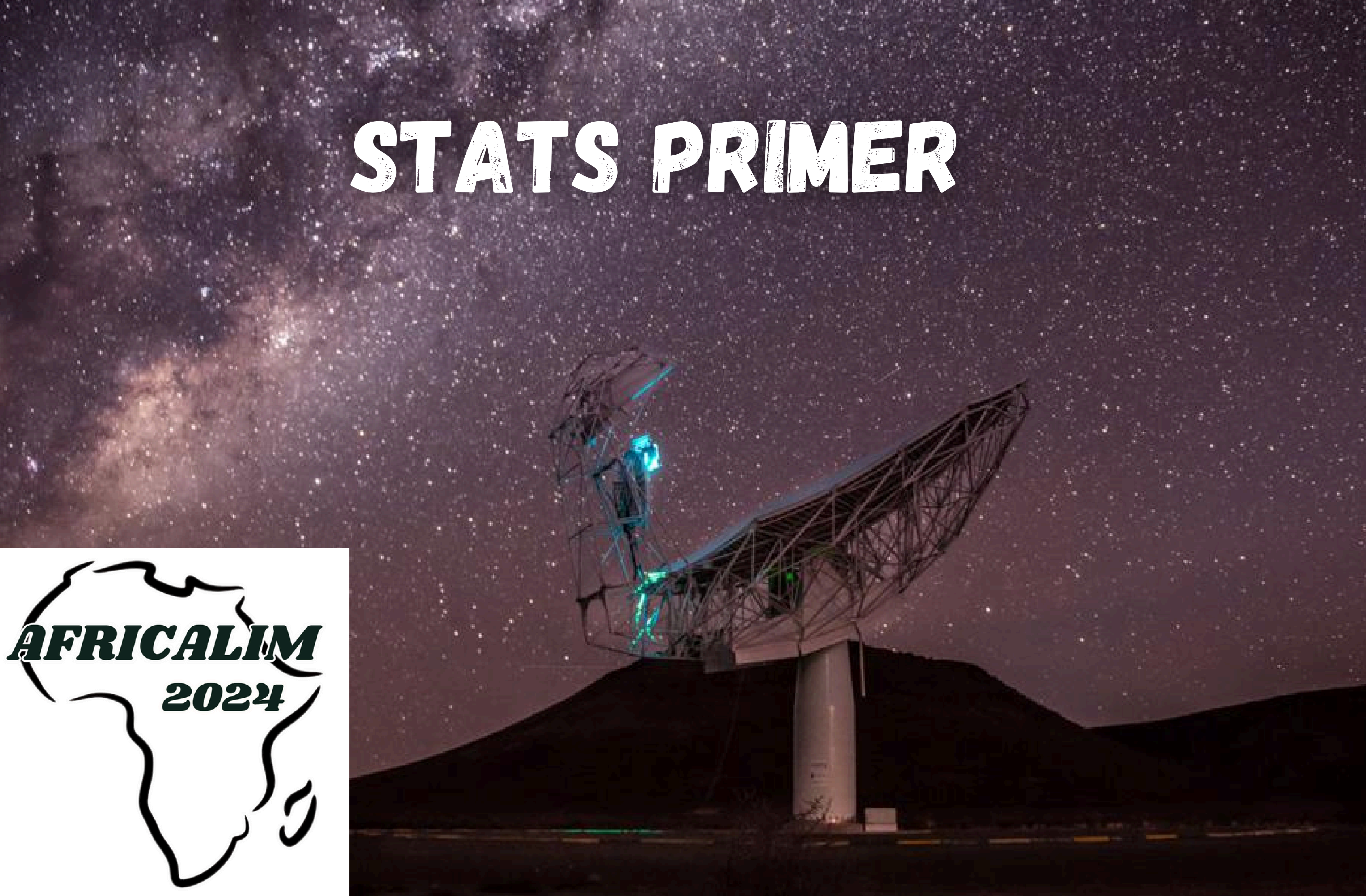
STATS PRIMER



AFRICALIM 2024

NRF National Research Foundation | SARAO South African Radio Astronomy Observatory

