Exercises FOR SECTION 4-2

Problem available in *WileyPLUS* at instructor's discretion.

Go Tutorial Tutoring problem available in *WileyPLUS* at instructor's discretion.

4-1. Suppose that $f(x)=e^{-x}$ for $0<x$. Determine the following:

(a) $P(1<X)$ (b) $P(1<X<2.5)$
(c) $P(X=3)$ (d) $P(X<4)$ (e) $P(3\le X)$
(f) x such that $P(x<X)=0.10$
(g) x such that $P(X\le x)=0.10$

4-2. Suppose that $f(x)=3(8x-x^2)/256$ for $0<x<8$. Determine the following:

(a) $P(X<2)$ (b) $P(X<9)$ (c) $P(2<X<4)$
(d) $P(X>6)$ (e) x such that $P(X<x)=0.95$

4-3. Suppose that $f(x)=0.5\cos x$ for $-\pi/2<x<\pi/2$. Determine the following:

(a) $P(X<0)$ (b) $P(X<-\pi/4)$ (c) $P(-\pi/4<X<\pi/4)$
(d) $P(X>-\pi/4)$ (e) x such that $P(X<x)=0.95$

4-4. The diameter of a particle of contamination (in micrometers) is modeled with the probability density function $f(x)=2/x^3$ for $x>1$. Determine the following:

(a) $P(X<2)$ (b) $P(X>5)$ (c) $P(4<X<8)$
(d) $P(X<4 \text{ or } X>8)$ (e) x such that $P(X<x)=0.95$

4-5. Go Tutorial Suppose that $f(x)=x/8$ for $3<x<5$. Determine the following probabilities:

(a) $P(X<4)$ (b) $P(X>3.5)$ (c) $P(4<X<5)$
(d) $P(X<4.5)$ (e) $P(X<3.5 \text{ or } X>4.5)$

4-6. Suppose that $f(x)=e^{-(x-4)}$ for $4<x$. Determine the following:

(a) $P(1<X)$ (b) $P(2\le X<5)$ (c) $P(5<X)$
(d) $P(8<X<12)$ (e) x such that $P(X<x)=0.90$

4-7. Suppose that $f(x)=1.5x^2$ for $-1<x<1$. Determine the following:

(a) $P(0<X)$ (b) $P(0.5<X)$
(c) $P(-0.5\le X\le 0.5)$ (d) $P(X<-2)$
(e) $P(X<0 \text{ or } X>-0.5)$ (f) x such that $P(x<X)=0.05$.

4-8. The probability density function of the time to failure of an electronic component in a copier (in hours) is $f(x)=e^{-x/1000}/1000$ for $x>0$. Determine the probability that

(a) A component lasts more than 3000 hours before failure.
(b) A component fails in the interval from 1000 to 2000 hours.
(c) A component fails before 1000 hours.
(d) The number of hours at which 10% of all components have failed.

4-9. The probability density function of the net weight in pounds of a packaged chemical herbicide is $f(x)=2.0$ for $49.75<x<50.25$ pounds.

(a) Determine the probability that a package weighs more than 50 pounds.
(b) How much chemical is contained in 90% of all packages?

4-10. The probability density function of the length of a cutting blade is $f(x)=1.25$ for $74.6<x<75.4$ millimeters. Determine the following:

(a) $P(X<74.8)$ (b) $P(X<74.8 \text{ or } X>75.2)$
(c) If the specifications for this process are from 74.7 to 75.3 millimeters, what proportion of blades meets specifications?

4-11. The probability density function of the length of a metal rod is $f(x)=2$ for $2.3<x<2.8$ meters.

(a) If the specifications for this process are from 2.25 to 2.75 meters, what proportion of rods fail to meet the specifications?
(b) Assume that the probability density function is $f(x)=2$ for an interval of length 0.5 meters. Over what value should the density be centered to achieve the greatest proportion of rods within specifications?

4-12. An article in Electric Power Systems Research ["Modeling Real-Time Balancing Power Demands in Wind Power Systems Using Stochastic Differential Equations" (2010, Vol. 80(8), pp. 966–974)] considered a new probabilistic model to balance power demand with large amounts of wind power. In this model, the power loss from shutdowns is assumed to have a triangular distribution with probability density function

$$f(x)=\begin{cases} -5.56\times10^{-4}+5.56\times10^{-6}x, & x\in[100,500] \\ 4.44\times10^{-3}-4.44\times10^{-6}x, & x\in[500,1000] \\ 0, & \text{otherwise} \end{cases}$$

Determine the following:

(a) $P(X<90)$ (b) $P(100<X\le 200)$
(c) $P(X>800)$ (d) Value exceeded with probability 0.1.

4-13. A test instrument needs to be calibrated periodically to prevent measurement errors. After some time of use without calibration, it is known that the probability density function of the measurement error is $f(x)=1-0.5x$ for $0<x<2$ millimeters.

(a) If the measurement error within 0.5 millimeters is acceptable, what is the probability that the error is not acceptable before calibration?
(b) What is the value of measurement error exceeded with probability 0.2 before calibration?
(c) What is the probability that the measurement error is exactly 0.22 millimeters before calibration?

4-14. The distribution of X is approximated with a triangular probability density function $f(x)=0.025x-0.0375$ for $30<x<50$ and $f(x)=-0.025x+0.0875$ for $50<x<70$. Determine the following:

(a) $P(X\le 40)$ (b) $P(40<X\le 60)$
(c) Value x exceeded with probability 0.99.

4-15. The waiting time for service at a hospital emergency department (in hours) follows a distribution with probability density function $f(x)=0.5\exp(-0.5x)$ for $0<x$. Determine the following:

(a) $P(X<0.5)$ (b) $P(X>2)$
(c) Value x (in hours) exceeded with probability 0.05.

4-16. If X is a continuous random variable, argue that $P(x_1\le X\le x_2)=P(x_1<X\le x_2)=P(x_1\le X<x_2)=P(x_1<X<x_2)$.

4-3 Cumulative Distribution Functions

An alternative method to describe the distribution of a discrete random variable can also be used for continuous random variables.

Cumulative Distribution Function

The **cumulative distribution function** of a continuous random variable X is

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)\, du \tag{4-3}$$

for $-\infty < x < \infty$.

The cumulative distribution function is defined for all real numbers. The following example illustrates the definition.

Example 4-3

Electric Current For the copper current measurement in Example 4-1, the cumulative distribution function of the random variable X consists of three expressions. If $x < 4.9$, $f(x) = 0$. Therefore,

$$F(x) = 0, \quad \text{for} \quad x < 4.9$$

and

$$F(x) = \int_{4.9}^{x} f(u)\, du = 5x - 24.5, \quad \text{for} \quad 4.9 \le x < 5.1$$

Finally,

$$F(x) = \int_{4.9}^{x} f(u)\, du = 1, \quad \text{for} \quad 5.1 \le x$$

Therefore,

$$F(x) = \begin{cases} 0 & x < 4.9 \\ 5x - 24.5 & 4.9 \le x < 5.1 \\ 1 & 5.1 \le x \end{cases}$$

The plot of $F(x)$ is shown in Fig. 4-6.

Notice that in the definition of $F(x)$, any $<$ can be changed to $\le$ and vice versa. That is, in Example 4-3 $F(x)$ can be defined as either $5x - 24.5$ or 0 at the end-point $x = 4.9$, and $F(x)$ can be defined as either $5x - 24.5$ or 1 at the end-point $x = 5.1$. In other words, $F(x)$ is a continuous function. For a discrete random variable, $F(x)$ is not a continuous function. Sometimes a continuous random variable is defined as one that has a continuous cumulative distribution function.

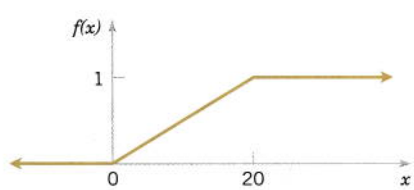

FIGURE 4-6 Cumulative distribution function for Example 4-3.

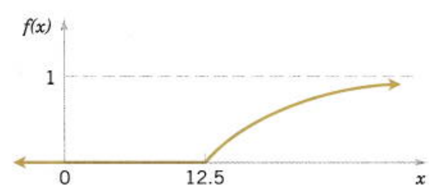

FIGURE 4-7 Cumulative distribution function for Example 4-4.

