1   Word Learning as Network Growth: A Cross-linguistic Analysis

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14   Abstract

15 Children tend to produce words earlier when they are connected to a variety of other words

16 along both the phonological and semantic dimensions. Though this connectivity effect has

17 been extensively documented, little is known about the underlying developmental

18 mechanism. One view suggests that learning is primarily driven by a network growth model

19 where highly connected words in the child's early lexicon attract similar words. Another

20 view suggests that learning is driven by highly connected words in the external learning

21 environment instead of highly connected words in the early internal lexicon. The present

22 study tests both scenarios systematically in both the phonological and semantic domains,

23 and across 8 languages. We show that external connectivity in the learning environment

24 drives growth in both the semantic and the phonological networks, and that this pattern is

25 consistent cross-linguistically. The findings suggest a word learning mechanism where

26 children harness their statistical learning abilities to (indirectly) detect and learn highly

27 connected words in the learning environment.

28   *Keywords:* Language understanding; audio-visual processing; word learning; speech

29 perception; computational modeling.

Word Learning as Network Growth: A Cross-linguistic Analysis

Introduction

What factors shape vocabulary learning over the course of early childhood? To investigate this question, scientists have adopted multiple research strategies, from conducting controlled laboratory experiments (e.g. Markman, 1990) to analyzing dense corpora capturing language learning in context (e.g., B. C. Roy, Frank, DeCamp, Miller, & Roy, 2015). One strategy consists in documenting the timeline of words' acquisition, and studying the properties that make words easy or hard to learn. For example, within a lexical category, words that are more frequent in child-directed speech are acquired earlier (J. C. Goodman, Dale, & Li, 2008). Other factors include word length, the mean length of utterances in which the word occurs, and concreteness (see Braginsky, Yurovsky, Marchman, & Frank, 2016).

Besides these word-level properties, the lexical structure (that is, how words relate to each other) also influences the age of acquisition of words. The lexical structure is best characterized in terms of a network where each node represents a word in the vocabulary, and each link between two nodes represents a relationship between the corresponding pair of words. Previous studies have investigated early vocabulary structure by constructing networks using a variety of word-word relations including shared semantic features, target-cue relationships in free association norms, co-occurrence in child directed speech, and phonological similarity. These studies have found that children tend to produce words that have higher neighborhood density (i.e., high connectivity in the network) earlier, both at the phonological and the semantic level (Engelthaler & Hills, 2017; Hills, Maouene, Riordan, & Smith, 2010; Hills, Maouene, Maouene, Sheya, & Smith, 2009; Stella, Beckage, & Brede, 2017; Storkel, 2009).

While most studies have focused on the static properties of the lexical network, a few have investigated the underlying developmental process. In particular, Steyvers and Tenenbaum (2005) suggested that the observed effects of connectivity are the consequence of

how the lexical network gets constructed in the child's mind. According to this explanation,
known as Preferential Attachment (PAT), highly connected words in the child's lexicon tend
to "attract" more words over time, in a rich-get-richer scenario (Barabasi & Albert, 1999).
In other words, what predicts word learning is the *internal* connectivity in the child's early
lexicon. In contrast, Hills et al. (2009) suggested that what biases the learning is not the
connectivity in the child's internal lexicon but, rather, *external* connectivity in the learning
environment. They called this alternative explanation Preferential Acquisition (PAC).
Figure 1 shows an illustration of both growth scenarios with the same simplified network.
These two proposals represent two divergent ideas about the role of lexical networks in
acquisition. On the PAT proposal, network structure is a causal factor in early word
learning; in contrast, on the PAC approach, network structure is not internally represented
and, therefore, might be an epiphenomenon of the statistics of the linguistic input.

Studies that investigate lexical network growth have focused on semantic networks
using English data (Hills et al., 2010, 2009; Steyvers & Tenenbaum, 2005). The novelty of
the current study is threefold: First, it investigates whether phonological networks, like
semantic networks, grow by PAC, or if they rather grow by PAT. Second, it provides a
systematic comparison of both network growth scenarios in the phonological and the
semantic domains and assesses their relative contribution to the learning process. Third, it
tests the generality of the findings across eight languages.

