Problem Set 2

Submit Hard Copy in Class To the TA on the Wednesday of Week 8.

Problems: Labor Supply

1. Suppose utility function is given by $U = U(l,c) = l^{\alpha} \times c^{\beta}$ and the budget constraint is given by $c = \frac{wT+V}{p} + \frac{-w}{p} l$ & $T = h + l$.
   a. Please follow the class notes to get the parametric equations and find the equilibrium **consumption, utility and labor supply** functions. Set up the Lagrangian and follow class notes.

$$U = l^{\alpha} c^{\beta}$$

$$T = h + l$$

$$pc = wh + V$$

$$\frac{\partial U}{\partial l} = \alpha l^{(\alpha-1)} c^{\beta} = MU_l \quad \frac{\partial U}{\partial c} = \beta l^{\alpha} c^{(\beta-1)} = MU_c$$

$$\Gamma = U + \lambda[w(T-l) + V - pc]$$

$$\frac{\partial \Gamma}{\partial l} = MU_l - \lambda w = 0....(1) \rightarrow pMU_l = \lambda w$$

$$\rightarrow \frac{\alpha c^*}{\beta l^*} = \frac{w}{p}$$

$$\frac{\partial \Gamma}{\partial c} = MU_c - \lambda p = 0....(2) \rightarrow MU_c = \lambda p \text{ ....(4)}$$

$$\frac{\partial \Gamma}{\partial \lambda} = 0 \Rightarrow w(T - l^*) + V - pc^* = 0........(3)$$

$$pc^* = w(T - l^*) + V$$

$$c^* = \frac{\beta w}{\alpha p} l^*$$

$$p \frac{\beta w}{\alpha p} l^* = w(T - l^*) + V = wT + V - wl^*$$

$$l^* = \frac{(wT+V)}{w\left(1+\frac{\beta}{\alpha}\right)} = \frac{T}{\left(1+\frac{\beta}{\alpha}\right)} + \frac{V}{w\left(1+\frac{\beta}{\alpha}\right)}$$

$$c^* = \frac{\beta w}{\alpha p}\left[\left(\frac{T}{1+\frac{\beta}{\alpha}}\right) + \frac{V}{w\left(1+\frac{\beta}{\alpha}\right)}\right]$$

$$U^* = l^{*\alpha} c^{*\beta}$$

h* = T-l* will give Labor Supply Function.

$$h^* = T - l^* = T - \frac{(wT+V)}{w\left(1+\frac{\beta}{\alpha}\right)} = T - \frac{T}{\left(1+\frac{\beta}{\alpha}\right)} - \frac{V}{w\left(1+\frac{\beta}{\alpha}\right)}$$

$$\frac{\partial h^*}{\partial w} = \frac{V}{w^2\left(1+\frac{\beta}{\alpha}\right)} > 0$$

$$\varepsilon^S = \frac{w}{h^*}\frac{\partial h^*}{\partial w} = \frac{w}{h^*} \times \frac{V}{w^2\left(1+\frac{\beta}{\alpha}\right)} > 0$$

b. Suppose there is a lump sum tax (**L**) imposed on the consumers's income. Draw and compare the vertical and horizontal intercepts of the budget lines from part a and part b.

$$pc = wh + V = w(T-l) + V = wT + V - wl$$

$$pc = wh + V - L = w(T-l) + (V-L) = wT + \hat{V} - wl$$

Where $\hat{V} = V - L$

The vertical intercept for l=T falls/reduces

c. Suppose there is a lump sum tax (**L**) imposed on the consumers's income. Find the new equilibrium consumption, utility and labor supply functions. Set up the Lagrangian and follow class notes.

