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A theory of spatial system archetypes

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Abstract

Historic reference behavior and archetypes of system structure are key tools for creating rigorous system dynamics (SD) models. Modelers often delineate causal relationships by employing common archetypes of dynamic system structure, which produce behaviors such as growth and decline, oscillation, and complex combinations thereof. We extend archetypes to spatial-dynamic models, focusing on structural archetypes that exhibit changing spatial patterns in two-dimensional landscapes. Although many fields employ spatial modeling techniques, analogy-based, causally focused system archetypes remain confined to non-spatial SD models. We draw on spatial analysis literature to explore the influence of space on dynamic relationships and archetypes, including methods for articulating "space" and expressing feedback. We offer simple examples of spatial system archetypes and explore network structures for spatially extending SD models. By doing this, we argue for spatial modeling techniques that parallel the learn-by-analogy environment that archetypes have promoted in aspatial SD research.
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Introduction

As system dynamics (SD) has grown in popularity and range of application over the last fifty years, its explicit emphasis on understanding causal relationships, along with its use of scientifically rigorous and iterative modeling processes, has differentiated it from other modeling methods (Sterman, 2000). Alongside SD, a vibrant literature on spatial-dynamic modeling has emerged in the last two decades, offering compelling arguments for explicitly considering detailed spatial effects within models. Developed in fields as diverse as ecology (e.g. tools for assessing the spatial fragmentation of wildlife habitat; McGarigal and Marks, 1995), economics (e.g. spatial econometrics; Anselin, 2002), and network analysis (Barabasi and Albert, 1999), spatial modeling involves the use of disaggregated spatial data and relationships in order to understand spatial forms and processes. By "spatializing" SD models, modelers can explicitly (i) simulate system structure that is heterogeneous over space, as well as (ii) consider how spatial interactions affect systems themselves (see examples in Figure 1).

However, although substantial work has established "best practices" for the aspatial SD modeling process that promote rigorous and causally focused models (Sterman, 2000), very limited work has applied the rigorous and transparent elements of the SD methodology in a spatial context. Likewise, while the independent development of spatial modeling platforms has driven individual spatial applications of SD (particularly using

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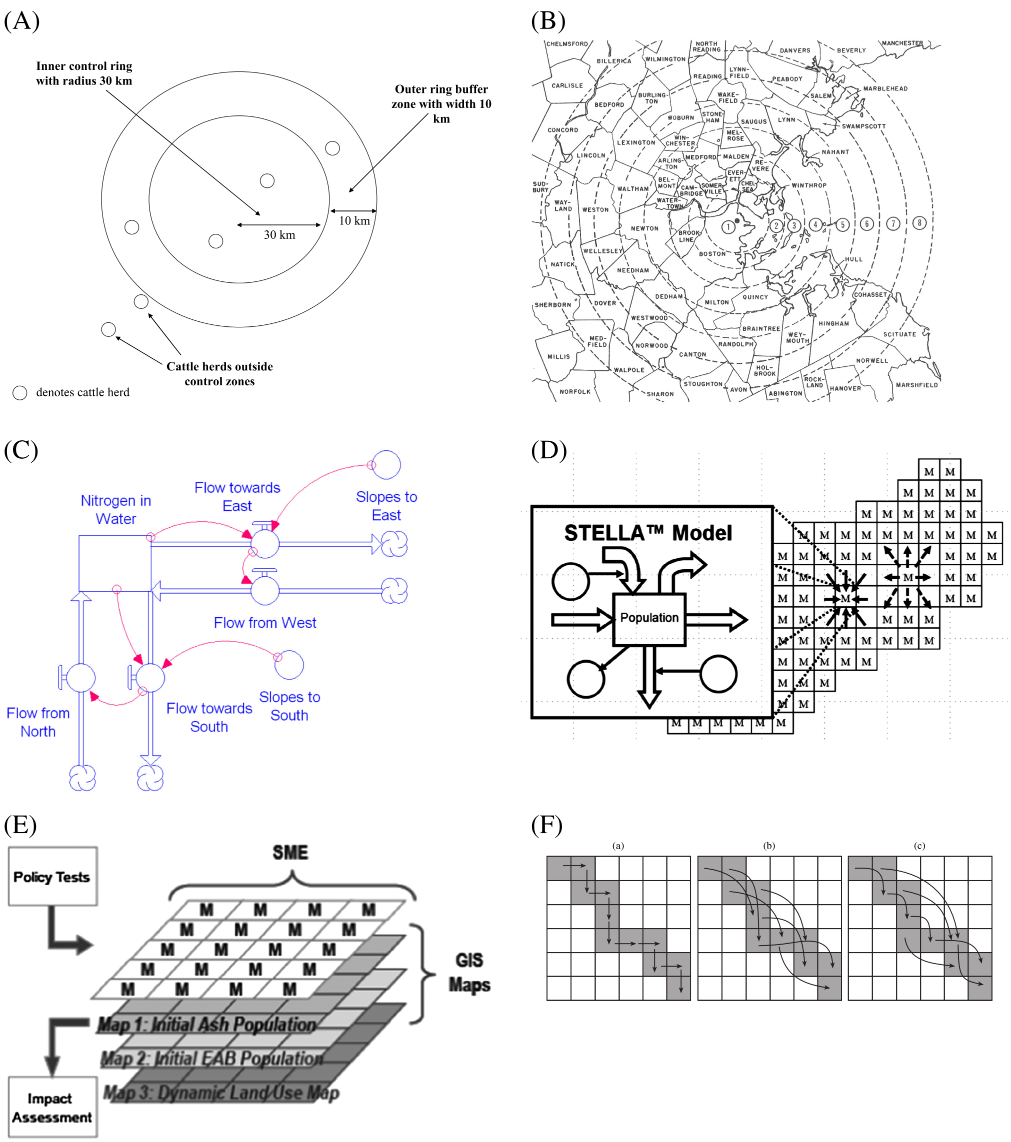

Fig. 1. Examples of spatial representation in system dynamics models. (A) Local spatial spread zones in model of South American foot-and-mouth disease spread (Rich, 2008). (B) Wils' (1974) zonal extension of the Forrester (1969) urban dynamics model. (C) Adapted from Ford's (1999) model of nitrogen flowing through a drainage basin. (D) Spatial Modeling Environment (SME) implementation of SD models in each grid cell (adapted from Voinov et al., 1999). (E) BenDor and Metcalf (2006) invasive species spread (emerald ash borer) model, implemented in SME. (F) Hydrological routing schemes used to model water moving (a) from one cell to the next one, (b) over several cells in one time step, and (c) under variable path length algorithm, the amount of water in the donor cell determines how far it travels. From Voinov et al. (2007), Patuxent watershed landscape model

well-developed spatial analytical frameworks such as cellular automata and geographic information systems), little work has actually extended the rigorous, scientific, analogy-driven, and learning-centered theory of system dynamics into spatial modeling.

