

# Chapter 8

## Thresholds and Tipping Points

*There is a tide in the affairs of men, Which taken at the flood, leads on to fortune. Omitted, all the voyage of their life is bound in shallows and in miseries. On such a full sea are we now afloat. And we must take the current when it serves, or lose our ventures.*

*Julius Ceaser* by William Shakespeare

Linearity implies a constant slope, a path that continues on the same trajectory. The most extreme form of nonlinearity would therefore be a trend that takes a ninety degree turn. In this chapter, we study models that produce these extreme nonlinear effects. The term “tipping point” has become ubiquitous in the media. Newspaper headlines proclaim “polar ice cap hits tipping point,” or “former aides say Katrina was tipping point for Bush’s repuation.” Scholars, reporters, and laypeople alike apply the concept of tipping points to physical, ecological, political, economic, and social processes. And yet, most people can’t give a precise definition of what constitutes a tipping point. More problematic, many of the canonical examples of tipping points probably wouldn’t satisfy a definition if we had one.

Here, I care less about coming up with a definition, than about understanding what

produces tipping points. However, to do the latter, to gain some foothold on what sorts of systems tip, I need a definition of tipping. Malcom Gladwell, a journalist who wrote a book titled “The Tipping Point” relies on the following definition: *The name given to that one dramatic moment in an epidemic when everything can change all at once is the Tipping Point.* I’ll begin with that as a working definition and then refine it a bit. Tipping points are when the system takes the sharp right turn.

Examples of these ninety degree turns include *phase transitions* and unstable points. A phase transition occurs when a small change in a variable suddenly has an enormous effect on some property of the system. Take something as basic as water. So long as the temperature of water stays above thirty-two degrees and less than two hundred and twelve degrees Fahrenheit it remains a liquid. But once it crosses that threshold those thresholds it changes it’s state: below thirty-two, it becomes ice, and above two hundred and twelve, its steam.

An *unstable points* is a concept taken from mathematics. The figure below shows a ball perched on top of a hill. A push in either direction will send the ball down a hill and into a basin. This picture represents an unstable point and it’s one way to conceptualize tipping points. The ball will either end up in position *A* or in position *B*. The tipping point, when the ball sits atop the hill, represents the moment at which a single event will determine the course of history. In the figure, that end result will be the bottom of a valley. In the real word, tipping points are that full sea on which the correct choice can lead to freedom and the incorrect one, tyranny.

To put tipping points in context, I like to contrast systems with tipping points with linear systems using two graphs. The graph on the left shows the weight of a camel and it’s pack as a function of the number of pieces of straw on the camel’s back. The graph on the right shows the height of a camel’s head above the ground also as a function of the number of pieces of straw on the camel’s back. The effect on the camel’s weight is linear:

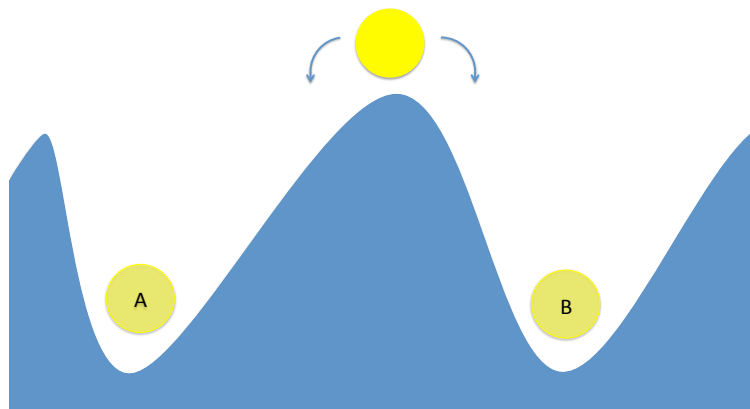


Figure 8.1: A representation of a tipping point

each additional piece of straw has the same effect on the camel's weight. The effect on the camel's height is not linear. Up to the tipping point, the camel's height increases linearly. At that point, the straw breaks the camel's back, and the camel becomes much shorter.

Notice the abrupt change in the graph on the right. The effect of changing the value of a variable has no effect up to some point and then has a large effect.<sup>1</sup>

Before I get into the details, I want to make the case why understanding tipping points is so important. Think back to the first chapter on linear models. Fitting a linear model to a system that's about to tip won't be informative about the future or about how the system performs given changes in variables. Consider three examples. The first example involves housing prices in the United States. The graph below shows an approximately linear rise in housing prices from 1963 to 2007. Then, there's a right turn. A tipping point. If you thought that prices would continue as they would and invested in that way, you lost a lot of money. If you anticipated that tip (and a few people did), you made a lot of money.

