IMAGING

AFRICALIM 2024

NRF National Research Foundation | SARAO South African Radio Astronomy Observatory

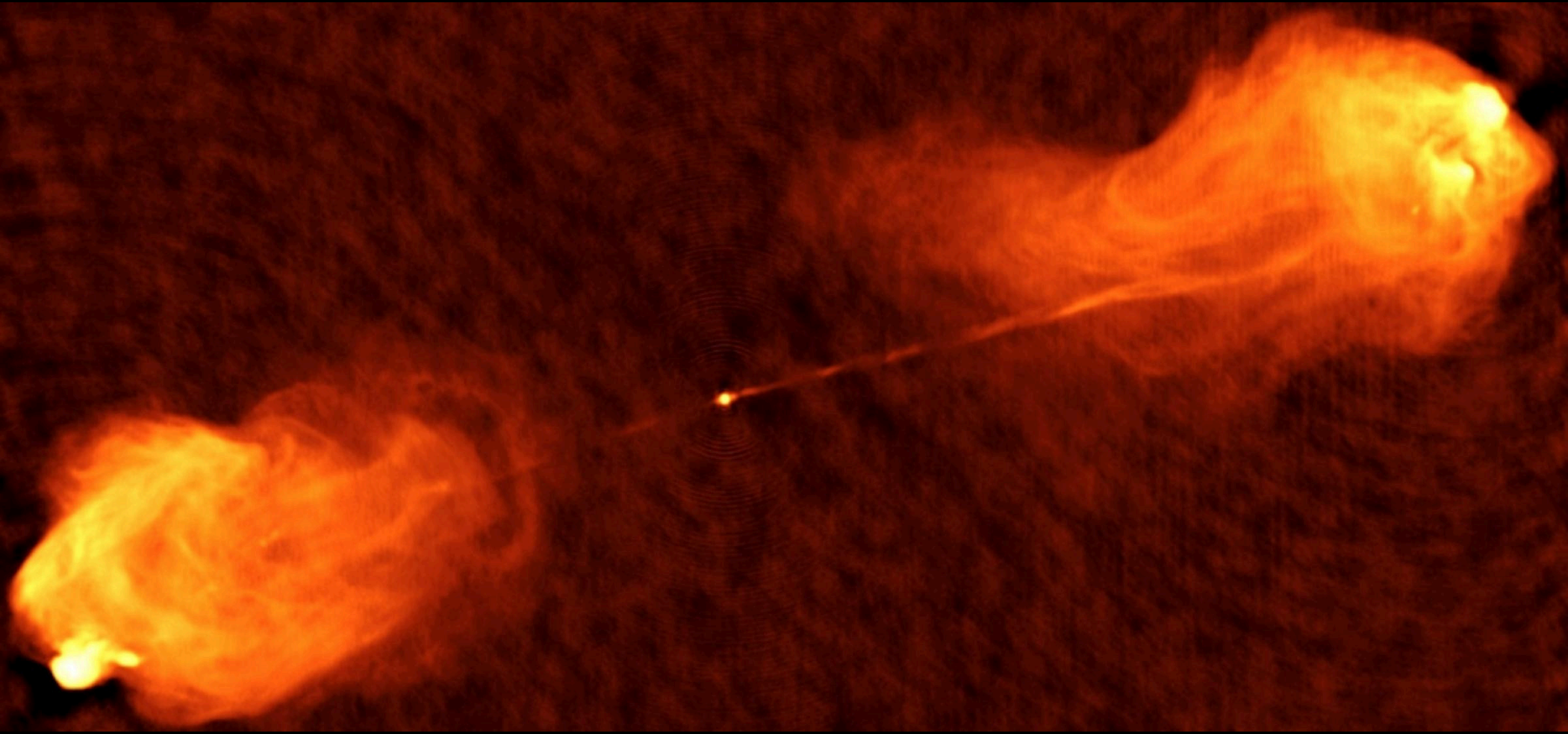
WITS CfA CENTRE FOR ASTROPHYSICS

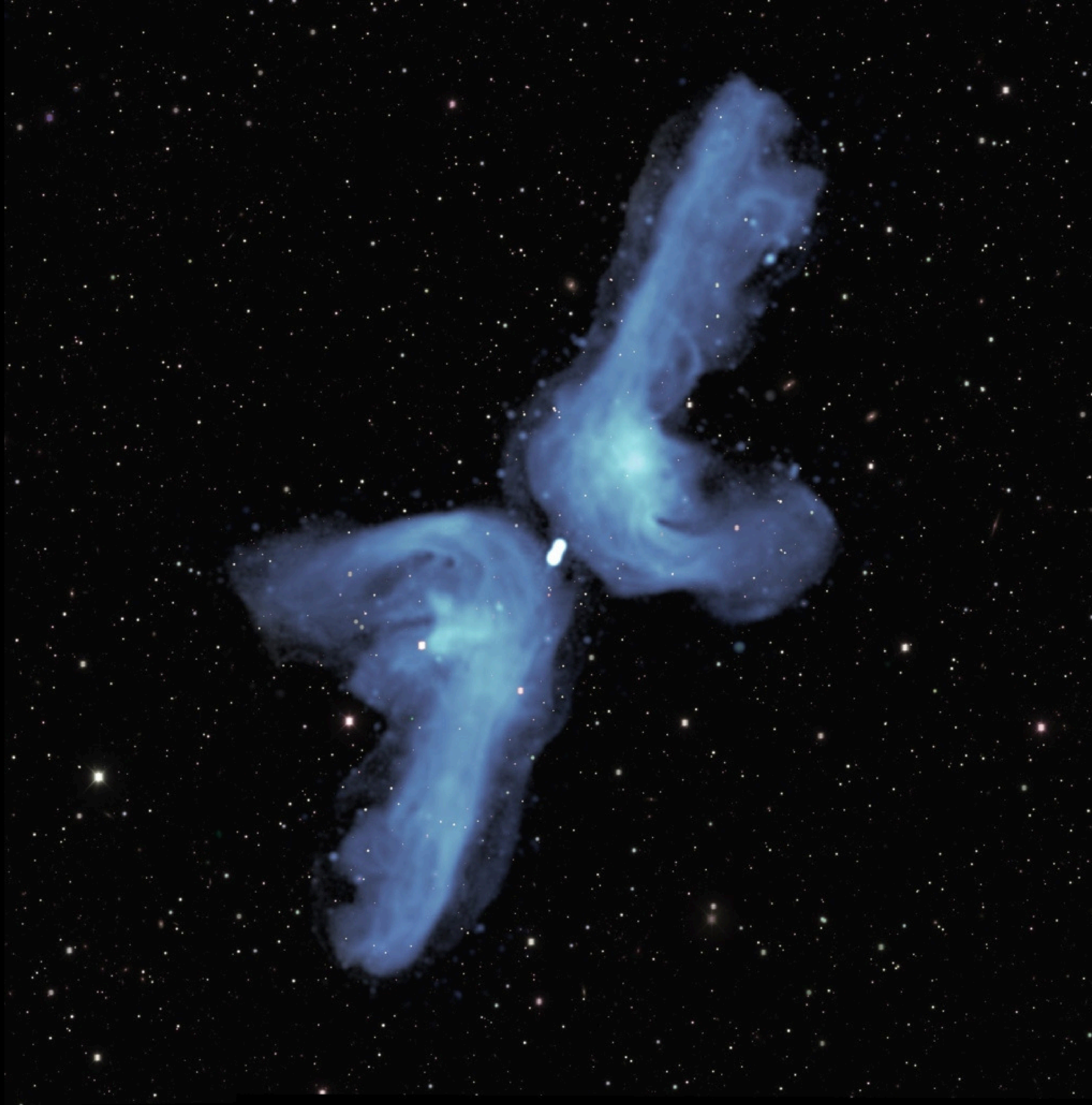
https://ratt.center RATT

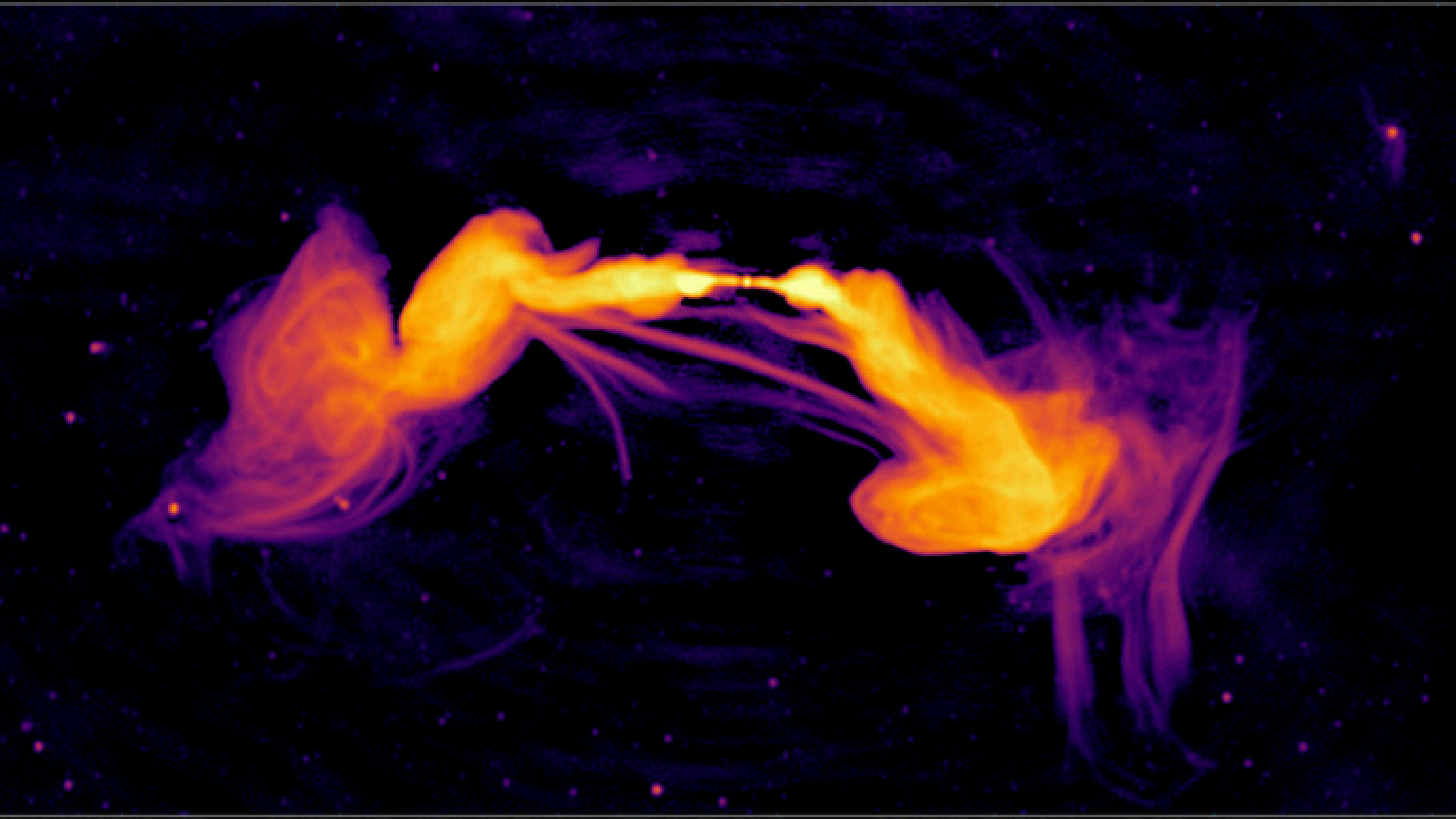
Deloitte.