The equivalence of the two formulas for variance can be derived from the same approach used for discrete random variables.

Example 4-6 **Electric Current** For the copper current measurement in Example 4-1, the mean of X is

$$E(X)=\int_{4.9}^{5.1} xf(x)\,dx = 5x^2/2\Big|_{4.9}^{5.1}=5$$

The variance of X is

$$V(X)=\int_{4.9}^{5.1}(x-10)^2 f(x)\,dx = 5(x-10)^3/3\Big|_{4.9}^{5.1}=0.0033$$

The **expected value of a function** $h(X)$ of a continuous random variable is also defined in a straightforward manner.

Expected Value of a Function of a Continuous Random Variable

If X is a continuous random variable with probability density function $f(x)$,

$$E\left[h(X)\right]=\int_{-\infty}^{\infty} h(x)f(x)dx \tag{4-5}$$

In the special case that $h(X)=aX+b$ for any constants a and b, $E\left[h(X)\right]=aE(X)+b$. This can be shown from the properties of integrals.

Example 4-7 In Example 4-1, X is the current measured in milliamperes. What is the expected value of power when the resistance is 100 ohms? Use the result that power in watts $P=10^{-6}RI^2$, where I is the current in milliamperes and R is the resistance in ohms. Now, $h(X)=10^{-6}100X^2$. Therefore,

$$E\left[h(X)\right]=10^{-4}\int_{4.9}^{5.1}(x)^2dx=0.0001\frac{x^3}{3}\Big|_{4.9}^{5.1}=0.00050\text{ watts}$$

Example 4-8 **Hole Diameter** For the drilling operation in Example 4-2, the mean of x is

$$E(X)=\int_{12.5}^{\infty} xf(x)\,dx=\int_{12.5}^{\infty} x\,20e^{-20(x-12.5)}\,dx$$

Integration by parts can be used to show that

$$E(X)=-xe^{-20(x-12.5)}-\frac{e^{-20(x-12.5)}}{20}\Big|_{12.5}^{\infty}=12.5+0.05=12.55$$

The variance of X is

$$V(X)=\int_{12.5}^{\infty}(x-12.55)^2 f(x)dx$$

Although more difficult, integration by parts can be used twice to show that $V(X)=0.0025$ and $\sigma=0.05$.

Practical Interpretation: The scrap limit at 12.60 mm is only 1 standard deviation greater than the mean. This is generally a warning that the scrap may be unacceptably high.

Exercises FOR SECTION 4-4

⊕ Problem available in *WileyPLUS* at instructor's discretion.
⊕ Go Tutorial Tutoring problem available in *WileyPLUS* at instructor's discretion.

4-35. ⊕ Suppose that $f(x)=0.25$ for $0<x<4$. Determine the mean and variance of X.

4-36. ⊕ Suppose that $f(x)=0.125x$ for $0<x<4$. Determine the mean and variance of X.

4-37. ⊕ Suppose that $f(x)=1.5x^2$ for $-1<x<1$. Determine the mean and variance of X.

4-38. ⊕ Suppose that $f(x)=x/8$ for $3<x<5$. Determine the mean and variance of x.

4-39. Determine the mean and variance of the random variable in Exercise 4-1.

4-40. Determine the mean and variance of the random variable in Exercise 4-2.

4.41 Determine the mean and variance of the random variable in Exercise 4-13.

4.42 Determine the mean and variance of the random variable in Exercise 4-14.

4.43 Determine the mean and variance of the random variable in Exercise 4-15.

4.44 Determine the mean and variance of the random variable in Exercise 4-16.

4-45. ⊕ Suppose that contamination particle size (in micrometers) can be modeled as $f(x)=2x^{-3}$ for $1<x$. Determine the mean of X. What can you conclude about the variance of X?

4-46. ⊕ Suppose that the probability density function of the length of computer cables is $f(x)=0.1$ from 1200 to 1210 millimeters.

(a) Determine the mean and standard deviation of the cable length.

(b) If the length specifications are $1195<x<1205$ millimeters, what proportion of cables is within specifications?

4-47. ⊕ The thickness of a conductive coating in micrometers has a density function of $600x^{-2}$ for $100\ \mu\text{m}<x<120\ \mu\text{m}$.

(a) Determine the mean and variance of the coating thickness.

(b) If the coating costs \$0.50 per micrometer of thickness on each part, what is the average cost of the coating per part?

4-48. ⊕ The probability density function of the weight of packages delivered by a post office is $f(x)=70/(69x^2)$ for $1<x<70$ pounds.

(a) Determine the mean and variance of weight.

(b) If the shipping cost is \$2.50 per pound, what is the average shipping cost of a package?

(c) Determine the probability that the weight of a package exceeds 50 pounds.

4-49. ⊕ Integration by parts is required. The probability density function for the diameter of a drilled hole in millimeters is $10e^{-10(x-5)}$ for $x>5$ mm. Although the target diameter is 5 millimeters, vibrations, tool wear, and other nuisances produce diameters greater than 5 millimeters.

(a) Determine the mean and variance of the diameter of the holes.

(b) Determine the probability that a diameter exceeds 5.1 millimeters.

4-5 Continuous Uniform Distribution

The simplest continuous distribution is analogous to its discrete counterpart.

Continuous Uniform Distribution

A continuous random variable X with probability density function

$$f(x)=1(b-a), \qquad a\le x\le b \tag{4-6}$$

is a **continuous uniform random variable.**

The probability density function of a continuous uniform random variable is shown in Fig. 4-8. The mean of the continuous uniform random variable X is

$$E(X)=\int_a^b \frac{x}{b-a}\,dx=\frac{0.5x^2}{b-a}\Big|_a^b=\frac{(a+b)}{2}$$

The variance of X is

$$V(X)=\int_a^b \frac{\left(x-\frac{(a+b)}{2}\right)^2}{b-a}\,dx=\frac{\left(x-\frac{(a+b)}{2}\right)^3}{3(b-a)}\Bigg|_a^b=\frac{(b-a)^2}{12}$$

(b) What is the probability that a line width is between 0.47 and 0.63 micrometer?

(c) The line width of 90% of samples is below what value?

4-76. The fill volume of an automated filling machine used for filling cans of carbonated beverage is normally distributed with a mean of 12.4 fluid ounces and a standard deviation of 0.1 fluid ounce.

(a) What is the probability that a fill volume is less than 12 fluid ounces?

(b) If all cans less than 12.1 or more than 12.6 ounces are scrapped, what proportion of cans is scrapped?

(c) Determine specifications that are symmetric about the mean that include 99% of all cans.

4-77. In the previous exercise, suppose that the mean of the filling operation can be adjusted easily, but the standard deviation remains at 0.1 fluid ounce.

(a) At what value should the mean be set so that 99.9% of all cans exceed 12 fluid ounces?

(b) At what value should the mean be set so that 99.9% of all cans exceed 12 fluid ounces if the standard deviation can be reduced to 0.05 fluid ounce?

4-78. A driver's reaction time to visual stimulus is normally distributed with a mean of 0.4 seconds and a standard deviation of 0.05 seconds.

(a) What is the probability that a reaction requires more than 0.5 seconds?

(b) What is the probability that a reaction requires between 0.4 and 0.5 seconds?

(c) What reaction time is exceeded 90% of the time?

4-79. The speed of a file transfer from a server on campus to a personal computer at a student's home on a weekday evening is normally distributed with a mean of 60 kilobits per second and a standard deviation of four kilobits per second.

(a) What is the probability that the file will transfer at a speed of 70 kilobits per second or more?

(b) What is the probability that the file will transfer at a speed of less than 58 kilobits per second?

(c) If the file is one megabyte, what is the average time it will take to transfer the file? (Assume eight bits per byte.)

4-80. In 2002, the average height of a woman aged 20–74 years was 64 inches with an increase of approximately 1 inch from 1960 (http://usgovinfo.about.com/od/healthcare). Suppose the height of a woman is normally distributed with a standard deviation of two inches.

(a) What is the probability that a randomly selected woman in this population is between 58 inches and 70 inches?

