Uncertainty in LCA via Monte Carlo

Introduction

We need to calculate the CO_2-equivalent emissions for a shower using the assumptions and data provided. The calculation is defined in three parts.

- **Shower length:** Assumed to follow a log-normal distribution with the provided parameters. This determines the number of liters of water used per shower, assuming a constant flow rate.

- **CO_2 emissions from water heating:** The energy required is calculated by assuming a fixed heat energy per liter of water (the shower temperature is not considered). The CO_2 equivalent factor depends on the energy source, which is modeled using a multinomial distribution. The volume of water to heat is given by the shower length.

- **CO_2 emissions from tap water production:** The CO_2-equivalent factor depends on the water source, with each of the seven sources having an equal probability (approximately 14.3%). The volume of water used is again determined by the shower length.

Alongside the provided data and assumptions, we have made several hypotheses.

- **CO_2 factors for heat sources:** The CO_2 factor is defined in kg CO_2 per MJ of heat, rather than as kg CO_2 for a specific amount of MJ for each heat source listed in the table. This assumption is based on the example calculation provided. Since we did not have access to the ecoinvent database, we were unable to verify this hypothesis.

- **Country assumption:** The calculations assume Switzerland. For this exercise, CO_2 factors from other European countries were extrapolated to Switzerland, based on the GEO column in both tables.

- **Shower length distribution:** We assumed that the provided parameters correspond to a mean shower length of 10 minutes and a standard deviation of 3 minutes. Using these values, we calculated the mean (μ) and standard deviation (σ) of the log-normal distribution with the formulas:

$$\mu = \ln\left(\frac{m^2}{\sqrt{v+m^2}}\right) \text{ and } \sigma = \sqrt{\ln\left(1+\frac{v}{m^2}\right)}$$

  where m is the mean shower length and v is the variance ($v =$ standard deviation^2). If, instead, the provided parameters were assumed to be the actual μ and σ, the resulting distribution would be unrealistic, yielding a mean shower time of 1,982,759 minutes.

Provided files

A Streamlit app has been made, and can be accessed here:

https://montecarloapp-xxtdmphppyingzcy7wzeff.streamlit.app/

Alongside this report, several files are provided to reproduce the calculations if needed.

In the folder Monte Carlo:

- A Jupyter Notebook containing the Python code.
- Two CSV files with the tables for heat and water emission factors. The source names have been modified to ensure uniqueness.

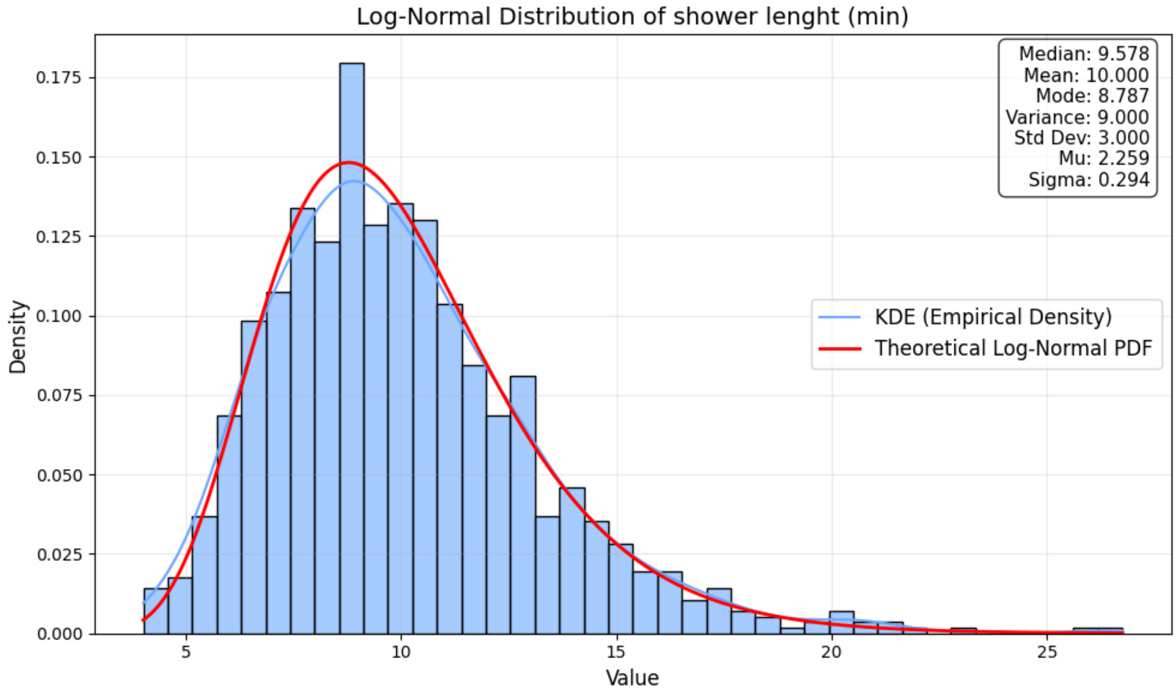
- An Excel file (Shower log normal distribution parameter.xlsx) to manually calculate μ and σ for the shower length log-normal distribution.
- A YAML file listing the dependencies and packages for the Conda environment.

