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CSCE355
October 10 2017
Test 2

CSCE 355, Fall 2017, Midterm 2, October 10, 2017

Please write your exam on the blank, white 8.5×11 paper provided. This exam lasts 45 minutes and is closed book, closed notes, and no electronic devices allowed. There are 70 points total, of which 65 points is full credit and the rest is extra credit.

Write your name on each sheet, and do not staple, fold, or tear your paper.

Generally, you need not justify a correct answer unless I explicitly say you must.

We let $\Sigma = \{a, b, c\}$ throughout.

1. (10 points) Prove via simple induction that $n < 2^n$ for every natural number n. You may use the fact that $2^n \geq 1$ for all natural numbers n.

2. (15 points) Define languages $L_1, L_2 \subseteq \Sigma^*$ as follows:

$$L_1 = \{w \in \Sigma^* \mid w \text{ has } ab \text{ as a substring}\},$$
$$L_2 = \{w \in \Sigma^* \mid w \text{ has } bc \text{ as a substring}\}.$$

   Using the product construction, give a DFA D recognizing $L_1 \cap L_2$ built from DFAs recognizing L_1 and L_2, respectively. Draw D as a transition diagram with states organized on a grid, similar to the ones drawn in class. You need not give transitions from states unreachable from the start state. D should have no more than 9 states.

3. (10 points) Give the transition diagram of a six-state DFA that recognizes the language L_3 of strings $w \in \Sigma^*$ that have $abaca$ as a substring.

4. (10 points) Give the transition diagram of a six-state DFA that recognizes the language L_4 of strings $w \in \Sigma^*$ that have $abaca$ as a subsequence, that is, the sequence of letters $\langle a, b, a, c, a \rangle$ occur in w in order, but not necessarily contiguously. For example, $bbaccbcabacccbabb \in L_4$ (and in fact, $L_3 \subseteq L_4$) whereas $acaba \notin L_4$.

5. (15 points total) Consider the following NFA N:
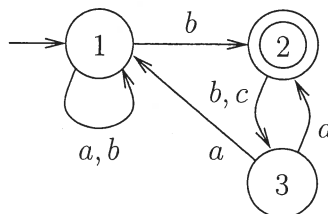
(a) (5 points) Give an accepting path for the input string *bbbabba* (as a sequence of states).

(b) (10 points) Using the sets-of-states (subset) construction, give the transition diagram of an equivalent DFA D. Label the states of D with the states of N that they contain. You only need to show those states of D reachable from its start state, and you may omit the dead state (corresponding to $\emptyset$).

6. (10 points) Give the transition diagram for an NFA that recognizes the language

$$L_5 := \{a^n \mid n \text{ is a multiple of either 3 or 5}\} .$$

Note that 0 is a multiple of every natural number. Your NFA should have no more than nine states. (Recall that a^n is the string of n many a's.)

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$\Sigma = \{a, b, c\}$

$2^3 = 2 \cdot 2 \cdot 2 = 8$

$2^2 \cdot 2 = 8$

1) Prove $n < 2^n$, $n \in \mathbb{N}$; $2^n \geq 1$ $n \in \mathbb{N}$.

Base case: ~~$2^n \geq 1$~~

Assume $n < 2^n$.

For all $n \in \mathbb{N}$, $2^n \geq 1$, $n < 2^n$ by induction.

$n+1 < 2^{n+1}$

$n+1 < 2^n(2^1)$

$n < 2(2^n) - 1$ *(red: ind. hyp?)*

case $n=0$ case $n=1$ case $n=2$ ...

$0 < 2(2^0)-1$ $1 < 2(2^1)-1$ $2 < 2(2^2)-1$

$0 < 1$ ✓ $1 < 3$ ✓ $2 < 7$ ✓

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2) $L_1 = \{w \in \Sigma^* \mid w \text{ has } ab \text{ as a substring}\}$ $D = L_1 \cap L_2$ *(red: transition diagram?)*