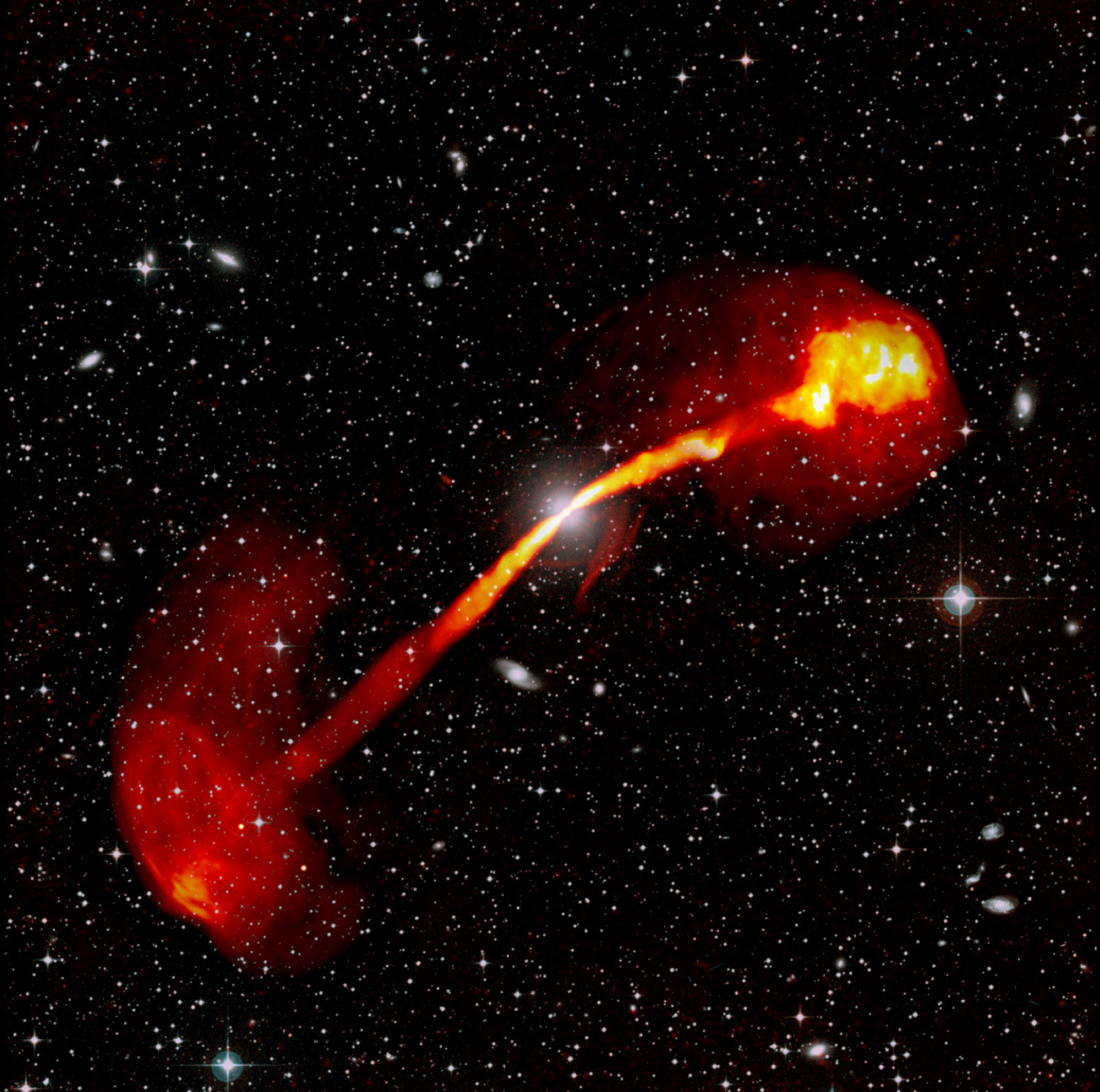
BREAKTHROUGH INITIATIVES BREAKTHROUGH PRIZE FOUNDATION

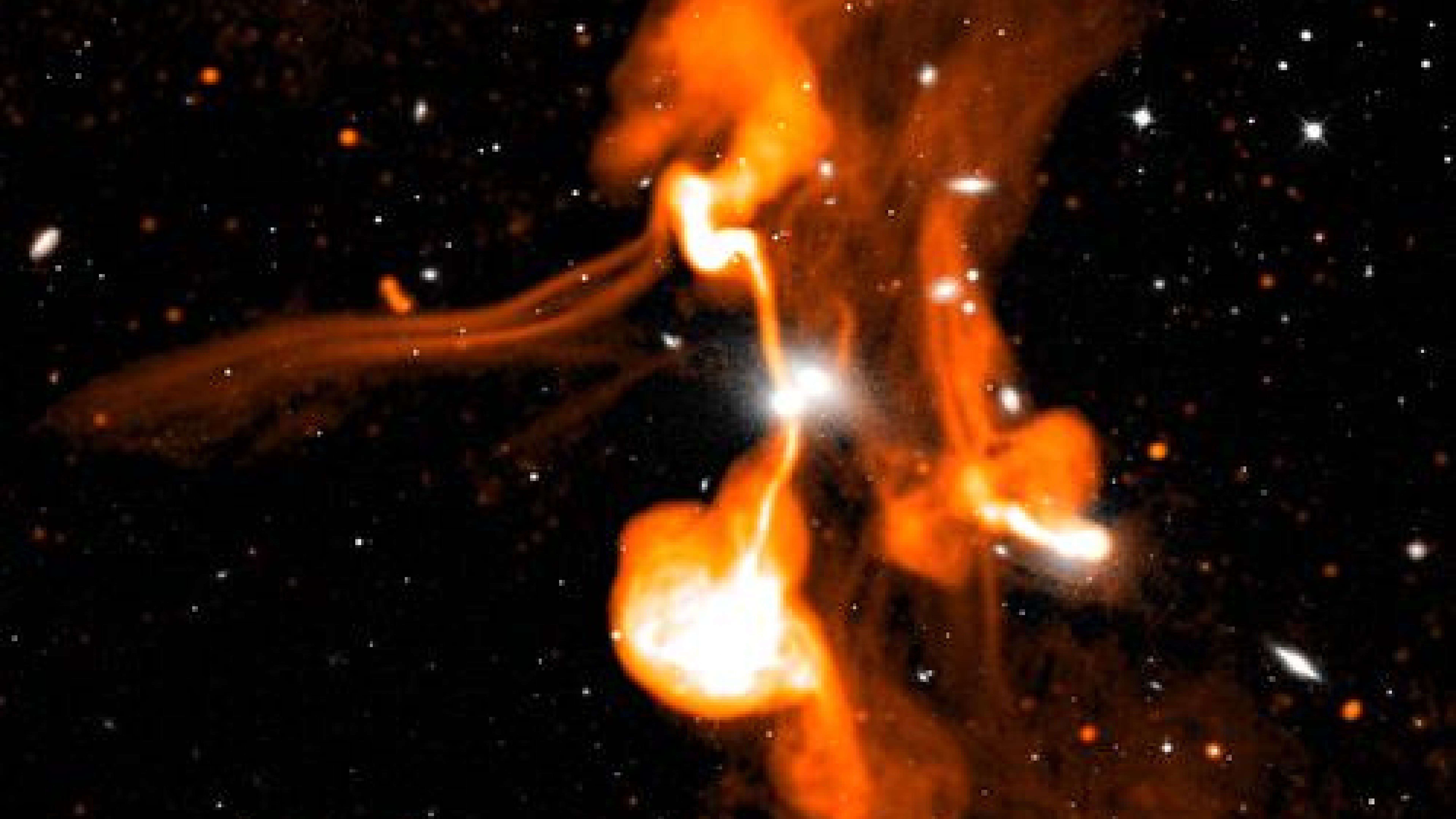
DARA DEVELOPMENT IN AFRICA WITH RADIO ASTRONOMY



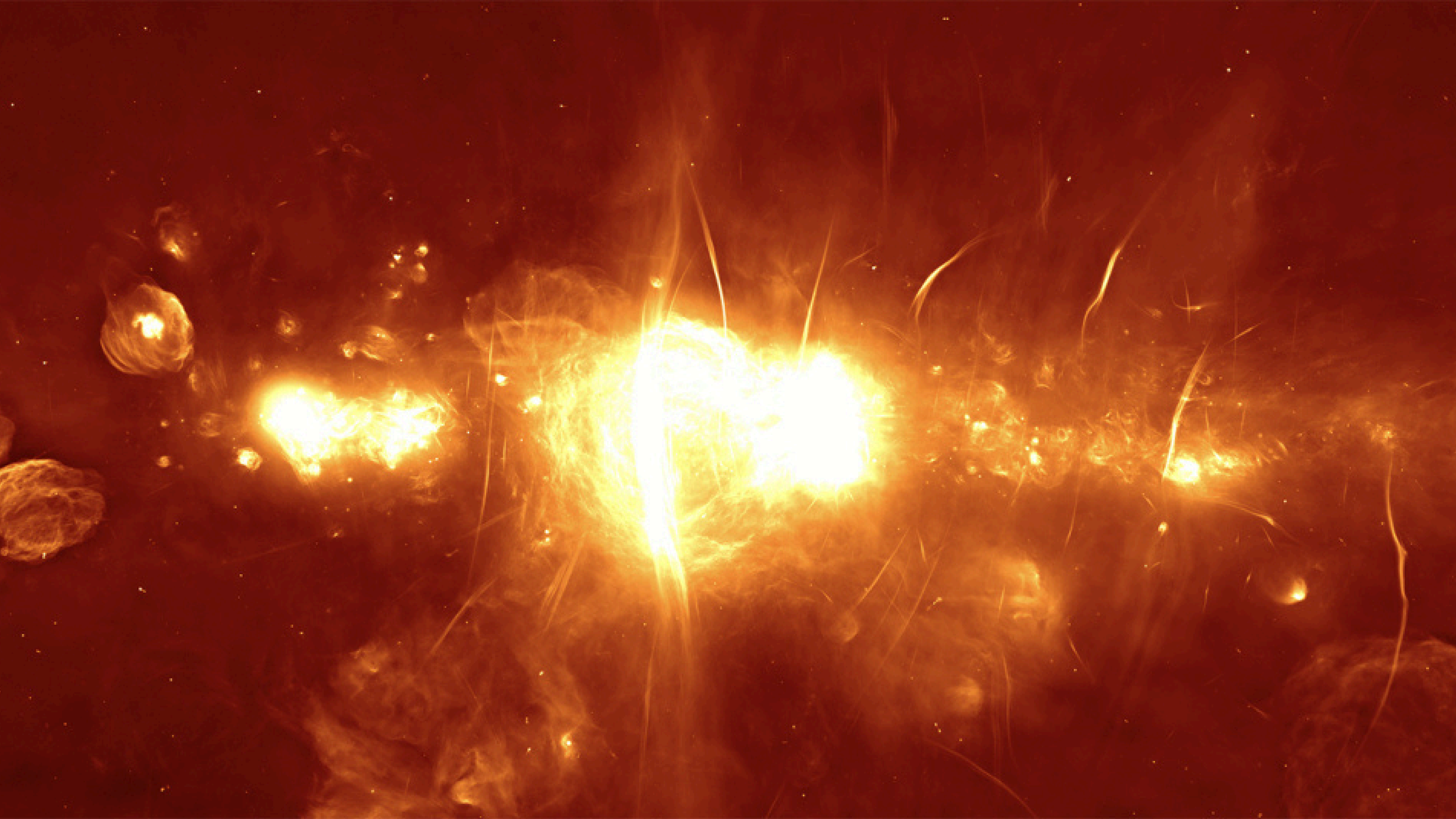






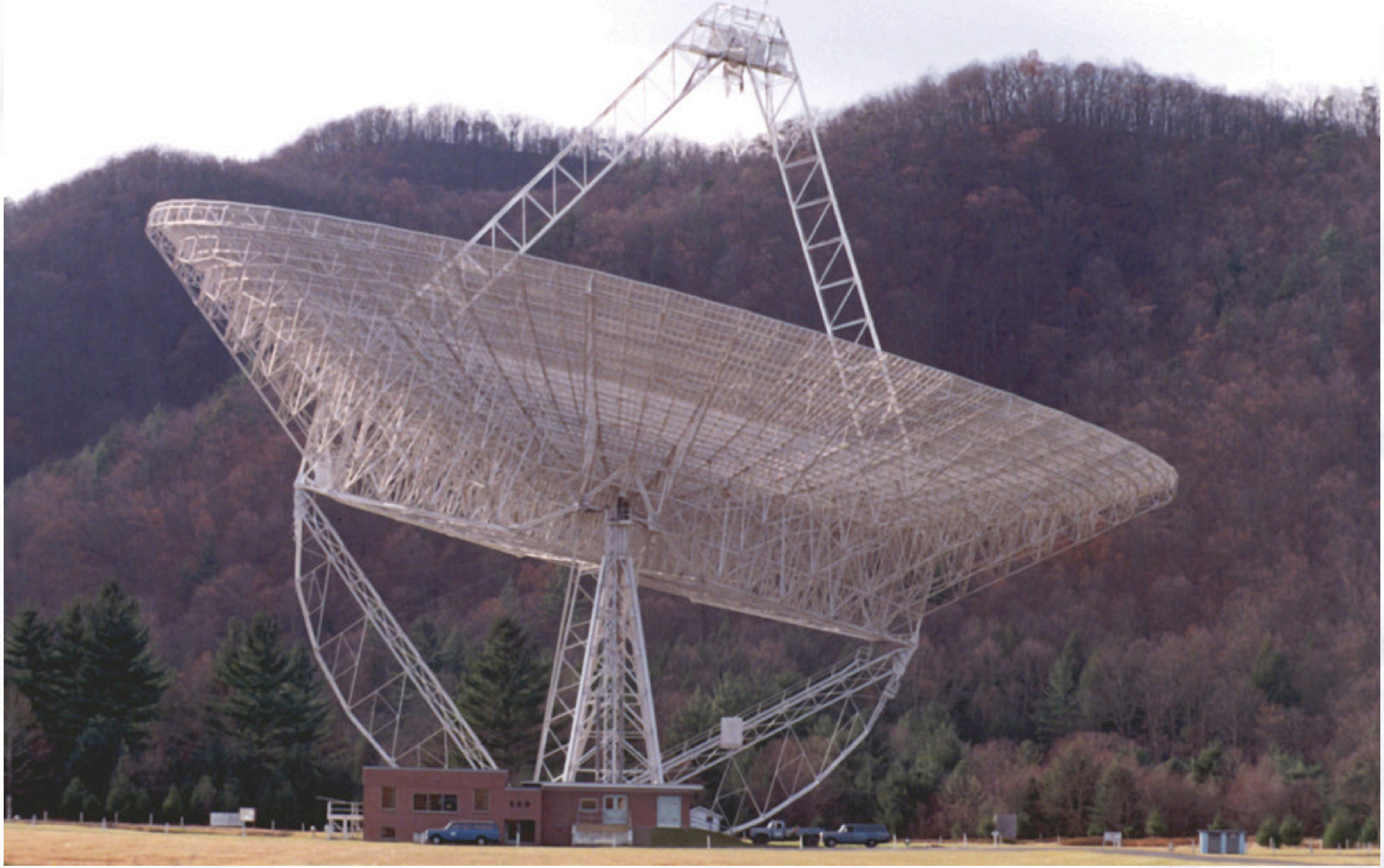


“From The Cradle To The Grave” (Ben Hugo, SARAO & Rhodes)



MOTIVATION FOR INTERFEROMETRY :

Green Bank Transit Telescope (100 m)



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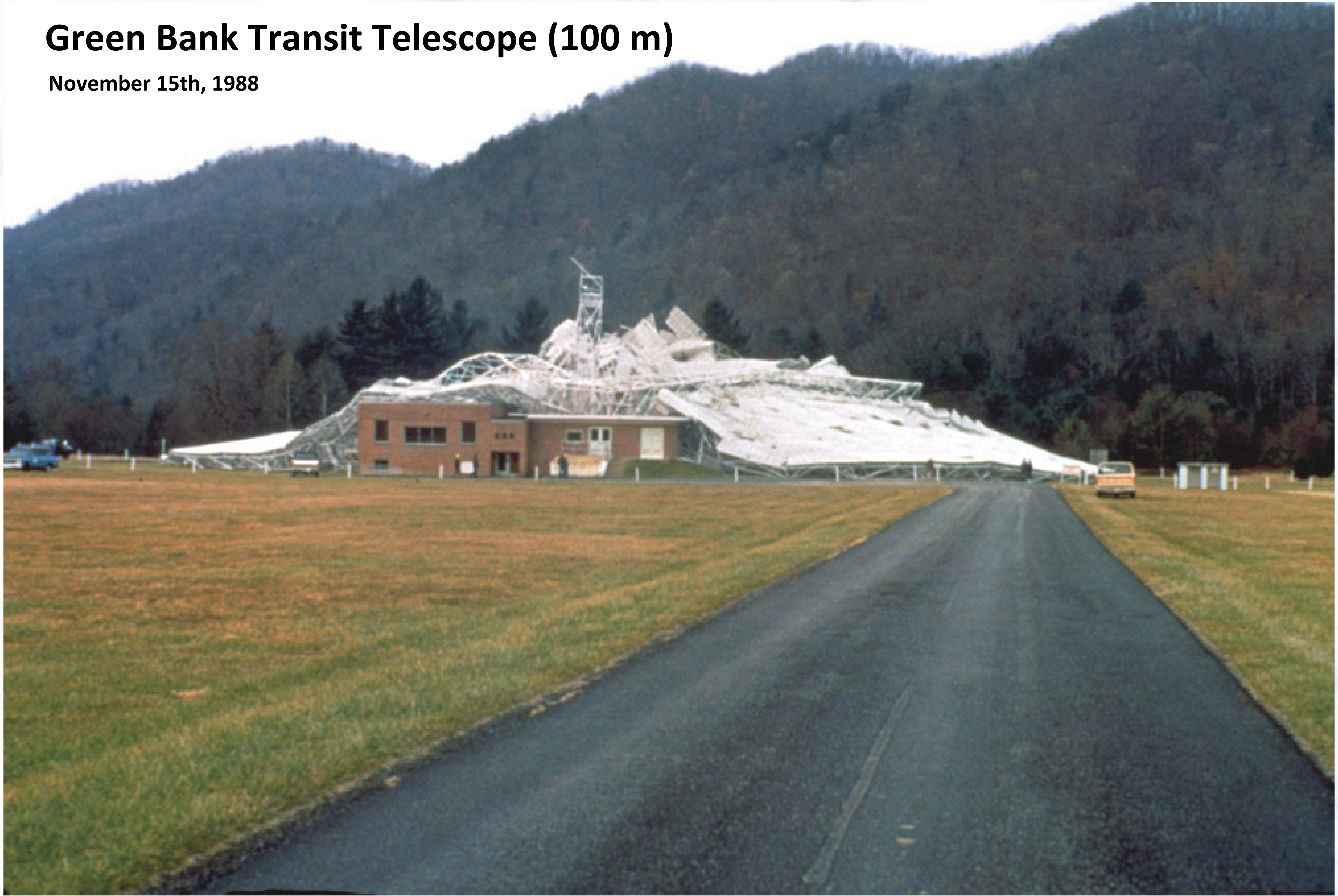

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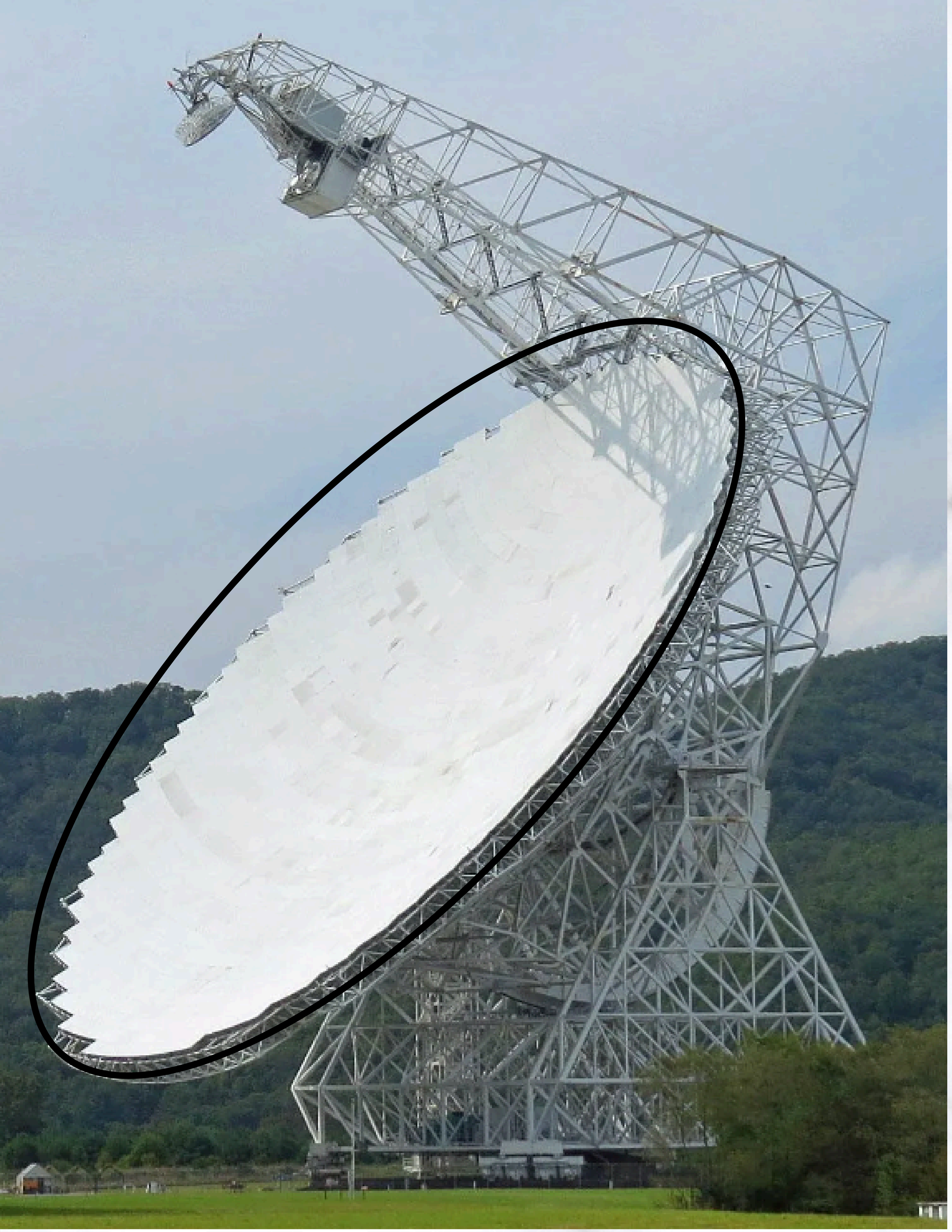
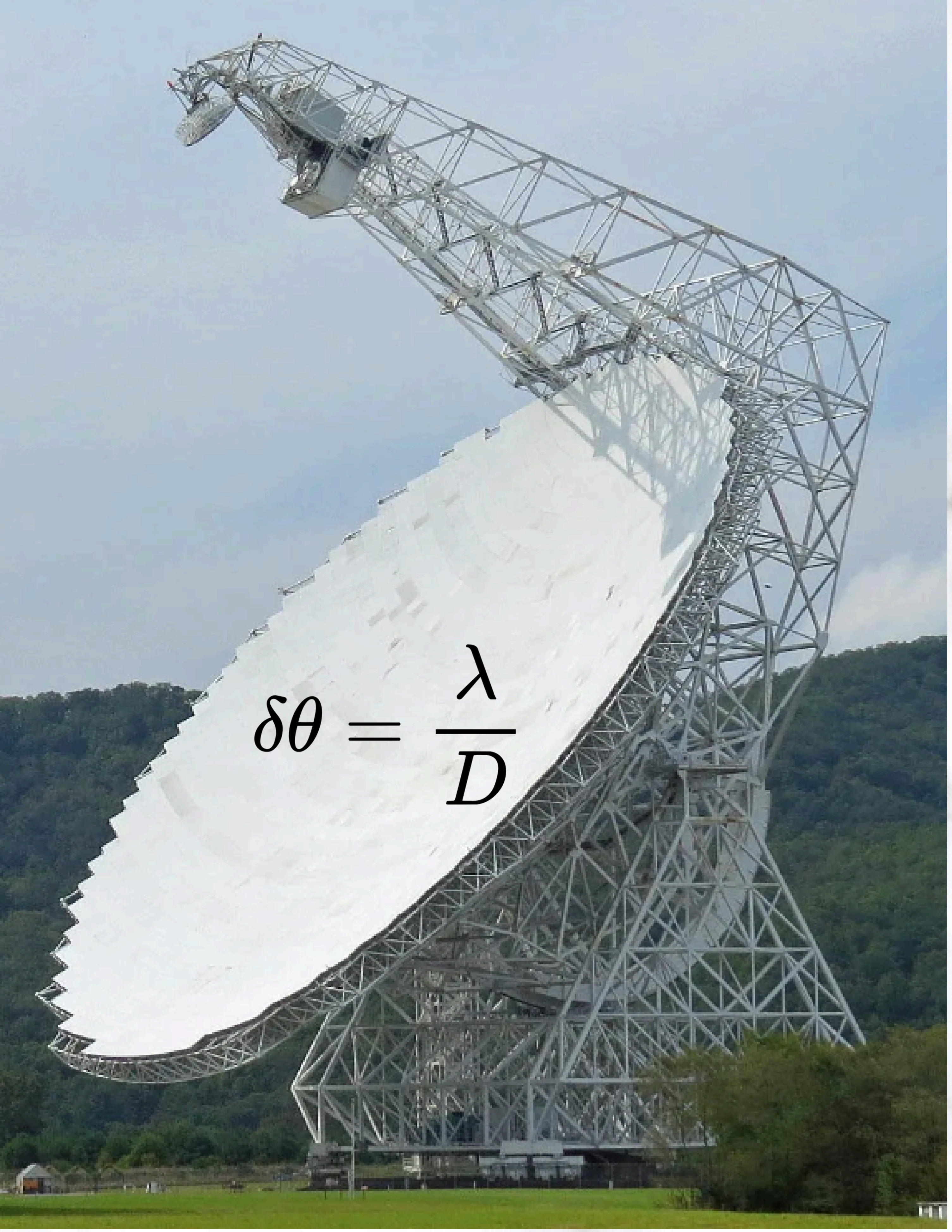
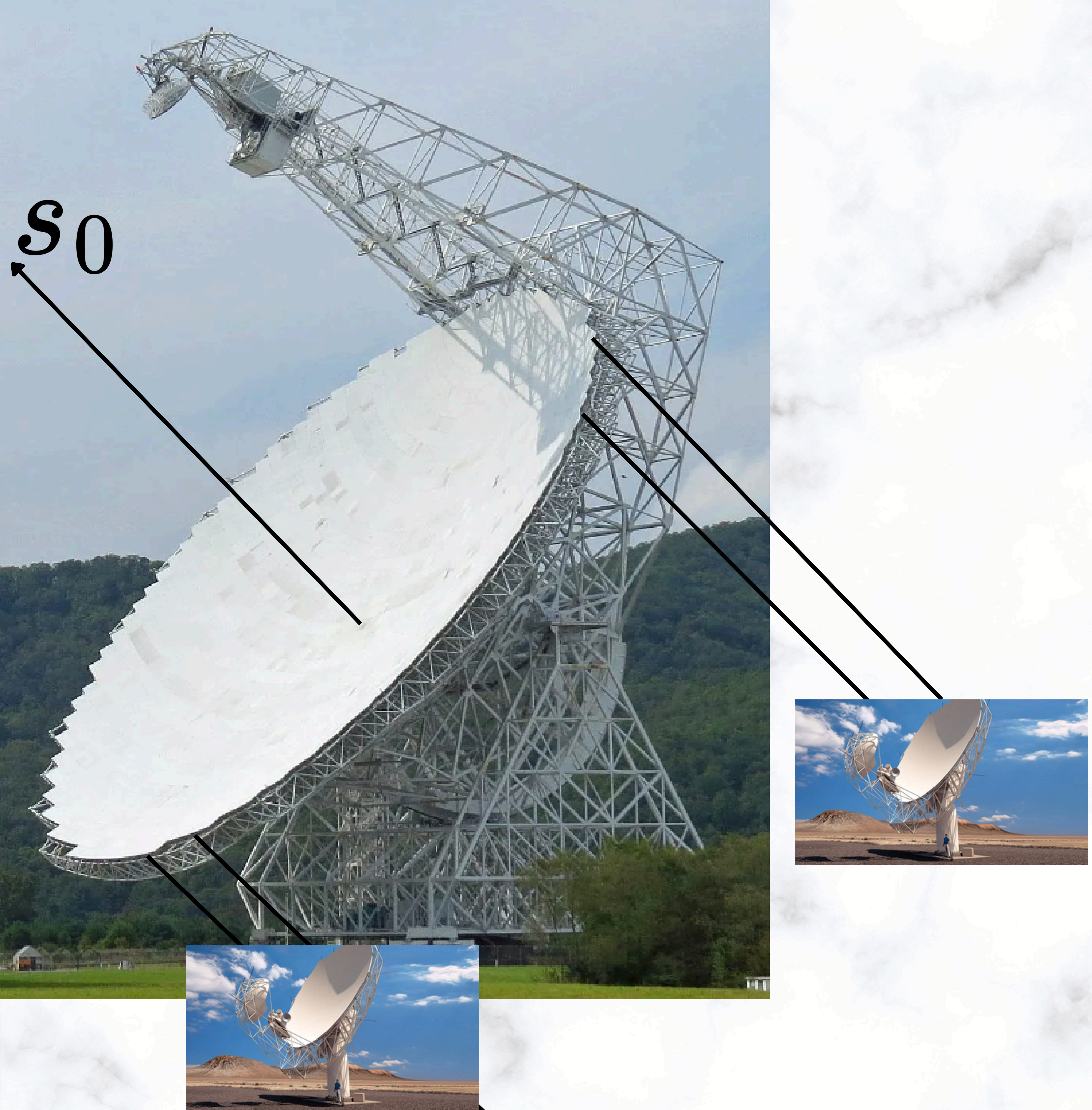
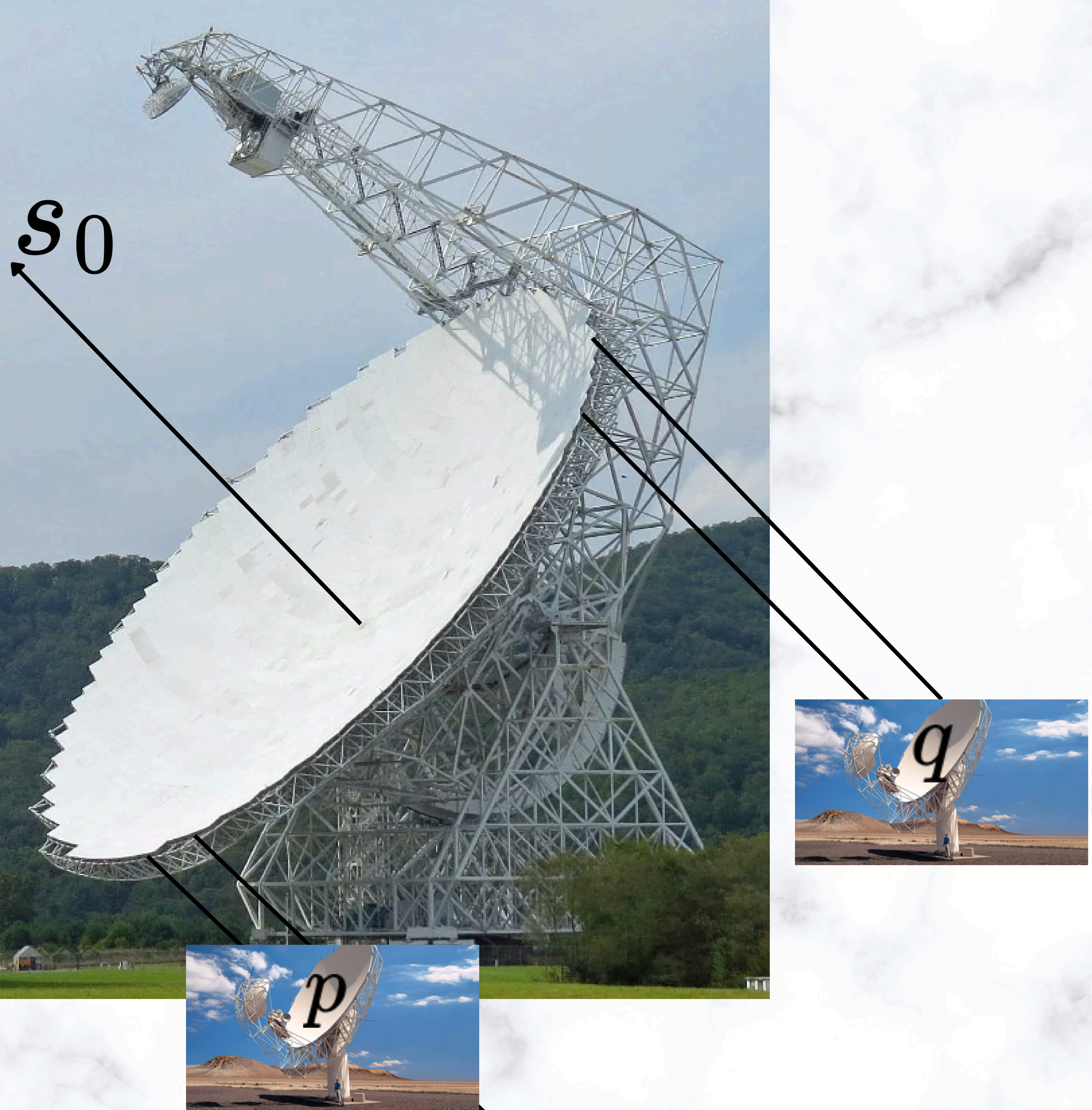
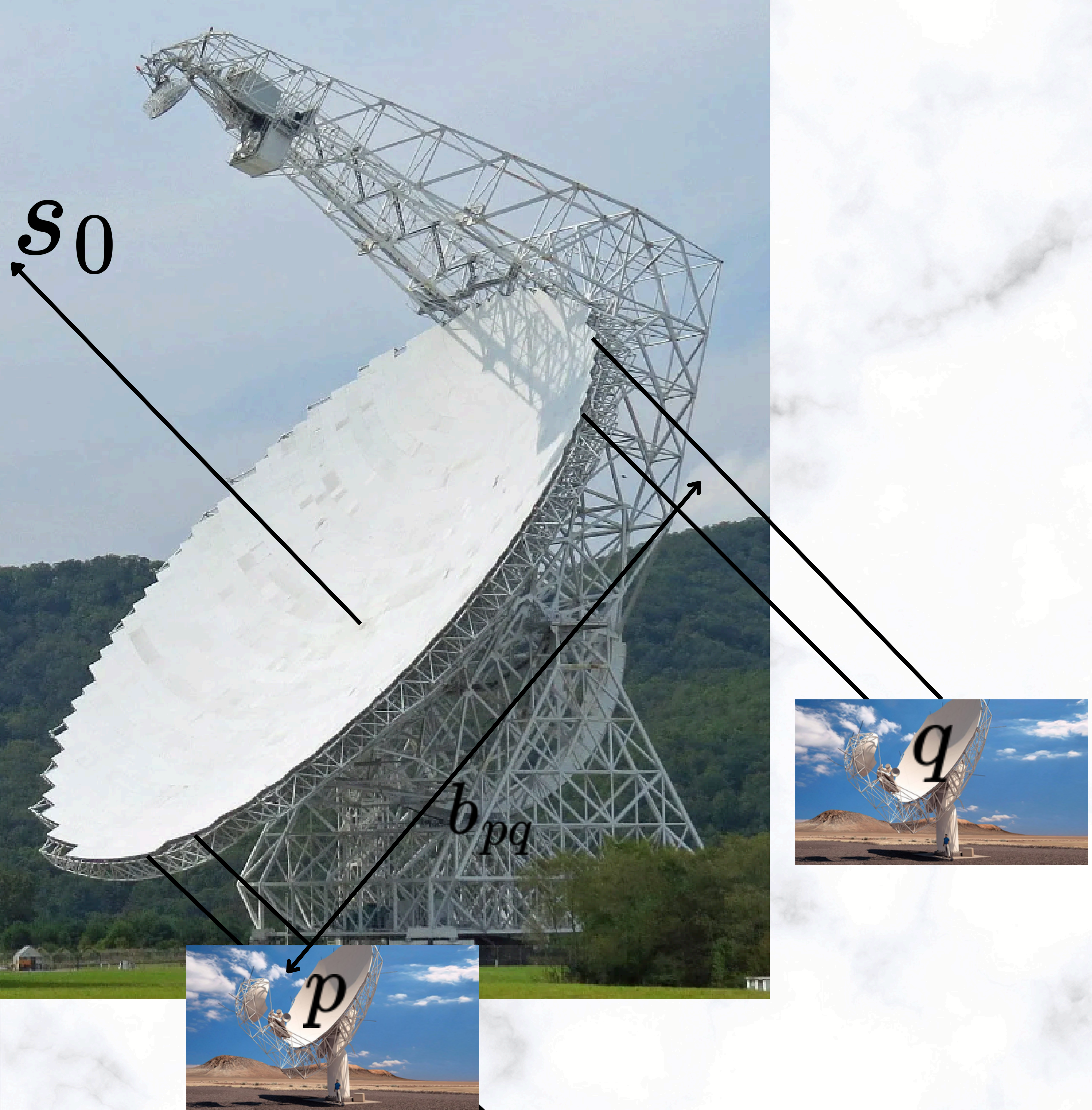




