

National and Kapodistrian University of Athens,  
Panepistimiou 30, Athens, Greece, 106-79.

# Partial Differential Equations Conference

Programme &  
Book of Abstracts



10-12 of June, 2025



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# DAY 1

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## Tuesday 10<sup>th</sup> of June, 2025

WELCOME: 09:15-09:30

### 1.1 Front propagation through a perforated wall (Hiroshi Matano)

**Time:** 09:30-10:25

**Location:** Amph. Argiriadis, NKUA, Historical Building, Panepistimiou 30, Athens, 106-79

**Speaker:** Hiroshi Matano (MEIJI INSTITUTE FOR ADVANCED STUDY OF MATHEMATICAL SCIENCES)

**Abstract:** In recent years, the behavior of solution fronts of reaction-diffusion equations in the presence of obstacles has attracted attention among many researchers.

In this talk, I will consider the case where the obstacle is a wall with many holes and discuss whether the front can pass through the wall and continue to propagate (“propagation”) or is blocked by the wall (“blocking”). The answer depends largely on the size and the geometric configuration of the holes.

This problem has led to a variety of interesting mathematical questions that are far richer than we had originally anticipated. Many questions still remain open. This is joint work with Henri Berestycki and Francois Hamel.

### 1.2 Singular harmonic maps into singular manifolds (Fabrice Bethuel)

**Time:** 10:30-11:25

**Location:** Amph. Argiriadis, NKUA, Historical Building, Panepistimiou 30, Athens, 106-79

**Speaker:** Fabrice Bethuel (SORBONNE UNIVERSITY)

**Abstract:** The purpose of the talk is to present a few problems and some results concerning the existence and properties of maps from a puncture plane with values in a one-dimensional singular manifold. A typical example is given by the ”eight-shaped”

manifold, which will be central in the talk. I will present some new results obtained in the thesis of Mehdi Trencse, in particular how they convert the problems as questions on Riemann surfaces.

COFFEE BREAK: 11:30-12:00

### **1.3 On propagation of electrical pulses in neurons (Michael Sigal)**

**Time:** 12:00-12:55

**Location:** Amph. Argiriadis, NKUA, Historical Building, Panepistimiou 30, Athens, 106-79

**Speaker:** Michael Sigal (UNIVERSITY OF TORONTO)

**Abstract:** Alongside the Nobel prize winning Hodgkin-Huxley system (HHS), the FitzHugh-Nagumo (FHN) one is at the foundation of quantitative neuroscience, giving a qualitatively, and often quantitatively, faithful description of the propagation of electrical impulses (pulses) in neurons.

Though pulses propagate on a surface of neural axons which are cylindrical surfaces of a complicated geometry, in computations and theoretical work, the latter are modelled by the zero thickness infinite straight line.

In this talk I will describe the recent mathematical results on propagation of pulses in a more realistic model of neural axons as cylindrical surfaces of variable radii. The talk is based on the recent joint work with Afroditi Talidou and Almut Burchard and with Georgia Karali and Kostas Tzirakis

LAUNCH BREAK: 13:00-14:30

### **1.4 Overhanging solitary waves in a fluid with vorticity (Juan Dávila)**

**Time:** 14:30-15:25

**Location:** Amph. Argiriadis, NKUA, Historical Building, Panepistimiou 30, Athens, 106-79

**Speaker:** Juan Dávila (UNIVERSITY OF BATH)

**Abstract:** We obtain solutions for an overdetermined elliptic problem, yielding solitary waves, which are non-graphical, in a fluid at constant vorticity. Despite numerical evidence for their existence, the construction of these nearly singular solutions proves challenging using complex variables or bifurcation theory. Our approach is reminiscent of the desingularization process applied to constant mean curvature surfaces. This is work in with Manuel del Pino, Monica Musso, and Miles Wheeler from the University of Bath.

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## 1.5 Triple Junction Solution for the Allen-Cahn System (Zhiyuan Geng)

**Time:** 15:30-16:25

**Location:** Amph. Argiriadis, NKUA, Historical Building, Panepistimiou 30, Athens, 106-79

**Speaker:** Zhiyuan Geng (PURDUE UNIVERSITY)

**Abstract:** In this talk, we will discuss recent results on the 2D Allen-Cahn system with a triple-well potential. By studying the blow-up limit of solutions near the junction of three phases, we construct an entire minimizing solution that asymptotically converges at infinity to a unique triple junction, corresponding to a planar minimal cone. We further establish the almost 1D symmetry of the solution along the sharp interface. A key estimate in our analysis is the sharp energy lower and upper bounds, which enable the localization of the diffuse interface within a small neighborhood of the limiting interface. The results don't rely on any symmetry assumptions.