(b) What are the quartiles of this distribution?

(c) Determine the height that is symmetric about the mean that includes 90% of this population.

(d) What is the probability that five women selected at random from this population all exceed 68 inches?

4-81. In an accelerator center, an experiment needs a 1.41-cm-thick aluminum cylinder (http://puhep1.princeton.edu/mumu/target/Solenoid_Coil.pdf). Suppose that the thickness of a cylinder has a normal distribution with a mean of 1.41 cm and a standard deviation of 0.01 cm.

(a) What is the probability that a thickness is greater than 1.42 cm?

(b) What thickness is exceeded by 95% of the samples?

(c) If the specifications require that the thickness is between 1.39 cm and 1.43 cm, what proportion of the samples meets specifications?

4-82. **Go Tutorial** The demand for water use in Phoenix in 2003 hit a high of about 442 million gallons per day on June 27 (http://phoenix.gov/WATER/wtrfacts.html). Water use in the summer is normally distributed with a mean of 310 million gallons per day and a standard deviation of 45 million gallons per day. City reservoirs have a combined storage capacity of nearly 350 million gallons.

(a) What is the probability that a day requires more water than is stored in city reservoirs?

(b) What reservoir capacity is needed so that the probability that it is exceeded is 1%?

(c) What amount of water use is exceeded with 95% probability?

(d) Water is provided to approximately 1.4 million people. What is the mean daily consumption per person at which the probability that the demand exceeds the current reservoir capacity is 1%? Assume that the standard deviation of demand remains the same.

4-83. The life of a semiconductor laser at a constant power is normally distributed with a mean of 7000 hours and a standard deviation of 600 hours.

(a) What is the probability that a laser fails before 5000 hours?

(b) What is the life in hours that 95% of the lasers exceed?

(c) If three lasers are used in a product and they are assumed to fail independently, what is the probability that all three are still operating after 7000 hours?

4-84. The diameter of the dot produced by a printer is normally distributed with a mean diameter of 0.002 inch and a standard deviation of 0.0004 inch.

(a) What is the probability that the diameter of a dot exceeds 0.0026?

(b) What is the probability that a diameter is between 0.0014 and 0.0026?

(c) What standard deviation of diameters is needed so that the probability in part (b) is 0.995?

4-85. The weight of a sophisticated running shoe is normally distributed with a mean of 12 ounces and a standard deviation of 0.5 ounce.

(a) What is the probability that a shoe weighs more than 13 ounces?

(b) What must the standard deviation of weight be in order for the company to state that 99.9% of its shoes weighs less than 13 ounces?

(c) If the standard deviation remains at 0.5 ounce, what must the mean weight be for the company to state that 99.9% of its shoes weighs less than 13 ounces?

4-86. Measurement error that is normally distributed with a mean of 0 and a standard deviation of 0.5 gram is added to the true weight of a sample. Then the measurement is rounded to the nearest gram. Suppose that the true weight of a sample is 165.5 grams.

(a) What is the probability that the rounded result is 167 grams?

(b) What is the probability that the rounded result is 167 grams or more?

4-87. Assume that a random variable is normally distributed with a mean of 24 and a standard deviation of 2. Consider an interval of length one unit that starts at the value a so that the interval is$[a, a + 1]$. For what value of a is the probability of the interval greatest? Does the standard deviation affect that choice of interval?

4-88. A study by Bechtel et al., 2009, described in the *Archives of Environmental & Occupational Health* considered polycyclic aromatic hydrocarbons and immune system function in beef cattle. Some cattle were near major oil- and gas-producing areas of western Canada. The mean monthly exposure to PM1.0 (particulate matter that is $< 1\ \mu m$ in diameter) was approximately 7.1 $\mu g/m^3$with standard deviation 1.5. Assume that the monthly exposure is normally distributed.

(a) What is the probability of a monthly exposure greater than 9 $\mu g/m^3$?

(b) What is the probability of a monthly exposure between 3 and 8 $\mu g/m^3$?

(c) What is the monthly exposure level that is exceeded with probability 0.05?

(d) What value of mean monthly exposure is needed so that the probability of a monthly exposure more than 9 $\mu g/m^3$ is 0.01?

4-89. An article in *Atmospheric Chemistry and Physics* "Relationship Between Particulate Matter and Childhood Asthma—Basis of a Future Warning System for Central Phoenix" (2012, Vol. 12, pp. 2479–2490)] reported the use of PM10 (particulate matter $< 10\ \mu m$ diameter) air quality data measured hourly from sensors in Phoenix, Arizona. The 24-hour (daily) mean PM10 for a centrally located sensor was 50.9 $\mu g/m^3$ with a standard deviation of 25.0. Assume that the daily mean of PM10 is normally distributed.

(a) What is the probability of a daily mean of PM10 greater than 100 $\mu g/m^3$?

(b) What is the probability of a daily mean of PM10 less than 25 $\mu g/m^3$?

(c) What daily mean of PM10 value is exceeded with probability 5%?

4-90. The length of stay at a specific emergency department in Phoenix, Arizona, in 2009 had a mean of 4.6 hours with a standard deviation of 2.9. Assume that the length of stay is normally distributed.

(a) What is the probability of a length of stay greater than 10 hours?

(b) What length of stay is exceeded by 25% of the visits?

(c) From the normally distributed model, what is the probability of a length of stay less than 0 hours? Comment on the normally distributed assumption in this example.

4-91. A signal in a communication channel is detected when the voltage is higher than 1.5 volts in absolute value. Assume that the voltage is normally distributed with a mean of 0. What is the standard deviation of voltage such that the probability of a false signal is 0.005?

4-92. An article in *Microelectronics Reliability* ["Advanced Electronic Prognostics through System Telemetry and Pattern Recognition Methods" (2007, Vol.47(12), pp. 1865–1873)] presented an example of electronic prognosis. The objective was to detect faults to decrease the system downtime and the number of unplanned repairs in high-reliability systems. Previous measurements of the power supply indicated that the signal is normally distributed with a mean of 1.5 V and a standard deviation of 0.02 V.

(a) Suppose that lower and upper limits of the predetermined specifications are 1.45 V and 1.55 V, respectively. What is the probability that a signal is within these specifications?

(b) What is the signal value that is exceeded with 95% probability?

(c) What is the probability that a signal value exceeds the mean by two or more standard deviations?

4-93. An article in *International Journal of Electrical Power & Energy Systems* ["Stochastic Optimal Load Flow Using a Combined Quasi–Newton and Conjugate Gradient Technique" (1989, Vol.11(2), pp. 85–93)] considered the problem of optimal power flow in electric power systems and included the effects of uncertain variables in the problem formulation. The method treats the system power demand as a normal random variable with 0 mean and unit variance.

(a) What is the power demand value exceeded with 95% probability?

(b) What is the probability that the power demand is positive?

(c) What is the probability that the power demand is more than -1 and less than 1?

4-94. An article in the *Journal of Cardiovascular Magnetic Resonance* ["Right Ventricular Ejection Fraction Is Better Reflected by Transverse Rather Than Longitudinal Wall Motion in Pulmonary Hypertension" (2010, Vol.12(35)] discussed a study of the regional right ventricle transverse wall motion in patients with pulmonary hypertension (PH). The right ventricle ejection fraction (EF) was approximately normally distributed with a mean and a standard deviation of 36 and 12, respectively, for PH subjects, and with mean and standard deviation of 56 and 8, respectively, for control subjects.

(a) What is the EF for PH subjects exceeded with 5% probability?

(b) What is the probability that the EF of a control subject is less than the value in part (a)?

(c) Comment on how well the control and PH subjects can be distinguished by EF measurements.

4-7 Normal Approximation to the Binomial and Poisson Distributions

We began our section on the normal distribution with the central limit theorem and the normal distribution as an approximation to a random variable with a large number of trials. Consequently, it should not be surprising to learn that the normal distribution can be used

The exponential distribution is often used in reliability studies as the model for the time until failure of a device. For example, the lifetime of a semiconductor chip might be modeled as an exponential random variable with a mean of 40,000 hours. The lack of memory property of the exponential distribution implies that the device does not wear out. That is, regardless of how long the device has been operating, the probability of a failure in the next 1000 hours is the same as the probability of a failure in the first 1000 hours of operation. The lifetime L of a device with failures caused by random shocks might be appropriately modeled as an exponential random variable.

