Word-Referent Identification Under Multimodal Uncertainty

Abdellah Fourtassi<sup>1</sup> & Michael C. Frank<sup>1</sup>

<sup>1</sup> Department of Psychology, Stanford University

Author Note

- Abdellah Fourtassi
- 6 Department of Psychology
- 5 Stanford University
- s 50 Serra Mall

3

- Jordan Hall, Building 420
- Stanford, CA 94301
- 11 Correspondence concerning this article should be addressed to Abdellah Fourtassi,
- Postal address. E-mail: afourtas@stanford.edu

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Abstract

Identifying a spoken word in a referential context requires both the ability to integrate 14 multimodal input and the ability to reason under uncertainty. How do these tasks interact 15 with one another? We introduce a task that allows us to examine how adults identify words 16 under joint uncertainty in the auditory and visual modalities. We propose an ideal observer 17 model which provides an account of how auditory and visual cues are combined optimally. 18 Model predictions are tested in three experiments where word recognition is made under two 19 kinds of uncertainty: category ambiguity and/or distorting noise. In all cases, the optimal model explains much of the variance in human mean judgments. In particular, when the 21 signal is not distorted with noise, participants weight the auditory and visual cues optimally, 22 that is, according to the relative reliability of each modality. But when one modality has noise added to it, human perceivers systematically prefer the unperturbed modality to a greater extent than the optimal model does. The study provides a formal framework which helps us understand precisely how word form and word meaning interact in word recognition under uncertainty. Moreover it offers a first step towards a model that accounts for form-meaning synergy in early word learning. 28

Keywords: Language understanding; audio-visual processing; word learning; speech perception; computational modeling.

Word count: X

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## Word-Referent Identification Under Multimodal Uncertainty

Language uses symbols expressed in one modality, e.g., the auditory modality, in the 33 case of speech, to communicate about the world, which we perceive through many different 34 sensory modalities. Consider hearing someone yell "bee!" at a picnic, as a honey bee buzzes 35 around the food. Identifying a word involves processing the auditory information as well as 36 other perceptual signals (e.g., the visual image of the bee, the sound of its wings, the 37 sensation of the bee flying by your arm). A word is successfully identified when information from these modalities provide convergent evidence. However, word identification takes place in a noisy world, and the cues received through each modality may not provide a definitive answer. On the auditory side, individual acoustic word tokens are almost always ambiguous with respect to the particular sequence of phonemes they represent, which is due to the inherent variability of how a phonetic category is realized acoustically (Hillenbrand, Getty, Clark, & Wheeler, 1995). And some tokens may be distorted additionally by mispronunciation or ambient noise. Perhaps the speaker was yelling "pea" and not "bee". Similarly, a sensory impression may not be enough to make a definitive identification of a visual category. Perhaps the insect was a beetle or a fly instead. How does the listener deal with such multimodal uncertainty to recognize the speaker's intended word? 48 The task of matching the sound to the corresponding visual object has been extensively 49 studied in the developmental literature since it is considered to be an crucial instance of early word learning. For example, many studies focused on how children might succeed in this task despite high referential ambiguity (Medina, Snedeker, Trueswell, & Gleitman, 2011; Pinker, 1989; Smith & Yu, 2008; Suanda, Mugwanya, & Namy, 2014; Vlach & Johnson, 2013; 53 Vouloumanos, 2008; Yurovsky & Frank, 2015). However, even when they know the exact meanings of the words, listeners (both children and adults) often face the task of recognizing which word the speaker utters, especially under noisy circumstances (Mattys, Davis, Bradlow,

<sup>&</sup>lt;sup>1</sup>In the general case, language can of course be visual as well as auditory, and object identification can be done through many modalities. For simplicity, we focus on audio-visual matching here.

& Scott, 2012; Peelle, 2018). The purpose of the current study is to explore novel word
 recognition by adults under multimodal uncertainty. We focus on the special case where
 people have access to multimodal cues from the auditory speech and the visual referent.

One rigorous way to approach this question is through conducting an *ideal observer* 60 analysis. This research strategy provides a characterization of the task/goal and shows what the optimal performance should be under this characterization.<sup>2</sup> When there is uncertainty in the input, the ideal observer performs an optimal probabilistic inference. For example, in order to recognize an ambiguous linguistic input, the model uses all available probabilistic knowledge in order to maximize the accuracy of this recognition. The ideal observer model can be seen as a theoretical upper limit on performance. It is not so much a realistic model 66 of human performance, as much as a baseline against which human performance can be 67 compared (Geisler, 2003; Rahnev & Denison, 2018). When there is a deviation from the ideal, it can reveal extra constraints on human cognition, such as limitations on the working memory or attentional resources. The ideal observer analysis has had a tremendous impact not only on speech related research (Clayards, Tanenhaus, Aslin, & Jacobs, 2008; Feldman, 71 Griffiths, & Morgan, 2009; Kleinschmidt & Jaeger, 2015; Norris & McQueen, 2008), but also 72 on many other disciplines in the cognitive sciences (for reviews, see Chater & Manning, 2006; 73 Knill & Pouget, 2004; Tenenbaum, Kemp, Griffiths, & Goodman, 2011)

Some of these ideal-observer-based studies are closely related to the question we are addressing in the current work. For instance, Clayards et al. (2008) simulated auditory uncertainty by manipulating the probability distribution of a cue (Voice Onset Time) that differentiated similar words (e.g., "beach" and "peach"). They found that humans were sensitive to these probabilistic cues and their judgments closely reflected the optimal predictions. In another work, Feldman et al. (2009) studied the perceptual magnet effect, a phenomenon that involves reduced discriminability near prototypical sounds in the native

<sup>&</sup>lt;sup>2</sup>It is, thus, a general instance of the rational approach to cognition (Anderson, 1990). It can also be seen as an instance of Marr's computational level of analysis (Marr, 1982).

language (Kuhl, 1991). They showed that this effect can be explained as the consequence of optimally solving the problem of perception under uncertainty.

Besides the acoustic cues explored in Clayards et al. (2008) and Feldman et al. (2009), 84 there is extensive evidence that information from the visual modality, such as the speaker's facial features, also influences speech understanding (see Campbell, 2008 for a review). Bejjanki, Clayards, Knill, and Aslin (2011) offered a mathematical characterization of how probabilistic cues from speech and lip movements can be optimally combined. They showed that human performance during audio-visual phonemic labeling was consistent (at least at the qualitative level) with the behavior of the ideal observer. This previous research. however, did not systematically study speech understanding when the visual information is 91 obtained, not through the speaker's facial features—as in audio-visual speech perception, but through the referential context. In fact, experimental findings showed that information about the identity of the semantic referent can be integrated with linguistic information to resolve lexical and syntactic ambiguities in speech (e.g., Eberhard, Spivey-Knowlton, Sedivy, & Tanenhaus, 1995; Spivey, Tanenhaus, Eberhard, & Sedivy, 2002; Tanenhaus, Spivey-Knowlton, Eberhard, & Sedivy, 1995). To our knowledge, however, no study offered an ideal observer analysis of word identification in such context, that is, when the listener has to combine cues from the sound and the referent.

On the face of it, the question of combining information from the sound and the visual 100 referent might seem similar to that of audio-visual speech integration. Nevertheless, there 101 are at least two fundamental differences between these two cases, and both can influence the 102 way the auditory and visual cues are combined: First, in the case of audio-visual speech, 103 both modalities offer information about the same underlying speech category. They may 104 differ only in terms of their informational reliability. In a referential context, however, the auditory and visual modalities play different roles in the referential process—in addition to 106 possible differences in informational reliability. Indeed, the auditory input represents the 107 symbol whereas the visual input represents the meaning. It has been shown that speech has a 108

privileged status compared to other sensory stimuli (Edmiston & Lupyan, 2015; Lupyan & Thompson-Schill, 2012; Vouloumanos & Waxman, 2014; Waxman & Gelman, 2009; Waxman & Markow, 1995), and that this privilege is indeed related to the speech's ability to refer (Waxman & Gelman, 2009).<sup>3</sup> Thus, in a referential context, it is possible that listeners do not treat the auditory and visual modalities as equivalent sources of information. Instead, there could be a sub-optimal bias for the auditory modality beyond what is expected from informational reliability alone.