$L_2 = \{w \in \Sigma^* \mid w \text{ has } bc \text{ as a substring}\}$

(red: why!?)   *(red: Why?!)*   *(red: Is this A-BC or AB-C?)*
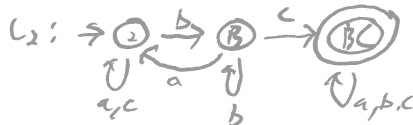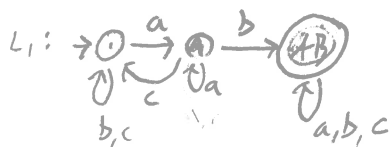
	a	b	c
→ 12	A2	1B	12
A2	A2	ABB	12
1B	A2	1B	1BC
ABB	AB2	ABB	ABBC
1BC	ABC	1BC	1BC
AB2	AB2	ABB	AB2
*ABBC	ABBC	ABBC	ABBC
ABC	ABC	ABBC	1BC

3) $L_3 = \{w \in \Sigma^* \mid w \text{ has } abaca \text{ as a substring}\}$ (six states)

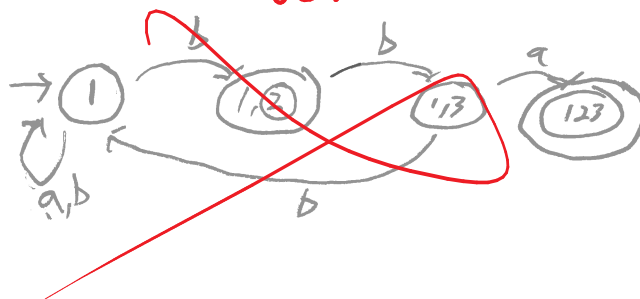*(red: +9)*

4)

→ ◯ —a→ (A) —b→ (AB) —a→ (ABA) —c→ (ABAC) —a→ ((ABACA))   **+W**

with loops: ◯ b,c ; A a,c ; AB b,c ; ABA a,b ; ABAC b,c ; ABACA a,b,c
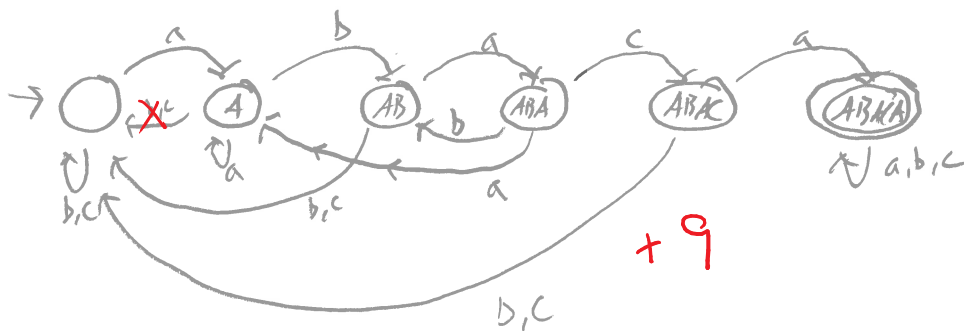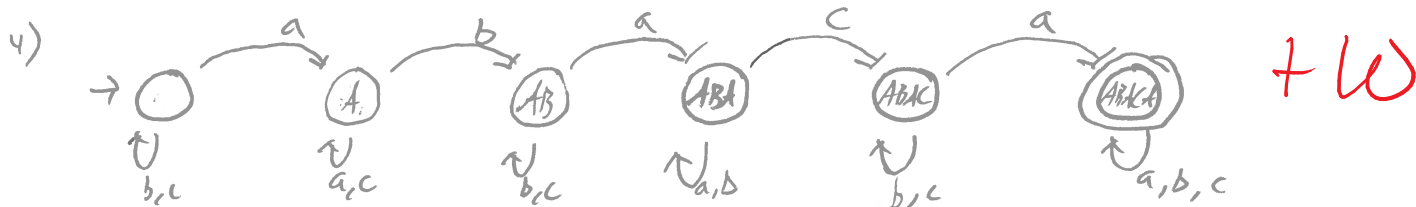
L4: w ∈ Σ* | abaca is a subsequence, not necessarily continuous

5)

a)

	states
→1	
b	1,2
bb	1,3
bbb	1
bbba	1
bbbab	1,2
bbbabb	1,3
bbbabba	1,2,3

?

b) DFA D: **Not a DFA**

a) $L_5 := \{a^n \mid n \text{ is a multiple of 3 or 5}\}$

0, 3, 5, 6, 9, 10, 12, 15, 18, 20, 21

accepts a^8