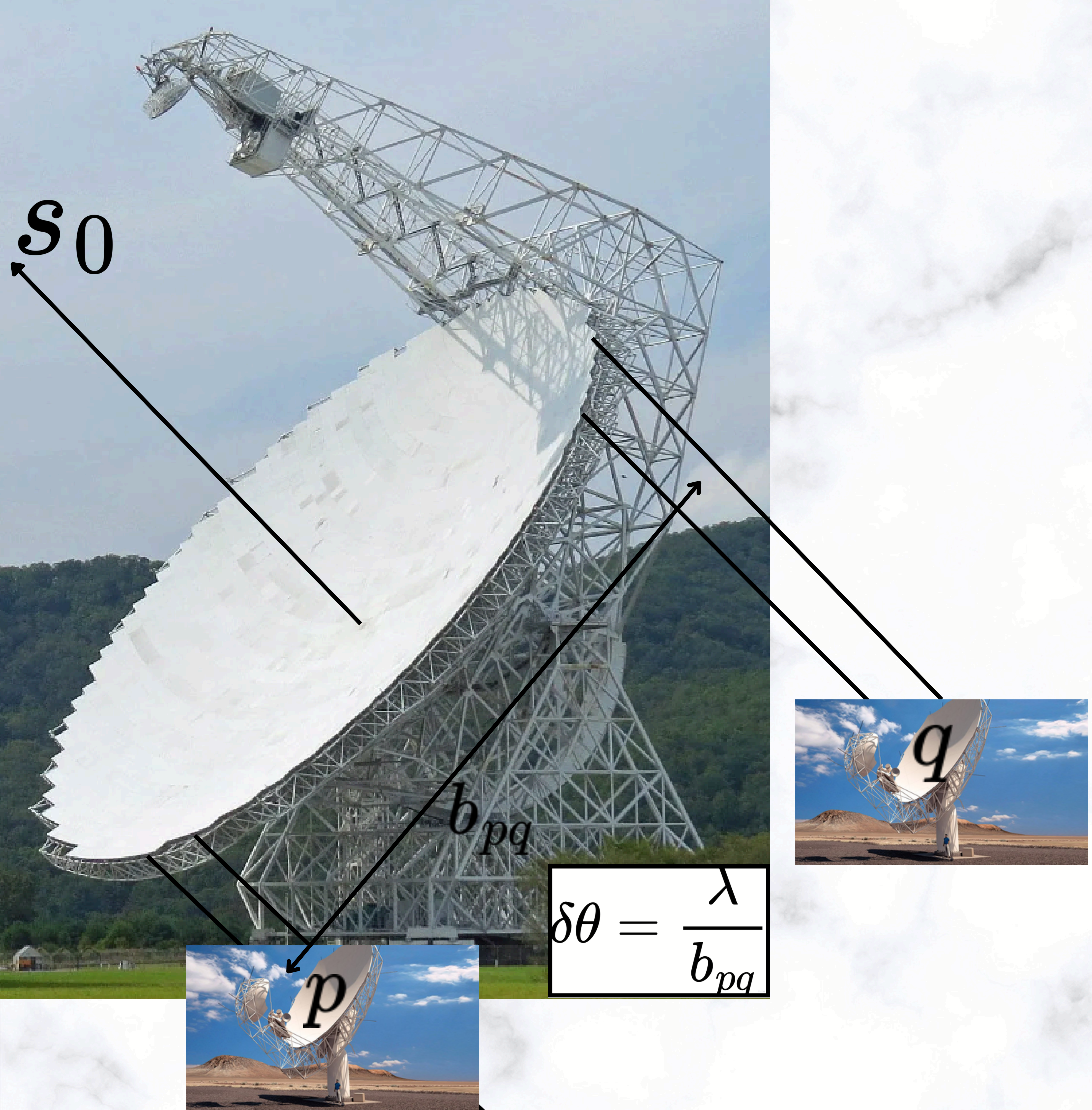
$$\delta\theta = \frac{\lambda}{D}$$

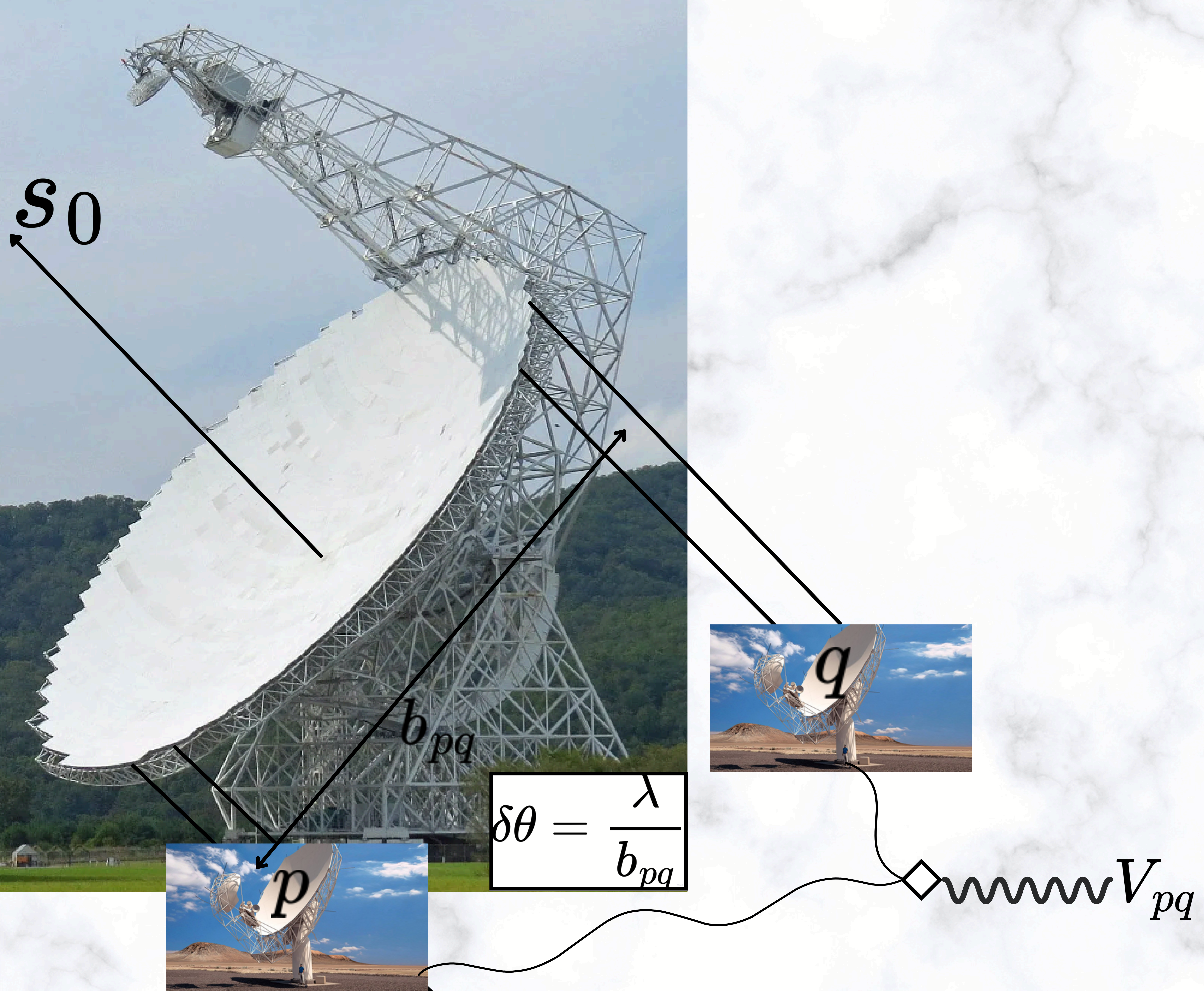

s_0


s_0
q
p



s_0
b_{pq}
q
p

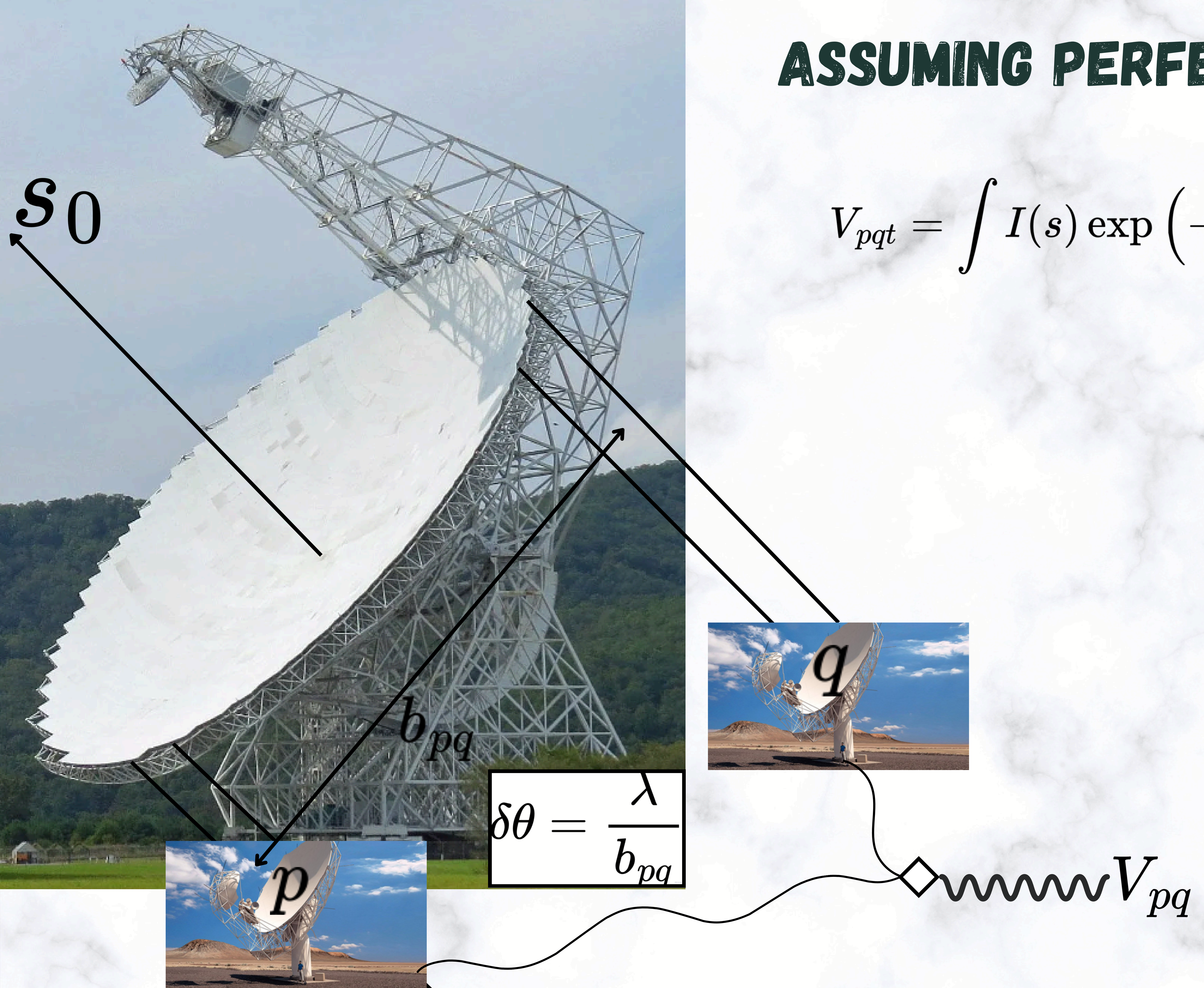

s_0
b_{pq}
q
p
$\delta\theta = \frac{\lambda}{b_{pq}}$



s_0
b_{pq}
q
p
$\delta\theta = \frac{\lambda}{b_{pq}}$
V_{pq}

ASSUMING PERFECT CALIBRATION

$$V_{pqt} = \int I(s) \exp\left(-2\pi\imath\frac{\nu}{c}(b_{pqt}^T(s - s_0))\right)ds$$



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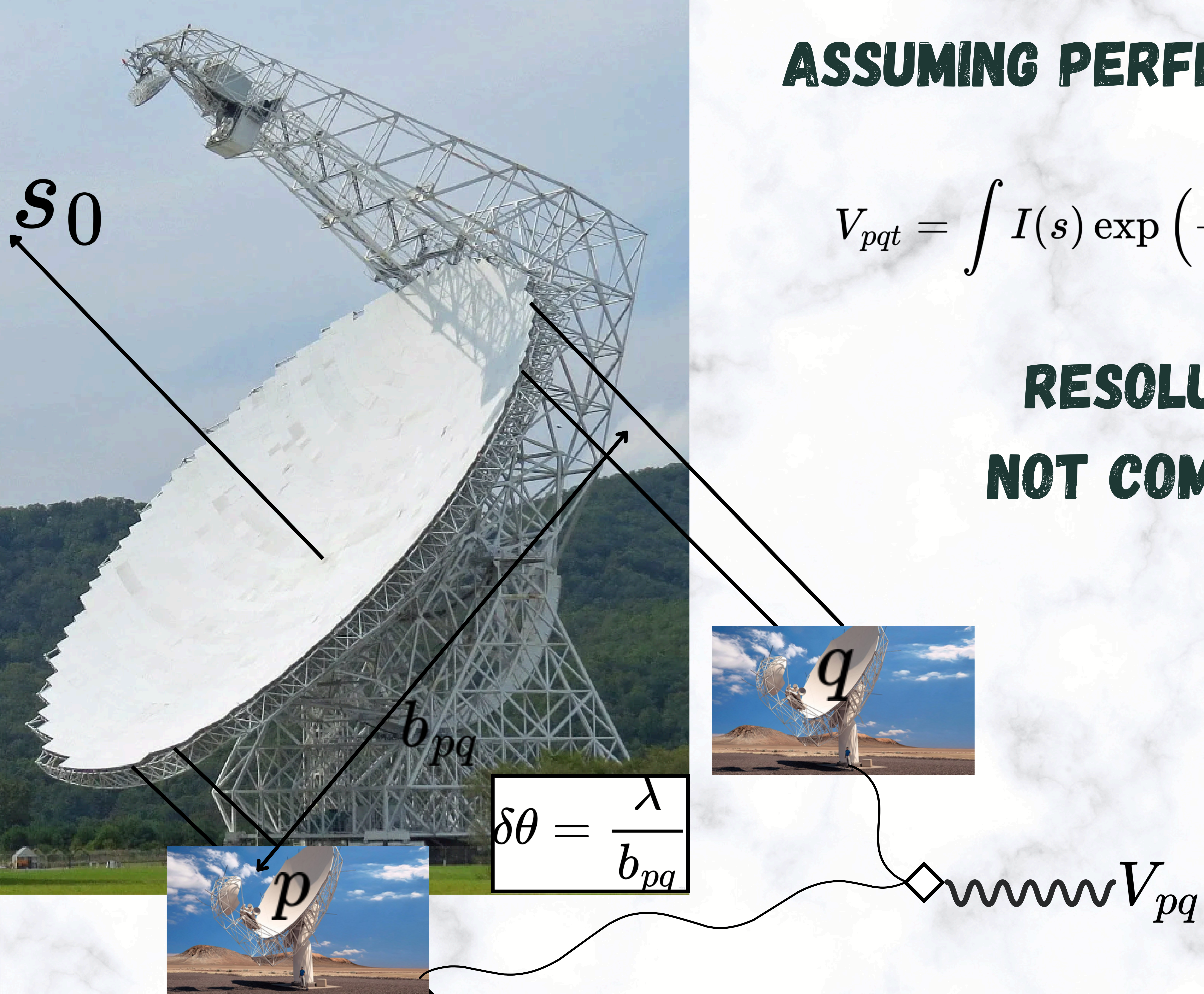
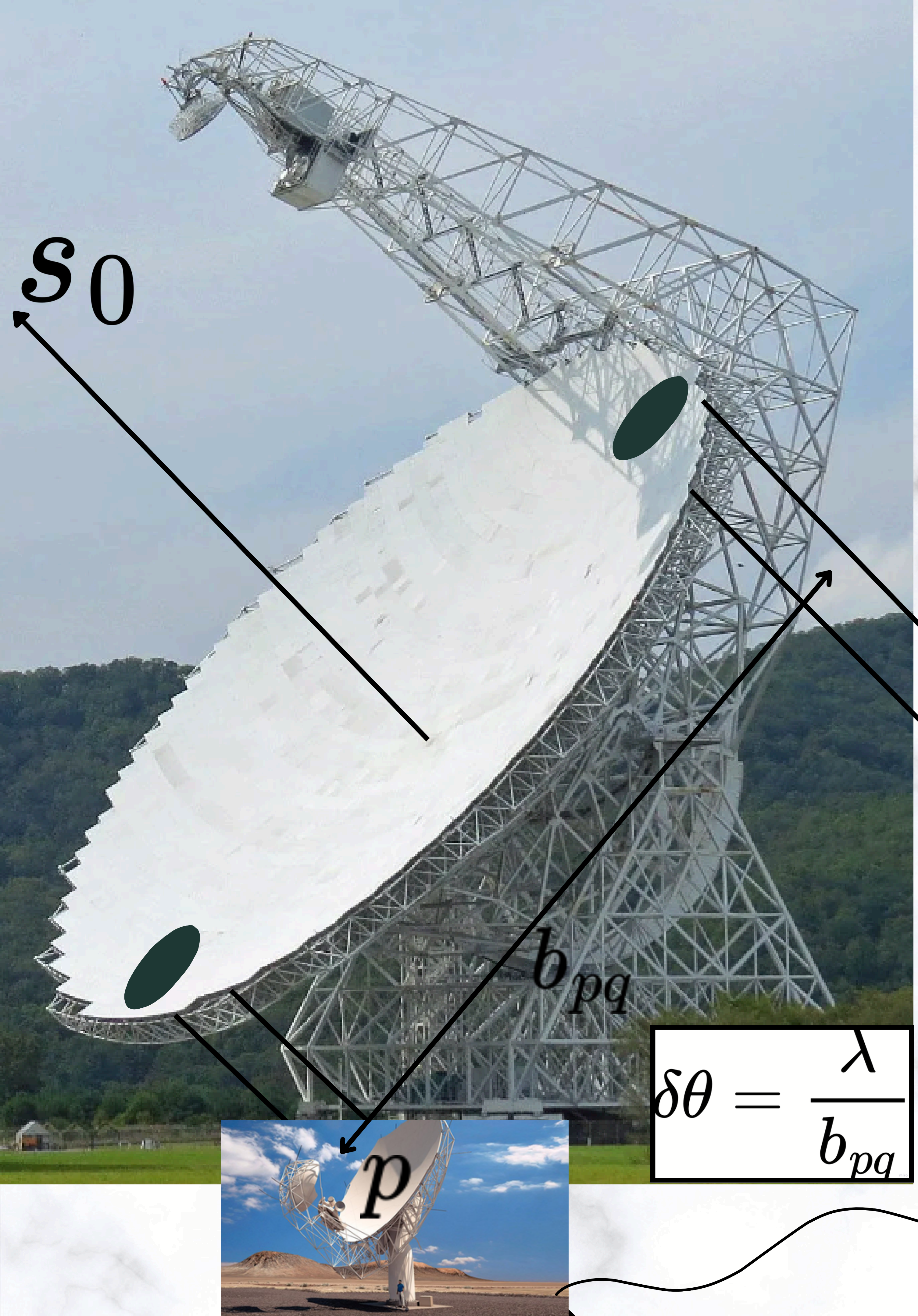
RESOLUTION DOES NOT COME FOR FREE!