$$U = l^{\alpha} c^{\beta}$$

$$T = h + l$$

$$pc = wh + \hat{V}$$

$$\frac{\partial U}{\partial l} = \alpha l^{(\alpha-1)} c^{\beta} = MU_l \quad \frac{\partial U}{\partial c} = \beta l^{\alpha} c^{(\beta-1)} = MU_c$$

$$\Gamma = U + \lambda[w(T-l) + \hat{V} - pc]$$

$$\frac{\partial \Gamma}{\partial l} = MU_l - \lambda w = 0 \ldots(1) \rightarrow pMU_l = \lambda w$$

$$\rightarrow \frac{\alpha c^*}{\beta l^*} = \frac{w}{p}$$

$$\frac{\partial \Gamma}{\partial c} = MU_c - \lambda p = 0 \ldots(2) \rightarrow MU_c = \lambda p \ \ldots\ldots\ldots\ldots(4)$$

$$\frac{\partial \Gamma}{\partial \lambda} = 0 \Rightarrow w(T - l^*) + \hat{V} - pc^* = 0 \ldots\ldots\ldots(3)$$

$$pc^* = w(T - l^*) + \hat{V}$$

$$c^* = \frac{\beta w}{\alpha p} l^*$$

$$p \frac{\beta w}{\alpha p} l^* = w(T - l^*) + \hat{V} = wT + \hat{V} - wl^*$$

$$l^* = \frac{\left(wT + \hat{V}\right)}{w\left(1 + \frac{\beta}{\alpha}\right)} = \frac{T}{\left(1 + \frac{\beta}{\alpha}\right)} + \frac{\hat{V}}{w\left(1 + \frac{\beta}{\alpha}\right)}$$

$$c^* = \frac{\beta w}{\alpha p}\left[\left(\frac{T}{1+\frac{\beta}{\alpha}}\right) + \frac{\hat{V}}{w\left(1+\frac{\beta}{\alpha}\right)}\right]$$

$$U^* = l^{*\alpha} c^{*\beta}$$

h* = T-l* will give Labor Supply Function.

$$h^* = T - l^* = T - \frac{\left(wT + \hat{V}\right)}{w\left(1+\frac{\beta}{\alpha}\right)} = T - \frac{T}{\left(1+\frac{\beta}{\alpha}\right)} - \frac{\hat{V}}{w\left(1+\frac{\beta}{\alpha}\right)}$$

$$\frac{\partial h^*}{\partial w} = \frac{\hat{V}}{w^2\left(1+\frac{\beta}{\alpha}\right)} > 0$$

$$\varepsilon^S = \frac{w}{h^*}\frac{\partial h^*}{\partial w} = \frac{w}{h^*} \times \frac{\hat{V}}{w^2\left(1+\frac{\beta}{\alpha}\right)} > 0$$

2. If Utility function is given by $U = (l - \bar{l})^{\alpha}(c - \bar{c})^{\beta}$; where $\bar{l}$ & $\bar{c}$ represent subsistence leisure and subsistence consumption and α & β are utility weights for leisure and consumption. A higher α would suggest that the person is leisure loving; while a lower α would suggest that the person is work loving. Similarly, higher β suggests a materialistic person & a lower β suggests that the person does not care about material comfort all that much. Suppose p is the price of the consumption good. This person earns w dollars an hour and receives a non-labor income in form of assets worth V. The government taxes hourly wages at a rate of τ As a result, this person's total labor earning is $(w-\tau)\times h$ and non-labor income is V. This person's total expenditure is pc. Assume this person has finite time T to be divided into labor versus leisure.

My answer below assumes that there is a lumpsum tax L in addition to the per unit tax. This was not part of the question. To get the exact answers to this part, you need to put $L = 0$ such that $\hat{V} = V$

a. Write the equation for the budget line & find the slope of the budget line. What does it mean?

