Strip Telescope Alignment

1 Gemoetry

The coordinate system used to describe the sensor orientation has the unit vector $\hat{u}$ parallel to the strips and directed away from the ROC's. The unit vector $\hat{v}$ is in the plane of the sensor, perpendicular to the strips and pointing in the direction of increasing strip number. The unit vector $\hat{n}$ is normal to the surface of the sensor as shown in Figure 1. The nominal center of the sensor is located on strip 320.

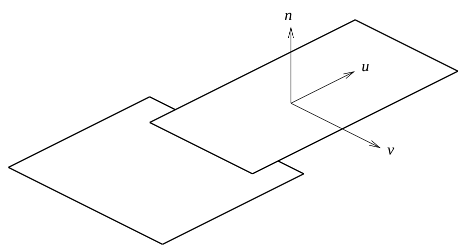

Figure 1: Coordinate system used to define the position and orientation of a sensor.

The telescope system uses a coordinate system in which the z-axis is in the direction of the beam, the y-axis points up and the x-axis is in the direction that forms a right-handed coordiate system. When looking at the telescope along the direction of the x-axis, the beam will traverse the telescope from left to right.

For this analysis, the sensors in the telescope are numbered 0..13 with even numbered sensors oriented with $\hat{u}$ vertical and with $\hat{n}$ pointing upstream and with odd-numbered sensors oriented with $\hat{u}$ horizontal and $\hat{n}$ facing in the direction of the beam.

2 Global Alignment

The first step in the alignment procedure is to use the average beam envelope to define the direction of the z-axis. This is achieved by examining the distribution of cluster positions in each sensor and defining the peak of this distribution to occur at $x = y = 0$. The peak position is identified by fitting a 4th order polynomial to the distribution of cluster positions and recording the strip position that maximizes the fitted function. Examples of such fits to the first and last planes in the telescope are shown in Figure 2.

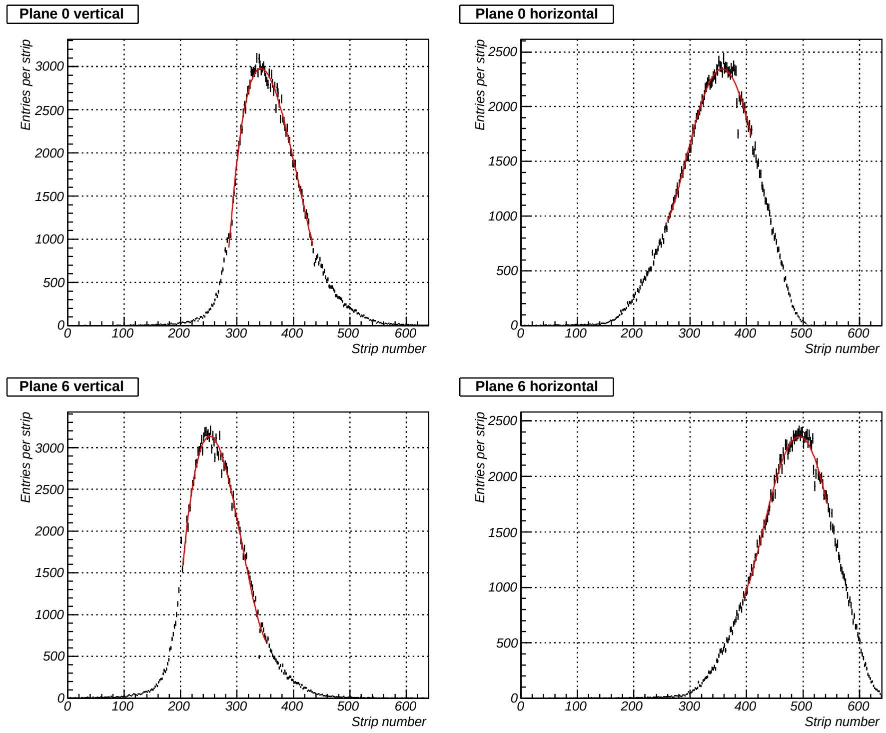

Figure 2: Fitted cluster positions in the first (plane 0) and last (plane 6) planes of the telescope.

We assume that gross misalignment at this level is due to the positioning of the upstream and downstream pieces of the telescope. This is corrected for by fitting a straight line through the upstream and downstream displacements separately to determine the orientation of each part with respect to the beam axis. Figure 3 shows the transverse displacements of each sensor before repositioning and the position of the cluster distribution maximum after realignment.