Second, in the case of audio-visual speech, the auditory and visual stimuli are expected 116 to be perceptually correlated. The expectation for this correlation is such that when there is 117 a mismatch between the auditory and visual input, people still integrate them into a unified 118 (but illusory) percept (McGurk & MacDonald, 1976). In the case of referential language, 119 however, the multimodal association is by nature arbitrary (Greenberg, 1957; Saussure, 120 1916). For instance, there is no logical/perceptual connection between the sound "bee" and 121 the corresponding insect. Moreover, variation in the way the sound "bee" is pronounced is 122 generally not expected to correlate perceptually with variation in the shape (or any other 123 visual property) in the category of bees. In sum, cue combination in the case of arbitrary 124 audio-visual associations (word-referent) is likely to be less automatic, more effortful, and 125 therefore less conducive to optimal integration than it is in the case of perceptually 126 correlated associations (as in audio-visual speech perception). 127

### 128 The current study

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We investigate how people combine cues from the auditory and the visual modality to recognize words in a referential context. In particular, we study how this combination is

<sup>&</sup>lt;sup>3</sup>There is, however, a debate as to whether speech is privileged for children and adults for similar reasons. Whereas some researchers suggest that speech is privileged for both children and adults because of its ability to refer (e.g., Waxman & Gelman, 2009), others suggest that speech might *not* have a referential status from the start. Rather, speech might be preferred by children only because of a low level auditory "overshadowing" (e.g., Sloutsky & Napolitano, 2003).

performed under various degrees of uncertainty in both the auditory and the visual modality. Imagine, for example, that someone is uncertain whether they heard "pea" or "bee", does 132 this uncertainty make them rely more on the referent (e.g., the object being pointed at)? 133 Vice versa, if they are not sure if they saw a bee or a fly, does it make them rely more on the 134 sound? More importantly, when input in both modalities is uncertain to varying degrees, do 135 they weight each modality according to its relative reliability, which is the optimal strategy, 136 or do they over-rely on a particular modality, which is a sub-optimal strategy? 137

We perform a rational analysis of the task. First we propose an ideal observer model 138 that performs the combination in an optimal fashion. Second we compare the predictions of 139 the optimal model to human responses. Humans can deviate from the ideal for several 140 reasons. For instance, as mentioned above, a sub-optimality can be induced by the suggested privileged status of speech or by the arbitrariness of the referential association. In order to 142 study possible patterns of sub-optimality, we compare the optimal model (which provides a 143 normative benchmark) to a descriptive model (which is fit to human responses). Comparing parameter estimates between these two formulations allows us to quantify the degree of 145 deviation from optimality. 146

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We tested the ideal observer model's predictions in three behavioral experiments where 147 we varied the source of uncertainty. In Experiment 1, audio-visual tokens were ambiguous 148 with respect to their category membership only. In Experiment 2, we intervened by adding 149 background noise to the auditory modality, and in Experiment 3, we intervened by adding 150 background noise to the visual modality. In all experiments, participants were quantitatively 151 near-optimal, though overall response precision was slightly lower than expected. Moreover, in Experiment 1 where neither of the modalities was perturbed with background noise, 153 participants weighted auditory and visual cues according to the relative reliability predicted 154 by the optimal model. In other words, we found no evidence for a modality bias towards 155 either the auditory or the visual modality. However, in Experiment 2 and 3, participants 156 over-relied on one modality when the other modality was perturbed with additional noise. 157

## Paradigm and Models

In this section we, first, briefly introduce the multimodal combination task. Then we explain how behavior in this paradigm can be characterized optimally with an ideal observer model.

## 162 The Audio-Visual Word Recognition Task

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We introduce a task adapted from Sloutsky and Napolitano (2003). The original task 163 has been used with both children and adults to probe audio-visual encoding (see Robinson & 164 Sloutsky, 2010 for a review). Here we use a slightly different version to test word recognition in a referential context. We use two visual categories (cat and dog) and two auditory categories (/b/ and /d/ embedded in the minimal pair /aba/-/ada/). For each participant, 167 an arbitrary pairing is set between the auditory and the visual categories, leading to two 168 audio-visual word categories (e.g., dog-/aba/, cat-/ada/). In each trial, participants are 169 presented with an audio-visual target (the prototype of the target category), immediately 170 followed by an audio-visual test stimulus (Figure 1). The test stimulus may differ from the 171 target in both the auditory and the visual components. After these two presentations, 172 participants press "same" or "different." 173

In the testing phase of the original task (Sloutsky & Napolitano, 2003), participants 174 are asked whether or not the two audio-visual presentations are *identical*. In the current 175 study, we are interested, rather, in the categorization, i.e., determining whether or not two 176 similar tokens are members of the same phonological/semantic category. Therefore, testing 177 in our task is category-based: Participants are asked to press "same" if they think the second item (the test) belongs to the same category as the first (target) (e.g., dog-/aba/), even if 179 there is a slight difference in the sound, in the referent, or in both. They are instructed to 180 press "different" only if they think that the second stimulus was an instance of the other 181 category (cat-/ada/). The task also includes trials where pictures are hidden (audio-only) or 182 where sounds are muted (visual-only). These unimodal trials provide us with the 183

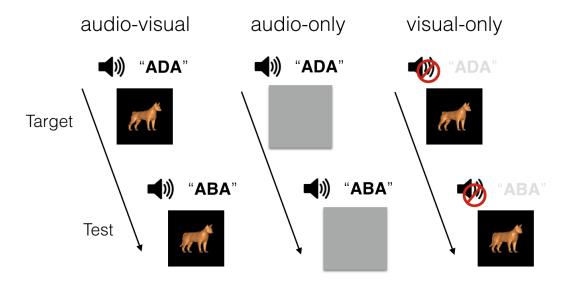


Figure 1. Overview of the task. In the audio-visual condition, participants are first presented with an audio-visual target (the prototype of the target category), immediately followed by an audio-visual test. The test may differ from the target in both the auditory and the visual components. After these two presentations, participants press 'same' (i.e., the same category as the target) or 'different' (not the same category). The auditory-only and visual-only conditions are similar to the audio-visual condition, except that only the sounds are heard, or only the pictures are shown, respectively.

participants' evaluation of the probabilistic information present in the auditory and visual categories. As we shall see, these unimodal distributions are used as inputs to the optimal cue combination model.

## 187 Optimal Model

We construct an ideal observer model that combines probabilistic information from the auditory and visual modalities. In contrast to the model used in most research on multisensory integration (e.g., Ernst & Banks, 2002)—which typically studies continuous stimuli (e.g., size, location)—the probabilistic information in our case cannot be characterized with *sensory noise*, only. Indeed, our task involves responses over categorical variables (phonemes and concepts), and therefore, the optimal model should take into

account, not only the noise variability around an individual perceptual estimate, but also its

categorical variability, i.e., the uncertainty related to whether this perceptual estimate

belongs to a given category (see also Bankieris, Bejjanki, & Aslin, 2017; Bejjanki et al.,

2011). In what follows, we describe a model that accounts for both types of variability. First,

we describe the model in the simplified case of categorical variability only. Second, we

augment this simplified model to account for sensory noise.

Categorical variability. We assume that both the auditory categories (i.e., /aba/ and /ada/) and the visual categories (cat and dog) are distributed along a single acoustic and semantic dimension, respectively (Figure 2). Moreover, we assume that all categories are normally distributed. Formally speaking, if A denotes an auditory category (/ada/ or /aba/), then the probability that a point a along the acoustic dimension belongs to the category A is

$$p(a|A) \sim N(\mu_A, \sigma_A^2)$$

where  $\mu_A$  and  $\sigma_A^2$  are respectively the mean and the variance of the auditory category.

Similarly, the probability that a point v along the visual dimension belongs to the category V is

$$p(v|V) \sim N(\mu_V, \sigma_V^2)$$

where  $\mu_V$  and  $\sigma_V^2$  are the mean and the variance of the visual category. An audio-visual signal w = (a, v) can be represented as a point in the audio-visual space. These audio-visual tokens define bivariate distributions in the bi-dimentional space. We call these bivariate distributions  $Word\ categories$ , noted W, and are distributed as follows:

$$p(w|W) \sim N(M_W, \Sigma_W)$$

where  $M_W = (\mu_A, \mu_V)$  and  $\Sigma_W$  are the mean and the covariance matrix of the word category.

The main assumption of the model is that the auditory and visual variables are independent

(i.e., uncorrelated), so the covariance matrix is simply:

$$\Sigma_W = \left[ \begin{array}{cc} \sigma_A^2 & 0 \\ 0 & \sigma_V^2 \end{array} \right]$$

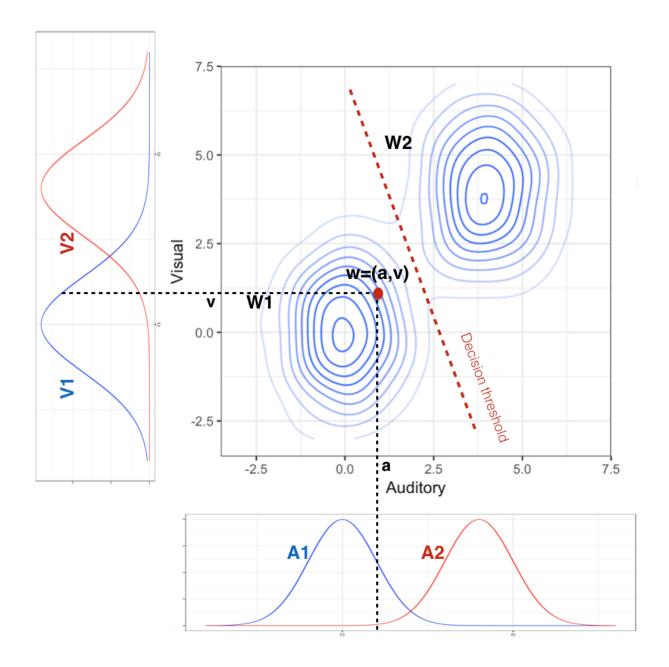


Figure 2. Illustration of the model using simulated data. A word category is defined as the joint bivariate distribution of an auditory category (horizontal, bottom panel) and a visual semantic category (vertical, left panel). Upon the presentation of a word token w, participants guess whether it is sampled from the word type  $W_1$  or from the word type  $W_2$ . Decision threshold is where the guessing probability is 0.5.