However, the lifetime L of a device that suffers slow mechanical wear, such as bearing wear, is better modeled by a distribution such that $P(L < t + \Delta t \mid L > t)$ increases with t. Distributions such as the Weibull distribution are often used in practice to model the failure time of this type of device. The Weibull distribution is presented in a later section.

Exercises FOR SECTION 4-8

⊕ Problem available in *WileyPLUS* at instructor's discretion.

⊕ **Go Tutorial** Tutoring problem available in *WileyPLUS* at instructor's discretion.

4-112. ⊕ Suppose that X has an exponential distribution with $\lambda = 2$. Determine the following:

(a) $P(X \le 0)$ (b) $P(X \ge 2)$

(c) $P(X \le 1)$ (d) $P(1 < X < 2)$

(e) Find the value of x such that $P(X < x) = 0.05$.

4-113. ⊕ Suppose that X has an exponential distribution with mean equal to 10. Determine the following:

(a) $P(X > 10)$ (b) $P(X > 20)$ (c) $P(X < 30)$

(d) Find the value of x such that $P(X < x) = 0.95$.

4-114. ⊕ Suppose that X has an exponential distribution with a mean of 10. Determine the following:

(a) $P(X < 5)$ (b) $P(X < 15 \mid X > 10)$

(c) Compare the results in parts (a) and (b) and comment on the role of the lack of memory property.

4-115. ⊕ Suppose that the counts recorded by a Geiger counter follow a Poisson process with an average of two counts per minute.

(a) What is the probability that there are no counts in a 30-second interval?

(b) What is the probability that the first count occurs in less than 10 seconds?

(c) What is the probability that the first count occurs between one and two minutes after start-up?

4-116. ⊕ Suppose that the log-ons to a computer network follow a Poisson process with an average of three counts per minute.

(a) What is the mean time between counts?

(b) What is the standard deviation of the time between counts?

(c) Determine x such that the probability that at least one count occurs before time x minutes is 0.95.

4-117. ⊕ The time between calls to a plumbing supply business is exponentially distributed with a mean time between calls of 15 minutes.

(a) What is the probability that there are no calls within a 30-minute interval?

(b) What is the probability that at least one call arrives within a 10-minute interval?

(c) What is the probability that the first call arrives within 5 and 10 minutes after opening?

(d) Determine the length of an interval of time such that the probability of at least one call in the interval is 0.90.

4-118. ⊕ The life of automobile voltage regulators has an exponential distribution with a mean life of six years. You purchase a six-year-old automobile, with a working voltage regulator and plan to own it for six years.

(a) What is the probability that the voltage regulator fails during your ownership?

(b) If your regulator fails after you own the automobile three years and it is replaced, what is the mean time until the next failure?

4-119. ⊕ Suppose that the time to failure (in hours) of fans in a personal computer can be modeled by an exponential distribution with $\lambda = 0.0003$.

(a) What proportion of the fans will last at least 10,000 hours?

(b) What proportion of the fans will last at most 7000 hours?

4-120. ⊕ The time between the arrival of electronic messages at your computer is exponentially distributed with a mean of two hours.

(a) What is the probability that you do not receive a message during a two-hour period?

(b) If you have not had a message in the last four hours, what is the probability that you do not receive a message in the next two hours?

(c) What is the expected time between your fifth and sixth messages?

4-121. ⊕ The time between arrivals of taxis at a busy intersection is exponentially distributed with a mean of 10 minutes.

(a) What is the probability that you wait longer than one hour for a taxi?

(b) Suppose that you have already been waiting for one hour for a taxi. What is the probability that one arrives within the next 10 minutes?

(c) Determine x such that the probability that you wait more than x minutes is 0.10.

(d) Determine x such that the probability that you wait less than x minutes is 0.90.

(e) Determine x such that the probability that you wait less than x minutes is 0.50.

4-122. The number of stork sightings on a route in South Carolina follows a Poisson process with a mean of 2.3 per year.

(a) What is the mean time between sightings?
(b) What is the probability that there are no sightings within three months (0.25 years)?
(c) What is the probability that the time until the first sighting exceeds six months?
(d) What is the probability of no sighting within three years?

4-123. According to results from the analysis of chocolate bars in Chapter 3, the mean number of insect fragments was 14.4 in 225 grams. Assume that the number of fragments follows a Poisson distribution.

(a) What is the mean number of grams of chocolate until a fragment is detected?
(b) What is the probability that there are no fragments in a 28.35-gram (one-ounce) chocolate bar?
(c) Suppose you consume seven one-ounce (28.35-gram) bars this week. What is the probability of no insect fragments?

4-124. ⊕ The distance between major cracks in a highway follows an exponential distribution with a mean of five miles.

(a) What is the probability that there are no major cracks in a 10-mile stretch of the highway?
(b) What is the probability that there are two major cracks in a 10-mile stretch of the highway?
(c) What is the standard deviation of the distance between major cracks?
(d) What is the probability that the first major crack occurs between 12 and 15 miles of the start of inspection?
(e) What is the probability that there are no major cracks in two separate five-mile stretches of the highway?
(f) Given that there are no cracks in the first five miles inspected, what is the probability that there are no major cracks in the next 10 miles inspected?

4-125. ⊕ The lifetime of a mechanical assembly in a vibration test is exponentially distributed with a mean of 400 hours.

(a) What is the probability that an assembly on test fails in less than 100 hours?
(b) What is the probability that an assembly operates for more than 500 hours before failure?
(c) If an assembly has been on test for 400 hours without a failure, what is the probability of a failure in the next 100 hours?
(d) If 10 assemblies are tested, what is the probability that at least one fails in less than 100 hours? Assume that the assemblies fail independently.
(e) If 10 assemblies are tested, what is the probability that all have failed by 800 hours? Assume that the assemblies fail independently.

4-126. ⊕ The time between arrivals of small aircraft at a county airport is exponentially distributed with a mean of one hour.

(a) What is the probability that more than three aircraft arrive within an hour?
(b) If 30 separate one-hour intervals are chosen, what is the probability that no interval contains more than three arrivals?
(c) Determine the length of an interval of time (in hours) such that the probability that no arrivals occur during the interval is 0.10.

4-127. The time between calls to a corporate office is exponentially distributed with a mean of 10 minutes.

(a) What is the probability that there are more than three calls in one-half hour?
(b) What is the probability that there are no calls within one-half hour?
(c) Determine x such that the probability that there are no calls within x hours is 0.01.
(d) What is the probability that there are no calls within a two-hour interval?
(e) If four nonoverlapping one-half-hour intervals are selected, what is the probability that none of these intervals contains any call?
(f) Explain the relationship between the results in part (a) and (b).

4-128. ⊕ Assume that the flaws along a magnetic tape follow a Poisson distribution with a mean of 0.2 flaw per meter. Let X denote the distance between two successive flaws.

(a) What is the mean of X?
(b) What is the probability that there are no flaws in 10 consecutive meters of tape?
(c) Does your answer to part (b) change if the 10 meters are not consecutive?
(d) How many meters of tape need to be inspected so that the probability that at least one flaw is found is 90%?
(e) What is the probability that the first time the distance between two flaws exceeds eight meters is at the fifth flaw?
(f) What is the mean number of flaws before a distance between two flaws exceeds eight meters?

4-129. If the random variable X has an exponential distribution with mean θ, determine the following:

(a) $P(X > \theta)$ (b) $P(X > 2\theta)$ (c) $P(X > 3\theta)$
(d) How do the results depend on θ?

4-130. ⊕ Derive the formula for the mean and variance of an exponential random variable.

4-131. Web crawlers need to estimate the frequency of changes to Web sites to maintain a current index for Web searches. Assume that the changes to a Web site follow a Poisson process with a mean of 3.5 days.

(a) What is the probability that the next change occurs in less than 2.0 days?
(b) What is the probability that the time until the next change is greater 7.0 days?
(c) What is the time of the next change that is exceeded with probability 90%?
(d) What is the probability that the next change occurs in less than 10.0 days, given that it has not yet occurred after 3.0 days?