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As a result, although many applications of SD models rely on spatial analysis techniques, very few spatial-SD models are created ground up using classic SD techniques extended to spatial systems—i.e., spatial reference modes, spatial-dynamic hypotheses, and rigorous selection and treatment of space itself. The application of reference modes and systemic archetypes in the spatial realm is very much a new frontier for SD research with substantial implications for the rigor and communicability of spatial-dynamic models.

In this article, our goal is to present a theoretical platform that encourages modelers to rigorously choose their representation of space, while offering a strategy for extending the system archetype concept to dynamic systems whose structure and behavior are determined by spatially explicit processes. This is important because, like the choice of modeling boundaries and timescale in all SD models, different manners of representing space (e.g. grids, networks, or zones) can completely alter the structure, composition, and functioning of spatial SD models.

We focus on several issues, including (i) the theoretical and topological arrangements of space necessary for rigorous spatial-dynamic hypothesis construction and modeling, (ii) the expansion of current two-dimensional reference modes (point data mapped through time) into four/five-dimensional modes (point data mapped over a two- or three-dimensional spatial surface and through time), (iii) the extension of system archetypes (discussed in the next section) into a spatial context, and (iv) the use of various tessellations for representing spatial structure.

While partial differential equations (PDEs) are the natural, spatial extension of the differential equation systems underlying SD models, they are only one of many approaches for modeling spatial-dynamic processes. Instead, we argue that space should be considered not as an inert container over which processes occur, but as a crucial variable whose structural modifications are integral to dynamic processes. Moreover, spatial system dynamics models must explicitly consider how space is represented in order to best suit the system being studied.

This article is organized into several substantive sections, beginning with discussions of spatial reasoning in SD and other fields, temporal and spatial feedback, and a proposed taxonomy of continuous spatial-dynamic processes. This section provides important background information on spatial modeling and the different theories of space at a modeler's disposal. Using this information, we then offer very simple, illustrative examples of spatial extensions to basic system archetypes, followed by more complex examples of spatial diffusion, simple disease spread, and more complex disease spread across a dynamic spatial network. These examples will demonstrate the manner by which space can effectively be integrated into models and offer a guide for thinking about different types of spatial reference modes and system archetypes. Finally, we conclude with a discussion of the implications of this research on the larger system dynamics research agenda.

The SD modeling process and system archetypes

As SD modeling has spread, patterns have been observed in the behavior of different systems; modelers have recognized that similar system structures often produce similar behavior, even in very different systems. As a means of avoiding wasted model-building effort and enhancing transferability of basic modeling concepts (Paich, 1985; Wolstenholme, 2003), modelers have explored archetypical dynamic hypotheses known to produce frequently encountered reference mode behaviors. These "systemic archetypes" help to explain a variety of generic system behaviors.[1] SD texts such as Sterman (2000) and Lane (1998) define numerous system archetypes, including those grouped around growth, decline and more complex combinations of simpler archetypes, such as oscillation, damped oscillation, and overshoot and collapse behavior (Breierova, 1997; Chung, 2001; these can be seen in Table 1).[2] Whether simple or complex, systemic archetypes are important and useful because they represent analogous system structures that span multiple fields, thereby aiding modeler learning processes (see Nokes and Ohlsson, 2005, for a discussion of the importance of analogies in learning). To define system archetypes more precisely,[3] we draw on the definition of system archetypes presented by Wolstenholme, who views archetypes as

> [A] formal and free-standing way of classifying structures responsible for generic patterns of behaviour over time, particularly counter-intuitive behaviour ... (Wolstenholme, 2003, p. 8)

Space in system dynamics

SD has made significant inroads into spatial modeling over the last 35 years. Zonal models, such as the one created by Wilbert Wils (1974) to extend the Forrester (1969) urban dynamics study, have attempted to disaggregate areas, such as cities, by replicating model structures to represent varying characteristics of the landscape (e.g. central business district, inner ring suburbs, outer ring exurban areas). However, although urban dynamics offered sophisticated dynamic representations of urban development processes (even in today's terms, more than 40 years later), representation of spatial heterogeneity was limited, leading to later criticisms and extensions (see, for example, Burdekin, 1979).

More recent zonal models include work by Ruth (1995), who modeled fisheries management through across four regions, Mosekilde *et al.* (1988), who modeled chaotic behavior in a two-zoned city, Rich (2008), who modeled the movement of foot and mouth disease between zones throughout South America (Figure 1A), and Pfaffenbichler *et al.* (2010), who modeled land use–transportation interactions in the city of Leeds, UK. These studies are similar in their attempts to spatially disaggregate the area of analysis in order to more accurately parameterize models, understand interactions, and improve model usability and accuracy.

The problem in confining SD spatial reasoning to this technique relates to the manner in which zonal models treat space. For example, in the Wils (1974) model, Zone 2 lies between Zones 1 and 3 and possesses some spatial extent (Figure 1B). However, within the model itself, that extent and location are largely irrelevant in determining dynamic structure; zones are modeled as interacting entities without any specific location or spatial structure. Zones, like aggregate models, continue to represent spatial areas as points, which fail to convey any information about relationships across or within space, or information about spaces themselves.

Although zonal representations may often be sufficient, they can be limiting in scenarios where substantial environmental or spatial heterogeneity determines or influences system structure and behavior (e.g. Anselin, 2002). As Douglass Lee (1973) discussed in his seminal "Requiem for Large-scale Models", much of the usefulness in modeling arises when models are used to represent sophisticated problems in usable ways. For many

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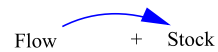
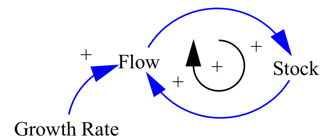
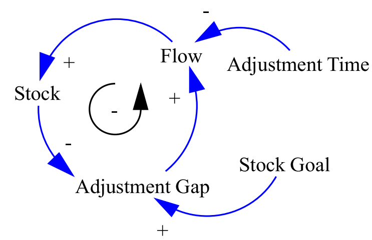
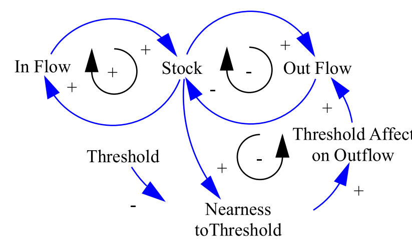
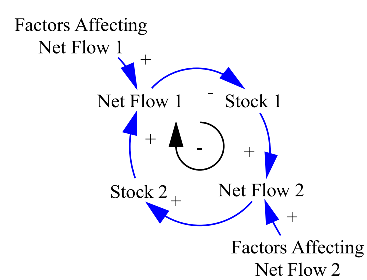
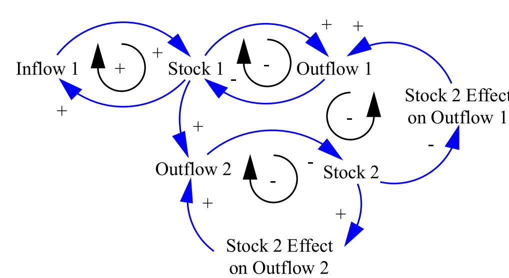
Table 1. Examples of non-spatial systemic archetypes

