STAT 509 Section E01 Homework 4 Solution

Instruction: Let me know if you find any error in the solution.

1. A textile fiber manufacturer is investigating a new drapery yarn, which the company claims has a mean thread elongation of 12 kilograms with a standard deviation of 0.5 kilograms. The company wishes to test the hypothesis $H_0 : \mu = 12$ against $H_a : \mu \neq 12$ using a random sample of four specimens. Suppose the random sample is from a normal population.

   (a) Follow the 4 steps of conducting a hypothesis test, what is your conclusion if the sample mean $\overline{y} = 11.3$ and we use $\alpha = 0.05$?
   Solution: The test statistic is

   $$z_0 = \frac{\overline{y} - \mu_0}{\sigma/\sqrt{n}} = \frac{11.3 - 12}{0.5/\sqrt{4}} = -2.8$$

   The p-value of the test is

   $$2[1 - \phi(|z_0|)] = P(Z < -2.8) + P(Z > 2.8) = 0.00511$$

   ```
   > 2*(1-pnorm(2.8))
   [1] 0.005110261
   ```

   We reject H_0 at $\alpha = 0.05$ level of significance, since p-value is less than 0.05 and conclude that there is sufficient evidence to support $\mu \neq 12$.

   (b) Calculate a 95% two-sided confidence interval for μ. Does the confidence interval cover 12?
   Solution: The 95% two-sided confidence interval for μ is

   $$\left(\overline{y} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \overline{y} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right) = \left(11.3 - 1.96\frac{0.5}{\sqrt{4}}, 11.3 - 1.96\frac{0.5}{\sqrt{4}}\right) = (10.81, 11.79).$$

   This interval does not contain $\mu_0 = 12$.
   Remark: As mentioned in the class, there is a close relationship between confidence interval and hypothesis testing. Suppose we use the same α, if the two-sided $(1-\alpha)$100% C.I. for μ covers $\mu = \mu_0$ under the null hypothesis, then we fail to reject the two-sided test at α significance level. If the two-sided $(1-\alpha)$100% C.I. for μ does not cover $\mu = \mu_0$, we will reject H_0. Similarly, you can draw conclusion for one-sided test using one-sided confidence interval. More details will be given in the lecture.

2. A manufacturing firm claims that the batteries used in their electronic games will last an average of at least 30 hours. To maintain this average, 16 randomly selected batteries are tested each month. Suppose the battery lifetime follows a normal distribution.

   `20,25,21,28,21,30,23,27,26,26,28,31,26,32,33,35`

   (a) State the null and alternative hypotheses.
   Solution: $H_0 : \mu \geq 30$ against $H_a : \mu < 30$.

   (b) Calculate the appropriate test statistic.
   Solution: The sample mean is $\overline{y} = 27$ and the sample standard deviation is $s = 4.442$ from

R. The test statistic is

$$t_0 = \frac{\overline{y} - \mu_0}{s/\sqrt{n}} = \frac{27 - 30}{4.442/\sqrt{16}} = -2.7$$

(c) What is the p-value of the test? What is your conclusion based on p-value ($\alpha = 0.01$).
Solution: p-value $= P(t_{15} > -2.7) = 0.008$.

```
> 1-pt( -2.7014,15,lower.tail=F)
[1] 0.008206322
```

(d) If we want to draw the conclusion based on a 99% confidence interval, what is the proper form of the confidence interval? What is your conclusion based on the confidence interval approach?
Solution: For $H_a : \mu < 30$, we use the upper confidence bound

$$\left(-\infty, \overline{y} + t_{\alpha,n-1}\frac{s}{\sqrt{n}}\right) = (0, 27 + 2.60248\frac{4.442}{4}) = (0, 29.9).$$

```
> qt(0.01,15,lower.tail=F)
[1] 2.60248
```

Remark: Lifetime can not be negative.
Again, we reject H_0 since this interval fails to include "30".

(e) Redo the above analysis using t.test in R, does your calculation by hand match the R output? Attach your R output.

```
> t.test(battery,alternative="less",conf.level=0.99,mu=30)
        One Sample t-test
data: battery
t = -2.7014, df = 15, p-value = 0.008207
alternative hypothesis: true mean is less than 30
99 percent confidence interval:
    -Inf 29.8902
sample estimates:
mean of x
       27
```

You can read p-value, test statistic, and lower confidence bound directly from above output.

