_{\pi,t}\left(\beta\gamma x_t + \kappa x_t + e_t - \pi_t\right) \end{bmatrix} + E_t Future \tag{3.9}$$

- $\pi_t: \ \pi_t + \ell_{\pi,t} = 0 \Rightarrow \pi_t = \ell_{\pi,t}$ (3.10)
- $x_t: \ \lambda x_t + (\kappa + \beta\gamma)\ell_{\pi,t} = 0 \Rightarrow x_t + (\kappa + \beta\gamma)\ell_{\pi,t} = 0$ (3.11)
- (2.9) in (2.10): $\boxed{\lambda x_t + (\kappa + \beta\gamma)\pi_t = 0}$ (3.12)

(c) On the blackboard: Phillips (3.2) Vs Rule (3.8) or (3.12). Assume e_t is not persistent.

Similar to Q4

Consider (3.1) and (3.2) and a CB that minimices (3.3)

(a) Find FOCs for a fully committed CB.

(a) Lagrangian: Max (3.3) subject to (3.1) and (3.2)

$$\mathcal{L} = E_t \sum_{i=1}^{\infty} \beta^i \begin{bmatrix} \frac{1}{2}\left(\pi_{t+i}^2 + \lambda x_{t+i}^2\right) + \\ \ell_{x,t+i}\left(x_{t+i+1} - \frac{1}{\sigma}(i_{t+i} - \pi_{t+i+1} - r_{t+i}^f) - x_{t+i}\right) + \\ \ell_{\pi,t+i}\left(\beta\pi_{t+i+1} + \kappa x_{t+i} + e_{t+i} - \pi_{t+i}\right) \end{bmatrix} \tag{4.1}$$

- $\pi_t: \;\; \pi_t - \ell_{\pi,t} = 0 \Rightarrow \pi_t = \ell_{\pi,t}$ (4.2)
- $x_t: \;\; \lambda x_t - \ell_{x,t} + \kappa\ell_{\pi,t} = 0 \Rightarrow x_t + \kappa\ell_{\pi,t} = \ell_{x,t}$ (4.3)
- $i_{t+i}: \;\; -E_t\beta^i \frac{1}{\sigma}\ell_{x,t+i} = 0 \Rightarrow E_t\ell_{x,t+i} = 0$ (i.e. not binding $\forall i$) (4.4)
- (4.4) and (4.2) in (4.3): $\boxed{\lambda x_t + \kappa\pi_t = 0}$ (4.5)

Similar to Q4

(a) Continue ...

$$\mathcal{L} = E_t \sum_{i=1}^{\infty} \beta^i \begin{bmatrix} \frac{1}{2}\left(\pi_{t+i}^2 + \lambda x_{t+i}^2\right) + \\ \ell_{x,t+i}\left(x_{t+i+1} - \frac{1}{\sigma}(i_{t+i} - \pi_{t+i+1} - r_{t+i}^f) - x_{t+i}\right) + \\ \ell_{\pi,t+i}\left(\beta\pi_{t+i+1} + \kappa x_{t+i} + e_{t+i} - \pi_{t+i}\right) \end{bmatrix} \quad (4.1)$$

- $\pi_{t+i}: \;\; E_t\beta^i\left(\pi_{t+i} - \ell_{\pi,t+i}\right) + E_t\beta^{i-1}\left(\frac{1}{\sigma}\ell_{x,t+i-1} + \beta\ell_{\pi,t+i-1}\right) = 0$
  $\Rightarrow E_t\pi_{t+i} + \frac{1}{\beta\sigma}E_t\ell_{x,t+i-1} + E_t\ell_{\pi,t+i-1} - E_t\ell_{\pi,t+i} = 0 \quad (4.6)$
- $x_{t+i}: \;\; E_t\beta^i\left(\lambda x_{t+i} - \ell_{x,t+i} + \kappa\ell_{\pi,t+i}\right) + E_t\beta^{i-1}\left(\ell_{x,t+i-1}\right) = 0$
  $\Rightarrow \lambda E_t x_{t+i} + \kappa E_t\ell_{\pi,t+i} = E_t\ell_{x,t+i} - \beta^{-1}E_t\ell_{x,t+i-1} \quad (4.7)$
- (4.4) in (4.7): $E_t\ell_{\pi,t+i} = -\frac{\lambda}{\kappa}E_t x_{t+i} \quad (4.8)$
- (4.4) and (4.8) in (4.6): $\boxed{\lambda\left(E_t x_{t+i} - E_t x_{t+i-1}\right) + \kappa E_t\pi_{t+i} = 0} \quad (4.9)$

Somewhat similar to Q5

The dynare program `NKM_s7.dyn` solves

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r_t^f) \qquad \text{Dynamic IS (5.1)}$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \qquad \text{Phillips curve (5.2)}$$
$$i_t = r_t^f + \phi_\pi \pi_t + \phi_x x_t + v_t \qquad \text{Taylor rule (5.3)}$$
$$r_t^f = \rho_r r_{t-1}^f + e_{r,t} \qquad \text{Demand shock (5.4)}$$
$$u_t = \rho_u u_{t-1} + e_{u,t} \qquad \text{Cost shock (5.5)}$$
$$v_t = \rho_v v_{t-1} + e_{v,t} \qquad \text{Policy shock (5.6)}$$

(a) Modify the Dynare file to exclude x_t from the Taylor rule.
(b) Explain IRFs for all shocks in: (i) an economy where the Taylor rule reacts to π and x, and (ii) an if the Taylor rule reacts only to π.

