- How Optimal is Word-Referent Identification Under Multimodal Uncertainty?
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Abstract

Identifying a spoken word in a referential context requires both the ability to integrate 14 multimodal input and the ability to reason under uncertainty. How do these tasks interact 15 with one another? We introduce a paradigm that allows us to examine how adults identify words under joint uncertainty in the auditory and visual modalities and propose an ideal observer model of how cues in these modalities are combined optimally. Model predictions are tested in three experiments where word recognition is made under two kinds of 19 uncertainty: category ambiguity and perceptual noise. In all cases, the optimal model 20 explains much of the variance in human mean judgments. When the signal is not distorted 21 with noise, participants weight the auditory and visual cues optimally, that is, according to 22 the relative reliability of each modality. But when one modality has noise added to it, 23 human perceivers systematically prefer the unperturbed modality to a greater extent than the optimal model does. The study provides a formal framework which helps to quantify 25 how word form and word meaning interact in word recognition under uncertainty. Moreover it offers a first step towards a model that accounts for form-meaning synergies in early word 27 learning. 28

Keywords: Language understanding; audio-visual processing; word learning; speech perception; computational modeling.

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Introduction

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Language uses symbols expressed in one modality – the auditory modality, in the case 34 of speech – to communicate about the world, which we perceive through many different 35 sensory modalities. Consider hearing someone yell "bee!" at a picnic, as a honey bee buzzes 36 around the food. Identifying a word involves processing the auditory information as well as 37 other perceptual signals (e.g., the visual image of the bee, the sound of its wings, the 38 sensation of the bee flying by your arm). A word is successfully identified when information from these modalities provide convergent evidence. However, word identification takes place in a noisy world, and the cues received through each modality may not provide a definitive answer. On the auditory side, individual acoustic word tokens are almost always ambiguous with respect to the particular sequence of phonemes they represent, which is due to the inherent variability of how a phonetic category is realized acoustically (Hillenbrand, Getty, Clark, & Wheeler, 1995). And some tokens may be distorted additionally by mispronunciation or ambient noise. Perhaps the speaker was yelling "pea" and not "bee." Similarly, a sensory impression may not be enough to make a definitive identification of a visual category. Perhaps the insect was a beetle or a fly instead. How does the listener deal with such multimodal uncertainty to recognize the speaker's intended word? 49

As a simplified case study of early word learning, the task of matching sounds to corresponding visual objects has been studied extensively in the developmental literature.

For example, many studies focus on how children might succeed in this type of task despite

<sup>&</sup>lt;sup>1</sup>In the general case, language can of course be visual as well as auditory, and object identification can be done through many modalities. For simplicity, we focus on audio-visual matching here.

referential ambiguity (Medina, Snedeker, Trueswell, & Gleitman, 2011; Pinker, 1989; Smith & Yu, 2008; Suanda, Mugwanya, & Namy, 2014; Vlach & Johnson, 2013; Vouloumanos, 2008; Yurovsky & Frank, 2015). However, even when they *know* the meanings of a word, listeners (both children and adults) often still find it challenging to recognize which word the speaker has uttered, especially under noise (Mattys, Davis, Bradlow, & Scott, 2012; Peelle, 2018). The purpose of the current study is thus to explore word recognition by adults under multimodal uncertainty. We focus on the special case where people have access to multimodal cues from the auditory speech and the visual referent. In the General Discussion, we return to the question of how these findings relate to questions about word learning.

One rigorous way to approach this question is through conducting an *ideal observer* 62 analysis. This research strategy provides a characterization of the task/goal and shows what 63 the optimal performance should be under this characterization.<sup>2</sup> When there is uncertainty in the input, the ideal observer performs an optimal probabilistic inference. For example, in 65 order to recognize an ambiguous linguistic input, the model uses all available probabilistic knowledge in order to maximize the accuracy of this recognition. The ideal observer model can be seen as a theoretical upper limit on performance. It is not so much a realistic model of human performance, as much as a baseline against which human performance can be compared (Geisler, 2003; Rahnev & Denison, 2018). When there is a deviation from the ideal, it can reveal extra constraints on human cognition, such as limitations on the working memory or attentional resources. This approach has had a tremendous impact not only on speech-related research (Clayards, Tanenhaus, Aslin, & Jacobs, 2008; Feldman, Griffiths, & Morgan, 2009; Kleinschmidt & Jaeger, 2015; Norris & McQueen, 2008), but also on many other disciplines in the cognitive sciences (for reviews, see Chater & Manning, 2006; Knill & 75 Pouget, 2004; Tenenbaum, Kemp, Griffiths, & Goodman, 2011)

<sup>&</sup>lt;sup>2</sup>It is, thus, a general instance of the rational approach to cognition (Anderson, 1990), instantiating Marr's computational level of analysis (Marr, 1982).

Some prior ideal observer studies are closely related to the question we are addressing in the current work. For instance, Clayards et al. (2008) simulated auditory uncertainty by manipulating the probability distribution of a cue (Voice Onset Time) that differentiated similar words (e.g., "beach" and "peach"). They found that humans were sensitive to these probabilistic cues and their judgments closely reflected the optimal predictions. And Feldman et al. (2009) studied the perceptual magnet effect, a phenomenon that involves reduced discriminability near prototypical sounds in the native language (Kuhl, 1991), showing that this effect can be explained as the consequence of optimally solving the problem of perception under uncertainty.

Besides the acoustic cues explored in Clayards et al. (2008) and Feldman et al. (2009), there is extensive evidence that information from the visual modality, such as the speaker's facial features, also influences speech understanding (see Campbell, 2008 for a review). 88 Bejjanki, Clayards, Knill, and Aslin (2011) offered a mathematical characterization of how probabilistic cues from speech and lip movements can be optimally combined. They showed that human performance during audio-visual phonemic labeling was consistent (at least at 91 the qualitative level) with the predictions of an ideal observer. This previous research did 92 not, however, systematically study speech understanding when visual information was obtained through the referential context rather than through observation of speaker's face. Although some experimental findings show that information about the identity of a referent can be integrated with linguistic information to resolve lexical and syntactic ambiguities in speech (e.g., Eberhard, Spivey-Knowlton, Sedivy, & Tanenhaus, 1995; Spivey, Tanenhaus, Eberhard, & Sedivy, 2002; Tanenhaus, Spivey-Knowlton, Eberhard, & Sedivy, 1995), to our knowledge no study has offered an ideal observer analysis of this task.

Combining information between words and visual referents might seem similar to audio-visual speech integration, but there are at least two fundamental differences between these two cases, and both can influence the way the auditory and visual cues are combined.

First, in the case of audio-visual speech, both modalities offer information about the 103 same underlying speech category. They differ only in terms of their informational reliability. 104 In a referential context, however, the auditory and visual modalities play different roles in 105 the referential process – the auditory input represents the symbol whereas the visual input 106 represents the meaning (and these differences are in addition to possible differences in 107 informational reliability). Further, speech is claimed to have a privileged status compared to 108 other sensory stimuli (Edmiston & Lupyan, 2015; Lupyan & Thompson-Schill, 2012; 109 Vouloumanos & Waxman, 2014; Waxman & Gelman, 2009; Waxman & Markow, 1995), and 110 that this privilege is suggested to be specifically related to the ability to refer (Waxman & 111 Gelman, 2009).<sup>3</sup> Thus, in a referential context, it is possible that listeners do not treat the 112 auditory and visual modalities as equivalent sources of information. Instead, there could be a 113 sub-optimal bias for the auditory modality beyond what is expected from informational reliability alone. 115

Second, in the case of audio-visual speech, the auditory and visual stimuli are expected 116 to be perceptually correlated. The expectation for this correlation is strong enough that 117 when there is a mismatch between the auditory and visual input, they are still integrated 118 into a unified (but illusory) percept (e.g., the McGurk Effect; McGurk & MacDonald, 1976). 119 In the case of referential language, however, the multimodal association is by nature 120 arbitrary (Greenberg, 1957; Saussure, 1916). For instance, there is no logical or perceptual 121 connection between the sound "bee" and the corresponding insect. Moreover, variation in 122 the way the sound "bee" is pronounced is generally not expected to correlate perceptually 123 with variation in the shape (or any other visual property) in the category of bees. In sum, 124

<sup>&</sup>lt;sup>3</sup>There is, however, a debate as to whether speech is privileged for children and adults for similar reasons. Whereas some researchers suggest that speech is privileged for both children and adults because of its ability to refer (e.g., Waxman & Gelman, 2009), others suggest that speech might *not* have a referential status from the start. Rather, speech might be preferred by children only because of a low level auditory "overshadowing" (e.g., Sloutsky & Napolitano, 2003).

cue combination in the case of arbitrary audio-visual associations (word-referent) is likely to be less automatic, more effortful, and therefore less conducive to optimal integration than it is in the case of perceptually correlated associations (as in audio-visual speech perception).

