

# Auctions, I

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# Auction Types

- Ascending auctions (English)
- Descending auctions (Dutch)
- Vickrey Auctions
- Multi-unit Auctions
- Others to be covered later
  - Combinatorial auctions
  - Spectrum auctions

# Auction goals

- Maximize revenues for auctioneer.
- Reward bidder with the highest valuation for the product.

# Ascending (English) auctions

- All bidders are initially *active*.
- Start price and increment are fixed.
- At each stage of the bidding:
  - Auctioneer calls out last price + increment
  - Zero or more bidders may become inactive
  - If at least 2 bidders are still active, auction proceeds to the next stage.
  - If only one auctioneer is active, then he wins at the current price.

# Ascending auction example

- John's willing to pay \$50 for item A.
- Jill's willing to pay \$40 for item A.
- Mary's willing to pay \$45.
- Start price = \$30, increment = \$10.
- \$30: John, Jill, Mary active.
- \$40: John, Jill, Mary active
- \$50: Only John is active => WINS and PAYS \$50.

# Descending auctions

- All bidders are initially *inactive*.
- Start price and decrement are fixed.
- At each stage of the bidding:
  - Auctioneer calls out last price - decrement
  - If at least one bidder says yes, then the first bidder to respond wins at the current price.
  - Else auctioneer proceeds to the next round.

# Descending auction example

- John's willing to pay \$50 for item A.
- Jill's willing to pay \$40 for item A.
- Mary's willing to pay \$45.
- Start price = \$60, increment = \$10.
- \$60: John, Jill, Mary inactive.
- \$50: John active
- John WINS and PAYS \$50.

# Observations

- In both cases, the person willing to bid the most wins.
- In both cases, the winner pays the current (winning) price.



# Vickrey auctions

- Same as ascending auction except for one difference.
- Winner pays the amount bid by the 2<sup>nd</sup> highest bidder.

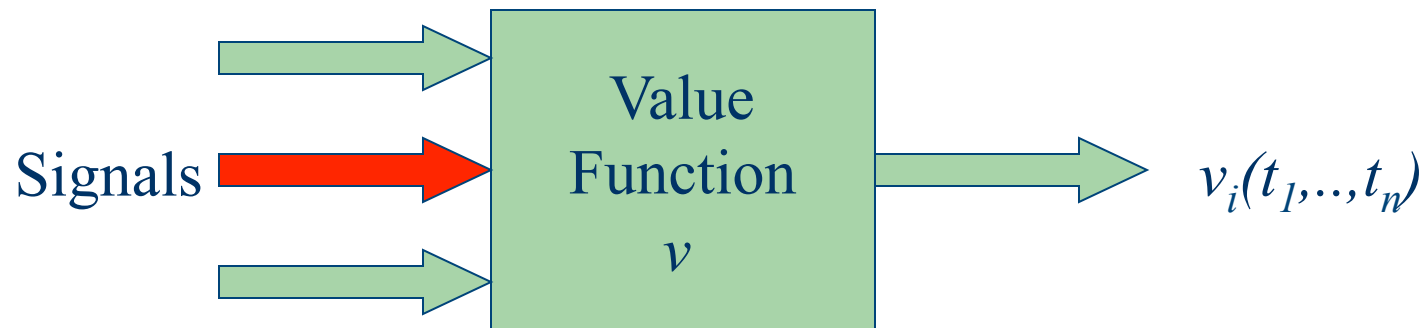
# Vickrey auction example

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- Jill's willing to pay \$40 for item A.
- Mary's willing to pay \$45.
- Start price = \$30, increment = \$10.
- \$30: John, Jill, Mary active.
- \$40: John, Jill, Mary active
- \$50: Only John is active => WINS and PAYS \$40.
- (If the increment had been \$5, John would have paid \$45).

# Private/Public values

- *Private value model:* Each bidder knows the value he places on the commodity. But he does not share this with others.
- *Pure common-value model:*
  - The item has a single value.
  - But different bidders have different perceptions of what that value is.

# General model



$t_i$  is the private signal of bidder  $i$

# Models

- Pure private model:  $v_i(t_1, \dots, t_n) = f(t_i)$  for some function  $f$ .
- Pure common value model:  $v_i(t_1, \dots, t_n) = v_j(t_1, \dots, t_n)$  for all  $i, j$ .

