Submission for the Test Flight Project for Introduction to Mathematical Thinking.

Proposition: For any $n \in \mathbb{Z}$, at least one of the integers n, $n+2$, or $n+4$ is divisible by 3.

Proof: We can show this to be true directly by observing that any number can be written in one of the forms $3k$, $3k+1$, or $3k+2$ and then substituting it into the appropriate n-term.

Case 1: Let $n = 3k$. Then we see that n is a multiple of 3.

Case 2: Let $n = 3k+1$. Then, by inserting it into $n+2$ we have

$$\begin{aligned} n+2 &= 3k+1+2 \\ &= 3k+3 \\ &= 3(k+1) \end{aligned}$$

which is also a multiple of 3.

Case 3: Let $n = 3k+2$. Then, by substituting into the final term we have

$$\begin{aligned} n+4 &= 3k+2+4 \\ &= 3k+6 \\ &= 3(k+2) \end{aligned}$$

which, finally, is also a multiple of 3.

We see that for any $n \in \mathbb{Z}$ it must be true that n or $n+2$ or $n+4$ is a multiple of 3. $\square$