Automatic Transactional Systems: Project

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Introduction

Forecasting stock prices has been a well-known challenge for financial analysts. As part of our final project we forecasted returns and close prices of a time series of a chosen stock (Johnson & Johnson). We developed two models: AR-GARCH and LSTM for return/price prediction. We built a long-short trading setup based on signals from these predictions. We assessed performance of each of the models using Sharpe ratio on the out-of-sample period, which consistuted the last 10% of the time series.

Global variables were defined to be used across both trading strategies. They were initialized with capital letters.

In [ ]:
```
TRANSACTION_COSTS = 0.0005 # cost is 5bps of trade value
```

Literature Review

AR (Autoregressive) models are frequently used across statistics; also in trading stratgies, as they provide a framework for explaining variables with lagged iterations of the variables. GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models are widely used in trading strategies for their ability to model and forecast volatility in financial time series, capturing time-varying volatility clustering effectively. LSTM (Long Short-Term Memory) models, a type of recurrent neural network, excel in handling sequential data and capturing long-term dependencies, making them suitable for predicting stock prices and trends based on historical data. Recent studies highlight that while GARCH models are robust in volatility prediction, LSTM models often outperform in price prediction due to their non-linear processing capabilities. Combining both models can leverage GARCH's strength in volatility estimation and LSTM's prowess in trend prediction, potentially enhancing trading strategy performance.

Data: AR-GARCH

In [ ]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
from datetime import datetime
from statsmodels.tsa.stattools import acf
from statsmodels.tsa.stattools import pacf
from statsmodels.graphics.tsaplots import plot_acf
from statsmodels.stats.descriptivestats import describe
from statsmodels.stats.diagnostic import het_arch
from statsmodels.stats.diagnostic import acorr_ljungbox
import scipy as scipy
import arch
```

Johnson & Johnson, a pharmaceutical company listed on the NYSE, was chosen for this project. It is a well established company contributing to the S&P500 index with no long-term directional movement in price over the last several years. It is a good candidate for testing the true power of autoregressive and machine learning models and displays high liquidity.

Instead of 6 years of close prices, 2 months of 5-minute prices from 19/04/2024-07/06/2024 were used. This translated to 2574 observations, more than the required 1512 (252 trading days x 6 years alternative).

As a fallback, missing values were populated with last available observations. Log returns were calculated on close prices and scaled by 1000 for model calibration to prevent obtaining inaccurate and spurious results. These were converted back for calculating P&L.

In [ ]:
```
jnj_raw = pd.read_csv('../jnj.us.txt')[["<DATE>", "<TIME>", "<OPEN>", "<CLOSE>"]].rename(columns={'<DATE>' :  '
                                                                                                    '<TIME>':
                                                                                                    '<OPEN>':
                                                                                                    '<CLOSE>':
jnj_raw
```

Out[ ]:

	Date	Time	Open	Close
0	20240419	153000	146.150	144.8000
1	20240419	153500	144.800	145.4200
2	20240419	154000	145.440	145.6200
3	20240419	154500	145.660	145.9858
4	20240419	155000	145.975	146.1800
...	...	...	...	...
2569	20240607	213500	147.250	147.1700
2570	20240607	214000	147.170	147.0500
2571	20240607	214500	147.050	146.9300
2572	20240607	215000	146.930	147.1700
2573	20240607	215500	147.175	147.0800

2574 rows × 4 columns

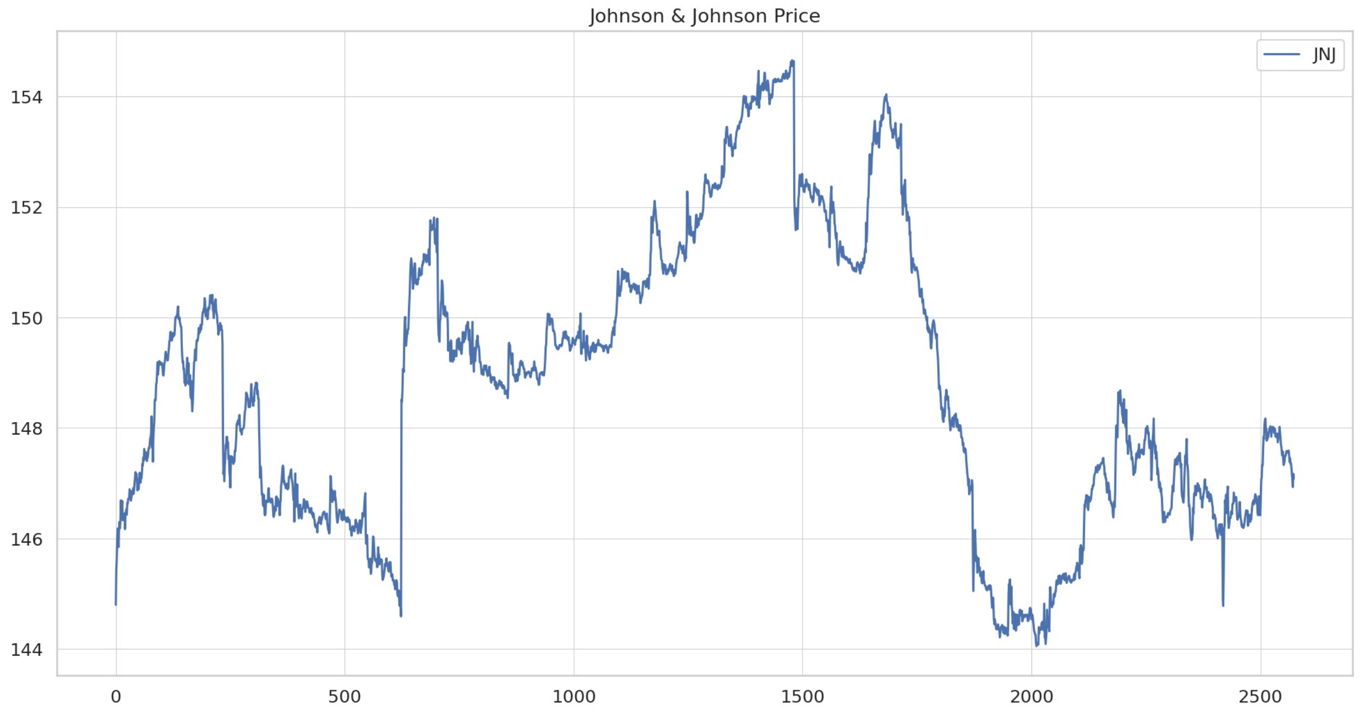
In [ ]:

```
jnj_raw['DateStr'] = jnj_raw['Date'].astype(str)
jnj_raw['TimeStr'] = jnj_raw['Time'].astype(str)

jnj_raw['Date'] = jnj_raw['DateStr'].apply(lambda x: f"{x[:4]}-{x[4:6]}-{x[6:]}")
jnj_raw['Time'] = jnj_raw['TimeStr'].apply(lambda x: f"{x[:2]}:{x[2:4]}:{x[4:]}")

jnj_raw['Datetime'] = pd.to_datetime(jnj_raw['DateStr'] + ' ' + jnj_raw['TimeStr'])
jnj_raw = jnj_raw.drop(columns = ['Open'])
jnj_raw = jnj_raw.rename(columns = {'Close': 'JNJ'})
```

In [ ]:

```
plt.figure(figsize=(16, 8), dpi=150)

jnj_raw['JNJ'].plot(label='JNJ')
plt.title('Johnson & Johnson Price')
plt.legend()
```

Out[ ]: <matplotlib.legend.Legend at 0x7fe85179dd50>



In [ ]:

```
returns = jnj_raw.fillna(method='ffill')
tickers = ['JNJ']

for ticker in tickers:
    returns['Ret_' + ticker] = (np.log(returns[ticker]) - np.log(returns[ticker].shift(1))) * 1000 # series sca

returns = returns.dropna() # only nulls left are the starting returns; will also have an issue with negative pr

returns.head()
```

```
/tmp/ipykernel_6274/2988824415.py:1: FutureWarning: DataFrame.fillna with 'method' is deprecated and will raise
in a future version. Use obj.ffill() or obj.bfill() instead.
  returns = jnj_raw.fillna(method='ffill')
```

Out[ ]:

	Date	Time	JNJ	DateStr	TimeStr	Datetime	Ret_JNJ
1	2024-04-19	15:35:00	145.4200	20240419	153500	2024-04-19 15:35:00	4.272627
2	2024-04-19	15:40:00	145.6200	20240419	154000	2024-04-19 15:40:00	1.374382
3	2024-04-19	15:45:00	145.9858	20240419	154500	2024-04-19 15:45:00	2.508868
4	2024-04-19	15:50:00	146.1800	20240419	155000	2024-04-19 15:50:00	1.329382
5	2024-04-19	15:55:00	146.0800	20240419	155500	2024-04-19 15:55:00	-0.684322

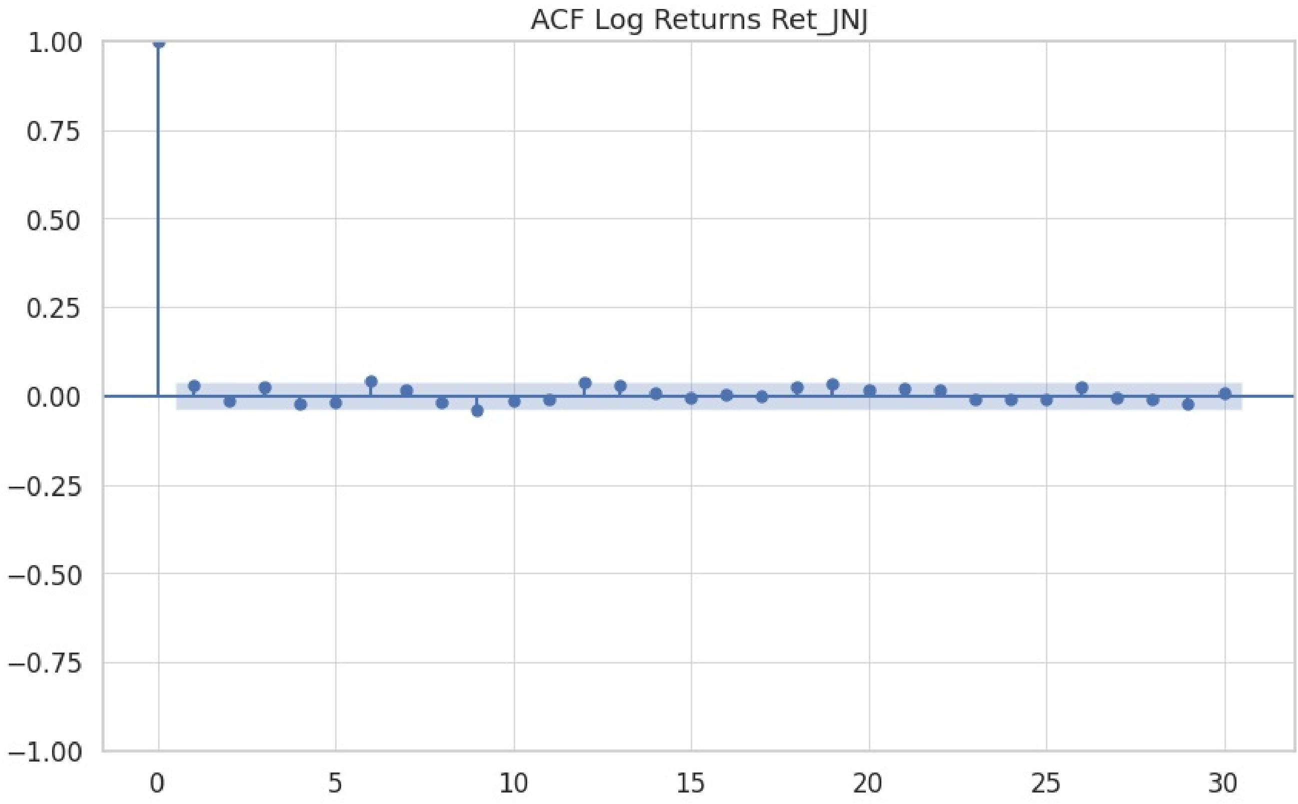
Statistics

In [ ]:
```
class data_extraction:
    def __init__(self, data, asset):
        self.data = data
        self.asset = asset
        self.series = self.extract_series()

    def extract_series(self):
        return self.data[self.asset]
```

In [ ]:
```
class analysis(data_extraction):
    def __init__(self, data, asset):
        data_extraction.__init__(self, data, asset)
        self.x = self.norm_dist()

    def acf_log_ret(self):
        plot_acf(self.series, lags=30, title='ACF Log Returns ' + self.asset)
        plt.show()

    def acf_sq_log_ret(self):
        plot_acf(self.series**2, lags=30, title='ACF Squared Log Returns ' + self.asset)
        plt.show()

    def describe(self):
        return describe(self.series)

    def norm_dist(self):
        mu, sigma = np.mean(self.series), np.std(self.series)
        x = np.random.normal(mu, sigma, 1000)
        x = pd.Series(x, name='Normal Distribution')
        return x

    def asset_hist_v_norm(self):
        fig, ax1 = plt.subplots()
        ax2 = ax1.twinx()
        ax2.grid(False)
        ax1.hist(self.series, bins=30)
        g1 = sns.kdeplot(self.x, ax=ax2, color='r')
        g1.set(ylabel=None)
        g1.set(yticklabels=[])
        plt.title('Histogram vs Normal Distribution ' + self.asset);

    def asset_dist_v_norm(self):
        sns.set_style('whitegrid')
        series_norm = pd.concat([self.series, self.x], axis=1)
        sns.kdeplot(data=series_norm, bw_method=0.5)
        plt.title('Distribution vs Normal Distribution ' + self.asset);