This is joint work with Nicholas Alikakos.

## 1.6 The nonlinear p-curl-curl problem (Jarosław Mederski)

**Time:** 16:30-17:25

**Location:** Amph. Argiriadis, NKUA, Historical Building, Panepistimiou 30, Athens, 106-79

**Speaker:** Jarosław Mederski (POLISH ACADEMY OF SCIENCES)

**Abstract:** Nonlinear curl-curl problems have recently emerged in the study of exact electromagnetic wave propagation in nonlinear media modeled by Maxwell's equations. For example, the quintic effect leads to a critical partial differential equation involving the curl-curl operator. Ground state solutions of this problem are related to the optimizers of a new Sobolev-type inequality. We present recent results concerning the existence of ground state solutions and discuss certain symmetry properties of the problem. Applications to zero modes of the Dirac equations will also be considered.

END OF DAY 1



# DAY 2

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## Wednesday 11<sup>th</sup> of June, 2025

### 2.1 Minimality of level sets in phase transitions (Dimitrios Gazoulis )

**Time:** 10:30-11:00

**Location:** Amph. Argiriadis, NKUA, Historical Building, Panepistimiou 30, Athens, 106-79

**Speaker:** Dimitrios Gazoulis (NKUA)

**Abstract:** In this talk we study the level sets of solutions of the Allen-Cahn equation and we prove local minimality of the zero level set with respect to certain perimeter functional with density. This provides a direct relationship between phase transition type problems and minimal surfaces with some weight. In particular, we establish that if  $u$  is a solution to the Allen-Cahn equation that satisfy  $u_{x_n} > 0$  and  $\lim_{x_n \rightarrow \pm\infty} u(x', x_n) = \pm 1$ , then the zero level set of locally minimizes a perimeter type functional. As an application, we establish the De Giorgi conjecture, proved by O. Savin, by reducing it to a Bernstein type result for anisotropic perimeter functionals obtained by L. Simon, thus directly linking it to the geometric problem.

### 2.2 Ground State of Some Variational Problems in Hilbert Spaces and Applications to PDEs (Ioannis Arkoudis)

**Time:** 11:00-11:30

**Location:** Amph. Argiriadis, NKUA, Historical Building, Panepistimiou 30, Athens, 106-79

**Speaker:** Ioannis Arkoudis (NKUA)

**Abstract:** We prove the existence of a ground state for some variational problems in Hilbert spaces, following the approach of Berestycki and Lions. Next, we examine the problem of constructing ground state solutions  $u : \mathbb{R}^{d+k} \rightarrow \mathbb{R}^m$  of the system  $\Delta u(x) = \nabla W(u(x))$  (with  $W : \mathbb{R}^m \rightarrow \mathbb{R}$ ), corresponding to some nontrivial stable solutions  $e : \mathbb{R}^k \rightarrow \mathbb{R}^m$ . The method we propose is based on a reduction to a ground state problem in a space of functions  $\mathcal{H}$ , where  $e$  is viewed as a local minimum of an

effective potential defined in  $\mathcal{H}$ . As an application, by considering a heteroclinic orbit  $e : \mathbb{R} \rightarrow \mathbb{R}^m$ , we obtain nontrivial solutions  $u : \mathbb{R}^{d+1} \rightarrow \mathbb{R}^m$  ( $d \geq 2$ ), converging asymptotically to  $e$ , which can be seen as the homoclinic analogs of the heteroclinic double layers, initially constructed by Alama-Bronsard-Gui and Schatzman.

COFFEE BREAK: 11:30-12:00

## **2.3 On a classification of steady solutions to two-dimensional Euler equations (Changfeng Gui)**

**Time:** 12:00-12:55

**Location:** Amph. Argiriadis, NKUA, Historical Building, Panepistimiou 30, Athens, 106-79

**Speaker:** Changfeng Gui (UNIVERSITY OF MACAU and ZHUHAI UM SCIENCE AND TECHNOLOGY RESEARCH INSTITUTE)

**Abstract:** In this talk, I shall provide a classification of steady solutions to two-dimensional incompressible Euler equations in terms of the set of flow angles. The first main result asserts that the set of flow angles of any bounded steady flow in the whole plane must be the whole circle unless the flow is a parallel shear flow. In an infinitely long horizontal strip or the upper half-plane supplemented with slip boundary conditions, besides the two types of flows appeared in the whole space case, there exists an additional class of steady flows for which the set of flow angles is either the upper or lower closed semicircles. This type of flows is proved to be the class of non-shear flows that have the least total curvature. A further classification of this type of solutions will also be discussed. As consequences, we obtain Liouville-type theorems for two-dimensional semilinear elliptic equations with only bounded and measurable non-linearity, and the structural stability of shear flows whose all stagnation points are not inflection points, including Poiseuille flow as a special case. Our proof relies on the analysis of some quantities related to the curvature of the streamlines.