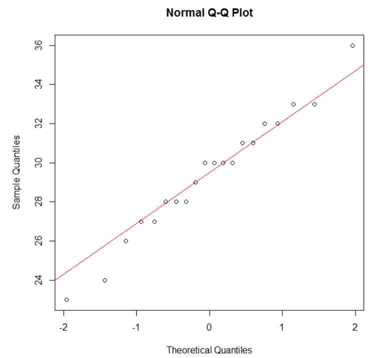
3. An industrial engineer has randomly chosen 20 experienced operators and timed them as they assembled a given piece of equipment. The times are in minutes:

```
30 33 31 24 28 27 33 28 32 30 27 30 28 32 36 30 29 26 31 23
```

The industrial engineer are interested in whether the mean assembly time equals to 28 or not. Conduct a complete data analysis in R.
Solution: We want to test $H_0 : \mu = 28$ against $H_a : \mu \neq 28$. We need to look at Q-Q plot to check the normality assumption before using a t-test.

```
time<-c(30,33,31,24,28,27,33,28,32,30,27,30,28,32,36,30,29,26,31,23)
qqnorm(time);qqline(time,col="2")
```

The Q-Q plot suggests that it is reasonable to assume normality. Let us look at R output of the t-test




```
> t.test(time,alternative="two.sided",mu=28)

        One Sample t-test

data:  time
t = 1.9862, df = 19, p-value = 0.06163
alternative hypothesis: true mean is not equal to 28
95 percent confidence interval:
 27.92469 30.87531
sample estimates:
mean of x
     29.4
```

The p-value of the test is 0.06163, which is very close to 0.05. We will reject H_0 at 0.1 significance level, we have evidence to support that the mean assembly time is not equal to 28. Recall that the smaller the p-value is, the stronger evidence against H_0.

4. Inexperienced data analysts often erroneously place too much faith in qq plots when assessing whether a distribution adequately represents a data set (especially when the sample size is small).The purpose of this problem is to illustrate to you the dangers that can arise. In this problem, you will use R to simulate the process of drawing repeated random samples from a given population distribution and then creating normal probability plots (Q-Q plots). Follow the code provided

   (a) Generate your own data and create a qq plot for each sample using this R code:

```
# create 2 by 2 figure
par(mfrow = c(2,2))
B = 4
n = 10
# create matrix to hold all data
data = matrix(round(rnorm(n*B,0,1),4), nrow = B, ncol = n)
# this creates a qq plot for each sample of data
for (i in 1:B){
   qqnorm(data[i,],pch=16,main="")
   qqline(data[i,])
}
```

   mark the qq plot that appears to violate the normal assumption the most. Note: In theory, all of these plots should display perfect linearity! Why? Because we are generating the data from a normal distribution! **Therefore, even when we create normal qq**

plots with normally distributed data, we can get plots that don't look perfectly linear. This is a byproduct of sampling variability. This is why you don't want to rush to discount a distribution as being plausible based on a single plot, especially when the sample size n is small (like $n = 10$).
Solution: The plot one the top left corner apperently violates the normal assumption. Note: your plot should different from mine plot due to the nature of random number generatation.

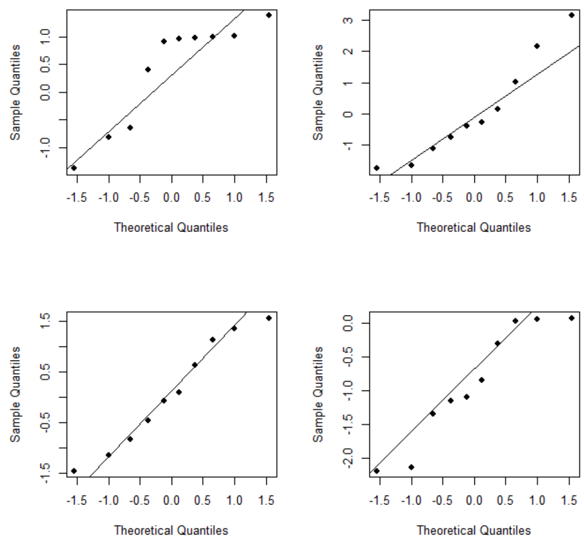


Figure 1: Part (a)

(b) Increase your sample size to $n = 100$ and repeat. What happens? What if $n = 1000$? Just change n in the R code and re-run.
Solution: Let us look at the case when $n = 1000$. Now, all the Q-Q plots works almost perfectly! It suggests that we can trust Q-Q plot if our sample size is large.

