Submission for the Test Flight Project for Introduction to Mathematical Thinking.

Problem 7:

Proposition: For any $n \in \mathbb{N}$,

$$2 + 2^2 + 2^3 + ... + 2^n = 2^{n+1} - 2$$

Proof: We will use the principle of mathematical induction to prove this statement. First we establish a base case:

Let $n = 1$:

$$\begin{aligned} 2^1 &= 2^{1+1} - 2 \\ 2 &= 4 - 2 \\ 2 &= 2 \end{aligned}$$

Let $n = 2$:

$$\begin{aligned} 2^1 + 2^2 &= 2^{2+1} - 2 \\ 2 + 4 &= 8 - 2 \\ 6 &= 6 \end{aligned}$$

We see that for the first few cases this is true. Next, assuming that the statement is true for n we add the $(n+1)$-th term:

$$\begin{aligned} 2^1 + 2^2 + ... + 2^n &= 2^{n+1} - 2 \\ 2^1 + 2^2 + ... + 2^n + 2^{n+1} &= 2^{n+1} - 2 + 2^{n+1} \\ &= 2 \cdot 2^{n+1} - 2 \\ &= 2^{n+2} - 2 \end{aligned}$$

This shows that our proposition was in fact true and concludes the proof. $\square$