This assumption says that, given a word-object mapping, e.g., W = (``cat"-CAT'), variation in the way "cat" is pronounced does not correlate with changes in any visual property of the object CAT, which is a valid assumption in the context of our task.<sup>4</sup>

Now we turn to the crucial question of modeling how the optimal decision should proceed given the probabilistic (categorical) information in the auditory and the visual modalities, as characterized above. We have two word categories:  $dog-/aba/(W_1)$  and  $cat-/ada/(W_2)$ . When making decisions, participants can be understood as choosing one of these two word categories (Figure 2). For an ideal observer, the probability of choosing category 2 when presented with an audio-visual instance w=(a,v) is the posterior probability of this category:

$$p(W_2|w) = \frac{p(w|W_2)p(W_2)}{p(w|W_2)p(W_2) + p(w|W_1)p(W_1)}$$

Using our assumption that the cues are uncorrelated, we have:

$$p(w|W) = p(a, v|W) = p(a|A)p(v|V)$$

Under this assumption, the posterior probability reduces to the following formula (see Appendix 1 for the details of the derivation):

$$p(W_2|w) = \frac{1}{1 + (1+b)\exp(\beta_0 + \beta_a a + \beta_v v)}$$
(1)

where

$$1 + b = \frac{p(W_1)}{p(W_2)}$$
$$\beta_0 = \frac{\mu_{A2}^2 - \mu_{A1}^2}{2\sigma_A^2} + \frac{\mu_{V2}^2 - \mu_{V1}^2}{2\sigma_V^2}$$

$$\beta_a = \frac{\mu_{A1} - \mu_{A2}}{\sigma_A^2}$$

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<sup>&</sup>lt;sup>4</sup>Note that this assumptions is more adequate in the case of arbitrary associations such as ours, and less so in the case of redundant association such as audio-visual speech. In the latter, variation in the pronunciation is expected to correlate, at least to some extent, with lip movements.

<sup>&</sup>lt;sup>5</sup>This mapping is randomized in the experiments.

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$$\beta_v = \frac{\mu_{V1} - \mu_{V2}}{\sigma_V^2}.$$

The parameter b represents the differential between the categories' prior probabilities. 231 However, since the identity of word categories is randomized across participants, b measures, 232 rather, a response bias to "same" if b > 0, and a response bias to "different" if b < 0. We 233 expect a general bias towards answering "different" because of the categorical nature of our 234 same-different task: When two items are ambiguous but perceptually different, participants 235 might have a slight preference for "different" over "same". As for the means, their values are fixed, and they correspond to the most typical tokens in our stimuli. Finally, observations 237 from each modality (a and v) are weighted in Equation 1 according to their reliability (that 238 is, according to the *inverse* of their variance): 230

$$\beta_a \propto \frac{1}{\sigma_A^2}$$

$$\beta_v \propto \frac{1}{\sigma_V^2}.$$

**Sensory variability.** So far, we only accounted for categorical variability. For 241 instance, if the speaker generates a target production  $a_t$  from an auditory category 242  $p(a_t|A) \sim N(\mu_A, \sigma_A^2)$ , the ideal model assumes that it has direct access to this production 243 token (i.e.,  $a = a_t$ ), and that all uncertainty is about the category membership of this token. 244 However, we might also want to account for internal noise in the brain and/or external noise 245 in the environment. For example, the observer might not have access to the exact produced 246 target, but only to the target perturbed by noise. If we assume this noise to be normally 247 distributed, that is,  $p(a|a_t) \sim N(a_t, \sigma_{N_A}^2)$ , then integrating over  $a_t$  leads to this new 248 expression of the probability distribution: 240

$$p(a|A) \sim N(\mu_A, \sigma_A^2 + \sigma_{N_A}^2)$$

Similarly, in the case of sensory noise in the visual modality, we get:

$$p(a|V) \sim N(\mu_V, \sigma_V^2 + \sigma_{N_V}^2)$$

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Finally, using exactly the same derivation as above, we end up with the following multimodal weighting scheme in the optimal combination model (Equation 1) which takes into account 252 both categorical and sensory variability: 253

$$\beta_a \propto \frac{1}{\sigma_A^2 + \sigma_{N_A}^2}$$

$$\beta_v \propto \frac{1}{\sigma_V^2 + \sigma_{N_V}^2}.$$

**Optimal cue combination.** Equation 1 provides the optimal model's predictions 255 for how probabilities that characterize uncertainty in the auditory and the visual modalities 256 can be combined to make categorical decisions. Parameter estimates of the probability 257 distributions in each modality are derived by fitting unimodal posteriors to the participants' 258 responses in the unimodal conditions, i.e., the condition where only the sounds are heard or 259 only the pictures are seen (Figure 1).<sup>6</sup> Using these derived parameters, the optimal model 260 makes predictions about responses in the bimodal (i.e., audio-visual) condition where participants both hear the sounds and see the pictures. 262

Auditory and Visual baselines. The predictions of the optimal model will be 263 compared to two baselines. The first baseline is a visual model which assumes that 264 participants rely only on visual information, and an auditory model, which assumes that 265 participants rely only on auditory information. More precisely, these baseline models assume 266 that the participants' responses in the bimodal condition will not be different from their 267 response in either the visual-only or the auditory-only condition. However, if the participants 268 rely on both the auditory and the visual modalities to make decision in the bimodal 269 condition, the optimal model would explain more variance in human responses than the 270 visual or the auditory model do.

<sup>&</sup>lt;sup>6</sup>Further technical detail about model fitting in the unimodal conditions will be given in the method section of Experiment 1

## Descriptive model and analysis of sub-optimality

The optimal model (as well as the auditory and visual baselines) are *normative* models. 273 Their predictions are made about human data in the bimodal condition, but their crucial 274 parameters (i.e., variances associated with the visual and auditory modalities) are derived 275 from data in the unimodal conditions. In addition to these normative models, we consider a descriptive model. It is formally identical to the normative optimal model (Equation 1), 277 except that the parameters are fit to actual responses in the bimodal condition. If the 278 referential task induces sub-optimality (due, for instance, to the arbitrary nature of the 279 sound-object association), then the descriptive model should explain more variance than the 280 optimal model does. 281

Comparison of the optimal and the descriptive models allows us, not only to quantify 282 how much people deviate from optimality, but also to understand precisely the nature of this 283 deviation. Let  $\sigma_A^2$  and  $\sigma_V^2$  be the values of the variances used in the optimal model (derived 284 from the unimodal conditions), and  $\sigma_{Ab}^2$  and  $\sigma_{Vb}^2$  be the values observed through the descriptive model in the bimodal condition. Deviation from optimality is measured in two ways. First, we measure the change in the values of the variance specific to each modality, that is, how  $\sigma_A^2$  compares to  $\sigma_{Ab}^2$ , and how  $\sigma_V^2$  compares to  $\sigma_{Vb}^2$ . Second, we measure changes 288 in the proportion of the visual and auditory variances, i.e., we examine how  $\frac{\sigma_A^2}{\sigma_V^2}$  compares to 280  $\frac{\sigma_{Ab}^2}{\sigma_{Vb}^2}$ . The first measure allows us to test if response precision changes for each modality when 290 we move from the unimodal to the bimodal conditions. The second allows us to test the 291 extent to which the weighting scheme follows the prediction of the optimal model. The 292 reason we used the proportion of the variances as a measure of cross-modal weighting is 293 because this proportion corresponds to the slope<sup>7</sup> of the decision threshold in the 294 audio-visual space (Figure 2). The decision threshold is defined as the set of values in this 295 audio-visual space along which the posterior is equal to 0.5. Formally speaking, the decision 296 threshold has the following form: 297

<sup>&</sup>lt;sup>7</sup>Or more precisely the absolute value of the slope.

$$v = -\frac{\sigma_V^2}{\sigma_A^2} a + v_0$$

If the absolute value of the slope derived from the descriptive model is greater than
that of the optimal model, the corresponding shift in the decision threshold indicates that
participants have a preference for the auditory modality in the bimodal case. Similarly, a
smaller absolute value of the slope would lead to a preference for the visual modality. The
limit cases are when there is exclusive reliance on the auditory cue (a vertical line), and
where there is exclusive reliance on the visual (a horizontal line).