    def arch_test(self):
        return het_arch(self.series)

    def arch_test_lag5(self):
        return het_arch(self.series, nlags=5)
```

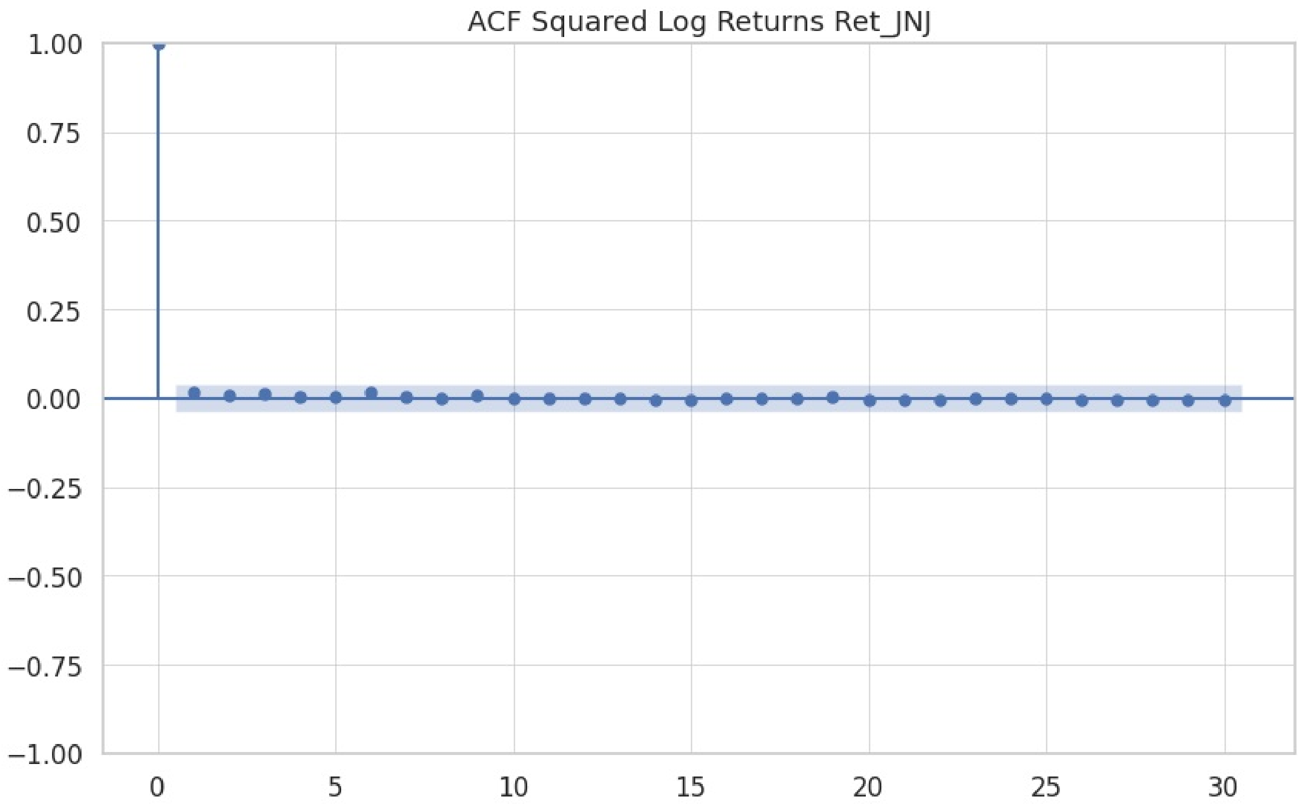
In [ ]:
```
analysis_jnj = analysis(returns, 'Ret_JNJ')
```

In [ ]:
```
analysis_jnj.acf_log_ret()
```



ACF of log returns indicates some AR/MA property among returns with lags 1, 4, 6, 6 being or almost being statistically significant at 5% confidence level.

In [ ]:
```
analysis_jnj.acf_sq_log_ret()
```



ACF of squared log returns indicates low likelihood of AR property among squared returns for (G)ARCH with lags not being statistically significant.

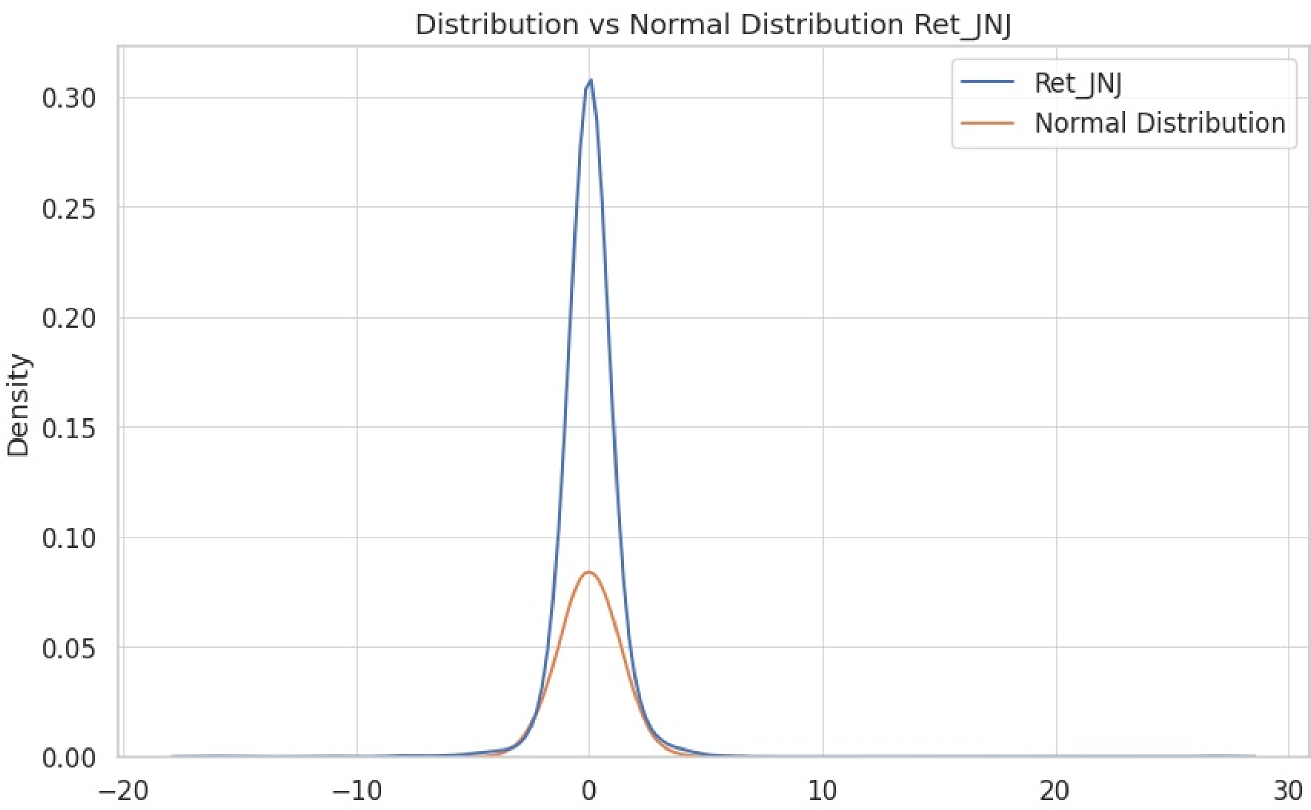
In [ ]:
```
analysis_jnj.describe()
```

Out[ ]:

	Ret_JNJ
nobs	2.573000e+03
missing	0.000000e+00
mean	6.071969e-03
std_err	2.346566e-02
upper_ci	5.206382e-02
lower_ci	-3.991988e-02
std	1.190290e+00
iqr	8.986146e-01
iqr_normal	6.661440e-01
mad	6.749249e-01
mad_normal	8.458929e-01
coef_var	1.960303e+02
range	4.288533e+01
max	2.678381e+01
min	-1.610152e+01
skew	2.789157e+00
kurtosis	1.216685e+02
jarque_bera	1.513068e+06
jarque_bera_pval	0.000000e+00
mode	0.000000e+00
mode_freq	2.409639e-02
median	0.000000e+00
1%	-3.127120e+00
5%	-1.379044e+00
10%	-9.634844e-01
25%	-4.444967e-01
50%	0.000000e+00
75%	4.541179e-01
90%	9.688703e-01
95%	1.474602e+00
99%	3.028375e+00

- Positive skewness: larger positive returns than negative ones are prevalent, which is unusual for equities
- Very high kurtosis: fluctuations in returns resulting in much fatter tails than expected of a normal distribution; somewhat expected given the frequency of the data
- jarque_bera_pval close to zero: H0 normality rejected

In [ ]:
```
analysis_jnj.asset_dist_v_norm()
```



In [ ]:
```
analysis_jnj.arch_test()
```

Out[ ]:
```
(2.035973237564, 0.9960569359573243, 0.202884681235908, 0.9960952426298892)
```

In [ ]:
```
analysis_jnj.arch_test_lag5()
```

Out[ ]:
```
(1.0866108654935305,
 0.9552819430219457,
 0.2169061916290882,
 0.9554283519084472)
```

- H0 no arch effects not rejected (large p-value): likely no arch effects in data

There were some mixed results regarding persistence of returns of Johnson & Johnson equity price. Whilst there were no arch effects, there was some indication of autoregression in the mean equation and an opportunity to exploit the fat tails of the leptokurtic distribution.

Methodology: AR-GARCH

Model

The first model was AR-GARCH. The AR (autoregressive) component defines the mean equation and regresses the variable on lags of itself with the lags specified denoting the persistence. The GARCH (generalised autoregressive conditional heteroskedasticity) component defines the variance equation and regresses variance on lagged variance and lagged squared error terms.

AR orders of 1, 4, 6 and 9 were selected for different specifications of the model in line with the ACF plot from the Statistics section.

GARCH(p,q) variations of (1,1), (1,2) and (2,1) were tested in the absence of statistically significant squared log returns and general lack of model improvement beyond two lags.

The model variations were run on 90% of the dataset as the train set. The remaining 10% was left for the test set. A rolling window approach was incorporated, whereby the parameters calibrated during training were applied to datetimes in the test set sequentially, forecasting only the next period return and volatility at a time.

In [ ]:
```
class garch_base(data_extraction):
    def __init__(self, approach, proportion, p, q, mean, data, asset, lags=0, o=0):
        data_extraction.__init__(self, data, asset)
        self.approach = approach
        self.proportion = proportion
        self.p = p
        self.o = o
        self.q = q
        self.mean = mean
        self.lags = lags
```

```
        self.train = self.series.iloc[:int(self.proportion*len(self.series))]
        self.test = self.series.iloc[int(self.proportion*len(self.series)):]
#         self.last_train_date = datetime.strftime(self.train.index[-1], '%Y-%m-%d')
        self.last_train_date = self.train.index[-1]
        self.res_garch = self.fit()

    def specs(self):
        return 'ar' + str(self.lags) + 'garch' + str(self.p) + str(self.q)

    def train_out(self):
        return self.train
    def fit(self):
        if self.approach == 'fixed':
            garch = arch.arch_model(self.train, mean=self.mean, lags=self.lags, vol='GARCH', p=self.p, o=self.o
            res_garch = garch.fit()
        elif self.approach == 'roll1d':
            garch = arch.arch_model(self.series, mean=self.mean, lags=self.lags, vol='GARCH', p=self.p, o=self.
            res_garch = garch.fit(last_obs=self.last_train_date)

        return res_garch

    def summary(self):
        res_garch_summary = self.res_garch.summary()

        return res_garch_summary

    def plot(self):
        fig = self.res_garch.plot()
        plt.show()

    def autocorr(self):
        lb = acorr_ljungbox(self.res_garch.std_resid**2, [10, 15, 20])

        return lb

    def info_crit(self):
        ic_out = {'AIC': [self.res_garch.aic], 'BIC': [self.res_garch.bic]}
        ic = pd.DataFrame(data=ic_out)

        return ic

    def acf_stan(self):
        plot_acf(self.res_garch.std_resid, lags=30, title='ACF of Standardized Residuals')
        plt.show()

    def acf_sq_stan(self):
        plot_acf(self.res_garch.std_resid**2, lags=30, title='ACF of Squared Standardized Residuals')
        plt.show()

class garch(garch_base):
    def __init__(self, approach, proportion, p, q, mean, data, asset, lags=0, o=0):
        garch_base.__init__(self, approach, proportion, p, q, mean, data, asset, lags, o)
        self.forecast = self.forecast()
        self.exclusion_dates = self.get_exclusion_dates()
        self.forecast_mean_values = self.forecast_mean_values()
        self.forecast_std_values = self.forecast_std_values()
        self.forecast_vs_test = self.forecast_vs_test()

    def forecast(self):
        forecast = self.res_garch.forecast(horizon=len(self.test))

        return forecast

#     def forecast_out(self):
#         return self.forecast

    def get_exclusion_dates(self):
#         return [date for date in self.forecast.mean.index.strftime('%Y-%m-%d').tolist() if date <= self.last_
        return [i for i in self.forecast.mean.index.tolist() if i <= self.last_train_date]

    def forecast_mean_values(self):
        if self.approach == 'fixed':
            forecast_mean_values = self.forecast.mean.iloc[0]
        elif self.approach == 'roll1d':
            forecast_mean_values = self.forecast.mean.drop(self.exclusion_dates)
            forecast_mean_values = forecast_mean_values.iloc[:,0]

        return forecast_mean_values

    def forecast_mean(self):
#         forecast_mean = self.forecast.mean
#         forecast_mean.iloc[0].plot()
        self.forecast_mean_values.plot()
```

