DAA – Assignment 1 Afraz Ul Huq (8886) Syed Hamza Haider (62610)

Prove or disprove the following using sound mathematical techniques. Briefly explain each step of your working.

1.
$$7n-2=O(n)$$

$$f(n) \le g(n) \ \forall \ n \ge k$$

$$f(n) = 7n - 2$$

$$g(n) = n$$

$$c = 7$$

$$7n = 2 \leq 7n$$

if
$$n = 1$$

$$7(1) - 2 \le 7(1)$$

$$5 \le 7 \ \forall \ n \ge 1$$

if
$$n = 2$$

$$7(2) - 2 \le 7(2)$$

$$12 \le 14 \ \forall \ n \ge 1$$

2. $7n-2=\Theta(n)$

$$C_1 g(n) \le f(n) \le C_2 g(n) \forall n \ge k$$

$$f(n) = 7n - 2$$

$$g(n) = n$$

$$C^1 = 5$$

$$C_2 = 7$$

$$5n \le 7n - 2 \le 7n \forall n \ge k$$

$$if \ n = 1$$

 $5(1) \le 7(1) - 2 \le 7(1) \ \forall \ n \ge 1$
 $5 \le 5 \le 7 \ \forall \ n \ge 1$

if
$$n = 2$$

 $5(2) \le 7(2) - 2 \le 7(2) \ \forall \ n \ge 1$
 $10 \le 12 \le 14 \ \forall \ n \ge 1$

3.
$$7n-2=\Theta(n^2)$$

$$C_1 g(n) \le f(n) \le C_2 g(n) \ \forall \ n \ge k$$

$$f(n) = 7n - 2$$

$$g(n) = n^2$$

$$C^1 = 5$$

$$C_2 = 7$$

if
$$n = 1$$

$$5n^2 \le 7n - 2 \le 7n^2$$

$$5(1)^2 \le 7(1) - 2 \le 7(1)^2$$

$$5 \le 5 \le 7$$

if
$$n = 2$$

$$5n^2 \le 7n - 2 \le 7n^2$$

$$5(2)^2 \le 7(2) - 2 \le 7(2)^2$$

$$20 \le 12 \le 28$$

4. $3n^3+20n^2+5=O(n^6)$

$$f(n) = 3n^{3} + 20n^{2} + 5$$

$$g(n) = n^{6}n^{2}$$

$$f(n) \le g(n) \ \forall \ n \ge k$$

$$3n^{3} + 20n^{2} + 5 \le n^{6} \ \forall \ n \ge k$$

if
$$n = 1$$

 $3(1)^3 + 20(1)^2 + 5$
 $3(1)^3 + 20(1)^2 + 5 \le 28(1)^6 \ \forall \ n \ge 1$
 $28 \le 28 \ \forall \ n \ge 1$

if
$$n = 2$$

 $3(2)^3 + 20(2)^2 + 5$
 $3(2)^3 + 20(2)^2 + 5 \le 28(2)^6 \,\forall \, n \ge 1$
 $109 \le 1792 \,\forall \, n \ge 1$