Networks

Data

We used data from Wordbank (Frank, Braginsky, Yurovsky, & Marchman, 2017), an
open repository aggregating cross-linguistic language developmental data of the
MacArthur-Bates Communicative Development Inventory (CDI), a parent report vocabulary
checklist. Parent report is a reliable and valid measure of children's vocabulary that allows
for the cost-effective collection of datasets large enough to test network-based models of

Preferential Attachment (PAT)
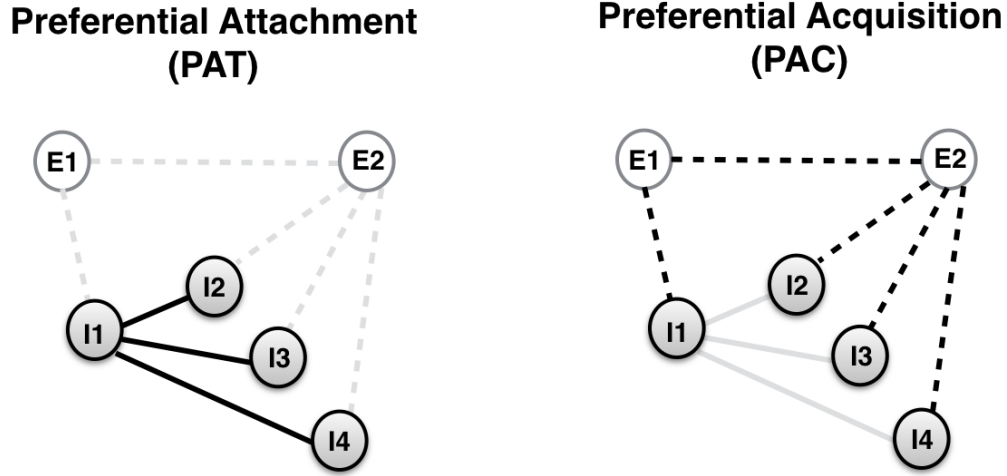
Preferential Acquisition (PAC)

Figure 1. Illustration of the growth scenarios. Filled circles (I1-I4) represent known words (internal), and empty circles (E1 and E2) represent words that have not been learned yet (external). Black lines represent links that are relevant in each growth scenario, and gray lines represent links that are irrelevant. For PAT, the utility of a candidate, external node is the average degree (i.e., number of links) of the internal nodes that it would attach to. Thus, according to PAT, the node E1 is more likely to enter the lexicon first. For PAC, the utility of a candidate node is its degree in the entire network. According to PAC, the node E2 is more likely to enter the lexicon first.

acquisition (Fenson et al., 1994). We used the *Words and Sentences* version of the CDI which contains the productive vocabulary of toddlers (age varied between 16 to 36 months). Following previous studies (Hills et al., 2009; Storkel, 2009), we restricted our analysis to nouns. We defined the age of acquisition of a given word by the month at which this word was produced by at least 50% of children (J. C. Goodman et al., 2008), and we excluded nouns that have not been learned (according to this criterion) by the last month for which we have CDI data.

We obtained these nouns in eight languages: Croatian, Danish, English, Italian, Norwegian, Russian, Spanish, and Turkish. We used the subset of nouns that had entries in

the Florida Association Norms (see below). Since these norms are available only in English,
we used the hand-checked translation equivalents provided by Braginsky et al. (2016),
allowing us to use the English association norms across languages. Table 1 gives an overview
of the data used. Translation equivalents were originally constructed for a subset of words
appearing on the toddler CDI form, and so not all words are currently available. Note,
however, that all languages have at least 60% of nouns translated.

	language	total	translated	normed
1	Croatian	253	177	170
2	Danish	295	198	187
3	English	296	296	274
4	Italian	311	203	194
5	Norwegian	305	193	186
6	Russian	311	311	285
7	Spanish	240	173	163
8	Turkish	293	175	164

Table 1

Total number of nouns produced by toddlers in the CDI (left). We included in our study the subset of these nouns that had available English translations (middle). The final set consisted of nouns that had both available translations as well entries in the Free Association Norms (right).

Semantic networks

We constructed semantic networks following the procedure outlined in Hills et al.
(2009). We used as an index of semantic relatedness the Florida Free Association Norms
(Nelson, McEvoy, & Schreiber, 1998). This dataset was collected by giving adult participants
a word (the cue), and asking them to write the first word that comes to mind (the target).
For example, when given the word "ball", they might answer with the word "game". A pair

of nodes were connected by a directed link from the cue to the target if there was a cue-target relationship between these nodes in the association norms. The connectivity of a given node was characterized by its *indegree*: the number of links for which the word was the target. To model growth from month to month, we constructed a different network at each month, based on the words that have been acquired by that month.

Phonological networks

We generated approximate International Phonetic Alphabet (IPA) transcriptions from the orthographic transcription, across languages, using the open source text-to-speech software **Espeak.** We used the Levenshtein distance (also known as edit distance) as a measure of phonological relatedness between two nodes. The measure counts the minimum number of operations (insertions, deletions, substitutions) required to change one string into another.

In previous studies, two nodes were linked if they had an edit distance of 1 (e.g., Storkel, 2009). However, in these previous studies the network was built using an adult vocabulary. In the current study, however, network growth models are based on the children's early vocabulary which contains very few word pairs with an edit distance of 1. When using this threshold, the resulting networks were too sparse and uninformative. Thus, we increased the threshold from 1 to 2, that is, two nodes were related if their edit distance was equal to 1 or 2. The connectivity of a given node was characterized with its *degree*: the number of links it shares with other words.

