(Over) Simplified Model

Assuming scalar, direction independent gains simplifies this to

$$\mathbf{V}_{pq,t\nu} = \mathbf{J}_{p,t\nu}\mathbf{X}_{pq,t\nu}\mathbf{J}^{\dagger}_{q,t\nu} + \epsilon_{pq,t\nu}$$

where $\mathbf{X}_{pq,t\nu}$ are given by the vCZ relation

$$\mathbf{X}_{pq,t\nu} = \int I(l,m,\nu)e^{-2\pi\imath\frac{\nu}{c}(u_{pq,t}l + v_{pq,t}m + w_{pq,t}(n-1))}\frac{dldm}{n}$$

- Usually what goes into the MODEL_DATA column
- This is what a degridder computes
- Tends to a non-uniform Fourier transform when
  1. array is coplanar
  2. narrow field of view

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model coherencies

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Why is this an over-simplification?

Calibration - Optimisation

Assuming:

1. sky is known $\rightarrow$ visibilities only depend on gains
2. noise is Gaussian $\epsilon \sim \mathcal{G}(0, \Sigma) \rightarrow$ form of likelihood
3. problem is well posed $\rightarrow$ ML solution is unique

it is possible to solve for gains by minimising

$$\begin{aligned} \hat{\mathbf{J}}_{p,t\nu} &= \underset{\mathbf{J}_{p,t\nu}}{\operatorname{argmin}} \sum_{q;q\neq p} r^\dagger_{pq,t\nu} \Sigma^{-1}_{pq,t\nu} r_{pq,t\nu}, \quad \text{where} \\ r_{pq,t\nu} &= \mathbf{V}_{pq,t\nu} - \mathbf{J}_{p,t\nu} \mathbf{X}_{pq,t\nu} \mathbf{J}^\dagger_{q,t\nu} \end{aligned}$$

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Where does this functional form come from?

"Correcting" the data

Once the gains are known, as long as they are non-zero, we can attempt to get back the model coherencies as

$$\mathbf{V}^C_{pq,t\nu} = \hat{\mathbf{J}}^{-1}_{p,t\nu}\mathbf{V}_{pq,t\nu}\hat{\mathbf{J}}^{-\dagger}_{q,t\nu} = \mathbf{X}_{pq,t\nu} + \hat{\mathbf{J}}^{-1}_{p,t\nu}\epsilon_{pqt\nu}\hat{\mathbf{J}}^{-\dagger}_{q,t\nu}$$

Questions:

- What is the distribution of the noise after this transform?
- How would you correct the data when the gains are direction dependent?
- What should you expect to see if you plot the ratio of the amplitudes $\mathbf{V}^C_{pq,t\nu}/\mathbf{X}_{pq,t\nu}$?
- What would you see if you plot the real vs. imaginary parts of $\mathbf{V}^C_{pq,t\nu}$ if the gains are correct and we are observing a point source at the phase center?

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corrected data

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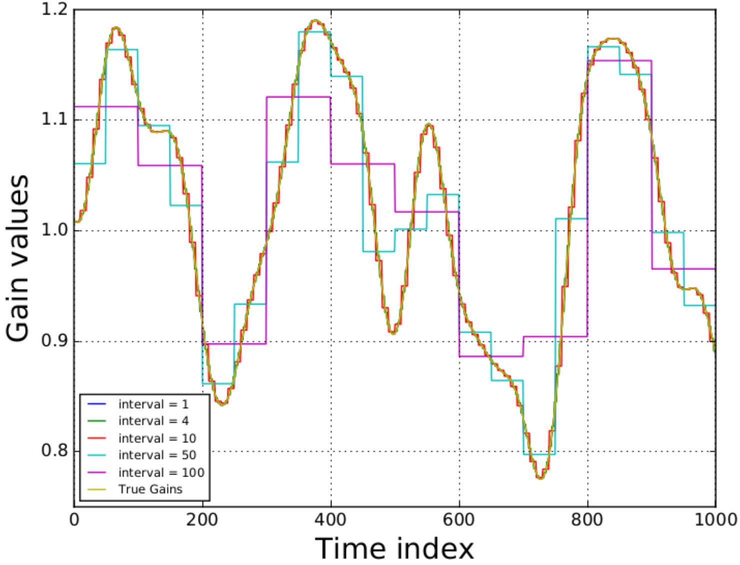
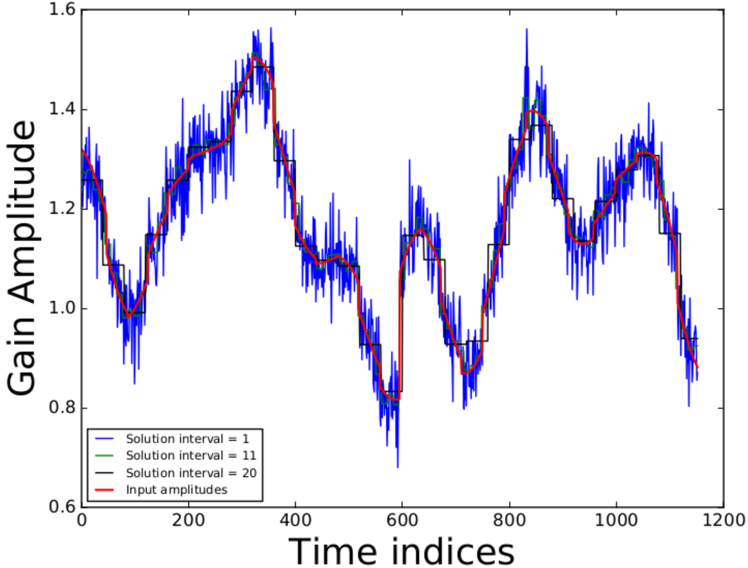
Calibration Caveats