4-132. The length of stay at a specific emergency department in a hospital in Phoenix, Arizona had a mean of 4.6 hours. Assume that the length of stay is exponentially distributed.

(a) What is the standard deviation of the length of stay?
(b) What is the probability of a length of stay of more than 10 hours?
(c) What length of stay is exceeded by 25% of the visits?

4-133. An article in *Journal of National Cancer Institute* ["Breast Cancer Screening Policies in Developing Countries: A Cost-Effectiveness Analysis for India" (2008, Vol.100(18),

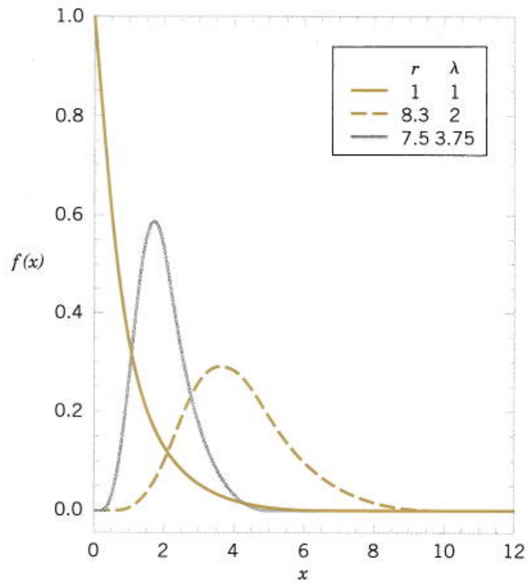

FIGURE 4-25 Gamma probability density functions for selected values of λ and r.

and variance of a gamma random variable multiply the exponential results by r. This motivates the following conclusions. Repeated integration by parts can be used to derive these, but the details are lengthy and omitted.

Mean and Variance

If X is a **gamma random variable** with parameters λ and r,

$$\mu = E(X) = r/\lambda \quad \text{and} \quad \sigma^2 = V(X) = r/\lambda^2$$

Example 4-24 The time to prepare a micro-array slide for high-throughput genomics is a Poisson process with a mean of two hours per slide. What is the probability that 10 slides require more than 25 hours to prepare?

Let X denote the time to prepare 10 slides. Because of the assumption of a Poisson process, X has a gamma distribution with $\lambda = 1/2, r = 10$, and the requested probability is $P(X > 25)$. The probability can be obtained from software that provides cumulative Poisson probabilities or gamma probabilities. For the cumulative Poisson probabilities, we use the method in Example 4-23 to obtain

$$P(X > 25) = \sum_{k=0}^{9} \frac{e^{-12.5}(12.5)^k}{k!}$$

In software we set the mean = 12.5 and the input = 9 to obtain $P(X > 25) = 0.2014$.

As a check, we use the gamma cumulative probability function in Minitab. Set the shape parameter to 10, the scale parameter to 0.5, and the input to 25. The probability computed is $P(X \le 25) = 0.7986$, and when this is subtracted from one we match with the previous result that $P(X > 25) = 0.2014$.

What are the mean and standard deviation of the time to prepare 10 slides? The mean time is

$$E(X) = r/\lambda = 10/0.5 = 20$$

The variance of time is

$$V(X) = r/\lambda^2 = 10/0.5^2 = 40$$

so that the standard deviation is $40^{1/2} = 6.32$ hours.

The slides will be completed by what length of time with probability equal to 0.95? The question asks for x such that

$$P(X \le x) = 0.95$$

where X is gamma with $\lambda = 0.5$ and $r = 10$. In software, we use the gamma inverse cumulative probability function and set the shape parameter to 10, the scale parameter to 0.5, and the probability to 0.95. The solution is

$$P(X \le 31.41) = 0.95$$

Practical Interpretation: Based on this result, a schedule that allows 31.41 hours to prepare 10 slides should be met 95% of the time.

Furthermore, the **chi-squared distribution** is a special case of the gamma distribution in which $\lambda = 1/2$ and r equals one of the values 1/2, 1, 3/2, 2, ... This distribution is used extensively in interval estimation and tests of hypotheses that are discussed in subsequent chapters. The chi-squared distribution is discussed in Chapter 7.

EXERCISES FOR SECTION 4-9

⊕ Problem available in *WileyPLUS* at instructor's discretion.

⊕ Go Tutorial Tutoring problem available in *WileyPLUS* at instructor's discretion.

4-137. ⊕ Use the properties of the gamma function to evaluate the following:
(a) $\Gamma(6)$ (b) $\Gamma(5/2)$ (c) $\Gamma(9/2)$

4-138. ⊕ Given the probability density function $f(x) = 0.01^3 x^2 e^{-0.01x} / \Gamma(3)$, determine the mean and variance of the distribution.

4-139. ⊕ Calls to a telephone system follow a Poisson distribution with a mean of five calls per minute.
(a) What is the name applied to the distribution and parameter values of the time until the 10th call?
(b) What is the mean time until the 10th call?
(c) What is the mean time between the 9th and 10th calls?
(d) What is the probability that exactly four calls occur within one minute?
(e) If 10 separate one-minute intervals are chosen, what is the probability that all intervals contain more than two calls?

4-140. ⊕ Raw materials are studied for contamination. Suppose that the number of particles of contamination per pound of material is a Poisson random variable with a mean of 0.01 particle per pound.
(a) What is the expected number of pounds of material required to obtain 15 particles of contamination?
(b) What is the standard deviation of the pounds of materials required to obtain 15 particles of contamination?

4-141. ⊕ The time between failures of a laser in a cytogenics machine is exponentially distributed with a mean of 25,000 hours.
(a) What is the expected time until the second failure?
(b) What is the probability that the time until the third failure exceeds 50,000 hours?

4-142. ⊕ In a data communication system, several messages that arrive at a node are bundled into a packet before they are transmitted over the network. Assume that the messages arrive at the node according to a Poisson process with $\tau = 30$ messages per minute. Five messages are used to form a packet.
(a) What is the mean time until a packet is formed, that is, until five messages have arrived at the node?
(b) What is the standard deviation of the time until a packet is formed?
(c) What is the probability that a packet is formed in less than 10 seconds?
(d) What is the probability that a packet is formed in less than five seconds?

4-143. ⊕ Errors caused by contamination on optical disks occur at the rate of one error every 10^5 bits. Assume that the errors follow a Poisson distribution.
(a) What is the mean number of bits until five errors occur?
(b) What is the standard deviation of the number of bits until five errors occur?
(c) The error-correcting code might be ineffective if there are three or more errors within 10^5 bits. What is the probability of this event?

4-144. ⊕ Calls to the help line of a large computer distributor follow a Poisson distribution with a mean of 20 calls per minute. Determine the following:
(a) Mean time until the one-hundredth call
(b) Mean time between call numbers 50 and 80
(c) Probability that three or more calls occur within 15 seconds

4-145. ⊕ The time between arrivals of customers at an automatic teller machine is an exponential random variable with a mean of five minutes.
(a) What is the probability that more than three customers arrive in 10 minutes?
(b) What is the probability that the time until the fifth customer arrives is less than 15 minutes?

4-146. Use integration by parts to show that $\Gamma(r) = (r-1)\Gamma(r-1)$.

4-147. Show that the gamma density function $f(x, \lambda, r)$ integrates to 1.

4-148. ⊕ Use the result for the gamma distribution to determine the mean and variance of a chi-square distribution with $r = 7/2$.

4-149. Patients arrive at a hospital emergency department according to a Poisson process with a mean of 6.5 per hour.

(a) What is the mean time until the 10th arrival?
(b) What is the probability that more than 20 minutes is required for the third arrival?

4-150. The total service time of a multistep manufacturing operation has a gamma distribution with mean 18 minutes and standard deviation 6.
(a) Determine the parameters λ and r of the distribution.
(b) Assume that each step has the same distribution for service time. What distribution for each step and how many steps produce this gamma distribution of total service time?