Please find in my website the Dyn-file `NKM_s7.dyn`. The solution for this question is posted in two files:
Dyn-file in `NKM_s7mod.dyn` and
M-file in `Get_Nice_Graphs.m`.
Understanding `NKM_s7mod.dyn` is more than enough for PS3. I will not spend much time explaining `Get_Nice_Graphs.m`.

Somewhat similar to Q5

(a) Add new coefficient: `Include_x,`

Modify Taylor rule to: `i = rn + phi_pi*pi+Include_x*phi_x*x+v;`

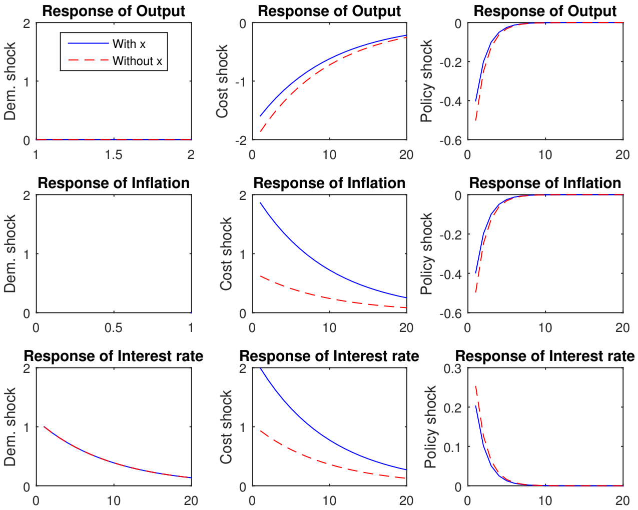
Run simulations:

```
Include_x=1;
steady;
check;
stoch_simul(dr_algo=0,irf=20) x pi i;
With_x = oo_;
save With_x.mat With_x;

Include_x=0;
steady;
check;
stoch_simul(dr_algo=0,irf=20) x pi i;
Without_x = oo_;
save Without_x.mat Without_x;
```

Somewhat similar to Q5

(b) I used `Get_Nice_Graphs.m` to generate the following graphs:



Somewhat similar to Q5

Shock to $\mathbf{r_t^f}$

x_t in Taylor rule:
The CB perfectly offset the effects of the shock.
x_t out of Taylor rule:
The CB perfectly offset the effects of the shock.

Shock to $\mathbf{u_t}$

x_t in Taylor rule:
1. Shock $\uparrow mc_t \Rightarrow \uparrow \pi_t$ and $\uparrow E_t\pi_t$.
2. Household $\downarrow y_t^d \Rightarrow$ firms $\downarrow y_t^s$ and $\downarrow n_t^d$.
3. CB $\uparrow i_t$ ($\uparrow \pi_t$ but $\downarrow x_t$)$\Rightarrow$ ($\downarrow x_t, \downarrow E_t\pi_t$).

x_t out of Taylor rule:
1. Shock $\uparrow mc_t \Rightarrow \uparrow \pi_t$ (markup) and $\uparrow E_t\pi_t$.
2. Household $\downarrow y_t^d \Rightarrow$ firms $\downarrow y_t^s$ and $\downarrow n_t^d$.
3. CB $\uparrow i_t$ (since $\uparrow \pi_t$)$\Rightarrow$ ($\downarrow\downarrow x_t, \downarrow\downarrow E_t\pi_t$).

i.e., In the first case the CB reduces the negative response of x_t, at the cost of a larger and more persistent π_t.

Shock to $\mathbf{v_t}$

x_t in Taylor rule:
1. Household $\downarrow y_t^d \Rightarrow$ firms $\downarrow y_t^s$ and $\downarrow n_t^d$.
2. $\downarrow x_t$ leads to $\downarrow \pi_t$
3. CB follows the persistent shock, but it is diminished by the i_t response to $\downarrow \pi_t$ and $\downarrow x_t$ ($v_0 = 1$ but $i_0 \approx 0.2$).

x_t out of Taylor rule:
1. Household $\downarrow y_t^d \Rightarrow$ firms $\downarrow y_t^s$ and $\downarrow n_t^d$.
2. $\downarrow x_t$ leads to $\downarrow \pi_t$
3. CB follows the persistent shock, but it is diminished by the i_t response to $\downarrow \pi_t$ only ($v_0 = 1$ but $i_0 \approx 0.25$).

Help on Q2

(Incomplete answer)

Why is inflation costly in the NK model?

Welfare cost: For the same intensity of labor, the household consume less than total production if there is inflation.

- Household demand for variety i: $c_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} C_t$ (2.1)
- Household total demand: $\int_0^1 c_t(i)di = C_t \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta} di$ (2.2)
- Market clearing: aggregated supply = aggregated demand; therefore $Y_t = C_t \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta} di$ (2.3)
- $\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta} di \begin{cases} = 1 & \text{if } \pi_t = 0 \\ > 1 & \text{if } \pi_t \neq 0 \end{cases}$ (2.4)

$\Rightarrow$ $\boxed{Y_t = Z_t N_t <,>,= C_t}$ ... complete the answer yourself on how N_t and C_t affect household's welfare given a level of Y_t.

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Comments on the Midterm

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202 - Winter 2016
Working on PS4

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PS3: Topics

Q1. FRED+Excel or FRED+Matlab: Evidence on NK model.

Q2. ZLB.

Q3. Dynare: NK + term structure.

Q4. Financial friction: Costly verification.

Q5. Financial friction: Moral hazard.

Questions?

Any burning question on assignments/lectures?

Today:

1. (In details) Similar exercise to Q1.
2. (In details) Similar exercise to Q2.