In the folder Monte Carlo STREAMLIT:

- Two Python files: one for the Streamlit app, and the other for the calculations.
- The Streamlit app has been published on Streamlit Community Cloud, but it can also be run locally using VS Code with a venv environment.

Log normal distribution of shower time

Assuming the hypotheses described above, the following is an example of the probability density function obtained by randomly generating 1,000 points:

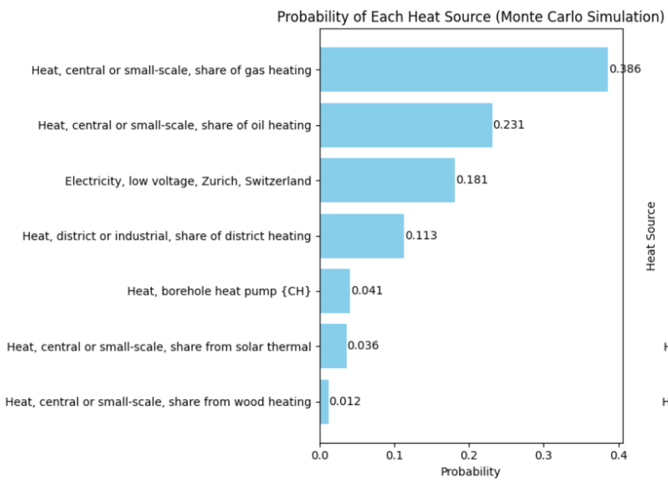


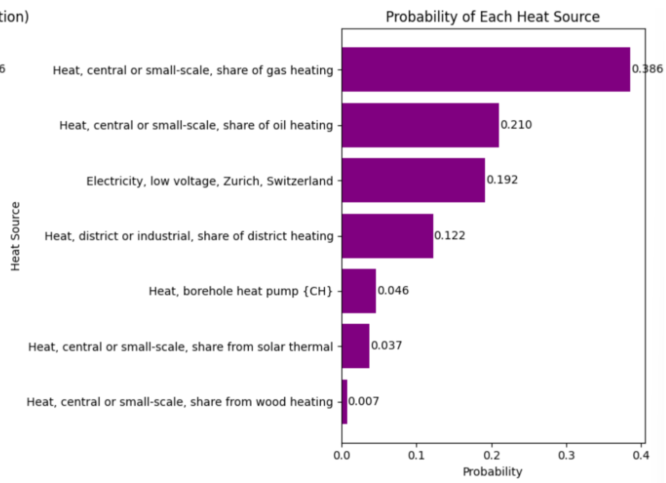
This confirms that the log-normal distribution respects the specified mean and standard deviation.

Simulations

After running 1,000 iterations, we can analyze the resulting simulation data.