The second example considers social change. Suppose that you're trying to rebuild a local

---

<sup>1</sup>Mathematicians and physicists might call the tipping point, the *critical threshold*.

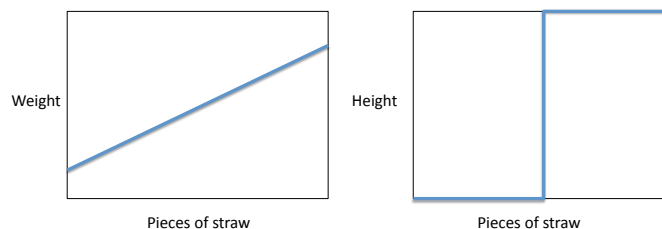


Figure 8.2: A linear effect and a tipping point

community, or even a country. You might have some measure of total effect that aggregates economic and social variables. If you do, you're likely to find that you're not having a huge effect. Changing society takes effort. The initial payoff from your efforts might be rather modest. The system though may be poised to take off once you pass certain thresholds of education levels, property rights, trust, and so on.

The final example comes from the world of medicine and touches on a sensitive topic: Down's Syndrome. Data on the probability of having a child with Down's syndrome reveals something pretty close to a tipping point at age forty. The trend is not linear.

In each case, if we assume linearity, we're going to be proven wrong. Ideally, we'd be able to anticipate these tips. In some cases, such as with the medical data, we can. We know where the tip occurs. In other cases, we perhaps cannot know with much certainty, but we can have some expectation about whether the system will tip or whether current trends will continue.

Now that we have some sense of the stakes, I want to take a moment to formally define

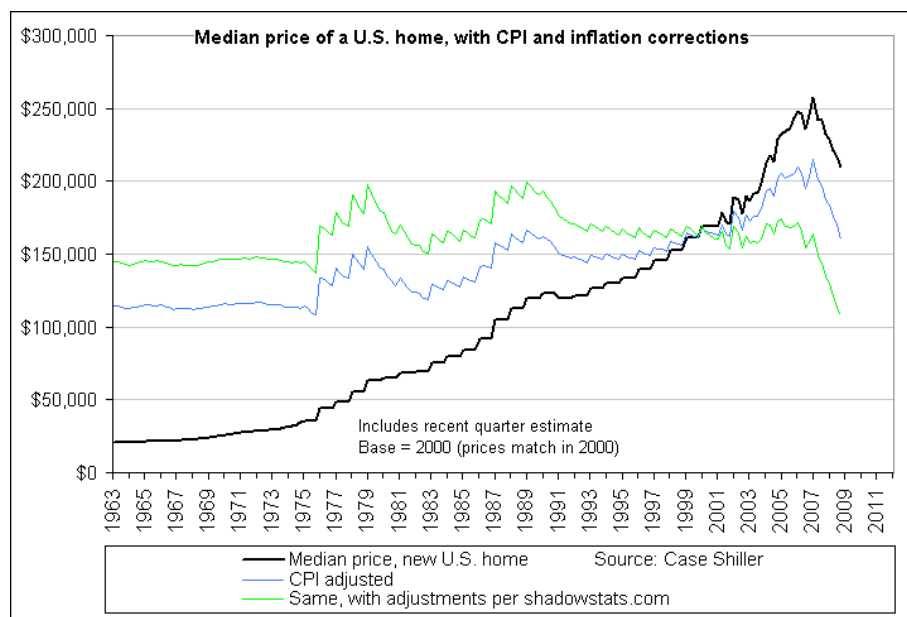


Figure 8.3: The Tipping Point in US Housing Prices

tipping points. After I do that, I'll present two models that produce tipping points. The first will be a percolation model. The second will be a model of diffusion. The first model originated in physics. The second model originated in epidemiology. Yet, like most of our models, we'll see how each can be used in a variety of settings.

## A First Definition of a Tipping Point

I want to propose a definition of a tipping point as a value of variable at which small changes can induce large changes in the future state of the system. Later on in this chapter, I'll offer a second, more subtle definition that characterizes a tipping point as an action or outcome that dramatically reduces uncertainty about the future. Both definitions borrow logical structure from our Markov model. They assume that the world can end up in some finite number of