Similar to Q1

Using the Excel "`FREDdata.xlsx`" do the following

(a) Calculate:

$$x_t = \log GDP_t - \log GDP_t^{pot} \text{ and} \quad (1)$$

$$\widehat{\pi}_t = [(\log PCE_t - \log PCE_{t-4}) - 0.02] \times 100 \quad (2)$$

(b) Scatter plot of x_t Vs. $\widehat{\pi}_t$ for 1980-2004, 2005-2007 and 2008-2016.

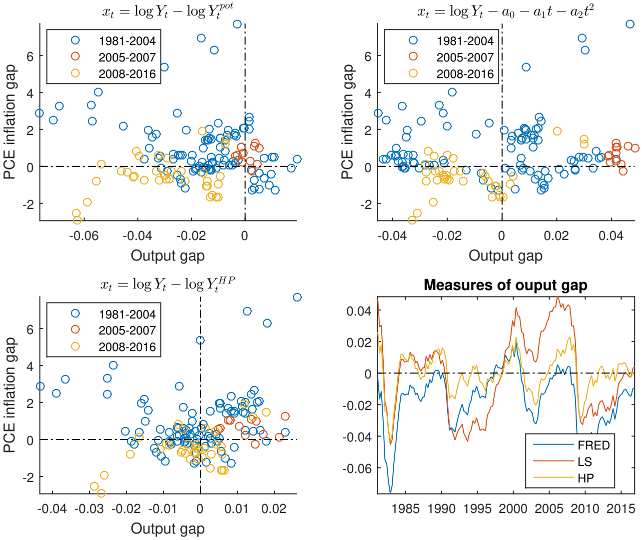
(c) Repeat (b), but

 (i) Change x_t by $x_t^{ls} = \log GDP_t - (\widehat{a}_0 + \widehat{a}_1 t + \widehat{a}_2 t^2)$, where $(\widehat{a}_0, \widehat{a}_1, \widehat{a}_2)$ are the LS estimates of $\log GDP_t$ over $(1, t, t^2)$.

 (ii) Change x_t by $x_t^{hp} = \log GDP_t - \log GDP_t^{HPTrend}$, where $\log GDP_t^{HPTrend}$ is the HP-trend of $\log GDP_t$ with weight $\lambda = 1400$.

Solution in Excel:
`FREDdata_S8sol.xlsx`. This requires the installation of the excel Add-In `HPFilter.xla` which was developed by Kurt Annen and is available to download at `http://www.web-reg.de/hp_addin.html`.
Solution in Matlab:
Master file `S8_Q1sol.m`. This program calls two auxiliary functions `LoadData.m` and `HP.m`.

Similar to Q1



Similar to Q2

Consider the following NK model

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left[i_t - E_t \pi_{t+1} - r_t^f\right] \quad \text{Dynamic IS (2.1)}$$

$$\pi_t = \beta \pi_{t+1} + \kappa x_t \quad \text{Phillips curve (2.2)}$$

$$i_t = \begin{cases} r_t^f + g(\pi_t, x_t), \text{ if } r_t^f > 0 \\ 0, \text{ if } r_t^f \leq 0 \end{cases} \quad \text{Policy rule (2.3)}$$

where the shock r_t^f follows a Markovian process

$$\begin{cases} r_t^f = r^f + u_t \text{ were } u_t \sim \mathcal{N}(0, \sigma_u) \text{ with probability } 1 - q \\ r_t^f = r^z + u_t \text{ were } u_t \sim \mathcal{N}(0, \sigma_u), \text{ with probability } q \end{cases} \quad (2.4)$$

with $r^f > 0$ and $r^z << 0$.
Let's assume that in the regime out of the ZLB, the policy rule $i_t = r_t^f + g(\pi_t, x_t)$ leads to $x_t = 0 = \pi_t$.

(a) Solve for x_t^z and π_t^z.

(b) Explains the effects of a negative demand shock u_t in and out of the ZLB.

Similar to Q2

(a) In the ZLB: $i_t = 0$ and $r_t^f = r^z + u_t$. (2.5)

(2.5) in (2.1): $x_t^z = E_t x_{t+1}^z + \frac{1}{\sigma}\left[E_t \pi_{t+1}^z + r^z + u_t\right]$. (2.6)

Take the following conjecture w.r.t. the solution

$x_t^z = A_x r^z + B_x u_t$ and $\pi_t^z = A_\pi r^z + B_\pi u_t$; (2.7)

therefore, $E_t x_{t+1}^z = q A_x r^z$ and $E_t \pi_t^z = q A_\pi r^z$. (2.8)

(2.7) and (2.8) in (2.6): $A_x r^z + B_x u_t = q A_x r^z + \frac{1}{\sigma}\left[q A_\pi r^z + r^z + u_t\right]$

$\Rightarrow (1-q)A_x - \frac{q}{\sigma}A_\pi = \frac{1}{\sigma}$ and $B_x = \frac{1}{\sigma}$. (2.9)

(2.7) and (2.8) in (2.2): $A_\pi r^z + B_\pi u_t = \beta q A_\pi r^z + \kappa\left(A_x r^z + B_x u_t\right)$

$\Rightarrow (1-\beta q)A_\pi = \kappa A_x$ and $B_\pi = \kappa B_x$. (2.10)

Combining (2.9) with (2.10) leads to

$A_x = \frac{1-\beta q}{\sigma(1-q)(1-\beta q)-q\kappa}$, $A_\pi = \frac{\kappa}{\sigma(1-q)(1-\beta q)-q\kappa}$, $B_x = \frac{1}{\sigma}$ & $B_\pi = \frac{\kappa}{\sigma}$ (2.11)

(2.11) in (2.7):

$$\boxed{x_t^z = \frac{1-\beta q}{\sigma(1-q)(1-\beta q)-q\kappa} r^z + \frac{1}{\sigma} u_t} \tag{2.12}$$

$$\boxed{\pi_t^z = \frac{\kappa}{\sigma(1-q)(1-\beta q)-q\kappa} r^z + \frac{\kappa}{\sigma} u_t} \tag{2.13}$$

Similar to Q2

(b) Effects of $u_t < 0$.