### The current study

We investigate how cues from the auditory and the visual modality are combined in 129 recognizing words in a referential context. In particular, we study how this combination is 130 performed under various degrees of uncertainty in both the auditory and the visual modality. 131 Imagine, for example, that someone is uncertain whether they heard "pea" or "bee." Does 132 this uncertainty make them rely more on the referent (e.g., the object being pointed at)? Or, 133 if they are not sure if they saw a bee or a fly, does this uncertainty make them rely more on 134 the sound? More importantly, when input in both modalities is uncertain to varying degrees, 135 do they weight each modality according to its relative reliability (the optimal strategy), or 136 do they over-rely on a particular modality? 137

We begin by proposing an ideal observer model that performs the combination in an 138 optimal fashion. We then compare the predictions of the optimal model to human responses. 139 Humans can deviate from the ideal for several reasons. For instance, as mentioned above, a 140 sub-optimality can be induced by the privileged status of a particular modality or by the 141 arbitrariness of the referential association. In order to study possible patterns of 142 sub-optimality, we compare the optimal model (which provides a normative benchmark) to a 143 descriptive model (which is fit to human responses). Comparing parameter estimates 144 between these two formulations allows us to quantify the degree of deviation from optimality. 145

We tested the ideal observer model's predictions in three behavioral experiments where
we varied the source of uncertainty. In Experiment 1, audio-visual tokens were ambiguous
with respect to their category membership only. In Experiment 2, we intervened by adding

perceptual noise to the auditory modality, and in Experiment 3, we intervened by adding
perceptual noise to the visual modality. In all experiments, participants were quantitatively
near-optimal, though overall response precision was slightly lower than expected. In
Experiment 1 – where neither of the modalities was perturbed with background noise –
participants weighted auditory and visual cues according to the relative reliability predicted
by the optimal model. However, in Experiment 2 and 3, participants over-relied on one
modality when the other modality was perturbed with additional noise.

# Paradigm and Models

In this section we first briefly introduce the multimodal combination task. Then we explain how behavior in this paradigm can be characterized optimally with an ideal observer model.

# The Audio-Visual Word Recognition Task

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We introduce a paradigm adapted from a task used by Sloutsky and Napolitano (2003). 161 The original was used with both children and adults to probe audio-visual encoding (see 162 Robinson & Sloutsky, 2010 for review). Here we use a slightly different version to test word 163 recognition in a referential context. We use two visual categories (cat and dog) and two 164 auditory categories (/b/ and /d/ embedded in the minimal pair /aba/-/ada/). For each 165 participant, an arbitrary pairing is set between the auditory and the visual categories, leading to two audio-visual word categories (e.g., dog-/aba/, cat-/ada/). In each trial, participants are presented with an audio-visual target (the prototype of the target category), 168 immediately followed by an audio-visual test stimulus (Figure 1). The test stimulus may 169 differ from the target in both the auditory and the visual components. After these two 170 presentations, participants press "same" or "different." 171

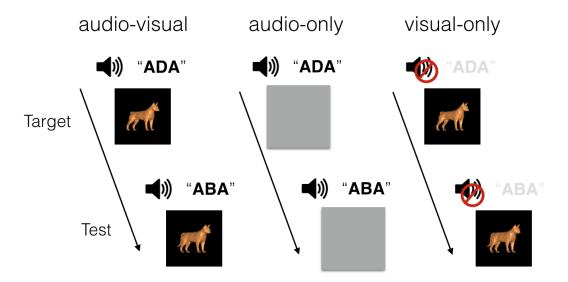


Figure 1. Overview of the task. In the audio-visual condition, participants are first presented with an audio-visual target (the prototype of the target category), immediately followed by an audio-visual test. The test may differ from the target in both the auditory and the visual components. After these two presentations, participants press 'same' (i.e., the same category as the target) or 'different' (not the same category). The auditory-only and visual-only conditions are similar to the audio-visual condition, except that only the sounds are heard, or only the pictures are shown, respectively.

In the testing phase of the original task (Sloutsky & Napolitano, 2003), participants 172 were asked whether or not the two audio-visual presentations are *identical*. In the current 173 study, we are interested, rather, in the categorization, i.e., determining whether or not two 174 similar tokens are members of the same phonological/semantic category. Therefore, testing 175 in our task is category-based: Participants are asked to press "same" if they think the second item (the test) belongs to the same category as the first (target) (e.g., dog-/aba/), even if 177 there is a slight difference in the sound, in the referent, or in both. They are instructed to 178 press "different" only if they think that the second stimulus was an instance of the other 179 category (cat-/ada/). The task also includes trials where pictures are hidden (audio-only) or 180 where sounds are muted (visual-only). These unimodal trials provide us with the 181

participants' evaluation of the probabilistic information present in the auditory and visual categories. As we shall see, these unimodal distributions are used as inputs to the optimal cue combination model.

# 185 Optimal Model

We construct an ideal observer model that combines probabilistic information from the 186 auditory and visual modalities. In contrast to the model used in most research on 187 multisensory integration (e.g., Ernst & Banks, 2002), which typically studies continuous 188 stimuli (e.g., size, location), the probabilistic information in our case cannot be characterized 189 with sensory noise only. Since our task involves responses over categorical variables 190 (phonemes and concepts), the optimal model should take into account not only the noise variability around an individual perceptual estimate but also its categorical variability, i.e., the uncertainty related to whether this perceptual estimate belongs to a given category (see 193 also Bankieris, Bejjanki, & Aslin, 2017; Bejjanki et al., 2011). In what follows, we describe a 194 model that accounts for both types of variability. First, we describe the model in the 195 simplified case of categorical variability only. Second, we augment this simplified model to 196 account for sensory noise. 197

Categorical variability. We assume that both the auditory categories (i.e., /aba/ and /ada/) and the visual categories (cat and dog) are distributed along a single acoustic and semantic dimension, respectively (Figure 2). Moreover, we assume that all categories are normally distributed. Formally speaking, if A denotes an auditory category (/ada/ or /aba/), then the probability that a point a along the acoustic dimension belongs to the category A is

$$p(a|A) \sim N(\mu_A, \sigma_A^2)$$

where  $\mu_A$  and  $\sigma_A^2$  are respectively the mean and the variance of the auditory category. Similarly, the probability that a point v along the visual dimension belongs to the category  $_{205}$  V is

$$p(v|V) \sim N(\mu_V, \sigma_V^2)$$

where  $\mu_V$  and  $\sigma_V^2$  are the mean and the variance of the visual category. An audio-visual signal w=(a,v) can be represented as a point in the audio-visual space. These audio-visual tokens define bivariate distributions in the bi-dimentional space. We call these bivariate distributions  $Word\ categories$ , noted W, and are distributed as follows:

$$p(w|W) \sim N(M_W, \Sigma_W)$$

where  $M_W = (\mu_A, \mu_V)$  and  $\Sigma_W$  are the mean and the covariance matrix of the word category.

The main assumption of the model is that the auditory and visual variables are independent

(i.e., uncorrelated), so the covariance matrix is simply:

$$\Sigma_W = \left[ \begin{array}{cc} \sigma_A^2 & 0 \\ 0 & \sigma_V^2 \end{array} \right]$$

This assumption says that, given a word-object mapping, e.g., W = (``cat"-CAT'), variation in the way "cat" is pronounced does not correlate with changes in any visual property of the object CAT, which is a valid assumption in the context of our task.<sup>4</sup>

Now we turn to the crucial question of modeling how the optimal decision should proceed given the probabilistic (categorical) information in the auditory and the visual modalities, as characterized above. We have two word categories: dog-/aba/  $(W_1)$  and cat-/ada/  $(W_2)$ . When making decisions, participants can be understood as choosing one of these two word categories (Figure 2). For an ideal observer, the probability of choosing category 2 when presented with an audio-visual instance w = (a, v) is the posterior

<sup>&</sup>lt;sup>4</sup>Note that this assumptions is more adequate in the case of arbitrary associations such as ours, and less so in the case of redundant association such as audio-visual speech. In the latter, variation in the pronunciation is expected to correlate, at least to some extent, with lip movements.

<sup>&</sup>lt;sup>5</sup>This mapping is randomized in the experiments.