Systemic archetype	Governing equations	Causal loop diagram
1. Linear growth	$\frac{dS}{dt} = k$	
2. Exponential growth	$\frac{dS}{dt} = kS$	
3. Goal-seeking growth	$\frac{dS}{dt} = \frac{C-S}{k}$	
4. Logistic growth	$\frac{dS}{dt} = kS\left(1 - \frac{S}{C}\right)$	
5. Sustained oscillations	$\frac{dS}{dt} = k_S$ $\frac{dR}{dt} = k_R S$	
6. Overshoot and collapse	$\frac{dS}{dt} = k_i S - k_o S \frac{R}{C_R}$ $\frac{dR}{dt} = -k_R S$	

Note: S, R = stocks, C = goal, carrying capacity, or “normal condition” (C_R = R-related goal or normal condition), and k = constant or adjustment time [k_i = inflow, k_o = outflow, k_R and k_S = R-related and S-related constants.

problems, spatial detail greatly enhances model accuracy, visualization and communication ability, and usability.

Ford (1999) demonstrates a more precise spatial application of a simple SD model that incorporates heterogeneity in a drainage basin. This model uses a gridded landscape where stocks are represented as water levels in connected areas. (Figure 1C). Ford's (1999) model demonstrates the laborious difficulty in replicating system dynamics models in each grid cell, similar to zonal applications. Efforts to overcome this difficulty have emerged in several efforts to spatialize system dynamics models (Maxwell and Costanza, 1997b), including a study by Ruth and Pieper (1994) which loosely coupled STELLA to a geographic information system (GIS) in order to spatially model the coastal dynamics of sea-level rise. However, recent research has begun to show that tessellating space into arbitrary patterns like grids can make models highly susceptible to artifacts of grid geometry which is likely to go undetected in SD modeling (Chen and Pontius, 2011). It is extremely difficult to perform sensitivity analyses on grid resolution and size, particularly when spatial data are available at low spatial resolution.

Spatial thinking in other disciplines

Partial differential equations and cellular automata

Partial differential equations (PDEs) are perhaps the most popular method of capturing the spatial dynamics of systems across many fields (Holmes *et al.*, 1994). For example, classical equations describing motion, heat, and fluid dynamics (among others) all use space that has a structure diffeomorphic to R^n (Euclidean space). In particular, the continuity property of the real numbers, along with its ordering, is crucial to the existence and construction of solutions to these differential equations. Numerical solutions to these differential equations rely on discretizing the space–time structure using techniques such as finite difference equations and finite element method. However, as Le Gall (1999) argues, PDEs are but one of many approaches for modeling spatial-dynamic processes, and can actually be highly limiting for systems where movements across space are non-continuous or if space itself is modified during the model runs.

In characterizing this type of modeling and its limitations, Toffoli (1984, p. 121, italics in original) writes: "(a) we stylize physics into differential equations then (b) we force these equations into the mold of discrete space and time and truncate the resulting power series, so as to arrive at *finite-difference equations* and finally in order to commit the latter to *algorithms* (c) we project real-valued variables onto finite computer words ('round-off')". Instead, he argues that cellular automata (CA) form a more straightforward method of simulating physical phenomena.

CA models have long been a staple of various fields that are interested in self-replicating, pattern-based models and therefore have long history of spatial-dynamic modeling (see, for example, Couclelis, 1997). In these models, neighborhoods of active cells act according to simple behavior rules, resulting in complex, emergent behavior. While the underlying spatial framework does not need to be specified, CA models typically tessellate space into regular grids, hexagons, triangles, or other geometric shapes.

Perhaps the most sophisticated effort to tightly couple SD techniques to traditional cellular automata have emerged in systems such as the Spatial Modeling Environment

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(SME), a platform for "spatializing" SD models by replicating them into gridded cells (see Figure 1D) and parametrizing them with GIS spatial data (Maxwell and Costanza, 1997a, 1997b).

While SME, and similar frameworks are useful for a variety of applications, none of the efforts to spatialize SD modeling have attempted to "spatialize" the rigorous elements of the SD modeling process or incorporate SD's theoretical and scientific underpinnings. How modelers choose to represent space within models has deep implications for the model's ability to accurately replicate system structure and behavior. Therefore, it becomes important to understand the different ways that space can be structured.

The nature of space

The treatment of "space" within simulation models continues to be an ongoing debate within the geographic information science literature, which offers two vastly different conceptions of space: Newtonian and Leibnitzian (Galton, 2001). Newtonian space requires the underlying geography to be absolute and act as an inert container; objects acquire properties, such as position and velocity, within this geography. Newtonian space is specified independently and prior to the description of objects that inhabit it and is therefore an absolute view of space. Contrasting this is the relativist Leibnitzian model, which asserts that space is constructed through relations between arrangements of objects; that is, space is simply a way of arranging the relative positions of things that we care about. Therefore, space is not absolute, and is instead a construct generated from relative locational attributes of objects of interest. For example, the type of space that we all perceive through lines of sight only provides connection to the areas around us until the next obstruction is reached. We are not aware of the world beyond that obstruction, and our perception of distance is dependent on the location and height of the observer. The distance metric in Newtonian space is unhindered by these considerations.

While both views have different merits and problems, we argue that, for the purposes of this article and like many CA and PDE applications, Newtonian space is more readily amenable for use in most SD modeling practices. However, this is not always true, as many of the emerging SD applications in agent-based and network modeling may quickly expand beyond this absolutist treatment of spatial structure (Borshchev and Filippov, 2004). For an example of this, refer to the dynamic network model of disease spread that we show in Figure 6 (discussed below). Here, "space" is largely expressed through the relationships (or physical connections) between individuals that are susceptible, infected, or recovered from an illness, rather than the actual physical positions of individuals, which may have no bearing on their disease status or communicability.

Vectors, rasters, fields and objects

Understanding the different theoretical representations of space helps us to better understand different topological constructions of space that are available for modeling. While space has been defined topologically in a variety of ways, the spatial science literature has primarily articulated space through *vector* or *raster* frameworks. In vectorized space, objects are depicted as points, lines (connected points), and polygons (area enclosed by connected lines). In rasterized space, which is more common for spatial modeling applications, space is tessellated into a collection of plane shapes with no overlaps or gaps (sometimes squares, rectangles, or hexagons of equal shape and size, as in a grid).

The vector/raster comparison is similar to that of continuous and discretized models of time in classical SD modeling treatments. While the vector representation of space may be more accurate, it is often computationally and theoretically intractable for modeling applications. Conversely, raster representations, like discretized time steps, approximate spatial processes given the spatial resolution of a model.

