1. If $t_1(n) \in O(g_1(n))$ and $t_2(n) \in O(g_2(n))$, then $t_1(n) + t_2(n) \in O(\max\{g_1(n), g_2(n)\})$. prove that assertions.

A.) To prove the assertion, we will derive an upper bond for $t_1(n) + t_2(n)$ in terms of $\max\{g_1(n), g_2(n)\}$

* since $t_1(n) \in O(g_1(n))$, there exist c_1 and n_1 such that-

$$t_1(n) \leq c_1 \cdot g_1(n) \quad \text{for all } n \geq n_1$$

similarly, since $t_2(n) \in O(g_2(n))$, there exist c_2 and n_2 such that:

$$t_2(n) \leq c_2 \cdot g_2(n) \quad \text{for all } n \geq n_2.$$

combine the bounds:

let $n_0 = \max(n_1, n_2)$. for all $n > n_0$:

$$t_1(n) \leq c_1 \cdot g_1(n) \text{ and } t_2(n) \leq c_2 \cdot g_2(n)$$

therefore, for all $n > n_0$.

$$t_1(n) + t_2(n) \leq c_1 \cdot g_1(n) + c_2 \cdot g_2(n).$$

simplify the expression:

combine the terms on right-hand side:

$$t_1(n) + t_2(n) \leq (c_1 + c_2) \cdot \max\{g_1(n), g_2(n)\}.$$

conclude Big-O notations:

let $c = c_1 + c_2$. we have show that

$$t_1(n) + t_2(n) \leq c \cdot \max\{g_1(n), g_2(n)\} \text{ for all } n \geq n_0.$$

Big O-notation. Hence,

$$t_1(n) + t_2(n) \in O(\max\{g_1(n), g_2(n)\}).$$

2. Find the time complexity of the below recurrence equation?

A.) Identify the constants:

$$a = 2 \quad b = 2 \quad f(n) = 1.$$

compute $\log_b a$

$$\log_b a = \log_2 2 = 1.$$

* compare $f(n)$ with $n^{\log_b a}$:

$$f(n) = 1 \text{ and } n^{\log_b a} = n^1 = n.$$

Apply the master's theorem:

* $f(n) = O(n^c)$ where $c < \log_b a$, then $T(n) = O(n^{\log_b a})$.
* $f(n) = O(n^{\log_b a})$, then $T(n) = O(n^{\log_b a} \log n)$.
* $f(n) = \Omega(n^c)$ where $c > \log_b a$, and $af\left(\frac{n}{b}\right) \leq k f(n)$ for some $k < 1$ and sufficiently large n, then $T(n) = O(n)$

$\therefore$ time complexity $T(n) = O(n^{\log_b a})$

$$= O(n^1).$$

$$= O(n).$$

3. show that $f(n) = n^2 + 3n + 5$ is $O(n^2)$. gi.

A) Identify the dominat term:

* the dominat term in $f(n)$ is n^2, since it grows faster than the terms, n, become large.

* compare each term to n^2:

$$n^2 \leq n^2$$

$$3n \leq 3n^2 \text{ (for } n \geq 1)$$

$$5 \leq 5n^2 \text{ (for } n \geq 1).$$

combaining. these, we get

$$n^2 + 3n + 5 \leq n^2 + 3n^2 + 5n^2$$

$$n^2 + 3n + 5 \leq 9n^2$$

choose Appropriate constants:

let $c = 9$ and $n_0 = 1$. then for all $n \geq n_0$

$$f(n) = n^2 + 3n + 5 \leq 9n^2.$$

$f(n) \leq c.n^2$

it follows that:

$f(n) = n^2 + 3n + 5 \in O(n^2)$.

thus, we have show that $f(n) = n^2 + 3n + 5$ is $O(n^2)$.

4.) prove that $g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^3)$.

A.) * Identify the dominat term.

the dominat term in $g(n)$ is n^3, since it grows faster than the other terms as n become, large.

combine and compare terms:

$$n^3 + 2n^2 + 4n \geq n^3$$

this is true because $2n^2 + 4n$ is always non-negative for all $n \geq 0$.

* choose an appropriate constant:

we can see that $n^3 + 2n^2 + 4n \geq n^3$ for any $n \geq 0$.

Hence we can choose $c = 1$ and $n_0 = 0$. therefore, for all $n \geq n_0$:

$$g(n) = n^3 + 2n^2 + 4n \geq 1.n^3.$$

conclusion:

$g(n) \geq c.n^3$

it follows that

$g(n) = n^3 + 2n^2 + 4n \in \Omega(n^3)$

$\therefore g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^3)$.

5. Determine wether $h(n) = 4n^2 + 3n$ is $O(n^2)$ or not.

A.) upper Bond (Big O)

* find c_2 and n_0:

$h(n) = 4n^2 + 3n$

$4n^2 + 3n \le 4n^2 + 3n^2 = 7n^2$ (for $n \ge 1$).

so, we can choose $c_2 = 7$ and $n_0 = 1$. then, for all $n \ge 1$:

$h(n) \le 7n^2$.

lower bound (Big omega) :

* find c_1 and n_0.

$h(n) = 4n^2 + 3n$

$4n^2 + 3n \ge 4n^2$..

so, we can choose $c_1 = 4$ and $n_0 = 1$. then, for all $n \ge 1$:

$h(n) \ge 4n^2$.

$\therefore c_1 = 4$, $c_2 = 7$ and $n_0 = 1$.

$h(n) = 4n^2 + 3n \in \theta(n^2)$.

$h(n) = 4n^2 + 3n$ is $\theta(n^2)$..