C++ in Quantitative Finance part #2
Individual Home Project for:
Adam Foster (457898)

1st deadline: 2024-07-04, 23:59:59
2nd deadline: 2024-09-15, 23:59:59

You are considering a **long call butterfly** option strategy for **arithmetic Asian options** with a **barrier**. Such strategy is based on a portfolio consisting of one long ITM call, two short ATM calls, and one long OTM call. All options have the same time to maturity. Assume that call options (if you have any) are of down-and-out flavour and that put options (if you have any) are of up-and-out flavour.

Specifically, you are interested in the relationship between the theoretical price of this portfolio and two factors (simultaneously): (1) price of the underlying instrument (after the portfolio is constructed) and (2) level of the barrier.

Your task is to:

1. Perform Monte Carlo simulations which will find prices of appropriate options and illustrate this relationship

   - **either in Excel:** using the option pricer we have developed in `cpp09/cpp10` and the `xlw` wrapper software to import your C++ function to the Excel. Basically, calling the function for appropriate options with given spot prices and maturities will be enough (see `cpp10`). Create **one** appropriate plot which will visualize the relationship described above in a clear and readable way.
   - **or in R:** as above, but using the `Rcpp` package (see `cpp07/cpp09`). Build your own `R` **source package**, install and load the package in the script in the separate R session, call appropriate function/functions to find out prices of options, and visualize the relationship with **one** plot using the `ggplot2` package.

   Include in your plots the appropriate range of two analyzed factors.

   Hint: do not overcomplicate things! Remember that value of the portfolio consisting of two options is given by the sum of values of single options (with appropriate signs, according to the position of the option in the portfolio).

2. Write a report on it.

Your code should be ready to price options for any values of their characteristics. Nevertheless consider the following values for options parameters:

- price of the underyling at the moment of option pricing: $S_0 = 115$
- annualized volatility rate $\sigma = 25\%$
- annualized risk-free rate $r = 5\%$
- annualized dividend rate $d = 0\%$
- time to maturity $t = 5$ months
- barrier is 20% below S_0 for down-and-out options, and 20% above S_0 for up-and-out options

RULES

1. Each project is to be prepared **individually**.
2. Use comments in your code as much as possible.
3. Your submission should consist of the following elements:
   (a) one `zip` file which includes
      - either: the `xlw` solution (see below) together with the resulting `xll` file and the `xls/xlsx/xlsm` file with the application of the function to the problem,
      - or: a complete `R` **source package** which is ready to be installed (see below) and a separate `R` project with a script or `Rmd` file with the application of the developed function from the package to the problem (simulations visualisations, etc.),

   (b) a `pdf` file with a short report (see below).
4. Rules to prepare the `zip` file with the `xlw` solution and the `xls/xlsx/xlsm` workbook:
   (a) Double-check if your code compiles without errors.
   (b) Delete all `\debug` and `\Debug` directories/subdirectories.
   (c) Delete any `*.suo` and `*.sdf` files, if you find any.
   (d) Place your `xls/xlsx/xlsm` Excel workbook with the application a the top of your project folder.
   (e) ZIP it all.
5. Rules to prepare the `R` **source package** and application of the `R` function:
   (a) Produce a **source package** using the `Rcpp` package as discussed during `cpp07` classes.
   (b) Place next to the package your `R` project with the script file or the `Rmd` file with the application of the developed function to the problem.
   (c) ZIP it all (the package and the `R` project with the script/`Rmd` file with the application).
6. Your report (a `pdf` file) should include short information about:
   (a) the objective of the project,
   (b) assumptions (for example: dynamics of the prices of underlying, number of iterations in Monte Carlo, etc.),
   (c) the portfolio to be priced and its characteristics,
   (d) code elements used (short explanations),
   (e) results of your study: prices of the portfolio, a plot representing the relationship and your comments to that.

   Please note, that the report is a very important part of your submission. Report of bad quality may significantly contribute to your overall score.
7. Your submission files should have following names: `XXXXX.zip` and `XXXXX.pdf`, where `XXXXX` is your surname. Examples: `sakowski.zip`, `sakowski.pdf`.
8. Share your solution (one `zip` file and one `pdf` file) with `p.sakowski@uw.edu.pl` via google drive, one drive, dropbox or similar services with a clickable link. You should get a confirmation of this from me within a couple of hours.
9. You can earn 40 pts for this assignment. Together with max of 40 pts from the final exam abd 20 pts for graded homeworks this sums up to 100 pts. As agreed at the beginning of the semester (see `cpp01.pdf`) a minimum to get a pass grade is 60 pts, which will be lowered to 50 pts for active students.
10. You must agree to abide by the following Honor Code:

(a) My solutions of home project will be my own work.