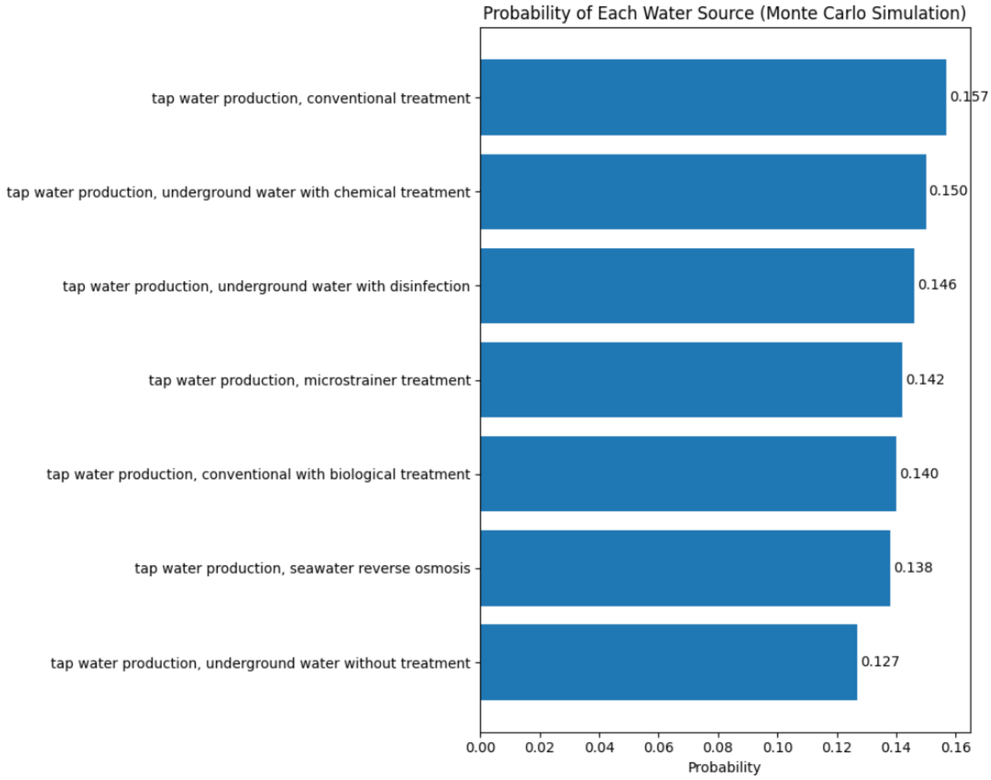
Heat source probability





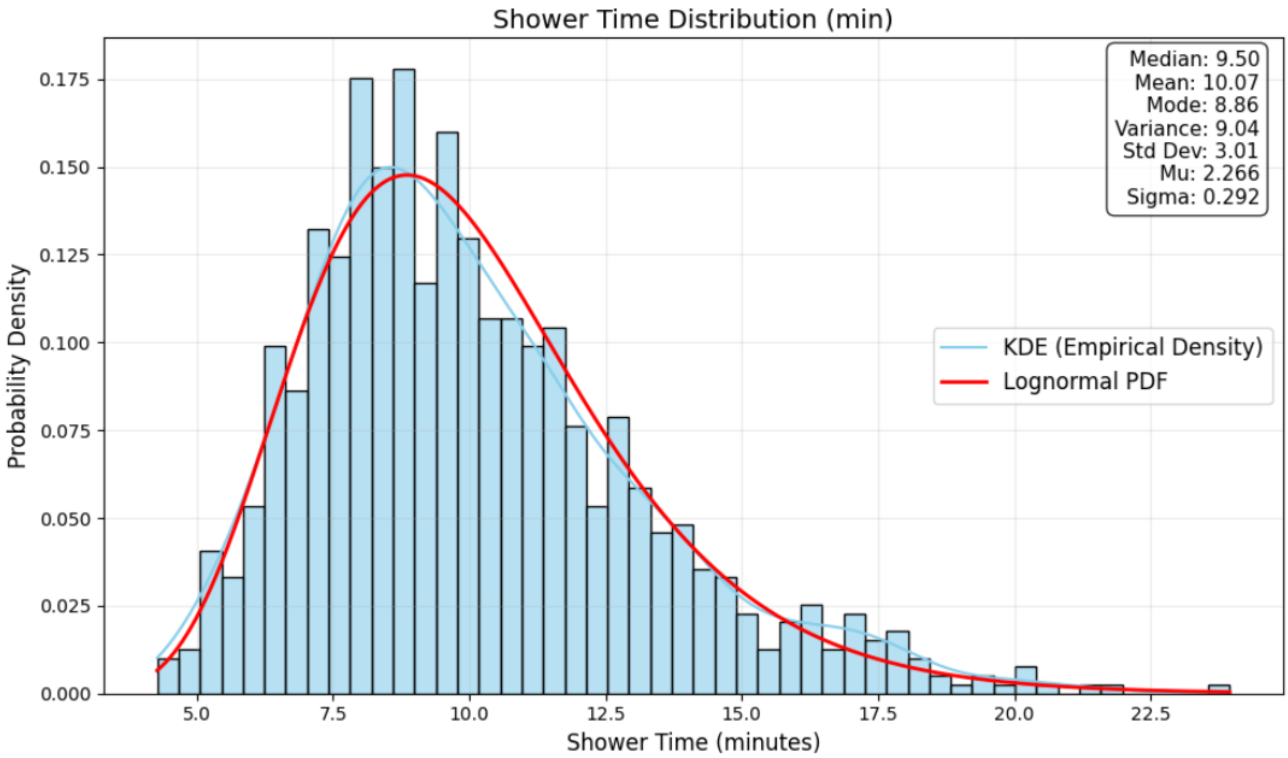
By examining the probability of each heat source (*left*), we see that the simulated probabilities closely match the provided data (*right*). Small discrepancies are due to the limited number of simulations and the inherent randomness of the method; with an infinite number of iterations, the simulated probabilities would converge to the true values.