There are three possible ways human responses can deviate from optimality. These scenarios are illustrated in Figure 3, and are as follows:

1) Both variances may increase, but their proportion remains the same. That is,  $\sigma_{Ab}^2 \geqslant \sigma_A^2$  and  $\sigma_{Vb}^2 \geqslant \sigma_V^2$ , but  $\frac{\sigma_{Ab}^2}{\sigma_{Vb}^2} \approx \frac{\sigma_A^2}{\sigma_V^2}$ . In this case, sub-optimality would be due to increased randomness in human responses in the bimodal condition. However, this randomness would not affect the relative weighting of both modalities, i.e., participants would still weigh modalities according to the relative reliability predicted by the optimal model.

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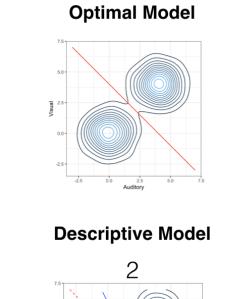
- 2) The auditory variance increases at a higher rate. That is,  $\sigma_{Ab}^2 \gg \sigma_A^2$  and  $\sigma_{Vb}^2 \geqslant \sigma_V^2$ , leading to  $\frac{\sigma_{Ab}^2}{\sigma_{Vb}^2} > \frac{\sigma_A^2}{\sigma_V^2}$ . In this case, sub-optimally would consist not only in participants being more random in the bimodal condition, but also in having a systematic preference for the visual modality, even after accounting for informational reliability.
- 315 3) The visual variance increases at a higher rate. That is,  $\sigma_{Vb}^2 \gg \sigma_V^2$ , and  $\sigma_{Ab}^2 \geqslant \sigma_A^2$ ,

  leading to  $\frac{\sigma_{Ab}^2}{\sigma_{Vb}^2} > \frac{\sigma_A^2}{\sigma_V^2}$ . This case is the reverse of case 2, i.e., in addition to increased

  randomness in the bimodal condition, there is a systematic preference for the auditory

  modality, even after accounting for informational reliability.

We compared these models to human responses in three experiments. In Experiment 1, we studied the case where bimodal uncertainty was due to categorical variability, only. In



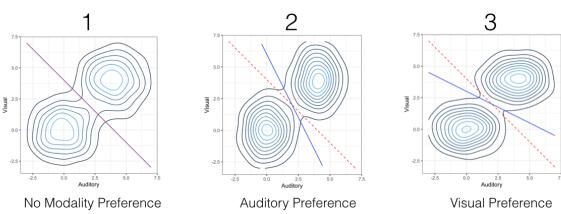


Figure 3. Illustration using simulated data showing the example of a prediction made by the optimal model (top), and the three possible ways human participants can deviate from this prediction (bottom). These cases are the following: 1) The variance increases equally for both modalities, but the weighting scheme (characterized by the decision threshold) is optimal, 2) The auditory variance increases at a higher rate, leading to a preference for the auditory modality, and 3) The visual variance increases at a higher rate, leading to a preference for the visual modality.

Experiment 2 and 3 we added auditory and visual noise, respectively, on top of categorical variability.

## Experiment 1

In this Experiment, we test the predictions of the model in the case where uncertainty 324 is due to categorical variability only (i.e., ambiguity in terms of category membership). We 325 do not add any external noise to the background and we assume that internal sensory noise 326 is negligible compared to categorical variability ( $\sigma_A^2 \gg \sigma_{N_A}^2$  and  $\sigma_V^2 \gg \sigma_{N_V}^2$ ). Thus, we use 327 the following cue weighting scheme: 328

$$\beta_a \propto \frac{1}{\sigma_A^2 + \sigma_{N_A}^2} \approx \frac{1}{\sigma_A^2}$$

$$\beta_v \propto \frac{1}{\sigma_V^2 + \sigma_{N_V}^2} \approx \frac{1}{\sigma_V^2}.$$

#### Methods

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We recruited a planned sample of 100 participants from Amazon Mechanical Turk. Only participants with US IP addresses and a task approval rate above 332 85% were allowed to participate. They were paid at an hourly rate of \$6/hour. Participants 333 were excluded if they reported having experienced a technical problem of any sort during the 334 online experiment (N=14), or if they had less than 50% accurate responses on the 335 unambiguous training trials (N=6). The final sample consisted of N=80 participants.<sup>8</sup> 336 For auditory stimuli, we used the continuum introduced in Vroomen, 337 Linden, Keetels, Gelder, and Bertelson (2004), a 9-point /aba/-/ada/ speech continuum 338 created by varying the frequency of the second (F2) formant in equal steps. We selected 5 339 equally spaced points from the original continuum by keeping the endpoints (prototypes) 1 340 and 9, as well as points 3, 5, and 7 along the continuum. For visual stimuli, we used a 341 cat/dog morph continuum introduced in Freedman, Riesenhuber, Poggio, and Miller (2001). 342 From the original 14 points, we selected 5 points as follows: we kept the item that seemed 343 most ambiguous (point 8), the 2 preceding points (i.e., 7 and 6) and the 2 following points

 $<sup>^8</sup>$ The sample size and exclusion criteria were specified in the pre-registration at https://osf.io/h7mzp/.

(i.e., 9 and 10). The 6 and 10 points along the morph were quite distinguishable, and we took them to be our prototypes.

**Design and Procedure.** We told participants that an alien was naming two 347 objects: a dog, called "aba" in the alien language, and a cat, called "ada". In each trial, we 348 presented the first object (the target) on the left side of the screen simultaneously with the corresponding sound. For each participant, the target was always the same (e.g., dog-/aba/). The second sound-object pair (the test) followed on the other side of the screen after 500ms 351 and varied in its category membership. For both the target and the test, visual stimuli were 352 present for the duration of the sound clip ( $\sim 800 \text{ms}$ ). We instructed participants to press "S" 353 for same if they thought the alien was naming another dog-/aba/, and "D" for different if 354 they thought the alien was naming a cat-/ada/. We randomized the sound-object mapping 355 (e.g., dog-/aba/, cat-/ada/) as well as the identity of the target (dog or cat) across 356 participants. 357

The first part of the experiment trained participants using only the prototype pictures 358 and the prototype sounds (12 trials, 4 each from the bimodal, audio-only, and visual-only 359 conditions). After completing training, we instructed participants on the structure of the 360 task and encouraged them to base their answers on both the sounds and the pictures (in the 361 bimodal condition). There were a total of 25 possible combinations in the bimodal condition, 362 and 5 in each of the unimodal conditions. Each participant saw each possible trial twice, for 363 a total of 70 trials/participant. Trials were blocked by condition and blocks were presented 364 in random order. The experiment lasted around 15 minutes.<sup>9</sup> 365

# Model fitting details.

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Unimodal conditions. Remember that data in these conditions allows us to derive the variances of both the auditory and the visual categories, and that these variances are used to make predictions about bimodal data (in the visual and auditory baselines as well as

<sup>&</sup>lt;sup>9</sup>The experiment can be accessed and played from the github repository: https://github.com/afourtassi/ WordRec/

in the optimal model). These individual variances were derived as follows (we explain the
derivation for the auditory-only case, but the same applies for the visual-only case). We use
the same Bayesian reasoning as we did in the derivation of the bimodal model: When
presented with an audio instance a, the probability of choosing the sound category 2 (that is,
to answer "different") is the posterior probability of this category  $p(A_2|a)$ . If we assume that
both sound categories have equal variances, the posterior probability reduces to:

$$p(A_2|a) = \frac{1}{1 + (1 + b_A) \exp(\beta_{a0} + \beta_a a)}$$

with  $\beta_a = \frac{\mu_{A_1} - \mu_{A_2}}{\sigma_A^2}$  and  $\beta_{a0} = \frac{\mu_{A_2}^2 - \mu_{A_1}^2}{2\sigma_A^2}$ .  $b_A$  is the response bias in the auditory-only condition. For this model (as well as all other models in this study), we fixed the values of the means to be the end-points of the corresponding continuum, since these points are the most typical instances in our stimuli. Thus, we have  $\mu_{A1} = 0$  and  $\mu_{A2} = 4$  (and similarly  $\mu_{V1} = 0$ , and  $\mu_{V2} = 4$ ). This leaves us with two free parameters: the bias  $b_A$  and the variance  $\sigma_A^2$ . To determine the values of these parameters, we fit the unimodal posterior to human data in the unimodal case.