The discussion of rasterized and vectorized space expresses an important difference in the way in which space can be conceptualized. The geographic modeling literature has historically characterized space by distinguishing "fields" from "objects" (Couclelis, 1992; Egenhofer *et al.*, 1999). Field-based representations of space completely and exhaustively tessellate space either into rectangular or other polygonal entities. Once a tessellation is specified (e.g. a rectangular grid or zones comprising cities or suburban regions), each location is endowed with continuous (e.g. temperature) or discrete (e.g. population) attributes, which are subject to change over time due to influence of the attributes of neighboring cells.[4]

Contrasting the field representation of space, we can think about *objects* as entities that have attributes, one of which can be location. Therefore, objects can potentially move in space and acquire new attributes. The object/field dichotomy is important to distinguish when constructing models that have (i) objects that change locations or (ii) locations that happen to have attributes. As we will see, this distinction becomes important when modeling systems where space itself can change as certain objects change locations, such as quarantine protocols that change as disease spreads across a landscape, the co-evolution of cities and their transportation networks, or flight schedules that change as individual economies grow. Irrespective of the vector and raster debate, we argue that network representations of topological relationships are an important technique for SD modeling. We motivate this in the next section.

Network representations of space

Representing space through network "topologies", which replace information about exact object locations with a network representation of object relationships, can be a very powerful tool for modelers. Network topologies can include information about the neighborhood around objects, the strength of network relationships through the imposition of weights on network links (e.g. strong social relationships, or speed limits determining rate of movement between cities), and abstract, but potentially vital, information about space itself that often cannot be captured by spatial grids (e.g. a disease spreading across a series of seemingly disconnected valleys, or a flow of information from one local economy to another nearby).

Defining relationships in spatial-dynamic systems commonly relies on measures of distance in a landscape or between system elements. Distance is often measured as simple proximity, but under network characterizations distance can also be modeled in a more sophisticated manner through the use of "spatial weights matrices" (Anselin, 2003), which are arrays that define "adjacency" in space, or reduce the bulk of information about spatial

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arrangement in a landscape to a simple representation of neighboring relationships between landscape elements. Importantly, under network topologies, space need not necessarily be linear or contiguous, as is often the case in systems where "leapfrogging" patterns are seen across space.

SD research has made several forays into network analysis including explorations of complexity and chaos, network marketing, material transfer, and immuno-dependent tumors (Alekseeva and Kirzhner, 1994; Reggiani and Nijkamp, 1995; Cruz and Olaya, 2008). These applications are excellent examples of the merged use of SD and network analysis. Although these studies did not discuss spatial-dynamic hypotheses or the justifications for spatial designs around networks explicitly, each of these studies shows how networks can facilitate construction and use of irregular tessellations of space, accommodating diverse spatial representations, including raster and vector models of landscapes, social connections and networks, and diffusion vectors. All of these common issues and concerns over spatial characterization can be unified under standard network topology.

Spatial reference modes and systemic archetypes

Extending the reference mode concept spatially would mean that reference modes would represent descriptive patterns of *spatial change* over time. Usually, these are based on historical observation, and rely, like classical reference modes, on pattern recognition to understand the type of dynamic observed. We give an example of a spatial reference mode for urban expansion of the Charleston region of South Carolina USA in Figure 2, where the spatial extent of urban land area is seen growing over three past decades (Allen and Lu, 2003). This dataset yields complex patterns through both space and time; for example, note the initial expansion to the north and west, before expansion along the coast. Spatial systemic archetypes could draw on this type of data to describe, in part or in whole, the system structure creating observed reference patterns. In order to understand how we can create spatial-dynamic hypotheses from observed data patterns, it is important first to understand some issues associated with feedback effects in spatial models.

Temporal and spatial feedback

In George Richardson's (1999) landmark work on feedback theory, he proposes that in modeling dynamic systems the direct or indirect influence of a system element on itself is based on contiguous temporal relationships. Spatial analysts often assign causal relationships to spatial behavior, but this is not possible without time. Spatial "causality" does not exist; time mediates spatial relationships, determining whether one object, affecting another across space, forms a causal influence with respect to time. This means that the causal focus of SD becomes more complex when time establishes causal relationships that form patterns across space. Since time relentlessly marches forward, the past can only influence the future and not vice versa. We can consider spatial feedback to be "bidirectional", in the sense that neighborhood relationships are, more often than not, bidirectional relationships. Unidirectional topological relationships are certainly possible and are useful in some cases, such as flow of water from higher elevation to lower elevation, and one-way streets (network representations allow for directed networks to be constructed). However, undirected networks represent topological relationships whereby processes at

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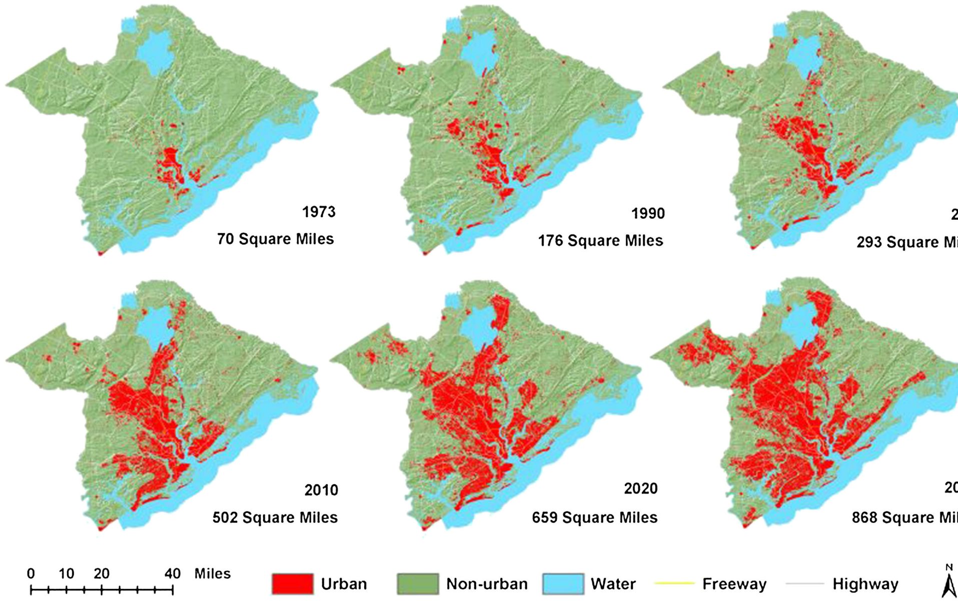

Fig. 2. Example of spatial reference mode for the urban expansion of Charleston, South Carolina. Source: Allen and Lu (2003)

one point (or node, or cell) not only influence all of surrounding neighbors in the next time step, but simultaneously all the neighbors influence the process at that point in that time step. It is therefore important to differentiate between concurrent dynamics and sequential dynamics; that is, determining how fast given dynamic processes occur versus how fast those processes influence surrounding neighbors (e.g. spread or diffusion).[5]

In BenDor and Metcalf's (2006) study of the spread of an invasive insect (Figure 1E), creating rules for spread dynamics involved specifying the neighborhood into which insects could travel. This neighborhood was partly dependent on the size of the dynamic time step chosen. Choice of a large time step would necessitate a larger spread neighborhood, or else spread would be artificially slowed, as insects would be technically unable to move great distances in successive time steps.