$$Total\ \ Expenditure = Total\ \ Income = Labor\ \ Income + NonLabor\ \ Income$$

$$pc = wh - \tau wh + V - L = wh(1-\tau) + V - L$$

$$T = h + l \Rightarrow h = T - l$$

$$pc = w(T-l)(1-\tau) + V - L = w(1-\tau)T + V - L - w(1-\tau)l$$

$$c = \frac{w(1-\tau)T + V - L}{p} - \frac{w(1-\tau)}{p}l = \frac{\hat{w}T + \hat{V}}{p} - \frac{\hat{w}}{p}l$$

$$\hat{V} = V - L$$

$$\hat{w} = w(1-\tau)$$

$$VI\big|_{l=T} = \frac{\hat{V}}{p}$$

$$VI_{l=0} = \frac{\hat{w}T + \hat{V}}{p}$$

$$|Slope| = |\mathrm{Re}al\ \ Wage| = \frac{\hat{w}}{p}$$

b. Find the equation for Reservation Wage using the definition $RW = |MRS|_{l=T, c=\frac{\hat{V}}{p}}$

$$U = U(l,c) = \left(l - \bar{l}\right)^{\alpha} \times \left(c - \bar{c}\right)^{\beta}$$

$$MU_l = \frac{\partial U(l,c)}{\partial l} = \alpha\left(l - \bar{l}\right)^{\alpha-1}\left(c - \bar{c}\right)^{\beta}$$

$$MU_c = \frac{\partial U(l,c)}{\partial c} = \beta\left(l - \bar{l}\right)^{\alpha}\left(c - \bar{c}\right)^{\beta-1}$$

$$\left|Slope\right| = \left|MRS\right| = \frac{MU_l}{MU_c} = \frac{\alpha\left(c-\bar{c}\right)}{\beta\left(l-\bar{l}\right)}$$

$$RW = \left|MRS\right|_{l=T,c=\frac{\hat{V}}{p}}$$

$$RW = \frac{\alpha\left(c-\bar{c}\right)}{\beta\left(l-\bar{l}\right)}\Big|_{l=T,c=\frac{V}{p}} = \frac{\alpha\left(\frac{\hat{V}}{p}-\bar{c}\right)}{\beta\left(T-\bar{l}\right)} = \frac{\alpha\left(\hat{V}-p\bar{c}\right)}{p\beta\left(T-\bar{l}\right)}$$

$$RW = \frac{\alpha\left(\hat{V}-p\bar{c}\right)}{p\beta\left(T-\bar{l}\right)}$$

c. Mathematically Compute $\frac{\partial RW}{\partial \bar{l}}$; $\frac{\partial RW}{\partial \alpha}$; $\frac{\partial RW}{\partial \bar{c}}$; $\frac{\partial RW}{\partial V}$; $\frac{\partial RW}{\partial \tau}$.

$$RW = \frac{\alpha\left(\hat{V}-p\bar{c}\right)}{p\beta\left(T-\bar{l}\right)}$$

$$\Rightarrow \ln RW = \ln[\alpha] + \ln\left[\left(\hat{V}-p\bar{c}\right)\right] - \ln[p] - \ln[\beta] - \ln\left[\left(T-\bar{l}\right)\right]$$

$$\frac{\partial RW}{\partial \bar{l}} = \frac{RW}{\left(T-\bar{l}\right)} > 0$$

$$\frac{\partial RW}{\partial \alpha} = \frac{RW}{\alpha} > 0$$

$$\frac{\partial RW}{\partial \bar{c}} = \frac{-p \times RW}{V-p\bar{c}} < 0$$

$$\frac{\partial RW}{\partial V} = \frac{RW}{\hat{V}-p\bar{c}} > 0$$

$$\frac{\partial RW}{\partial \tau} = 0$$

d. Suppose initially $RW = Market\ Wage = \hat{w}$; and the person involved is indifferent between working and not working. Now, when each of these parameters change individually (i.e. one by one with everything else remaining constant); is this person more likely to work or less likely to work?