- Out of the ZLB:
  $x_t = 0 = \pi_t$ regardless of the shock, because $i_t = r_t^f + g(\pi_t, x_t)$.
- In the ZLB:
  From (2.12): $\frac{\partial x_t}{\partial u_t} = \frac{1}{\sigma} > 0 \Rightarrow \downarrow u_t \rightarrow \downarrow x_t^z$
  From (2.13): $\frac{\partial \pi_t}{\partial u_t} = \frac{\kappa}{\sigma} > 0 \Rightarrow \downarrow u_t \rightarrow \downarrow x_t^z$.

Extra: In order to show this result graphically, lets combine (2.7) with (2.8) and (2.11) to get $E_t x_{t+1}^z = q x_t^z - \frac{q}{\sigma} u_t$ and $E_t \pi_{t+1}^z = q \pi_t^z - \frac{q\kappa}{\sigma} u_t$ (2.14)

(2.14) in (2.6): $\pi_t^z = -\frac{1}{q} r^z + \sigma \left(\frac{1}{q} - 1 \right) x_t^z - \left(\frac{1}{q} - 1 - \frac{\kappa}{\sigma} \right) u_t$ (2.15)

(2.14) in (2.2): $\pi_t = \frac{\kappa}{1-\beta q} x_t - \frac{\beta q \kappa}{\sigma(1-\beta q)} u_t$ (2.16)

In the blackboard: Graph π_t^z (y-axis) Vs. x_t^z (x-axis).

In PS4: Notice that Q2 in PS4 is much simpler than the exercise solved here. The conjecture in (2.7) is not necessary at all. For Q2 in PS4 it is enough to notice that $E_t x_{t+1}^z = q x_t^z$ and $E_t \pi_{t+1}^z = q \pi_t^z$ (i.e., (2.14) with $u_t = 0$); therefore, after plugging these expectations in (2.2) and (2.6) you get an easy-to-solve system of two equations ((2.15) and (2.16) with $u_t = 0$) with two unknown (x_t^z and π_t^z).

Help on Q3

Everybody has acces to the Dyn-file for the canonical NK model used for PS3: `NKM_ps3.dyn`. With some minor modifications, we can get the Dyn-file to solve the model introduced for Q3:

1st Declare additional endogenous variable: `var ...`
`rL,` % LR real interest rate
`phi,` % Term premium shock
`r;` % 1-period real interest rate

2nd Declare additional exogenous variable: `varexo ...`
`e_phi;` % Term premium innovation

3rd Declare additional parameters: `parameters ...`
`rho_i,` % Policy inertia
`rho_phi,` % Persistence of term premium shock
`sde_phi;` % St. Dev. of term premium innovation

4th Verify that calibration coincide with PS and add:
`rho_i=0.5; rho_phi=0.9; sde_phi=0.01;`.

Help on Q3

5^{th} Change and add equations:

```
x=x(+1)-(1/sigma)*(rL - rn); %Change Euler
i=rn+rho_i*i(-1)+(1-rho_i)*(phi_pi*pi+phi_x*x)+v;%Ch. Taylor
rL=(1/4)*(r+r(+1)+r(+2))+phi; % Add: Long term interest rate
phi=rho_phi*phi(-1)+e_phi;% Add: Term premium
r=i-pi(+1);% Add: Fisher equation
```

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PS3: Topics

Q1. FRED+Excel or FRED+Matlab: Evidence on NK model. ✓

Q2. ZLB. ✓

Q3. Dynare: NK + term structure. ✓

Q4. Financial friction: Costly verification.

Q5. Financial friction: Moral hazard.

Questions?

Any burning question on assignments/lectures?

Today:

1. (In details) Similar exercise to Q4.
2. (In details) Similar exercise to Q5.

Similar to Q4: Costly verification

you are the credit manager of Bank of America, Santa Cruz branch.

- N investment projects $i \in \{1, ..., N\}$ to fund.
- 3 equally likely states of nature: projects' returns are $R_i = R + x_i$, $R_i = R$ and $R_i = R - x_i$ in each state.
- Identical borrowers: collateral C, they can borrow L to fund one project.
- Your opportunity cost of funds is r.
- You and borrowers are risk neutral
- Defaults: $R - x_i$ occurs $\Rightarrow$ you, as lender, get $R - x_i + C < (1 + r^l)L$ where r^l is the interest rate on the loan.

Answer the following

(a) If you can pay μ to monitor which project is the borrower undertaking. How much are your expected profits per project? How much are borrower's profits?

(b) How much should you charge to fund each project? How do r, μ, x_i and C affect the interest rate?

(c) If it is no possible to monitor a borrower, what should you expect?

(d) If it is no possible to monitor a borrower and you cannot set interest rates, what is the maximum level of risk that you can tolerate?