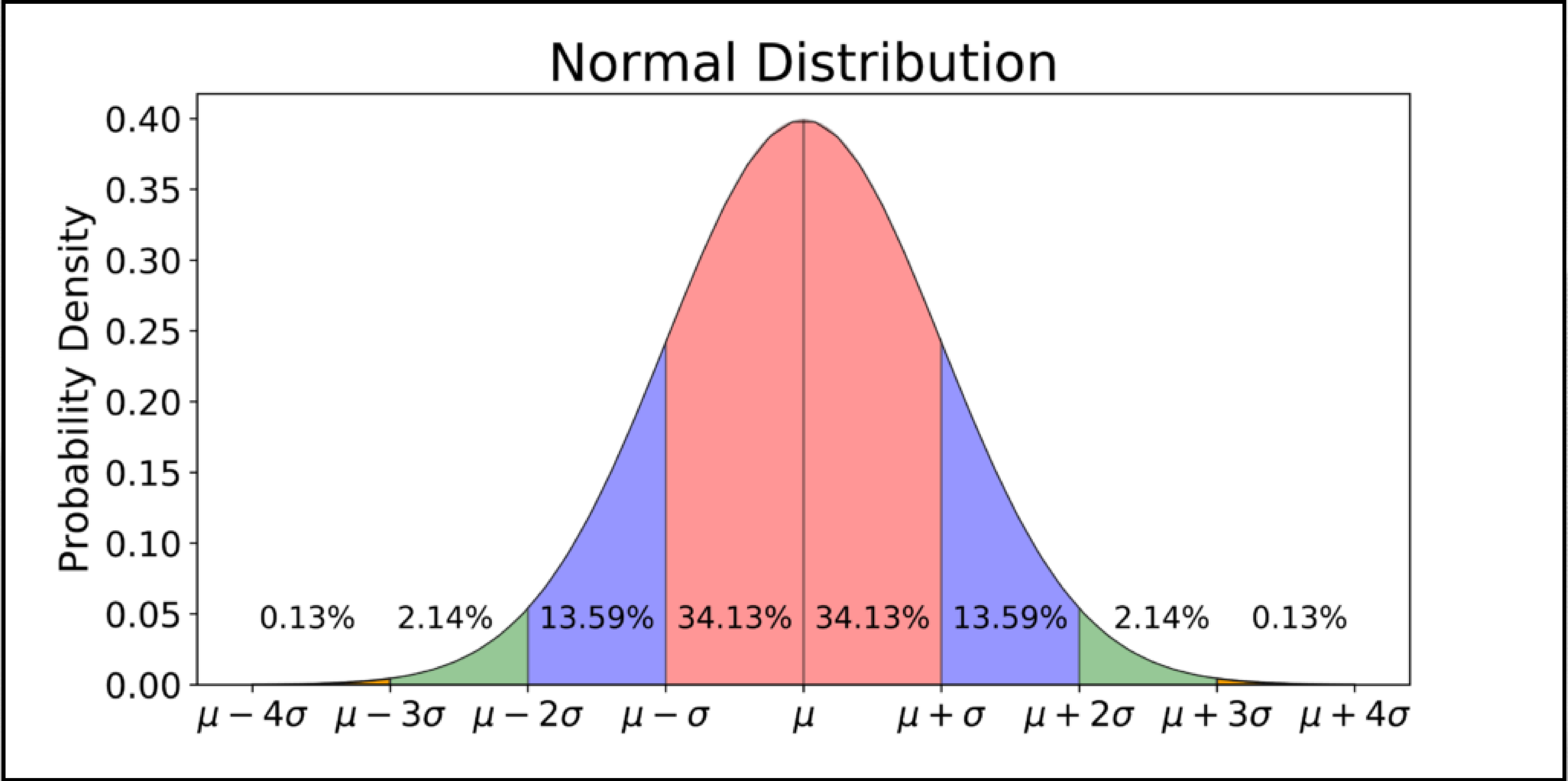
WITS CfA CENTRE FOR ASTROPHYSICS