Subsistence leisure: **All else remaining equal, increase in subsistence leisure reduces probability of working by raising RW**

Elasticity of leisure: **All else remaining equal, increase in elasticity of leisure in utility reduces probability of working by raising RW**

Subsistence consumption: **All else remaining equal, increase in subsistence consumption increases probability of working by reducing RW**

Non Labor Income: **All else remaining equal, increase in non labor income reduces probability of working by raising RW**

Tax Rate: **Suppose initially, RW = (1-tax) Market Wage. If tax rate increases, (1-tax) Market wage decreases relative to RW. This will reduce incentive to work**

e. Find out the tangency condition (i.e. when the slope of Budget Line and Indifference Curve are tangent to one another i.e. $|MRS| = |\mathrm{Re}\,al\ Wage|$)

$$\underset{\{l,c\}}{Max} \quad U = (l-\bar{l})^{\alpha}(c-\bar{c})^{\beta}$$

$$s.t. \quad c = \frac{\hat{w}T + \hat{V}}{p} - \frac{\hat{w}}{p}l$$

$$where\ \hat{V} = V - L \ \ \& \ \hat{w} = w(1-\tau)$$

$$\frac{\partial U}{\partial l} = \alpha\left(l-\bar{l}\right)^{(\alpha-1)} c^{\beta} = MU_l$$

$$\frac{\partial U}{\partial c} = \beta l^{\alpha}\left(c-\bar{c}\right)^{(\beta-1)} = MU_c$$

$$\Gamma = U + \lambda[\hat{w}(T-l) + \hat{V} - pc]$$

$$\frac{\partial \Gamma}{\partial l} = MU_l - \lambda\hat{w} = 0....(1) \rightarrow pMU_l = \lambda\hat{w}$$

$$\frac{\partial \Gamma}{\partial c} = MU_c - \lambda p = 0 \ldots .(2) \rightarrow MU_c = \lambda p \rightarrow \frac{\alpha(c^* - \bar{c})}{\beta(l^* - \bar{l})} = \frac{\hat{w}}{p}$$

$$\ldots\ldots\ldots\ldots(4)$$

$$\frac{\partial \Gamma}{\partial \lambda} = 0 \Rightarrow \hat{w}(T - l^*) + \hat{V} - pc^* = 0 \ldots\ldots\ldots(3)$$

$$pc^* = \hat{w}(T - l^*) + \hat{V}$$

$$\hat{c}^* = \frac{\beta w}{\alpha p} \hat{l}^*$$

f. Solve for the equilibrium **consumption, utility and labor supply** functions

$$p \frac{\beta w}{\alpha p} \hat{l}^* = w(T - \hat{l}^*) + \hat{V} = \hat{w}T + \overleftarrow{V} - \hat{w} l^*$$

$$\hat{l}^* = \frac{(\hat{w}T + \hat{V})}{\hat{w}\left(1 + \frac{\beta}{\alpha}\right)} = \frac{T}{\left(1 + \frac{\beta}{\alpha}\right)} + \frac{\hat{V}}{\hat{w}\left(1 + \frac{\beta}{\alpha}\right)}$$

$$\hat{c}^* = \frac{\beta \hat{w}}{\alpha p}\left[\left(\frac{T}{1 + \frac{\beta}{\alpha}}\right) + \frac{\hat{V}}{\hat{w}\left(1 + \frac{\beta}{\alpha}\right)}\right]$$

$$\hat{U}^* = \hat{l}^{*\alpha} \hat{c}^{*\beta}$$

h* = T-l* will give Labor Supply Function.

$$\hat{h}^* = T - \hat{l}^* = T - \frac{(\hat{w}T + \hat{V})}{\hat{w}\left(1 + \frac{\beta}{\alpha}\right)} = T - \frac{T}{\left(1 + \frac{\beta}{\alpha}\right)} - \frac{\hat{V}}{\hat{w}\left(1 + \frac{\beta}{\alpha}\right)}$$

$$\frac{\partial \hat{h}^*}{\partial w} = \frac{(1 - \tau)\hat{V}}{\hat{w}^2\left(1 + \frac{\beta}{\alpha}\right)} > 0$$

$$\varepsilon^S = \frac{w}{h^*} \frac{\partial \hat{h}^*}{\partial w} = \frac{w}{h^*} \times \frac{(1 - \tau)\hat{V}}{w^2\left(1 + \frac{\beta}{\alpha}\right)} > 0$$

<u>Problems: Returns to Education:</u> In Vive Le Revolution

a. What is the problem with estimating wages of individuals on their years of schooling?

