Submission for the Test Flight Project for Introduction to Mathematical Thinking.

Problem 6:

Proposition: There exists only one "prime triple" (i.e. three primes, each 2 from the next) and it is 3, 5, 7.

Proof: Choose any odd natural number $2k+1$, $k \in \mathbb{N}$. Let this be the least of three consecutive odd numbers, $2k+1$, $2(k+1)+1$, and $2(k+2)+1$.

Observing that one of the k-terms must be a multiple of three (or k-term $= 3m$ for some $m \in \mathbb{N}$), we can break the argument into three cases:

Case 1: k is a multiple of 3. We choose the odd number $2(k+1)+1$:

$$\begin{aligned} 2(k+1)+1 &= \\ 2(3m+1)+1 &= 6m+2+1 \\ &= 6m+3 \\ &= 3(2m+1) \end{aligned}$$

We see that $2(k+1)+1$ is a multiple of 3.

Case 2: $k+1$ is a multiple of 3. We choose the odd number $2(k+2)+1$:

$$\begin{aligned} 2(k+2)+1 &= \\ 2(3m+1)+1 &= 6m+2+1 \\ &= 6m+3 \\ &= 3(2m+1) \end{aligned}$$

We see that $2(k+2)+1$ is a multiple of 3.

Case 3: $k+2$ is a multiple of 3. We choose the odd number $2k+1$:

$$\begin{aligned} 2k+1 &= \\ 2(k+2-2)+1 &= \\ 2(3m-2)+1 &= 6m-4+1 \\ &= 6m-3 \\ &= 3(2m-1) \end{aligned}$$

And finally we see that $2k+1$ is a multiple of 3.

For any case, at least one of the three consecutive odd numbers must be a multiple of 3.

The only multiple of 3 which is prime is 3 itself, which leaves only two potential "prime triples" left to examine: 1, 3, 5, and 3, 5, 7.

By definition 1 is not a prime, so any triple including it cannot be a "prime triple". Thus we have concluded that 3, 5, 7 is the only possible "prime triple". $\square$