Submission for the Test Flight Project for Introduction to Mathematical Thinking.

Problem 4:

Proposition: Every odd natural number is one of the forms $4n + 1$ or $4n + 3$, where $n \in \mathbb{Z}$.

Proof: We can rewrite the two forms as $4n + 1 = 2(2n) + 1$ and $4n + 3 = 2(2n + 1) + 1$. Because $2n$ and $2n + 1$ are the standard form for every even and every odd number, we know that $\{2n | n \in \mathbb{Z}\} \cup \{2m + 1 | m \in \mathbb{Z}\} \equiv \mathbb{Z}$.

Then we can simplify our initial claim to say that every odd number is of the form $2k + 1$ where $k \in \mathbb{Z}$. This is by definition an odd number, which concludes the proof. $\square$