Analysis

Static properties of the global network

We start by analyzing word connectivity in the global (static) network. We constructed this network using nouns learned by the oldest age for which we have CDI data (e.g., in English this corresponds to the network by 30 months). This global network is the end-state

129 towards which both PAT and PAC should converge by the last month of learning. Moreover,

130 following Hills et al. (2009), we used this end-state network as a proxy for the external

131 connectivity in the learning environment. Below we analyze properties of this global

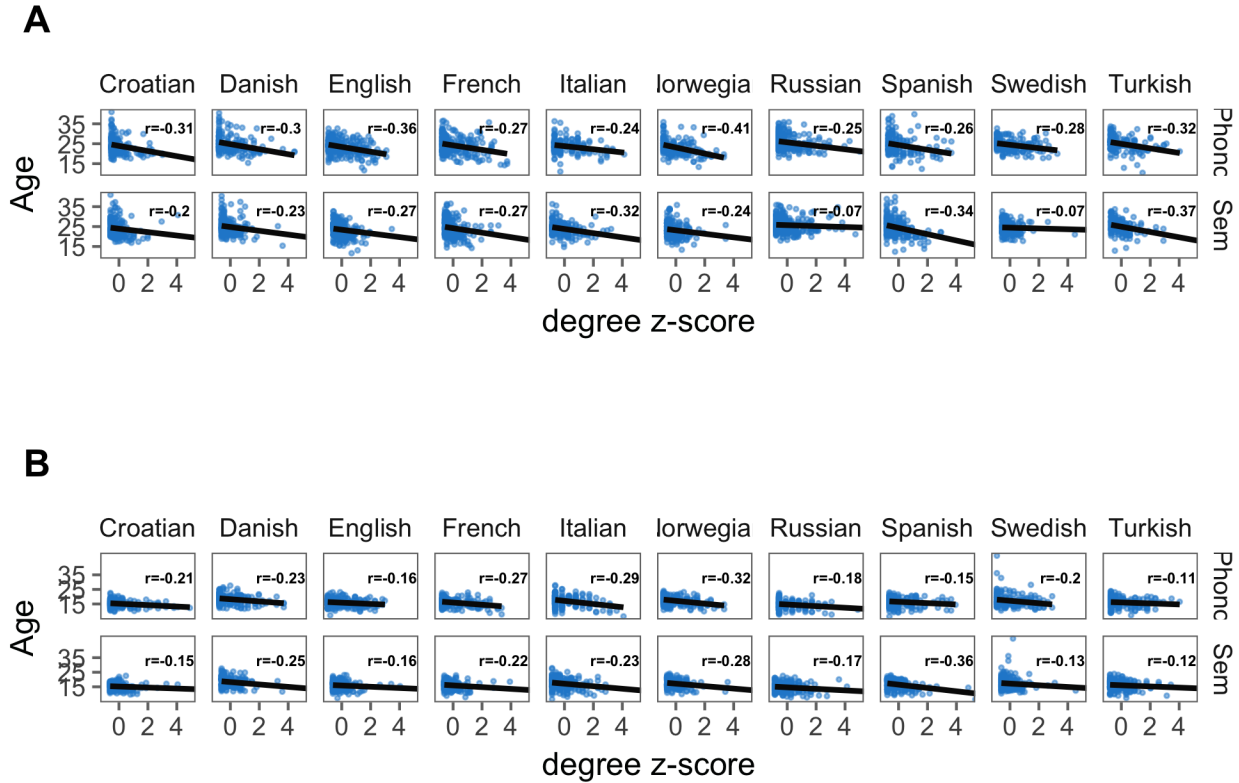132 networks that are relevant to PAC and/or PAT.

Figure 2. Age of production (A) and comprehension (B) in the global network as predicted by the degree in this network. Results are shown in each language for phonological and semantic networks. Each point is a word, with lines indicating linear model fits.

133 **Connectivity predicts the age of acquisition.**   Connectivity in the global

134 network is directly related to PAC as it represents the explicit criterion PAC uses to

135 determine what words should be learned first (Figure 1). Therefore, a direct consequence of

136 a PAC-like growth scenario is a correlation between connectivity in the global network and

137 the age of acquisition.[1] Figure 2 shows how the age of production (A) and comprehension

138 (B) for each word varies as a function of its degree (or indegree for the semantic network).