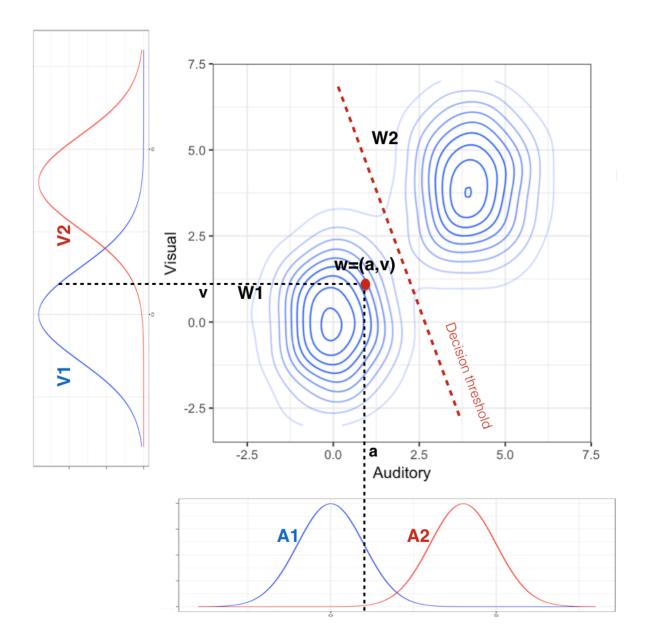


Figure 2. Illustration of the model using simulated data. A word category is defined as the joint bivariate distribution of an auditory category (horizontal, bottom panel) and a visual semantic category (vertical, left panel). Upon the presentation of a word token w, participants guess whether it is sampled from the word type  $W_1$  or from the word type  $W_2$ . Decision threshold is where the guessing probability is 0.5.

probability of this category:

$$p(W_2|w) = \frac{p(w|W_2)p(W_2)}{p(w|W_2)p(W_2) + p(w|W_1)p(W_1)}$$

Using our assumption that the cues are uncorrelated, we have:

$$p(w|W) = p(a, v|W) = p(a|A)p(v|V)$$

Under this assumption, the posterior probability reduces to the following formula (see

225 Appendix 1 for the details of the derivation):

$$p(W_2|w) = \frac{1}{1 + (1+b)\exp(\beta_0 + \beta_a a + \beta_v v)}$$
(1)

226 where

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$$1 + b = \frac{p(W_1)}{p(W_2)}$$
$$\beta_0 = \frac{\mu_{A2}^2 - \mu_{A1}^2}{2\sigma_A^2} + \frac{\mu_{V2}^2 - \mu_{V1}^2}{2\sigma_V^2}$$

$$\beta_a = \frac{\mu_{A1} - \mu_{A2}}{\sigma_A^2}$$
$$\beta_v = \frac{\mu_{V1} - \mu_{V2}}{\sigma_V^2}.$$

The parameter b represents the differential between the categories' prior probabilities. 229 However, since the identity of word categories is randomized across participants, b measures, 230 rather, a response bias to "same" if b > 0, and a response bias to "different" if b < 0. We 231 expect a general bias towards answering "different" because of the categorical nature of our 232 same-different task: When two items are ambiguous but perceptually different, participants 233 might have a slight preference for "different" over "same". As for the means, their values are fixed, and they correspond to the most typical tokens in our stimuli. Finally, observations from each modality (a and v) are weighted in Equation 1 according to their reliability (that 236 is, according to the *inverse* of their variance): 237

$$\beta_a \propto \frac{1}{\sigma_A^2}$$
 $\beta_v \propto \frac{1}{\sigma_V^2}$ .

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**Sensory variability.** So far, we have only accounted for categorical variability. For 239 instance, if the speaker generates a target production  $a_t$  from an auditory category 240  $p(a_t|A) \sim N(\mu_A, \sigma_A^2)$ , the ideal model assumes that it has direct access to this production 241 token (i.e.,  $a = a_t$ ), and that all uncertainty is about the category membership of this token. 242 However, we might also want to account for internal noise in the brain and/or external noise 243 in the environment. For example, the observer might not have access to the exact produced 244 target, but only to the target perturbed by noise. If we assume this noise to be normally 245 distributed, that is,  $p(a|a_t) \sim N(a_t, \sigma_{N_A}^2)$ , then integrating over  $a_t$  leads to this new 246 expression of the probability distribution: 247

$$p(a|A) \sim N(\mu_A, \sigma_A^2 + \sigma_{N_A}^2)$$

Similarly, in the case of sensory noise in the visual modality, we get:

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$$p(a|V) \sim N(\mu_V, \sigma_V^2 + \sigma_{N_V}^2)$$

Finally, using exactly the same derivation as above, we end up with the following multimodal weighting scheme in the optimal combination model (Equation 1) which takes into account both categorical and sensory variability:

$$\beta_a \propto \frac{1}{\sigma_A^2 + \sigma_{N_A}^2}$$

$$\beta_v \propto \frac{1}{\sigma_V^2 + \sigma_{N_V}^2}.$$

Optimal cue combination. Equation 1 provides the optimal model's predictions
for how probabilities that characterize uncertainty in the auditory and the visual modalities
can be combined to make categorical decisions. Parameter estimates of the probability
distributions in each modality are derived by fitting unimodal posteriors to the participants'
responses in the unimodal conditions, i.e., the condition where only the sounds are heard or

only the pictures are seen (Figure 1).<sup>6</sup> Using these derived parameters, the optimal model makes predictions about responses in the bimodal (i.e., audio-visual) condition where participants both hear the sounds and see the pictures.

Auditory and Visual baselines. The predictions of the optimal model will be 261 compared to two baselines. The first baseline is a visual model which assumes that 262 participants rely only on visual information, and an auditory model, which assumes that 263 participants rely only on auditory information. More precisely, these baseline models assume 264 that the participants' responses in the bimodal condition will not be different from their 265 response in either the visual-only or the auditory-only condition. However, if the participants 266 rely on both the auditory and the visual modalities to make decision in the bimodal 267 condition, the optimal model would explain more variance in human responses than the 268 visual or the auditory model do. 269

# 270 Descriptive model and analysis of sub-optimality

The optimal model (as well as the auditory and visual baselines) are *normative* models. 271 Their predictions are made about human data in the bimodal condition, but their crucial 272 parameters (i.e., variances associated with the visual and auditory modalities) are derived 273 from data in the unimodal conditions. In addition to these normative models, we consider a 274 descriptive model. It is formally identical to the normative optimal model (Equation 1), 275 except that the parameters are fit to actual responses in the bimodal condition. If the 276 referential task induces sub-optimality (due, for instance, to the arbitrary nature of the sound-object association), then the descriptive model should explain more variance than the 278 optimal model does.

how much people deviate from optimality, but also to understand precisely the nature of this 281 deviation. Let  $\sigma_A^2$  and  $\sigma_V^2$  be the values of the variances used in the optimal model (derived 282 from the unimodal conditions), and  $\sigma_{Ab}^2$  and  $\sigma_{Vb}^2$  be the values observed through the 283 descriptive model in the bimodal condition. Deviation from optimality is measured in two 284 ways. First, we measure the change in the values of the variance specific to each modality, 285 that is, how  $\sigma_A^2$  compares to  $\sigma_{Ab}^2$ , and how  $\sigma_V^2$  compares to  $\sigma_{Vb}^2$ . Second, we measure changes 286 in the proportion of the visual and auditory variances, i.e., we examine how  $\frac{\sigma_A^2}{\sigma_V^2}$  compares to 287  $\frac{\sigma_{Ab}^2}{\sigma_{Vb}^2}$ . The first measure allows us to test if response precision changes for each modality when 288 we move from the unimodal to the bimodal conditions. The second allows us to test the 289 extent to which the weighting scheme follows the prediction of the optimal model. The 290 reason we used the proportion of the variances as a measure of cross-modal weighting is 291 because this proportion corresponds to the slope<sup>7</sup> of the decision threshold in the audio-visual space (Figure 2). The decision threshold is defined as the set of values in this 293 audio-visual space along which the posterior is equal to 0.5. Formally speaking, the decision threshold has the following form:

$$v = -\frac{\sigma_V^2}{\sigma_A^2}a + v_0$$

If the absolute value of the slope derived from the descriptive model is greater than
that of the optimal model, the corresponding shift in the decision threshold indicates that
participants have a preference for the auditory modality in the bimodal case. Similarly, a
smaller absolute value of the slope would lead to a preference for the visual modality. The
limit cases are when there is exclusive reliance on the auditory cue (a vertical line), and
where there is exclusive reliance on the visual (a horizontal line).