(c) Take $n = 100$, replace

```
data = matrix(round(rnorm(n*B,0,1),4), nrow = B, ncol = n)
```

with

```
data = matrix(round(rexp(n*B,1),4), nrow = B, ncol = n)
```

and re-run. By doing this, you are changing the underlying population distribution from $\mathcal{N}(0, 1)$ to exponential(1). What do these normal qq plots look like? Are you surprised?
Solution:

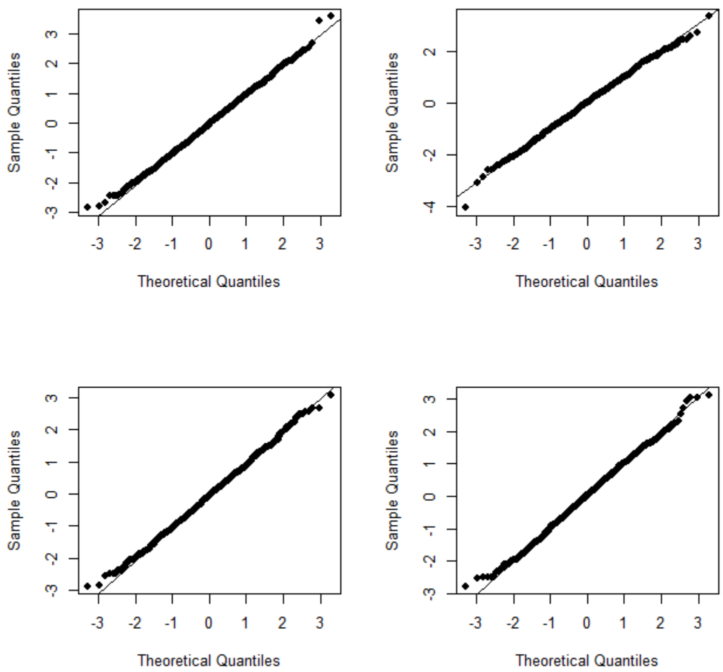


Figure 2: Part (b)

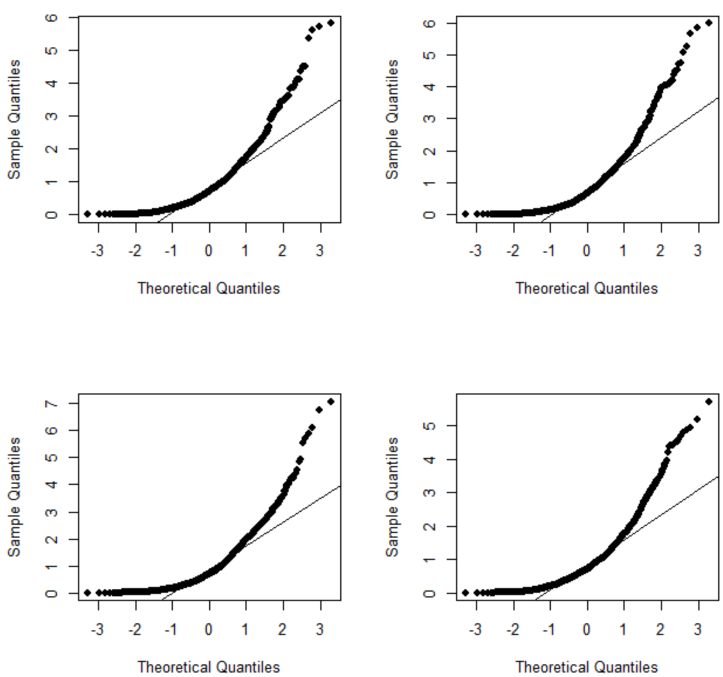


Figure 3: Part (c)

No Surprise, we expect to see curvature in the Q-Q plots, since we generate our data from exponential distribution instead of normal.

5. A study to characterize the physical behavior of steel concrete beams under periodic load was carried out at the National Institute of Standards and Technology. The response variable is deflection (from rest point) of the steel concrete beam. The variance should be less than 70. A sample of fifteen beams gave the following summary statistics:

Variable	n	mean	variance	std. Dev.
size	15	569.4	8.829	7.670

(a) Estimate the variance with a 95% confidence interval.
Solution: A two-sided 95% confidence interval is

$$\left(\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}\right) = \left(\frac{14(8.829)}{26.12}, \frac{14(8.829)}{5.63}\right) = (4.73, 21.95).$$

The chi-squared upper percentage point can be found via R

```
> qchisq(0.975,14,lower.tail=F)
[1] 5.628726
> qchisq(0.025,14,lower.tail=F)
[1] 26.11895
```

(b) Perform a hypothesis test that tests the alternative hypothesis that the variance is less than 70. Be sure to include the hypotheses, calculation of the test statistic, calculation of the p-value, and statement of conclusion.
Solution: We want to test $H_0 : \sigma^2 = 70$ against $H_a : \sigma^2 < 70$. The test statistic is

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{14(8.829)}{70} = 1.7658.$$

The p-value of the test is $P(\chi^2_{14} < \chi^2_0) = P(\chi^2_{14} < 1.7658) < 0.001$

```
> pchisq(1.7658,14)
[1] 3.851168e-05
```

We will reject H_0 at any reasonable significance level, we have **strong** evidence that the variance is less than 70.