```
    def forecast_std_values(self):
        forecast_vol_raw = self.forecast.variance**0.5

        if self.approach == 'fixed':
            forecast_std_values = forecast_vol_raw.iloc[0]
        elif self.approach == 'roll1d':
            forecast_std_values = forecast_vol_raw.drop(self.exclusion_dates)
            forecast_std_values = forecast_std_values.iloc[:,0]

        return forecast_std_values
#         std3 = std2.transpose()
#         for i, col in enumerate(std3.columns):
#             std3[col] = std3[col].shift(i)
#         std3
#         std3.plot()

    def forecast_std(self):
#         forecast_stdev = self.forecast.variance**0.5
#         forecast_stdev.iloc[0].plot()
        self.forecast_std_values.plot()

    def forecast_vs_test(self):
#         if self.approach == 'fixed':
#             forecast_mean_values = self.forecast.mean.iloc[0].values
#         elif self.approach == 'roll1d':
#             exclusion_dates = [date for date in self.forecast.mean.index.strftime('%Y-%m-%d').tolist() if date
#             forecast_mean_values = self.forecast.mean.drop(exclusion_dates)
#             forecast_mean_values = forecast_mean_values.iloc[:,0].values

        forecast_mean_values = self.forecast_mean_values.values

        forecast_mean = pd.Series(forecast_mean_values, index=self.test.index)
        self.train.rename('Ret_Train', inplace=True)
        self.test.rename('Ret_Test', inplace=True)
        forecast_mean.rename('Ret_Forecast', inplace=True)
        forecast_vs_test = pd.concat([self.train, self.test, forecast_mean], axis=1)

        forecast_vs_test['Ret_Test_Broad'] = forecast_vs_test[['Ret_Train','Ret_Test']].max(axis=1)

        forecast_vs_test['Vol_Train'] = forecast_vs_test[['Ret_Train']].rolling(30).std().shift(1) #30d std dev
        forecast_vs_test['Vol_Test_Broad'] = forecast_vs_test[['Ret_Test_Broad']].rolling(30).std().shift(1) #3
        forecast_vs_test['Vol_Test'] = np.where(forecast_vs_test['Ret_Test'].isna(), forecast_vs_test['Ret_Test

#         forecast_vol_raw = self.forecast.variance**0.5
#         if self.approach == 'fixed':
#             forecast_std_values = forecast_vol_raw.iloc[0].values
#         elif self.approach == 'roll1d':
#             forecast_std_values = forecast_vol_raw.drop(exclusion_dates)
#             forecast_std_values = forecast_std_values.iloc[:,0].values

        forecast_std_values = self.forecast_std_values.values

        forecast_vs_test_condvol = pd.Series(forecast_std_values, index=self.test.index) #conditional vol
        forecast_vs_test_condvol.rename('Vol_Forecast', inplace=True)
        forecast_vs_test = pd.concat([forecast_vs_test, forecast_vs_test_condvol], axis=1)

        return forecast_vs_test

    def forecast_vs_test_mean_plot(self):
        self.forecast_vs_test[['Ret_Train','Ret_Test','Ret_Forecast']].plot()

    def forecast_vs_test_vol_plot(self):
        self.forecast_vs_test[['Vol_Train','Vol_Test','Vol_Forecast']].plot()
```

In [ ]:

```
ar1garch11 = garch('roll1d', 0.9, 1, 1, 'AR', returns, 'Ret_JNJ', 1)
ar1garch12 = garch('roll1d', 0.9, 1, 2, 'AR', returns, 'Ret_JNJ', 1)
ar1garch21 = garch('roll1d', 0.9, 2, 1, 'AR', returns, 'Ret_JNJ', 1)
ar4garch11 = garch('roll1d', 0.9, 1, 1, 'AR', returns, 'Ret_JNJ', 4)
ar4garch12 = garch('roll1d', 0.9, 1, 2, 'AR', returns, 'Ret_JNJ', 4)
ar4garch21 = garch('roll1d', 0.9, 2, 1, 'AR', returns, 'Ret_JNJ', 4)
ar6garch11 = garch('roll1d', 0.9, 1, 1, 'AR', returns, 'Ret_JNJ', 6)
ar6garch12 = garch('roll1d', 0.9, 1, 2, 'AR', returns, 'Ret_JNJ', 6)
ar6garch21 = garch('roll1d', 0.9, 2, 1, 'AR', returns, 'Ret_JNJ', 6)
ar9garch11 = garch('roll1d', 0.9, 1, 1, 'AR', returns, 'Ret_JNJ', 9)
ar9garch12 = garch('roll1d', 0.9, 1, 2, 'AR', returns, 'Ret_JNJ', 9)
ar9garch21 = garch('roll1d', 0.9, 2, 1, 'AR', returns, 'Ret_JNJ', 9)
```

```
Iteration:      1,   Func. Count:      7,   Neg. LLF: 61052.79280867893
Iteration:      2,   Func. Count:     17,   Neg. LLF: 6453202.524870855
Iteration:      3,   Func. Count:     26,   Neg. LLF: 2149143.382759834
Iteration:      4,   Func. Count:     33,   Neg. LLF: 3517.4363591318224
Iteration:      5,   Func. Count:     40,   Neg. LLF: 3726.2849913105483
```

```
Iteration:      6,   Func. Count:     47,   Neg. LLF: 3474.4284763938213
Iteration:      7,   Func. Count:     53,   Neg. LLF: 3468.286028036266
Iteration:      8,   Func. Count:     59,   Neg. LLF: 3466.453192727503
Iteration:      9,   Func. Count:     65,   Neg. LLF: 3466.4588223039746
Iteration:     10,   Func. Count:     72,   Neg. LLF: 3465.8531836189095
Iteration:     11,   Func. Count:     78,   Neg. LLF: 3465.8516812260423
Iteration:     12,   Func. Count:     84,   Neg. LLF: 3465.851670324504
Iteration:     13,   Func. Count:     90,   Neg. LLF: 3465.8516689144944
Iteration:     14,   Func. Count:     95,   Neg. LLF: 3465.8516682923864
Optimization terminated successfully    (Exit mode 0)
            Current function value: 3465.8516689144944
            Iterations: 14
            Function evaluations: 95
            Gradient evaluations: 14
Iteration:      1,   Func. Count:      8,   Neg. LLF: 69083.69358667244
Iteration:      2,   Func. Count:     19,   Neg. LLF: 653012.6043595799
Iteration:      3,   Func. Count:     29,   Neg. LLF: 194456.95020774924
Iteration:      4,   Func. Count:     37,   Neg. LLF: 3501.82492811266
Iteration:      5,   Func. Count:     44,   Neg. LLF: 4090.229723472999
Iteration:      6,   Func. Count:     55,   Neg. LLF: 5942.46722337079
Iteration:      7,   Func. Count:     63,   Neg. LLF: 3509.8033411977813
Iteration:      8,   Func. Count:     71,   Neg. LLF: 6261.042107955817
Iteration:      9,   Func. Count:     79,   Neg. LLF: 3539.4626232689297
Iteration:     10,   Func. Count:     87,   Neg. LLF: 3467.9994882246
Iteration:     11,   Func. Count:     94,   Neg. LLF: 3465.858098309489
Iteration:     12,   Func. Count:    101,   Neg. LLF: 3464.918364053072
Iteration:     13,   Func. Count:    108,   Neg. LLF: 3464.641273723206
Iteration:     14,   Func. Count:    115,   Neg. LLF: 3464.271482345731
Iteration:     15,   Func. Count:    122,   Neg. LLF: 3469.1688783718078
Iteration:     16,   Func. Count:    130,   Neg. LLF: 3468.759623816978
Iteration:     17,   Func. Count:    138,   Neg. LLF: 3464.1262951008443
Iteration:     18,   Func. Count:    146,   Neg. LLF: 3463.3881920539093
Iteration:     19,   Func. Count:    153,   Neg. LLF: 3463.3784688239493
Iteration:     20,   Func. Count:    160,   Neg. LLF: 3463.3778556887087
Iteration:     21,   Func. Count:    167,   Neg. LLF: 3463.377790240138
Iteration:     22,   Func. Count:    174,   Neg. LLF: 3463.377759874371
Iteration:     23,   Func. Count:    181,   Neg. LLF: 3463.377758124896
Iteration:     24,   Func. Count:    187,   Neg. LLF: 3463.377757729458
Optimization terminated successfully    (Exit mode 0)
            Current function value: 3463.377758124896
            Iterations: 24
            Function evaluations: 187
            Gradient evaluations: 24
Iteration:      1,   Func. Count:      8,   Neg. LLF: 17937.57810219515
Iteration:      2,   Func. Count:     19,   Neg. LLF: 12067.049581800402
Iteration:      3,   Func. Count:     29,   Neg. LLF: 4308684.068809033
Iteration:      4,   Func. Count:     37,   Neg. LLF: 4324.496639023809
Iteration:      5,   Func. Count:     45,   Neg. LLF: 4085.5710164266698
Iteration:      6,   Func. Count:     53,   Neg. LLF: 3477.0671584381316
Iteration:      7,   Func. Count:     60,   Neg. LLF: 3557.145130122208
Iteration:      8,   Func. Count:     68,   Neg. LLF: 3468.412369435724
Iteration:      9,   Func. Count:     75,   Neg. LLF: 3466.3265758692205
Iteration:     10,   Func. Count:     82,   Neg. LLF: 3465.89949425693
Iteration:     11,   Func. Count:     89,   Neg. LLF: 3465.859659552012
Iteration:     12,   Func. Count:     96,   Neg. LLF: 3465.8520416145147
Iteration:     13,   Func. Count:    103,   Neg. LLF: 3465.851669786107
Iteration:     14,   Func. Count:    110,   Neg. LLF: 3465.8516689141215
Optimization terminated successfully    (Exit mode 0)
            Current function value: 3465.8516689141215
            Iterations: 14
            Function evaluations: 110
            Gradient evaluations: 14
Iteration:      1,   Func. Count:     10,   Neg. LLF: 107330.04160288133
Iteration:      2,   Func. Count:     23,   Neg. LLF: 266139.92923819047
Iteration:      3,   Func. Count:     35,   Neg. LLF: 2989524.886098926
Iteration:      4,   Func. Count:     47,   Neg. LLF: 3318250.735340111
Iteration:      5,   Func. Count:     57,   Neg. LLF: 536188.7902039221
Iteration:      6,   Func. Count:     67,   Neg. LLF: 9356.110124948276
Iteration:      7,   Func. Count:     78,   Neg. LLF: 33807.1634548685
Iteration:      8,   Func. Count:     90,   Neg. LLF: 3665.12416696479
Iteration:      9,   Func. Count:    100,   Neg. LLF: 3644.5712264644308
Iteration:     10,   Func. Count:    110,   Neg. LLF: 3449.0239295454876
Iteration:     11,   Func. Count:    119,   Neg. LLF: 3438.380658383642
Iteration:     12,   Func. Count:    128,   Neg. LLF: 3437.5974908116596
Iteration:     13,   Func. Count:    137,   Neg. LLF: 3437.5499037554855
Iteration:     14,   Func. Count:    146,   Neg. LLF: 3437.540667237796
Iteration:     15,   Func. Count:    155,   Neg. LLF: 3437.539538200621
Iteration:     16,   Func. Count:    164,   Neg. LLF: 3437.539503588682
Iteration:     17,   Func. Count:    173,   Neg. LLF: 3437.5395029441615
Optimization terminated successfully    (Exit mode 0)
            Current function value: 3437.5395029441615
            Iterations: 17
            Function evaluations: 173
```