139 For ease of visual comparison, the predictor (i.e., the degree) was centered and scaled across

140 languages. The plots show, overall, a negative correlation between the month of acquisition

141 and the degree, indicating that nouns with higher degrees are generally learned earlier.
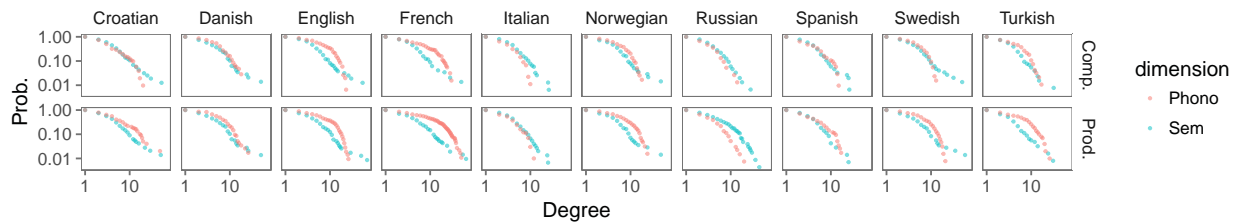

Figure 3. Log-log plot of the cumulative degree distribution function for the global phonological and semantic networks across languages. The figure shows the results for both production and comprehension data. A perfect power-law distribution should appear as a straight line in this graph.

142 **Power-law degree distribution?** We also analyzed the global network's degree

143 distribution. The shape of this distribution is particularly relevant to PAT as this growth

144 scenario is known to generate networks with a power-law degree distribution (i.e., a

145 distribution of the form $p(k) \propto \frac{1}{k^\alpha}$, Barabasi & Albert, 1999). If the network displays this

146 property, this fact would suggest a PAT-like generative process. Conversely, if the degree

147 distribution does not follow a power law, this fact would weaken the case for PAT. The

148 log-log plots are shown in Figure 3. We fit a power law to each empirical degree distribution

149 following the procedure outlined in Clauset, Shalizi, and Newman (2009) and using the

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[1]This correlation is also compatible with PAT, although the causality is reversed. Indeed, from the perspective of this growth scenario, higher connectivity in the global network is caused by earlier learning, not the other way around. Some words end up being highly connected in the global network precisely because they happen to be acquired earlier and, therefore, have a higher chance of accumulating more links over time.

150 related R package (poweRlaw, Gillespie, 2015). In brief, the analysis consisted in two steps.

151 First, we derived the optimal cut-off, k_{min}, above which the distribution is more likely to

152 follow a power law,[2] and we estimate the corresponding scaling parameter α. Second we

153 calculated the goodness-to-fit, which resulted in a p-value quantifying the plausibility of the

154 model. The results are shown in Appendix 1, table XX. Overall, we could not reject the null

155 hypothesis of a power-law distribution: the p-value was generally above 0.1.

156   In sum, the static properties of the global network are *a priori* compatible with both

157 PAT and PAC. In order to decide between these two developmental scenarios, we need to fit

158 explicit growth models to the data.

159 **Network growth models**

160   **How does each growth scenario predict noun development?**   To test the

161 network growth scenarios, we fit different growth models to the data. We calculated the

162 probability that a word w_i, with a growth value d_i would enter the lexicon at a given month,

163 using a softmax function:

$$p(w_i) = \frac{e^{\beta d_i}}{\sum_j e^{\beta d_j}} \tag{1}$$

164 where β is a fitted parameter that captures the magnitude of the relationship between

165 network parameters and growth (analogous to a regression coefficient). A positive value of β

166 means that words with higher growth values d_i are acquired first, and a negative value

167 means that words with lower growth values are acquired first (see Figure 1 for an illustration

168 of how growth values d_i are defined in each growth scenario). The normalization includes all

169 words that could be learned at that month.