(b) I will not make solutions to homeworks available to anyone else.

(c) I will not engage in any other activities that will dishonestly improve my results or dishonestly improve/hurt the results of others.

11. Cheating is a very serious offence. Please keep in mind that any evidence of copying of your home project solutions (the report as well as the code) will be treated as cheating and will be reported immediately to the Dean. Please take is seriously.

12. Additionally, you are asked to include in your report a following declaration:

    In accordance with the Honor Code, I certify that my answers here are my own work, and I did not make my solutions available to anyone else.

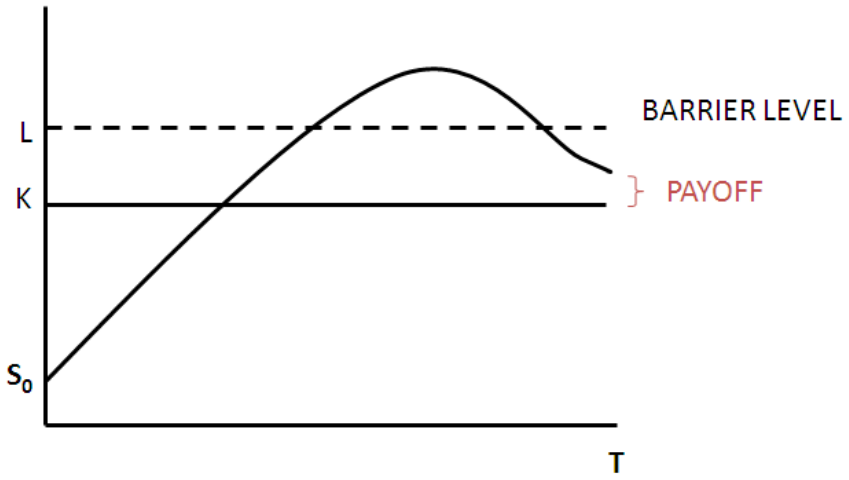
Good luck!

Paweł

A short refresher: payoffs of EUROPEAN barrier options with continuous barriers

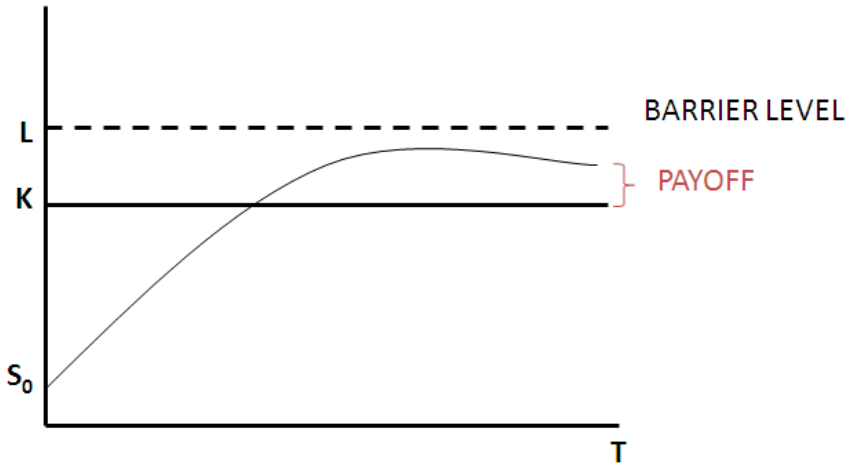
1. CALL UP-and-IN
   - the price has to increase to reach the barrier
   - activated when asset price reaches the barrier

$$\Phi_t = \max(S_T - X, 0) \quad \text{if} \quad \max_{0 \leqslant t \leqslant T}(S_t) \geqslant L \tag{1}$$



2. CALL UP-and-OUT
   - the price has to increase to reach the barrier
   - canceled when the asset price reaches the barrier