After repositioining the upstream and downstream ends of the telescope, the most probable beam position is measured again and the sensors are shifted in the $\hat{v}$ direction so that the beam coincides with $x = y = 0$ in the plane of each sensor.

3 Kalman Filter

Subsequent alignment steps rely on fitting clusters found on each sensor using a Kalman Filter. The track is parametrized by the equations

$$x(z) = x_0 + \alpha z \quad (1)$$

$$y(z) = y_0 + \beta z \quad (2)$$

and its state at plane k is represented by the vector

$$\mathbf{x}_k = \begin{pmatrix} x_k \\ y_k \\ \alpha_k \\ \beta_k \end{pmatrix}. \quad (3)$$

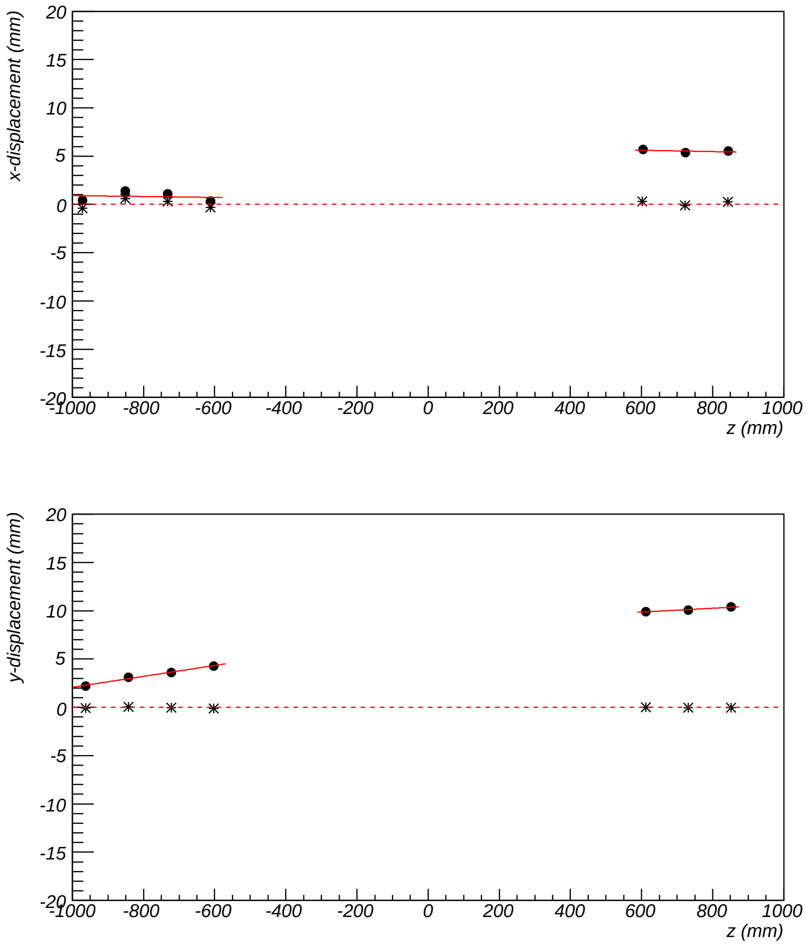

Figure 3: Transverse displacements of sensors with respect to the beam axis before (solid points) and after (asterisks) repositioning of the upstream and downstream parts of the telescope.

The position of each sensor is defined by the displacement vector $\vec{p}_k$ and the unit vectors $\hat{u}_k$, $\hat{v}_k$, and $\hat{n}_k$. The intersection point of a track with a sensor plane is determined by solving the equation

$$\hat{n}_k \cdot (\vec{p}_k - \vec{x}(z)) = 0 \tag{4}$$

for z and calculating the position of the intersected strip

$$s_k = \hat{v}_k \cdot (\vec{p}_k - \vec{x}(z)). \tag{5}$$

This equation is linearized and written in terms of the state vector as follows:

$$s_k = \mathbf{H}_k \mathbf{x}_{k,t} - d_k \tag{6}$$

The covariance matrix for the state vector which includes information all planes before k is written $\mathbf{C}_{k-1}$ and the predicted covaraince matrix for the state vector, extrapolated to plane k is written