170   We estimated the parameter β using a Bayesian approach. The inference was

171 performed using the probabilistic programming language WebPPL (N. Goodman &

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[2]In natural phenomena, it is often the case that the power law applies only for values above a certain minimum.
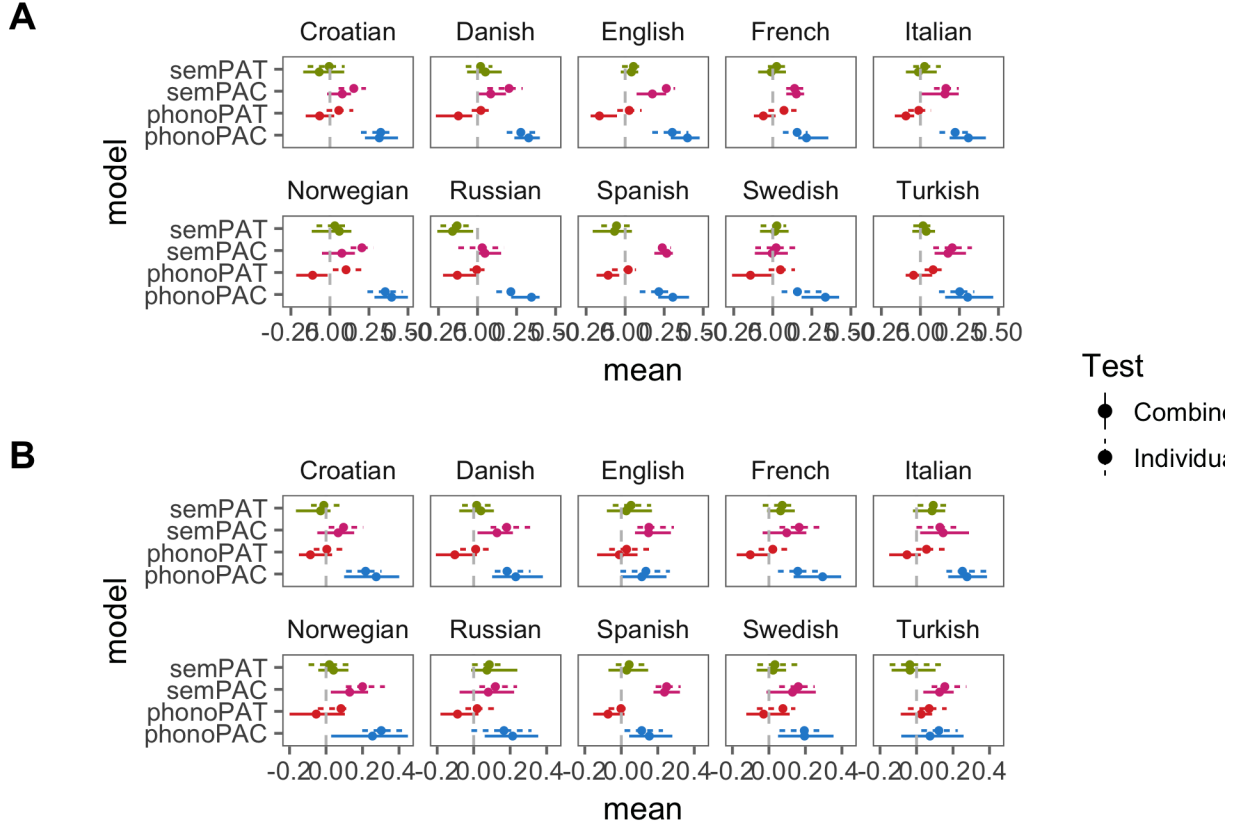
Figure 4. Evaluation of network growth scenarios, for both A) production and B) comprehension. We plot values when predictors are evaluted individually (dotted), and when combined in the same growth model (solid). Each dot represents the mean of the posterior distribution of the corresponding growth parameter, with ranges representing 95% credible intervals (computed using the highest density intervals). Positive values mean that learning proceeds according to the predictions of the growth scenario. Negative values mean that learning proceeds in opposition to the predictions of the growth scenario.

172 Stuhlmuller, 2014). We defined a uniform prior over β, and at each month, we computed the

173 likelihood function over words that could possibly enter the lexicon at that month, fit to the

174 words that have been learned at that month (using formula 1). Markov Chain Monte Carlo

175 sampling resulted in a posterior distribution over β, which we summarized in Figure 4.

176   For the semantic networks, the results replicate Hills et al.'s finding in English, which is

177 that the semantic network grows by PAC, not by PAT. Moreover, this finding holds in seven

178 of the eight languages we examined.[3] The PAC model also fits better than PAT for

179 phonological networks. We note however that PAT, though weaker, fares better for the

180 phonological networks (where it predicts part of the growth process in some languages such

181 as Croatian, English, Norwegian and Russian) than it does for the semantic networks (where