Bimodal condition. In this condition, only the descriptive model is fit to the data,
using the expression of the posterior (Equation 1). Since the values of the means are fixed,
we have 3 free parameters: the variances for the visual and the auditory modalities,
respectively, and b, the response bias. The visual and auditory baselines as well as the
optimal model are not fit to the bimodal data, but their predictions are tested against these
bimodal data. All these normative models use the variances derived from the unimodal data
and the bias term derived from the fit to bimodal data.

Although the paradigm is within-subjects, we did not have enough statistical power to fit a different model for each individual participant.<sup>10</sup> Instead, models were constructed with

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<sup>&</sup>lt;sup>10</sup>We had a relatively high number of trials spanning all possible audio-visual matchings. Getting enough data points per trial per participant would have required running the online experimental for a much longer time (more than an hour). This could have increased significantly the dropout rate and possibly affected the

data collapsed across all participants. That being said, we will also analyze the distribution of individual responses. The fit was done with a nonlinear least squares regression using the NLS package in R (Bates & Watts, 1988). We computed the values of the parameters, as well as their 95% confidence intervals, through non-parametric bootstrap (using 10000 iterations).

# 396 Results and analysis

Unimodal conditions. Average categorization judgments and best fits are shown in Figure 4. The categorization function of the auditory condition was slightly steeper than that of the visual condition, meaning that participants perceived the sound tokens slightly more categorically and while higher certainty than they did with the visual tokens. For the auditory modality, we obtained the following values:  $b_A = -0.20 \ [0.02, -0.38]$  and  $\sigma_A^2 = 2.04 \ [1.66, 2.53]$ . For the visual modality, we obtained  $b_V = -0.12 \ [0.06, -0.28]$  and  $\sigma_V^2 = 3.33 \ [2.83, 3.92]$ .

#### Bimodal condition.

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Figure 5 compares the predictions of the normative models Normative models. 405 against human responses. Remember that the normative models use variance estimates from 406 the unimodal conditions (where people see input from only one modality) to predict data in 407 the bimodal condition (where people see input from both modalities). Each point represents 408 data form a particular audio-visual matching (e.g., the visual token v whose distance from 409 the target  $v_t$  is  $|v - v_t| = 3$ , matched with the auditory token a whose distance from the 410 target  $a_t$  is  $|a - a_t| = 2$ ). The visual, auditory and optimal model explained, respectively, 411 30%, 67%, and 89% of total variance in mean responses. 412

Descriptive model. In the descriptive model, all parameters are fit to human responses in the bimodal condition. We found b = -0.34 [-0.28, -0.39],  $\sigma_{Ab}^2 = 4.96$  [4.58, 5.40] and  $\sigma_{Vb}^2 = 7.06$  [6.40, 7.84]. Note that the variance of both the auditory and visual modalities increased compared to the unimodal conditions.

quality of the data collected online.

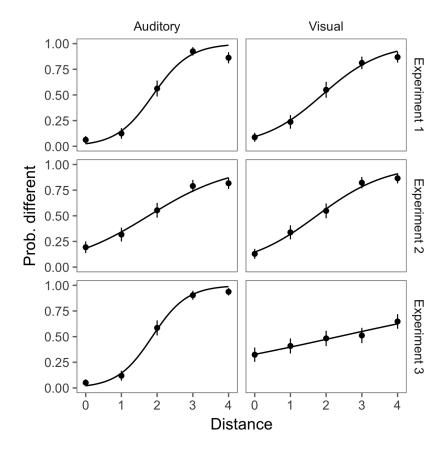


Figure 4. Human responses in the unimodal conditions across the three experiments. Points represent the proportion of 'different' to 'same' responses in the auditory-only condition (left), and visual-only condition (right). Error bars are 95% confidence intervals. Solid lines represent best unimodal posterior fits.

The descriptive model explained 95% of total variance. However, since the descriptive model was fit to the same data, there is a risk that this high correlation is due to overfitting. To examine this possibility, we cross-validated the model using half the responses to predict the other half (averaging across 10 random partitions). The predictive power of the model remained very high  $(r^2=0.93)$ .

Cue combination and Modality preference. We next analyzed if cue
combination was performed in an optimal way, or if there was a systematic preference for
one modality when making decisions in the bimodal condition. As explained above, modality

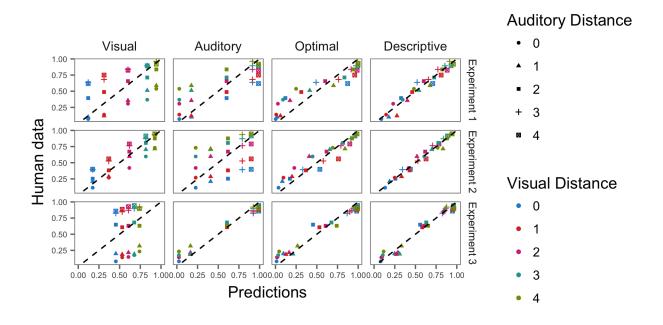


Figure 5. Human responses vs. Models' predictions in the bimodal condition across the three experiments. Each point represents data form a particular audio-visual matching. Shape represents auditory distance from the target, and color represents visual distance from the target. Thus, each point is characterized by both shape and color.

preference can be characterized formally as a deviation from the decision threshold predicted by the optimal model. Figure 7 (top) shows both the decision threshold derived from the descriptive model (in black) and the decision threshold predicted by the optimal model (in red). The deviation from optimality is compared to two hypothetical cases of modality preference (dotted lines). We found that the descriptive and optimal decision thresholds were almost identical. Indeed, non-parametric resampling of the data showed no evidence of a deviation from the optimal prediction (Figure 7, bottom).

### Discussion

Overall, we found that the optimal model explained much of the variance in the mean 433 judgments, and largely more than what can be explained with the auditory or the visual 434 models alone. Moreover, the high value of the coefficient of determination in the optimal model ( $r^2$ =0.89) suggests that the population was near-optimal. However, we see in Figure 5 that the mean responses deviated systematically from the optimal prediction in that they 437 were slightly pulled toward chance (i.e., the probability 0.5). This fact is due to the increase 438 in the value of the variance associated with each modality. Note however that, despite this 439 increase in randomness, our analysis of modality preference showed that the relative values 440 of these variances were not different (Figure 7), meaning that there was no evidence for a 441 modality preference. Thus, 1) There was a simultaneous increase in the values of the auditory 442 and visual variances in the bimodal condition compared to the unimodal condition, meaning 443 that the bimodal input lead to an increase in response randomness, and 2) this increased 444 randomness did not affect the relative weighting of both modalities, i.e., the participants was 445 weighting modalities according to the relative reliability predicted by the optimal model. This situation corresponds to the first case of sub-optimally described in Figure 3. 447 As we noted earlier, the model addresses the question of optimality at the population 448 level. However, it is important to know how individual responses are distributed. In fact, one 449 could think of an extreme case where optimality at the population level would be misleading. 450 Imagine, for instance, that in the bimodal condition half the participants relied exclusively 451 on the visual modality, whereas the other half relied exclusively on the auditory modality. 452 This case could still lead to an aggregate behavior which appears optimal, but this 453 optimality would be spurious. 454 To examine this possibility, we consider the distribution of individual cross-modal 455 weighting in the bimodal condition (i.e.,  $\frac{\sigma_{Vb}^2}{\sigma_{Ab}^2}$ ). Using a factor of 10 as a cut-off, we found 456 that 5 participants relied almost exclusively on the visual modality, and 12 relied almost 457 exclusively on the auditory modality. The percentage of both cases was relatively small 458

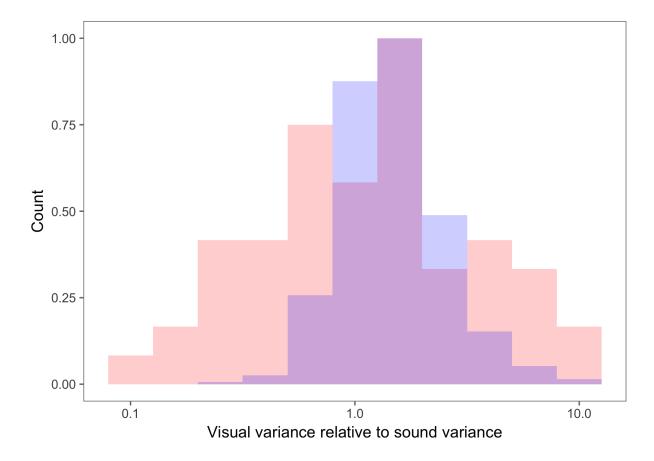


Figure 6. Distributions of individual values of the visual variance relative to the auditory variance in Experiment 1. Light color represents the real individual distribution, and dark color represents the simulated individual distribution sampled from the descriptive model.

compared to the total number of participants (21.25%). When these outliers were removed,
the distribution had a rather unimodal shape (Figure 6). This finding indicates that the
population's near optimality is not spurious, but based mostly on genuine cue combination
at the individual level.

