Our network is a graph (S, F) with S a set of switches and F a set of fibers. We associate a binary variable x_{ij} to each pair of adjacent switches $(i, j) \in S^2$, such that

$$x_{ij} = \begin{cases} 1 \text{ if the fiber } (i,j) \in F \text{ belongs to the path} \\ 0 \text{ otherwise} \end{cases}$$

We note d_{ij} the distance between i and j. The length L of a path is

$$\mathrm{L} = \sum_{(i,j)\in F} d_{ij} x_{ij}$$

Finding the shortest path comes down to minimizing L.

A few constraints must be introduced to ensure the flow conservation :

— The difference between the outgoing traffic and the incoming traffic at the source s must be equal to 1.

$$\sum_{j:(s,j)\in F} x_{sj} - \sum_{j:(j,s)\in F} x_{js} = 1$$

— For any switch other than the source and the destination, the outgoing traffic and the incoming traffic must be equal (flow conservation).

$$\sum_{j:(i,j)\in F} x_{ij} - \sum_{j:(j,i)\in F} x_{ji} = 0 \quad \forall i \in S \setminus \{s, d\}$$

— the x_{ij} are binary variables.

$$x_{ij} \in \{0, 1\} \quad \forall (i, j) \in F$$

Note : at the destination switch, we must have

$$\sum_{j:(d,j)\in F} x_{dj} - \sum_{j:(j,d)\in F} x_{jd} = -1$$

. However, it can be proven that this is a consequence of the two above-mentioned constraints combined : it doesn't need to be an explicit constraint.

$$\sum_{j:(s,j)\in F} x_{sj} - \sum_{j:(j,s)\in F} x_{js} = 1$$

We note $G(P, E)$ the transformed grah which nodes are traffic paths $p \in P$, and we define Λ a set of wavelengths such that $\Lambda = \{\lambda_1, \lambda_2, ..., \lambda_{|\Lambda|}\}$.

Given a wavelength λ and a path p, we define the binary variables

$$x_p^\lambda = \begin{cases} 1 \text{ if } \lambda \text{ is assigned to } p \\ 0 \text{ otherwise} \end{cases}$$

$$y_\lambda = \begin{cases} 1 \text{ if } \lambda \text{ is used at least once} \\ 0 \text{ otherwise} \end{cases}$$

Minimizing the number of assigned wavelengths in the optical network comes down to minimizing

$$\sum_{\lambda \in \Lambda} y_\lambda$$

To obtain a valid tour, we need to add the following constraints :

— For every path, there must one and only one wavelength assigned.

$$\sum_{\lambda \in \Lambda} x_p^\lambda = 1 \quad \forall p \in P$$

— If a wavelength λ is not used ($y_\lambda = 0$), it cannot be assigned to any path ($x_p^\lambda = 0 \quad \forall p \in P$), and if it is used ($y_\lambda = 1$), it cannot be assigned to more than one path (i.e $x_p^\lambda + x_{p'}^\lambda \leq 1 \quad \forall (p, p') \in P^2$). In other words,

$$x_p^\lambda + x_{p'}^\lambda \leq y_\lambda \quad \forall \lambda \in \Lambda, \quad \forall (p, p') \in P^2$$

— Wavelengths are assigned sequentially, in increasing order of indices.

$$y_{\lambda_k} \geq y_{\lambda_{k+1}} \quad \forall k \in [\![1, |\Lambda| - 1]\!]$$

— For every path p and every wavelength λ, x_p^λ and y_λ are binary variables.

$$y_\lambda \in \{0, 1\} \quad \forall \lambda \in \Lambda$$

$$x_p^\lambda \in \{0, 1\} \quad \forall p \in P, \quad \forall \lambda \in \Lambda$$