This talk is based on joint works with David Ruiz, Chunjing Xie and Huan Xu.

LAUNCH BREAK: 13:00-14:30

## **2.4 Particles almost in contact: the method of reduced functionals in 3D (Denis Bonheure)**

**Time:** 14:30-15:25

**Location:** Amph. Argiriadis, NKUA, Historical Building, Panepistimiou 30, Athens, 106-79

**Speaker:** Denis Bonheure (UNIVERSITÉ LIBRE DE BRUXELLES)

**Abstract:** The formal analysis of the forces and torques on two moving solid particles suspended in a laminar flow and almost in contact with each other (or on a particle almost in contact with the wall of a container) traces back at least to Brenner and Cox in the late 1960's by using lubrication theory. While the stream function is defined up

to a constant in 2D, the vector potential in 3D is defined up to a gradient and the choice of a gauge. I will show that by choosing an ad-hoc gauge, one can find the optimal potential by solving the dual formulation of a resulting Euler-Lagrange equation. This allows to compute (and fully justify) the asymptotic expansion of any Stokes solution when inclusions are close to isolated contacts. As a byproduct, we can derive the Stokes resistance matrix for a cloud of particles almost in contact. The construction is fully variational while the sharp asymptotics are basically based on estimates for a weighted elliptic operator in divergence form. I will start the talk by explaining the method on the easier problem of estimating the relative capacity of sets close to contact and showing a link with a missing weighted Hardy inequality.

The talk is based on a joint project with E. Bocchi (Pol. Milano) and M. Hillairet (Univ. Montpellier).

## 2.5 Normalised solutions to a fractional Schrödinger equation in the strongly sublinear regime (Jacopo Schino)

**Time:** 15:30-16:25

**Location:** Amph. Argiriadis, NKUA, Historical Building, Panepistimiou 30, Athens, 106-79

**Speaker:** Jacopo Schino (UNIVERSITY OF WARSAW)

**Abstract:** Schrödinger-type equations model a lot of natural phenomena and their solutions have interesting and important properties: one of them is the conservation of mass, which gives rise to the search for normalised solutions. In this talk, I will explain a possible approach to solve

$$\begin{cases} (-\Delta)^s u + \mu u = g(u) \\ \int_{\mathbb{R}^N} u^2 dx = m \\ (\mu, u) \in \mathbb{R} \times H^s(\mathbb{R}^N), \end{cases}$$

where  $N \geq 2$ ,  $0 < s < 1$ , and  $m > 0$  is prescribed, in cases that include the so-called strongly sublinear regime:

$$\lim_{t \rightarrow 0} \frac{g(t)}{t} = \infty \quad (2.1)$$

This makes a direct variational approach impossible because the energy functional is not well-defined in  $H^s(\mathbb{R}^N)$ . In the proposed approach, when  $m$  is sufficiently large, a family of approximating problems is considered so that the energy functional is of class  $\mathcal{C}^1$  and a corresponding family of solutions is obtained, which eventually converge to a solution to the original problem. When (2.1) holds, the previous result for a suitably translated problem is exploited to obtain a solution for any  $m > 0$ .

This is joint work with Marco Gallo (Catholic University of the Sacred Heart, Brescia, Italy).

DINNER: 20:00

END OF DAY 2



# DAY 3

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## Thursday 12<sup>th</sup> of June, 2025

### 3.1 Harmonic measure and Green function estimates for the Robin boundary value problem in rough domains (Guy David)

**Time:** 09:30-10:25

**Location:** Amph. Argiriadis, NKUA, Historical Building, Panepistimiou 30, Athens, 106-79

**Speaker:** Guy David (UNIVERSITÉ PARIS-SACLAY - UNIVERSITÉ DE PARIS SUD (ORSAY))

**Abstract:** The main point is to study the (mutual, quantitative) absolute continuity, with respect to a reference measure on the boundary (like surface or Hausdorff measure), of the Robin harmonic (or elliptic) measure in a domain, for solutions of elliptic divergence form equations, i.e. with the Robin boundary condition  $\partial_n u + au = f$  at the boundary (instead of the usual Dirichlet condition  $u = f$ ). Here  $\partial_n u$  is the normal derivative of a solution  $u$ . This is a very reasonable condition for application, with features of the Dirichlet and Neumann conditions (we should mention the lung). We also estimate the Green function.

Joint work with S. Decio, M. Engelstein, S. Mayboroda, M. Michetti, M. Filoche.