As a second analysis, we asked whether the observed variance in the individual distribution was due to mere sampling errors or whether it corresponded to a real between-subject variability. We simulated individual responses from the posterior distribution whose parameters were fit to the population as a whole (i.e., the descriptive posterior). The resulting distribution is shown in Figure 6. For ease of comparison, the

simulated distribution was superimposed to the real distribution. We found that the real distribution had a standard deviation of sd=2.24 which was larger than that of the simulated distribution (sd=1.10), indicating that there was real between-subject variation beyond sampling errors. This finding means that the participants varied in terms of how they weighted modalities: Compared to the predictions of the population-level model, some participants relied more on the auditory modality, whereas others relied more on the visual modality.

In Experiment 1, we tested word recognition when there was multimodal uncertainty in terms of category membership only. In real life, however, tokens can undergo distortions due to noisy factors in the environment (e.g., car noise in the background, blurry vision in a foggy weather). In Experiment 2 and 3, we explore this additional level of uncertainty.

## Experiment 2

In this Experiment, we explored the effect of added noise on performance. We tested a case where the background noise was added to the auditory modality. We were interested to know if participants would treat this new source of uncertainty as predicted by the optimal model, that is, according to the following weighting scheme

$$eta_a \propto rac{1}{\sigma_A^2 + \sigma_{N_A}^2}$$
  $eta_v \propto rac{1}{\sigma_V^2}.$ 

The alternative hypothesis is that noise in one modality leads to a systematic preference for the non-noisy modality.

#### 487 Methods

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Participants. A sample of 100 participants was recruited online through Amazon Mechanical Turk. We used the same exclusion criteria as in Experiment 1. 7 participants were excluded because they had less than 50% accurate responses on the unambiguous training trials. The final sample consisted of N=93 participants.

Stimuli and Procedure. We used the same visual stimuli as in Experiment 1. We also used the same auditory stimuli, but we convolved each item with Brown noise of amplitude 1 using the free sound editor Audacity (2.1.2). The average signal-to-noise ratio was - 4.4 dB. The procedure was exactly the same as in the previous experiment, except that the test stimuli (but not the target) were presented with the new noisy auditory stimuli.

## <sup>497</sup> Results and analysis

Unimodal conditions. We fit a model for each modality. For the auditory modality, our parameter estimates were  $b_A = -0.18$  [-0.05, -0.30] and  $\sigma_A^2 + \sigma_N^2 = 4.70$  [4.03, 5.55]. For the visual modality, we found  $b_V = -0.24$  [-0.10, -0.36] and  $\sigma_V^2 = 3.93$  [3.43, 4.55]. Figure 4 shows responses in the unimodal conditions as well as the corresponding best fits. The visual data is a replication of the visual data in Experiment 1. As for the auditory data, in contrast to Experiment 1, responses were flatter, showing more uncertainty.

### Bimodal condition.

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Normative models. Figure 5 compares the predictions of the visual, auditory and optimal models to human responses. These normative models explained, respectively, 77%, 21%, and 91% of total variance in mean judgements. Note that, in contrast to Experiment 1, the visual model explained more variance than the auditory model did.

Descriptive model. We estimated b = -0.38 [-0.33, -0.42],  $\sigma_{Ab}^2 + \sigma_{Nb}^2 = 9.84$  [8.75, 11.27], and  $\sigma_{Vb}^2 = 5.21$  [4.84, 5.64]. The fit explained 0.97% of total variance.

Cross-validation using half the responses to predict the other half yielded  $r^2 = 0.96$ .

Modality preferences. Figure 7 (top) shows that the participants' decision
threshold deviated from optimality, and that this deviation was biased towards the visual
modality (the non-noisy modality). Indeed non-parametric resampling of the data showed a
decrease in the value of the slope in the descriptive model compared to the optimal model
(Figure 7, bottom).

### Discussion

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We found, similar to Experiment 1, that the population was generally near optimal  $(r^2 = 0.91)$ , and that the optimal model explained more variance than the auditory or the visual models alone. We also found a similar discrepancy from the optimal model as precision dropped for both the auditory and the visual modalities. As for the weighting scheme used by participants, contrary to Experiment 1 where modalities were weighted according to their relative reliability, we found in this experiment that the visual modality had a greater weight than what was expected from its relative reliability. This situation corresponds to the second case of sub-optimally described in Figure 3.

We were also interested in whether noise in the auditory modality lead more 526 participants to rely exclusively on the visual modality at the individual level. Using the same 527 cut-off as in Experiment 1 (a factor of 10), the percentage of participants who relied 528 exclusively on either modality was 34.41%, which is much higher than the percentage 529 obtained in Experiment 1 (21.25%). Moreover, the subset of participants relying exclusively 530 on the visual modality (compared to those who relied exclusively on the auditory modality) 531 increased from 29.41% in Experiment 1 to 68.75% in Experiment 2, indicating that noise in 532 the auditory modality prompted more participants to rely exclusively and disproportionately 533 on the visual modality (see Table 1). 534

In Experiment 2, we tested the case of added background noise to the auditory modality. In Experiment 3, we test the case of added noise to the visual modality.

### Experiment 3

In this Experiment, we added background noise to the visual modality. Similar to
Experiment 2, we were interested to know if participants would treat this new source of
uncertainty as predicted by the optimal model, that is, according to the following weighting
scheme:

$$\beta_a \propto \frac{1}{\sigma_A^2}$$

$$\beta_v \propto \frac{1}{\sigma_V^2 + \sigma_{N_V}^2}.$$

The alternative hypothesis is that noise in the visual modality would lead to a preference for the auditory input, just like noise in the auditory modality lead to a preference for the visual input in Experiment 2.

## 545 Methods

Participants. A planned sample of 100 participants was recruited online through Amazon Mechanical Turk. We used the same exclusion criteria as in both previous experiments. N=2 participants were excluded because they reported having a technical problem, and N=10 participants were excluded because they had less than 50% accurate responses on the unambiguous training trials. The final sample consisted of N=88 participants.

Stimuli and Procedure. We used the same auditory stimuli as in Experiment 1.
We also used the same visual stimuli, but we blurred the tokens using the free image editor
GIMP (2.8.20). We used a Gaussian blur with a radius<sup>11</sup> of 10 pixels. The experimental
procedure was exactly the same as in the previous Experiments.

## Results and analysis

Unimodal conditions. For the auditory modality, our parameter estimates were  $b_A = -0.24$  [-0.04, -0.42] and  $\sigma_A^2 = 1.94$  [1.61, 2.33]. For the visual modality, we found  $b_V =$  0.11 [0.27, -0.03] and  $\sigma_V^2 + \sigma_N^2 = 13.00$  [9.92, 18.94]. Figure 4 shows responses in the unimodal conditions as well as the corresponding fits. The auditory data is a replication of the auditory data in Experiment 1. As for the visual data, we found that, in contrast to Experiment 1 and 2, responses were flatter, showing much more uncertainty.

## Bimodal condition.

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<sup>&</sup>lt;sup>11</sup>A features that modulates the intensity of the blur.

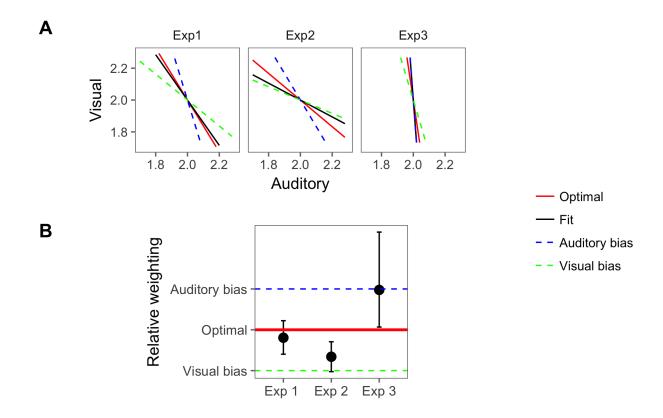


Figure 7. Modality preference is characterized as a deviation from the optimal decision threshold. A) The decision thresholds of both the optimal and the descriptive models (solid red and black lines, respectively). Deviation from optimality is compared to two hypothetical cases of modality preference. In these cases, deviation from optimality is due to over-lying on the visual or the auditory input by a factor of 2 (green and blue dotted lines, respectively). B) An alternative way to represent the same data. Each point represents the value of the decision threshold's slope derived from the descriptive model relative to that of the optimal model (log-scaled). The lines represent the optimal case as well as the two hypothetical cases of modality preference. Error bars represent 95% confidence intervals over the distribution obtained through non-parametric resampling.