There are three possible ways human responses can deviate from optimality. These

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<sup>&</sup>lt;sup>7</sup>Or more precisely the absolute value of the slope.

scenarios are illustrated in Figure 3, and are as follows:

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- 1) Both variances may increase, but their proportion remains the same. That is,  $\sigma_{Ab}^2 \geqslant \sigma_A^2$  and  $\sigma_{Vb}^2 \geqslant \sigma_V^2$ , but  $\frac{\sigma_{Ab}^2}{\sigma_{Vb}^2} \approx \frac{\sigma_A^2}{\sigma_V^2}$ . In this case, sub-optimality would be due to increased randomness in human responses in the bimodal condition. However, this randomness would not affect the relative weighting of both modalities, i.e., participants would still weigh modalities according to the relative reliability predicted by the optimal model.
- The auditory variance increases at a higher rate. That is,  $\sigma_{Ab}^2 \gg \sigma_A^2$  and  $\sigma_{Vb}^2 \geqslant \sigma_V^2$ ,
  leading to  $\frac{\sigma_{Ab}^2}{\sigma_{Vb}^2} > \frac{\sigma_A^2}{\sigma_V^2}$ . In this case, sub-optimality would consist not only in participants
  being more random in the bimodal condition, but also in having a systematic
  preference for the visual modality, even after accounting for informational reliability.
- 313 3) The visual variance increases at a higher rate. That is,  $\sigma_{Vb}^2 \gg \sigma_V^2$ , and  $\sigma_{Ab}^2 \geqslant \sigma_A^2$ ,

  134 leading to  $\frac{\sigma_{Ab}^2}{\sigma_{Vb}^2} > \frac{\sigma_A^2}{\sigma_V^2}$ . This case is the reverse of case 2, i.e., in addition to increased

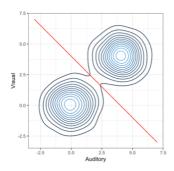
  135 randomness in the bimodal condition, there is a systematic preference for the auditory

  136 modality, even after accounting for informational reliability.
- We compared these models to human responses in three experiments. In Experiment 1,
  we studied the case where bimodal uncertainty was due to categorical variability, only. In
  Experiment 2 and 3 we added auditory and visual noise, respectively, on top of categorical
  variability.

### Experiment 1

In this Experiment, we test the predictions of the model in the case where uncertainty is due to categorical variability only (i.e., ambiguity in terms of category membership). We do not add any external noise to the background and we assume that internal sensory noise is negligible compared to categorical variability ( $\sigma_A^2 \gg \sigma_{N_A}^2$  and  $\sigma_V^2 \gg \sigma_{N_V}^2$ ). Thus, we use





# **Descriptive Model**

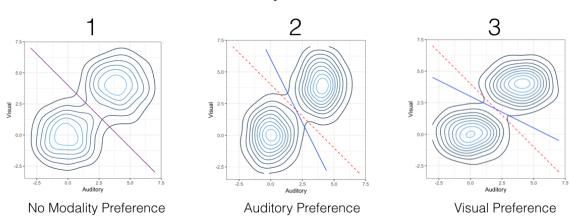


Figure 3. Illustration using simulated data showing the example of a prediction made by the optimal model (top), and the three possible ways human participants can deviate from this prediction (bottom). These cases are the following: 1) The variance increases equally for both modalities, but the weighting scheme (characterized by the decision threshold) is optimal, 2) The auditory variance increases at a higher rate, leading to a preference for the auditory modality, and 3) The visual variance increases at a higher rate, leading to a preference for the visual modality.

the following cue weighting scheme:

$$\beta_a \propto \frac{1}{\sigma_A^2 + \sigma_{N_A}^2} \approx \frac{1}{\sigma_A^2}$$

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$$\beta_v \propto \frac{1}{\sigma_V^2 + \sigma_{N_V}^2} \approx \frac{1}{\sigma_V^2}.$$

#### 8 Methods

Participants. We recruited a planned sample of 100 participants from Amazon

Mechanical Turk. Only participants with US IP addresses and a task approval rate above

85% were allowed to participate. They were paid at an hourly rate of \$6/hour. Participants

were excluded if they reported having experienced a technical problem of any sort during the

online experiment (N=14), or if they had less than 50% accurate responses on the

unambiguous training trials (N=6). The final sample consisted of N = 80 participants. All

participants provided informed consent before taking the experiment. 8

For auditory stimuli, we used the continuum introduced in Vroomen, 336 Linden, Keetels, Gelder, and Bertelson (2004), a 9-point /aba/-/ada/ speech continuum 337 created by varying the frequency of the second (F2) formant in equal steps. We selected 5 338 equally spaced points from the original continuum by keeping the endpoints (prototypes) 1 339 and 9, as well as points 3, 5, and 7 along the continuum. For visual stimuli, we used a 340 cat/dog morph continuum introduced in Freedman, Riesenhuber, Poggio, and Miller (2001). 341 From the original 14 points, we selected 5 points as follows: we kept the item that seemed 342 most ambiguous (point 8), the 2 preceding points (i.e., 7 and 6) and the 2 following points 343 (i.e., 9 and 10). The 6 and 10 points along the morph were quite distinguishable, and we 344 took them to be our prototypes.

Design and Procedure. We told participants that an alien was naming two
objects: a dog, called "aba" in the alien language, and a cat, called "ada". In each trial, we
presented the first object (the target) on the left side of the screen simultaneously with the
corresponding sound. For each participant, the target was always the same (e.g., dog-/aba/).

 $<sup>^8</sup>$ The sample size and exclusion criteria were specified in the pre-registration at https://osf.io/h7mzp/.

The second sound-object pair (the test) followed on the other side of the screen after 500ms and varied in its category membership. For both the target and the test, visual stimuli were present for the duration of the sound clip (~ 800ms). We instructed participants to press "S" for same if they thought the alien was naming another dog-/aba/, and "D" for different if they thought the alien was naming a cat-/ada/. We randomized the sound-object mapping (e.g., dog-/aba/, cat-/ada/) as well as the identity of the target (dog or cat) across participants.

The first part of the experiment trained participants using only the prototype pictures and the prototype sounds (12 trials, 4 each from the bimodal, audio-only, and visual-only conditions). After completing training, we instructed participants on the structure of the task and encouraged them to base their answers on both the sounds and the pictures (in the bimodal condition). There were a total of 25 possible combinations in the bimodal condition, and 5 in each of the unimodal conditions. Each participant saw each possible trial twice, for a total of 70 trials/participant. Trials were blocked by condition and blocks were presented in random order. The experiment lasted around 15 minutes.<sup>9</sup>

# Model fitting details.

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### $Unimodal\ conditions.$

Remember that data in these conditions allows us to derive the variances of both the auditory and the visual categories, and that these variances are used to make predictions about bimodal data (in the visual and auditory baselines as well as in the optimal model).

These individual variances were derived as follows (we explain the derivation for the auditory-only case, but the same applies for the visual-only case). We use the same Bayesian reasoning as we did in the derivation of the bimodal model: When presented with an audio

<sup>&</sup>lt;sup>9</sup>The experiment can be accessed and played from the github repository: https://github.com/afourtassi/

instance a, the probability of choosing the sound category 2 (that is, to answer "different") is the posterior probability of this category  $p(A_2|a)$ . If we assume that both sound categories have equal variances, the posterior probability reduces to:

$$p(A_2|a) = \frac{1}{1 + (1 + b_A)\exp(\beta_{a0} + \beta_a a)}$$

with  $\beta_a = \frac{\mu_{A_1} - \mu_{A_2}}{\sigma_A^2}$  and  $\beta_{a0} = \frac{\mu_{A_2}^2 - \mu_{A_1}^2}{2\sigma_A^2}$ .  $b_A$  is the response bias in the auditory-only condition. For this model (as well as all other models in this study), we fixed the values of the means to be the end-points of the corresponding continuum, since these points are the most typical instances in our stimuli. Thus, we have  $\mu_{A1} = 0$  and  $\mu_{A2} = 4$  (and similarly  $\mu_{V1} = 0$ , and  $\mu_{V2} = 4$ ). This leaves us with two free parameters: the bias  $b_A$  and the variance  $\sigma_A^2$ . To determine the values of these parameters, we fit the unimodal posterior to human data in the unimodal case.

# $Bimodal\ condition.$

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In this condition, only the descriptive model is fit to the data, using the expression of
the posterior (Equation 1). Since the values of the means are fixed, we have 3 free
parameters: the variances for the visual and the auditory modalities, respectively, and b, the
response bias. The visual and auditory baselines as well as the optimal model are not fit to
the bimodal data, but their predictions are tested against these bimodal data. All these
normative models use the variances derived from the unimodal data and the bias term
derived from the fit to bimodal data.

Although the paradigm is within-subjects, we did not have enough statistical power to
fit a different model for each individual participant. Instead, models were constructed with
data collapsed across all participants. The fit was done with a nonlinear least squares
regression using the NLS package in R (Bates & Watts, 1988). We computed the values of

the parameters, as well as their 95% confidence intervals, through non-parametric bootstrap (using 10000 iterations).

