

Problem 1:

Proposition: $(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m + 5n = 12)$

Proof: To rephrase the proposition: There exists a natural number m and a natural number n such that $3m + 5n = 12$. We observe that if $n > 2$ then $5n > 12$, so n must necessarily equal to 1 or 2.

Case 1: Let $n = 1$. The equation can be rewritten as

$$3m + (5)(1) = 12$$

Solving for m we have

$$3m = 7$$

or

$$m = \frac{7}{3}$$

which is not a natural number.

Case 2: Let $n = 2$. The equation can be rewritten as

$$3m + (5)(2) = 12$$

Solving for m we have

$$3m = 2$$

or

$$m = \frac{2}{3}$$

which is also not a natural number.

Neither of these cases produces an m which is a natural number, so the proposition must be false.

□