### 3.2 Liouville theorem for the one dimensional Gross-Pitaevskii equation (Michał Kowalczyk)

**Time:** 10:30-11:25

**Location:** Amph. Argiriadis, NKUA, Historical Building, Panepistimiou 30, Athens, 106-79

**Speaker:** Michał Kowalczyk (UNIVERSITY OF CHILE, SANTIAGO)

**Abstract:** The asymptotic stability of the black and dark solitons of the one-dimensional Gross-Pitaevskii equation was proved by Béthuel, Gravejat and Smets and Gravejat and Smets, using a rigidity property in the vicinity of solitons. We provide an alternate proof of their Liouville theorems using a factorization identity for the linearized operator which trivializes the spectral analysis.

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COFFEE BREAK: 11:30-12:00

### **3.3 Aronson-Bénilan estimates for the parabolic-elliptic Keller-Segel model (Filippo Santambrogio)**

**Time:** 12:00-12:55

**Location:** Amph. Argiriadis, NKUA, Historical Building, Panepistimiou 30, Athens, 106-79

**Speaker:** Filippo Santambrogio (UNIVERSITÉ CLAUDE BERNARD LYON 1)

**Abstract:** This talk, based on a joint work (still to be finished) with Charles Elbar (Lyon) and Alejandro Fernandez Jimenez (Oxford), lies at the intersection of two well-known phenomena in parabolic equations. The first concerns nonlinear estimates on the second derivatives of solutions to diffusion equations: by looking at the PDE satisfied by the Laplacian of the logarithm of the solution of the heat equation (or of suitable powers of the solution for non-linear diffusion such as porous media equations) one can obtain lower bounds in the form of  $\Delta(\log \rho) \geq -C/t$ . The second, instead, concerns the critical mass in the parabolic-elliptic Keller-Segel chemotactic system where linear diffusion is coupled with advection by a potential generated by the convolution of the solution with the Poisson Kernel: it is well-known for this nonlinear equation that explosion in finite time or global existence depends on the mass (which is preserved in time) and the best estimates are obtained for small mass. In the talk I will show how, under small mass assumptions, it is also possible to obtain estimates, with instantaneous regularization, on the Laplacian of the pressure for the critical case, i.e. for the case of linear diffusion in 2D where the pressure is logarithmic or in power-case in higher dimension. The proofs I will show will mainly focus on the 2D case, easier to explain.

LAUNCH BREAK: 13:00-14:30

### **3.4 Local minimisers in higher order Calculus of Variations in $L^\infty$ : existence, uniqueness and characterisation (Nikos Katzourakis)**

**Time:** 14:30-15:25

**Location:** Amph. Argiriadis, NKUA, Historical Building, Panepistimiou 30, Athens, 106-79

**Speaker:** Nikos Katzourakis (UNIVERSITY OF READING)

**Abstract:** Higher order problems are very novel in the Calculus of Variations in  $L^\infty$ , and exhibit a strikingly different behaviour compared to first order problems, for which there exists an established theory, pioneered by Aronsson in 1960s. In this talk I will discuss how a complete theory can be developed for second order functionals. Under appropriate conditions, “localised” minimisers can be characterised as solutions to a nonlinear system of PDEs, which is different from the corresponding Aronsson equation; the latter is only a necessary, but not a sufficient condition for minimality. I will also discuss the existence and uniqueness of localised minimisers subject to Dirichlet

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boundary conditions, and also their partial regularity outside a singular set of codimension one, which may be non-empty even in 1D. The talk will not assume any previous knowledge on the topic.

### **3.5 Semilinear approximations of quasilinear parabolic equations (Tomasz Dlotko)**

**Time:** 15:30-16:25

**Location:** Amph. Argiriadis, NKUA, Historical Building, Panepistimiou 30, Athens, 106-79

**Speaker:** Tomasz Dlotko (UNIVERSITY OF SILESIA IN KATOWICE)

**Abstract:** An approach to solvability of certain quasilinear parabolic equations will be presented by approximating the quasilinear equation under consideration with a parameter family of semilinear problems with stronger linear fractional diffusion term. Defined on arbitrarily long time intervals, solutions to the original problem are found as a suitable limit of global solutions to those semilinear approximations. The method is applied to the celebrated Navier-Stokes equations in 3D, nonlinear parabolic Kirchhoff equation and to critical 2D surface Quasi-geostrophic equation with Dirichlet boundary conditions.

The above approach was presented in the recent publication:

- R. Czaja, T. Dlotko, Semilinear approximations of quasilinear parabolic equations with applications, *Math. Methods. Appl. Sci.* 48 vol. 1 (2025), 435-462.

END OF DAY 3