### 397 Results and analysis

Unimodal conditions. Average categorization judgments and best fits are shown in Figure 4. The categorization function of the auditory condition was slightly steeper than that of the visual condition, meaning that participants perceived the sound tokens slightly more categorically and with higher certainty than they did with the visual tokens. For the auditory modality, we obtained the following values:  $b_A = -0.20$  [0.02, -0.38] and  $\sigma_A^2 = 2.04$  [1.66, 2.53]. For the visual modality, we obtained  $b_V = -0.12$  [0.06, -0.28] and  $\sigma_V^2 = 3.33$  [2.83, 3.92].

### Bimodal condition.

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#### $Normative \ models.$

Figure 5 compares the predictions of the normative models against human responses. 407 Remember that the normative models use variance estimates from the unimodal conditions 408 (where people see input from only one modality) to predict data in the bimodal condition 409 (where people see input from both modalities). Each point represents data form a particular 410 audio-visual matching (e.g., the visual token v whose distance from the target  $v_t$  is 411  $|v-v_t|=3$ , matched with the auditory token a whose distance from the target  $a_t$  is 412  $|a - a_t| = 2$ ). The visual, auditory and optimal model explained, respectively, 30%, 67%, and 413 89% of total variance in mean responses. 414

#### $Descriptive \ model.$

In the descriptive model, all parameters are fit to human responses in the bimodal condition. We found b = -0.34 [-0.28, -0.39],  $\sigma_{Ab}^2 = 4.96$  [4.58, 5.40] and  $\sigma_{Vb}^2 = 7.06$  [6.40,

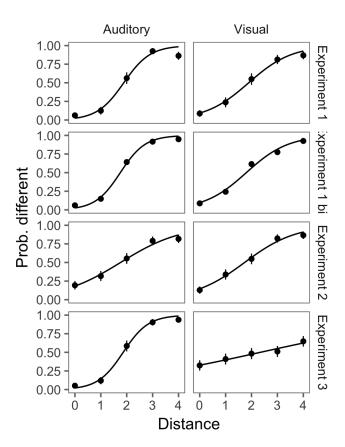


Figure 4. Human responses in the unimodal conditions across the three experiments. Points represent the proportion of 'different' to 'same' responses in the auditory-only condition (left), and visual-only condition (right). Error bars are 95% confidence intervals. Solid lines represent best unimodal posterior fits.

7.84]. Note that the variance of both the auditory and visual modalities increased compared to the unimodal conditions.

The descriptive model explained 95% of total variance. However, since the descriptive model was fit to the same data, there is a risk that this high correlation is due to overfitting. To examine this possibility, we cross-validated the model using half the responses to predict the other half (averaging across 10 random partitions). The predictive power of the model remained very high  $(r^2=0.93)$ .

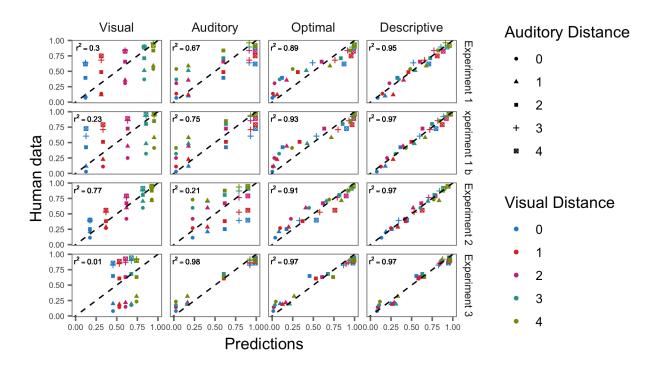


Figure 5. Human responses vs. Models' predictions in the bimodal condition across the three experiments. Each point represents data form a particular audio-visual matching. Shape represents auditory distance from the target, and color represents visual distance from the target. Thus, each point is characterized by both shape and color.

# Cue combination and Modality preference.

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We next analyzed if cue combination was performed in an optimal way, or if there was
a systematic preference for one modality when making decisions in the bimodal condition.
As explained above, modality preference can be characterized formally as a deviation from
the decision threshold predicted by the optimal model. Figure 7 (top) shows both the
decision threshold derived from the descriptive model (in black) and the decision threshold
predicted by the optimal model (in red). The deviation from optimality is compared to two
hypothetical cases of modality preference (dotted lines). We found that the descriptive and

optimal decision thresholds were almost identical. Indeed, non-parametric resampling of the
data showed no evidence of a deviation from the optimal prediction (Figure 7, bottom).

### Discussion

Overall, we found that the optimal model explained much of the variance in the mean 436 judgments, and largely more than what can be explained with the auditory or the visual 437 models alone. Moreover, the high value of the coefficient of determination in the optimal 438 model ( $r^2$ =0.89) suggests that the population was near-optimal. However, we see in Figure 5 439 that the mean responses deviated systematically from the optimal prediction in that they 440 were slightly pulled toward chance (i.e., the probability 0.5). This fact is due to the increase 441 in the value of the variance associated with each modality. Note however that, despite this 442 increase in randomness, our analysis of modality preference showed that the relative values 443 of these variances were not different (Figure 7), meaning that there was no evidence for a 444 modality preference. Thus, 1) There was a simultaneous increase in the values of the auditory 445 and visual variances in the bimodal condition compared to the unimodal condition, meaning that the bimodal input lead to an increase in response randomness, and 2) this increased randomness did not affect the relative weighting of both modalities, i.e., the participants was weighting modalities according to the relative reliability predicted by the optimal model. This situation corresponds to the first case of sub-optimality described in Figure 3. 450

As we noted earlier, we did not have enough statistical power to fit a different model for each individual participant. Instead, we fit one model using the data aggregated across all participants. This analysis is informative of behavior at the population level, only. However, optimality at the population level could be misleading. Imagine, for instance, that in the bimodal condition half the participants relied exclusively on the visual modality, whereas the other half relied exclusively on the auditory modality. This case could still lead to an aggregate behavior which appears optimal, but this optimality would be spurious. This, in

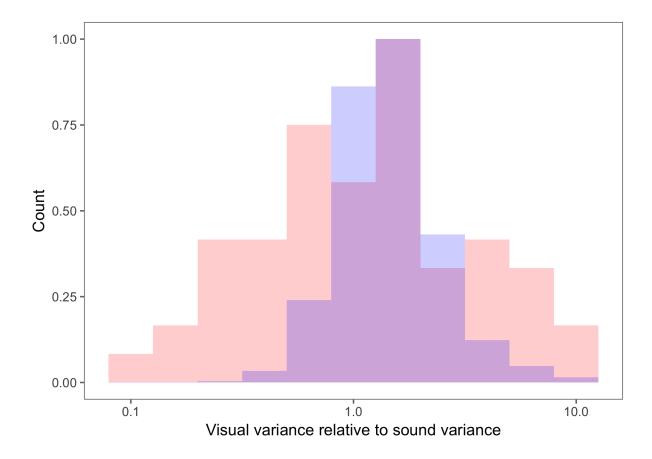


Figure 6. Distributions of individual values of the visual variance relative to the auditory variance in Experiment 1. Light color represents the real individual distribution, and dark color represents the simulated individual distribution sampled from the descriptive model.

the experiemnt below, we test the cue integration strategy at the individual level. To this end, we increase the statistical power by extending the length of the experiment, allowing us to collect the number of datapoints necessary to fit the models for each participant.

# Experiment 1 bis

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Participants. We recruited a planned sample of N=XX participants from Amazon Mechanical Turk. Only participants with US IP addresses and a task approval rate above 99% were allowed to participate. They were paid at an hourly rate of \$6/hour. Participants were excluded if they reported having experienced a technical problem of any sort during the

online experiment (N=XX), or if they had less than 75% accurate responses on the
unambiguous training trials (N=XX). The final sample consisted of N = XX participants. [I
should also add how many participants I excluded because their models did not converge].
All participants provided informed consent before taking the experiment. 10

**Stimuli.** We used the same stimuli as in Experiment 1.

Design and Procedure. The design and procedure were similar to Experiment 1.

The only difference was that we increased the number of datapoints elicited per subject from 70 to 300.

### Results and analysis

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Unimodal conditions. We fit both population-level models to the aggregated data
(as in Experiment 1), and individual models to data from each participant. For the
populaton-level models, the results (shown in Figure 4) replicated the results obtained in
Experiment 1. As for the individual-level analysis, the models showed there to be
between-subject variability as seen in the paramters's distribution in Figure XX.

Bimodal condition and Cue combination models. We first generated
poupulation-level normative and descriptive models. As shown in Figure 5, the predicted
patterns of these models mirrored closely the results obtained in Experiment 1. In addition,
the optimality analysis (i.e., done through comparing the optimal and descriptive models)
shows that the fitted mean decision threshold was indisguisable form the optimal threshold,
thus replicating the results of Experiment 1 (Figure 7 shows that).