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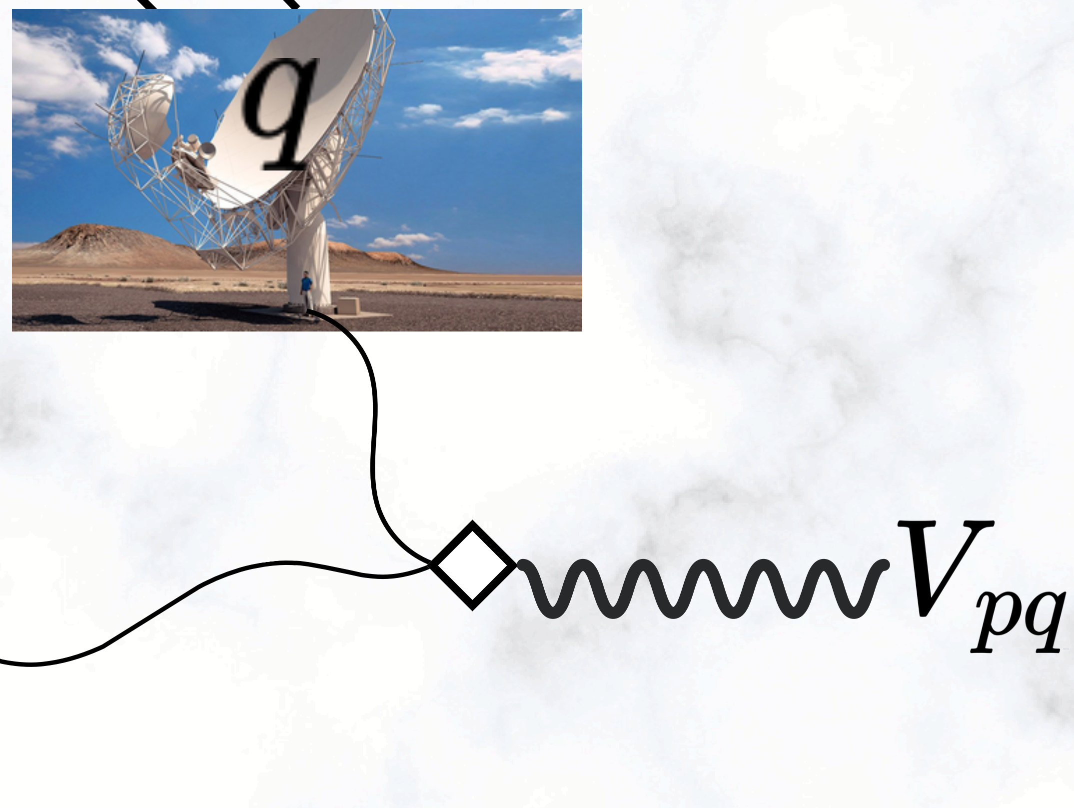

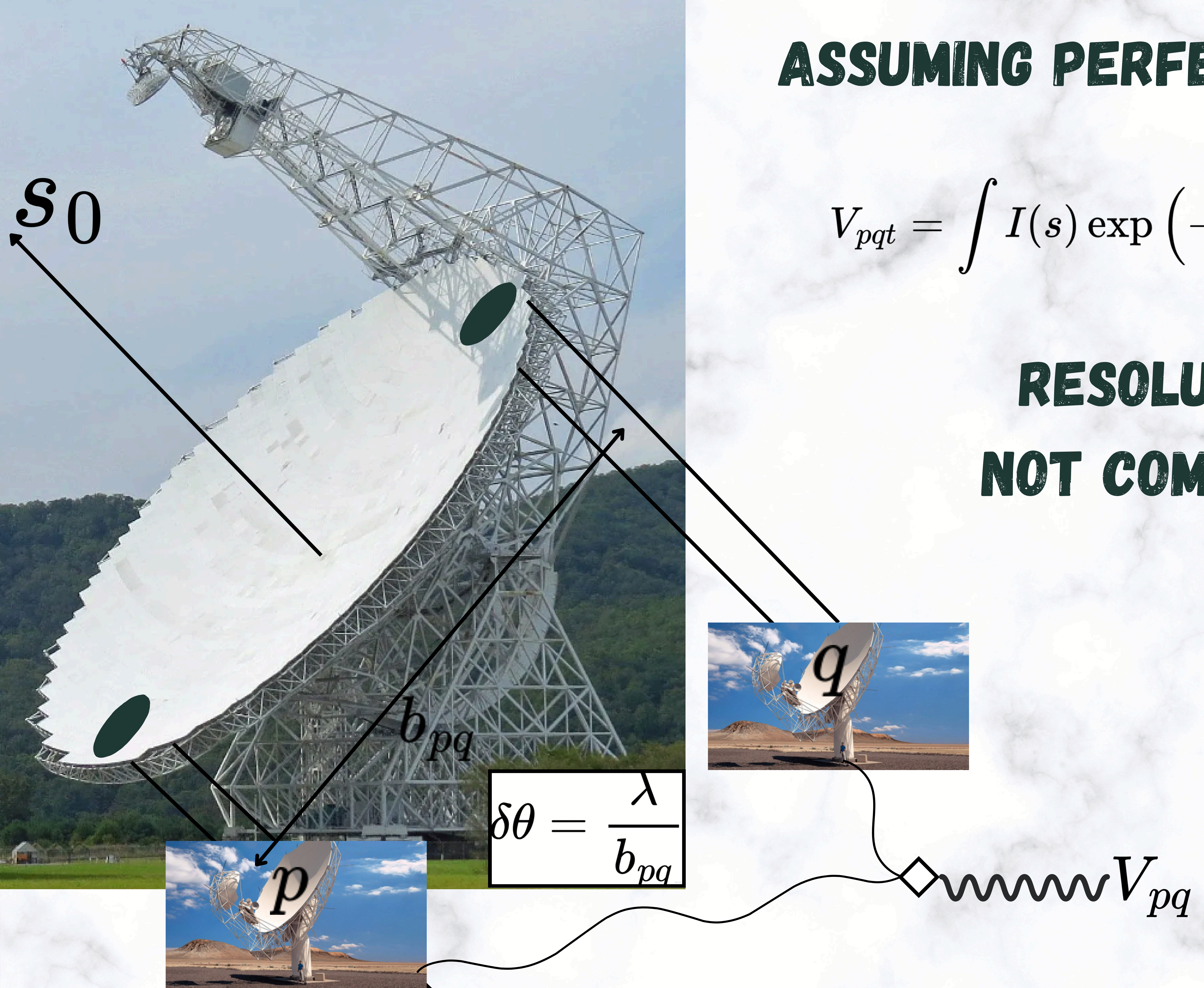

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MORE ANTENNAS

+

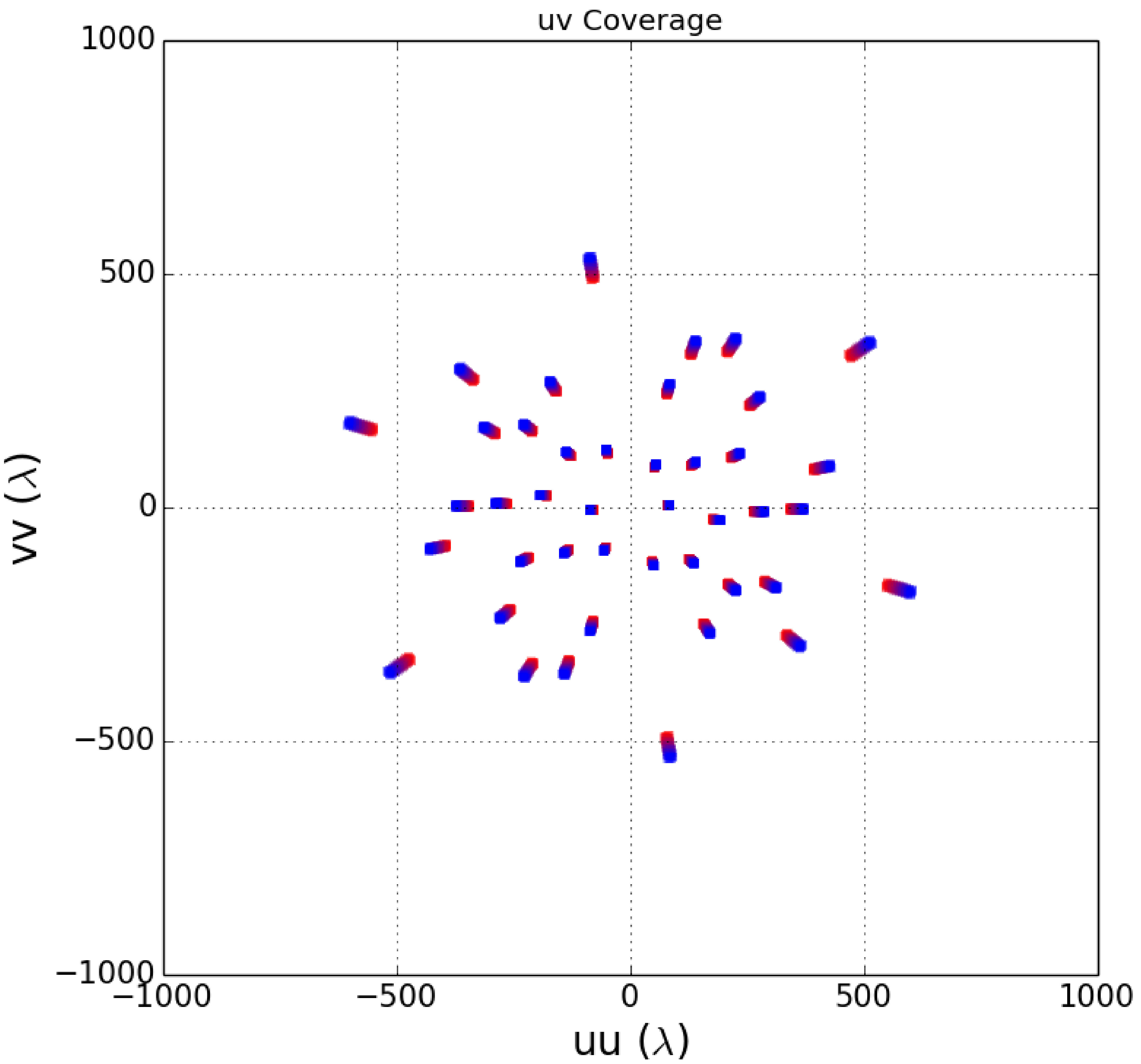
EARTH ROTATION

+

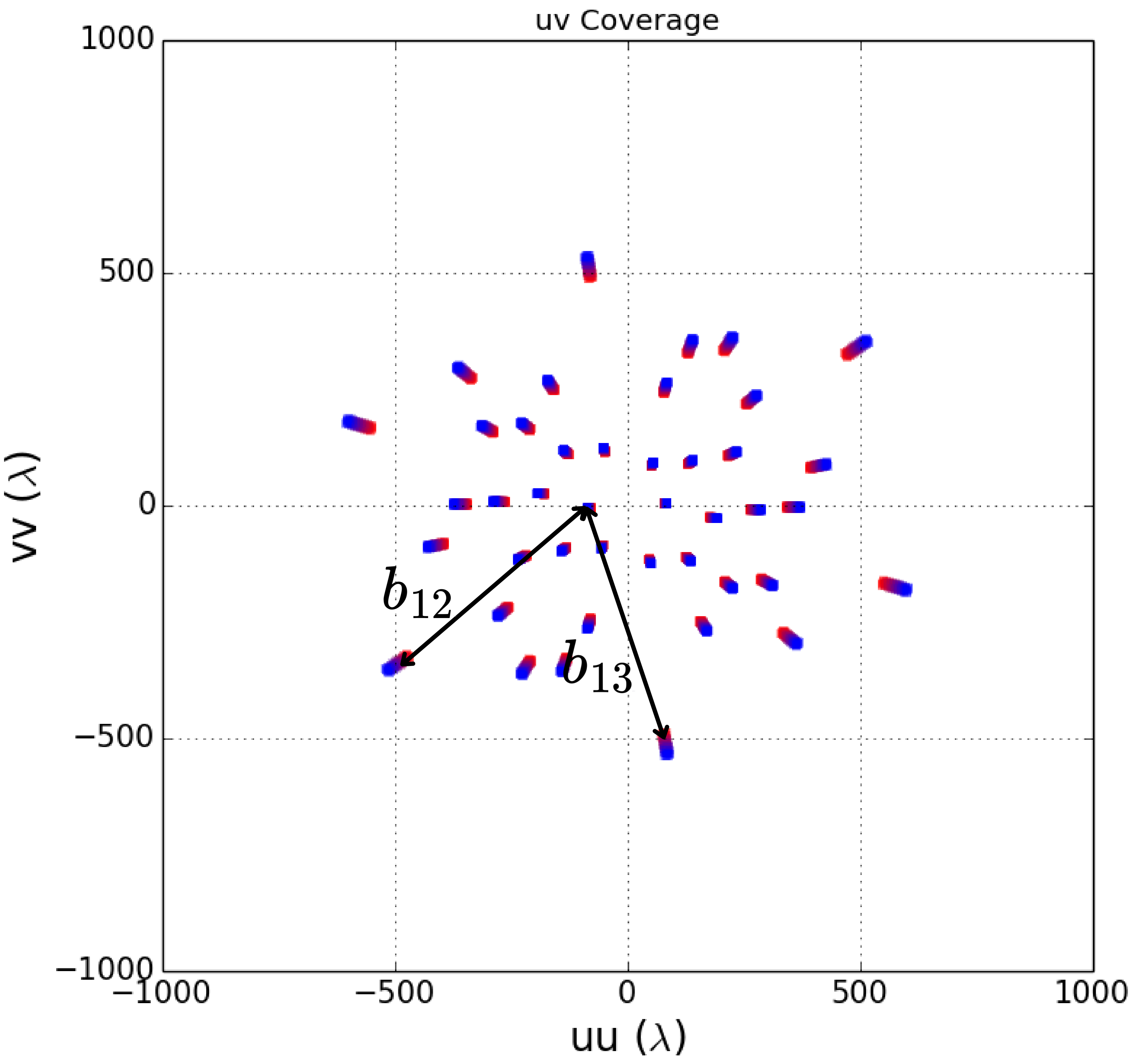
FREQUENCY COVERAGE

=

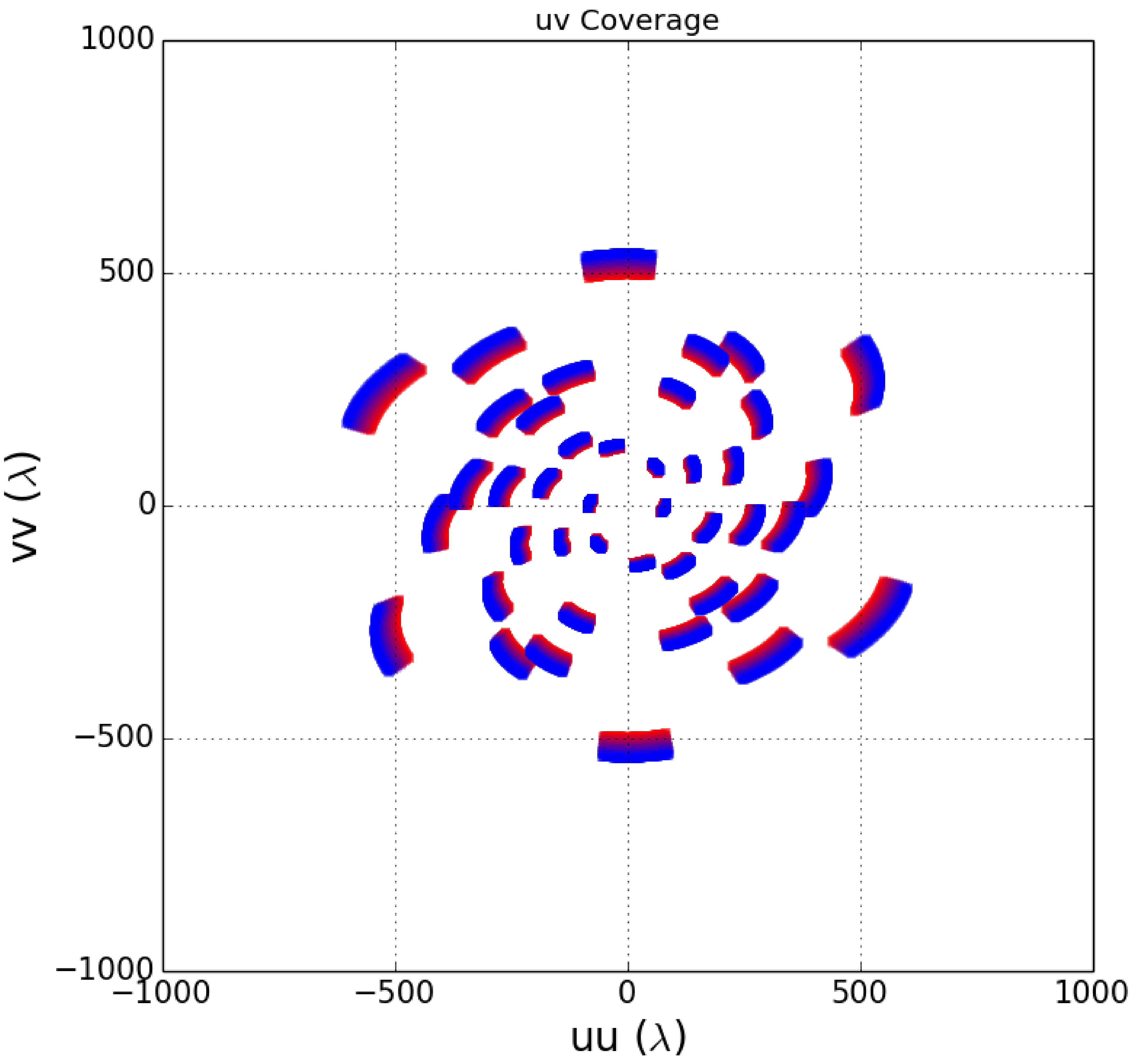
FEWER HOLES


uv Coverage
1000
500
0
−500
−1000
−1000
−500
0
500
1000
vv (λ)
uu (λ)

- SAMPLING PATTERN DEFINED BY RELATIVE POSITION OF ANTENNAS



- SAMPLING PATTERN DEFINED BY RELATIVE POSITION OF ANTENNAS
- EARTH ROTATION CAUSES BASELINES TO TRACE OUT ELLIPTICAL TRACKS