Similar to Q4: Costly verification

(a) The lender:

$$E\Pi_\ell(i) = \overbrace{(1/3)(1+r_i^l)L + (1/3)(1+r_i^l)L}^{\text{No default}} + \overbrace{(1/3)[R - x_i + C]}^{\text{Default}} - \mu$$
$$= (2/3)(1+r_i^l)L + (1/3)[R+C] - \mu - (1/3)x_i$$

- The borrower

$$E\Pi_b(i) = \overbrace{(1/3)[R + x_i - (1+r_i^l)L] + (1/3)[R - (1+r_i^l)L]}^{\text{No default}} + \overbrace{(1/3)[-C]}^{\text{Default}}$$
$$= (2/3)R - (2/3)(1+r_i^l)L - (1/3)C + (1/3)x_i$$

(b) Indifference: $E\Pi_\ell(i) = (1+r)L$

$$(1+r_i^l) = \frac{3}{2}(1+r) + \frac{3}{2}\frac{\mu}{L} + \frac{1}{2}\frac{x_i - R - C}{L}$$
$$\Rightarrow \frac{\partial r_i^l}{\partial r} > 0,\ \frac{\partial r_i^l}{\partial \mu} > 0,\ \frac{\partial r_i^l}{\partial x_i} > 0 \text{ and } \frac{\partial r_i^l}{\partial C} < 0$$

Similar to Q4: Costly verification

(c) Notice that $\frac{\partial E\Pi_b(i)}{\partial x_i} = 1/3 > 0$; hence, borrowers prefer to undertake the riskiest project.

(d) In this case $r_i^l = r^l$ $\forall i$ and $\mu = 0$; therefore, expected profits become

$$E\Pi_\ell(x_i) = (2/3)(1 + r^l)L + (1/3)[R + C] - (1/3)x_i$$

Projects with $x_i > x^*$ are not desirable if $E\Pi_\ell(x_i) \leq E\Pi_\ell(x^*) = (1+r)L$; hence,

$$(2/3)(1 + r^l)L + (1/3)[R + C] - (1/3)x^* = (1 + r)L$$
$$\Rightarrow x^* = 2(1 + r^l)L + [R + C] - 3(1 + r)L$$

From where

$$\frac{\partial x^*}{\partial r^l} > 0, \ \frac{\partial x^*}{\partial r} < 0, \ \frac{\partial x^*}{\partial R} > 0 \text{ and } \frac{\partial x^*}{\partial C} > 0$$

Similar to Q5: Moral hazard

Supose a Bank collects funds from household deposits d_t and net worth n_t. The bank can use these funds to purshase private securities a_t and goverment bonds b_t. Therefore, the bank balance sheet is

$$a_t + b_t = d_t + n_t \tag{1}$$

Let's define the leverage ratio as $\ell_t = (a_t + b_t)/n_t \geq 1$ and the bond share in assets as $\omega_t = b_t/(a_t + b_t) \in (0, 1)$. Let's also assume that the agent can default on its debts and capture (or divert) a fraction θ of a_t and the fraction $\theta\gamma$ of b_t ($\gamma \leq 1$, i.e., it is harder to divert goberment bonds).

(a) In which situation does the banker has no incentive to default?

(b) Suppose that a contract is designed such that the bank is never defaulting. Should this contract imply a borrowing constraint?

(c) If yes, what is the maximum leverage ratio?

Similar to Q5: Moral hazard

(a) The bank's market-value is given by its equity n_t. On the other hand, by defaulting deposits the banker can divert $\theta a_t + \theta\gamma b_t$. That is, the banker loses all n_t but may divert $\theta a_t + \theta\gamma b_t$. Therefore, the restriction to avoid such a situation is

$$\theta a_t + \theta\gamma b_t \leq n_t \tag{2}$$

(b) Yes. The definition $\omega_t = \frac{b_t}{a_t+b_t}$ implies that $b_t = \frac{\omega_t}{1-\omega_t} a_t$. (4)

(4) in (1): $\frac{1}{1-\omega_t} a_t = d_t + n_t$ (5)

(4) in (2): $\theta \left[\frac{1-(1-\gamma)\omega_t}{1-\omega_t} \right] a_t \leq n_t$ (6)

a_t from (5) in (6): $\boxed{d_t \leq \frac{1-\theta+\theta(1-\gamma)\omega_t}{\theta-\theta(1-\gamma)\omega_t} n_t}$ (7)

(7) is how much can the banker borrow from households as deposits.

(c) (7)+n_t: $d_t + n_t \leq \frac{1}{\theta-\theta(1-\gamma)\omega_t} n_t$ (8)

(1) in (8): $a_t + b_t \leq \frac{1}{\theta-\theta(1-\gamma)\omega_t} n_t$ (9)

(9)$\div n_t$: $\boxed{\ell_t \equiv \frac{a_t+b_t}{n_t} \leq \frac{1}{\theta[1-(1-\gamma)\omega_t]}}$ (10)

As a result the maximum leverage ratio is: $\boxed{\ell_t^* = \frac{1}{\theta[1-(1-\gamma)\omega_t]}}$

Notice that $\frac{\partial \ell_t^*}{\partial \theta} < 0$, $\frac{\partial \ell_t^*}{\partial \gamma} < 0$ and $\frac{\partial \ell_t^*}{\partial \omega_t} > 0$.