Ability varies between different individuals. If years of schooling is correlated with ability, then the estimate of returns to schooling is endogenous i.e. biased (upwards if cov (ability, schooling)>0 & downwards if cov(ability, schooling)<0)

b. What is the exogenous shock that is uncorrelated with error but affects years of schooling?

1968 French student revolution, which made universities suspend the written part of the exam temporarily (the written exam was started again from next year). So cohorts born in 1948 or 1949 or 1950 would get a little more years of schooling compared to the neighboring cohorts

c. Which evidence in the paper convinces you that the exogenous shock is strongly correlated with years of schooling?

It shows that cohorts born in 1948, 1949 went to university more and graduated from university more than the other cohorts before and after the change was made.

d. Which evidence in the paper convinces you that the shock is exogenous?

It happens once, no one knew that university authorities would buckle under the pressure of the students; or that students would come together (in absence of social media) was a little bit of a shock.

e. Can you think of issues to this identification strategy which will compromise the unbiased nature of the IV estimate

i. IV estimates are about 60% larger than OLS estimates. This points to the fact that ability and years of schooling are negatively related. This is subject to verification in France.

ii. Is it possible that bar for getting into well paid jobs were increased (either by the employers or by the government) independent of the skills developed by the universities (signaling argument rather than human capital argument)

f. Look at table 4 from Vive Le Revolution: which column gives you first stage and which columns give you the second stage result. What are the dependent variables and what are the independent variables & dependent variables in column 4 and column 3 of this table?

Table 4
Impact of Birth Cohort on the Education and Labor Market Outcomes of Male Workers

	Baccalauréat Only (1)	At Least University Diploma (*Bac* + 2 or More) (2)	At Least University Degree (*Bac* + 3) (3)	Years of Higher Education (4)	Log Wage (5)	*Cadre* (Upper-White-Collar Occupation) (6)
1947	−.009 (.006)	.014 (.008)	.008 (.006)	.060 (.050)	.006 (.010)	.001 (.008)
1948	.007 (.006)	.015 (.008)	.012 (.006)	.080 (.050)	.031 (.010)	.008 (.008)
1949	−.001 (.006)	.027 (.008)	.009 (.006)	.150 (.050)	.021 (.010)	.016 (.008)
1950	−.001 (.006)	.008 (.008)	−.002 (.006)	.030 (.050)	.005 (.010)	.000 (.008)
1951	−.005 (.006)	.002 (.008)	−.001 (.006)	.010 (.050)	.003 (.010)	.003 (.008)
Trend	−.000 (.001)	.001 (.008)	−.001 (.001)	.005 (.010)	.010 (.002)	−.005 (.001)
Age	−.000 (.001)	.001 (.008)	.000 (.001)	.004 (.005)	.023 (.001)	.003 (.001)
N	26,370	26,370	26,370	26,370	26,370	26,370

Source.—Labor Force Survey 1990, 1993, 1996, and 1999.
Note.—Sample is male wage earners born between 1946 and 1952. Coefficients for the worker's cohort dummy are relative to the comparison cohorts of 1946 and 1952. Standard deviation is in parentheses.

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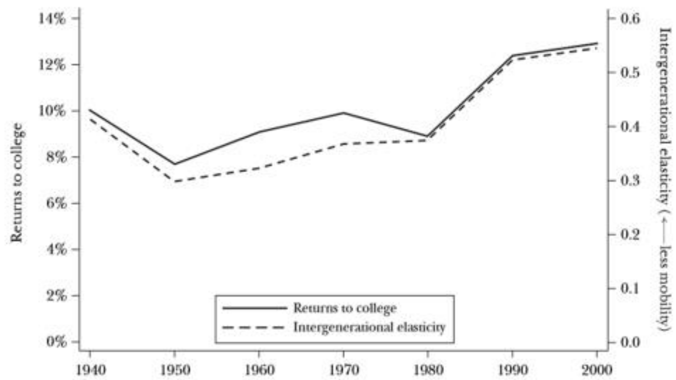
Columns, 2, 3 and 4 talks about the first stage. There is no column, which shows the second stage. Column 5 is called the reduced form estimation – it shows that the IVs do not have any direct effect on log wages – sometimes this is used as a way to demonstrate exogeneity.