Water source probability



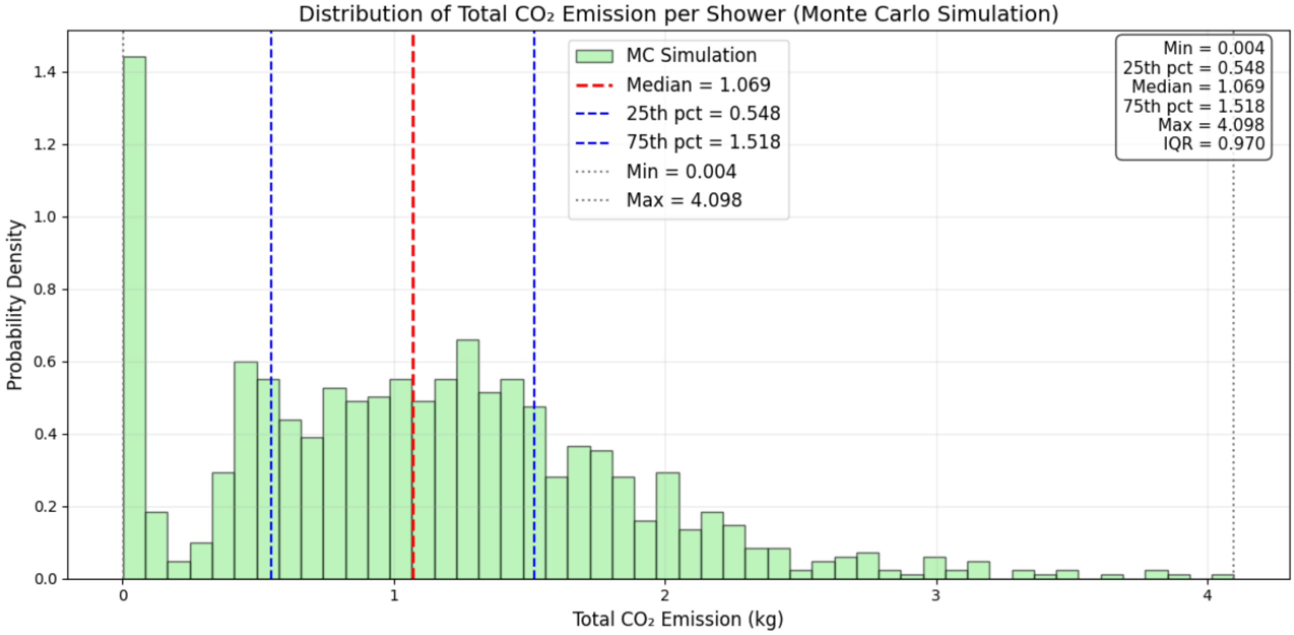
Similarly, the probability distribution of water sources is not perfectly uniform for the same reasons as for heat sources.