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Working on PS4

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202 - Winter 2016
Final review

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Some comments on PS4

Q1. Monetary policy:

Scatter	‘too expansionary’	‘too tight’
X-axis: $\widehat{x}$ Vs. Y-axis: $\widehat{\pi}$	Quadrant I	Quadrant III
X-axis: $\widehat{\mu}$ Vs. Y-axis: $\widehat{\pi}$	Quadrant II	Quadrant IV

Q3. IRFs Interpretation:

$$x_t = E_t x_{t+1} - \sigma^{-1}\left(r_t^L - r_t^n\right), \qquad \text{Euler (1.1)}$$

$$\pi = \beta E_t \pi_{t+1} + \kappa x_t + u_t, \qquad \text{Phillips (1.2)}$$

$$r_t^L = (1/3)\left(r_t + E_t r_{t+1} + E_t r_{t+2}\right) + \phi_t, \qquad \text{LT real Int. rate (1.3)}$$

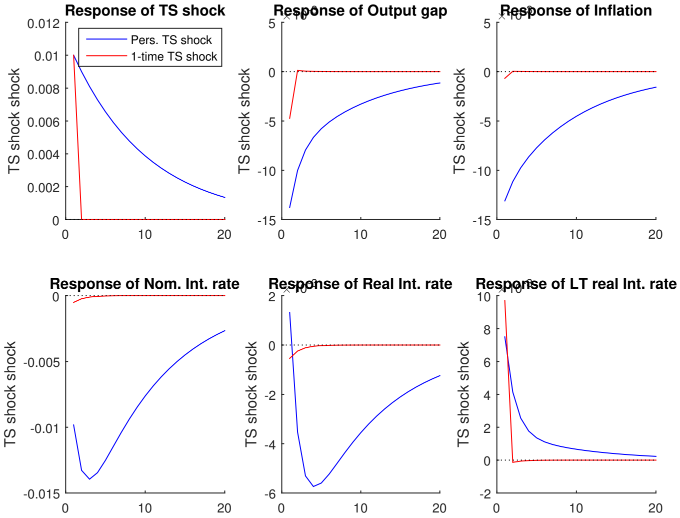
$$r_t = i_t - E_t \pi_{t+1} \qquad \text{Fisher (1.4)}$$

$$i_t = r_t^n + \rho_i i_{t-1} + (1-\rho_i)\left(\phi_\pi \pi_t + \phi_x x_t\right) \qquad \text{Taylor (1.5)}$$

$$\phi_t = \rho_\phi \phi_\pi t - 1 + e_{\phi,t} \qquad \text{Shocks (1.6)}$$

u_t and r_t^n are exogenous shocks with a similar evolution as (1.6).

Some comments on PS4



Some comments on PS4

Q3a. $\rho_\phi \phi = 0.9$ i.e., persistent TS shock.

1^{st} Direct effects: The shock rises r_t^L.

Goods market: Households would prefer to increase saving and reduce consumption (which diminish aggregated demand). Therefore, x_t becomes negative.

Labor market: Firms are required to reduce production (to match a lower demand). Given that productivity is not affected by the shock, the only way to cut production is by hiring fewer hours. This reduces labor demand (which at equilibrium reduces labor and wages). Smaller wages reduce real marginal cost. Hence, π_t becomes negative.

Monetary policy: The Taylor rule commands the CB to cut i_t as x_t and π_t are both negative.

2^{nd} Secondary effects: The shock is persistent, given a rational household, expected inflation should be negative (the shock will stay in the economy for some periods).

Interest rates: As $E_t \pi_{t+1} < 0$, it undercuts the effects of the smaller i_t, because it increases r_t (Eq. (1.4)). At the moment of the shock, r_t^L barely reacts to the MP. Consequently, expectations prevent MP to regulate the economy.

3^{nd} Convergence: MP needs to be persistent in order to moderate expectations and to eventually reduce the effects of the shock as it fades away.

Some comments on PS4

Q3a. $\rho_\phi \phi = 0$ i.e., persistent TS shock.

1^{st} Direct effects: Similar to Q3a.

2^{nd} Secondary effects: This is a 1-period shock, given a rational household, expected inflation should be quite close to zero (the shock completely fades away at the next period).
Interest rates: As $E_t \pi_{t+1} \to 0$, the smaller i_t quickly reduces r_t (Eq. (1.4)). Although, the effect on r_t^L is still small, expectations help MP to regulate the economy.

3^{nd} Convergence: MP is more effective and the economy quickly returns to the steady state.

Q4b. If lenders cannot monitor borrowers activities, they should presume that borrowers always undertake the riskiest projects. This happens because borrowers' expected returns increases with risk (i.e. $\partial E \Pi_b / \partial x_i > 0$). Consequently, it is rational for the lender to charge the interest rate associated to the riskiest project (the highest one), regardless of the actual project undertaken by the borrower.
Most likely, less risky projects will not get funding as the cost of credit becomes too high. For borrowers, the expensive credit could be seen as an additional incentive to undertake the riskiest project.

Structure of review questions

Q	Topic	Q	Topic
1	Solow growth model.	2	RBC.
$\boxed{20}$, $\boxed{21}$, $\boxed{24}$	fiscal policy.	3,6	Fisher equation.
4	Rational expectation.	8,11,22	NK equilibrium.
7,9,10, $\boxed{\mathbf{13}}$	Optimal MP.	12, $\boxed{\mathbf{18}}$	ZLB.
16	Systemic bank run.	23	NK + wage stickiness.
5,14, $\boxed{17}$	Costly verification.	15, $\boxed{\mathbf{19}}$	Moral Hazard.

Today I will solve bold and framed questions, and if time allows it the rest of framed questions.

Blue numbers were on the midterm.

Any burning question on assignments/lectures?