This issue can be also seen in Figure 1(E), which depicts varying possibilities in Voinov *et al.*'s (2007) Patuxent landscape model, which models water flow between surrounding grid cells, where neighborhoods consist of (a) contiguous cells only, (b) a larger, second ring of cells, and (c) a dynamic structure where distance of flow from a cell is based on water depth. This example illustrates the complexities of linking neighborhood size, model resolution, and time step to the dynamic processes modeled.

Using networks for system archetypes

A graph G is a collection of edges E, which define topological relationships between a set of vertices V. For a given vertex v, $N(v)$ is the set of all neighbors of v. It is now sufficient to re-characterize tessellated space as a graph, where vertices represent polygons and edges represent spatial proximity (topological connections; see Figure 3). In this graph-based

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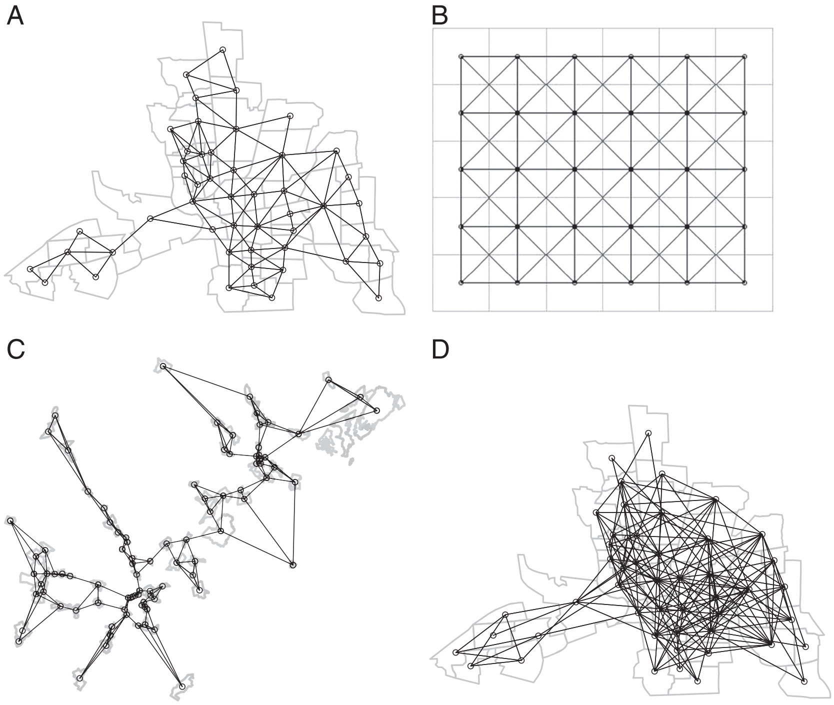

Fig. 3. Network representations of space. (A) Network representation of complex, non-uniform polygon map (Columbus, Ohio, neighborhoods; Anselin, 2003). (B) Nearly "regular" graph as network representation of a grid—each node is equally connected to all contiguous neighbors. (C) Non-contiguous neighborhood connections among spatially disconnected parcels. (D) Example of "second-order" connections, where polygons are connected to all neighbors of their neighbors. Note that the number of connections have increased geometrically from that of panel A.

spatial structure, a dynamic process can occur on the nodes, edges, or a combination of both.

Many different kinds of underlying spatial entities can be represented using a network. Figure 3(A) is representation of contiguous polygons that affect one another through their neighborhood spillovers. Similarly, a regular grid translates to a near-regular graph (a graph is "regular" when the "degree" of all vertices, defined as the size of a given vertex's neighborhood, is equal; Figure 3B). Figure 3(C), on the other hand, is a representation of non-contiguous polygons. However, the processes in one of these polygons may affect its nearest neighbors, irrespective of whether those neighbors share a boundary.[6]

It is therefore important to realize that contiguity does not guarantee connectivity. Rather, connectivity is determined by the problem in question and the particular spillover effects that necessitate modeling. For example, "second-order" contiguity can be represented in a simple graph even though it necessitates links between polygons that are typically one link removed from each other (i.e. a neighborhood constructed entirely out of your neighbors' neighbors; see Figure 3(D) and note that neighboring polygons are not connected in the network).

Once such a network is constructed, archetypical patterns created by spatial-dynamic models are fairly straightforward to observe; we can characterize the behavior of any given node s_i as a function of its own dynamics, and the dynamics of its neighboring nodes $s_{N(i)}$, respectively: $\frac{ds_i}{dt} = f(s_i, s_{N(i)})$.

The advantage of this articulation is that explicit spatial connections and feedbacks are represented through links. Directional relationships can be easily mediated through the direction of the links. For example, a hydrological model that is accounting for the flow of the runoff can have directional link between the nodes at higher elevation to the nodes at lower elevations, in addition to the neighborhood relationship. Such directional relationships also help simplify the model articulations as opposed to grid frameworks, which necessitate additional information (e.g. elevation data that allows flow direction calculations).

In order to explore the systemic archetype structures underlying different types of continuous spatial-dynamic behavior, we divide spatial-dynamic processes into two different categories, extensive and intensive.[7] Extensive spatial processes involve change at the margins (i.e. processes that flip a point/region in space from being within a domain of the process to outside the domain, or vice versa). Under intensive spatial processes, on the other hand, the value of the process at each point affects the value of its neighbors. While it may seem that extensive processes are a subset of the intensive processes, it will be useful to think of them separately while formulating spatial system archetypes.

Extensive processes

Extensive processes describe the extent of boundaries or characterize changes in boundaries over time. An example of this would be a model of the region into which a given technology has diffused, with the edges of the region gradually changing as new areas adopt the technology. Extensive spatial processes are analogous to Markov processes, where the value at the next time step is dependent only on the current value, not on history (Bhat and Miller, 2002). In a sense, these are strictly binary processes, where Newtonian space is divided into regions either inside or outside the domain of the process. Under these conditions, the process of expansion of the boundary can be described by archetypes that are very similar to those of aspatial process.

For example, Table 2 provides a series of example analogues to the basic system archetypes discussed in Table 1. In these extensive process examples, rather than describing changes to a bank account or population stock, as in aspatial system dynamics models, the equations describe changes in the area of a circle, as expressed in changes to the radius of that circle. We can visualize these very simple examples of extensive processes as changes in the extent of a circle over time, which we depict in the final column as a series of nested circles that depict the circle's growth through time (numbers in each graph show connections to the aspatial archetypical behavior over time). We should note that, although these circles could move towards representing the spatial expansion of an entity like a city (e.g. urban land use), they are not in any way typical "spatial patterns", but rather are examples of patterns created by simple spatial-dynamic processes. More complex examples directed at city growth, for example, could include edge directed or constrained growth (e.g. urban growth hugging a highway) that exhibit linear, exponential, or other kinds of dynamic patterns, which could be visualized in similar ways.