https://ratt.center RATT

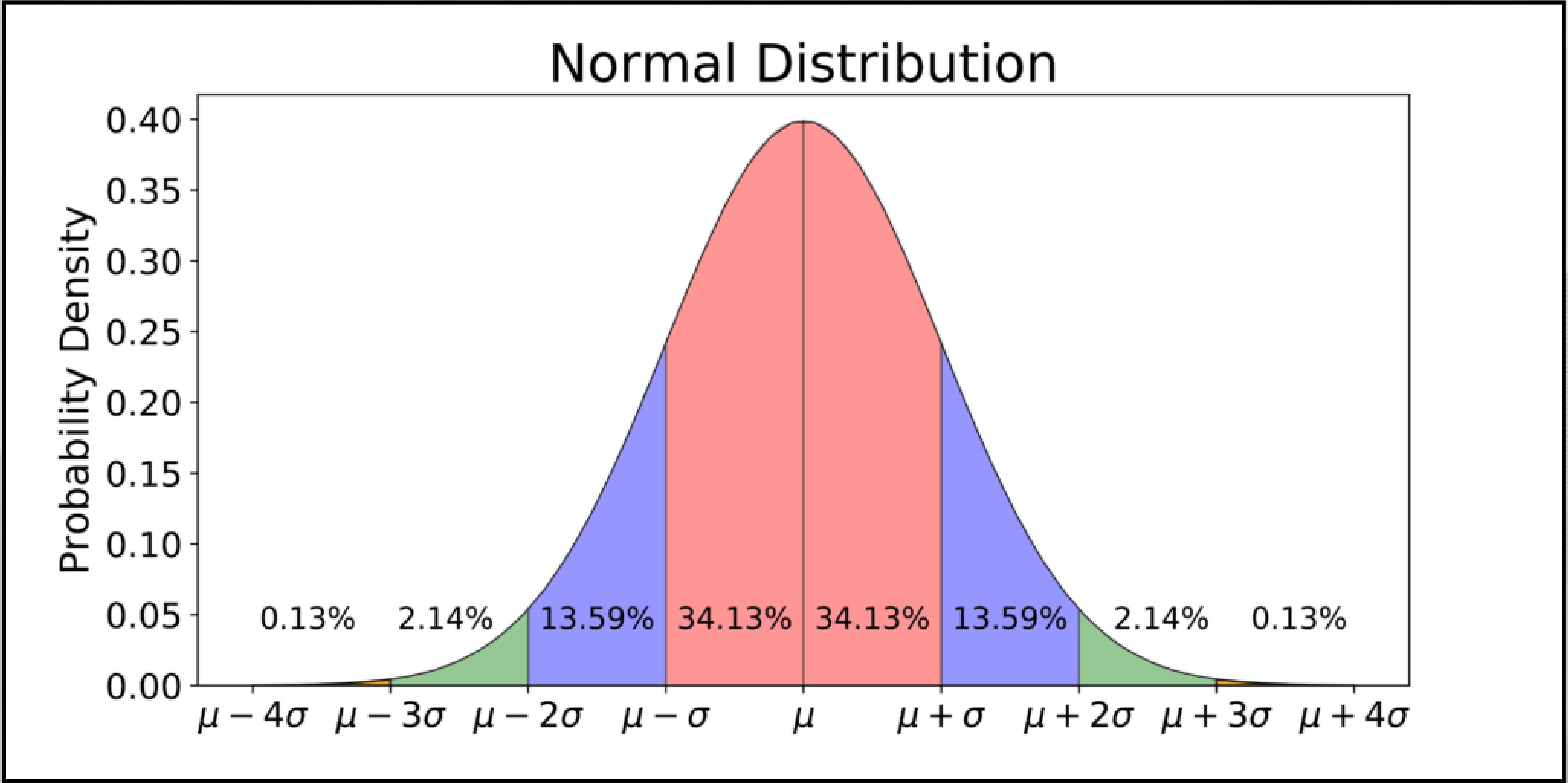
Deloitte.