4-151. An article in *Sensors and Actuators A: Physical* ["Characterization and Simulation of Avalanche PhotoDiodes for Next-Generation Colliders" (2011, Vol.172(1), pp.181–188)] considered an avalanche photodiode (APD) to detect charged particles in a photo. The number of arrivals in each detection window was modeled with a Poisson distribution with a mean depending on the intensity of beam. For one beam intensity, the number of electrons arriving at an APD follows a Poisson distribution with a mean of 1.74 particles per detection window of 200 nanoseconds.
(a) What is the mean and variance of the time for 100 arrivals?
(b) What is the probability that the time until the fifth particle arrives is greater than 1.0 nanosecond?

4-152. An article in *Mathematical Biosciences* ["Influence of Delayed Viral Production on Viral Dynamics in HIV-1 Infected Patients" (1998, Vol.152(2), pp. 143–163)] considered the time delay between the initial infection by immunodeficiency virus type 1 (HIV-1) and the formation of productively infected cells. In the simulation model, the time delay is approximated by a gamma distribution with parameters $r = 4$ and $1/\lambda = 0.25$ days. Determine the following:
(a) Mean and variance of time delay
(b) Probability that a time delay is more than half a day
(c) Probability that a time delay is between one-half and one day

4-10 Weibull Distribution

As mentioned previously, the Weibull distribution is often used to model the time until failure of many different physical systems. The parameters in the distribution provide a great deal of flexibility to model systems in which the number of failures increases with time (bearing wear), decreases with time (some semiconductors), or remains constant with time (failures caused by external shocks to the system).

Weibull Distribution

The random variable X with probability density function

$$f(x) = \frac{\beta}{\delta}\left(\frac{x}{\delta}\right)^{\beta-1} \exp\left[-\left(\frac{x}{\delta}\right)^{\beta}\right], \quad \text{for } x > 0 \tag{4-20}$$

is a **Weibull random variable** with scale parameter $\delta > 0$ and shape parameter $\beta > 0$.

The graphs of selected probability density functions in Fig. 4-26 illustrate the flexibility of the Weibull distribution. By inspecting the probability density function, we can see that when $\beta = 1$, the Weibull distribution is identical to the exponential distribution. Also, the **Raleigh distribution** is a special case when the shape parameter is 2.

The cumulative distribution function is often used to compute probabilities. The following result can be obtained.

Cumulative Distribution Function

If X has a Weibull distribution with parameters δ and β, then the cumulative distribution function of X is

$$F(x) = 1 - e^{-\left(\frac{x}{\delta}\right)^{\beta}}$$

Also, the following results can be obtained.

Mean and Variance

If X has a Weibull distribution with parameters δ and β,

$$\mu = E(X) = \delta\Gamma\left(1+\frac{1}{\beta}\right) \quad \text{and} \quad \sigma^2 = V(X) = \delta^2\Gamma\left(1+\frac{2}{\beta}\right) - \delta^2\left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2 \tag{4-21}$$

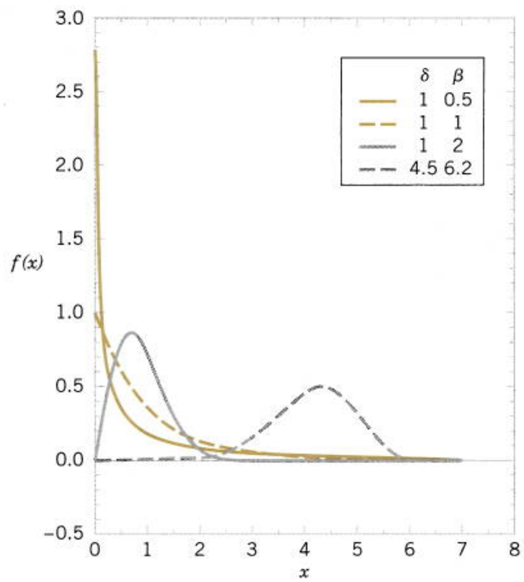

FIGURE 4-26 Weibull probability density functions for selected values of δ and β.

Example 4-25 **Bearing Wear** The time to failure (in hours) of a bearing in a mechanical shaft is satisfactorily modeled as a Weibull random variable with $\beta = 1/2$ and $\delta = 5000$ hours. Determine the mean time until failure.

From the expression for the mean,

$$E(X) = 5000\Gamma\left[1 + (1/2)\right] = 5000\Gamma\left[1.5\right] = 5000 \times 0.5\sqrt{\pi} = 4431.1 \text{ hours}$$

Determine the probability that a bearing lasts at least 6000 hours. Now,

$$P(X > 6000) = 1 - F(6000) = \exp\left[-\left(\frac{6000}{5000}\right)^2\right] = e^{-1.44} = 0.237$$

Practical Interpretation: Consequently, only 23.7% of all bearings last at least 6000 hours.

EXERCISES FOR SECTION 4-10

⊕ Problem available in *WileyPLUS* at instructor's discretion.
⊕ Go Tutorial Tutoring problem available in *WileyPLUS* at instructor's discretion.

4-153. ⊕ Suppose that X has a Weibull distribution with $\beta = 0.2$ and $\delta = 100$ hours. Determine the mean and variance of X.

4-154. ⊕ Suppose that X has a Weibull distribution with $\beta = 0.2$ and $\delta = 100$ hours. Determine the following:
(a) $P(X < 10{,}000)$ (b) $P(X > 5000)$

4-155. ⊕ If X is a Weibull random variable with $\beta = 1$ and $\delta = 1000$, what is another name for the distribution of X, and what is the mean of X?

4-156. ⊕ Assume that the life of a roller bearing follows a Weibull distribution with parameters $\beta = 2$ and $\delta = 10{,}000$ hours.
(a) Determine the probability that a bearing lasts at least 8000 hours.
(b) Determine the mean time until failure of a bearing.
(c) If 10 bearings are in use and failures occur independently, what is the probability that all 10 bearings last at least 8000 hours?

4-157. ⊕ The life (in hours) of a computer processing unit (CPU) is modeled by a Weibull distribution with parameters $\beta = 3$ and $\delta = 900$ hours. Determine (a) and (b):
(a) Mean life of the CPU. (b) Variance of the life of the CPU.
(c) What is the probability that the CPU fails before 500 hours?

4-158. Assume that the life of a packaged magnetic disk exposed to corrosive gases has a Weibull distribution with $\beta = 0.5$ and the mean life is 600 hours. Determine the following:

(a) Probability that a disk lasts at least 500 hours.

(b) Probability that a disk fails before 400 hours.

4-159. The life (in hours) of a magnetic resonance imaging machine (MRI) is modeled by a Weibull distribution with parameters $\beta = 2$ and $\delta = 500$ hours. Determine the following:

(a) Mean life of the MRI

(b) Variance of the life of the MRI

(c) Probability that the MRI fails before 250 hours.

4-160. An article in the *Journal of the Indian Geophysical Union* titled "Weibull and Gamma Distributions for Wave Parameter Predictions" (2005, Vol. 9, pp. 55–64) described the use of the Weibull distribution to model ocean wave heights. Assume that the mean wave height at the observation station is 2.5 m and the shape parameter equals 2. Determine the standard deviation of wave height.

4-161. An article in the *Journal of Geophysical Research* ["Spatial and Temporal Distributions of U.S. of Winds and Wind Power at 80 m Derived from Measurements" (2003, vol. 108)] considered wind speed at stations throughout the United States. A Weibull distribution can be used to model the distribution of wind speeds at a given location. Every location is characterized by a particular shape and scale parameter. For a station at Amarillo, Texas, the mean wind speed at 80 m (the hub height of large wind turbines) in 2000 was 10.3 m/s with a standard deviation of 4.9 m/s. Determine the shape and scale parameters of a Weibull distribution with these properties.

4-162. Suppose that X has a Weibull distribution with $\beta = 2$ and $\delta = 8.6$. Determine the following:

(a) $P(X < 10)$ (b) $P(X > 9)$ (c) $P(8 < X < 11)$

(d) Value for x such that $P(X > x) = 0.9$

4-163. Suppose that the lifetime of a component (in hours) is modeled with a Weibull distribution with $\beta = 2$ and $\delta = 4000$. Determine the following in parts (a) and (b):

(a) $P(X > 3000)$ (b) $P(X > 6000 \mid X > 3000)$

(c) Comment on the probabilities in the previous parts compared to the results for an exponential distribution.