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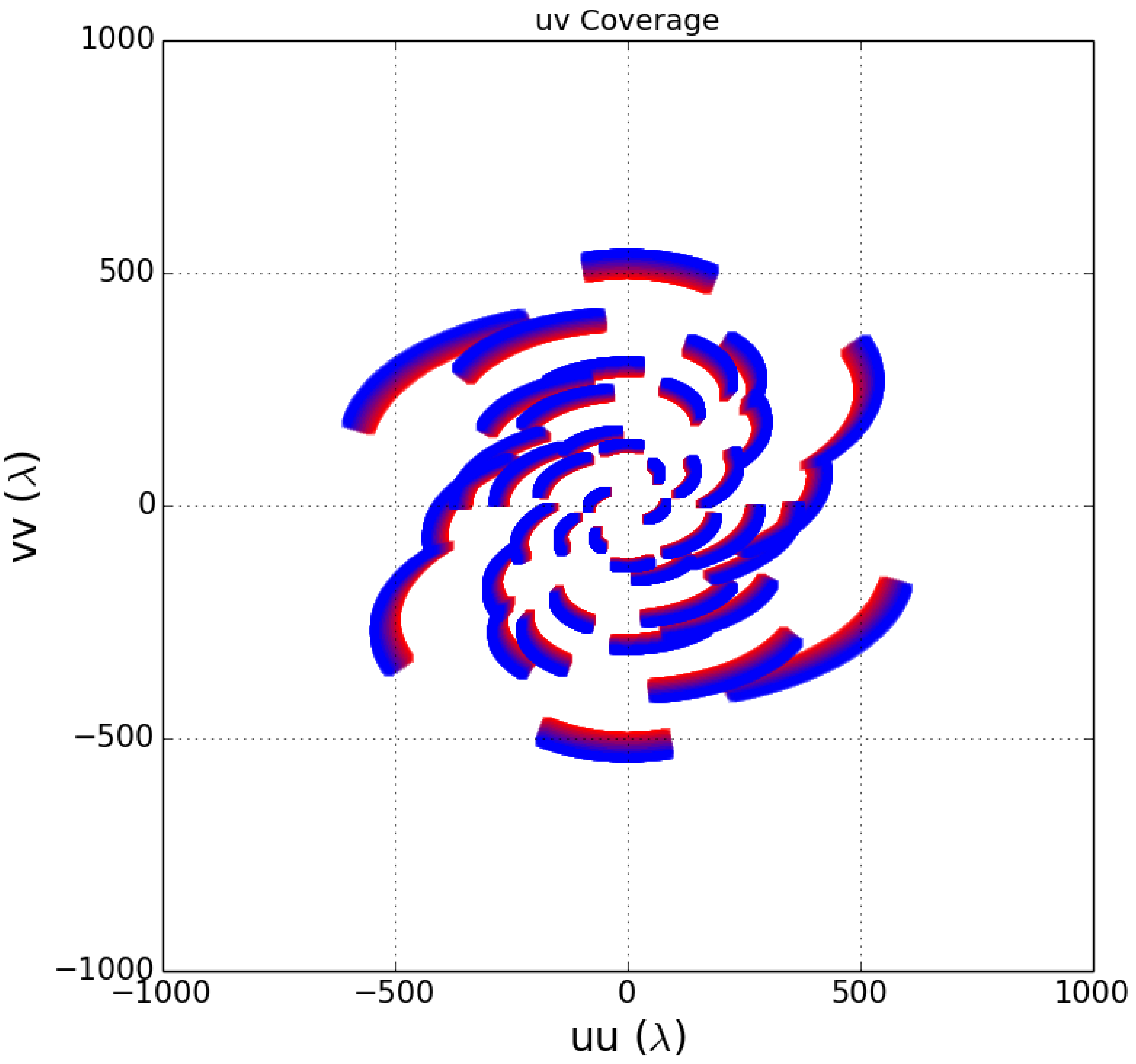

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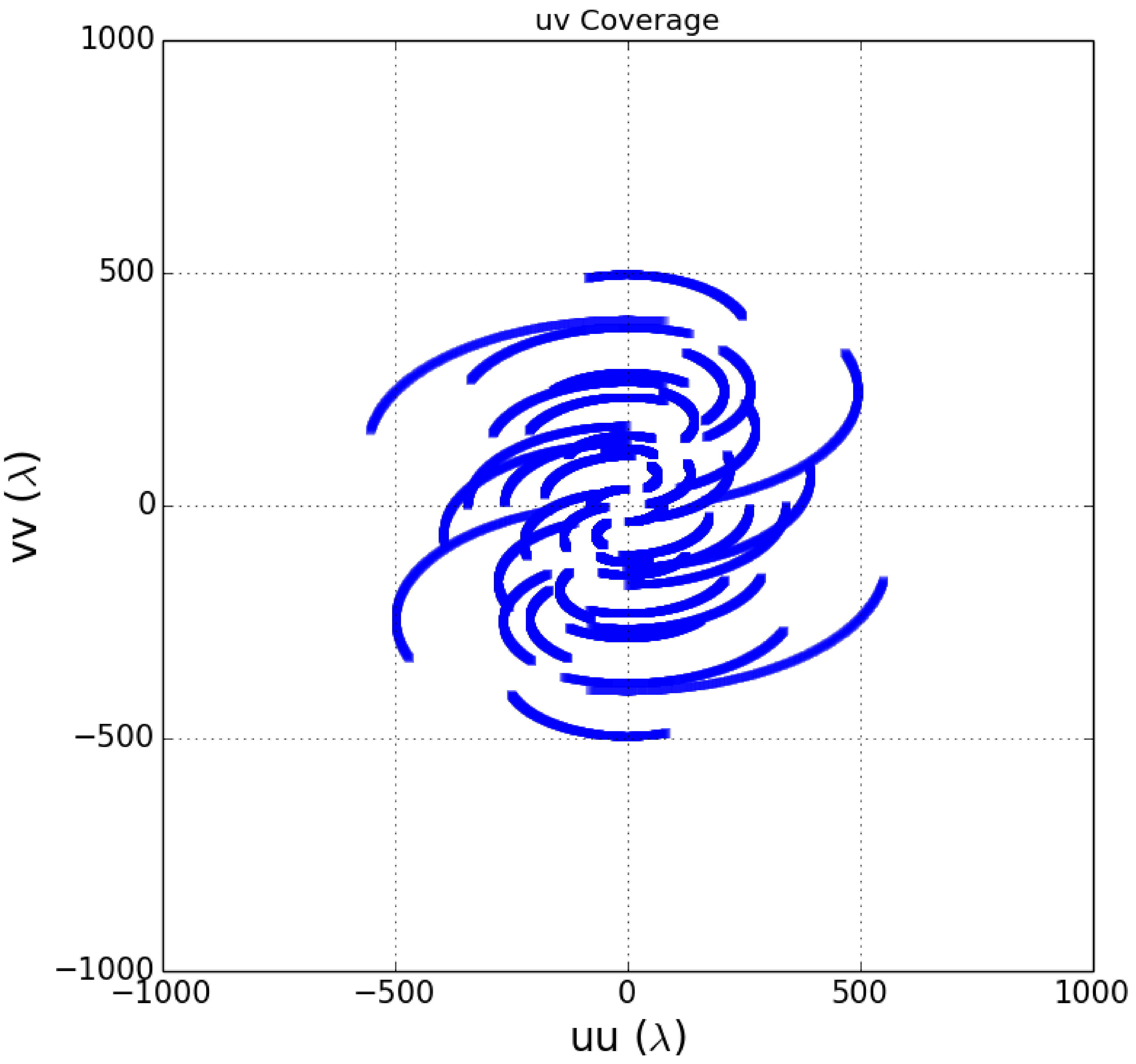
- FREQUENCY COVERAGE FILLS THE UV-PLANE RADIALLY



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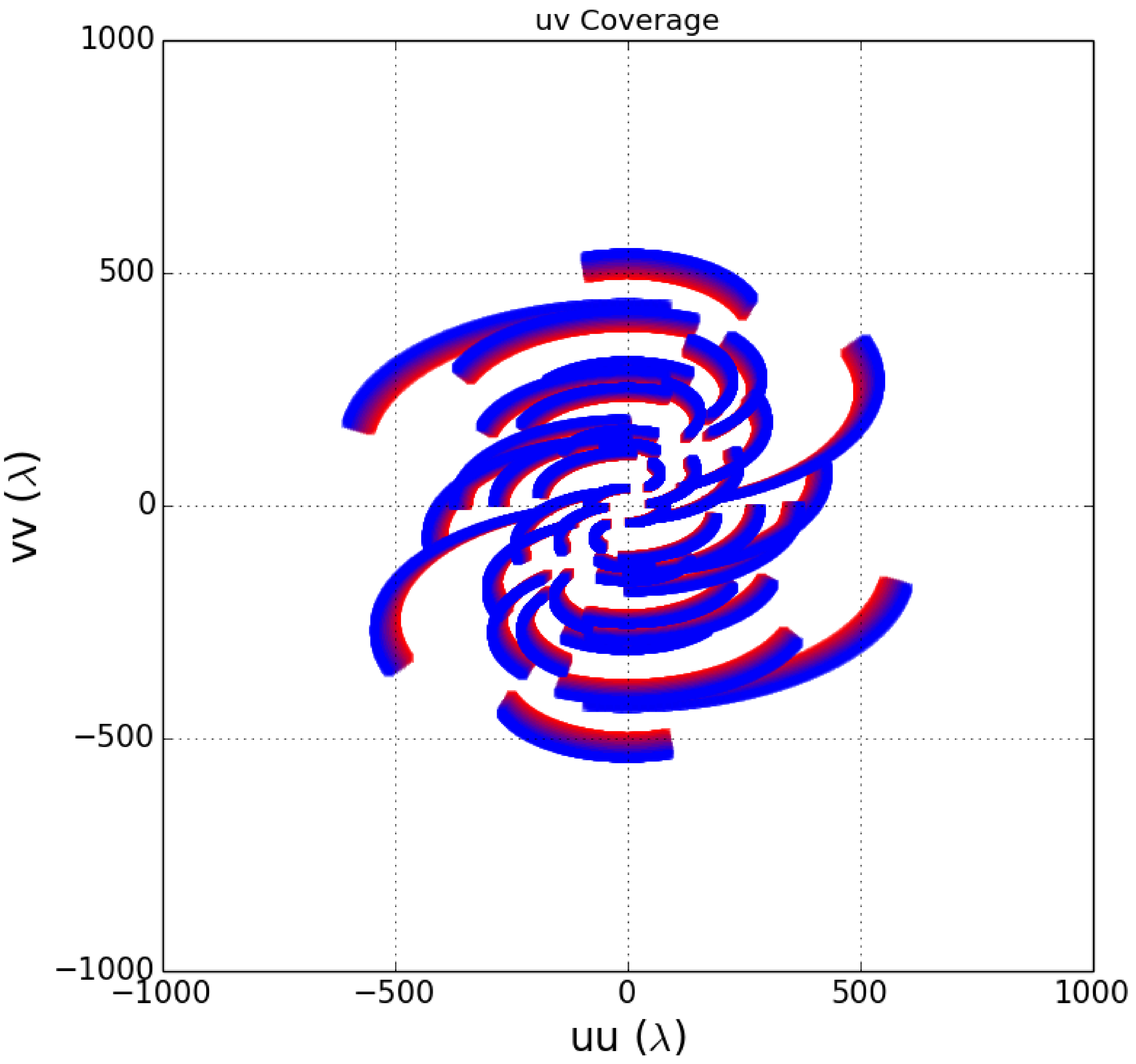

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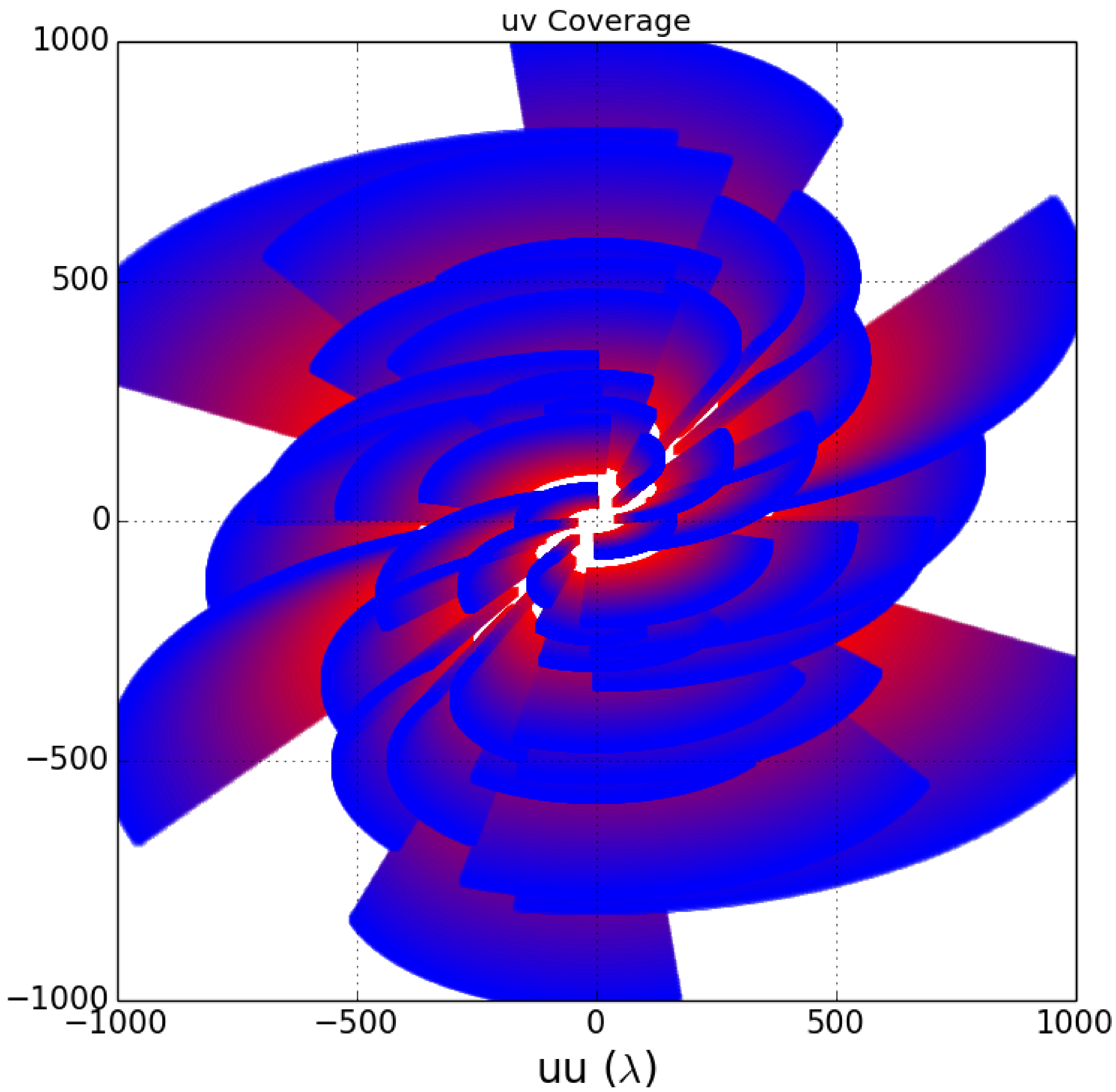

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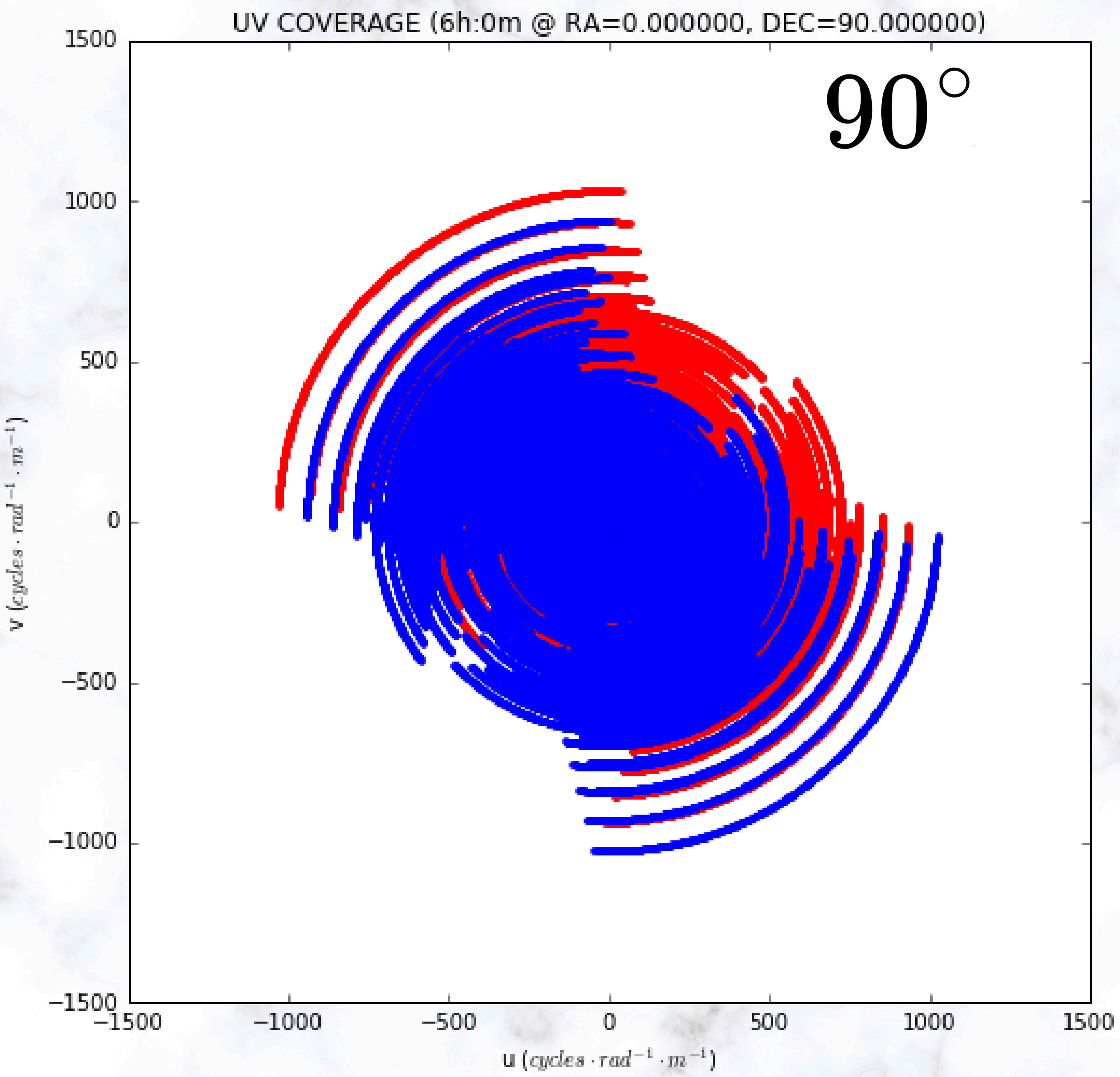
- SAMPLING PATTERN DEPENDS ON DECLINATION



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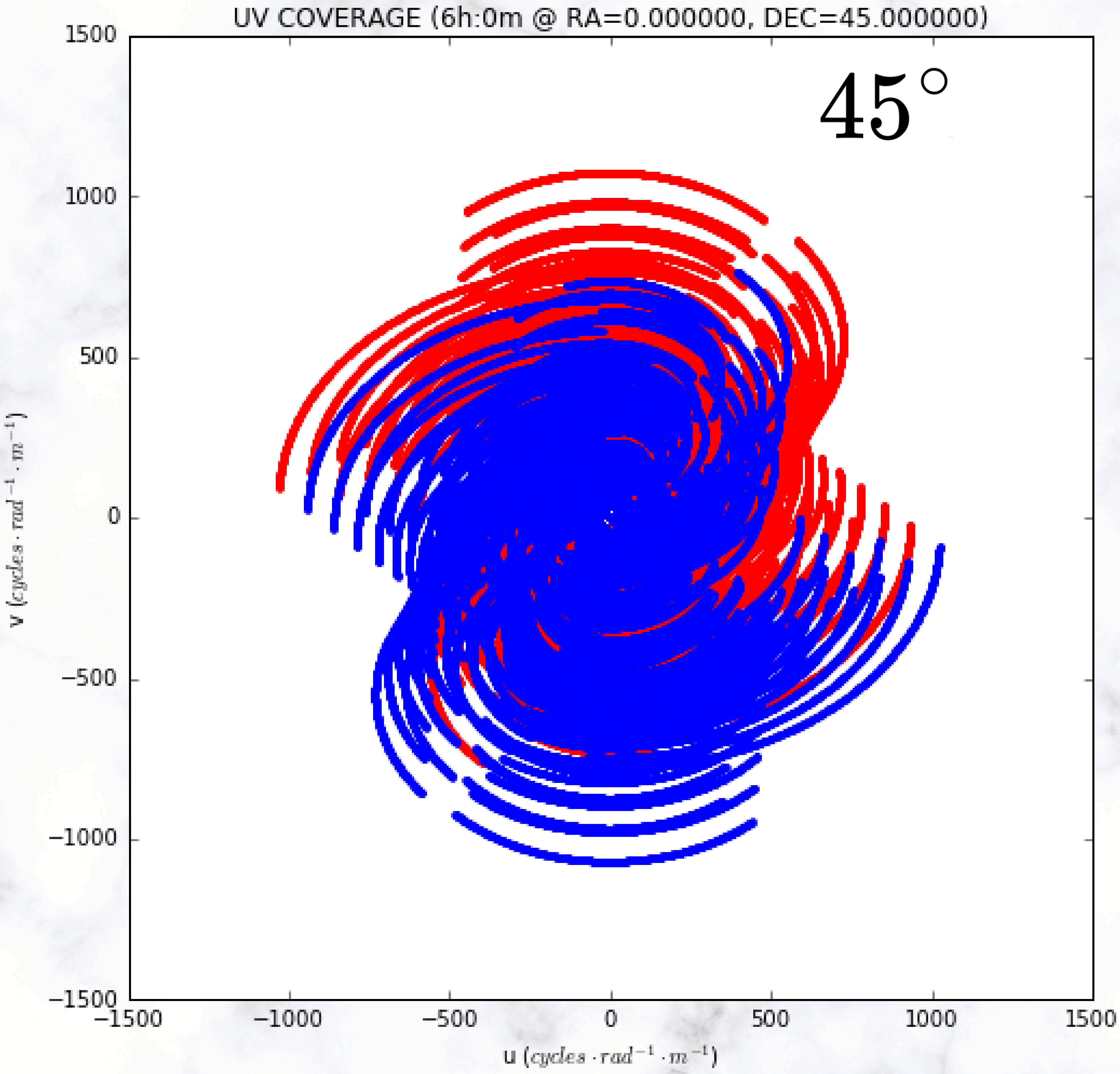