BREAKTHROUGH INITIATIVES BREAKTHROUGH PRIZE FOUNDATION

DARA DEVELOPMENT IN AFRICA WITH RADIO ASTRONOMY


Normal Distribution
Probability Density
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.13%
2.14%
13.59%
34.13%
34.13%
13.59%
2.14%
0.13%
$\mu - 4\sigma$
$\mu - 3\sigma$
$\mu - 2\sigma$
$\mu - \sigma$
μ
$\mu + \sigma$
$\mu + 2\sigma$
$\mu + 3\sigma$
$\mu + 4\sigma$

$$p(x|\mu) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(-\frac{1}{2\sigma^2}(x-\mu)^\dagger(x-\mu)\right)}$$



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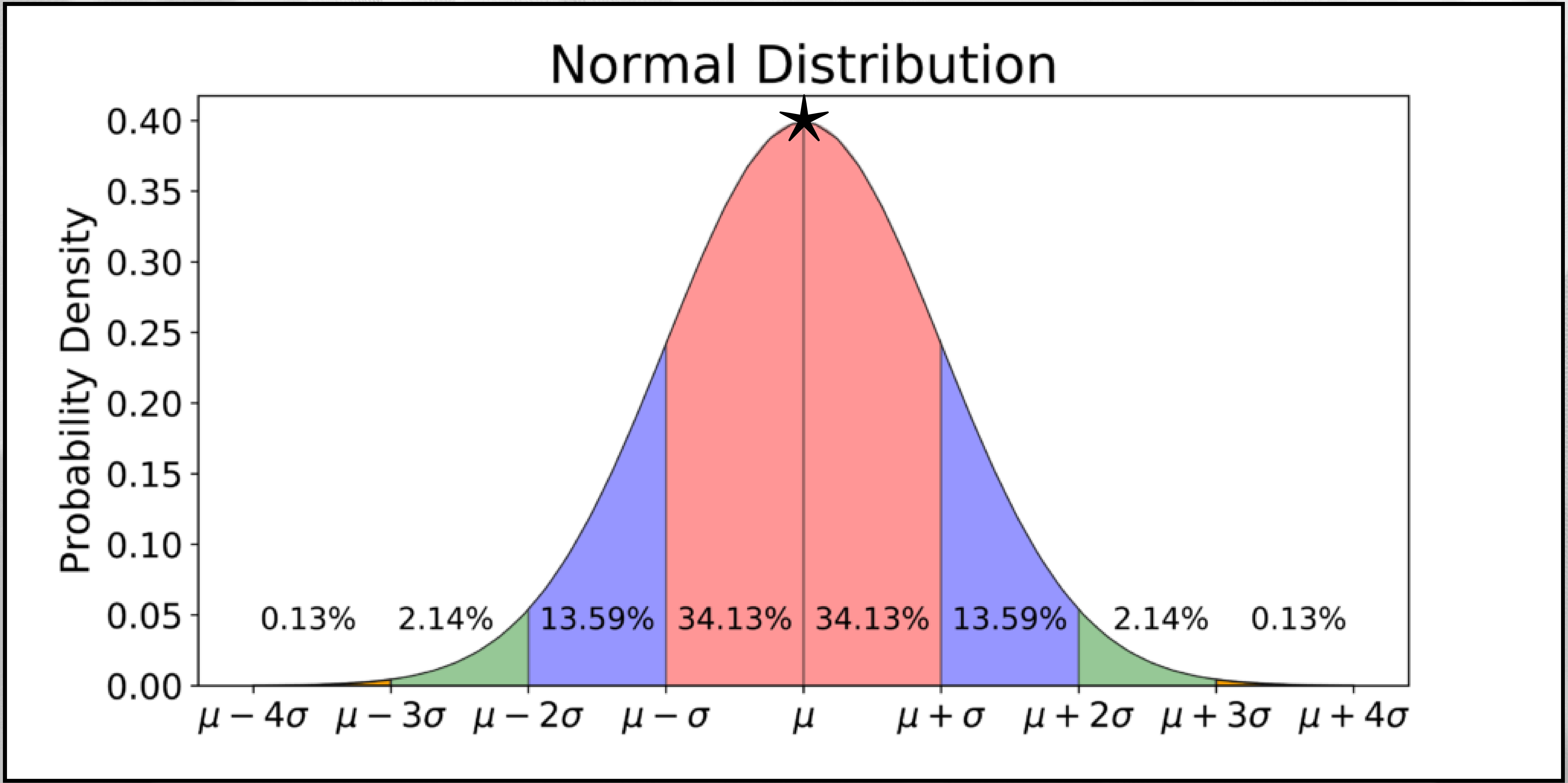