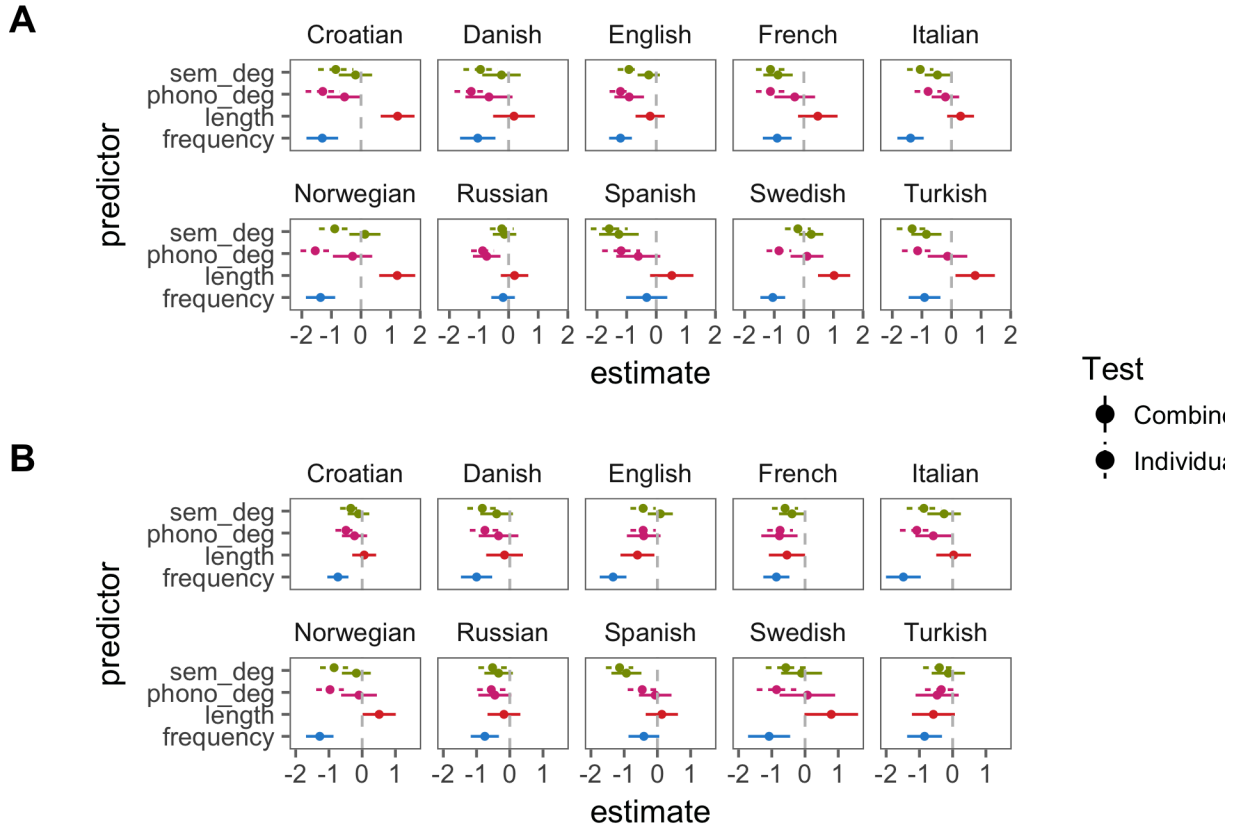182 it is rather universally unpredictive).

Figure 5. Estimates of predictor coefficients by language in production (A) and comprehension (B), with ranges indicating 95% confidence intervals. Positive values indicate a positive relationship (e.g. longer words tend to have a higher AoA), while negative values indicate a negative relationship (e.g. words with higher frequency tend to have a lower AoA).

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[3]One could imagine that the fact of using English free association norms cross-linguistically would decrease the effect of non-English semantic networks because of possible cultural differences. However, our findings do not support this assumption as the effects were generally similar in magnitude cross-linguistically.

183 **What is the relative contribution of each growth model?** Above we

184 evaluated the network growth scenarios individually. As a next step, we analyzed their

185 relative contribution to the learning process. This was done through adding more fitted

186 parameters to the model, that is, by substituting βd_i in formula (1) with:

$$\beta_1 d_{i,1} + \beta_2 d_{i,2} + \beta_3 d_{i,3} + \beta_4 d_{i,4}$$

187 where the indices represent the 4 networks: semPAT, semPAC, phonoPAT and PhonoPAC.

188 Using the same fitting technique, we obtained the values shown in Figure **??**. PAC

189 dominates the learning. Both phonological and semantic networks contribute to lexical

190 growth, but the phonological network appears to be stronger and more consistent across

191 languages. In summary, the findings show that both semantic and phonological networks

192 contribute to the learning process, and that they both grow primarily by PAC, relying on the

193 external connectivity in the learning environment, rather than the internal connectivity in

194 the acquired lexicon.

Comparison to other predictors of age of acquisition

196 We saw that the way semantic and phonological information is structured in the

197 learning environment (i.e., PAC) contributes to noun learning across languages. However, we

198 know that other factors influence learning as well (e.g., Braginsky et al., 2016). Next we

199 investigated how semantic and phonological connectivity interact with two other factors.

200 The first one is word frequency, a well studied factor shown to predict the age of acquisition

201 in a reliable fashion (e.g. J. C. Goodman et al., 2008). The second factor is word length,

202 which correlates with phonological connectivity.

203 Since PAT was uninformative, we dropped it from this analysis, keeping only PAC.

204 This simplified the model because we no longer needed to fit growth month-by-month.[4] A

205 more direct way to assess and compare the contribution of PAC in relation to other