At the individual level, we computed the distribution of optimal weighing relative to the fitted (i.e., descriptive) weighing and we show this distribution in Figure XX. The

 $<sup>^{10}</sup>$ The sample size and exclusion criteria were specified in the pre-registration at https://osf.io/h7mzp/.

distribution centers around the optimal prediction (i.e., 1) with a median value of XX [XX,XX]. The variance of the distribution indicates that, while most participants were near optimal in their cue combination, part of the participant tended to rely predominantly on the auditory modality and another part tended to rely predominantly on the visual modality.

[Here compare to random model to see whether the resuls were due to measurement errors or to genuine betwee-subject variability, but isn't that what the CI around the median show?]

### Discussion

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This experiment was an extension to Experiment 1. Through collecting larger-size data per subject, we were able to analyze the cue combination optimlity, not only at the population level, but also at the individual level. The population-level analysis replicated the results of Experiment 1. The individual-level analysis showed that participants' performance approximated a normal distribution centered around the optimal combination. The large variance of this distribution indicate that some participants tend to over-rely on the auditory modality and others tend to over-rely on the visual modality.

In Experiment 1 and Experiment 1 bis, we tested word recognition when there was multimodal uncertainty in terms of category membership and inherent perceptual noise, only.

In real life, however, both sound and visual tokens can undergo distortions due to noisy factors in the environment (e.g., car noise in the background, blurry vision in a foggy weather). In Experiment 2 and 3, we explore this additional level of uncertainty.

#### Experiment 2

In this Experiment, we explored the effect of added noise on performance. We tested a case where the background noise was added to the auditory modality. We were interested to

know if participants would treat this new source of uncertainty as predicted by the optimal model, that is, according to the following weighting scheme

$$\beta_a \propto \frac{1}{\sigma_A^2 + \sigma_{N_A}^2}$$

$$\beta_v \propto \frac{1}{\sigma_V^2}$$
.

The alternative hypothesis is that noise in one modality leads to a systematic preference for the non-noisy modality.

#### 516 Methods

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Participants. A sample of 100 participants was recruited online through Amazon Mechanical Turk. We used the same exclusion criteria as in Experiment 1. 7 participants were excluded because they had less than 50% accurate responses on the unambiguous training trials. The final sample consisted of N=93 participants.

Stimuli and Procedure. We used the same visual stimuli as in Experiment 1. We also used the same auditory stimuli, but we convolved each item with Brown noise of amplitude 1 using the free sound editor Audacity (2.1.2). The average signal-to-noise ratio was - 4.4 dB. The procedure was exactly the same as in the previous experiment, except that the test stimuli (but not the target) were presented with the new noisy auditory stimuli.

### Results and analysis

Unimodal conditions. We fit a model for each modality. For the auditory modality, our parameter estimates were  $b_A = -0.18$  [-0.05, -0.30] and  $\sigma_A^2 + \sigma_N^2 = 4.70$  [4.03, 529 5.55]. For the visual modality, we found  $b_V = -0.24$  [-0.10, -0.36] and  $\sigma_V^2 = 3.93$  [3.43, 4.55]. Figure 4 shows responses in the unimodal conditions as well as the corresponding best fits.

The visual data is a replication of the visual data in Experiment 1. As for the auditory data, in contrast to Experiment 1, responses were flatter, showing more uncertainty.

### Bimodal condition.

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#### $Normative \ models.$

Figure 5 compares the predictions of the visual, auditory and optimal models to human responses. These normative models explained, respectively, 77%, 21%, and 91% of total variance in mean judgements. Note that, in contrast to Experiment 1, the visual model explained more variance than the auditory model did.

### $Descriptive \ model.$

We estimated b= -0.38 [-0.33, -0.42],  $\sigma_{Ab}^2+\sigma_{Nb}^2=9.84$  [8.75, 11.27], and  $\sigma_{Vb}^2=5.21$  [4.84, 5.64]. The fit explained 0.97% of total variance. Cross-validation using half the responses to predict the other half yielded  $r^2=0.96$ .

# Modality preferences.

Figure 7 (top) shows that the participants' decision threshold deviated from optimality, and that this deviation was biased towards the visual modality (the non-noisy modality). Indeed non-parametric resampling of the data showed a decrease in the value of the slope in the descriptive model compared to the optimal model (Figure 7, bottom).

### 548 Discussion

We found, similar to Experiment 1, that the population was generally near optimal  $(r^2 = 0.91)$ , and that the optimal model explained more variance than the auditory or the visual models alone. We also found a similar discrepancy from the optimal model as precision dropped for both the auditory and the visual modalities. As for the weighting

scheme used by participants, contrary to Experiment 1 where modalities were weighted according to their relative reliability, we found in this experiment that the visual modality had a greater weight than what was expected from its relative reliability. This situation corresponds to the second case of sub-optimality described in Figure 3.

We were also interested in whether noise in the auditory modality lead more 557 participants to rely exclusively on the visual modality at the individual level. Using the same 558 cut-off as in Experiment 1 (a factor of 10), the percentage of participants who relied 559 exclusively on either modality was 34.41%, which is much higher than the percentage 560 obtained in Experiment 1 (21.25%). Moreover, the subset of participants relying exclusively 561 on the visual modality (compared to those who relied exclusively on the auditory modality) 562 increased from 29.41% in Experiment 1 to 68.75% in Experiment 2, indicating that noise in 563 the auditory modality prompted more participants to rely exclusively and disproportionately 564 on the visual modality (see Table 1). 565

In Experiment 2, we tested the case of added background noise to the auditory modality. In Experiment 3, we test the case of added noise to the visual modality.

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### Experiment 3

In this Experiment, we added background noise to the visual modality. Similar to
Experiment 2, we were interested to know if participants would treat this new source of
uncertainty as predicted by the optimal model, that is, according to the following weighting
scheme:

$$\beta_a \propto \frac{1}{\sigma_A^2}$$
$$\beta_v \propto \frac{1}{\sigma_V^2 + \sigma_{N_V}^2}.$$

The alternative hypothesis is that noise in the visual modality would lead to a preference for the auditory input, just like noise in the auditory modality lead to a preference for the visual 576 input in Experiment 2.

#### $^{577}$ Methods

Participants. A planned sample of 100 participants was recruited online through
Amazon Mechanical Turk. We used the same exclusion criteria as in both previous
experiments. N=2 participants were excluded because they reported having a technical
problem, and N=10 participants were excluded because they had less than 50% accurate
responses on the unambiguous training trials. The final sample consisted of N = 88
participants.

Stimuli and Procedure. We used the same auditory stimuli as in Experiment 1.

We also used the same visual stimuli, but we blurred the tokens using the free image editor

GIMP (2.8.20). We used a Gaussian blur with a radius<sup>11</sup> of 10 pixels. The experimental

procedure was exactly the same as in the previous Experiments.

# Results and analysis

Unimodal conditions. For the auditory modality, our parameter estimates were  $b_A = -0.24$  [-0.04, -0.42] and  $\sigma_A^2 = 1.94$  [1.61, 2.33]. For the visual modality, we found  $b_V =$  0.11 [0.27, -0.03] and  $\sigma_V^2 + \sigma_N^2 = 13.00$  [9.92, 18.94]. Figure 4 shows responses in the unimodal conditions as well as the corresponding fits. The auditory data is a replication of the auditory data in Experiment 1. As for the visual data, we found that, in contrast to Experiment 1 and 2, responses were flatter, showing much more uncertainty.

### Bimodal condition.

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<sup>&</sup>lt;sup>11</sup>A features that modulates the intensity of the blur.

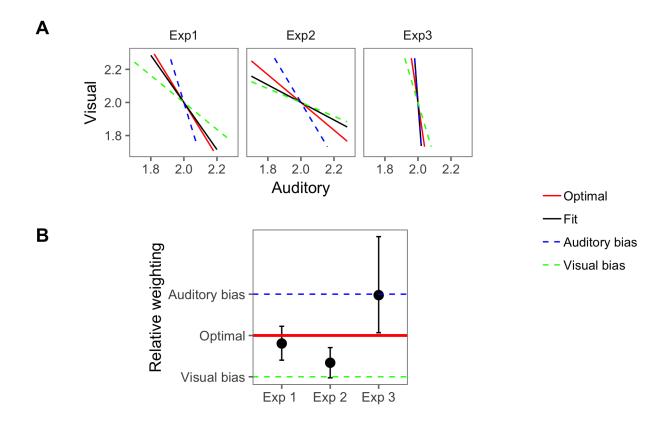


Figure 7. Modality preference is characterized as a deviation from the optimal decision threshold. A) The decision thresholds of both the optimal and the descriptive models (solid red and black lines, respectively). Deviation from optimality is compared to two hypothetical cases of modality preference. In these cases, deviation from optimality is due to over-lying on the visual or the auditory input by a factor of 2 (green and blue dotted lines, respectively). B) An alternative way to represent the same data. Each point represents the value of the decision threshold's slope derived from the descriptive model relative to that of the optimal model (log-scaled). The lines represent the optimal case as well as the two hypothetical cases of modality preference. Error bars represent 95% confidence intervals over the distribution obtained through non-parametric resampling.