Shower time probability



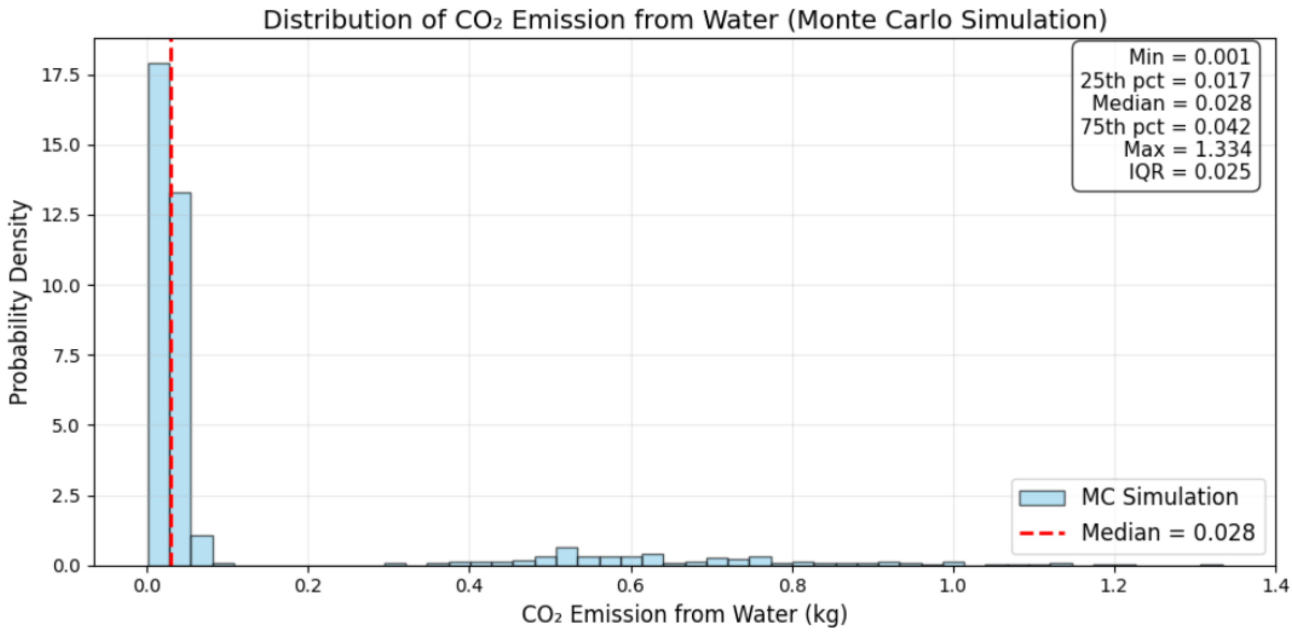
Checking the shower time distribution, we observe that the mean and standard deviation of the initial distribution are preserved, confirming that the log-normal shape is maintained.