- Model is not always known at the outset
- Σ is not known for all data (eg. RFI)
- Failing to keep track of the noise properties of $\mathbf{V}^{C}_{pq,t\nu}$ potentially biases the imaging step
- Non-linear optimisation (eg. Guass-Newton) requires a good starting guess (eg. FFT based delay estimates)
- Straightforward ML complex-valued solution not optimal $\rightarrow$ **need for regularisation!**

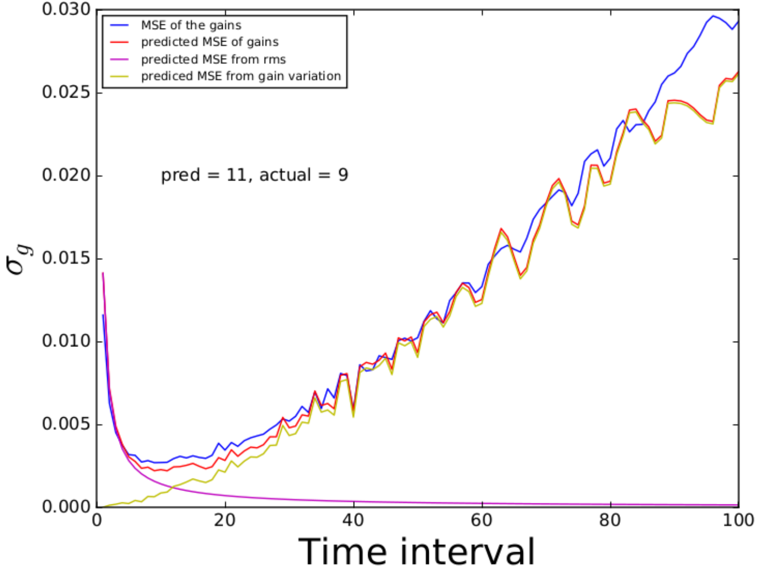
Solution intervals - noise free case



Solution intervals - noisy



Solution intervals - SNR vs. Gain Variability



Calibration - KGB

Can also "regularise" the gains using knowledge of the physical system i.e. impose

$$\mathbf{J} = KGB \quad \text{where}$$

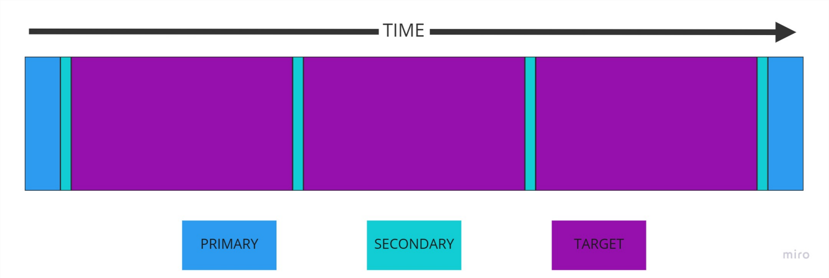
1. $K = e^{\imath\delta(t)\nu}$ captures the time variable delay caused by differences in the path length

2. $G = G(t)$ captures the time variable drift caused by instabilities in the electronic receivers

3. $B = B(\nu)$ is the bandpass of the receiver, should be stable in time

These are the first order effects that we need to correct for in almost any observation. Requires a very good model of the calibrator i.e. a primary calibrator!

Ideal Observation Structure



Calibrators

Primary:

- bright and dominant over surrounding sources
- stable with very high SNR
- known shape (preferably compact) and spectrum
- provides amplitude and phase solutions
- few and far between (especially in southern sky) so can be far away from the target

Secondary:

- often assumed to be a point source at the phase center
- can bootstrap model from selfcal as long as it's a simple field with good SNR
- only trust phase solutions
- closer to target

Typical 1GC recipe (post flagging)

1. set primary model flux (setjy)
2. delay (gaincal with gaintype=‘K’) on primary (per scan)
3. amplitude (gaincal with gaintype=‘G’) on primary (per scan)
4. bandpass on primary (global)
5. transfer solutions to secondary (applycal)
6. delay (gaincal with gaintype=‘K’) on secondary (per scan)
7. phase (gaincal with gaintype=‘G’ and calmode=‘p’) on secondary (per scan)
8. calibrator diagnostics
9. transfer solutions to target (applycal)