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[4]This was a requirement only for PAT where the words' utilities varied from month to month, depending on how connectivity changed in the growing internal network.
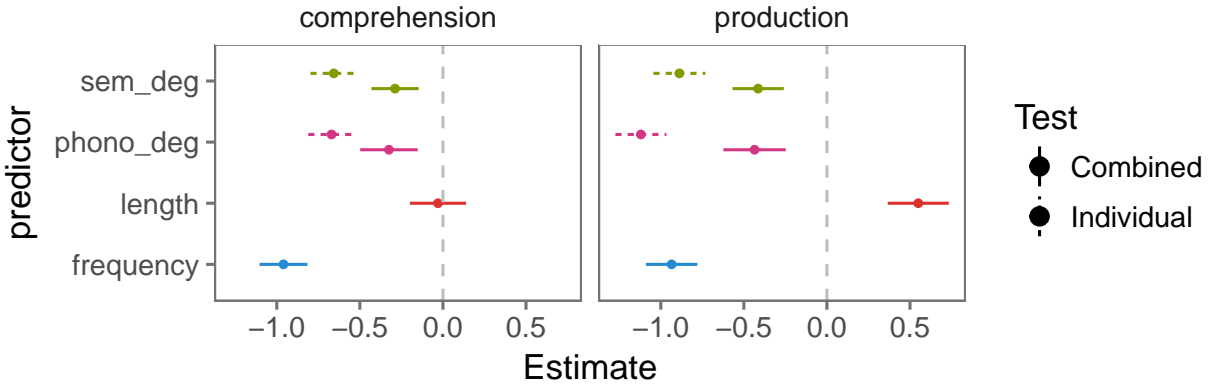
Figure 6. Estimates of predictor coefficients in the combined mixed-effect model with language as a random effect for both production and comprehension. Ranges indicate 95% confidence intervals. Lighter points indicate estimates of PAC predictors in a model that does not include frequency and length as covariates.

word-level factors is through conducting linear regressions, where connectivity in the learning environment, frequency and length predict the age of acquisition.

We used the frequency estimates from Braginsky et al. (2016) where unigram counts were derived based on CHILDES corpora in each language.[5] For each word, counts included words that shared the same stem (e.g., "cats" counts as "cat"), or words that were synonymous (e.g. "father" counts as "daddy"). For word length, we counted the number of phonemes in our generated IPA transcription.

We conducted two analyses. We fit a linear regression for each language, and we fit a linear mixed-effect model to all the data pooled across languages, with language as a random effect. Figure 5 shows the coefficient estimate for each predictor in each language for

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[5]Note that these frequency counts are based on transcripts from independent sets of children and represent a general estimate of environmental frequency across children.

²¹⁶ production and comprehension data. Figure 6 shows the coefficient estimates for all

²¹⁷ languages combined (all predictors were centered and scaled). The findings were as follows.

²¹⁸ Overall, frequency is the largest and most consistent predictor of age of acquisition,

²¹⁹ replicating results for nouns across a variety of analyses (Braginsky et al., 2016; J. C.

²²⁰ Goodman et al., 2008; B. C. Roy et al., 2015). Word length predicts learning in some

²²¹ languages such as Croatian and Norwegian, but not in others (including English). It remains,

²²² however, a significant predictor in the global model. As for the factors of interest, i.e.,

²²³ semantic and phonological connectivity, we also found cross-linguistic differences.

²²⁴ Phonological connectivity contributes to learning in languages such as Croatian, English and

²²⁵ Russian, whereas semantic connectivity contributes to learning in Turkish, Spanish and to

²²⁶ some extent in Croatian, but not in English.[6] Despite these cross-linguistic differences, both

²²⁷ phonological and semantic connectivity are significant predictors in the combined model.

Discussion

²²⁹ The present study provided a comprehensive analysis of how lexical connectivity

²³⁰ influences the age of acquisition of nouns in toddlers. We compared two network growth

²³¹ scenarios and assessed their relative contributions across eight languages. One scenario, PAT,

²³² described a rich-get-richer network growth model in which the structure of the learner's

²³³ internal network determines future growth; the other, PAC, described a model in which the

²³⁴ external, global environmental network structure determines learners' growth patterns. Our

²³⁵ findings largely replicate the results obtained by Hills et al. (2009): Semantic networks grow

²³⁶ by preferential acquisition, not by preferential attachment. A novel finding is that

²³⁷ phonological networks also grow primarily by preferential acquisition. Moreover, both

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[6]Semantic connectivity does not explain variance in English data beyond that explained by phonological connectivity, frequency and length. This contrasts with the original finding in Hills et al. 2009. However, in this previous study, semantic connectivity was not tested in a model that included frequency, length and phonological connectivity as covariates. Another important difference is the number of words tested: Our study uses a larger set of nouns.

semantic and phonological connectivity in the learning environment predict growth. These findings generalize well across languages. When pitted against other known predictors of age of acquisition (word frequency and length), the effect of word connectivity shows a cross-linguistic variation, predicting learning in some languages, but not in others. Nevertheless, this cross-linguistic variability is to be taken with a grain of salt as it might be exaggerated in our study by the limited and partially-overlapping sample of nouns for each language. In fact, both phonological and semantic connectivity are significant predictors when data are pooled across languages.

Children start by learning words that have high semantic and phonological similarity to a variety of other words in the learning environment, not in the child's available lexicon. This result suggests that children are sensitive to connectivity even without having first acquired the connected words. How can children indirectly detect highly connected words, and why would such words be more readily learned?