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$$-\log p(x|\mu) \propto \frac{1}{2\sigma^2}(x-\mu)^\dagger(x-\mu)$$

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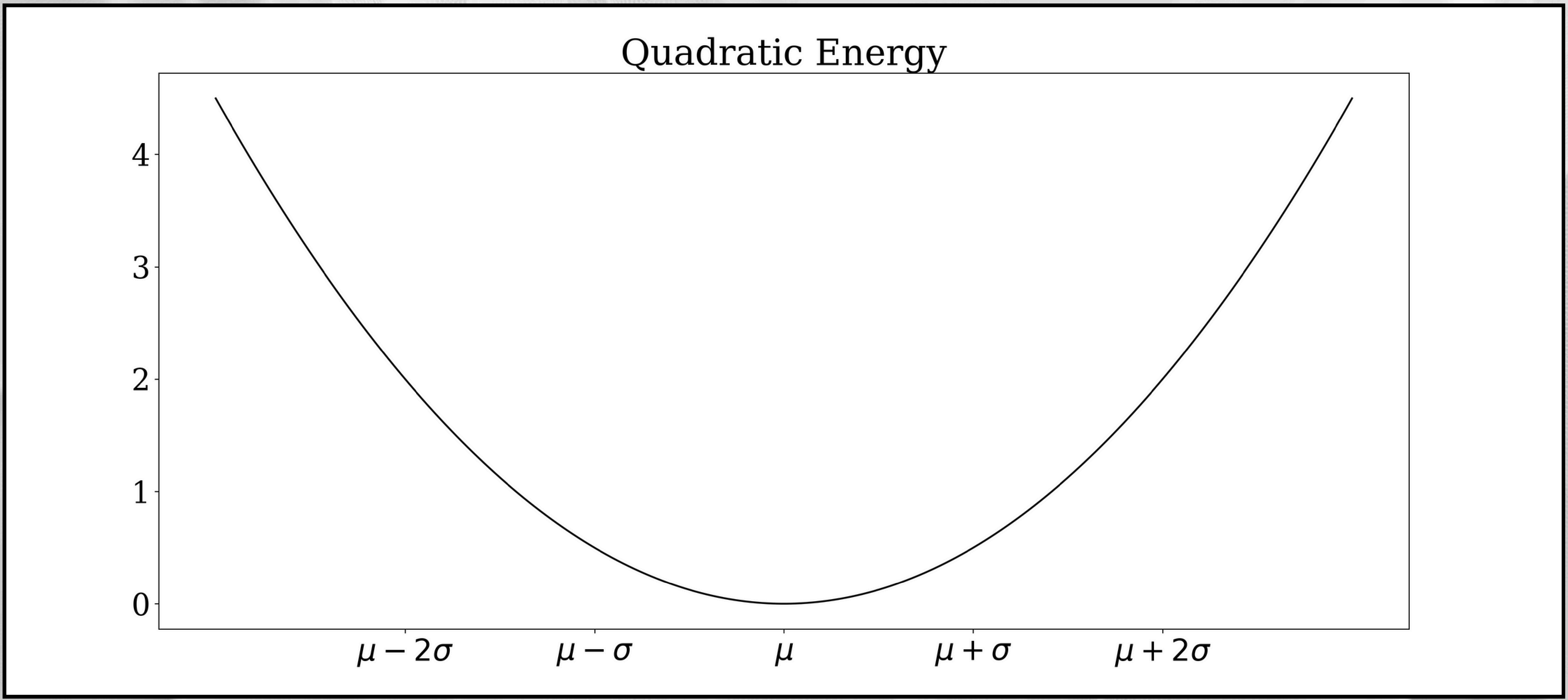
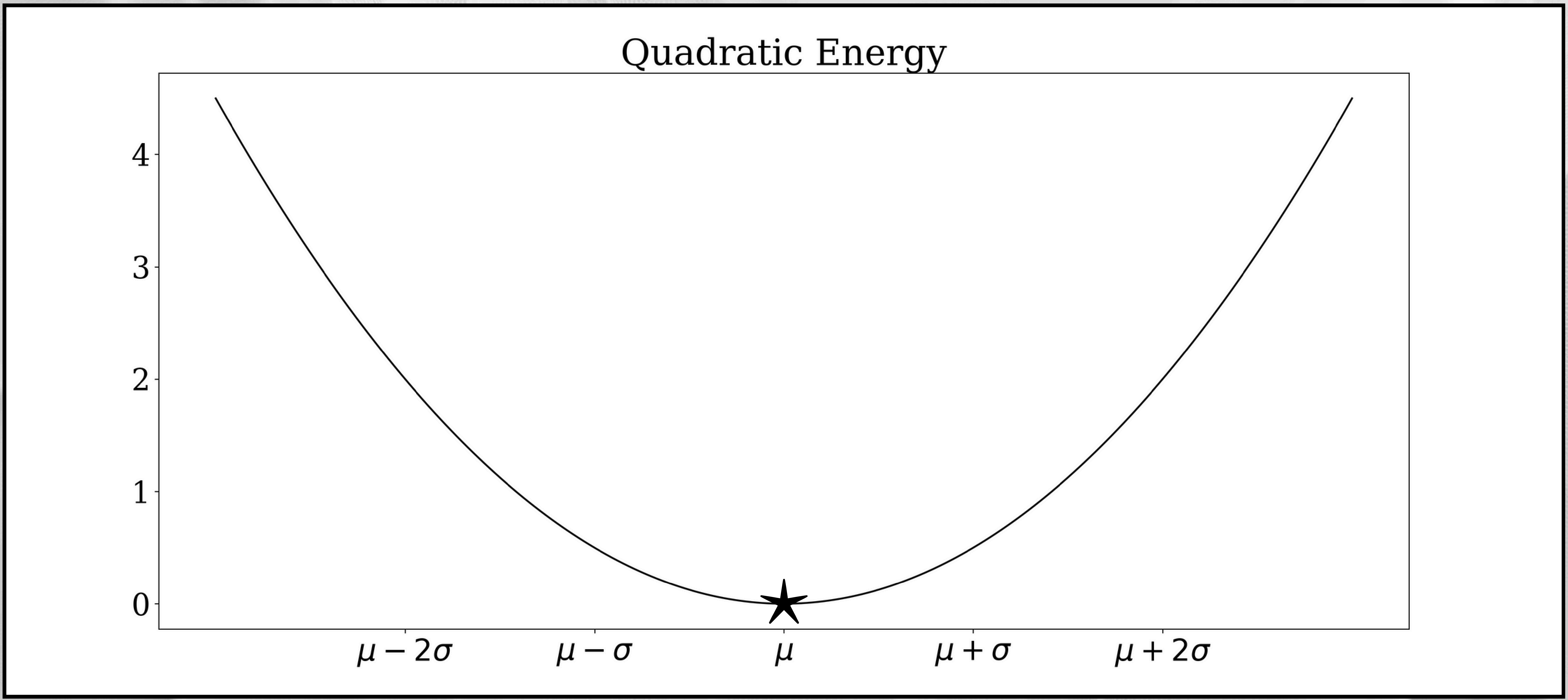


$$\chi^2 = \frac{1}{2\sigma^2}(x-\mu)^\dagger(x-\mu)$$



MULTI-VARIATE GAUSSIAN

MULTI-VARIATE GAUSSIAN

NORMALISED PROBABILITY DENSITY

$$p(x|\mu,\Sigma) = \frac{1}{\sqrt{(2\pi)^n|\Sigma|}}\exp\left(\frac{1}{2}(x-\mu)^\dagger\Sigma^{-1}(x-\mu)\right)$$