Distribution of total CO_2 Emission

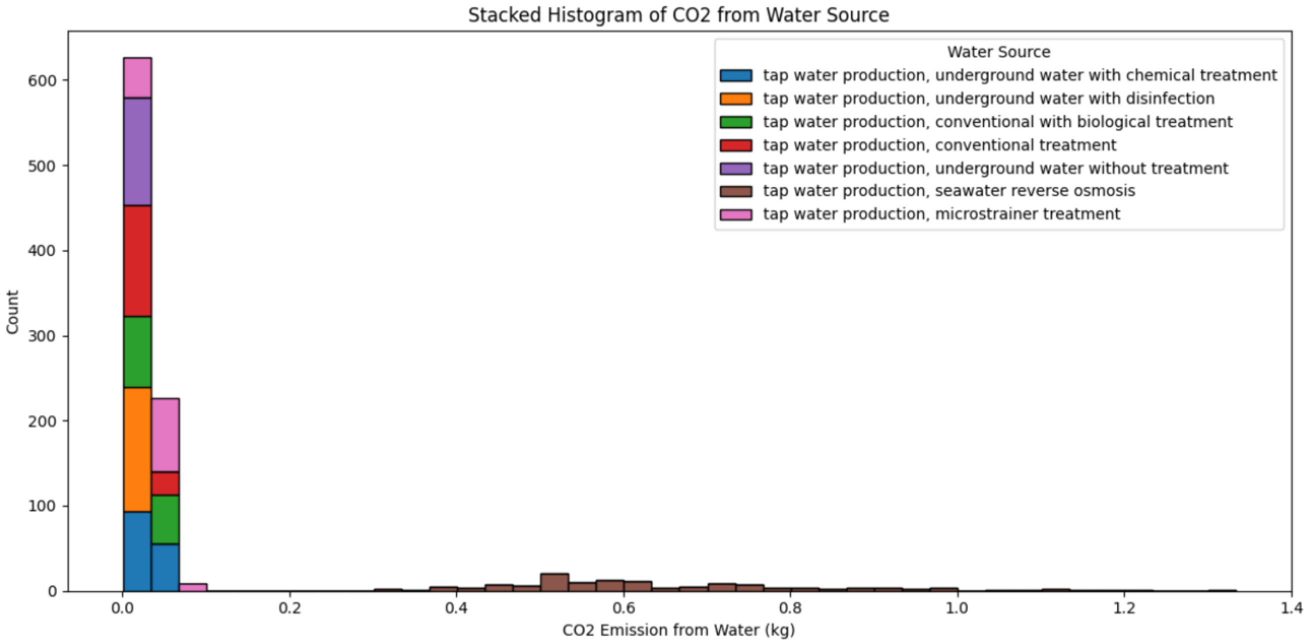


Next, we plot the total CO_2 emissions per shower. Most values are concentrated near 0 kg, with a median around 1 kg and an interquartile range reflecting typical variability.

Distribution of CO_2 Emission - Water

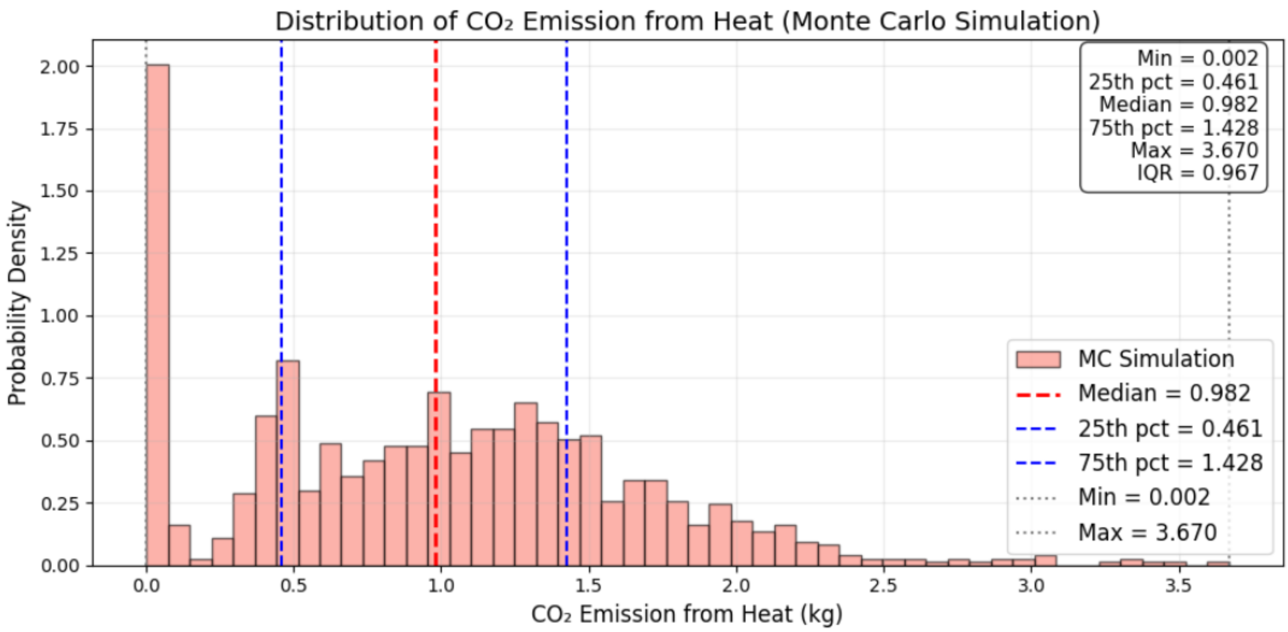


To assess the contribution of each source to CO_2 emissions, we separate water and heat sources. The plot shows that water contributes minimally to the overall carbon footprint.