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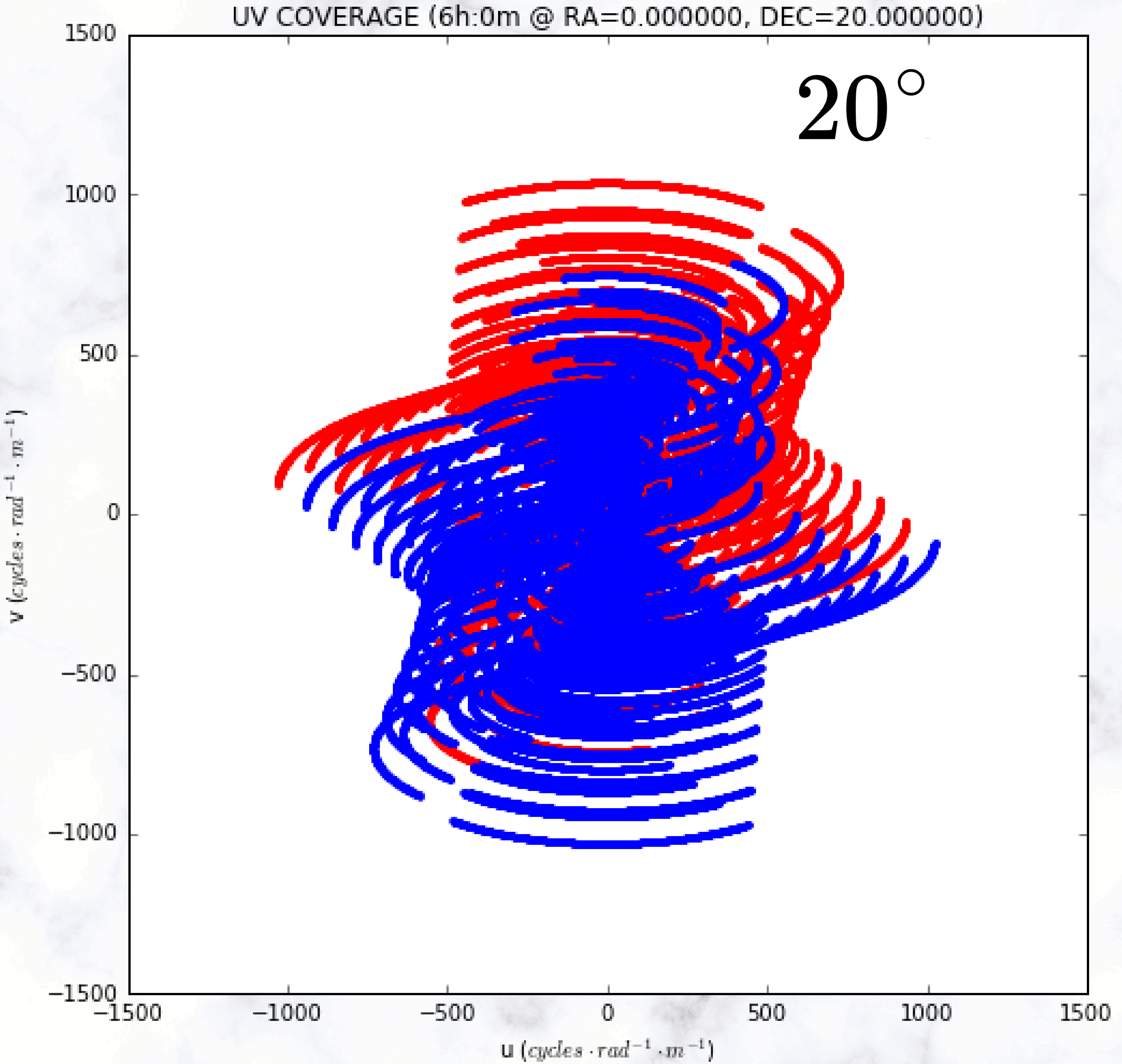

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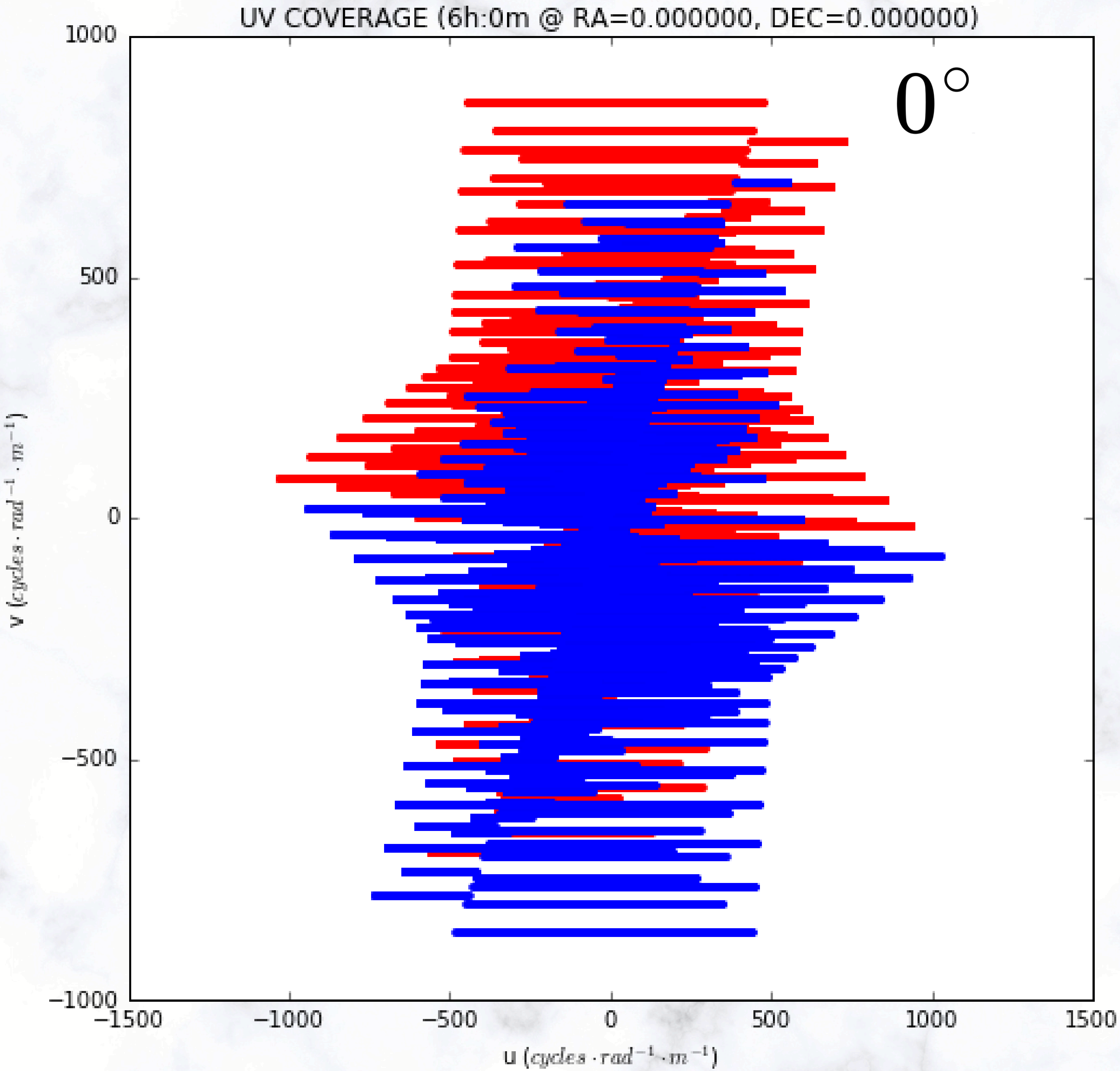

VAN CITTERT ZERNIKE

$$V_{pq} = \int I(l, m) \exp\left(-2\pi\imath\frac{\nu}{c}(u_{pq}l + v_{pq}m + w_{pq}(n-1)\right)\frac{dldm}{n}$$

VAN CITTERT ZERNIKE

VISIBILITIES (DATA)

$$V_{pq} = \int I(l,m) \exp\left(-2\pi\imath\frac{\nu}{c}(u_{pq}l + v_{pq}m + w_{pq}(n-1))\right)\frac{dldm}{n}$$

miro

VAN CITTERT ZERNIKE

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TOTAL INTENSITY IMAGE

VAN CITTERT ZERNIKE

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FOURIER SPACE COORDINATES

VAN CITTERT ZERNIKE

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IMAGE
SPACE
COORDINATES

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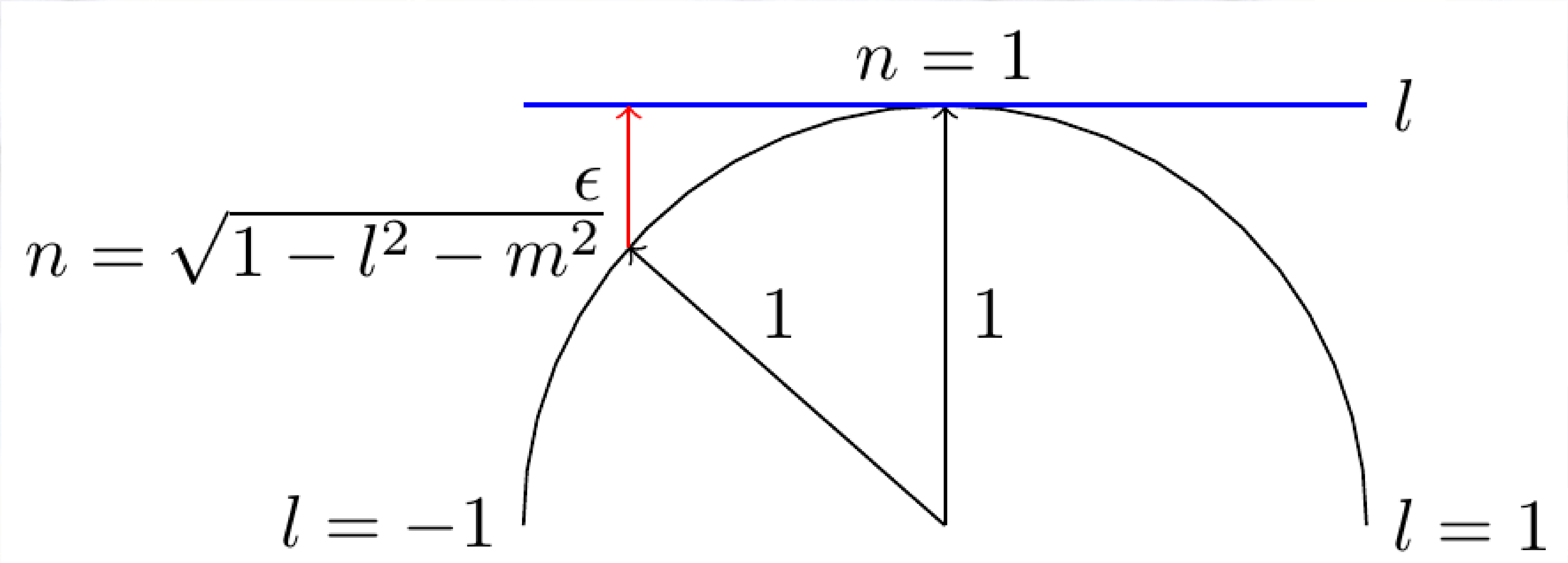
IMAGE SPACE COORDINATES



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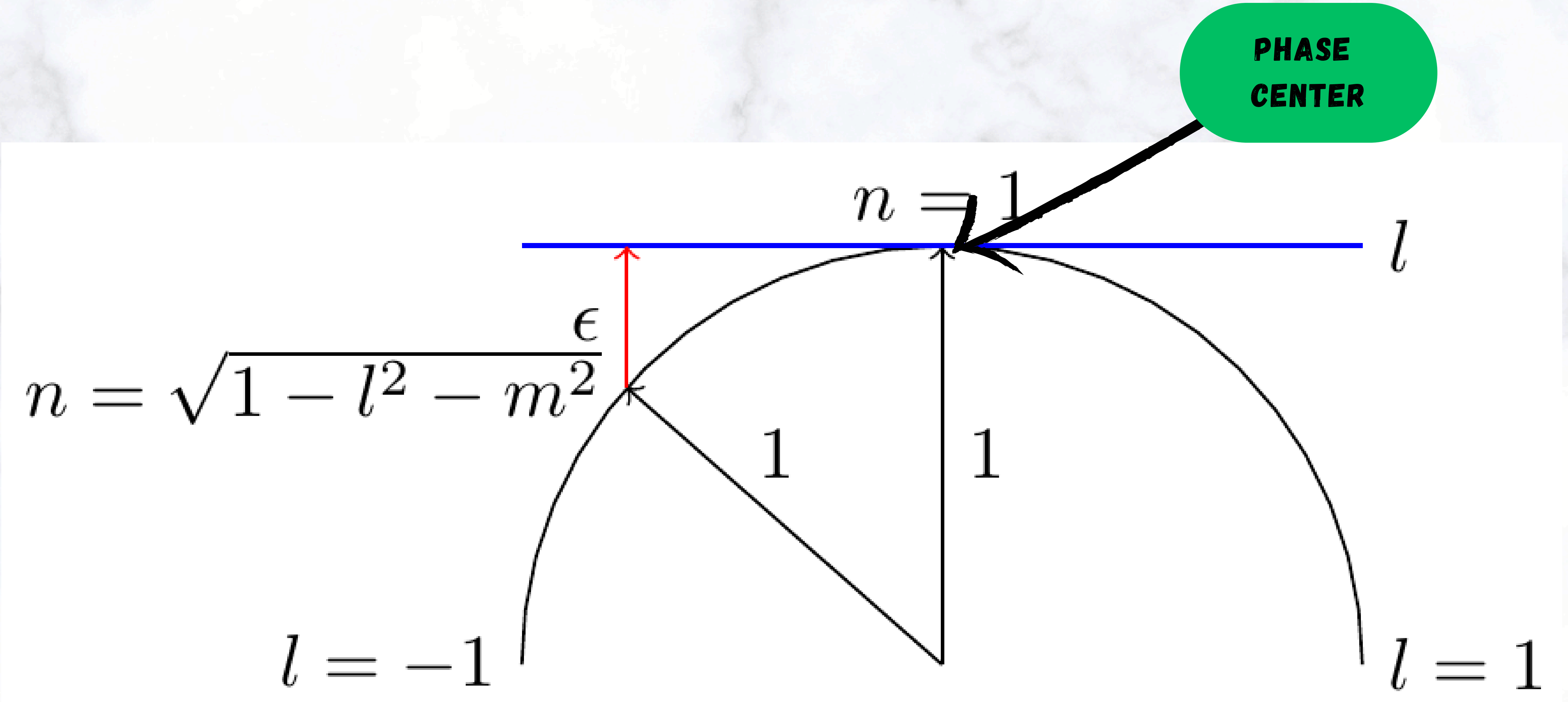

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IMAGING -> HOW TO OBTAIN IMAGE FROM MEASURED VISIBILITIES?

Some Fourier Transform Pairs:

$x(t)$	FT	$X(\nu)$
$A_0\,\delta(t-t_0)$		$A_0\,e^{-2\pi\imath\nu t_0}$
$\sqcap(t)$		$\mathrm{sinc}\,(\nu)$
$\exp(-at^2)$		$\sqrt{\frac{\pi}{a}}\,\exp(-\pi^2\nu^2/a)$
$\vdots$		$\vdots$

Fourier Theorems

Linearity	$\mathcal{F}[ax(t)+by(t)](s) = a\mathcal{F}[x(t)](s) + b\mathcal{F}[y(t)](s)$
Shift	$\mathcal{F}[x(t-t_0)] = e^{-2\pi \imath s t_0}\mathcal{F}[x(t)]$
Similarity	$\mathcal{F}[x(at)](s) = \frac{1}{\lvert a\rvert}\mathcal{F}[x(t)](\frac{s}{a})$
Convolution	$\mathcal{F}[x(t)y(t)](s) = \mathcal{F}[x(t)](s) * \mathcal{F}[y(t)](s)$
Parseval	$\langle x, y\rangle = x^\dagger y = \langle \mathcal{F}[x], \mathcal{F}[y]\rangle$

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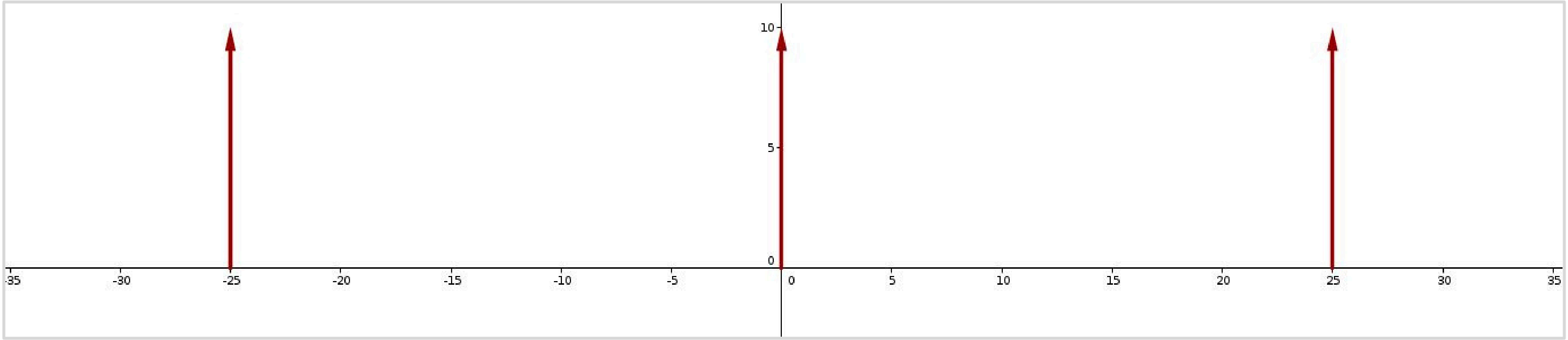
Example - Bed of nails convolved with sinc:



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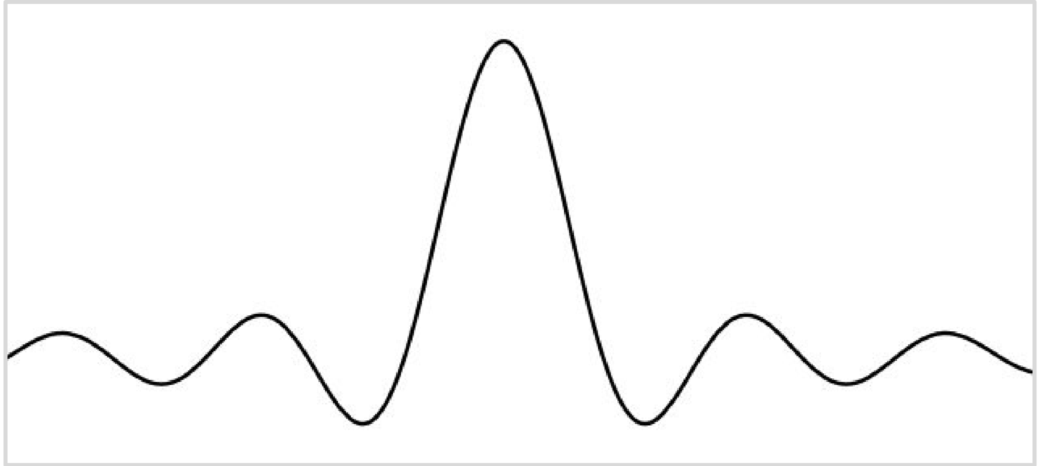

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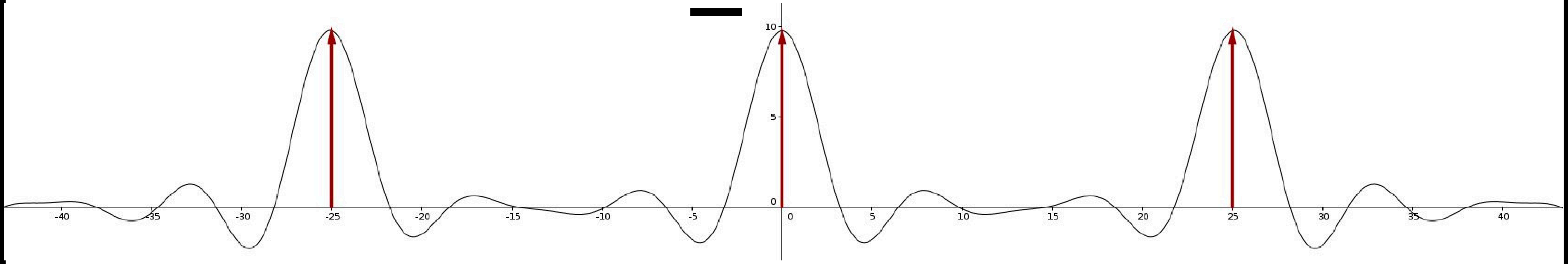
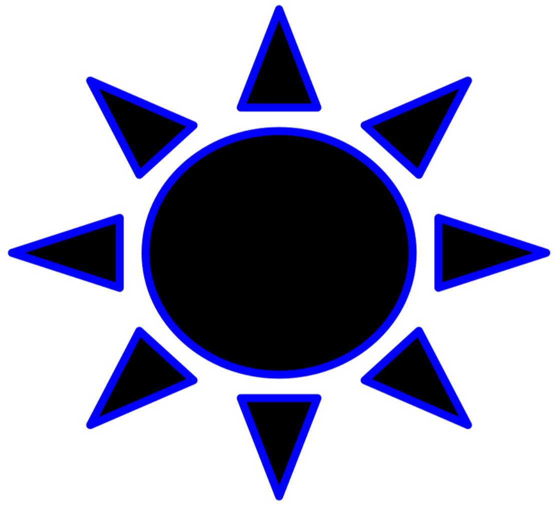