```
            Gradient evaluations: 17
Iteration:      1,   Func. Count:     11,   Neg. LLF: 105720.44470005757
Iteration:      2,   Func. Count:     25,   Neg. LLF: 683766.6679308289
Iteration:      3,   Func. Count:     38,   Neg. LLF: 5462395.862088518
Iteration:      4,   Func. Count:     51,   Neg. LLF: 1444737.1997767761
Iteration:      5,   Func. Count:     62,   Neg. LLF: 638258.2652303372
Iteration:      6,   Func. Count:     73,   Neg. LLF: 29198.0853324265
Iteration:      7,   Func. Count:     86,   Neg. LLF: 22402.62799317786
Iteration:      8,   Func. Count:     99,   Neg. LLF: 3623.1679622507577
Iteration:      9,   Func. Count:    110,   Neg. LLF: 3714.667514908565
Iteration:     10,   Func. Count:    121,   Neg. LLF: 3526.277068510915
Iteration:     11,   Func. Count:    132,   Neg. LLF: 3433.865890560791
Iteration:     12,   Func. Count:    142,   Neg. LLF: 3512.7631667158885
Iteration:     13,   Func. Count:    153,   Neg. LLF: 7170.226133384282
Iteration:     14,   Func. Count:    164,   Neg. LLF: 3427.9876264156064
Iteration:     15,   Func. Count:    174,   Neg. LLF: 3427.692363021994
Iteration:     16,   Func. Count:    184,   Neg. LLF: 3427.681634449081
Iteration:     17,   Func. Count:    194,   Neg. LLF: 3427.6800815500483
Iteration:     18,   Func. Count:    204,   Neg. LLF: 3427.6788795949988
Iteration:     19,   Func. Count:    214,   Neg. LLF: 3427.6787273519885
Iteration:     20,   Func. Count:    224,   Neg. LLF: 3427.678719087139
Iteration:     21,   Func. Count:    233,   Neg. LLF: 3427.6787182690528
Optimization terminated successfully    (Exit mode 0)
            Current function value: 3427.678719087139
            Iterations: 21
            Function evaluations: 233
            Gradient evaluations: 21
Iteration:      1,   Func. Count:     11,   Neg. LLF: 108299.77978454804
Iteration:      2,   Func. Count:     25,   Neg. LLF: 19340.934776461952
Iteration:      3,   Func. Count:     38,   Neg. LLF: 1210590.1341135735
Iteration:      4,   Func. Count:     50,   Neg. LLF: 1782770.466980549
Iteration:      5,   Func. Count:     61,   Neg. LLF: 7005.79502068715
Iteration:      6,   Func. Count:     73,   Neg. LLF: 10650.717901502567
Iteration:      7,   Func. Count:     85,   Neg. LLF: 14875.376959228373
Iteration:      8,   Func. Count:     98,   Neg. LLF: 4056.0141719267535
Iteration:      9,   Func. Count:    109,   Neg. LLF: 3461.7022728077327
Iteration:     10,   Func. Count:    119,   Neg. LLF: 3509.3295800203277
Iteration:     11,   Func. Count:    130,   Neg. LLF: 3459.7770224371625
Iteration:     12,   Func. Count:    141,   Neg. LLF: 3438.0374668686536
Iteration:     13,   Func. Count:    151,   Neg. LLF: 3438.575662515697
Iteration:     14,   Func. Count:    162,   Neg. LLF: 3437.592100089162
Iteration:     15,   Func. Count:    172,   Neg. LLF: 3437.547091149
Iteration:     16,   Func. Count:    182,   Neg. LLF: 3437.539624467403
Iteration:     17,   Func. Count:    192,   Neg. LLF: 3437.5395061532554
Iteration:     18,   Func. Count:    202,   Neg. LLF: 3437.5395029536576
Iteration:     19,   Func. Count:    211,   Neg. LLF: 3437.5395020398882
Optimization terminated successfully    (Exit mode 0)
            Current function value: 3437.5395029536576
            Iterations: 19
            Function evaluations: 211
            Gradient evaluations: 19
Iteration:      1,   Func. Count:     12,   Neg. LLF: 86800.32633580668
Iteration:      2,   Func. Count:     27,   Neg. LLF: 345174.4364261344
Iteration:      3,   Func. Count:     41,   Neg. LLF: 1823096.5120845647
Iteration:      4,   Func. Count:     55,   Neg. LLF: 991167.6355774499
Iteration:      5,   Func. Count:     67,   Neg. LLF: 2476266.307154471
Iteration:      6,   Func. Count:     79,   Neg. LLF: 23587.09144941823
Iteration:      7,   Func. Count:     93,   Neg. LLF: 524973.7651144749
Iteration:      8,   Func. Count:    107,   Neg. LLF: 258862.99512491532
Iteration:      9,   Func. Count:    120,   Neg. LLF: 654049.8307928008
Iteration:     10,   Func. Count:    134,   Neg. LLF: 8823.155759110101
Iteration:     11,   Func. Count:    148,   Neg. LLF: 3523.9009235776066
Iteration:     12,   Func. Count:    160,   Neg. LLF: 3460.7104580802793
Iteration:     13,   Func. Count:    172,   Neg. LLF: 3432.237765958468
Iteration:     14,   Func. Count:    183,   Neg. LLF: 3430.693247739154
Iteration:     15,   Func. Count:    194,   Neg. LLF: 3430.624392349625
Iteration:     16,   Func. Count:    205,   Neg. LLF: 3430.617922502993
Iteration:     17,   Func. Count:    216,   Neg. LLF: 3430.6175694173253
Iteration:     18,   Func. Count:    227,   Neg. LLF: 3430.6175498167263
Iteration:     19,   Func. Count:    238,   Neg. LLF: 3430.617547419066
Iteration:     20,   Func. Count:    248,   Neg. LLF: 3430.6175465758624
Optimization terminated successfully    (Exit mode 0)
            Current function value: 3430.617547419066
            Iterations: 20
            Function evaluations: 248
            Gradient evaluations: 20
Iteration:      1,   Func. Count:     13,   Neg. LLF: 594183.6739543932
Iteration:      2,   Func. Count:     29,   Neg. LLF: 548510.0915971808
Iteration:      3,   Func. Count:     44,   Neg. LLF: 652507.7977986201
Iteration:      4,   Func. Count:     59,   Neg. LLF: 1014958.531062253
Iteration:      5,   Func. Count:     72,   Neg. LLF: 1333871.4017855674
Iteration:      6,   Func. Count:     85,   Neg. LLF: 16055.98701418603
Iteration:      7,   Func. Count:     99,   Neg. LLF: 14393.760649319356
```

```
Iteration:      8,   Func. Count:    114,   Neg. LLF: 32802.56181942888
Iteration:      9,   Func. Count:    129,   Neg. LLF: 6811.624067239947
Iteration:     10,   Func. Count:    143,   Neg. LLF: 3672.198131222566
Iteration:     11,   Func. Count:    156,   Neg. LLF: 4748.203685468032
Iteration:     12,   Func. Count:    170,   Neg. LLF: 3438.05532187779
Iteration:     13,   Func. Count:    182,   Neg. LLF: 3439.04765217898
Iteration:     14,   Func. Count:    196,   Neg. LLF: 7731.157719820667
Iteration:     15,   Func. Count:    209,   Neg. LLF: 3452.7187814758854
Iteration:     16,   Func. Count:    222,   Neg. LLF: 3417.864598077047
Iteration:     17,   Func. Count:    234,   Neg. LLF: 3417.4373825750017
Iteration:     18,   Func. Count:    246,   Neg. LLF: 3417.264599118532
Iteration:     19,   Func. Count:    258,   Neg. LLF: 3417.2367763351685
Iteration:     20,   Func. Count:    270,   Neg. LLF: 3417.2340839065464
Iteration:     21,   Func. Count:    282,   Neg. LLF: 3417.234044639953
Iteration:     22,   Func. Count:    294,   Neg. LLF: 3417.2340426349983
Iteration:     23,   Func. Count:    305,   Neg. LLF: 3417.234041627248
Optimization terminated successfully    (Exit mode 0)
            Current function value: 3417.2340426349983
            Iterations: 23
            Function evaluations: 305
            Gradient evaluations: 23
Iteration:      1,   Func. Count:     13,   Neg. LLF: 75218.41200646257
Iteration:      2,   Func. Count:     29,   Neg. LLF: 19671.872310762952
Iteration:      3,   Func. Count:     44,   Neg. LLF: 2268448.796854481
Iteration:      4,   Func. Count:     58,   Neg. LLF: 3291596.9174673483
Iteration:      5,   Func. Count:     71,   Neg. LLF: 7430.407614743193
Iteration:      6,   Func. Count:     85,   Neg. LLF: 13140.885411368079
Iteration:      7,   Func. Count:     99,   Neg. LLF: 16859.28419801085
Iteration:      8,   Func. Count:    114,   Neg. LLF: 9295.661133562644
Iteration:      9,   Func. Count:    129,   Neg. LLF: 5161.525684388116
Iteration:     10,   Func. Count:    142,   Neg. LLF: 4590.076086454085
Iteration:     11,   Func. Count:    155,   Neg. LLF: 3702.851821141414
Iteration:     12,   Func. Count:    168,   Neg. LLF: 3512.3050201709857
Iteration:     13,   Func. Count:    181,   Neg. LLF: 3433.4192110069507
Iteration:     14,   Func. Count:    193,   Neg. LLF: 3433.777360688722
Iteration:     15,   Func. Count:    206,   Neg. LLF: 3430.697009006723
Iteration:     16,   Func. Count:    218,   Neg. LLF: 3430.6289925951
Iteration:     17,   Func. Count:    230,   Neg. LLF: 3430.619013641425
Iteration:     18,   Func. Count:    242,   Neg. LLF: 3430.6176832330766
Iteration:     19,   Func. Count:    254,   Neg. LLF: 3430.6175526901698
Iteration:     20,   Func. Count:    266,   Neg. LLF: 3430.6175476085923
Iteration:     21,   Func. Count:    277,   Neg. LLF: 3430.617546765311
Optimization terminated successfully    (Exit mode 0)
            Current function value: 3430.6175476085923
            Iterations: 21
            Function evaluations: 277
            Gradient evaluations: 21
Iteration:      1,   Func. Count:     15,   Neg. LLF: 88707.84092169083
Iteration:      2,   Func. Count:     34,   Neg. LLF: 24352265.54097928
Iteration:      3,   Func. Count:     51,   Neg. LLF: 7423026.486659714
Iteration:      4,   Func. Count:     68,   Neg. LLF: 2132723.1804170315
Iteration:      5,   Func. Count:     84,   Neg. LLF: 1128168.5976529364
Iteration:      6,   Func. Count:     99,   Neg. LLF: 11516930.5615177
Iteration:      7,   Func. Count:    115,   Neg. LLF: 5369893.44731659
Iteration:      8,   Func. Count:    132,   Neg. LLF: 24503.60825122107
Iteration:      9,   Func. Count:    149,   Neg. LLF: 17890.22923049912
Iteration:     10,   Func. Count:    166,   Neg. LLF: 24604.389959691376
Iteration:     11,   Func. Count:    183,   Neg. LLF: 5591.345220345723
Iteration:     12,   Func. Count:    199,   Neg. LLF: 5435.762395135499
Iteration:     13,   Func. Count:    215,   Neg. LLF: 15794.771730555112
Iteration:     14,   Func. Count:    232,   Neg. LLF: 3713.6022761414097
Iteration:     15,   Func. Count:    247,   Neg. LLF: 4216.03661643017
Iteration:     16,   Func. Count:    262,   Neg. LLF: 3409.5223577266306
Iteration:     17,   Func. Count:    276,   Neg. LLF: 3407.074053427291
Iteration:     18,   Func. Count:    290,   Neg. LLF: 3405.1217499163035
Iteration:     19,   Func. Count:    304,   Neg. LLF: 3405.018856448305
Iteration:     20,   Func. Count:    318,   Neg. LLF: 3405.015295212379
Iteration:     21,   Func. Count:    332,   Neg. LLF: 3405.014895329883
Iteration:     22,   Func. Count:    346,   Neg. LLF: 3405.0148767988976
Iteration:     23,   Func. Count:    360,   Neg. LLF: 3405.014874665576
Iteration:     24,   Func. Count:    373,   Neg. LLF: 3405.014873426846
Optimization terminated successfully    (Exit mode 0)
            Current function value: 3405.014874665576
            Iterations: 24
            Function evaluations: 373
            Gradient evaluations: 24
Iteration:      1,   Func. Count:     16,   Neg. LLF: 142408.81828828537
Iteration:      2,   Func. Count:     36,   Neg. LLF: 11821321.537696928
Iteration:      3,   Func. Count:     54,   Neg. LLF: 141619741.40669888
Iteration:      4,   Func. Count:     72,   Neg. LLF: 1199931.9879678087
Iteration:      5,   Func. Count:     89,   Neg. LLF: 2405569.4319773437
Iteration:      6,   Func. Count:    105,   Neg. LLF: 17263027.95102325
Iteration:      7,   Func. Count:    122,   Neg. LLF: 17937.852518408086
```

```
Iteration:      8,   Func. Count:    140,   Neg. LLF: 13584.148050944752
Iteration:      9,   Func. Count:    158,   Neg. LLF: 17029.57502359087
Iteration:     10,   Func. Count:    176,   Neg. LLF: 8388.460968971425
Iteration:     11,   Func. Count:    193,   Neg. LLF: 3867.7838429741555
Iteration:     12,   Func. Count:    209,   Neg. LLF: 6298.533612131359
Iteration:     13,   Func. Count:    226,   Neg. LLF: 11531.41715935291
Iteration:     14,   Func. Count:    244,   Neg. LLF: 4900353975.574669
Iteration:     15,   Func. Count:    261,   Neg. LLF: 3622.7469841542415
Iteration:     16,   Func. Count:    277,   Neg. LLF: 3405.480493787151
Iteration:     17,   Func. Count:    292,   Neg. LLF: 3433.6159799671827
Iteration:     18,   Func. Count:    308,   Neg. LLF: 4086.271309528178
Iteration:     19,   Func. Count:    325,   Neg. LLF: 3416.3648966115884
Iteration:     20,   Func. Count:    341,   Neg. LLF: 3395.121305412791
Iteration:     21,   Func. Count:    356,   Neg. LLF: 3395.0766436693
Iteration:     22,   Func. Count:    371,   Neg. LLF: 3395.064650251343
Iteration:     23,   Func. Count:    386,   Neg. LLF: 3395.0580011683596
Iteration:     24,   Func. Count:    401,   Neg. LLF: 3395.0577128220684
Iteration:     25,   Func. Count:    416,   Neg. LLF: 3395.057682108208
Iteration:     26,   Func. Count:    431,   Neg. LLF: 3395.05768077473
Iteration:     27,   Func. Count:    445,   Neg. LLF: 3395.0576794840154
Optimization terminated successfully    (Exit mode 0)
            Current function value: 3395.05768077473
            Iterations: 27
            Function evaluations: 445
            Gradient evaluations: 27
Iteration:      1,   Func. Count:     16,   Neg. LLF: 105829.2646551054
Iteration:      2,   Func. Count:     35,   Neg. LLF: 4254697.025616312
Iteration:      3,   Func. Count:     53,   Neg. LLF: 37468110.06297142
Iteration:      4,   Func. Count:     70,   Neg. LLF: 2235955.244480608
Iteration:      5,   Func. Count:     86,   Neg. LLF: 2801106.530496165
Iteration:      6,   Func. Count:    102,   Neg. LLF: 40680.993817821
Iteration:      7,   Func. Count:    120,   Neg. LLF: 22003.56627718916
Iteration:      8,   Func. Count:    138,   Neg. LLF: 126849658.46430635
Iteration:      9,   Func. Count:    156,   Neg. LLF: 3036136.989924136
Iteration:     10,   Func. Count:    173,   Neg. LLF: 53284242.370206356
Iteration:     11,   Func. Count:    190,   Neg. LLF: 15391.939605237068
Iteration:     12,   Func. Count:    208,   Neg. LLF: 17067.173799006396
Iteration:     13,   Func. Count:    226,   Neg. LLF: 6199.368876048974
Iteration:     14,   Func. Count:    243,   Neg. LLF: 3908.2730870805417
Iteration:     15,   Func. Count:    259,   Neg. LLF: 4032.453741903061
Iteration:     16,   Func. Count:    275,   Neg. LLF: 3414.4177276076225
Iteration:     17,   Func. Count:    290,   Neg. LLF: 3405.685118540223
Iteration:     18,   Func. Count:    305,   Neg. LLF: 3405.110160567734
Iteration:     19,   Func. Count:    320,   Neg. LLF: 3405.0275977411047
Iteration:     20,   Func. Count:    335,   Neg. LLF: 3405.0156115273357
Iteration:     21,   Func. Count:    350,   Neg. LLF: 3405.0149199813623
Iteration:     22,   Func. Count:    365,   Neg. LLF: 3405.014876521888
Iteration:     23,   Func. Count:    380,   Neg. LLF: 3405.0148746674645
Iteration:     24,   Func. Count:    394,   Neg. LLF: 3405.0148734289187
Optimization terminated successfully    (Exit mode 0)
            Current function value: 3405.0148746674645
            Iterations: 24
            Function evaluations: 394
            Gradient evaluations: 24
```

In [ ]:

