over a set of twenty alternatives are a mere drop in the bucket compared to the number of irrational preference relations that a collection of people might have.

Spatial Preferences

Up to now, the alternatives were arbitrary, so we had no reason to attach any significance to preferring A to B or B to A. But suppose that we construct a *perspective* of these alternatives. Sometimes creating a perspective is easy. If we were to analyze how much people enjoy work, play, and sleep, we might describe an outcome as a vector (*work*, *play*, *sleep*) where the three variables denote the time spent working, playing, and sleeping, respectively. Decompositions like this into separate dimensions are a common approach in economics and political science.

Other times, representing alternatives in a perspective becomes complicated. Consider someone's preferences for food. Listing the particular food items, such as nachos, sushi, and pretzels, would be cumbersome. We could create *dimensions* that characterize food items based on ingredients. In the Ben and Jerry's example, this worked great. The number and size of chunks characterized the pints of ice cream. These two dimensions allowed Ben and Jerry to make a *spatial* representation of the various pints. However, this won't always work. Many of the items at Taco Bell contain the same ingredients in the same proportions. A taco salad is just a taco in a new arrangement.

But let's suppose that we can map the alternatives to a single-dimensional perspective. We can then distinguish between three types of preferences along that dimension. In defining each type, we take the other dimensions as fixed and ask what happens to preferences as we vary the level on one dimension. The first type of preference applies to those dimensions for which more is better.

Preferences are **increasing** *if more is always preferred to less.*

Preferences about money are usually assumed to be increasing. More money is better. Preferences are also increasing about health, gas mileage, and computer speed.

The second type of preferences apply to things that people do not like, such as pollution or noise, for which less is better.

Preferences are **decreasing** *if less is always preferred to more.*

Preferences about pollution are decreasing, as are preferences about the amount of time spent doing our taxes.

For most things, including sleep, salmon, and software, more is not always better and neither is less. We like more up to a point, and then we like less. Consider the size of an ice cream cone. One scoop is nice. Two are better. Three may be a bit much. And four borders on outrageous (unless you happen to be fourteen years old). We call such preferences *single-peaked* because graphical representations of our happiness, or what economists call utility, have a single peak. We call the amount that provides the highest utility the *ideal point*.[5]

Preferences are **single-peaked** *if there exists an* **ideal point.** *If the current amount is less than the ideal point, more is preferred. If the current amount is more than the ideal point, less is preferred.*

The powerful implicit assumption in the spatial formulation is that the dimensions used to define the alternatives, as defined by the *perspective*, capture those attributes of the alternatives that drive preferences. Otherwise, the assumptions of increasing, decreasing, or single-peaked preferences do not make sense. Think back to the masticity-based perspective on ice cream. Masticity was a measure of how long it took to chew a spoonful of ice cream. Most people would not have increasing, decreasing, or single-peaked preferences about masticity. This discussion reiterates a point made at length earlier: making sense of the world, in this case making sense of preferences, requires a good perspective.

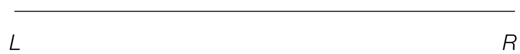

Figure 9.1 The Space of Sno-Cone Colors

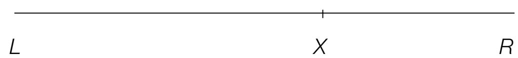

Figure 9.2 An Ideal Sno-Cone

Raspberry and Bubble Gum Sno-Cones

Spatial representations of the set of alternatives, combined with assumptions that structure preferences, limit the number of possible preference orderings. To see this, consider preferences about the color for raspberry sno-cones. First some background for those unfortunately not in the know about raspberries and sno-cones. In the wild, raspberries can be black, red, and even yellowish orange. Raspberry-flavored sno-cones vary in color as well. In some regions of the United States, you will find dark red raspberry sno-cones. In others, you will find that they are light blue. Had we the time and energy, we might even make a map of the country coloring some states red and other states blue depending on the more common color of their raspberry sno-cones. (Maps of red and blue states are important to political scientists.)