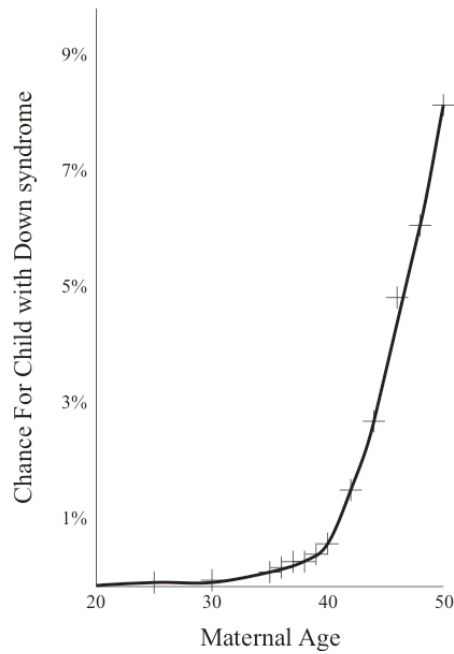


Figure 8.4: A Physiological Tipping Point

possible *states*. In the first, the state that the system ends up in depends upon the value of a parameter. In the second, we're unsure of which state that will be. After tipping point, the system's uncertainty goes away. For example, if we're not sure if a business will succeed or fail (those would be the two possible end states), a big new order could tip the business toward success.

Here though, I'll focus on the first, simpler definition. Let  $x$  denote the value of some variable, this could be the number of adopters of a new technology, the amount loans are leveraged, the communicability of a disease or any number of variables. Define, the *end state* given  $x$  as where the system will end up. Let  $x^-$  denote a value of that variable that's a tiny tiny amount less than  $x$ , and let  $x^+$  denote a value of that variable that an equally small amount above  $x$ .

A system has a **tipping point** at  $x$  if a small change in the value of  $x$  has large effects on the end state. In more formal language, if  $x$  is a tipping point, the end state given  $x^-$  differs by a large amount from the end state given  $x^+$ .

This more formal definitions captures formally the looser notion that  $x$  represents a dramatic moment. If we change the value of our variable just a little bit, we have huge effects on outcomes. This definition is nice, but it doesn't tell us why some systems have tipping points and some do not. To understand that, we need some models.

## A Percolation Model

The first tipping point model that I cover is called a *percolation model*. These are physical models designed to address the question of whether a resource can flow across a network or a grid. In engineering, percolation models are used to analyze whether fluids can flow through a solid. Soil that contains too much clay doesn't percolate and requires an engineered septic system.

The model that I describe here takes place on a grid of horizontal and vertical lines. Given the origin of the model, it's common to think of the grid as having a *top* and a *bottom*. We want to think of this grid as a network, so that each intersection of the lines is a *node* and each line between nodes is an *edge*.

In the percolation model, the edges can be thought of as pipes. Each of those pipes can be open or closed. Water can flow through open pipes, but it cannot flow through closed pipes. The grid percolates if it is possible to construct a path of open pipes that connect the top and bottom of the grid. In the figure below, I represent open pipes with darker lines and closed pipes with lighter lines. Of the two checkerboards on the grid, only the one on the left percolates.

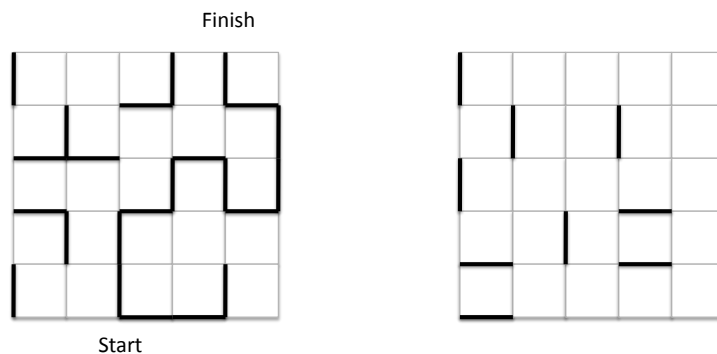


Figure 8.5: Percolation Model

Now, suppose that I have an enormous grid and that for each edge with probability  $p$  the pipe along that edge will be open. Clearly, if  $p$  equals zero, none of the pipes are open so the grid won't percolate. And if  $p$  equals one, every pipe is open, and the grid does percolate. But what about values in between? It turns out that the probability of percolation has a tipping point at  $p = 1/2$ . If I open less than half the pipes, then the system will almost certainly not percolate. If the probability of a pipe being open is even a little bit above one half, then the system will percolate almost for sure. Thus, the percolation model has a tipping point at  $p = 1/2$ . The end state of the system moves from no percolation to percolation as the probability of an open pipe crosses one-half.

This pipe based model is just one of many that can be written. You can also write a percolation model on the squares, so that in order to percolate you must jump from square to square. In that model, the percolation threshold is no longer one-half, it's a little over



59%. More generally, the percolation threshold depends on the network of connections.