```
runs = [ar1garch11, ar1garch12, ar1garch21, ar4garch11, ar4garch12, ar4garch21, ar6garch11, ar6garch12, ar6garc

for i, ic in enumerate(runs):
    ic_append = ic.info_crit()
    ic_append['Model'] = ic.specs()

    if i == 0:
        ics = ic_append
    else:
        ics = pd.concat([ics, ic_append])

cols = ics.columns.tolist()
cols = cols[-1:] + cols[:-1]
ics = ics[cols]
ics
```

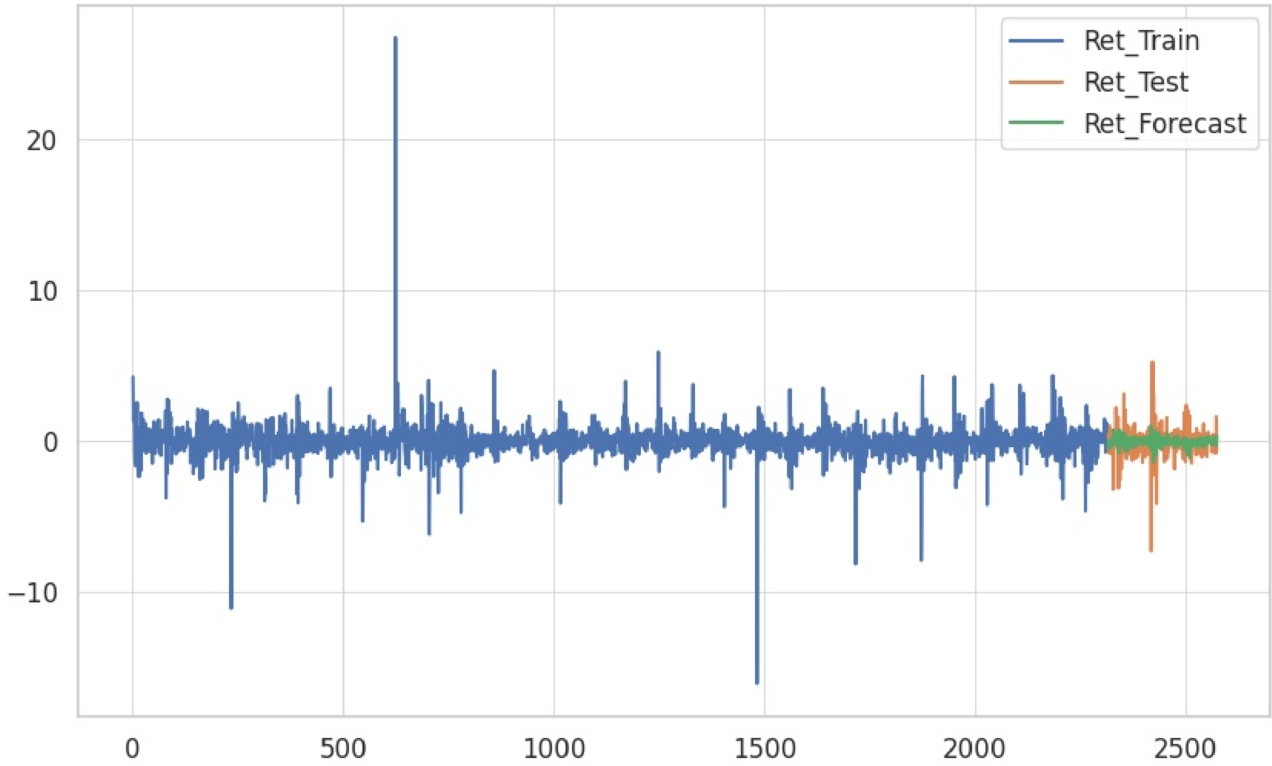
Out[ ]:

	Model	AIC	BIC
0	ar1garch11	6941.703338	6970.437002
0	ar1garch12	6938.755516	6973.235914
0	ar1garch21	6943.703338	6978.183735
0	ar4garch11	6891.079006	6937.042491
0	ar4garch12	6873.357438	6925.066359
0	ar4garch21	6893.079006	6944.787926
0	ar6garch11	6881.235095	6938.680793
0	ar6garch12	6856.468085	6919.658353
0	ar6garch21	6883.235095	6946.425363
0	ar9garch11	6836.029749	6910.692255
0	ar9garch12	6818.115362	6898.521137
0	ar9garch21	6838.029749	6918.435525

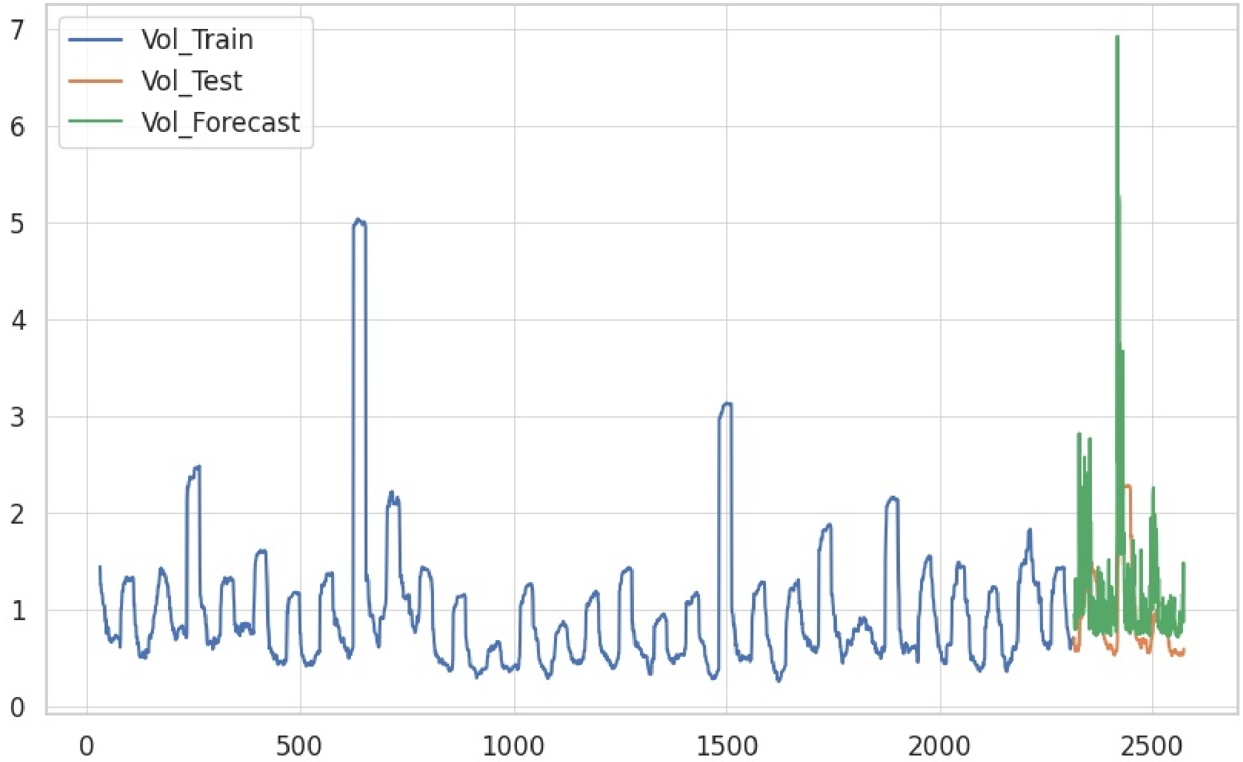
It can be seen that across all model variations AR(9)-GARCH(1,2) had the lowest Akaike Information Criterion (AIC) and was the model of choice.

Representations of its mean and volatility forecasts are shown below.

In [ ]:
```
ar9garch12.forecast_vs_test_mean_plot()
```



In [ ]:
```
ar9garch12.forecast_vs_test_vol_plot()
```



Strategy & Results

The forecasts were used to generate trading signals. Whenever the next period return was forecast to be positive, a long position was selected, otherwise a short position was selected. The position was only initiated if the next period volatility was forecast to be above the last observed volatility level, i.e. if a relatively large move in either direction was anticipated to increase the chances of profiting from the trade and avoiding unnecessary transaction costs.

Gross P&L was calculated based on the direction and realised return. Net P&L substracted trading costs (5 basis points of trade value) from this wherever there was a change in position day-on-day.

In [ ]:

```
class strategy(garch):
    def __init__(self, proportion, approach, p, q, mean, data, asset, lags, o = 0):
        garch.__init__(self, proportion, approach, p, q, mean, data, asset, lags, o = 0)
        self.trade = self.build()

    def build(self):
        trade = self.forecast_vs_test
        print(self.last_train_date)

        close =  self.data[['JNJ']]
        trade = pd.merge(trade, close, left_index = True, right_index = True)

        last_train = trade.iloc[lambda x: x.index == self.last_train_date]
        last_vol_train = last_train['Vol_Train'].iloc[0]

        trade['direction'] = np.where(trade['Ret_Train'].notna(), 0, np.where(trade['Ret_Forecast'] > 0, 1, -1)
        trade['amplifier'] = np.where(trade['Ret_Train'].notna(), 0, np.where(trade['Vol_Forecast'] > last_vol_
        trade['position'] = trade['direction'] * trade['amplifier']
        trade['pnl_gross'] = trade['Ret_Test'] / 1000 * trade['position'] # adjust returns back to normal level
        trade['pnl_gross_cum'] = trade['pnl_gross'].cumsum()
        trade['position_lag'] = trade['position'].shift(1)
        trade['cost'] = np.where(trade['position'] == trade['position_lag'], 0, TRANSACTION_COSTS * trade['JNJ'
        trade['pnl_net'] = trade['pnl_gross'] - trade['cost']
        trade['pnl_net_cum'] = trade['pnl_net'].cumsum()

        trade = trade.iloc[lambda x: x.index >= self.last_train_date]

        return trade

    def pos(self):
        plt.figure(figsize=(16, 8), dpi=150)

        self.trade['direction'].plot(label='Direction')
        self.trade['amplifier'].plot(label='Amplifier')
        self.trade['position'].plot(label='Position')
        self.trade['cost'].plot(label='Cost')

        plt.title('Trading Positions ' + self.asset + ' ' + self.specs())
        plt.legend()
```

```python
    def pnl_gross(self):
        plt.figure(figsize=(16, 8), dpi=150)

        self.trade['pnl_gross'].plot(label='Gross P&L')
        self.trade['pnl_gross_cum'].plot(label='Cumulative Gross P&L')

        plt.title('Trading Gross P&L ' + self.asset + ' ' + self.specs())
        plt.legend()

    def pnl_net(self):
        plt.figure(figsize=(16, 8), dpi=150)

        self.trade['pnl_net'].plot(label='Net P&L')
        self.trade['pnl_net_cum'].plot(label='Cumulative Net P&L')

        plt.title('Trading Net P&L ' + self.asset + ' ' + self.specs())
        plt.legend()

    def sr(self):
        sr_gross = 252 ** 0.5 * np.mean(self.trade['pnl_gross']) / np.std(self.trade['pnl_gross'])
        sr_net = 252 ** 0.5 * np.mean(self.trade['pnl_net']) / np.std(self.trade['pnl_net'])