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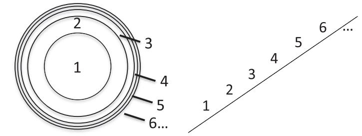
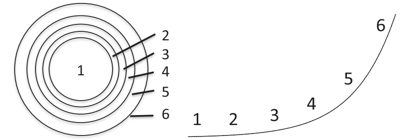
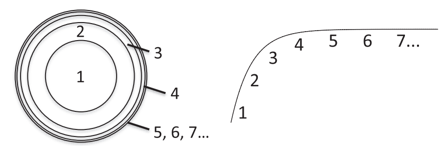
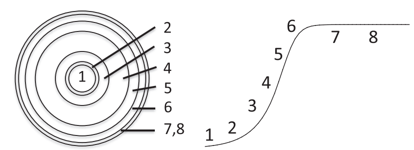
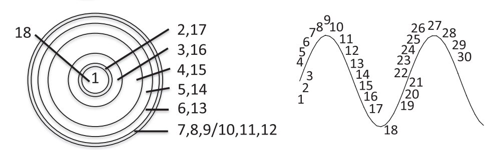
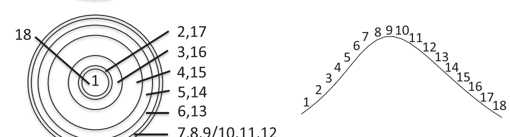
Table 2. Simple examples of spatial systemic archetypes where patterns are specified for change in the area of a circle (*not the radius*)

Systemic archetype	Governing equations	Spatial-dynamic visualization	Realization on a network
1. Linear growth	$\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{\mathrm{k}}{2\pi r}$		$\Delta n=\frac{k}{s}$
2. Exponential growth	$\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{kr}{2}$		$\Delta n=kn$
3. Goal-seeking growth	$\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{C}{2\pi kr}-\frac{r}{2k}$		$\Delta n=N-\frac{n}{k}$
4. Logistic growth	$\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{kr}{2}\left(1-\frac{\pi r^2}{C}\right)$		$\Delta n=kn\left(1-\frac{n}{N}\right)$
5. Sustained oscillations	$\frac{\mathrm{d}r_1}{\mathrm{d}t}=\frac{kr_2^2}{2r_1}\frac{dr_2}{dt}=\frac{kr_1^2}{2r_2}$		$\Delta n_1=kn_2$ $\Delta n_2=kn_1$
6. Overshoot and collapse	$\frac{\mathrm{d}r_1}{\mathrm{d}t}=r_1\left(k_i-\frac{k_o\pi r_2^2}{C_R}\right)\frac{dr_2}{dt}=\frac{k_R r_1^2}{2r_2}$		$\Delta n_1=k_i n_1-k_0 n_1\left(\frac{n_2}{N_2}\right)$ $\Delta n_2=-k_r n_1$

Note: S, R = stocks, C = goal, carrying capacity, or "normal condition" (C_R = R-related goal or normal condition), k = constant or adjustment time [k_i = inflow, k_o = outflow, k_R and k_S = R-related and S-related constants], Δn = new nodes inducted into a domain in a time step, s = area represented by a node, and N = total number of nodes

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Intensive processes

The extensive process characterization does not allow for multidirectional feedbacks that become important in spatial-dynamic processes. Under intensive processes, rather than only considering the boundaries covered by the spatial dynamics of spread or change, we focus instead on the multidirectional feedbacks that create complex dynamics internal to our domain of interest (e.g. density dependent growth and overshoot and collapse behavior within individual cells during invasive species spread; BenDor and Metcalf, 2006). To account for these feedbacks, we will explore how to characterize most forms of continuous and non-continuous space as a topological network.

It is more difficult to construct archetypes out of intensive processes, as we have to consider the behavior of each node, not just the nodes at the edge of the domain. Take a simple example on constant growth, where s_i is the stock in each node and $S = \sum_{i \in G} s_i$, where G is the network (see Figure 4) and S is the stock of the total system. If the rate of change of S is a constant k (i.e. $\mathrm{d}S/\mathrm{d}t = k$), then $\sum \mathrm{d}s_i/\mathrm{d}t = k$, i.e. the sum of individual node changes should add up to the constant. However, $\mathrm{d}s_i/\mathrm{d}t = f\left(s_i, s_{N(i)}\right)$, where $s_{N(i)}$ are the stocks in neighboring nodes. This means that it is very difficult to estimate exactly how k can be apportioned between each of the nodes.

We can think of this problem in the opposite direction by considering the system archetypes that occur in each node, rather than the whole system itself. When there are no neighbor interactions, and assuming constant growth within each node, $\mathrm{d}s_i/\mathrm{d}t = k \rightarrow \mathrm{d}S/\mathrm{d}t = Pk$, the system exhibits a constant growth archetype, where P is the total number of nodes. As soon as we start including neighbor interactions within the dynamic equation (e.g. $\mathrm{d}s_i/\mathrm{d}t = k + r\sum_{j \in N(i)} s_j$, where r is another constant), we cannot derive a global system archetype from the local archetype. However, although a general solution is not intuitively observable without running the simulation,[8] it is still possible to find archetypes in spatial network models that are very similar to aspatial archetypes as seen in the examples below.

Examples of system archetypes in network models

We can now characterize any of the basic spatial system archetypes listed in Table 2 using arbitrary graph structures to characterize tessellations of space. To do this, we create a random array of nodes, connected through a random graph using *Netlogo* 5, a spatial, dynamic, and agent-based modeling framework (Wilensky, 1999).

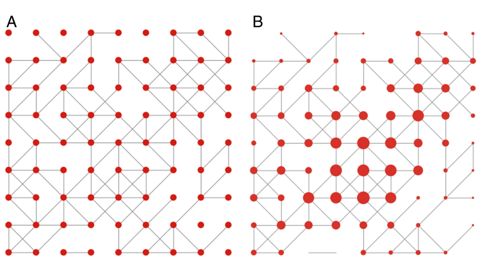

Fig. 4. Intensive process examples on random networks. Size indicates each node's relative stock size as determined from the size of the stocks in connected nodes. The nodes are initially equally sized (A), but change as their connections determine their sizes in the future (B)

Extensive processes on a network

One can begin imagining extensive processes on a network as essentially a representation of the nodes that are part of the process domain or not, at any given instant. The rate at which boundary nodes are inducted into the domain depends on the rate of conversion at the edges that determine the archetype. If each node represents a fixed spatial area, linear growth models essentially induct the same number of neighboring nodes into the domain at every time step, while an exponential growth inducts neighbors that are proportional to the number of nodes already in the domain (see Table 2).

As we can visualize in Figure 1(D–F), including only the "neighbors" of existing cells does not produce the requisite exponential growth archetype in a grid representation of space. Unfortunately, as is the current common practice in spatial SD models (BenDor and Metcalf, 2006; Voinov *et al.*, 2007), modelers must create sophisticated neighborhoods (e.g. Figure 1F), to simulate spatial growth that occurs at increasing rates. This example illustrates the importance of neighborhood choice and its affects on system behavior and system archetypes.

Intensive processes: SIR model on a static network

However, the real advantage of a network representation is in the co-evolution of spatial structure that accompanies an intensive process. We implement a simple disease spread model, commonly known as an SIR (susceptible-infected-recovered populations) model (Sterman, 2000) to demonstrate an intensive process.