MULTI-VARIATE GAUSSIAN

NORMALISED PROBABILITY DENSITY

$$p(x|\mu,\Sigma) = \frac{1}{\sqrt{(2\pi)^n|\Sigma|}}\exp\left(\frac{1}{2}(x-\mu)^\dagger\Sigma^{-1}(x-\mu)\right)$$

EVALUATE THE FOLLOWING

$$\int \exp\left(\frac{1}{2}(x-\mu)^\dagger\Sigma^{-1}(x-\mu)\right)dx = \ ?$$

LINEAR + GAUSSIAN NOISE

THE MEASUREMENT MODEL

$$y = Rx + \epsilon, \quad \epsilon \sim \mathcal{N}(0, \Sigma)$$

LINEAR + GAUSSIAN NOISE

THE MEASUREMENT MODEL

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DATA

LINEAR + GAUSSIAN NOISE

THE MEASUREMENT MODEL

$$y = Rx + \epsilon, \quad \epsilon \sim \mathcal{N}(0, \Sigma)$$

NOISE

LINEAR + GAUSSIAN NOISE

THE MEASUREMENT MODEL

$$y = Rx + \epsilon, \quad \epsilon \sim \mathcal{N}(0, \Sigma)$$

NOISE COVARIANCE

LINEAR + GAUSSIAN NOISE

THE MEASUREMENT MODEL

$$y = Rx + \epsilon, \quad \epsilon \sim \mathcal{N}(0, \Sigma)$$

SIGNAL

LINEAR + GAUSSIAN NOISE

THE MEASUREMENT MODEL

$$y = Rx + \epsilon, \quad \epsilon \sim \mathcal{N}(0, \Sigma)$$

MEASUREMENT OPERATOR

LINEAR + GAUSSIAN NOISE

THE MEASUREMENT MODEL

$$y = Rx + \epsilon, \quad \epsilon \sim \mathcal{N}(0, \Sigma)$$

GIVES RISE TO A LIKELIHOOD OF THE FORM

$$p(y|x) = \frac{1}{\sqrt{(2\pi)^n|\Sigma|}} \exp\left(-\frac{1}{2}(y - Rx)^\dagger \Sigma^{-1}(y - Rx)\right)$$

LINEAR + GAUSSIAN NOISE

THE MEASUREMENT MODEL

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CLOSED FORM SOLUTION

$$\nabla \chi^2 = 0 \ \rightarrow \hat{x} = (R^\dagger \Sigma^{-1} R)^{-1} R^\dagger \Sigma^{1} y$$

LINEAR + GAUSSIAN NOISE

THE MEASUREMENT MODEL

$$y = Rx + \epsilon, \quad \epsilon \sim \mathcal{N}(0, \Sigma)$$

GIVES RISE TO A LIKELIHOOD OF THE FORM

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CLOSED FORM SOLUTION

$$\boxed{\nabla \chi^2 = 0 \ \rightarrow \hat{x} = (R^\dagger \Sigma^{-1} R)^{-1} R^\dagger \Sigma^1 y}$$

NON-LINEAR + GAUSSIAN NOISE

THE MEASUREMENT MODEL

$$y = f(x) + \epsilon, \quad \epsilon \sim \mathcal{N}(0, \Sigma)$$

GIVES RISE TO

$$\chi^2 = \frac{1}{2}(y - f(x))^\dagger \Sigma^{-1}(y - f(x))$$

NO GENERAL CLOSED FORM SOLUTION

NON-LINEAR + GAUSSIAN NOISE

THE MEASUREMENT MODEL

$$y = f(x) + \epsilon, \quad \epsilon \sim \mathcal{N}(0, \Sigma)$$

GIVES RISE TO

$$\chi^2 = \frac{1}{2}(y - f(x))^\dagger \Sigma^{-1}(y - f(x))$$

NO GENERAL CLOSED FORM SOLUTION

GAUSS-NEWTON TO THE RESCUE

$$\boxed{x_{k+1} = x_k + (J_k^\dagger \Sigma^{-1} J_k)^{-1} J_k^\dagger \Sigma^{-1}(y - f(x_k))}$$