        return pd.DataFrame({'Metric': ['SR Gross', 'SR Net'], 'Value': [sr_gross, sr_net]})
```

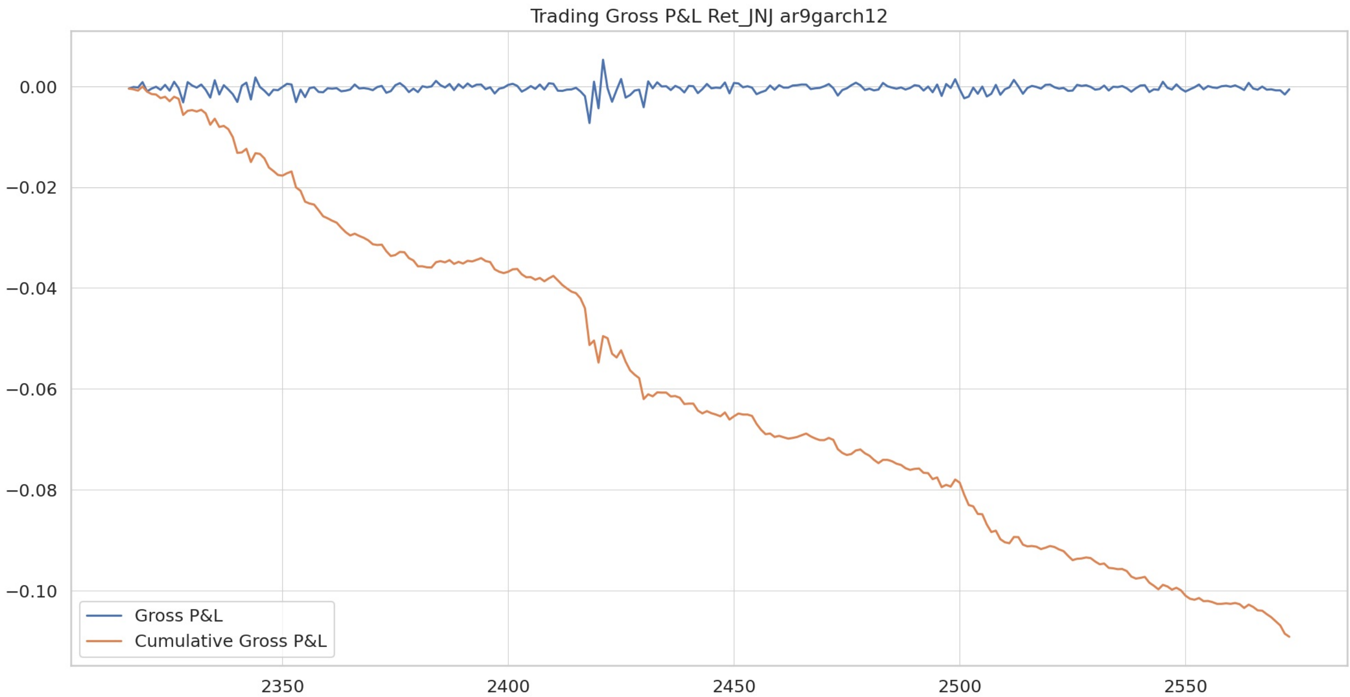
In [ ]:
```python
ar9garch12_strat = strategy('roll1d', 0.9, 1, 2, 'AR', returns, 'Ret_JNJ', 9)
```

```
Iteration:      1,   Func. Count:     16,   Neg. LLF: 142408.81828828537
Iteration:      2,   Func. Count:     36,   Neg. LLF: 11821321.537696928
Iteration:      3,   Func. Count:     54,   Neg. LLF: 141619741.40669888
Iteration:      4,   Func. Count:     72,   Neg. LLF: 1199931.9879678087
Iteration:      5,   Func. Count:     89,   Neg. LLF: 2405569.4319773437
Iteration:      6,   Func. Count:    105,   Neg. LLF: 17263027.95102325
Iteration:      7,   Func. Count:    122,   Neg. LLF: 17937.852518408086
Iteration:      8,   Func. Count:    140,   Neg. LLF: 13584.148050944752
Iteration:      9,   Func. Count:    158,   Neg. LLF: 17029.57502359087
Iteration:     10,   Func. Count:    176,   Neg. LLF: 8388.460968971425
Iteration:     11,   Func. Count:    193,   Neg. LLF: 3867.7838429741555
Iteration:     12,   Func. Count:    209,   Neg. LLF: 6298.533612131359
Iteration:     13,   Func. Count:    226,   Neg. LLF: 11531.41715935291
Iteration:     14,   Func. Count:    244,   Neg. LLF: 4900353975.574669
Iteration:     15,   Func. Count:    261,   Neg. LLF: 3622.7469841542415
Iteration:     16,   Func. Count:    277,   Neg. LLF: 3405.480493787151
Iteration:     17,   Func. Count:    292,   Neg. LLF: 3433.6159799671827
Iteration:     18,   Func. Count:    308,   Neg. LLF: 4086.271309528178
Iteration:     19,   Func. Count:    325,   Neg. LLF: 3416.3648966115884
Iteration:     20,   Func. Count:    341,   Neg. LLF: 3395.121305412791
Iteration:     21,   Func. Count:    356,   Neg. LLF: 3395.0766436693
Iteration:     22,   Func. Count:    371,   Neg. LLF: 3395.064650251343
Iteration:     23,   Func. Count:    386,   Neg. LLF: 3395.0580011683596
Iteration:     24,   Func. Count:    401,   Neg. LLF: 3395.0577128220684
Iteration:     25,   Func. Count:    416,   Neg. LLF: 3395.057682108208
Iteration:     26,   Func. Count:    431,   Neg. LLF: 3395.05768077473
Iteration:     27,   Func. Count:    445,   Neg. LLF: 3395.0576794840154
Optimization terminated successfully    (Exit mode 0)
            Current function value: 3395.05768077473
            Iterations: 27
            Function evaluations: 445
            Gradient evaluations: 27
2315
```

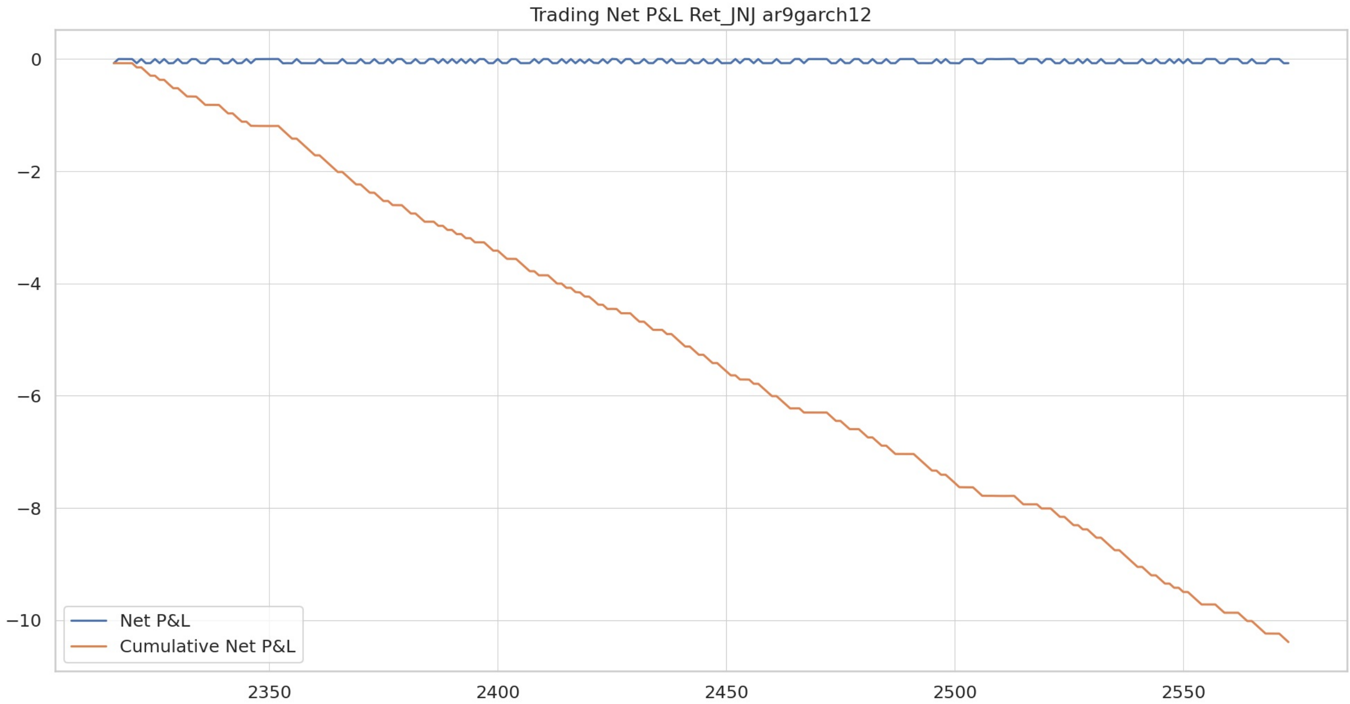
In [ ]:
```python
ar9garch12_strat.pos()
```

The strategy resulted in frequent trades, often switching positions between long and short. These were always triggered due to the volatility forecast always exceeding the last observed level of volatility. The volatility forecast was inaccurate in line with lack of arch effects and no statistically significant squared log returns observed earlier. The frequent execution of trades implied high trading costs.

In [ ]:
```
ar9garch12_strat.pnl_gross()
```



In [ ]:
```
ar9garch12_strat.pnl_net()
```



As expected, net cumulative P&L was negative. Gross cumulative P&L was also negative, which suggests being in the market (momentum strategy) based on recently observed returns is not a profitable strategy. A mean-reversion strategy may have performed better.

In [ ]:
```
ar9garch12_strat.sr()
```

Out[ ]:

	Metric	Value
0	SR Gross	-6.581488
1	SR Net	-17.505621

As a result, the net Sharpe ratio on the test set was -17.5.

Data: LSTM

LSTM (Long Short Term Memory) was the second model for performing univariate prediction of the time series of Johnson & Johnson equity price.

In [ ]:
```
# Shared processing imports start here...
import numpy as np                    # General purpose math
import pandas as pd                   # Dataframes
import matplotlib.pyplot as plt      # Simple plotting
# End of shared processing imports.

# LSTM processing imports start here...
import tensorflow as tf                                 # LSTM support
from tensorflow.keras.layers import LSTM                # LSTM model definition
from tensorflow.keras.models import Sequential          # Sequential LSTM layer
from tensorflow.keras.layers import Dense               # Dense LSTM layer
from sklearn.preprocessing import MinMaxScaler          # Transforming the observations to a specific range
from sklearn.model_selection import train_test_split    # Enabling out-of-sample validation
from sklearn.metrics import mean_squared_error          # Model evaluation
import quantstats as qs                                 # calculating ratios
# End of LSTM processing imports.
```

In [ ]:
```
jnj_close = pd.read_csv('../jnj.us.txt')[["<DATE>", "<TIME>", "<OPEN>", "<CLOSE>"]].rename(columns={'<DATE>' :
                                                                                                     '<TIME>':
                                                                                                     '<OPEN>':
                                                                                                     '<CLOSE>':

jnj_close.head()
```

Out[ ]:

	Date	Time	Open	Close
0	20240419	153000	146.150	144.8000
1	20240419	153500	144.800	145.4200
2	20240419	154000	145.440	145.6200
3	20240419	154500	145.660	145.9858
4	20240419	155000	145.975	146.1800

The date and time information was split between two columns. A single column unifying the two was created to allow for later construction of a timestamp.

In [ ]:
```
jnj_close['Datetime String'] = jnj_close.Date.map(str) + " " + jnj_close.Time.map(str)
jnj_close.head()
```

Out[ ]:

	Date	Time	Open	Close	Datetime String
0	20240419	153000	146.150	144.8000	20240419 153000
1	20240419	153500	144.800	145.4200	20240419 153500
2	20240419	154000	145.440	145.6200	20240419 154000
3	20240419	154500	145.660	145.9858	20240419 154500
4	20240419	155000	145.975	146.1800	20240419 155000

In [ ]:
```
jnj_close['Datetime'] = pd.to_datetime(jnj_close['Datetime String'], format='%Y%m%d %H%M%S')
jnj_close = jnj_close.set_index('Datetime').drop(columns=['Datetime String', 'Date', 'Time'])