We begin with a random graph representing connections between different nodes (e.g. road connections between neighboring towns; Figure 5C). Within each node, an individual SIR model (Figure 5A) diffuses sick individuals into nearby nodes based on (i) diffusion rate d, (ii) the number of sick individuals in the surrounding nodes (I_v; v is the neighborhood set of i.), and (iii) the number of susceptible individuals in the target node (S_i). As shown in the equation below, the diffusion rate (d) modifies the infection rate (r_f). The number of sick individuals in the target node is also influenced by the infection rate (r_f) multiplied by the susceptible (S_i) and infected (I_i) proportions of the population (P_i) and the rate of recovery (r_r):

$$I_{i_{t+1}} = I_{i_t} + \left(d \frac{\sum_{j \in N(i)} I_{j_t}}{\sum_{j \in N(i)} P_{j_t}} + r_f \frac{I_{i_t}}{P_{i_t}} \right) S_{i_t} - r_r I_{i_t}$$

The infection (signified by squares) begins near the lower right corner (panel D), spreading faster to more highly connected nodes (panel E), eventually hitting the upper left corner (further away, as measured by network distance), but completely missing non-connected nodes (see pocket of nodes at lower left, and two individual nodes on the right side of the graph). After the infection has swept through the network (panel E), infected individuals begin to recover (triangles), which sweep through the network as another wave (panel F). An aggregate measure of the infected and recovered populations mimics classic SIR model behavior (panel B; Sterman, 2000). The overshoot and collapse behavior in aspatial SIR models is depicted as a series of propagating waves as different stocks come to dominate each node sequentially across the space.

In a standard grid framework, spatial interactions are usually represented as ad hoc constraints on top of the Newtonian spatial structure (e.g. a river that obstructs migration

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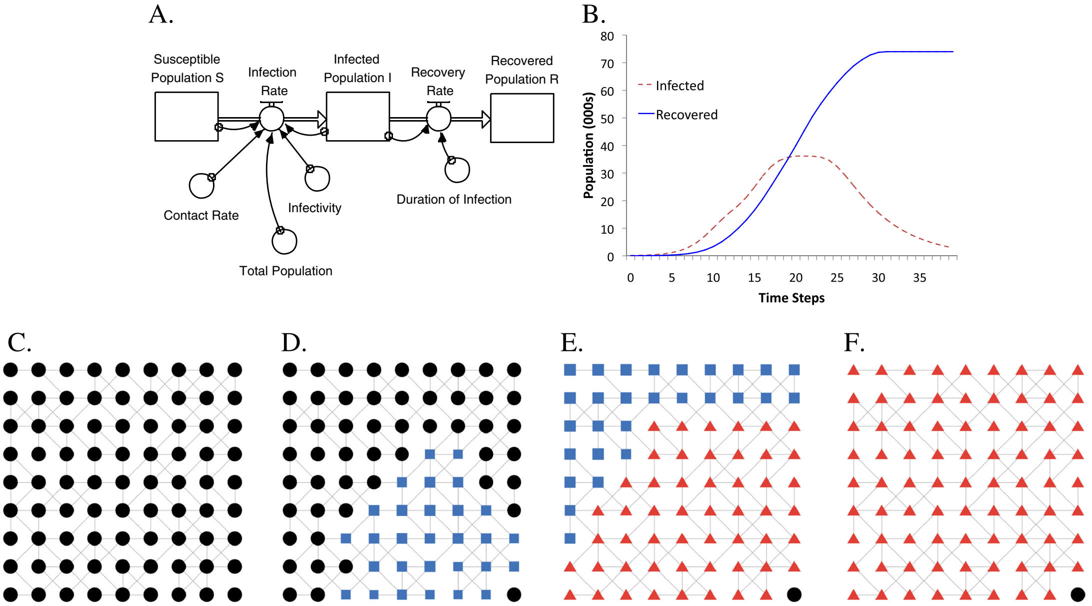

Fig. 5. Network representation of classic, a-spatial SIR model (A; Sterman 2000). Aggregate dynamic pattern of total population mirrors classic model (B). Shapes determine dominant type of population that changes through the wave of infection; black circles indicate initial susceptible populations, which become infected and turn into squares, and begin to recover as indicated by triangles. (C) SIR model initialization, (D) timestep = 10, (E) timestep = 20, (F) timestep = 30 (full recovery)

between adjacent grid cells). Under a grid structure, much of the dynamics would be lost as the uniform connections assumed between neighboring grid cells would not mimic the uneven diffusion patterns seen in the randomly connected network model. Furthermore, the network framework allows us to explore how the underlying spatial structure affects the progression of the disease in both space and time; that is, the logistic growth archetype exhibited by the infected population depends not only on the infected and recovery rate, but on the degree distribution (spatial structure) of the nodes as well.

While this is a key advantage over other approaches of representing space, the far greater utility of this network framework is seen when we explore what happens when we allow the model to influence the structure of space itself during the course of the simulation. To illustrate this we represent a quarantine policy in an SIR model that incorporates a dynamic network.

SIR model on a dynamic network

Although dynamic networks can add nearly infinite complexity to models (Breiger *et al.*, 2003), networks can be easily altered by adding weights that represent links as binary connections (e.g. on/off, social connection/no social connection), or as fuzzy-valued weights defining the strength and frequency of interaction (e.g. acquaintances, good friends, spouse). This network becomes dynamic when these weights can change over time, either independently (stochastically) or conditioned on the attributes of the nodes the links connect (Figure 6).

For example, instituting a quarantine policy (e.g. triggered when the infected population within a node reaches >30%) that attempts to shut down disease diffusion by eliminating