### $Normative \ models.$

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Figure 5 compares the predictions of the visual, auditory and optimal models to human

responses. These normative models explained, respectively, 1%, 98%, and 97% of total variance in the mean judgements.

### $Descriptive \ model.$

We estimated b= -0.35 [-0.29, -0.40],  $\sigma_{Ab}^2=3.00$  [2.75, 3.25], and  $\sigma_{Vb}^2+\sigma_{Nb}^2=39.42$  [25.06, 98.96]. The fit explained 97% of total variance. Cross-validation using half the responses to predict the other half yielded  $r^2=0.96$ .

# Modality preferences.

Participants' decision threshold suggested a preference for the auditory modality (the non-noisy modality). Indeed non-parametric resampling of the data showed an increase in the value of the slope in the descriptive model compared to the optimal model (Figure 7).

#### Discussion

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We found that the optimal model accounted for almost all the variance  $(r^2 = 0.97)$ . 600 However, whereas in previous experiments the optimal model explained more variance than 610 the auditory or the visual models, here the auditory model explained at least as much 611 variance  $(r^2 = 0.98)$ . Thus, though participants were still sensitive to variation in the noisy 612 visual data in the unimodal condition, they tended to ignore this information in the bimodal 613 condition, and relied almost exclusively on the non-noisy auditory modality. The reason why 614 we saw this (floor) effect when we added noise to the visual modality (Experiment 3), and not when we added noise to the auditory modality (Experiment 2), is the fact that our visual stimuli were originally perceived less categorically and with less certainty than the auditory 617 stimuli. This fact made it more likely for the visual categorization function to become flat 618 and uninformative after a few drops in precision due to noise on the one had, and to the 619 additional randomness induced by the bimodal presentation on the other hand. 620

The general finding corresponds to the third case of sub-optimality described in
Figure 3. Indeed, precision dropped for both modalities in the bimodal condition compared
to the unimodal condition. But the drop was much greater for the visual modality, resulting
in a much lower weight assigned to it than what is expected from its reliability. Therefore,
just like participants over-relied on the visual modality when the auditory modality was
noisy (Experiment 2), they also over-relied on the auditory modality when the visual
modality was noisy (Experiment 3).

The percentage of participants who relied exclusively on either the visual modality or
the auditory modality was 38.64%, which is closer to the percentage of Experiment 2, except
that now almost all of them relied on the auditory modality (94.12%). For ease of
comparison, Table 1 provides a summary of the numbers across the three experiments.

#### General Discussion

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In the current paper, we explored word identification under uncertainty about both 633 words and their referents. We conducted an ideal observer analysis of this task whereby a 634 model provided predictions about how information from each modality should be combined 635 in an optimal fashion. The predictions of the model were tested in a series of three 636 experiments where instances of both the form and the meaning were ambiguous with respect to their category membership only (Experiment 1), when instances of the form were perturbed with additional background noise (Experiment 2), and when instances of the 639 referent were perturbed with additional visual noise (Experiment 3). We discuss the findings of these studies first with respect to our ideal observer model and inferences about optimality 641 and second with respect to their implications for word identification more generally.

Table 1

The percentage of participants who relied exclusively on either the visual modality or the auditory modality, using a factor of 10 as a cut-off (e.g., we consider that a participant relied exclusively on the visual modality when their auditory variance is a at least 10 times larger than their visual variance). We show the percentage compared to the total number of participants in each Experiment ('of Total'). From this subset of participants, we show the percentage of those who relied on the auditory modality ('Auditory'), and the percentage of those who relied on the visual modality (Visual').

Experiment	ofTotal	Auditory	Visual
Exp1	21.25	70.59	29.41
Exp2	34.41	31.25	68.75
Exp3	38.64	94.12	5.88

### Patterns of optimality and sub-optimality

In all of our experiments, at the highest level, we found generally optimal behavior. 644 Quantitatively speaking, the optimal model accounted, respectively, for 89%, 91%, and 97% 645 of the variance in mean responses. When compared to the predictions of the visual or the 646 auditory models, participants generally relied on both modalities to make their decisions in the bimodal condition. Indeed, in Experiment 1 and 2, the optimal model accounted for more variance in mean responses than the auditory or the visual models did. In Experiment 3, participants appeared to rely on one modality, but this was likely a floor effect, due to the fact that noise made the visual input barely perceptible. Further, in Experiment 1, which did not involve background noise, participants not only relied on both modalities, but 652 generally weighted these modalities according to the predictions of the optimal model, that 653 is, according to their relative reliability. At the individual level, however, we found evidence 654 of a between-subject variation: Some participants relied more on the visual modality, 655 whereas others relied more on the auditory modality. 656

Despite this overall pattern, we documented two major cases of sub-optimality. First, 657 in all experiments, the variance associated with each modality increased in the bimodal 658 condition compared to the unimodal conditions. Participants responded slightly more 659 randomly in the bimodal condition than they did in the unimodal conditions. This finding 660 contrasts with research on multisensory integration where associations tend to lead to a 661 higher precision (e.g., Ernst & Banks, 2002). Nevertheless, there is a crucial difference 662 between these two situations (besides the obvious difference in terms of the models used). Research on multisensory integration (of which audio-visual speech is arguably an instance) deals with redundant multimodal cues, and these cues are integrated into a unified percept. In contrast, the word-referent association is usually arbitrary and, in particular, the cues are not expected to be correlated perceptually. Therefore the observer cannot form a unified 667 percept, rather, information must be encoded separately from both modalities and must 668

retain this encoding through the decision making process. Retaining two separate cues at the same time instead of forming one unified percept (as in multisensory integration of redundant cues), or instead of retaining only one cue (as in the unimodal case), is likely to place extra demands on cognitive resources, which, in turn, could cause general performance to drop. Indeed, there is evidence that cognitive load due to divided attention (e.g., when performing two tasks at the same time) has a detrimental effect on word recognition (Mattys & Wiget, 2011).

Some previous research bas found similar cases of suboptimal behavior. For instance, 676 studies that have explored the identification of ambiguous, newly learned pairs of 677 word-referent associations have reported what appears to be a decrease in speech perception acuity in both children (Stager & Werker, 1997) and adults (Pajak, Creel, & Levy, 2016). 679 Recently, Hofer and Levy (2017) provided a probabilistic model of this phenomenon. In 680 agreement with the findings of our study, Hofer and Levy (2017) characterized the apparent 681 reduction in perceptual acuity as an increase in the noise variance of the auditory modality. 682 Our findings, besides providing more evidence to this documented fact, suggest that the 683 reduction in perceptual acuity may occur simultaneously in both the auditory and the visual 684 modalities. 685

The second case of sub-optimality is related to how participants weighted the cues 686 from the visual and the auditory modalities in a noisy context. In contrast to Experiment 1 687 where the combination was indistinguishable from the optimal prediction, results of 688 Experiment 2 and 3 suggested that participants had a systematic preference for the other (non-noisy) modality. This finding aligns with previous work that suggests that when speech signals are degraded, participants compensate by relying more on other sources of 691 information such as the accompanying visual cues, the semantic/syntactic context, or the 692 top-down expectations. This kind of compensation has been observed with adults (Mattys et 693 al., 2012; Tanenhaus et al., 1995), and recent evidence suggests that it starts in childhood (K. 694

MacDonald, Marchman, Fernald, & Frank, 2018; Yurovsky, Case, & Frank, 2017). Generally 695 speaking, previous experimental studies have not differentiated between an optimal 696 compensatory strategy (i.e., relying more on the alternative source while using all 697 information still available in the distorted signal), and a sub-optimal strategy (i.e., relying 698 more on the alternative source while ignoring at least some of the information still available 690 in the distorted signal), however. The formal approach followed in this paper allowed us to 700 tease apart these two possibilities, and our analysis supports the sub-optimal compensatory 701 strategy: The preference for the non-noisy modality is above and beyond what can be 702 explained by the relative reliability alone, meaning that the participants tend to ignore at 703 least part of the information still available in the noisy modality.