4-164. Suppose that the lifetime of a component (in hours), X is modeled with a Weibull distribution with $\beta = 0.5$ and $\delta = 4000$. Determine the following in parts (a) and (b):

(a) $P(X > 3500)$ (b) $P(X > 6000 \mid X > 3000)$

(c) Comment on the probabilities in the previous parts compared to the results for an exponential distribution.

(d) Comment on the role of the parameter β in a lifetime model with the Weibull distribution.

4-165. Suppose that X has a Weibull distribution with $\beta = 2$ and $\delta = 2000$. Determine the following in parts (a) and (b):

(a) $P(X > 3500)$

(b) $P(X > 3500)$ for an exponential random variable with the same mean as the Weibull distribution

(c) Comment on the probability that the lifetime exceeds 3500 hours under the Weibull and exponential distributions.

4-166. An article in *Electronic Journal of Applied Statistical Analysis* ["Survival Analysis of Dialysis Patients Under Parametric and Non-Parametric Approaches" (2012, Vol. 5(2), pp. 271–288)] modeled the survival time of dialysis patients with chronic kidney disease with a Weibull distribution. The mean and standard deviation of survival time were 16.01 and 11.66 months, respectively. Determine the following:

(a) Shape and scale parameters of this Weibull distribution

(b) Probability that survival time is more than 48 months

(c) Survival time exceeded with 90% probability

4-167. An article in *Proceeding of the 33rd International ACM SIGIR Conference on Research and Development in Information Retrieval* ["Understanding Web Browsing Behaviors Through Weibull Analysis of Dwell Time" (2010, p. 3791–386)] proposed that a Weibull distribution can be used to model Web page dwell time (the length of time a Web visitor spends on a Web page). For a specific Web page, the shape and scale parameters are 1 and 300 seconds, respectively. Determine the following:

(a) Mean and variance of dwell time

(b) Probability that a Web user spends more than four minutes on this Web page

(c) Dwell time exceeded with probability 0.25

4-168. An article in *Financial Markets Institutions and Instruments* ["Pricing Reinsurance Contracts on FDIC Losses" (2008, Vol. 17(3)] modeled average annual losses (in billions of dollars) of the Federal Deposit Insurance Corporation (FDIC) with a Weibull distribution with parameters $\delta = 1.9317$ and $\beta = 0.8472$. Determine the following:

(a) Probability of a loss greater than \$2 billion

(b) Probability of a loss between \$2 and \$4 billion

(c) Value exceeded with probability 0.05

(d) Mean and standard deviation of loss

4-169. An article in *IEEE Transactions on Dielectrics and Electrical Insulation* ["Statistical Analysis of the AC Breakdown Voltages of Ester Based Transformer Oils" (2008, Vol. 15(4))] used Weibull distributions to model the breakdown voltage of insulators. The breakdown voltage is the minimum voltage at which the insulator conducts. For 1 mm of natural ester, the 1% probability of breakdown voltage is approximately 26 kV, and the 7% probability is approximately 31.6 kV. Determine the parameters δ and β of the Weibull distribution.

4-11 Lognormal Distribution

Variables in a system sometimes follow an exponential relationship as $x = \exp(w)$. If the exponent is a random variable W, then $X = \exp(W)$ is a random variable with a distribution of interest. An important special case occurs when W has a normal distribution. In that case, the distribution of X is called a **lognormal distribution**. The name follows from the transformation $\ln(X) = W$. That is, the natural logarithm of X is normally distributed.

Probabilities for X are obtained from the transform of the normal distribution. The range of X is $(0, \infty)$. Suppose that W is normally distributed with mean θ and variance ω^2; then the cumulative distribution function for X is

$$F(x) = P[X \le x] = P[\exp(W) \le x] = P[W \le \ln(x)]$$
$$= P\left[Z \le \frac{\ln(x) - \theta}{\omega}\right] = \Phi\left[\frac{\ln(x) - \theta}{\omega}\right]$$

for $x > 0$, where Z is a standard normal random variable and $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. Therefore, Appendix Table III can be used to determine the probability. Also, $F(x) = 0$ for $x \le 0$.

The probability density function of X can be obtained from the derivative of $F(x)$. This derivative is applied to the last term in the expression for $F(x)$. Because $\Phi(\cdot)$ is the integral of the standard normal density function, the fundamental theorem of calculus is used to calculate the derivative. Furthermore, from the probability density function, the mean and variance of X can be derived. The details are omitted, but a summary of results follows.

Lognormal Distribution

Let W have a normal distribution with mean θ and variance ω^2; then $X = \exp(W)$ is a **lognormal random variable** with probability density function

$$f(x) = \frac{1}{x\omega\sqrt{2\pi}} \exp\left[-\frac{(\ln(x) - \theta)^2}{2\omega^2}\right] \qquad 0 < x < \infty$$

The mean and variance of X are

$$E(X) = e^{\theta + \omega^2/2} \quad \text{and} \quad V(X) = e^{2\theta + \omega^2}\left(e^{\omega^2} - 1\right) \qquad \text{(4-22)}$$

The parameters of a lognormal distribution are θ and ω^2, but these are the mean and variance of the normal random variable W. The mean and variance of X are the functions of these parameters shown in Equation (4-22). Figure 4-27 illustrates lognormal distributions for selected values of the parameters.

The lifetime of a product that degrades over time is often modeled by a lognormal random variable. For example, this is a common distribution for the lifetime of a semiconductor laser. A Weibull distribution can also be used in this type of application, and with an appropriate choice for parameters, it can approximate a selected lognormal distribution. However, a lognormal distribution is derived from a simple exponential function of a normal random variable, so it is easy to understand and easy to evaluate probabilities.

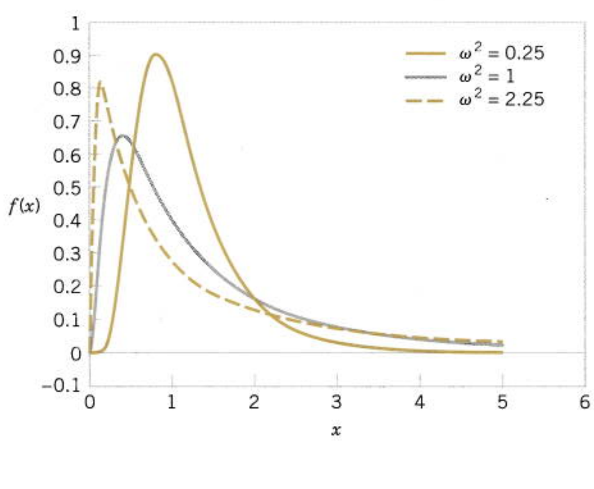

FIGURE 4-27 Lognormal probability density functions with $\theta = 0$ for selected values of ω^2.

4-190. An *allele* is an alternate form of a gene, and the proportion of alleles in a population is of interest in genetics. An article in *BMC Genetics* ["Calculating Expected DNA Remnants From Ancient Founding Events in Human Population Genetics" (2008, Vol. 9:66)] used a beta distribution with mean 0.3 and standard deviation 0.17 to model initial allele proportions in a genetic simulation. Determine the parameters α and β for this beta distribution.

4-191. Suppose that the construction of a solar power station is initiated. The project's completion time has not been set due to uncertainties in financial resources. The completion time for the first phase is modeled with a beta distribution and the minimum, most likely (mode), and maximum completion times for the first phase are 1.0, 1.25, and 2.0 years, respectively. Also, the mean time is assumed to equal $\mu = 1 + 4(1.25) + 2) / 6 = 1.333$. Determine the following in parts (a) and (b):

(a) Parameters α and β of the beta distribution.

(b) Standard deviation of the distribution.

(c) Sketch the probability density function.

Supplemental Exercises

⊕ Problem available in *WileyPLUS* at instructor's discretion.

⊕ Go Tutorial Tutoring problem available in *WileyPLUS* at instructor's discretion.

4-192. ⊕ The probability density function of the time it takes a hematology cell counter to complete a test on a blood sample is $f(x) = 0.04$ for $50 < x < 75$ seconds.

(a) What percentage of tests requires more than 70 seconds to complete?

(b) What percentage of tests requires less than one minute to complete?