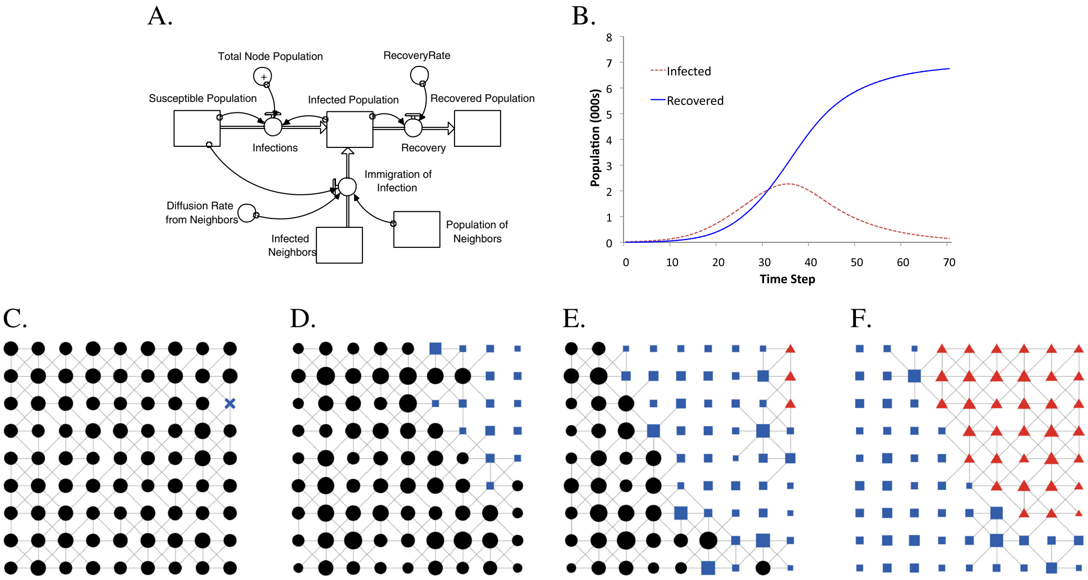

Fig. 6. Dynamic network representation of Classic SIR model (A; Sterman 2000), where aggregate behavior (all nodes) again mirrors classic model (B). Shapes determine dominant type of population (size determines relative number); circles indicate susceptible, squares indicate infected (infection origin noted with "X"), and triangles indicate recovered populations that dominate the node. (C) SIR model initialization. (D) timestep = 20, (E) timestep = 30, and (F) timestep = 40. Note that node links disappear during infection and re-establish after recovery

links will drastically alter the spread and recovery pattern (e.g. Figure 6E). In our example, the links are restored when the infected population proportion is less than 10% (see Figure 6F). Therefore, the space itself co-evolves with underlying dynamic processes, thus better representing the complex dynamics of quarantine policies and their spatial effects. As a result, we see a different representation of wave propagation across space as disease takes hold, nodes are quarantined, populations recover, and reconnections are made (see, for example, Barabasi and Albert, 1999). Similar to aspatial SIR models, quarantine and interaction policy changes do not affect the inherent structural dynamics of the model (a series of waves across space and time), but rather change the effect of where, and how quickly, those waves travel, and how many people get sick (Watts *et al.*, 2005).

In many SD models, spatial structure is not assumed to change during the processes. This is inadequate to represent many processes, such as nonlinear co-evolution of the road network with local urbanization (the placement of roads affects the spatial structure of cities and the structure of cities affects the future location of roads). The processes of interest dynamically affect space and its structure, and a network representation helps us move beyond the Newtonian representation of space towards a more Leibnitzian characterization.

Conclusions and discussion

Spatial system dynamics models are not new. However, contrasting the rigorous, scientific process of defining causal mechanisms in dynamic systems, little thought seems to be given to how and why we represent space in SD models.

In this article we posit three major arguments: (i) it is possible, and often absolutely necessary, to model spatially; SD models should not be limited to non-spatial applications as they now often are (see the frustrations in Ford, 1999, for an example of this); (ii) spatial SD modeling should be as rigorous as non-spatial modeling in using reference behavior, forming dynamic hypotheses, and being understandable to wide audiences for the purposes of peer review and checks on model integrity and performance; and (iii) the decision of how, when, where, and why to represent space (e.g. vector or raster, network or grid) is a choice that is as important as how many stocks and flows to include in a model.

This article pursues a unified, theoretical underpinning to inform how we can rigorously represent space in SD models, and how we can begin to create a library of archetypical spatial-SD structures that represent common spatial-dynamic behaviors. To do this, we argue that it is possible portray spatial processes as either extensive or intensive, depending on whether we are concerned with the behavior at the boundary, or within the boundary, of a given space.

Recent spatial-system dynamics research has articulated "space" as a tessellation into regular grids (BenDor and Metcalf, 2006). However, recent research has shown that this process is highly susceptible to artifacts of grid geometry (Chen and Pontius, 2011), which is likely to go undetected in SD modeling. We argue that in order to abstract away the artifacts of this tessellation, we should instead view spatial interactions as they occur across a topological network that defines the underlying structure of space. One way of doing this—by articulating space through networks—allows us to abstract away arbitrary grid representations and more rigorously (and easily) study how models are affected by particular spatial representations.

The weighted network model that we discuss in our final example endows attributes to both nodes and links, allowing us to model the co-evolution of space alongside dynamic disease processes. This contrasts with raster-based SD models, where spatial pattern is determined by collecting the homogenous values of the processes within a grid, requiring that the underlying spatial structure remains invariant. The network representation of space treats the spatial relationships themselves as dynamic and therefore allows for changes in the local spatial structure affecting the global process dynamics.

Building on years of visualization research in aspatial SD, future research must continue to explore spatial-dynamic visualization techniques. In Table 2, we depict some very simple examples of archetypical behavior of extensive processes and potential modes of visualization. However, intensive processes are not, in our experience, easily amenable to such visual representations. Extending models spatially means abandoning common, 2-D graphical visualizations of the behavior of system elements. Rather, methods and software need to be developed for exploring 4-D or 5-D (three dimensions, time, and value) representation of maps and networks.

As the system dynamics method evolves and becomes more sophisticated, strong theories informing model spatialization and the spatial-dynamic modeling process will become increasingly important. Many of the considerations that currently introduce rigor into the SD modeling process, including the use of historical behavior as reference mode information, dynamic hypothesis creation, and iteration in the model construction process, have spatial analogues. The same rigor should be used in (i) determining spatial representations (zonal, gridded, vector, network, etc.) and (ii) thinking through archetypical spatial processes. Expanding the scientific basis of SD into the spatial realm will enrich both the

SD and spatial science and enable modelers to create more accurate, useful, and usable spatial-dynamic models.

Notes

1. System archetypes are also sometimes referred to as "generic structures", "atoms of structure", or "micro-structures" (Paich, 1985; Lane, 1998; Wolstenholme, 2003, 2004).
2. A variety of more complex archetypes have been created that necessitate entire studies to fully explain, including Forrester's (1975) market growth archetype and Ulli-Beer *et al.*'s (2010) structure simulating the basic traits of acceptance–rejection dynamics.
3. See Lane and Smart (1996) for an extensive discussion of the evolution of generic structure and system archetype definitions.
4. Sensitivity to changes in spatial resolution is an important area of study in the spatial analysis and modeling fields. For more in-depth analysis of the impact of spatial resolution and neighborhood structures on CA and grid cell modeling output, we recommend Marceau and Moreno (2008).
5. Because SD models are constructed on a "serial" computer, it is imperative to understand the quirks of software in handling concurrency. For example, software can number cells/nodes/points and calculate dynamics in each sequentially, or it can move north to south and west to east calculating in order of cell/node/point position.
6. Conversely, it may be possible that the space could be represented as clusters of disconnected components (e.g. isolated areas), rather than a connected graph. However, this does not affect the construction of archetypes since the processes in each isolated component do not affect each other, allowing us to model processes in each component independently.
7. SD models classically do not consider discontinuous processes (e.g. discrete event modeling; Banks *et al.*, 2004). Under the same line of thinking, we also do not consider discrete event modeling (such as spatial Poisson processes; Cox and Isham, 1980) in the spatial context.
8. There are some graphs (e.g. in perfectly regular graph), where solutions can be empirically derived.

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