Problems: Inequality : MIles Corak

Figure 5
The Higher the Return to College, the Lower the Degree of Intergenerational Mobility: United States, 1940 to 2000



Source: Adapted by the author from Mazumder (2012, Figure 1).
Notes: Information on the returns to college and the intergenerational earnings elasticity were provided to the author by Bhashkar Mazumder. As reported in Mazumder (2012), these are respectively from Goldin and Katz (1999) and Aaronson and Mazumder (2008, table 1 column 2). The 1940 estimate of the elasticity is a projection using Aaronson and Mazumder (2008, table 2 column 2).

a. Look at Figure 5 of the paper on inequality. What is it telling you?

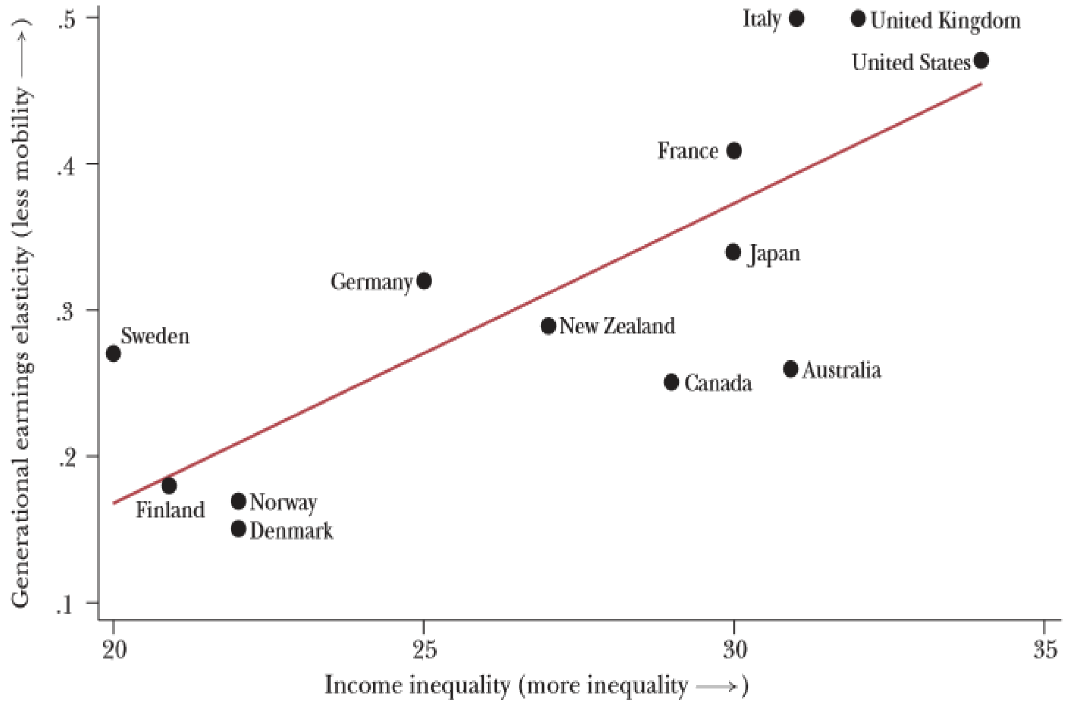
It tells you that intergenerational inequality in earnings and returns to college (skill premium) goes hand in hand and therefore, educational returns may be causing more inequality rather than curing inequality across generations.

b. What is the point of the graph below (Figure 1)

It compares the outcome in different countries based on how big of an income inequality exists in these countries, versus how high is the elasticity of intergenerational mobility. IT shows that in US, there is high inequality and high dependence between parents income and kid's income. In comparison, Denmark shows low inequality and low association between kids income and parent's income.