$$\mathbf{C}_k^{k-1} = \mathbf{C}_{k-1} + \mathbf{Q}_{k-1} \tag{7}$$

where $\mathbf{C}_{k-1}$ is the previous estimate of the covariance matrix and $\mathbf{Q}_{k-1}$ is the covariance of the track parameters due to the variable describing multiple scattering in plane $k-1$. In the case where the scattering angle is Gaussian with a width parameter of ρ_{k-1},

$$\mathbf{Q}_{k-1} = \begin{pmatrix} z_{k-1}^2\rho_{k-1}^2 & 0 & -z_{k-1}\rho_{k-1}^2 & 0 \\ 0 & z_{k-1}^2\rho_{k-1}^2 & 0 & -z_{k-1}\rho_{k-1}^2 \\ -z_{k-1}\rho_{k-1}^2 & 0 & \rho_{k-1}^2 & 0 \\ 0 & -z_{k-1}\rho_{k-1}^2 & 0 & \rho_{k-1}^2 \end{pmatrix}. \tag{8}$$

The residual of a measurement of a cluster position m_k with covariance $\mathbf{V}_k = (\sigma_k^2)$ is written

$$\mathbf{r}_k^{k-1} = m_k - \mathbf{H}_k\mathbf{x}_k^{k-1} + d_k. \tag{9}$$

and the correction to the state vector after incorporating the measuremnet on plane k is

$$\Delta\mathbf{x}_k = \mathbf{K}_k\mathbf{r}_k^{k-1} \tag{10}$$

where $\mathbf{K}_k$ is the Kalman gain matrix:

$$\mathbf{K}_k = \frac{\mathbf{C}_k^{k-1}\mathbf{H}_k^T}{r} \tag{11}$$

where

$$r = \mathbf{H}_k\mathbf{C}_k^{k-1}\mathbf{H}_k^T + \sigma_k^2. \tag{12}$$

The new covariance matrix for the state vector is

$$\mathbf{C}_k = \mathbf{C}_k^{k-1} - \frac{\mathbf{C}_k^{k-1}\mathbf{B}_k\mathbf{C}_k^{k-1}}{r} \tag{13}$$

where

$$\mathbf{B_k} = \mathbf{H}_k\mathbf{H}_k^T. \tag{14}$$

Smoothed track parameters can be calculated after incorporating information from all sensors. The last state $\mathbf{x}_n$ provides the initial estimate for the smoothed parameters. Smoothed parameter estimates at other points are calculated using

$$\mathbf{x}_k^n = \mathbf{x}_k + \mathbf{A}_k(\mathbf{x}_{k+1}^n - \mathbf{x}_{k+1}^k) \tag{15}$$

where

$$\mathbf{A}_k = \mathbf{C}_k(\mathbf{C}_{k+1}^k)^{-1}. \tag{16}$$

In practice, the matrices $\mathbf{C}_k$ and $\mathbf{C}_{k+1}^k$ and the predicted state $\mathbf{x}_{k+1}^k$ will have already been calculated in the initial filtering stage of the fit. The covariance matrix for the smoothed points is calculated using

$$\mathbf{C}_k^n = \mathbf{C}_k + \mathbf{A}_k(\mathbf{C}_{k+1}^n - \mathbf{C}_{k+1}^k)\mathbf{A}_k^T. \tag{17}$$

	Sensor	Carbon fiber
x	300 μm	300 μm
ρX_0	21.8 g/cm^2	42.7 g/cm^2
ρ	2.329 g/cm^3	$\sim$ 2.5 g/cm^3
X_0	9.36 cm	17.08 cm
x/X_0	3.21×10^{-3}	1.76×10^{-3}
θ_k	5.02 μrad	3.61 μrad
$\theta_k/\sqrt{2}$	3.55 μrad	2.55 μrad
ρ_k	4.37 μrad	

Table 1: Multiple scattering parameters for material in the telescope planes.

3.1 Multiple Scattering

The width of the distribution of multiple scattering angles due to material with thickness x and radiation length X_0 is calculated using

$$\theta_k = \frac{(13.6\text{ MeV})}{\beta c p}\sqrt{x/X_0}\,(1 + 0.038\log(x/X_0)) \tag{18}$$

$$\rho_k = \theta_k/\sqrt{2} \tag{19}$$

In this case, we assume that $\beta \approx 1$ and $p = 120$ GeV/c and that multiple scattering occurs only in the sensors and in the carbon fiber sheets that cover the telescope planes, both of which are 300 μm thick. Table 1 summarizes the determination of the width of the multiple scattering angle distribution.