Submission for the Test Flight Project for Introduction to Mathematical Thinking.

Problem 8:

Proposition: If the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \to \infty$, then for any $M > 0$, the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to limit ML.

Proof: Take the definition of the limit of a sequence: There exists $m \in \mathbb{N}$ such that for all $\epsilon > 0$ it is true that $|a_m - L| < \epsilon$. Then, because $M > 0$, we can multiply the inequality by M, and distribute it through the absolute value on the left-hand side:

$$\begin{aligned} |a_m - L| &< \epsilon \\ M \cdot |a_m - L| &< M \cdot \epsilon \\ |M \cdot a_m - M \cdot L| &< M \cdot \epsilon \end{aligned}$$

Because ϵ can be arbitrarily small and m can be arbitrarily large, this is sufficient to show that $\{Ma_n\}_{n=1}^{\infty}$ tends to limit ML as n tends to ∞. $\square$