In details: Q13

Consider:

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r_t^f) \qquad \text{Dynamic IS (13.1)}$$

$$\pi_t - \pi^T = \beta E_t \left(\pi_{t+1} - \pi^T\right) + \kappa x_t + e_t \qquad \text{Phillips curve (13.2)}$$

and a CB that minimices

$$\frac{1}{2} E_t \sum_{i=1}^{\infty} \beta^i \left[\left(\pi_{t+i} - \pi^T\right)^2 + \lambda x_{t+i}^2\right] \qquad \text{CB loss function (13.3)}$$

(a) Find FOCs for a discretionary CB.

(b) Which are the two conditions that determine x_t and π_t?

(c) Suppose r_t^f and e_t are 0-mean *iid* processes. Plot if $(r_t^f, e_t) = (0, 0)$.

(d) Explain in your graphs the effects of $e_t < 0$.

(e) Explain in your graphs the effects of $r_t^f < 0$.

In details: Q13

(a) Let's define $\widehat{\pi}_t = \pi_t - \pi^T$ (notice that $d\widehat{\pi}_t = d\pi_t$); then the Lagrangian for Max (13.3) subject to (13.1) and (13.2) is

$$\mathcal{L} = E_t \sum_{i=1}^{\infty} \beta^i \begin{bmatrix} \frac{1}{2}\left(\widehat{\pi}_{t+i}^2 + \lambda x_{t+i}^2\right) + \\ \ell_{x,t+i}\left(x_{t+i+1} - \frac{1}{\sigma}(i_{t+i} - \widehat{\pi}_{t+i+1} - r_{t+i}^f) - x_{t+i}\right) + \\ \ell_{\widehat{\pi},t+i}\left(\beta\widehat{\pi}_{t+i+1} + \kappa x_{t+i} + e_{t+i} - \widehat{\pi}_{t+i}\right) \end{bmatrix} \quad (13.3)$$

- Discretionary CB: 'future' taken as given

$$\mathcal{L} = E_t \begin{bmatrix} \frac{1}{2}\left(\widehat{\pi}_t^2 + \lambda x_t^2\right) + \\ \ell_{x,t}\left(-\frac{1}{\sigma}(i_t - r_t^f) - x_t\right) + \\ \ell_{\widehat{\pi},t}\left(\kappa x_t + e_t - \widehat{\pi}_t\right) \end{bmatrix} + E_t Future \quad (13.4)$$

- $\widehat{\pi}_t : \ \widehat{\pi}_t - \ell_{\widehat{\pi},t} = 0 \Rightarrow \widehat{\pi}_t = \ell_{\widehat{\pi},t}$ (13.5)
- $x_t : \ \lambda x_t - \ell_{x,t} + \kappa \ell_{\widehat{\pi},t} = 0 \Rightarrow x_t + \kappa \ell_{\widehat{\pi},t} = \ell_{x,t}$ (13.6)
- $i_t : \ -\frac{1}{\sigma}\ell_{x,t} = 0 \Rightarrow \ell_{x,t} = 0$ (i.e. not binding) (13.7)
- (13.7) and (13.5) in (13.6): $\lambda x_t + \kappa \widehat{\pi}_t = 0$ or

$$\boxed{\lambda x_t + \kappa\left(\pi_t - \pi^T\right) = 0} \quad (13.8)$$

In details: Q13

(b) Shocks are 0-mean *iid* $\Rightarrow E_t\pi_{t+1} = \pi^T$ and $E_t x_{t+1} = 0$. Furthermore, as restriction (13.1) is not binding (see Eq. (13.7)), the economy is fully described by (13.2) and (13.8); therefore:

$$\pi_t = \pi^T + \kappa x_t + e_t \qquad \text{Phillips curve (13.9)}$$

$$\pi_t = \pi^T - \frac{\lambda}{\kappa} x_t \qquad \text{CB FOC (13.10)}$$

(c) On the blackboard (Phillips curve has a positive slope and intercept, while CB FOC has a negative slope and intercept).

(d) On the blackboard (Phillips curve shifts down)

(e) On the blackboard (Nothing happens)

In details: Q18

Consider:

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r_t^f) \qquad \text{Dynamic IS (18.1)}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \qquad \text{Phillips curve (18.2)}$$

$$i_t = r_t^f + \phi E_t \pi_{t+1} \qquad \text{Taylor rule (18.3)}$$

$$r_t^f = r^f - a_1 z_t + a_2 g_t + \psi_t \qquad \text{Demand shock (18.4)}$$

(a) Effect of g_t?

(b) Assume $\psi_t \in \{\psi^z, 0\} \sim iid\mathcal{B}(q)$, $g = \bar{g}$ and $i_t = 0$ if $\psi_t = \psi^z << 0$. Solve for x and π.

(c) Graph this economy for $\bar{g} = 0$.

(d) Explain in your graphs the effects of $\bar{g} > 0$.

In details: Q18

(a) No effects of g_t as i_t matches r_t^f in a one-to-one ratio.