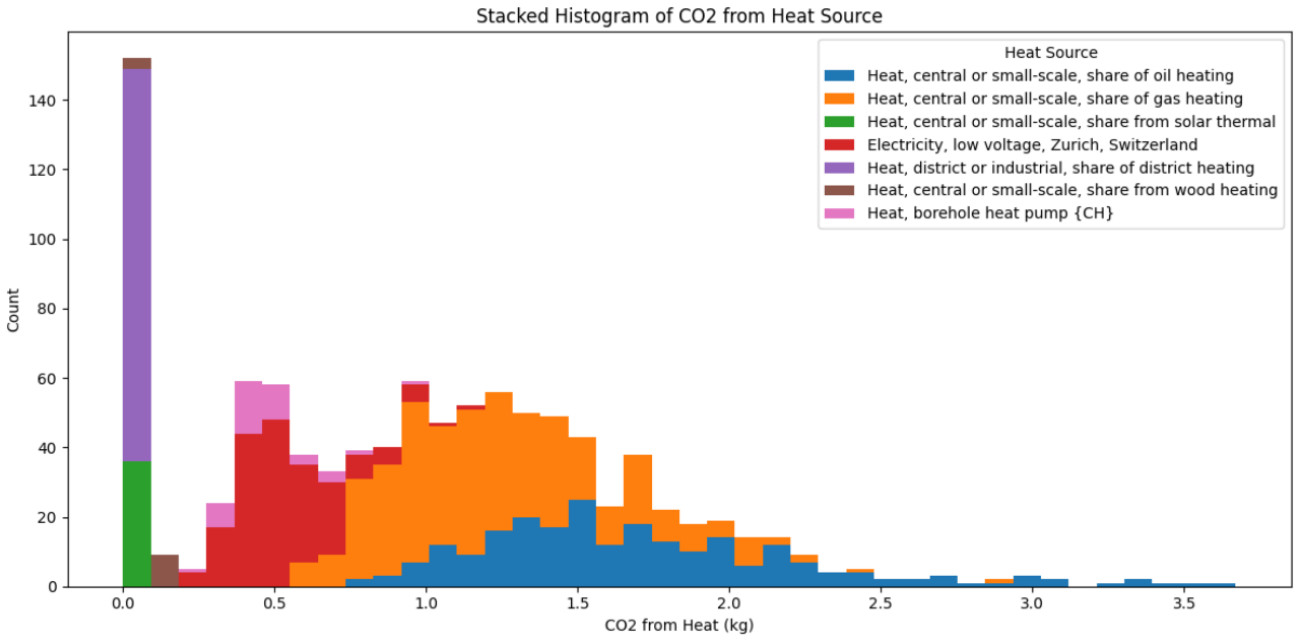


Examining water sources in detail, low-emission values are distributed across nearly all sources, while seawater reverse osmosis has the highest impact, consistent with its energy-intensive nature.

Distribution of CO_2 Emission - Heat



Similarly, for heat sources, it is clear that the variability in total CO_2 emissions is primarily driven by the type of heat source, with a comparable distribution pattern across the two datasets.



Renewable energy sources (e.g., solar, waste with district heating) and low-probability sources (e.g., wood) yield emissions near 0. Fossil fuels (gas, oil) account for most emissions. For electricity, without knowing the production source (mainly hydroelectric in Switzerland), it is difficult to draw an interpretation.

Conclusion

This study used Monte Carlo simulation to quantify uncertainty in shower-related carbon emissions by incorporating variability in shower duration, heating systems, and water sourcing methods. Shower durations follow a right-skewed log-normal distribution centered around 10 minutes, reflecting realistic day-to-day variability.

Heat-related CO_2 emissions are heavily concentrated near 0 kg when low-carbon energy sources such as solar or borehole heat pumps are used. Higher emissions (0.8–2.5 kg) are predominantly driven by gas and oil heating, identifying fossil-based systems as the main contributors to elevated carbon output.

Water-related emissions show a similar right-skewed pattern, with most impacts near zero, suggesting that either water treatment processes have a low carbon footprint or water quality is already sufficient to minimize treatment needs. Higher-emission cases are largely attributable to seawater reverse osmosis, which requires substantially more energy.

Overall, the findings highlight that most shower-related emissions in Switzerland remain low, although certain heating and water treatment systems can lead to significantly higher emissions.

References

Rydh, A., & Sandén, B. (2002). Environmental assessment of energy systems: A comparison of electricity generation technologies. *Journal of Cleaner Production, 10*, 409–421. https://doi.org/10.1016/S0959-6526(02)00028-8

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