

Problem 9:

Proposition: For the collection of intervals $A_n = (0, \frac{1}{n})$, where $n = 1, 2, \dots$, every $A_{n+1} \subset A_n$ and $\bigcup_{n=1}^{\infty} A_n = \emptyset$.

Proof: Because $\frac{1}{n}$ decreases as n increases but cannot be negative, we know that the limit of $\frac{1}{n}$ goes to 0 as n goes to ∞ . Thus, the limit of the right side of the interval will go to 0 as n goes to ∞ , so the interval will tend to $(0, 0)$. But because the interval is open on both sides, 0 is not included in that set, so the intersection of all of the intervals A_n will be \emptyset . \square