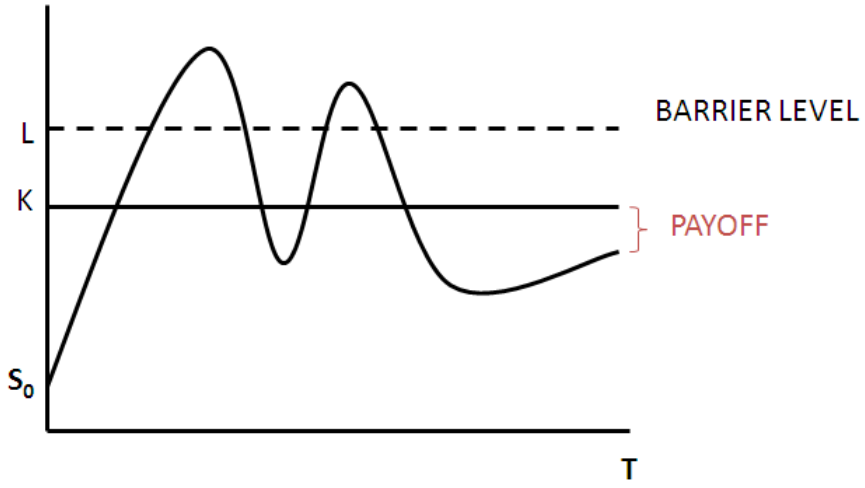
$$\Phi_t = \max(S_T - X, 0) \quad \text{if} \quad \max_{0 \leqslant t \leqslant T}(S_t) \leqslant L \tag{2}$$



3. PUT UP-and-IN

- the price has to increase to reach the barrier
- activated when asset price reaches the barrier

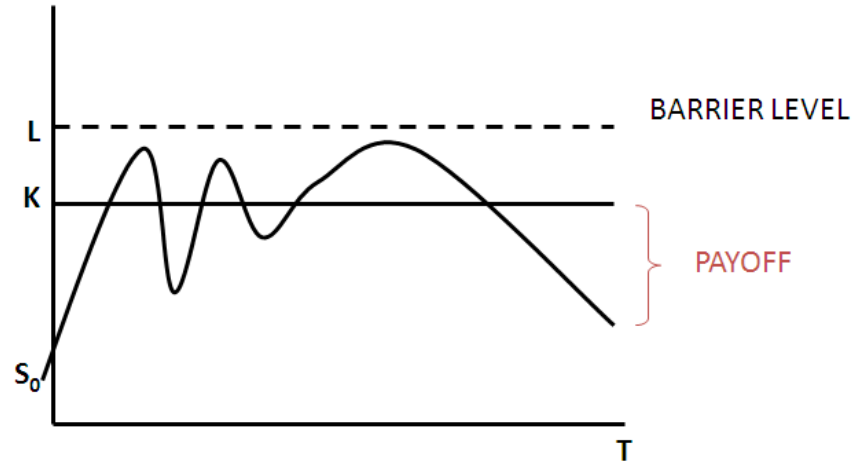
$$\Phi_t = \max(X - S_T, 0) \quad \text{if} \quad \max_{0 \leqslant t \leqslant T}(S_t) \geqslant L \tag{3}$$



4. PUT UP-and-OUT

- the price has to increase to reach the barrier
- canceled when the asset price reaches the barrier

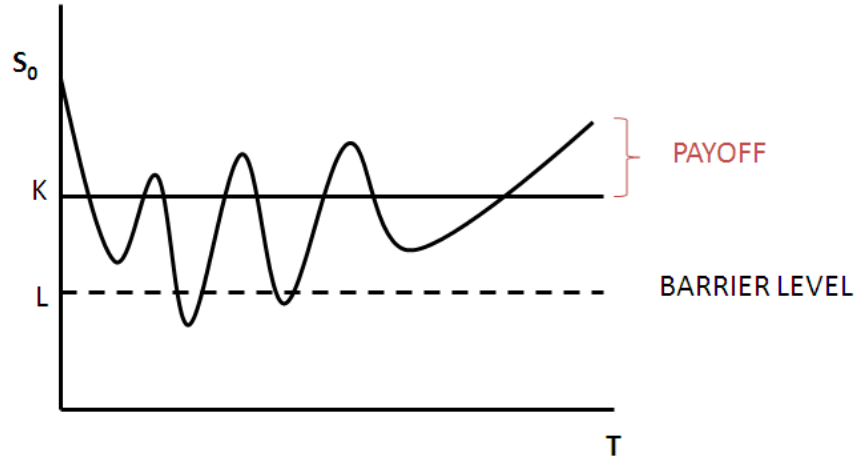
$$\Phi_t = \max(X - S_T, 0) \quad \text{if} \quad \max_{0 \leqslant t \leqslant T}(S_t) \leqslant L \tag{4}$$