In the semantic case, the networks are based on free association norms. These associations can be (partly) derived from the patterns of word-word co-occurrence (e.g., Griffiths, Steyvers, & Tenenbaum, 2007), i.e., two words are associated if they co-occur in many different contexts. In a network structure, highly connected words would be the words that co-occur with many other words in various contexts. Why would such words be easier to learn? One possibility, suggested by Hills et al. (2010), is that the referents of these words are more easily disambiguated from other potential referents because their presence in multiple contexts provides more cross-situational, disambiguating statistics about their true referents (Smith & Yu, 2008).

In the phonological case, connectivity is inherently correlated with phonotactic probability (Vitevitch, Luce, Pisoni, & Auer, 1999). That is, highly connected words tend to be made of frequent sound sequences. Even infants show a sensitivity for high frequency sound sequences in the ambient language (Jusczyk, Luce, & Charles-Luce, 1994). Moreover, phonotactic probability facilitates learning and recognition (e.g., Storkel, 2001). In other

words, children's sensitivity to local phonotactic regularities might lead them to learn higher-probability words more easily. This learning effect, in turn, would lead to an observed pattern of growth that would appear to follow the PAC growth model even though learners themselves would only be tracking local statistics.

Finally, while validating previous results using network growth models, our study suggests that these correlational patterns may emerge from the operation of simpler mechanisms in both the semantic and phonological domains. One question for future experimental work is whether such patterns of growth can be produced in controlled behavioral experiments.

> All data and code for these analyses are available at
>
> https://github.com/afourtassi/WordRec

Acknowledgements

Disclosure statement

None of the authors have any financial interest or a conflict of interest regarding this work and this submission.

Appendix 1: Power law model

	Kmin	alpha	pValue	dimension	language
1	4.00	2.55	0.88	Sem	Croatian
2	4.00	2.18	0.12	Phono	Croatian
3	4.00	2.38	0.00	Sem	Danish
4	11.00	4.55	0.86	Phono	Danish
5	5.00	2.66	0.13	Sem	English
6	20.00	9.14	0.51	Phono	English
7	8.00	2.81	0.13	Sem	French
8	20.00	3.75	0.11	Phono	French
9	4.00	2.93	0.61	Sem	Italian
10	9.00	9.45	0.78	Phono	Italian
11	5.00	2.88	0.20	Sem	Norwegian
12	15.00	6.28	0.74	Phono	Norwegian
13	24.00	5.61	0.72	Sem	Russian
14	8.00	4.20	0.54	Phono	Russian
15	4.00	2.98	0.46	Sem	Spanish
16	13.00	8.75	0.74	Phono	Spanish
17	4.00	2.49	0.17	Sem	Swedish
18	11.00	4.68	0.10	Phono	Swedish
19	4.00	2.87	0.93	Sem	Turkish
20	8.00	3.26	0.38	Phono	Turkish

Table 2

Results of fitting a power law model to the degree distribution in each model for production data. Kmin is the optimal degree cut-off, alpha is the scaling parameter, and pValue is the probability that quantifies the plausibility of the power law hypothesis. If pValue is close to 1, the power law model cannot be rejected as a plausible fit for the data. If, instead, pValue is small (e.g., $p < 0.05$) then the null hypothesis of a power law model can be rejected.

	Kmin	alpha	pValue	dimension	language
1	5.00	2.67	0.90	Sem	Croatian
2	2.00	2.06	0.02	Phono	Croatian
3	4.00	2.39	0.01	Sem	Danish
4	5.00	2.98	0.14	Phono	Danish
5	4.00	2.64	0.77	Sem	English
6	13.00	5.16	0.23	Phono	English
7	4.00	2.63	0.33	Sem	French
8	18.00	5.58	0.34	Phono	French
9	4.00	2.88	0.69	Sem	Italian
10	8.00	10.27	0.91	Phono	Italian
11	5.00	2.87	0.43	Sem	Norwegian
12	13.00	7.65	0.44	Phono	Norwegian
13	8.00	3.91	0.95	Sem	Russian
14	5.00	3.97	0.85	Phono	Russian
15	5.00	3.11	0.55	Sem	Spanish
16	5.00	3.01	0.09	Phono	Spanish
17	5.00	2.81	0.71	Sem	Swedish
18	9.00	6.75	0.10	Phono	Swedish
19	4.00	3.13	0.89	Sem	Turkish
20	9.00	5.73	0.96	Phono	Turkish

Table 3

Results of fitting a power law model to the degree distribution in each model for comprehension data. Kmin is the optimal degree cut-off, alpha is the scaling parameter, and pValue is the probability that quantifies the plausibility of the power law hypothesis. If pValue is close to 1, the power law model cannot be rejected as a plausible fit for the data. If, instead, pValue is small (e.g., $p < 0.05$) then the null hypothesis of a power law model can be rejected.

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