**Normative models.** Figure 5 compares the predictions of the visual, auditory and optimal models to human responses. These normative models explained, respectively, 1%,

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98%, and 97% of total variance in the mean judgements. 566

**Descriptive model.** We estimated b = -0.35 [-0.29, -0.40],  $\sigma_{Ab}^2 = 3.00$  [2.75, 3.25], 567 and  $\sigma_{Vb}^2+\sigma_{Nb}^2=$  39.42 [25.06, 98.96]. The fit explained 97% of total variance. 568 Cross-validation using half the responses to predict the other half yielded  $r^2 = 0.96$ . 569 **Modality preferences.** Participants' decision threshold suggested a preference for 570 the auditory modality (the non-noisy modality). Indeed non-parametric resampling of the 571 data showed an increase in the value of the slope in the descriptive model compared to the 572 optimal model (Figure 7). 573

#### Discussion

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We found that the optimal model accounted for almost all the variance  $(r^2 = 0.97)$ . 575 However, whereas in previous experiments the optimal model explained more variance than 576 the auditory or the visual models, here the auditory model explained at least as much 577 variance  $(r^2 = 0.98)$ . Thus, though participants were still sensitive to variation in the noisy 578 visual data in the unimodal condition, they tended to ignore this information in the bimodal 579 condition, and relied almost exclusively on the non-noisy auditory modality. The reason why 580 we saw this (floor) effect when we added noise to the visual modality (Experiment 3), and 581 not when we added noise to the auditory modality (Experiment 2), is the fact that our visual 582 stimuli were originally perceived less categorically and with less certainty than the auditory 583 stimuli. This fact made it more likely for the visual categorization function to become flat 584 and uninformative after a few drops in precision due to noise on the one had, and to the 585 additional randomness induced by the bimodal presentation on the other hand. 586 The general finding corresponds to the third case of sub-optimality described in 587 Figure 3. Indeed, precision dropped for both modalities in the bimodal condition compared 588 to the unimodal condition. But the drop was much greater for the visual modality, resulting 589 in a much lower weight assigned to it than what is expected from its reliability. Therefore, 590 just like participants over-relied on the visual modality when the auditory modality was

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noisy (Experiment 2), they also over-relied on the auditory modality when the visual modality was noisy (Experiment 3).

The percentage of participants who relied exclusively on either the visual modality or
the auditory modality was 38.64%, which is closer to the percentage of Experiment 2, except
that now almost all of them relied on the auditory modality (94.12%). For ease of
comparison, Table 1 provides a summary of the numbers across the three experiments.

#### General Discussion

When identifying a spoken word under uncertainty, one often needs to make the most 599 of the available cues. Some previous work studied optimal behavior under uncertainty from 600 the auditory input only (e.g., Clayards et al., 2008; Feldman et al., 2009), and others studied 601 optimality under multimodal uncertainty in auditory speech and visual facial features (e.g. 602 Bejjanki et al., 2011). The current work explored the case of word identification under 603 uncertainty in speech (word form) and the visual referent. More specifically, we conducted an ideal observer analysis of the task whereby a model provided predictions about how information from each modality should be combined in an optimal fashion. The predictions of the model were tested in a series of three experiments where instances of both the form 607 and the meaning were ambiguous with respect to their category membership only 608 (Experiment 1), when instances of the form were perturbed with additional background noise 609 (Experiment 2), and when instances of the referent were perturbed with additional visual 610 noise (Experiment 3). 611

In all Experiments, we found many patterns of optimal behavior. Quantitatively speaking, the optimal model accounted, respectively, for 89%, 91%, and 97% of the variance in mean responses. When compared to the predictions of the visual or the auditory models, participants generally relied on both modalities to make their decisions in the bimodal condition. Indeed, in Experiment 1 and 2, the optimal model accounted for more variance in mean responses than the auditory or the visual models did. In Experiment 3, participants

Table 1

The percentage of participants who relied exclusively on either the visual modality or the auditory modality, using a factor of 10 as a cut-off (e.g., we consider that a participant relied exclusively on the visual modality when their auditory variance is a at least 10 times larger than their visual variance). We show the percentage compared to the total number of participants in each Experiment ('of Total'). From this subset of participants, we show the percentage of those who relied on the auditory modality ('Auditory'), and the percentage of those who relied on the visual modality (Visual').

Experiment	ofTotal	Auditory	Visual
Exp1	21.25	70.59	29.41
Exp2	34.41	31.25	68.75
Exp3	38.64	94.12	5.88

appeared to rely on one modality, but this was likely a floor effect, due to the fact that noise
made the visual input barely perceivable. In Experiment 1, which did not involve
background noise, participants not only relied on both modalities, but generally weighted
these modalities according to the prediction of the optimal model, that is, according to their
relative reliability. At the individual level, however, we found evidence of a between-subject
variation: Some participants relied more on the visual modality, whereas others relied more
on the auditory modality.

We documented two major cases of sub-optimality. First, in all Experiments, the 625 variance associated with each modality increased in the bimodal condition compared to the 626 unimodal conditions. This fact means that participants responded slightly more randomly in 627 the bimodal condition than they did in the unimodal conditions. This finding contrasts with 628 research on multisensory integration where associations tend to lead to a higher precision 629 (e.g., Ernst & Banks, 2002). Nevertheless, there is a crucial difference between these two 630 situations (besides the obvious difference in terms of the models used). Research on 631 multisensory integration (of which audio-visual speech is arguably an instance) deals with 632 redundant multimodal cues, and these cues are integrated into a unified percept. In contrast, 633 the word-referent association is usually arbitrary and, in particular, the cues are not expected to be correlated perceptually. Therefore the observer cannot form a unified percept, rather, it must encode information separately from both modalities and retain this encoding 636 through the decision making process. Retaining two separate cues at the same time instead 637 of forming one unified percept (as in multisensory integration of redundant cues), or instead 638 of retaining only one cue (as in the unimodal case), is likely to place extra-demand on 639 cognitive resources, which, in turn, can cause general performance to drop. Indeed, there is 640 evidence that cognitive load due to divided attention (e.g., when performing two tasks at the 641 same time) has a detrimental effect on word recognition (Mattys & Wiget, 2011). 642

Some previous research found a similar case of suboptimal behavior. For instance, studies that explored the identification of ambiguous, newly learned pairs of word-referent

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associations all reported what appears to be a decrease in speech perception acuity in both 645 children (Stager & Werker, 1997) and adults (Pajak, Creel, & Levy, 2016). Recently, Hofer 646 and Levy (2017) provided a probabilistic model of this phenomenon. In agreement with the 647 method and finding in the current study, Hofer and Levy (2017) characterized the apparent 648 reduction in perceptual acuity as an increase in the noise variance of the auditory modality. 649 Our finding, besides providing more evidence to this documented fact, suggests that the 650 reduction in perceptual acuity may occur simultaneously in both the auditory and the visual 651 modalities. 652

The second case of sub-optimality is related to how participants weighted the cues 653 from the visual and the auditory modalities in a noisy context. In contrast to Experiment 1 654 where the combination was indistinguishable from the optimal prediction, results of 655 Experiment 2 and 3 which both involved background noise in one modality, showed that 656 participants had a systematic preference for the other (non-noisy) modality. When the 657 speech signal is degraded, people, in previous work, were shown to compensate by relying 658 more on other sources of information such as the accompanying visual cues, the 650 semantic/syntactic context, or the top-down expectations. This kind of compensation has 660 been observed with adults (Mattys et al., 2012; McClelland, Mirman, & Holt, 2006; 661 Tanenhaus et al., 1995), and recent evidence suggests that it starts in childhood (K. MacDonald, Marchman, Fernald, & Frank, 2018; Yurovsky, Case, & Frank, 2017). However, 663 and generally speaking, previous experimental studies have not differentiated between an 664 optimal compensatory strategy (i.e., relying more on the alternative source while using all 665 information still available in the distorted signal), and a sub-optimal strategy (i.e., relying more on the alternative source while ignoring at least some of the information still available in the distorted signal). The formal approach followed in this paper allowed us to tease apart these two possibilities, and our analysis supports the sub-optimal compensatory strategy: 669 The preference for the non-noisy modality is above and beyond what can be explained by the 670 relative reliability alone, meaning that the participants tend to ignore at least part of the 671

information still available in the noisy modality.