5. CALL DOWN-and-IN

- the price has to decrease to reach the barrier
- activated when asset price reaches the barrier

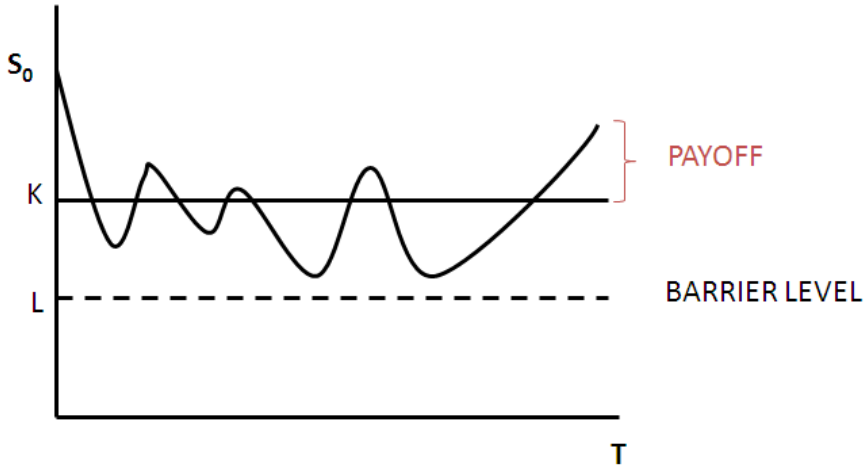
$$\Phi_t = \max(S_T - X, 0) \quad \text{if} \quad \min_{0 \leqslant t \leqslant T}(S_t) \leqslant L \tag{5}$$



6. CALL DOWN-and-OUT

- the price has to decrease to reach the barrier
- canceled when asset price reaches the barrier

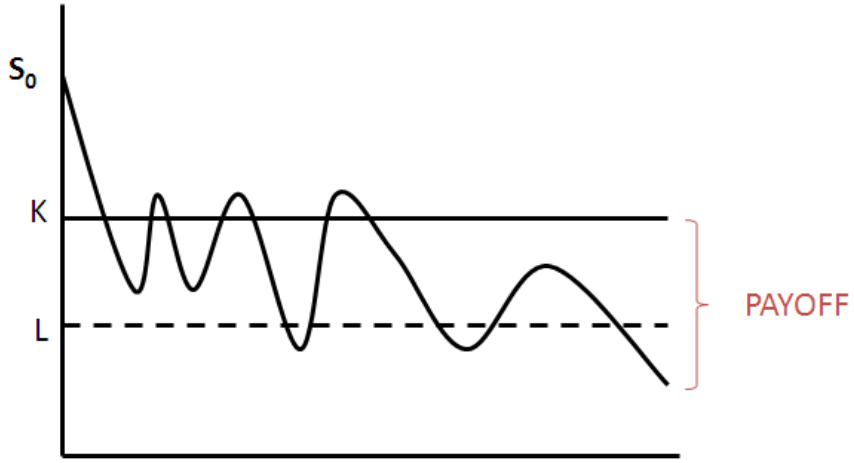
$$\Phi_t = \max(S_T - X, 0) \quad \text{if} \quad \min_{0 \leqslant t \leqslant T}(S_t) \geqslant L \tag{6}$$



7. PUT DOWN-and-IN

- the price has to decrease to reach the barrier
- activated when asset price reaches the barrier

$$\Phi_t = \max(X - S_T, 0) \quad \text{if} \quad \min_{0 \leqslant t \leqslant T}(S_t) \leqslant L \tag{7}$$



8. PUT DOWN-and-OUT

- the price has to decrease to reach the barrier
- canceled when asset price reaches the barrier

$$\Phi_t = \max(X - S_T, 0) \quad \text{if} \quad \min_{0 \leqslant t \leqslant T}(S_t) \geqslant L \tag{8}$$