(b) In the ZLB: $i_t = 0$ and $r_t^f = (r^f + \psi^z) + a_2\bar{g}$. (18.5)

(18.5) in (18.1): $x_t^z = E_t x_{t+1}^z + \frac{1}{\sigma}\left[E_t\pi_{t+1}^z + (r^f + \psi^z) + a_2\bar{g}\right]$. (18.6)

Expectations

$E_t x_{t+1}^z = q x_t^z + (1-q)0$ and $E_t\pi_t^z = q\pi_t^z + (1-q)0$. (18.7)

(18.7) in (18.6): $\boxed{\pi_t^z = -\frac{(r^f+\psi^z)+a_2\bar{g}}{q} + \sigma\left(\frac{1}{q} - 1\right)x_t^z}$ IS (18.8)

(18.7) in (18.2): $\boxed{\pi_t^z = \frac{\kappa}{1-\beta q}x_t^z}$ Phillips (18.9)

Combining (18.8) with (18.9), we get the solution for x and π:

$$\boxed{x_t^z = \frac{1-\beta q}{\sigma(1-q)(1-\beta q)-q\kappa}\left[(r^f + \psi^z) + a_2\bar{g}\right]} \tag{18.10}$$

$$\boxed{\pi_t^z = \frac{\kappa}{\sigma(1-q)(1-\beta q)-q\kappa}\left[(r^f + \psi^z) + a_2\bar{g}\right]} \tag{18.11}$$

(c) On the blackboard (Phillips curve has a positive slope and no intercept, while Aggregated demand has a positive slope and intercept).

(d) On the blackboard (Aggregated demand [or IS] shifts downwards)

In details: Q19

Suppose the BS of a bank consists of assets a, deposit d, loans from other banks b, and the bank's own capital n.

$$a = d + b + n. \qquad \text{Bank's BS (19.1)}$$

Let's define the divertable assets as $a - \omega_d d - \omega_b b$ where $0 \geq \omega_b, \omega_d \leq 1$ and $\omega_d < \omega_b$. If the bank defaults, it can divert the proportion $\theta \in (0, 1)$ of its divertible assets. Additionally, the bank wants to maximice its market value given by

$$V \equiv r_a a - r_d d - r_b b. \qquad \text{Bank's Value (19.2)}$$

(a) Under what condition does the bank have an incentive to divert assets?

(b) Find the bank's FOC (use λ as Lagrange multiplier).

(c) Why $(r_a - r_d) > (r_a - r_b)$?

In details: Q19

(a) If the bank defaults, it gets $\theta(a - \omega_d d - \omega_b b)$; if the bank does not, it keeps is market value V $\Rightarrow$ the bank has incentive to diver if

$$\boxed{\theta(a - \omega_d d - \omega_b b) > V} \tag{19.3}$$

(b) Problem: $\max V$ s.t. the incentive compatibility constraint $\theta(a - \omega_d d - \omega_b b) \leq V$ and the definition of $V \equiv r_a a - r_d d - r_b b$. Equivalently:

$$\max_{a,b,d} [r_a a - r_d d - r_b b] \text{ s.t. } \theta(a - \omega_d d - \omega_b b) \leq r_a a - r_d d - r_b b \tag{19.4}$$

Lagrangian: $\mathcal{L} = r_a a - r_d d - r_b b + \lambda [r_a a - r_d d - r_b b - \theta(a - \omega_d d - \omega_b b)]$
FOCs:

$$a: \; r_a + \lambda(r_a - \theta) = 0 \Rightarrow r_a = \tfrac{\lambda}{1+\lambda}\theta \tag{19.5}$$

$$b: \; -r_b + \lambda(-r_b + \theta\omega_b) = 0 \Rightarrow r_b = \tfrac{\lambda}{1+\lambda}\theta\omega_b \tag{19.6}$$

$$b: \; -r_d + \lambda(-r_d + \theta\omega_d) = 0 \Rightarrow r_d = \tfrac{\lambda}{1+\lambda}\theta\omega_d \tag{19.7}$$

Spreads (19.5)-(19.6 or 7): $$\boxed{r_a - r_j = \tfrac{\lambda\theta}{1+\lambda}[1 - \omega_j] \;\; \forall j \in \{b, d\}} \tag{19.8}$$

(c) Because $\omega_d < \omega_b$, i.e., it is easier to diver fund funded with deposits.

If time allows it: Q17

Q16 Solution in firsts 15 pages of "Slides_BankRuns_QE_and_FiscalPolicy.pdf".

Q17 Suppose the bank expected that it will need to audit a firm that it has lent to with probability q. The costs of auditing are c. Assume banks operate in a competitive environment. If there are no other frictions in the loan market and all borrowers are identical ex ante, explain why the spread between the interest rate on loans and the bank's opportunity cost of funds would be qc.

Expected profits:

$$E\Pi^b = \overbrace{(1-q)(1+r^L)L}^{\text{No auditing}} + \overbrace{q\left[(1+r^L)L - cL\right]}^{Auditing} = (1+r)L$$

Therefore: $r^L - r = qc$

If time allows it: Q20, Q21 and Q24

Q20. In the basic flexible price model, why does a temporary rise in government purchases financed by taxes increase output?
Household welfare: $W = \sum_{i=1}^{\infty} \frac{Income_{t+i} - Taxes_{t+i}}{1+r_{t+i}}$
Household labor supply: $V'(N_t) = U'(C_t)w_t$.
Therefore, a rise in G makes the household poorer (as lup-sum taxes increase), which reduces consumption. Therefore, the household increases labor supply. Consequently, output is increased.

Q21. In the basic flexible price model, what is the effect of a temporary rise in G on C that is financed by an increase in T?
A rise in G makes the household poorer (as lup-sum taxes increase), which reduces consumption.

Q24. Suppose thre is a temporary need for government purcases to rise. Explain why it can be optimal for the government to borrow to finance this temporary rise in G_t.
If taxes are discretionary rather than lump-sum, it is optimal to smooth taxation over the time. As a result, debt can be used to smooth taxation while financing sudden rises of G.

It was fun,
thank you guys and ...

in all your finals!

Macroeconomic Analysis
202 - Winter 2016
Final review

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UCSC

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https://sites.google.com/a/ucsc.edu/alanledesma/teaching/2017q1