Figure 1

The Great Gatsby Curve: More Inequality is Associated with Less Mobility across the Generations



c. Give 4 reasons using economic logic to explain about the reasons behind an increase in income inequality in the last 20 years. Make sure you have pointed clearly out how each reason you talks about affects inequality

i. Human capital investments differ between people, the wealthy have made more human capital investment in their kids

ii. Skill biased technological change, which have increased the returns to college education

iii. Higher supply of unskilled labor through immigration (legal + illegal) have depressed the wages of low skill people in the country

iv. Too many kids in low skilled families have reduced human capital investment in kids – **Quality of kids Quantity of kids trade off (Gary Becker)**

d. What are the policies in light of this paper would you recommend to reduce income inequality?

Assisting people in attaining higher levels of human capital would work. Stopping immigration may work if the externalities from human capital are not significant (we have talked about this while talking about Borjas & we concluded that human capital externalities would possibly be large!) Some kind of family planning at the low end of the income distribution is warranted – parents should have information of how to raise successful kids (for example staying away from corporeal punishment to motivate them!).

Problems: Discrimination Paper

a. In how many ways can discrimination arise? We talked about this in class.

There can be four different types of discrimination: employer, employee, customer and statistical discrimination

b. In the paper by Sendhil Mullianathan (Emily and Greg) what is the experiment? You want to talk about what is the treatment group, what is control group, if treatment and control come from similar distributions, what is the exogenous assignment rule, which parameter is identified & if there are significant challenges to the estimation strategy that the author talks about?

The experiment is to send out identical resumes with different names (i.e. names with higher probability of being white versus non-white) randomly to human resources people. Then the main outcome variable which captures the preferences of the human resources people would be the call backs for whites and blacks.

c. Talk about two other significant challenges to estimation strategy that the author fails to address or addresses inadequately. Comment on what these challenges mean for the estimates.

The call back can be measured with error depending on if the calls were missed or not. The race of the human resources guys may matter. The job offerings were picked from newspapers, maybe more racially prejudiced occupations put ads in the newspapers.

Problems: Crime & Sports

Intimate Partner Violence, Male on Female, at Home

	Assaults Occurring Between (Eastern Time):			
	12 PM to 3 PM (1)	3 PM to 6 PM (2)	6 PM to 9 PM (3)	9 PM to 12 AM (4)
Games starting at 1 PM				
Loss * Predicted Win	.024	.142	.042	.049
(Upset Loss)	(.071)	(.066)	(.060)	(.060)
Loss * Predicted Close	.004	-.022	.010	.094
(Close Loss)	(.061)	(.060)	(.052)	(.051)
Win * Predicted Loss	-.018	-.018	.055	.004
(Upset Win)	(.075)	(.069)	(.061)	(.060)
Predicted Win	-.031	-.107	.096	-.180
	(.107)	(.103)	(.088)	(.090)
Predicted Close	.011	-.098	.077	-.154
	(.104)	(.100)	(.086)	(.087)
Predicted Loss	-.057	-.020	.057	-.107
	(.098)	(.093)	(.080)	(.081)
Nielsen Rating	.000	.021	-.005	.020
	(.011)	(.011)	(.009)	(.009)

1. The regression table above comes from the “NFL and Domestic Violence” paper.

A. Which predictor variable provides a statistically significant estimate of domestic violence occurring between 3 PM and 6 PM? What is the magnitude (size) of the estimate of that predictor variable?

Loss* Predicted Win (Upset Loss) : .142 i.e. 14% more likely to have domestic violence

B. Suppose police watches/listens to NFL games while they are ongoing. They respond to domestic violence calls later than when the domestic violence occurs. If this effect is not picked up by city/municipality level Fixed Effects how will it bias the estimate from part A of this question & why?

Should lead to overestimated coefficients – just inflates the likelihood that calls will be answered in the 3 PM – 4 PM segment more than in 1 PM to 3 PM segment. The issue really is: when was the result known? Was it a close game or blowout?