Discrete Fourier Transform



$$X_k = \sum_{n=0}^{N-1} x_n e^{-2\pi \imath \frac{kn}{N}}$$

$$x_n = \sum_{k=0}^{N-1} X_k e^{2\pi \imath \frac{kn}{N}}$$

$$t_n = t_0 + n\Delta t$$

$$\nu_k = \frac{k f_s}{N} \quad f_s = \frac{1}{\Delta t}$$

Discrete Fourier Transform

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Sampling interval or cell size

$$t_n = t_0 + n\Delta t$$

$$\nu_k = \frac{kf_s}{N} \quad f_s = \frac{1}{\Delta t}$$

Discrete Fourier Transform

$$X_k = \sum_{n=0}^{N-1} x_n e^{-2\pi\imath\frac{kn}{N}}$$

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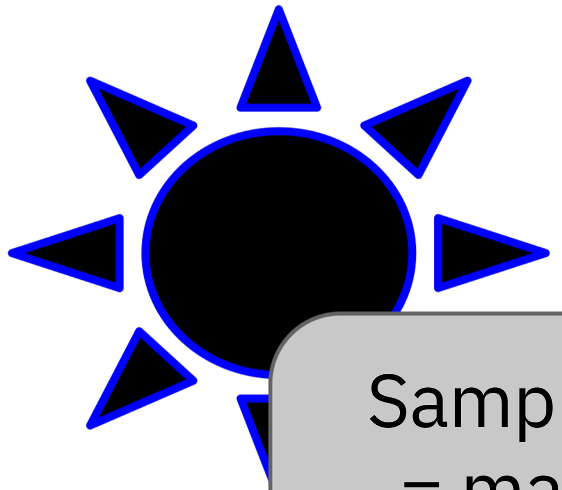
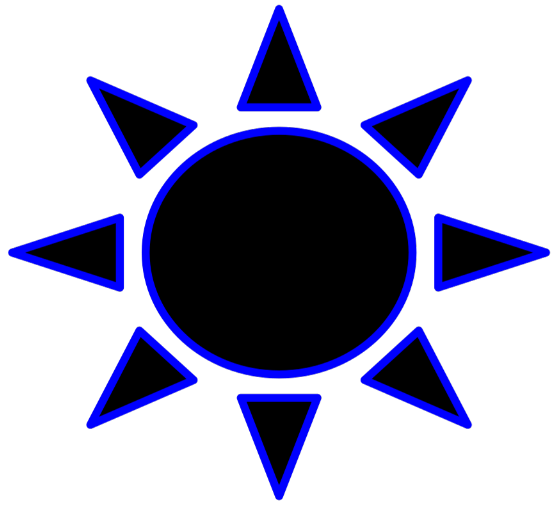


$$t_n = t_0 + n\Delta t$$

$$\nu_k = \frac{kf_s}{N} \quad f_s = \frac{1}{\Delta t}$$

Sampling rate = maximum frequency in spectrum

Discrete Fourier Transform



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$$t_n = t_0 + n\Delta t$$

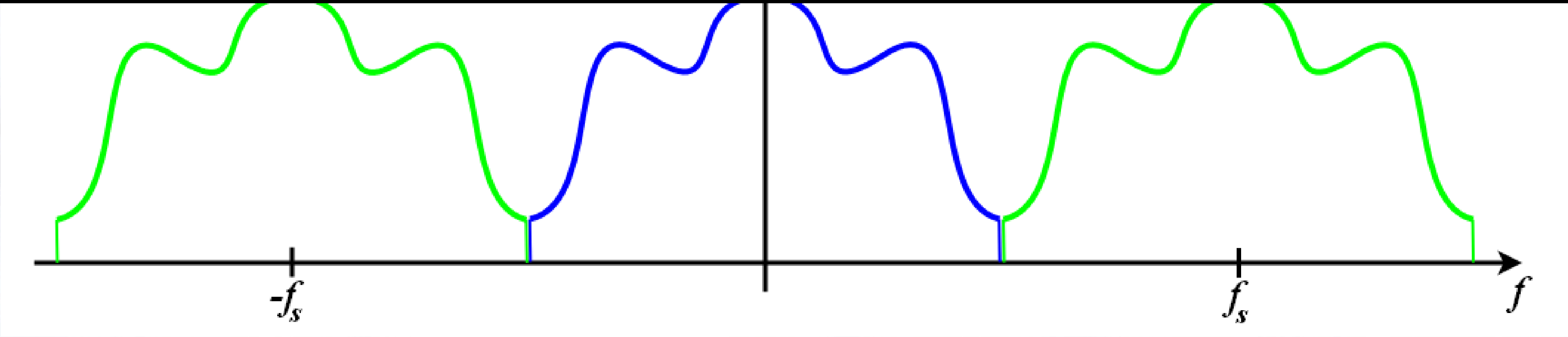
$$\nu_k = \frac{kf_s}{N} \quad f_s = \frac{1}{\Delta t}$$

Implemented using FFT which has computational complexity $\mathcal{O}(N \log_2 N)$

SAMPLING THEOREM AND ALIASING

FROM WIKIPEDIA

Theorem — If a ... om its
ordinates at a se...

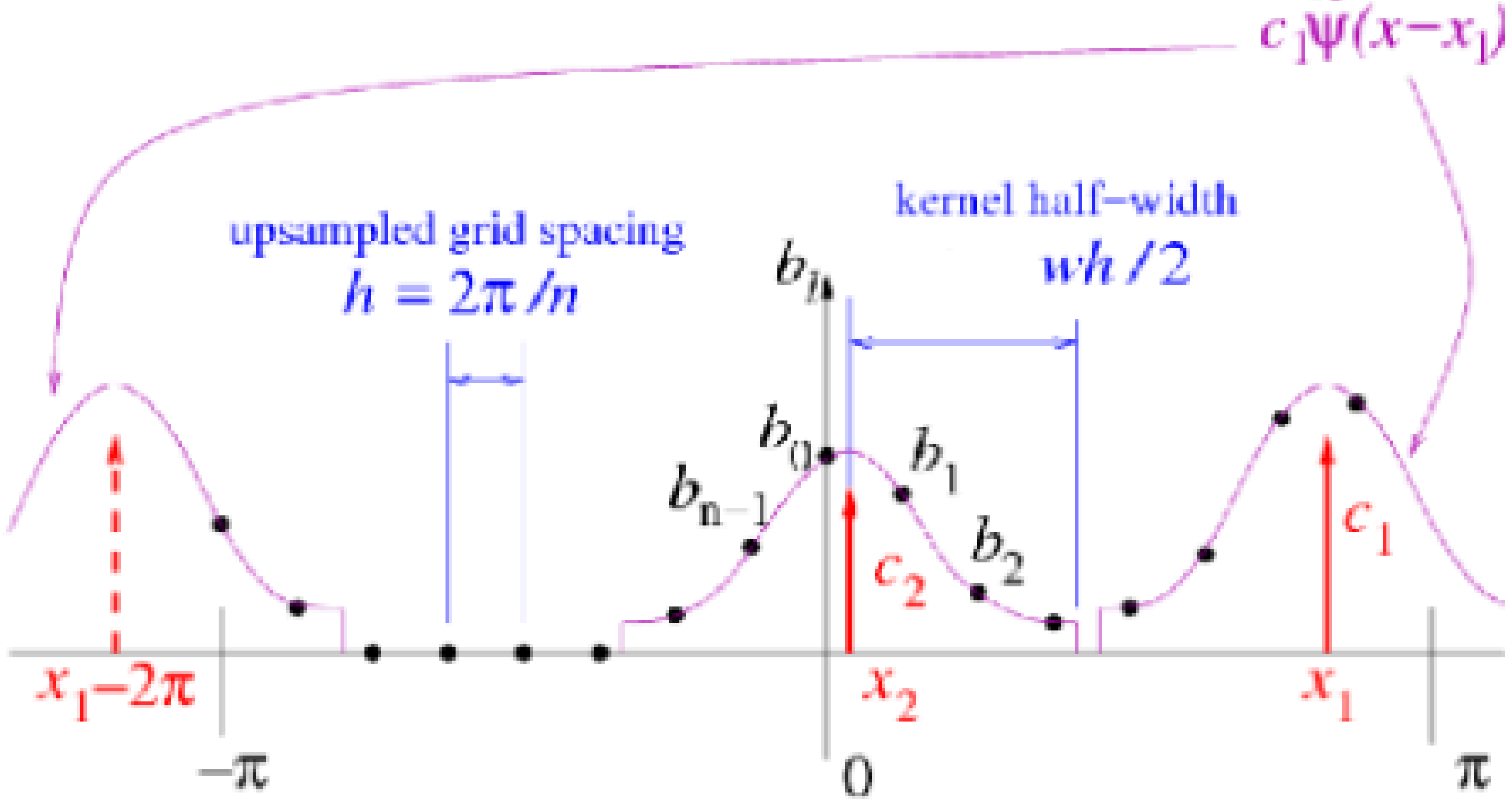


NON-UNIFORM FFT

SEE EG.

Flatiron Institute Nonuniform Fast Fourier Transform

fiNUFFT



IMPLEMENTED EFFICIENTLY USING

CONVOLUTIONAL GRIDDING + FFT

VAN CITTERT ZERNIKE

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NUFFT IN ABSENCE OF WIDE-FIELD EFFECTS

VAN CITTERT ZERNIKE

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NUFFT IN ABSENCE OF WIDE-FIELD EFFECTS

UV-COVERAGE DETERMINED BY INSTRUMENT -> SET CELL SIZE BASED ON NYQUIST CRITERION

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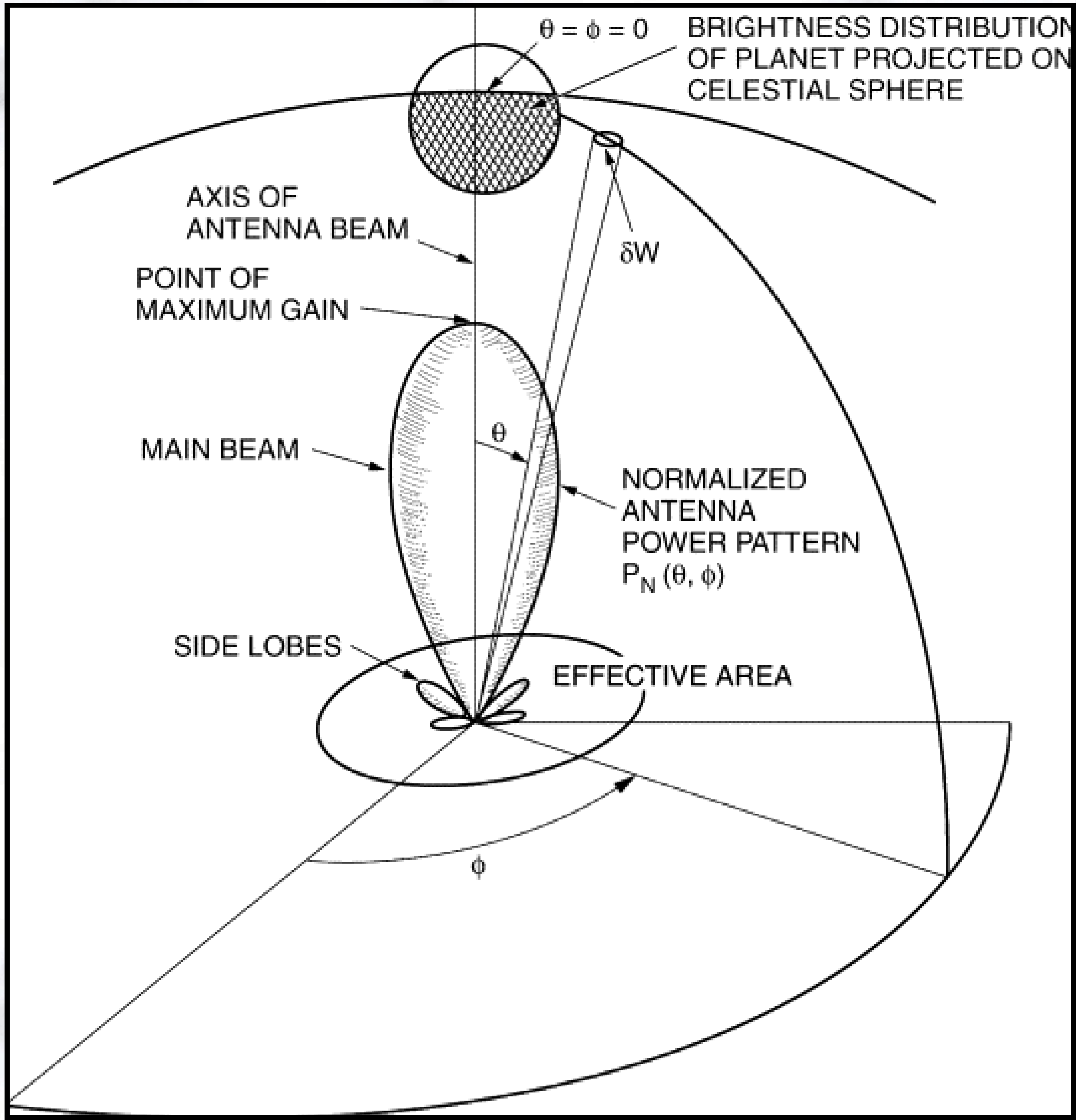
NUFFT IN ABSENCE OF WIDE-FIELD EFFECTS

UV-COVERAGE DETERMINED BY INSTRUMENT -> SET CELL SIZE BASED ON NYQUIST CRITERION

WHAT ABOUT THE FIELD OF VIEW (I.E. NPIX)?

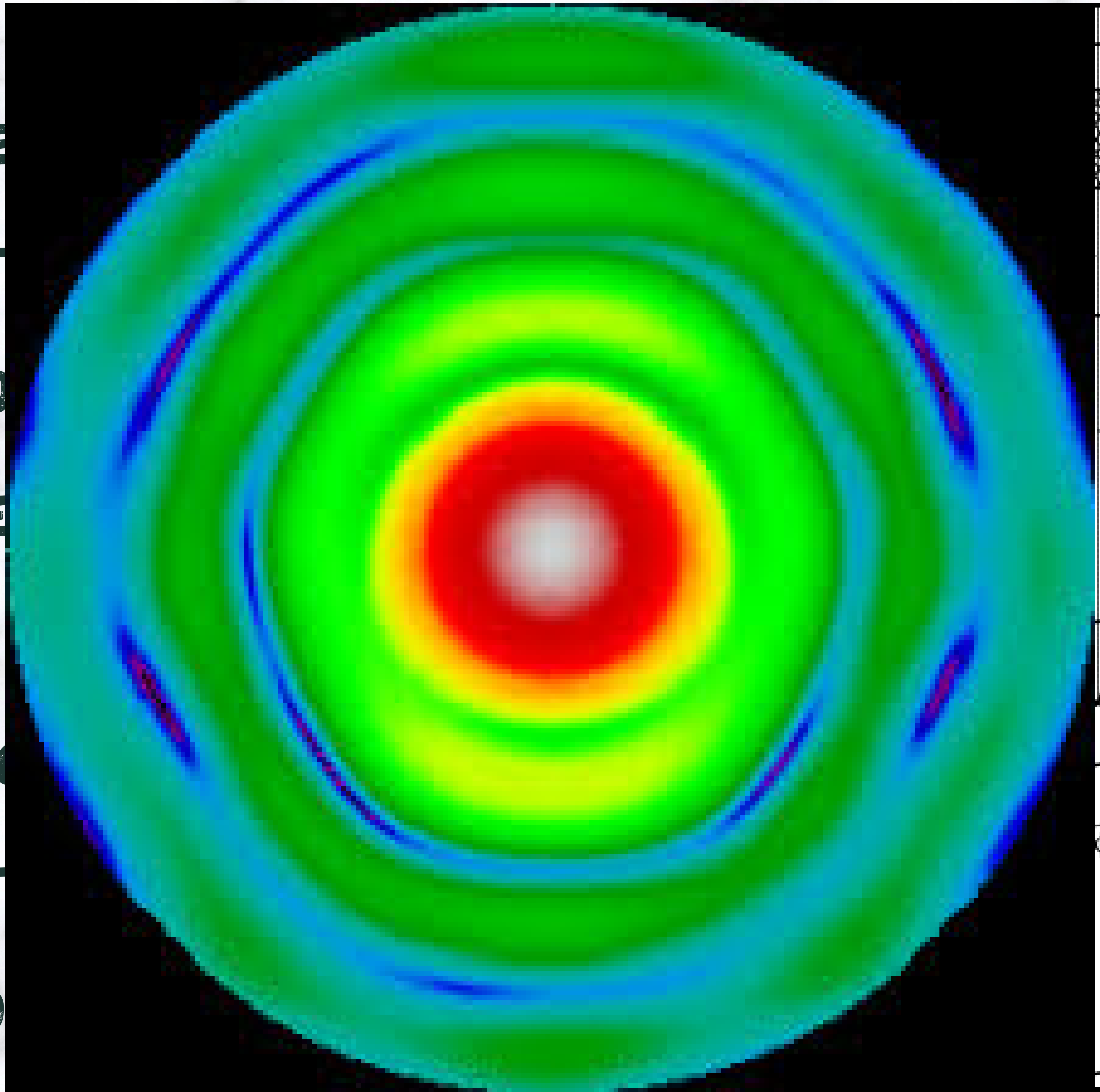
PRIMARY BEAM PATTERN

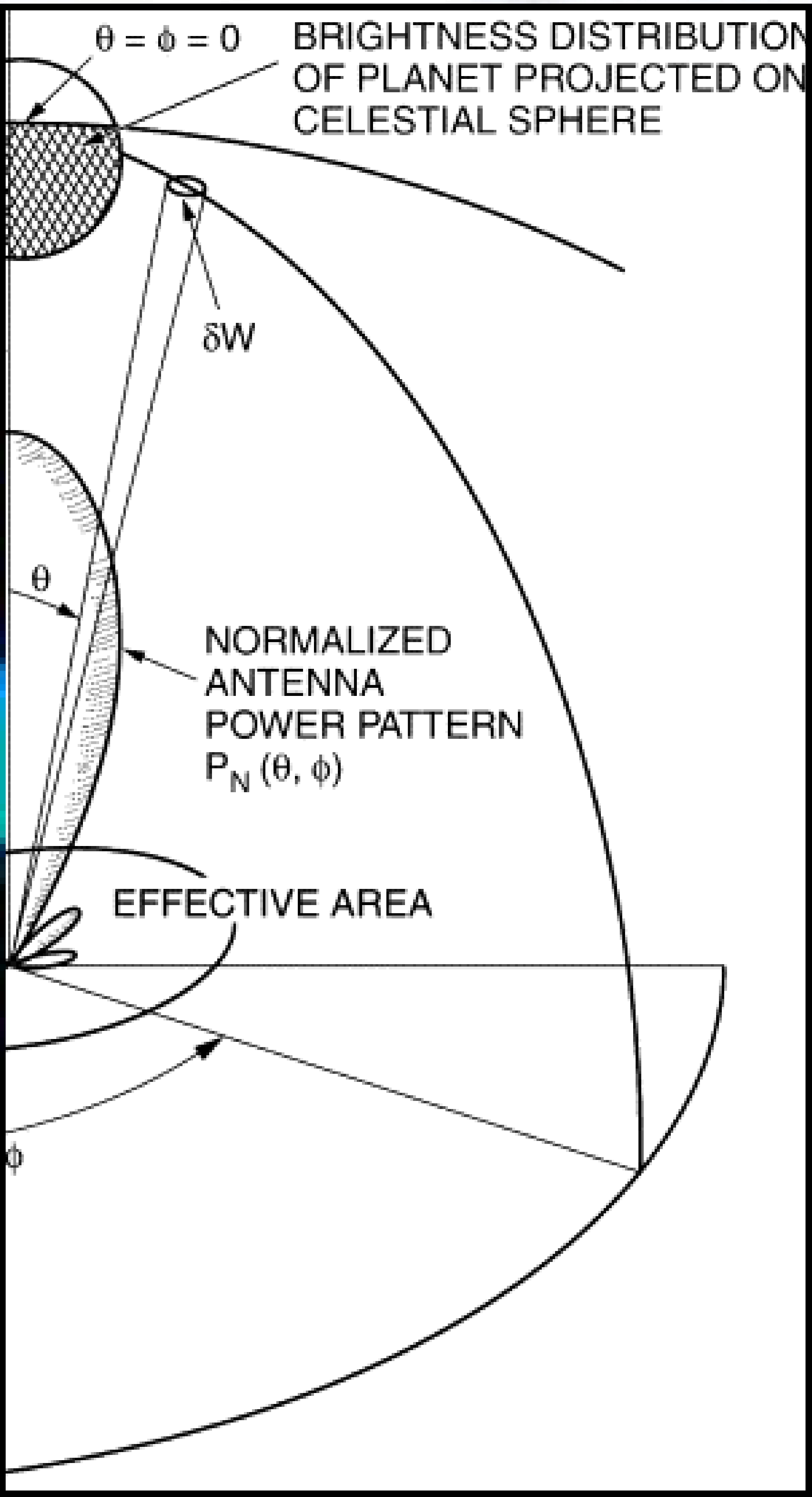
- SHAPE OF DISH DETERMINES ANTENNA SENSITIVITY PATTERN (YET MORE FOURIER TRANSFORMS)
- TIME AND FREQUENCY DEPENDENT!
- PRIMARY SOURCE OF DIRECTION DEPENDENT EFFECTS FOR MEERKAT
- THIS LARGELY DETERMINES THE REQUIRED FIELD OF VIEW



Primary Beam Pattern

- Shape of dish de
  sensitivity patt
  Fourier transfo
- Time and freque
- Primary source
  dependent effec
- This is what det
  required field o





IMAGING IN A NUTSHELL

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DISCRETIZED MEASUREMENT MODEL FOR IMAGING

$$y = Rx + \epsilon, \;\; \epsilon \sim \mathcal{N}(0, \Sigma)$$

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DISTRIBUTION OF THE NOISE GIVES

$$\chi^2(x) \propto (y - Rx)^\dagger \Sigma^{-1}(y - Rx) \;\rightarrow\; \nabla_x \chi^2 = R^\dagger \Sigma^{-1}(y - Rx)$$

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SETTING THE GRADIENT TO ZERO

$$\nabla_x \chi^2 = 0 \;\rightarrow\; R^\dagger \Sigma^{-1} y = R^\dagger \Sigma^{-1} R x \;\rightarrow\; I^D \approx I^{PSF} \star x$$