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This second case of sub-optimal behavior is possibly related to the fact that language 673 understanding under degraded conditions is cognitively more taxing than language 674 understanding under normal conditions (Mattys et al., 2012; Peelle, 2018; Rönnberg, Rudner, 675 Lunner, Zekveld, & others, 2010). This fact can lead to a sub-optomal behavior (i.e., 676 over-reliance on the less noisy cue) as participants would seek to minimized cognitive effort. 677 One could also explain this phenomenon in terms of the metacognitive experience about the 678 fluency with which information is processed. The perceived perceptual fluency (e.g., the ease 679 with which a stimulus' physical identity can be identified) can affect a wide variety of human 680 judgements (see Schwarz, 2004 for a review). In particular, variables that improve fluency 681 tends to increase liking/preference (Reber, Winkielman, & Schwarz, 1998). In our case, the 682 subjective experience of lower fluency in the noisy modality might cause people to 683 underestimate information that can be extracted from this modality, especially when presented simultaneously with a higher fluency alternative. 685

An important question to ask is how the combination mechanism—as revealed in our 686 controlled study—scales up to real life situations. Note that in order to test audio-visual cue 687 combination under uncertainty, we had to use a case of double ambiguity, that is, a case 688 where both the word forms ("ada"-"aba") and the referents (cat-dog) were similar and, thus, confusable. However, to what extent does such a case occur in real languages? 690 Cross-linguistic corpus analyses suggest that lexical encoding tends, surprisingly, towards 691 double ambiguity in many languages (Dautriche, Mahowald, Gibson, & Piantadosi, 2017; 692 Monaghan, Shillcock, Christiansen, & Kirby, 2014; Tamariz, 2008). For instance, Dautriche et al. (2017) analyzed 100 languages and found that words that are similar phonologically tend to be similar semantically as well. These studies suggest that the case of double uncertainty, though perhaps not pervasive, could be a real issue in language as it increase the probability of confusability for many words. 697 That being said, besides the case of double ambiguity *intrinsic* to language, our mechanism might play a more significant role when ambiguity in both the form and/or the referent is induced by an *external* noisy context even when these forms and referents are not confusable in normal situations.

Though we only tested adults in this paper, the problem of word recognition under 702 uncertainty, as well as the need to make the most of the available cues, is arguably more 703 pressing for children. In fact, children have greater difficulties differentiating the meanings of 704 novel similar-sounding words (e.g., "bin" vs. "din"), even when these words are uttered very 705 clearly (Creel, 2012; Merriman & Schuster, 1991; Stager & Werker, 1997; Swingley, 2016; 706 White & Morgan, 2008). Such similar-sounding words can be shown to be differentiated by 707 infants in simplified experimental settings (e.g., Yoshida, Fennell, Swingley, & Werker, 2009). 708 Nevertheless, Swingley (2007) suggested this differentiation is not mature and is probably 700 noisier than the adult-like representation and/or encoded with lower confidence. Thus, it 710 looks like children, even more than adults, may greatly benefit from additional 711 disambiguating cues during new word-referent encoding and recognition. For example, 712 Dautriche, Swingley, and Christophe (2015) showed that 18-month-olds can leverage the 713 difference in the syntactic category (noun vs. verb). Further work is needed to explore 714 whether salient referential cues also help children disambiguate similar sounding words. 715

The multi-modal cue combination strategy might help children not only recognize 716 words, but also refine the underlying phonological and semantic representations in the 717 process. Previous research in early word learning has—whether implicitly or 718 explicitly—largely treated the process of refining the word form and of refining the word 719 meaning as following a linear timeline However, recent developmental data shows that children do not wait to have completed the acquisition of forms to start learning the 721 meanings (Bergelson & Swingley, 2012; Tincoff & Jusczyk, 1999). Rather, both form and meaning representations develop in a parallel fashion. A few studies pointed to the 723 possibility of an interaction between sound and meaning in early acquisition. For instance, 724 Waxman and Markow (1995) showed that labeling various objects with the same name helps 725

infants form the underlying semantic category (but see Sloutsky & Napolitano, 2003). Vice versa, Yeung and Werker (2009) showed that pairing similar sounds with different objects help infants enhance their sensitivity to the subtle phonological contrasts in their native language. The present study proposes a first step towards a formal framework where isolated accounts of sound-meaning interaction in development can be unified and further explored.

One limitation of this work is that we used simplified stimuli. For the auditory 731 modality, we used speech categories that varied along a single acoustic dimension. While this 732 dimension might be sufficient to recognize words in our specific case, in general the speech 733 signal may be more complex, varying along several acoustic/phonetic dimensions. 734 Additionally, these dimensions may be highly variable due to various kinds of speaker and 735 context differences. The same thing can be said about the referential stimuli. Here we used a 736 continuum along a single morph dimension in order to construct a multimodal input where 737 the auditory and visual components have symmetrical properties. Though such morph is not 738 the exact visual variability that people would encounter in their daily lives, it allowed us to 730 precisely test the role of auditory and visual information in the cue combination process. 740 Parameterizing semantic dimensions is a notoriously difficult problem, but morphs have been 741 used in previous research as a reasonable proxy (Freedman et al., 2001; Havy & Waxman, 742 2016; Sloutsky & Fisher, 2004). It is an open question as to whether people use the same 743 strategy in controlled laboratory conditions, as in more naturalistic settings where they have 744 to deal with various levels of variability. An answer to this question is likely to involve a multifaceted research approach, involving—besides conducting laboratory experiments—testing computational mechanisms with an input that represents, more accurately, the complexity of multimodal variability in the learning environment (Dupoux, 2018; Fourtassi, Schatz, Varadarajan, & Dupoux, 2014; Harwath, Torralba, & Glass, 2016; B. 749 C. Roy, Frank, DeCamp, & Roy, 2015).

751 Conclusion

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This work studied the mechanism of spoken word identification under uncertainty in 752 both the form and the referent. We conducted an ideal observer analysis of this task. We 753 found people to be near optimal in their cue combination (at least at the population level): They weighted each modality according to its relative reliability. However, they also showed 755 patterns of sub-optimality especially when the stimuli were perturbed with additional 756 background noise. This work provides a formal framework where old and new questions 757 about word recognition in a referential context can be given a precise formulation. For 758 instance, of particular interest is the case of iconicity, that is, when there is a resemblance 759 between the sound of a word and its referent. Previous work has suggested that iconicity, 760 among other things, helps with learning (and generalizing the meaning of) new words (see 761 Dingemanse, Blasi, Lupyan, Christiansen, & Monaghan, 2015 for a review). Using the 762 research strategy in this paper, we can, for example, test whether iconicity has such an 763 advantage because it mitigates the sub-optimal patterns observed with more arbitrary 764 pairings. Finally, though the current framework only characterizes adult word recognition, it 765 provides a first step towards a model where developmental questions can also be investigated. 766 For instance, future work should explore whether children, like adults, use probabilistic cues 767 from both the auditory and the visual input to recognize ambiguous words, the extent to 768 which they combine these cues in an optimal fashion, and whether these combination help 769 them with refining their early phonological and semantic representations. 770

All data and code for these analyses are available at https://github.com/afourtassi/WordRec

# Appendix 1: derivation of the posterior (Equation 1)

For an ideal observer, the probability of choosing category 2 when presented with an audio-visual instance w = (a, v) is the posterior probability of this category:

$$p(W_2|w) = \frac{p(w|W_2)p(W_2)}{p(w|W_2)p(W_2) + p(w|W_1)p(W_1)}$$

Which reduces to:

$$p(W_2|w) = \frac{1}{1 + \frac{p(w|W_1)}{p(w|W_2)} \frac{p(W_1)}{p(W_2)}}$$

In order to further simplify the quantity  $\frac{p(w|W_1)}{p(w|W_2)}$ , we use our assumption that the cues are uncorrelated:

$$p(w|W) = p(a, v|W) = p(a|A)p(v|V)$$

Using the log transformation, we get:

$$\ln(\frac{p(w|W_1)}{p(w|W_2)}) = \ln(\frac{p(a|W_1)}{p(a|W_2)}) + \ln(\frac{p(v|W_1)}{p(v|W_2)})$$

Under the assumption that the categories are normally distributed and that, within each modality, the categories have equal variances, we get (after simplification):

$$\ln(\frac{p(a|W_1)}{p(a|W_2)}) = \frac{\mu_{A1} - \mu_{A2}}{\sigma_A^2} \times a + \frac{\mu_{A2}^2 - \mu_{A1}^2}{2\sigma_A^2}$$

and similarly:

$$\ln(\frac{p(v|W_1)}{p(v|W_2)}) = \frac{\mu_{V1} - \mu_{V2}}{\sigma_V^2} \times v + \frac{\mu_{V2}^2 - \mu_{V1}^2}{2\sigma_V^2}$$

When putting all these terms together, we obtain this final expression for the posterior:

$$p(W_2|w) = \frac{1}{1 + (1+b)\exp(\beta_0 + \beta_a a + \beta_v v)}$$

783 where

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$$1 + b = \frac{p(W_1)}{p(W_2)}$$
$$\beta_0 = \frac{\mu_{A2}^2 - \mu_{A1}^2}{2\sigma_A^2} + \frac{\mu_{V2}^2 - \mu_{V1}^2}{2\sigma_V^2}$$

$$\beta_a = \frac{\mu_{A1} - \mu_{A2}}{\sigma_A^2}$$

$$\beta_v = \frac{\mu_{V1} - \mu_{V2}}{\sigma_V^2}.$$

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