Here we consider preferences within Ohio, a blue state, at least for raspberry sno-cones. We represent the range of possible blue colors on a line with light blue (denoted by L) on the left, and royal blue, denoted by R, on the right (see figure 9.1).

Each Ohioan has an *ideal point*, a color that she most prefers. A person with an ideal point L (resp R) has a decreasing (increasing) preference, and a person with an ideal point in the interior has a single-peaked preference. In what follows, distance to the ideal point determines preference: the closer a color lies to a person's ideal color, the more the person prefers that color. Though not necessary, this assumption simplifies the presentation. Figure 9.2 shows a person's ideal point at X.

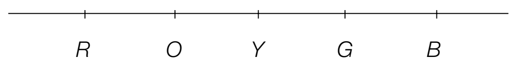

Figure 9.3 Five Alternative Sno-Cones

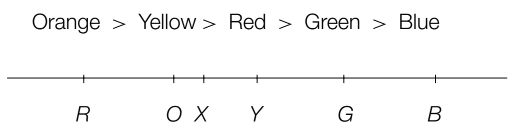

Figure 9.4 Brenda's Preferences

We now explore the implications of our three assumptions: (i) that the alternatives can be placed on a single line, (ii) that people have ideal points, and (iii) that preferences about alternatives are determined by the distance of the alternatives from the ideal point.

To see the restrictiveness of these assumptions, let's consider the task of assigning a color to bubblegum sno-cones. Bubblegum sno-cones could be any color. We won't use all of the colors—just the familiar ROYGB (red, orange, yellow, green, and blue) arranged along the spectrum. To make the comparison exact, we place these five colors on a line (see figure 9.3).

Suppose that Brenda most prefers the color orange. Her ideal point could be right at *O*; it could lie between *R* (red) and *O* or it could lie between *O* and *Y* (yellow). Let's suppose that her most preferred color lies between *O* and *Y*. For the purposes of this example, we assume that the colors are evenly spaced on the line and that Brenda's preferences about colors depend on the distance from the color to her ideal point. If we look at figure 9.4, we see that she must then prefer yellow (*Y*) to red (*R*), and she must also prefer red to green (*G*) and green to blue (*B*).

Thus, once we place her ideal point on the line, we uniquely define her preferences and limit preference diversity. We can calculate how many possible preference orderings can exist if we represent preferences on a single line. As before, we rule out ties. If a person most prefers red, she must like orange second best, then yellow, then green, and then blue. A person who most prefers blue must have preferences that go in the opposite order. Someone who most prefers orange could like yellow next best (as in our example above) or could like red next best. Either way, once we know her second favorite color, we know her full preferences. Therefore, two preference orderings have orange as the favorite color. The exact same logic applies to preferences that rank green or yellow first.[6]

Adding up all of these possibilities: only one preference ordering each for red and blue being ranked first, and two each for the other three colors, makes a total of only eight possible spatial preference orderings. If we relax the equal spacing assumption, then with a little effort we can see that only fifteen possible orderings exist. Either number is small when compared to the 120 possible rational orderings, the 1,024 intransitive relations, and the 59,049 relations that are neither complete nor transitive.

Thus, we see that assuming single-dimensional preferences reduces the number of possible preference relations—just as does imposing completeness and transitivity. If we increase the dimensionality of the perspective—say, moving from a line to a square—we allow for more preference orderings. The higher dimensional the perspective needed to make sense of preferences, the more diverse rational preferences can be. We might ask if we can always represent preferences spatially. We can, but we might have to make the number of dimensions large. It may even have to equal the number of alternatives.

Getting More Serious

We may disagree about what we believe to be pressing problems. Some of us believe it to be poverty, others believe it to be environmental sustainability, and still others believe it to be international stability. No one believes it to be selecting a type of beard or the color for a sno-cone. What we learn from fun examples, though, also applies to more serious contexts.[7] And we can think of the one dimension as representing an ideological spectrum from left to right. In fact, the sno-cone model provides a logical