jnj_close.head()
```

Out[ ]:

	Open	Close
Datetime		
2024-04-19 15:30:00	146.150	144.8000
2024-04-19 15:35:00	144.800	145.4200
2024-04-19 15:40:00	145.440	145.6200
2024-04-19 15:45:00	145.660	145.9858
2024-04-19 15:50:00	145.975	146.1800

The 'Close' variable was the target for predicting. The prediction model had to operate on lagged open/close variables to ensure look-ahead bias was eliminated.

In [ ]:
```
jnj_close['Open Lag']   = jnj_close['Open'].shift(6)
jnj_close['Close Lag']  = jnj_close['Close'].shift(6)
jnj_close.head(10)
```

Out[ ]:

	Open	Close	Open Lag	Close Lag
Datetime				
2024-04-19 15:30:00	146.150	144.8000	NaN	NaN
2024-04-19 15:35:00	144.800	145.4200	NaN	NaN
2024-04-19 15:40:00	145.440	145.6200	NaN	NaN
2024-04-19 15:45:00	145.660	145.9858	NaN	NaN
2024-04-19 15:50:00	145.975	146.1800	NaN	NaN
2024-04-19 15:55:00	146.180	146.0800	NaN	NaN
2024-04-19 16:00:00	146.090	145.8450	146.15	144.8000
2024-04-19 16:05:00	145.830	146.1700	144.80	145.4200
2024-04-19 16:10:00	146.170	146.3000	145.44	145.6200
2024-04-19 16:15:00	146.320	146.1800	145.66	145.9858

In [ ]:
```
jnj_close = jnj_close.dropna()
```

Methodology: LSTM

Model

The model building process was started by affixing the random seed for reproducibility.

In [ ]:
```
tf.random.set_seed(7)
```

LSTMs are sensitive to input data scale. Dataframe values were normalised before fitting the model to ensure best performance.

In [ ]:
```
scaler = MinMaxScaler(feature_range=(0, 1))
jnj_close = pd.DataFrame(scaler.fit_transform(jnj_close), index=jnj_close.index, columns=jnj_close.columns)
jnj_close.head()
```

Out[ ]:

	Open	Close	Open Lag	Close Lag
Datetime				
2024-04-19 16:00:00	0.209603	0.169242	0.215143	0.070714
2024-04-19 16:05:00	0.185596	0.199885	0.090489	0.129171
2024-04-19 16:10:00	0.216990	0.212142	0.149584	0.148028
2024-04-19 16:15:00	0.230840	0.200828	0.169898	0.182518
2024-04-19 16:20:00	0.218375	0.213556	0.198984	0.200828

The model was validated on a portion of the dataset which was not used for training. A "train-test" split was performed. Additionally, the focus was on close price prediction only (lagged open values remained as features).

In [ ]:
```
X = jnj_close[['Open Lag', 'Close Lag']] # Notice how the 'Open' column is implicitly dropped, unused.
y = jnj_close[['Close']]
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.10, random_state=42)
X_train.head()
```

Out[ ]:

	Open Lag	Close Lag
Datetime		
2024-06-07 17:30:00	0.384340	0.350742
2024-05-07 20:50:00	0.462604	0.453513
2024-04-24 18:05:00	0.332410	0.315385
2024-04-22 17:30:00	0.495845	0.482741
2024-04-24 17:25:00	0.286242	0.305956

LSTMs require 3-dimensional inputs in the form of [samples, timesteps, features]. Inputs were reshaped.

In [ ]:
```
X_train = X_train.values.reshape(X_train.shape[0], 1, X_train.shape[1])
X_test = X_test.values.reshape(X_test.shape[0], 1, X_test.shape[1])
```

An LSTM model was defined with multiple intermediate Dense layers. This increased both model accuracy and overall training time. It was then fitted.

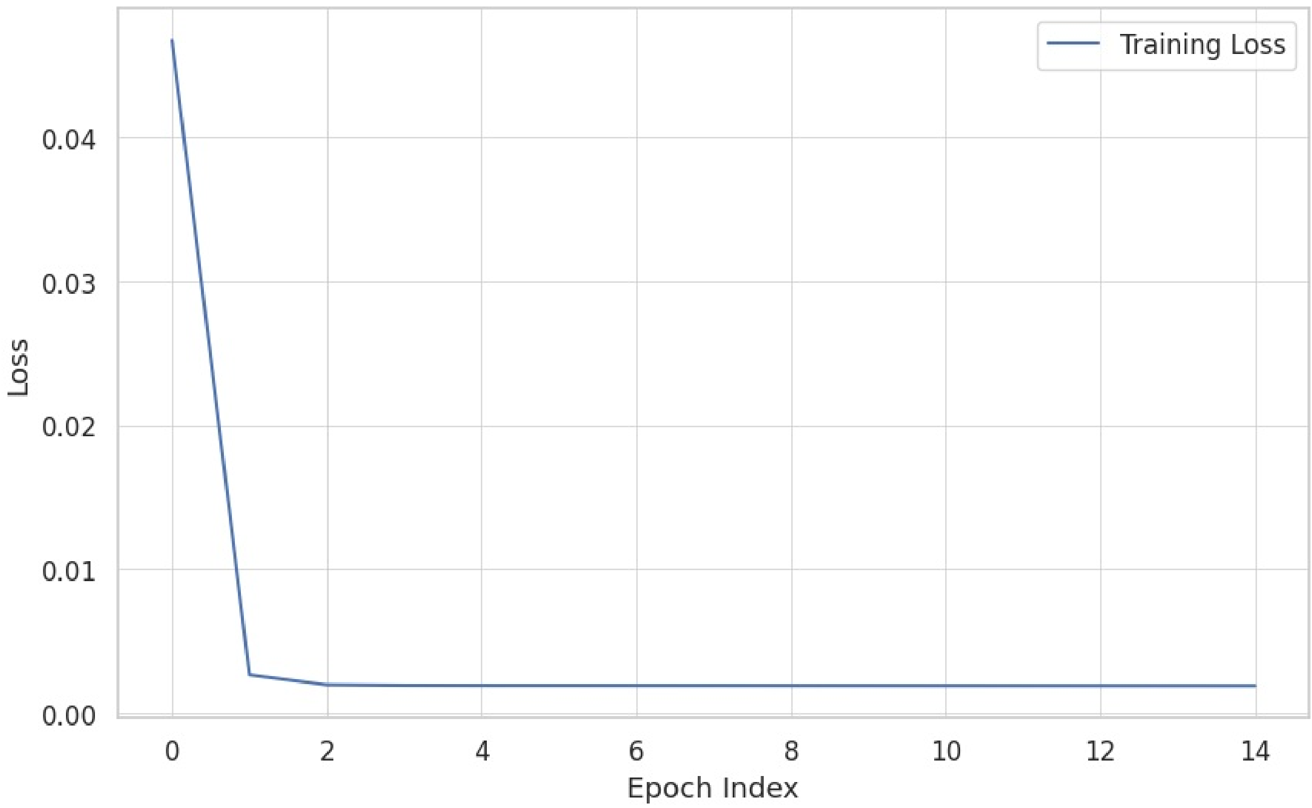
In [ ]:
```
model = Sequential()
model.add(LSTM(5, activation='relu', input_shape=(1,2)))
model.add(Dense(2))
model.add(Dense(2))
model.compile(optimizer='adam', loss='mse')
```

```
/home/stanisz/Documents/programming/ATS_project/venv/lib64/python3.11/site-packages/keras/src/layers/rnn/rnn.py:
204: UserWarning: Do not pass an `input_shape`/`input_dim` argument to a layer. When using Sequential models, pr
efer using an `Input(shape)` object as the first layer in the model instead.
  super().__init__(**kwargs)
```

In [ ]:
```
fitting_history = model.fit(X_train, y_train, epochs=15, batch_size=3, verbose=2)
```

```
Epoch 1/15
771/771 - 4s - 5ms/step - loss: 0.0467
Epoch 2/15
771/771 - 2s - 2ms/step - loss: 0.0027
Epoch 3/15
771/771 - 2s - 2ms/step - loss: 0.0020
Epoch 4/15
771/771 - 2s - 2ms/step - loss: 0.0019
Epoch 5/15
771/771 - 2s - 2ms/step - loss: 0.0019
Epoch 6/15
771/771 - 2s - 2ms/step - loss: 0.0019
Epoch 7/15
771/771 - 2s - 2ms/step - loss: 0.0019
Epoch 8/15
771/771 - 2s - 2ms/step - loss: 0.0019
Epoch 9/15
771/771 - 2s - 2ms/step - loss: 0.0019
Epoch 10/15
771/771 - 2s - 2ms/step - loss: 0.0019
Epoch 11/15
771/771 - 2s - 2ms/step - loss: 0.0019
Epoch 12/15
771/771 - 2s - 2ms/step - loss: 0.0019
Epoch 13/15
771/771 - 2s - 2ms/step - loss: 0.0019
Epoch 14/15
771/771 - 2s - 2ms/step - loss: 0.0019
Epoch 15/15
771/771 - 2s - 2ms/step - loss: 0.0019
```

In [ ]:
```
_, ax = plt.subplots()
ax.plot(fitting_history.history['loss'], label='Training Loss')
ax.set_xlabel('Epoch Index')
ax.set_ylabel('Loss')
plt.legend()
plt.show()
```



The model fitting history shows that the loss quickly diminished and remained constant after the initial epochs.

In [ ]:
```
testPredict = model.predict(X_test)[:,1]
testPredict = testPredict.reshape(len(testPredict), 1)

trainPredict = model.predict(X_train)[:,1]
trainPredict = trainPredict.reshape(len(trainPredict), 1)
```

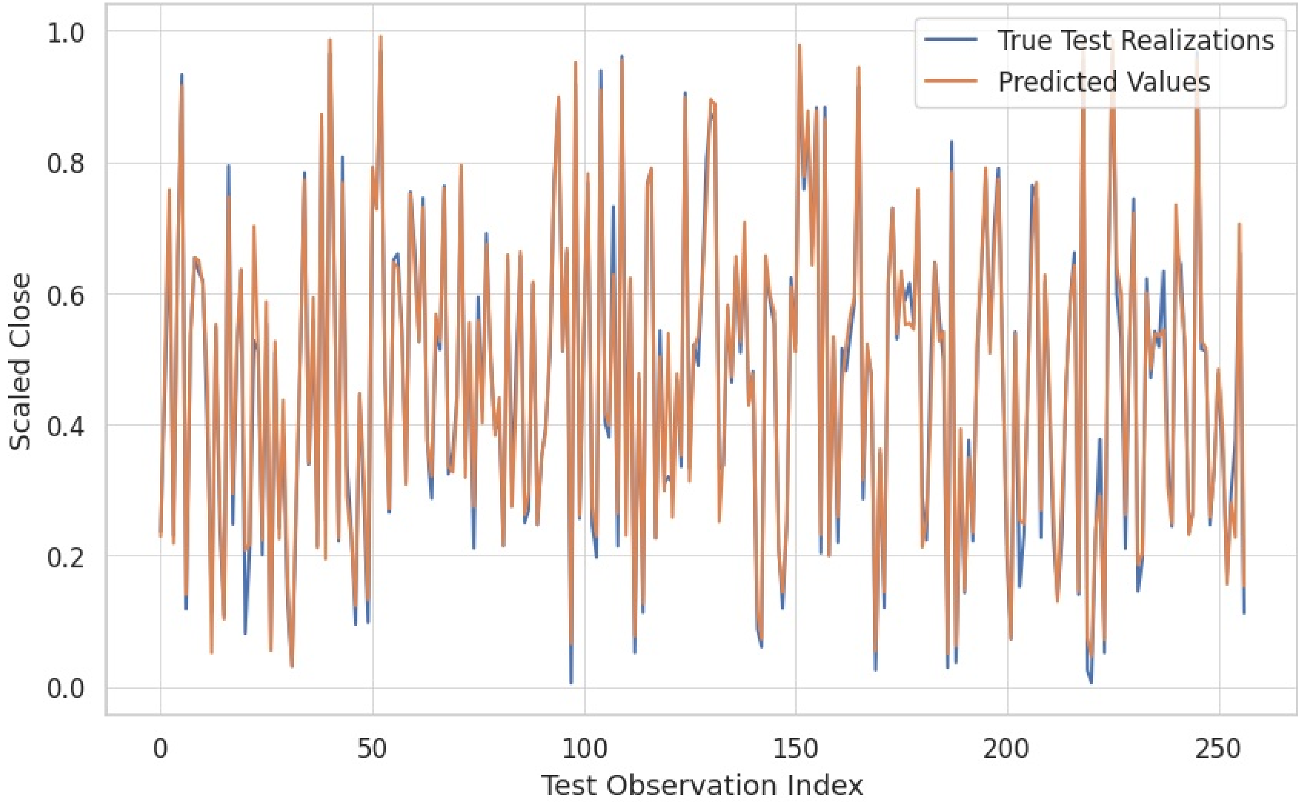
```
9/9 ━━━━━━━━━━━━━━━━━━━━ 0s 28ms/step
73/73 ━━━━━━━━━━━━━━━━━━━━ 0s 2ms/step
```

In [ ]:
```
trainScore = np.sqrt(mean_squared_error(y_train, trainPredict))
print('Train Score: %.4f RMSE' % (trainScore))
```

```
testScore = np.sqrt(mean_squared_error(y_test, testPredict))
print('Test Score: %.4f RMSE' % (testScore))
```

```
Train Score: 0.0435 RMSE
Test Score: 0.0371 RMSE
```

In [ ]:
```
_, ax = plt.subplots()
ax.plot(y_test.values, label='True Test Realizations')
ax.plot(testPredict, label='Predicted Values')
ax.set_xlabel('Test Observation Index')
ax.set_ylabel('Scaled Close')
plt.legend()
plt.show()
```



As seen on the figure above, predictions of the scaled close price were very accurate. The test score was only slightly worse than the train score (as expected), which meant that the procedure did not overfit the model.

Strategy & Results

The strategy was to set a long position when the model predicted a positive return. It did nothing if the predicted return was negative to limit trading frequency.

Firstly, the test sample data was arranged into a data frame, because LSTM forced data into numpy arrays above. The below dataframe still operated on scaled (to [0,1]) values.

In [ ]:
```
lstm_strategy = pd.DataFrame(data=[y_test.values.T[0], testPredict.T[0]]).transpose()
lstm_strategy.columns = columns=['True', 'Predicted']
lstm_strategy.head()
```

Out[ ]:

	True	Predicted
0	0.236232	0.228875
1	0.461055	0.493040
2	0.701483	0.757850
3	0.230056	0.218773
4	0.653398	0.653928

The scaling applied on the dataset was inverted to get the actual values of the close price.

In [ ]:
```
predicted = testPredict.T[0].reshape(-1, 1)
predicted = np.c_[ predicted, predicted, predicted, predicted ] # The original transform had 4 features, we only
predicted = scaler.inverse_transform(predicted)[:,1]

actual = y_test.values.T[0].reshape(-1, 1)
```

```
actual = np.c_[ actual, actual, actual, actual ]
actual = scaler.inverse_transform(actual)[:,1]
```

In [ ]:
```
lstm_strategy = pd.DataFrame(data=[actual, predicted]).transpose()
lstm_strategy.columns = columns=['True', 'Predicted']
lstm_strategy.head()
```

Out[ ]:

	True	Predicted
0	146.5555	146.477478
1	148.9400	149.279236
2	151.4900	152.087830
3	146.4900	146.370331
4	150.9800	150.985626

Predicted and true returns on the out-of-sample test period were calculated.

In [ ]:
```
lstm_strategy['Predicted Return'] = (lstm_strategy['Predicted'] - lstm_strategy['Predicted'].shift(1)).shift(-1
lstm_strategy['True Return'] = (lstm_strategy['True'] - lstm_strategy['True'].shift(1)).shift(-1)