This second case of sub-optimal behavior may be related to the fact that language 705 understanding under degraded conditions is cognitively more taxing than language 706 understanding under normal conditions (Mattys et al., 2012; Peelle, 2018; Rönnberg, Rudner, 707 Lunner, Zekveld, & others, 2010). Perhaps these demands lead to sub-optimal behavior (i.e., 708 over-reliance on the less noisy cue) as participants seek to minimized cognitive effort. One 709 could also explain this phenomenon in terms of the metacognitive experience about the 710 fluency with which information is processed. The perceived perceptual fluency (e.g., the ease 711 with which a stimulus' physical identity can be identified) can affect a wide variety of human judgements (see Schwarz, 2004 for a review). In particular, variables that improve fluency 713 tend to increase liking/preference (Reber, Winkielman, & Schwarz, 1998). In our case, the subjective experience of lower fluency in the noisy modality might cause people to 715 underestimate information that can be extracted from this modality, especially when 716 presented simultaneously with a higher fluency alternative. 717

#### Word recognition in the wild

An important question to ask is how the combination mechanism – as revealed in our 719 controlled study – scales up to real life situations. Note that in order to test audio-visual cue 720 combination under uncertainty, we had to use a case of double ambiguity, that is, a case 721 where both the word forms ("ada"-"aba") and the referents (cat-dog) were similar and, thus, 722 confusable. However, to what extent does such a case occur in real languages? 723 Cross-linguistic corpus analyses suggest that lexical encoding tends, surprisingly, towards double ambiguity in many languages (Dautriche, Mahowald, Gibson, & Piantadosi, 2017; 725 Monaghan, Shillcock, Christiansen, & Kirby, 2014; Tamariz, 2008). For instance, Dautriche et al. (2017) analyzed 100 languages and found that words that are similar phonologically 727 tend to be similar semantically as well. These studies suggest that the case of double 728 uncertainty, though perhaps not pervasive, could be a real issue in language as it increase 720 the probability of confusability for many words. That said, the inferences discussed here 730 might play a more significant role in naturalistic language comprehension when ambiguity in 731 both the form and/or the referent is induced by an external noisy context – e.g., a very noisy 732 party or a far away referent – even when these forms and referents are not confusable in 733 normal situations. 734

Though we only studied adult performance in this paper, the problem of word recognition under uncertainty is likely more pressing for children. In fact, children have greater difficulties differentiating the meanings of novel similar-sounding words (e.g., "bin" vs. "din"), even when these words are uttered very clearly (Creel, 2012; Merriman & Schuster, 1991; Stager & Werker, 1997; Swingley, 2016; White & Morgan, 2008). Such similar-sounding words can be shown to be differentiated by infants in simplified experimental settings (e.g., Yoshida, Fennell, Swingley, & Werker, 2009). Nevertheless, Swingley (2007) suggested that the ability to make this differentiation is likely not mature in early childhood; children's representations are almost certainly noisier than the adults' representations and may also be

encoded with lower confidence. Thus, children even more than adults might benefit from additional disambiguating cues during new word-referent encoding and recognition.

A multi-modal cue combination strategy might help children not only recognize words, 746 but also refine their underlying phonological and semantic representations in the process. 747 Previous research in early word learning has – whether implicitly or explicitly – largely 748 treated the process of refining the word form and of refining the word meaning as following a 749 linear timeline. However, developmental data reveal that children do not wait to have 750 complete acquisition of word forms before they start learning their meanings (Bergelson & Swingley, 2012; Tincoff & Jusczyk, 1999). Rather, both form and meaning representations develop in a parallel fashion. A few studies have already suggested the possibility of an interaction between sound and meaning in early acquisition. For instance, Waxman and 754 Markow (1995) showed that labeling various objects with the same name helps infants form 755 the underlying semantic category (but see Sloutsky & Napolitano, 2003). And in the opposite 756 direction, Yeung and Werker (2009) showed that pairing similar sounds with different objects 757 can helps infants enhance their sensitivity to subtle phonological contrasts in their native 758 language. The present study proposes a first step towards a formal framework where these 759 sorts of sound-meaning interactions in development can be unified and further explored. 760

One salient limitation of our current work is that we used a restricted and highly 761 simplified stimulus set. For the auditory modality, we used speech categories that varied 762 along a single acoustic dimension. While this dimension might be sufficient to recognize 763 words in our specific case, in general the speech signal is far more complex, varying along several acoustic/phonetic dimensions. Additionally, these dimensions may be highly variable 765 due to various kinds of speaker and context differences. The same thing can be said about our visual stimuli. Here we used a continuum along a single morph dimension in order to 767 construct a multimodal input where the auditory and visual components have symmetrical 768 properties. Though such morph is not the exact visual variability that people would 769

encounter in their daily lives, it allowed us to precisely test the role of auditory and visual information in the cue combination process. Parameterizing semantic dimensions is a 771 notoriously difficult problem, but morphs have been used in previous research as a reasonable 772 proxy (Freedman et al., 2001; Havy & Waxman, 2016; Sloutsky & Fisher, 2004). It is an 773 open question whether people use the same strategy in controlled laboratory conditions and 774 more naturalistic settings where they have to deal with various levels of variability. An 775 answer to this question is likely to involve a multifaceted research approach that goes beyond 776 controlled experimentation. We believe that one fruitful approach is to test computational 777 mechanisms with an input that more accurately represents the full extent of multimodal 778 variability in the learning environment (Dupoux, 2018; Fourtassi, Schatz, Varadarajan, & 779 Dupoux, 2014; Harwath, Torralba, & Glass, 2016; B. C. Roy, Frank, DeCamp, & Roy, 2015). 780

781 Conclusions

Our work provides a formal framework where old and new questions about word 782 recognition in a referential context can be given a precise formulation. While we focused on 783 the case of arbitrary associations, it is possible to use the same framework to study, for 784 instance, the case of *iconicity*, that is, when there is a resemblance between the sound of a 785 word and its referent. Previous work has suggested that iconicity, among other things, helps 786 with learning (and generalizing the meaning of) new words (see Dingemanse, Blasi, Lupyan, 787 Christiansen, & Monaghan, 2015 for a review). Using the research strategy in this paper, we 788 can, for example, test whether iconicity has such an advantage because it mitigates the sub-optimal patterns observed with more arbitrary pairings. Finally, though the current framework only characterizes adult word recognition, it provides a first step towards a model where developmental questions can also be investigated. For instance, future work should explore whether children, like adults, use probabilistic cues from both the auditory and the 793 visual input to recognize ambiguous words, the extent to which they combine these cues in

an optimal fashion, and whether this cue combination help them to refine their early phonological and semantic representations.

All data and code for these analyses are available at https://github.com/afourtassi/WordRec

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# Appendix 1: derivation of the posterior (Equation 1)

For an ideal observer, the probability of choosing category 2 when presented with an audio-visual instance w = (a, v) is the posterior probability of this category:

$$p(W_2|w) = \frac{p(w|W_2)p(W_2)}{p(w|W_2)p(W_2) + p(w|W_1)p(W_1)}$$

Which reduces to:

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$$p(W_2|w) = \frac{1}{1 + \frac{p(w|W_1)}{p(w|W_2)} \frac{p(W_1)}{p(W_2)}}$$

In order to further simplify the quantity  $\frac{p(w|W_1)}{p(w|W_2)}$ , we use our assumption that the cues are uncorrelated:

$$p(w|W) = p(a, v|W) = p(a|A)p(v|V)$$

809 Using the log transformation, we get:

$$\ln(\frac{p(w|W_1)}{p(w|W_2)}) = \ln(\frac{p(a|W_1)}{p(a|W_2)}) + \ln(\frac{p(v|W_1)}{p(v|W_2)})$$

Under the assumption that the categories are normally distributed and that, within each modality, the categories have equal variances, we get (after simplification):

$$\ln\left(\frac{p(a|W_1)}{p(a|W_2)}\right) = \frac{\mu_{A1} - \mu_{A2}}{\sigma_A^2} \times a + \frac{\mu_{A2}^2 - \mu_{A1}^2}{2\sigma_A^2}$$

and similarly:

$$\ln(\frac{p(v|W_1)}{p(v|W_2)}) = \frac{\mu_{V1} - \mu_{V2}}{\sigma_V^2} \times v + \frac{\mu_{V2}^2 - \mu_{V1}^2}{2\sigma_V^2}$$

When putting all these terms together, we obtain this final expression for the posterior:

$$p(W_2|w) = \frac{1}{1 + (1+b)\exp(\beta_0 + \beta_a a + \beta_v v)}$$

814 where

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$$1 + b = \frac{p(W_1)}{p(W_2)}$$
$$\beta_0 = \frac{\mu_{A2}^2 - \mu_{A1}^2}{2\sigma_A^2} + \frac{\mu_{V2}^2 - \mu_{V1}^2}{2\sigma_V^2}$$

$$\beta_a = \frac{\mu_{A1} - \mu_{A2}}{\sigma_A^2}$$

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$$\beta_v = \frac{\mu_{V1} - \mu_{V2}}{\sigma_V^2}.$$

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