(c) Determine the mean and variance of the time to complete a test on a sample.

4-193. ⊕ The tensile strength of paper is modeled by a normal distribution with a mean of 35 pounds per square inch and a standard deviation of 2 pounds per square inch.

(a) What is the probability that the strength of a sample is less than 40 lb/in^2?

(b) If the specifications require the tensile strength to exceed 30 lb/in^2, what proportion of the samples is scrapped?

4-194. ⊕ The time it takes a cell to divide (called *mitosis*) is normally distributed with an average time of one hour and a standard deviation of five minutes.

(a) What is the probability that a cell divides in less than 45 minutes?

(b) What is the probability that it takes a cell more than 65 minutes to divide?

(c) By what time have approximately 99% of all cells completed mitosis?

4-195. ⊕ The length of an injection-molded plastic case that holds magnetic tape is normally distributed with a length of 90.2 millimeters and a standard deviation of 0.1 millimeter.

(a) What is the probability that a part is longer than 90.3 millimeters or shorter than 89.7 millimeters?

(b) What should the process mean be set at to obtain the highest number of parts between 89.7 and 90.3 millimeters?

(c) If parts that are not between 89.7 and 90.3 millimeters are scrapped, what is the yield for the process mean that you selected in part (b)?

Assume that the process is centered so that the mean is 90 millimeters and the standard deviation is 0.1 millimeter. Suppose that 10 cases are measured, and they are assumed to be independent.

(d) What is the probability that all 10 cases are between 89.7 and 90.3 millimeters?

(e) What is the expected number of the 10 cases that are between 89.7 and 90.3 millimeters?

4-196. ⊕ The sick-leave time of employees in a firm in a month is normally distributed with a mean of 100 hours and a standard deviation of 20 hours.

(a) What is the probability that the sick-leave time for next month will be between 50 and 80 hours?

(b) How much time should be budgeted for sick leave if the budgeted amount should be exceeded with a probability of only 10%?

4-197. The percentage of people exposed to a bacteria who become ill is 20%. Assume that people are independent. Assume that 1000 people are exposed to the bacteria. Approximate each of the following:

(a) Probability that more than 225 become ill

(b) Probability that between 175 and 225 become ill

(c) Value such that the probability that the number of people who become ill exceeds the value is 0.01

4-198. ⊕ The time to failure (in hours) for a laser in a cytometry machine is modeled by an exponential distribution with $\lambda = 0.00004$. What is the probability that the time until failure is

(a) At least 20,000 hours? (b) At most 30,000 hours?

(c) Between 20,000 and 30,000 hours?

4-199. ⊕ When a bus service reduces fares, a particular trip from New York City to Albany, New York, is very popular. A small bus can carry four passengers. The time between calls for tickets is exponentially distributed with a mean of 30 minutes. Assume that each caller orders one ticket. What is the probability that the bus is filled in less than three hours from the time of the fare reduction?

4-200. ⊕ The time between process problems in a manufacturing line is exponentially distributed with a mean of 30 days.

(a) What is the expected time until the fourth problem?

(b) What is the probability that the time until the fourth problem exceeds 120 days?

4-201. The life of a recirculating pump follows a Weibull distribution with parameters $\beta = 2$ and $\delta = 700$ hours. Determine for parts (a) and (b):

(a) Mean life of a pump (b) Variance of the life of a pump

(c) What is the probability that a pump will last longer than its mean?

4-202. The size of silver particles in a photographic emulsion is known to have a log normal distribution with a mean of 0.001 mm and a standard deviation of 0.002 mm.

(a) Determine the parameter values for the lognormal distribution.

(b) What is the probability of a particle size greater than 0.005 mm?

4-203. Suppose that $f(x) = 0.5x - 1$ for $2 < x < 4$. Determine the following:

(a) $P(X < 2.5)$ (b) $P(X > 3)$ (c) $P(2.5 < X < 3.5)$

(d) Determine the cumulative distribution function of the random variable.

(e) Determine the mean and variance of the random variable.

4-204. The time between calls is exponentially distributed with a mean time between calls of 10 minutes.

(a) What is the probability that the time until the first call is less than five minutes?

(b) What is the probability that the time until the first call is between 5 and 15 minutes?

(c) Determine the length of an interval of time such that the probability of at least one call in the interval is 0.90.

(d) If there has not been a call in 10 minutes, what is the probability that the time until the next call is less than 5 minutes?

(e) What is the probability that there are no calls in the intervals from 10:00 to 10:05, from 11:30 to 11:35, and from 2:00 to 2:05?

(f) What is the probability that the time until the third call is greater than 30 minutes?

(g) What is the mean time until the fifth call?

4-205. The CPU of a personal computer has a lifetime that is exponentially distributed with a mean lifetime of six years. You have owned this CPU for three years.

(a) What is the probability that the CPU fails in the next three years?

(b) Assume that your corporation has owned 10 CPUs for three years, and assume that the CPUs fail independently. What is the probability that at least one fails within the next three years?

4-206. Suppose that X has a lognormal distribution with parameters $\theta = 0$ and $\omega^2 = 4$. Determine the following:

(a) $P(10 < X < 50)$

(b) Value for x such that $P(X < x) = 0.05$

(c) Mean and variance of X

4-207. Suppose that X has a lognormal distribution and that the mean and variance of X are 50 and 4000, respectively. Determine the following:

(a) Parameters θ and ω^2 of the lognormal distribution

(b) Probability that X is less than 150

4-208. Asbestos fibers in a dust sample are identified by an electron microscope after sample preparation. Suppose that the number of fibers is a Poisson random variable and the mean number of fibers per square centimeter of surface dust is 100. A sample of 800 square centimeters of dust is analyzed. Assume that a particular grid cell under the microscope represents 1/160,000 of the sample.

(a) What is the probability that at least one fiber is visible in the grid cell?

(b) What is the mean of the number of grid cells that need to be viewed to observe 10 that contain fibers?

(c) What is the standard deviation of the number of grid cells that need to be viewed to observe 10 that contain fibers?

4-209. Without an automated irrigation system, the height of plants two weeks after germination is normally distributed with a mean of 2.5 centimeters and a standard deviation of 0.5 centimeter.

(a) What is the probability that a plant's height is greater than 2.25 centimeters?

(b) What is the probability that a plant's height is between 2.0 and 3.0 centimeters?

(c) What height is exceeded by 90% of the plants?

4-210. With an automated irrigation system, a plant grows to a height of 3.5 centimeters two weeks after germination. Without an automated system, the height is normally distributed with mean and standard deviation 2.5 and 0.5 centimeters, respectively.

(a) What is the probability of obtaining a plant of this height or greater without an automated system?

(b) Do you think the automated irrigation system increases the plant height at two weeks after germination?

4-211. The thickness of a laminated covering for a wood surface is normally distributed with a mean of five millimeters and a standard deviation of 0.2 millimeter.

(a) What is the probability that a covering thickness is more than 5.5 millimeters?

(b) If the specifications require the thickness to be between 4.5 and 5.5 millimeters, what proportion of coverings does not meet specifications?

(c) The covering thickness of 95% of samples is below what value?

4-212. The diameter of the dot produced by a printer is normally distributed with a mean diameter of 0.002 inch.

(a) Suppose that the specifications require the dot diameter to be between 0.0014 and 0.0026 inch. If the probability that a dot meets specifications is to be 0.9973, what standard deviation is needed?

(b) Assume that the standard deviation of the size of a dot is 0.0004 inch. If the probability that a dot meets specifications is to be 0.9973, what specifications are needed? Assume that the specifications are to be chosen symmetrically around the mean of 0.002.

4-213. The waiting time for service at a hospital emergency department follows an exponential distribution with a mean of three hours. Determine the following:

(a) Waiting time is greater than four hours

(b) Waiting time is greater than six hours given that you have already waited two hours

(c) Value x (in hours) exceeded with probability 0.25

4-214. The life of a semiconductor laser at a constant power is normally distributed with a mean of 7000 hours and a standard deviation of 600 hours.

(a) What is the probability that a laser fails before 5800 hours?

(b) What is the life in hours that 90% of the lasers exceed?

(c) What should the mean life equal for 99% of the lasers to exceed 10,000 hours before failure?