lstm_strategy = lstm_strategy.dropna()
lstm_strategy.head()
```

Out[ ]:

	True	Predicted	Predicted Return	True Return
0	146.5555	146.477478	2.801758	2.3845
1	148.9400	149.279236	2.808594	2.5500
2	151.4900	152.087830	-5.717499	-5.0000
3	146.4900	146.370331	4.615295	4.4900
4	150.9800	150.985626	2.782455	2.9650

The signal was defined to equal 1 only if predicted one-day return was positive. The strategy did not trade otherwise.

In [ ]:
```
lstm_strategy = lstm_strategy.assign(signal = lambda row: row['Predicted Return'] > 0)
lstm_strategy.head()
```

Out[ ]:

	True	Predicted	Predicted Return	True Return	signal
0	146.5555	146.477478	2.801758	2.3845	True
1	148.9400	149.279236	2.808594	2.5500	True
2	151.4900	152.087830	-5.717499	-5.0000	False
3	146.4900	146.370331	4.615295	4.4900	True
4	150.9800	150.985626	2.782455	2.9650	True

Strategy returns were defined to equal the actual returns obtained in the market that time less transaction costs defined above. It should be noted that the time index represented 5-minute observations. The evaluation period was therefore short.

In [ ]:
```
lstm_strategy = lstm_strategy.assign(strategy_gross_returns = lambda row: row['signal'] * row['True Return'])
lstm_strategy['Did Position Change'] = (lstm_strategy['signal'].shift(1) == False) & (lstm_strategy['signal'] =
lstm_strategy = lstm_strategy.assign(strategy_net_returns = lambda row: row['strategy_gross_returns'] - row['Di
lstm_strategy['Cumulative Gross Returns'] = lstm_strategy.cumsum()['strategy_gross_returns']
lstm_strategy['Cumulative Net Returns'] = lstm_strategy.cumsum()['strategy_net_returns']
lstm_strategy.head(10)
```

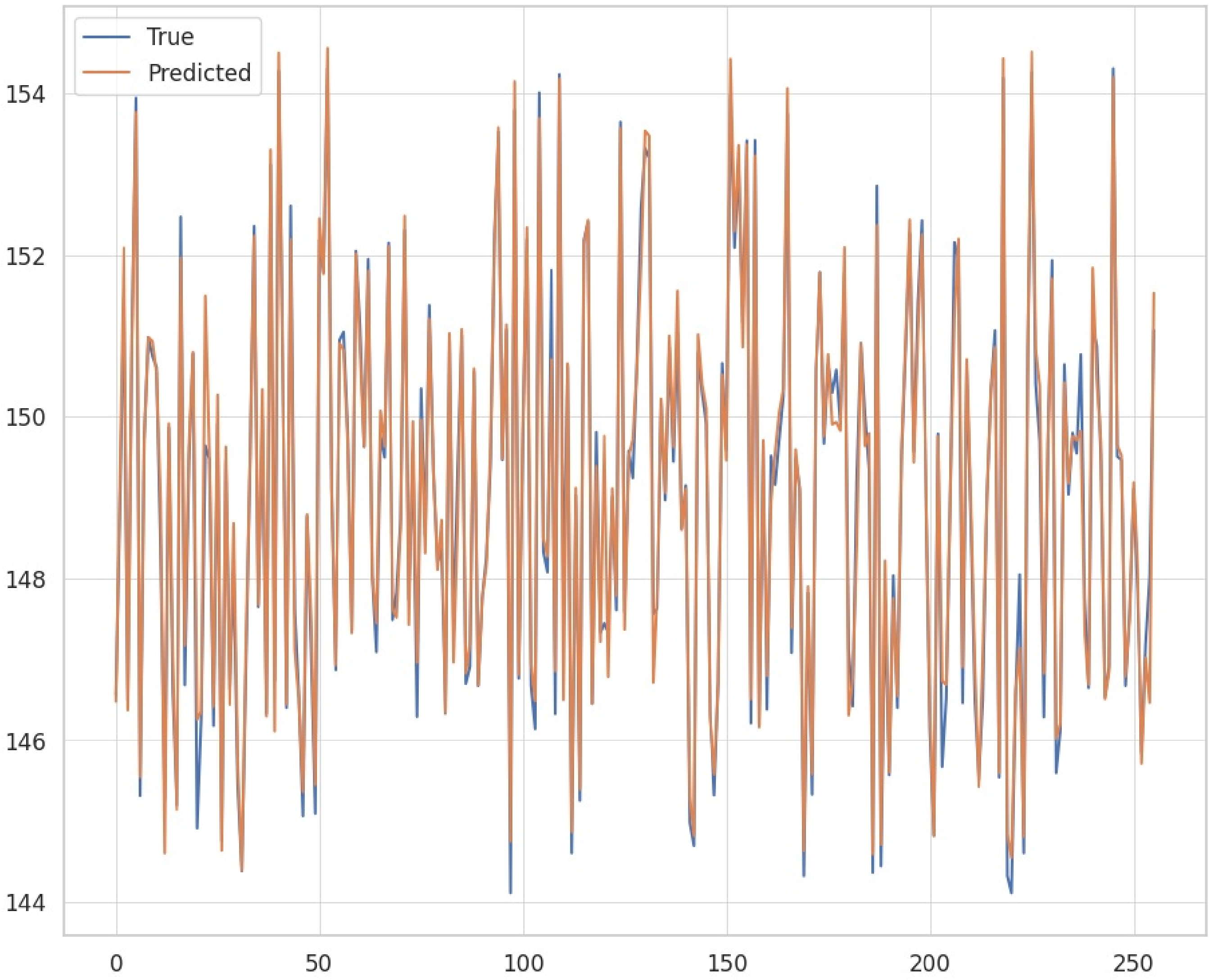
Out[ ]:

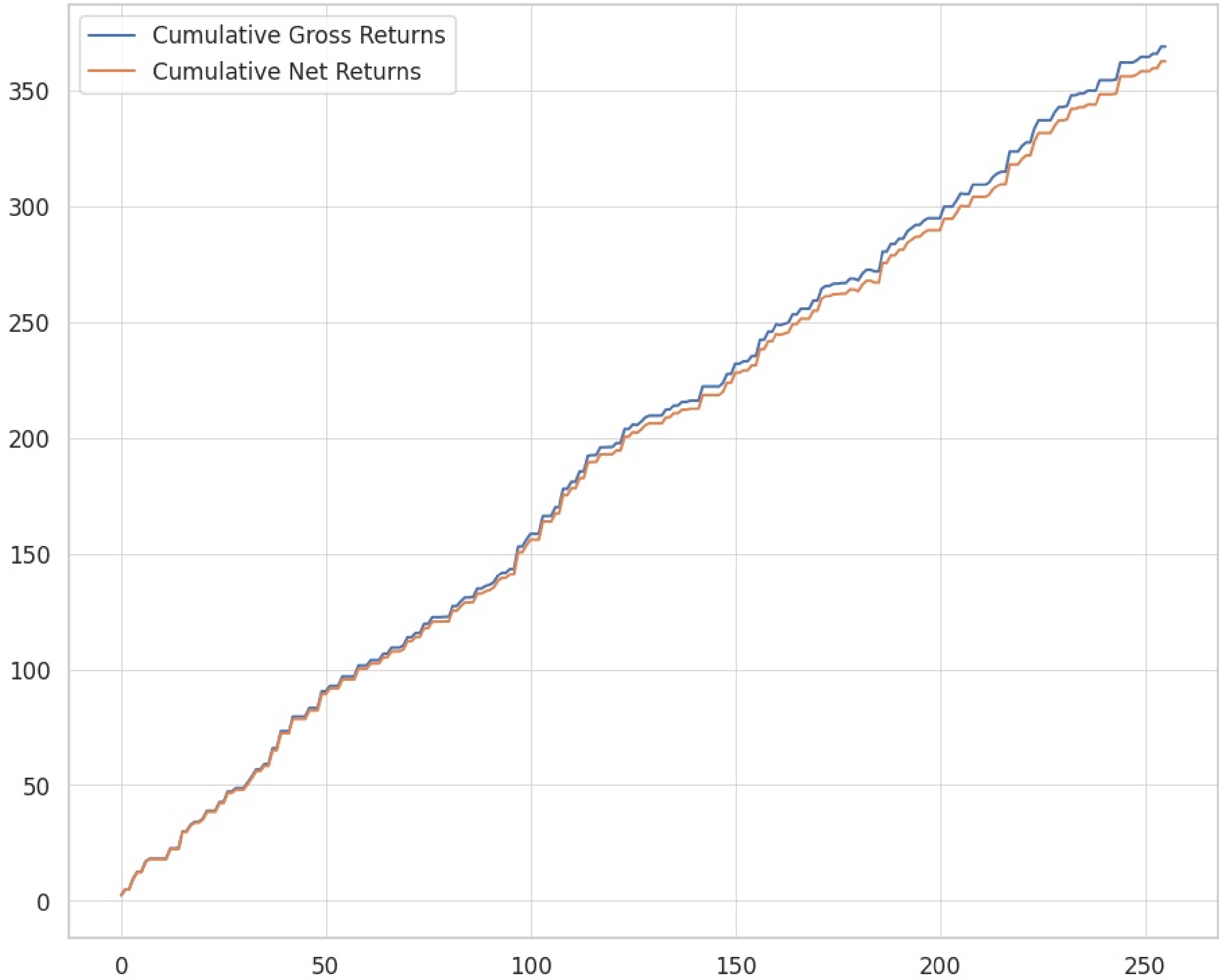
	True	Predicted	Predicted Return	True Return	signal	strategy_gross_returns	Did Position Change	strategy_net_returns	Cumulative Gross Returns	Cumulative Net Returns
0	146.5555	146.477478	2.801758	2.3845	True	2.3845	False	2.384500	2.3845	2.384500
1	148.9400	149.279236	2.808594	2.5500	True	2.5500	False	2.550000	4.9345	4.934500
2	151.4900	152.087830	-5.717499	-5.0000	False	-0.0000	False	-0.000000	4.9345	4.934500
3	146.4900	146.370331	4.615295	4.4900	True	4.4900	True	4.416755	9.4245	9.351255
4	150.9800	150.985626	2.782455	2.9650	True	2.9650	False	2.965000	12.3895	12.316255
5	153.9450	153.768082	-8.224365	-8.6350	False	-0.0000	False	-0.000000	12.3895	12.316255
6	145.3100	145.543716	4.159760	4.4300	True	4.4300	True	4.357345	16.8195	16.673600
7	149.7400	149.703476	1.281158	1.2450	True	1.2450	False	1.245000	18.0645	17.918600
8	150.9850	150.984634	-0.047119	-0.2350	False	-0.0000	False	-0.000000	18.0645	17.918600
9	150.7500	150.937515	-0.393753	-0.1400	False	-0.0000	False	-0.000000	18.0645	17.918600

Strategy results were plotted against true and predicted returns. They included transaction costs defined above.

In [ ]:
```
lstm_strategy[['True', 'Predicted']].plot(figsize=(11,9))
lstm_strategy[['Cumulative Gross Returns', 'Cumulative Net Returns']].plot(figsize=(11,9))
```

Out[ ]: <Axes: >





The LSTM model was assessed using the total Sharpe Ratio on the out-of-sample test period. Simple statistics on the number of performed trades were calculated.

In [ ]:
```
lstm_number_of_trades = lstm_strategy.cumsum()['signal'].iloc[-1]
lstm_proportion_of_trades = lstm_number_of_trades / len(lstm_strategy)
print(f"LSTM model made {lstm_number_of_trades} trades, it traded {lstm_proportion_of_trades:.{2}}% of times on
```

```
LSTM model made 130 trades, it traded 0.51% of times on average.
```

In [ ]:
```
lstm_sharpe = qs.stats.sharpe(lstm_strategy[['Cumulative Net Returns']])[0]
print(f"LSTM model total Net Sharpe Ratio is equal to {lstm_sharpe:.{3}}")
```

```
LSTM model total Net Sharpe Ratio is equal to 3.79
/tmp/ipykernel_6274/3507800921.py:1: FutureWarning: Series.__getitem__ treating keys as positions is deprecated.
In a future version, integer keys will always be treated as labels (consistent with DataFrame behavior). To acce
ss a value by position, use `ser.iloc[pos]`
  lstm_sharpe = qs.stats.sharpe(lstm_strategy[['Cumulative Net Returns']])[0]
```

The Sharpe ratio was very high. Even including transaction costs, the strategy generated profits steadily and, because it traded only long, the variance of cumulative returns was very low.

Conclusion

The underlying series of Johnson & Johnson equity returns displayed leptokurtic and autoregressive properties across the 19/04/2024-07/06/2024 period, assuming 5-minute prices.

The AR(9)-GARCH(1,2) model produced poor results despite the confirmed persistence of returns. The GARCH component was not able to forecast volatility sufficiently which triggered too many costly trades. The reaction to the signals was inappropriate for the frequency of data used - a mean-reversion strategy and a longer interval between observations may have been beneficial at increasing the -17.5 Sharpe ratio.

The LSTM model performed very well with an impressive Sharpe ratio of 3.79. The high result was attributed to the specification of the LSTM model, which captured short relationships in time very well. The diminishing gradients problem did not affect the prediction adversely. Additionally, the forecasting period of the stock was very stable and behaved like a mean-reverting process. Such a relationship was easily discovered by the model with a dense neural layer, as it effectively partially reduced to a scalar multiplication by -1. The model results could have been worse during a more volatile and trending period on the market. Nonetheless, the model results were impressive and were a good choice for a calm market.

References

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Distribution of Work

Distribution of work matches our initial division of work from the project proposal. In general:

Adam:

- GitHub repo setup
- Definition of classes used in the first trading strategy
- Descriptive statistics of the dataset
- Implementation and testing of AR-GARCH
- Evalutation of the AR-GARCH, plotting results and strategy returns

Maciek:

- Data download
- Reading and preparing the time series
- Implementation and testing of the LSTM model
- Evaluation of LSTM, plotting results and strategy returns