# Bidding in Vickrey auctions

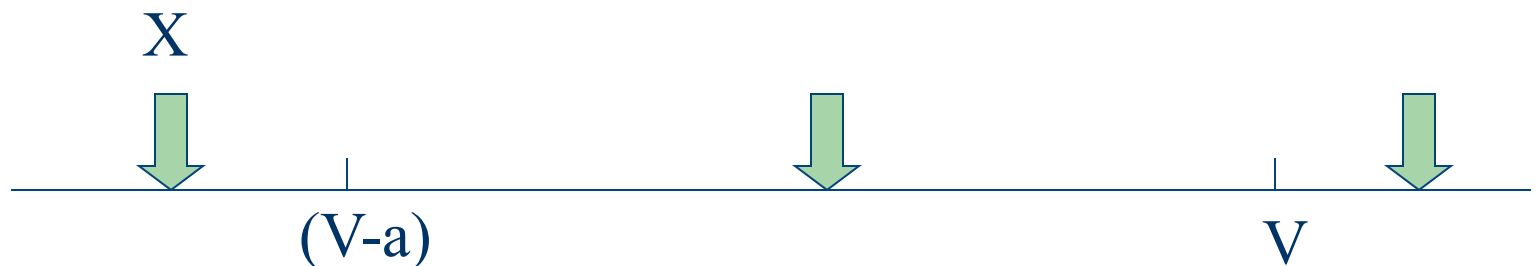
- Suppose my private value is  $V$ .
- What should I bid?

# Bidding in Vickrey auctions

- Suppose my private value is  $V$ .
- What should I bid?
- $V$ .

# Why?

- Suppose I bid  $(V-a)$ .
- Let the value of the highest bidder (other than mine) be  $X$ .
- Three cases:





# Vickrey auction bid analysis

- Case 1:  $X < (V-a)$ . In this case, I win and pay  $X$ . Had I bid  $V$ , I'd still have won and I'd still have paid  $X$ . So no benefit to me to bid  $V$  in this case.
- Case 2:  $X > V$ : In this case, I lose, and I'd have lost even if I'd bid  $V$ . The outcome does not change if I bid  $(V-a)$ .
- Case 3:  $(V-a) < X < V$ . In this case, I lose. Had I bid  $V$ , I'd have won and still paid less than my true value.

# Multiunit auctions

- One item, but lots of identical units for sale.
- E.g. 35 identical Rolex watches.
- Rules:
  - Auctioneer calls out a price per unit (e.g. \$500 per watch).
  - Bidders specify how many units they want (e.g. 5 watches).
  - As the unit price goes up, bidders cannot increase the units they want.
  - All bidders can hear the other bidders.

# Example multiunit auction

- Player 1: Has \$3000.
- Player 2: Has \$2500.
- Number of watches being sold: 6.
- Starting price: \$500 per watch.
- Bid Increments: \$100.
- Auction proceeds as follows.

# Example auction

Price/unit	Player 1' s bid	Player 2' s bid
\$500	6	5

# Example auction

Price/unit	Player 1' s bid	Player 2' s bid
\$500	6	5
\$600	5	4

# Example auction

Price/unit	Player 1' s bid	Player 2' s bid
\$500	6	5
\$600	5	4
\$700	4	3

# Example auction

Price/unit	Player 1' s bid	Player 2' s bid
\$500	6	5
\$600	5	4
\$700	4	3
\$800	3	3

# Net Result

- Auctioneer's revenue: GREAT.
- He takes in \$4800.
- But the bidders did no reason intelligently !!!!!
- Each knows something about the other's budget and interests.



# End of Round 1

- Player 1 knows Player 2 does not want to spend more than \$2500. Why?
- Player 2 knows Player 1 does not want to spend more than \$3000.
- Both players know that demand ( $6+5=11$  Rolexes) exceeds supply (6).
- By bidding as they just did, they both lost money. A better bidding approach would have been for *both* players to recognize that.
- Notice that the bidders never speak to each other. They are only drawing inferences from events they are entitled to know about.

# A better way of bidding

Price/unit	Player 1' s bid	Player 2' s bid
\$500	6	5
\$600	3	3

# Why is this good?

- Player 1 is safe. He knows he can't get more than 3 Rolexes anyway (as long as Player 2 really wants Rolexes).
- So he can sit tight with his bid for 3 Rolexes. Player 2 has no choice but to compromise.
- If Player 2 reasons the same way, then both can save dollars.

# Outcome

- Suppose each bidder can resell the Rolexes for \$1000.
- Player 1's profit is now  $\$400 \times 3 = \$1200$ .
- Same for Player 2.
- Auctioneer in trouble: His revenues are just \$3600 not \$4800 as before.
- Can build a game tree for this.