RESOLUTION ← ROBUSTNESS → **SENSITIVITY**


UNIFORM
RESOLUTION
ROBUSTNESS
SENSITIVITY

UNIFORM

NATURAL

RESOLUTION

ROBUSTNESS

SENSITIVITY

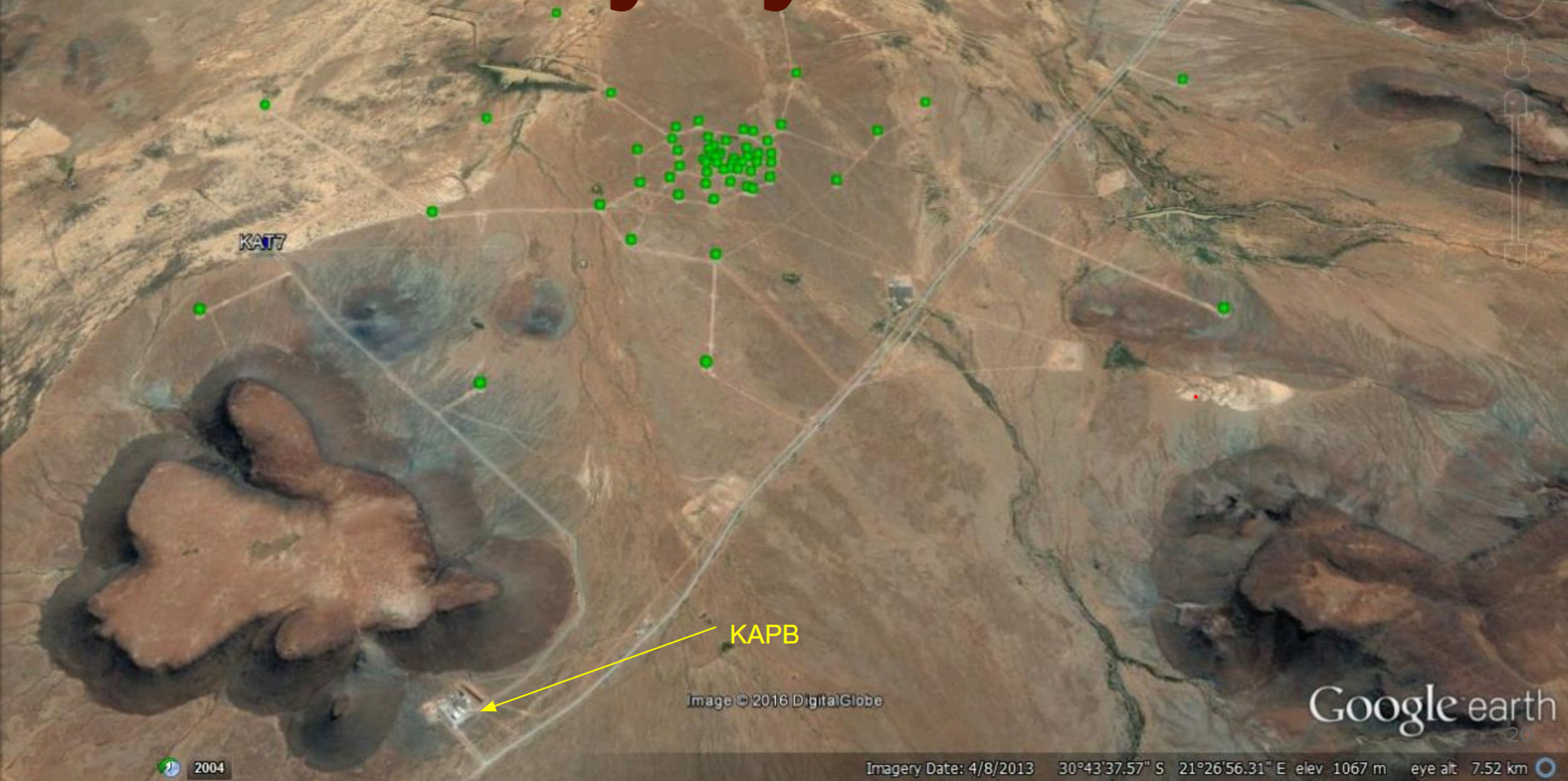
UNIFORM

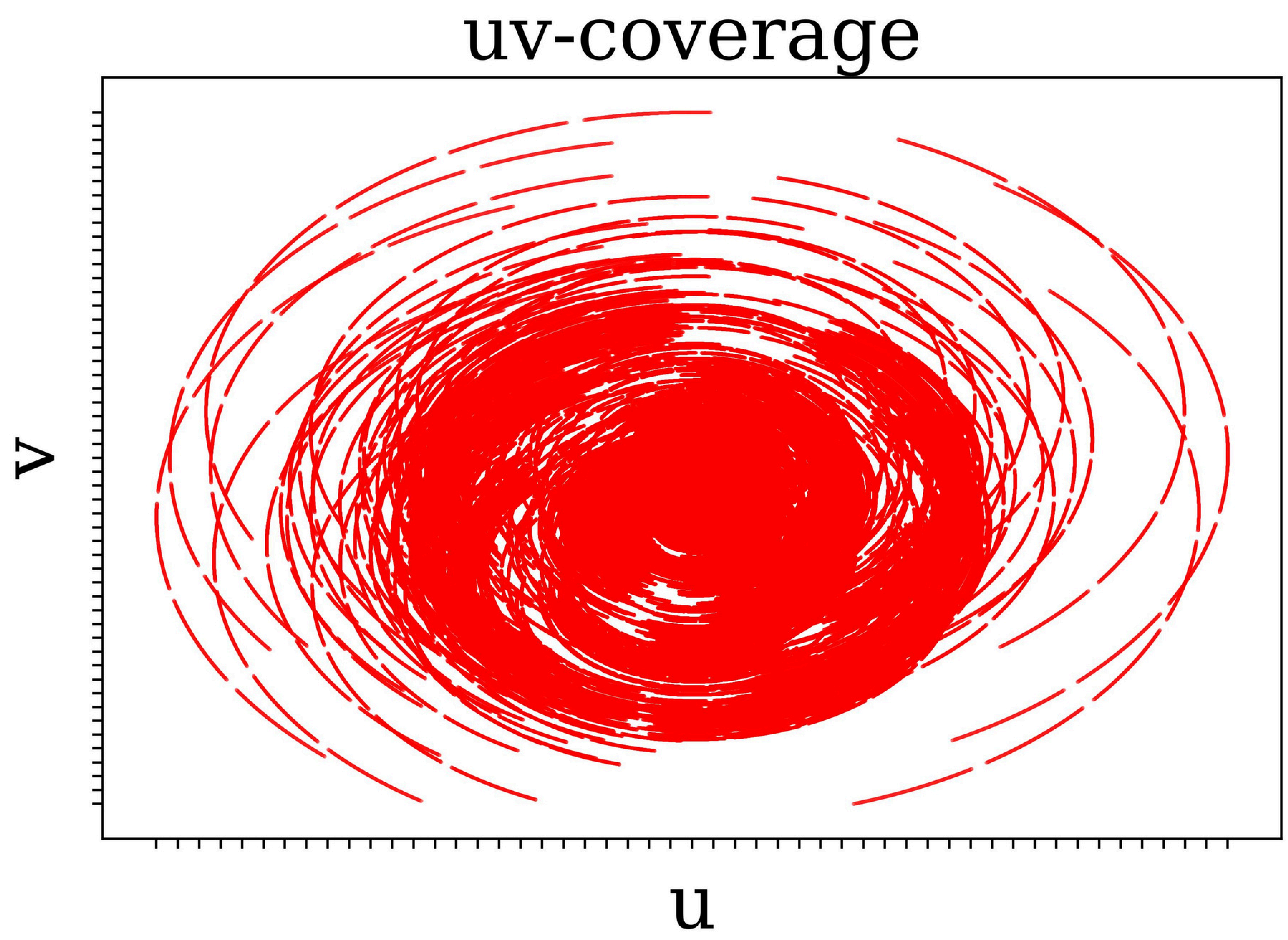
NATURAL

RESOLUTION ⟷ ROBUSTNESS ⟷ SENSITIVITY

WHY NOT BOTH?

MeerKAT Array Layout




uv-coverage
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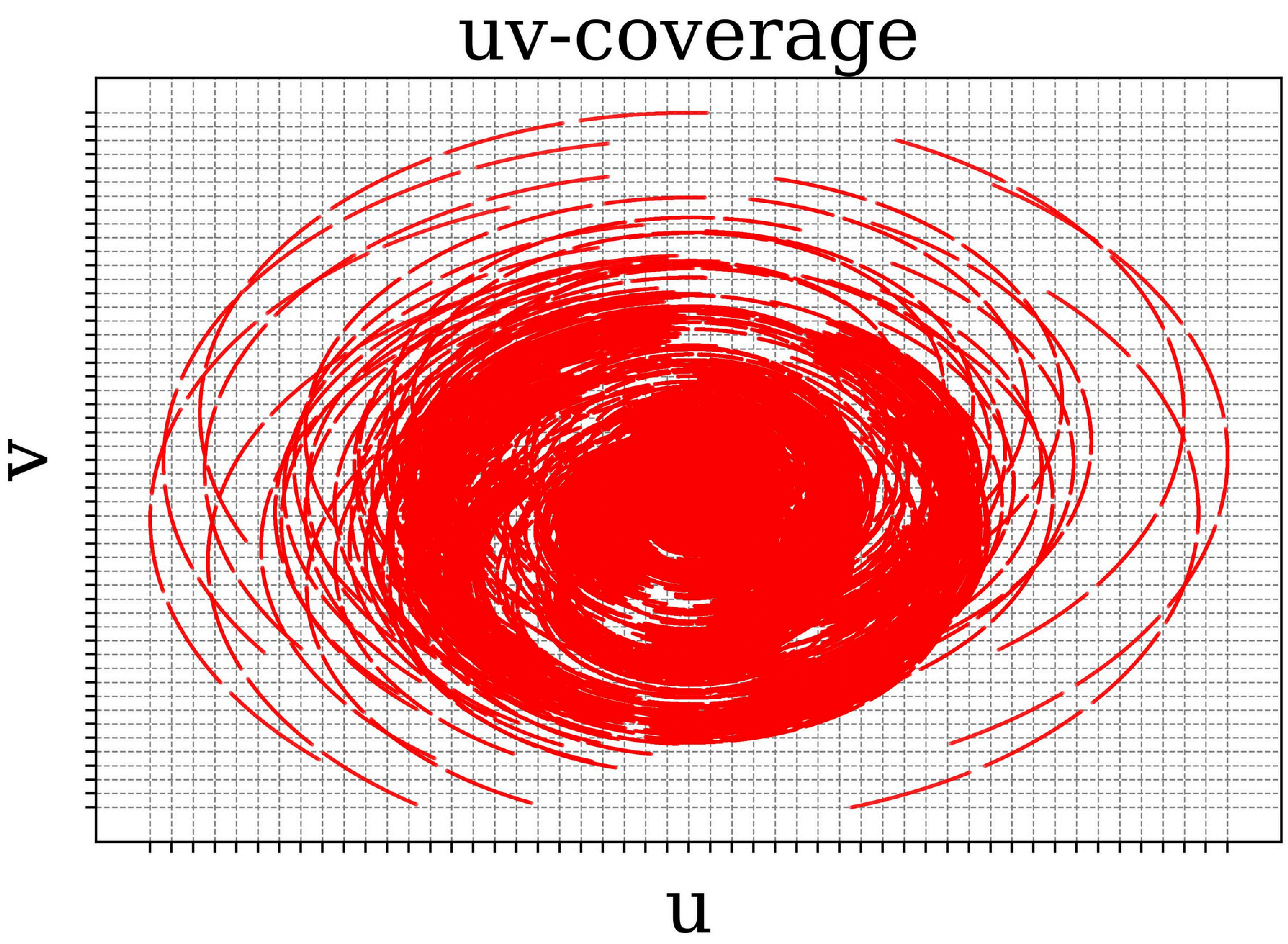
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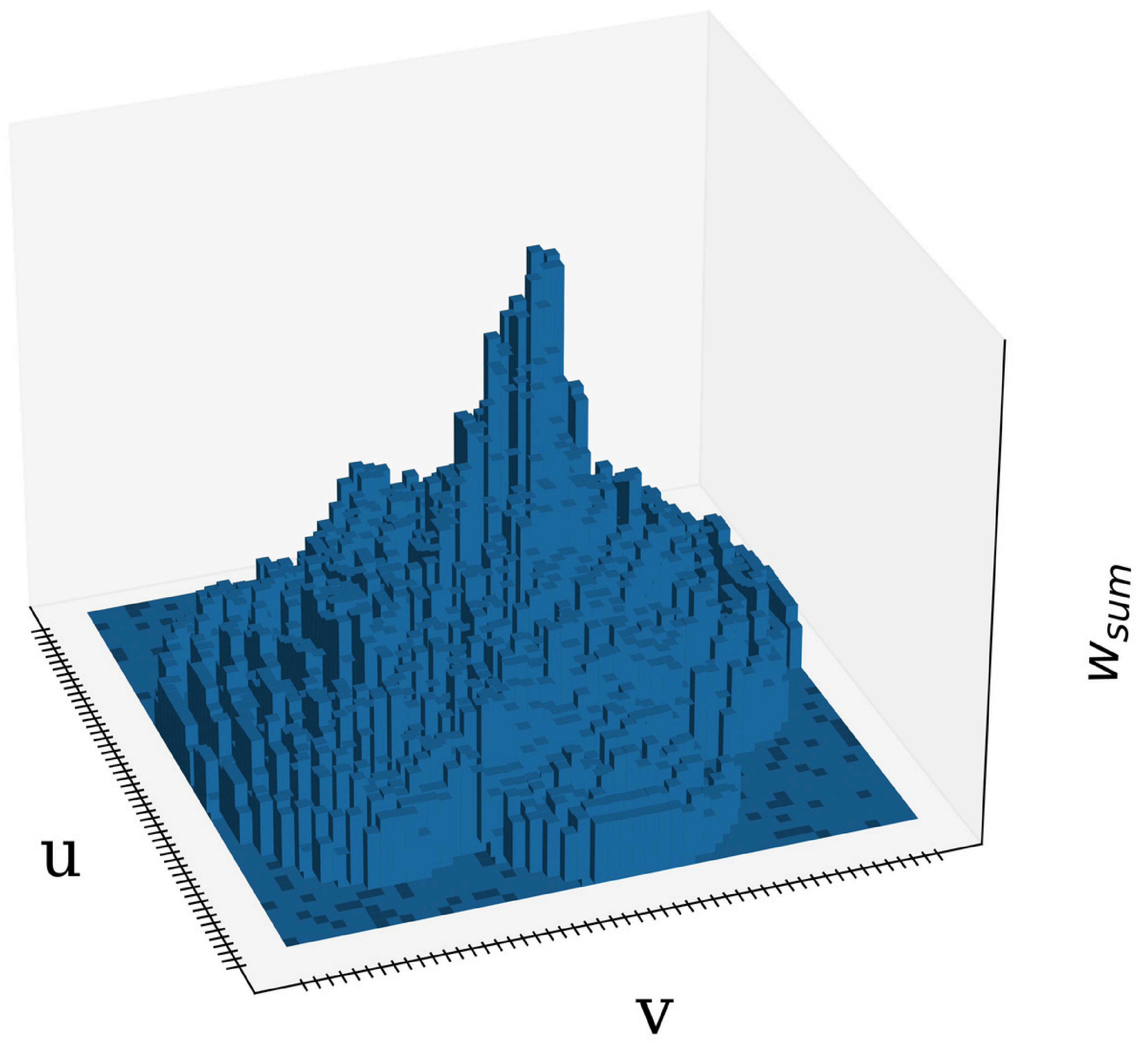
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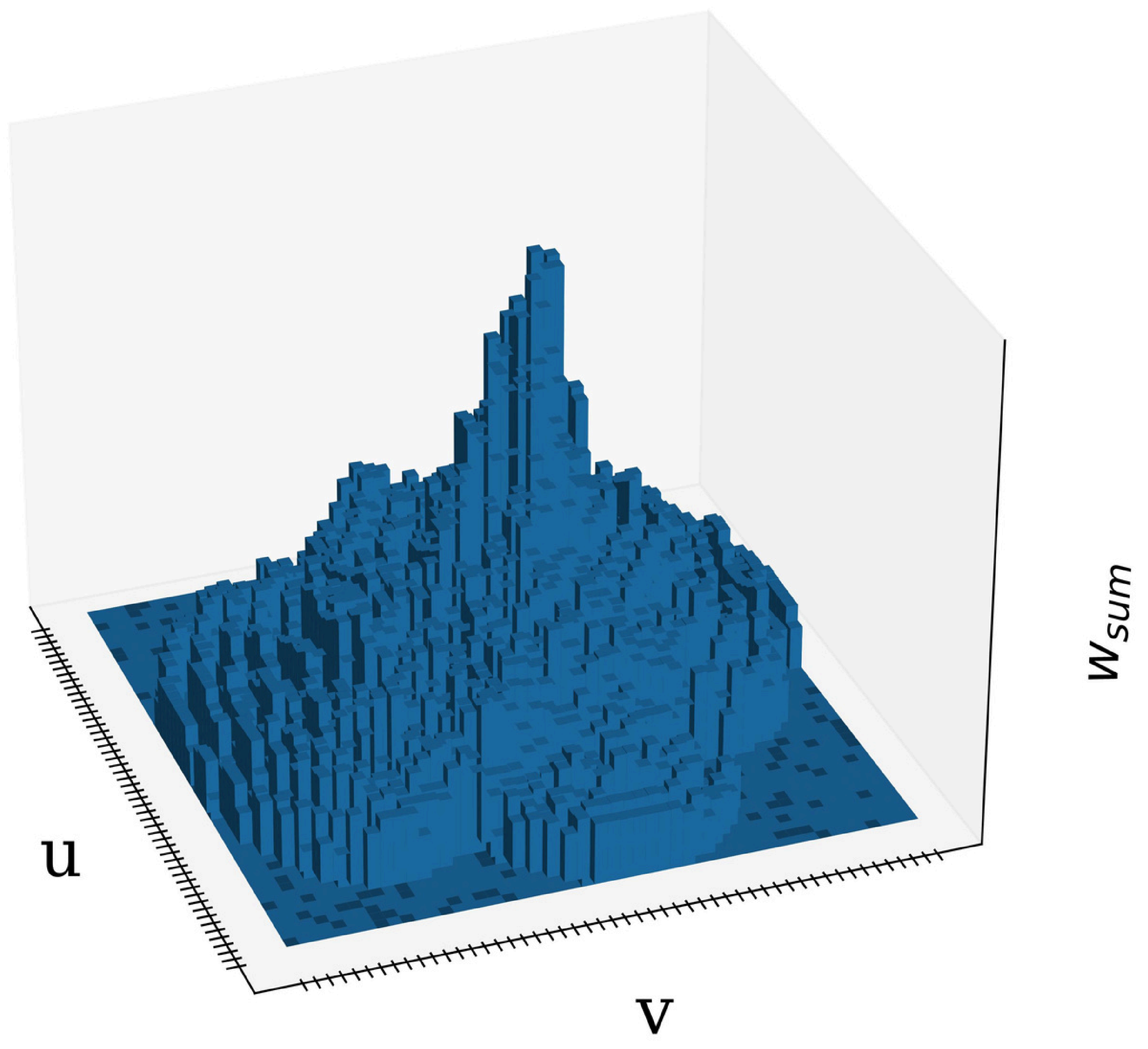
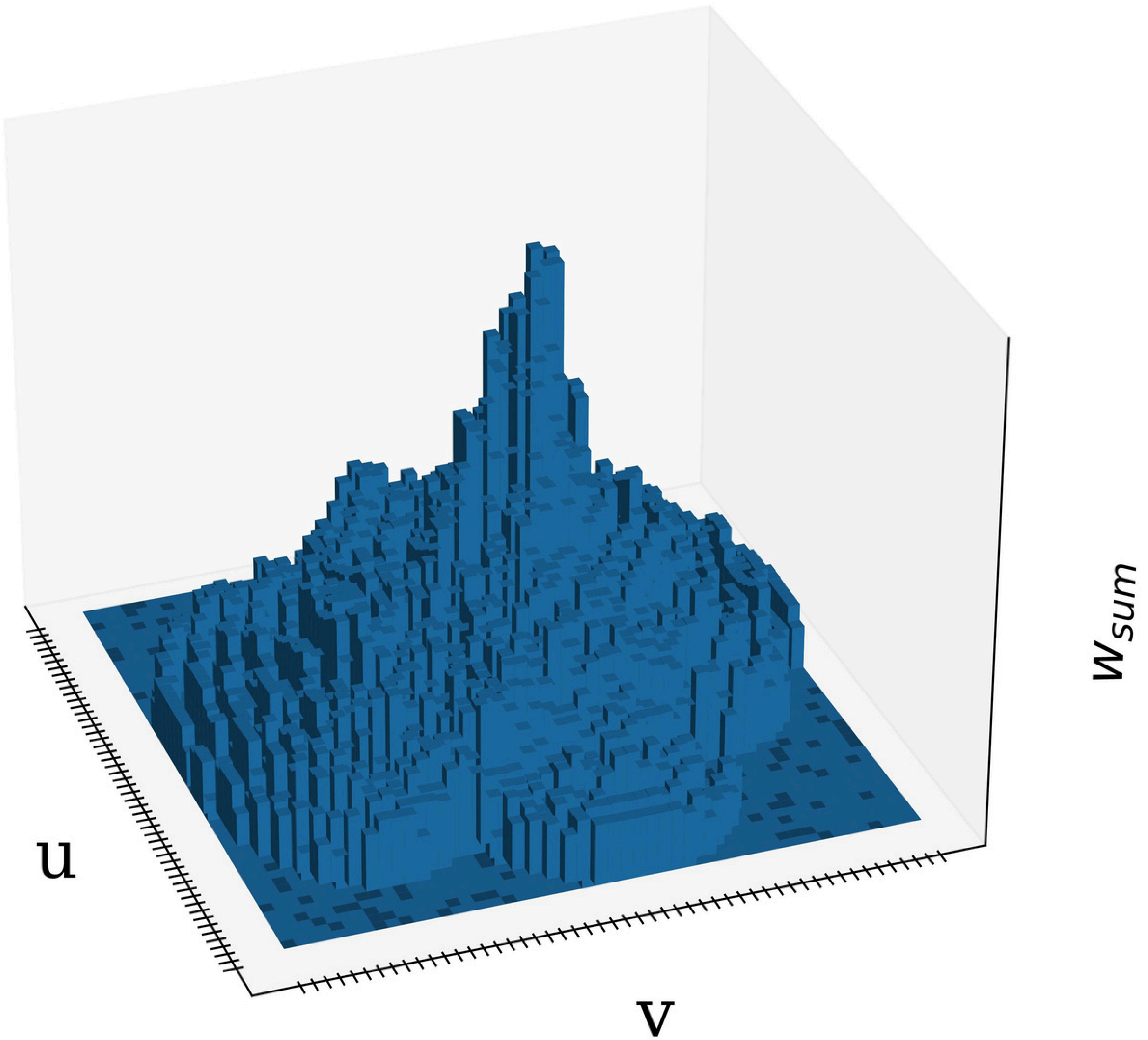
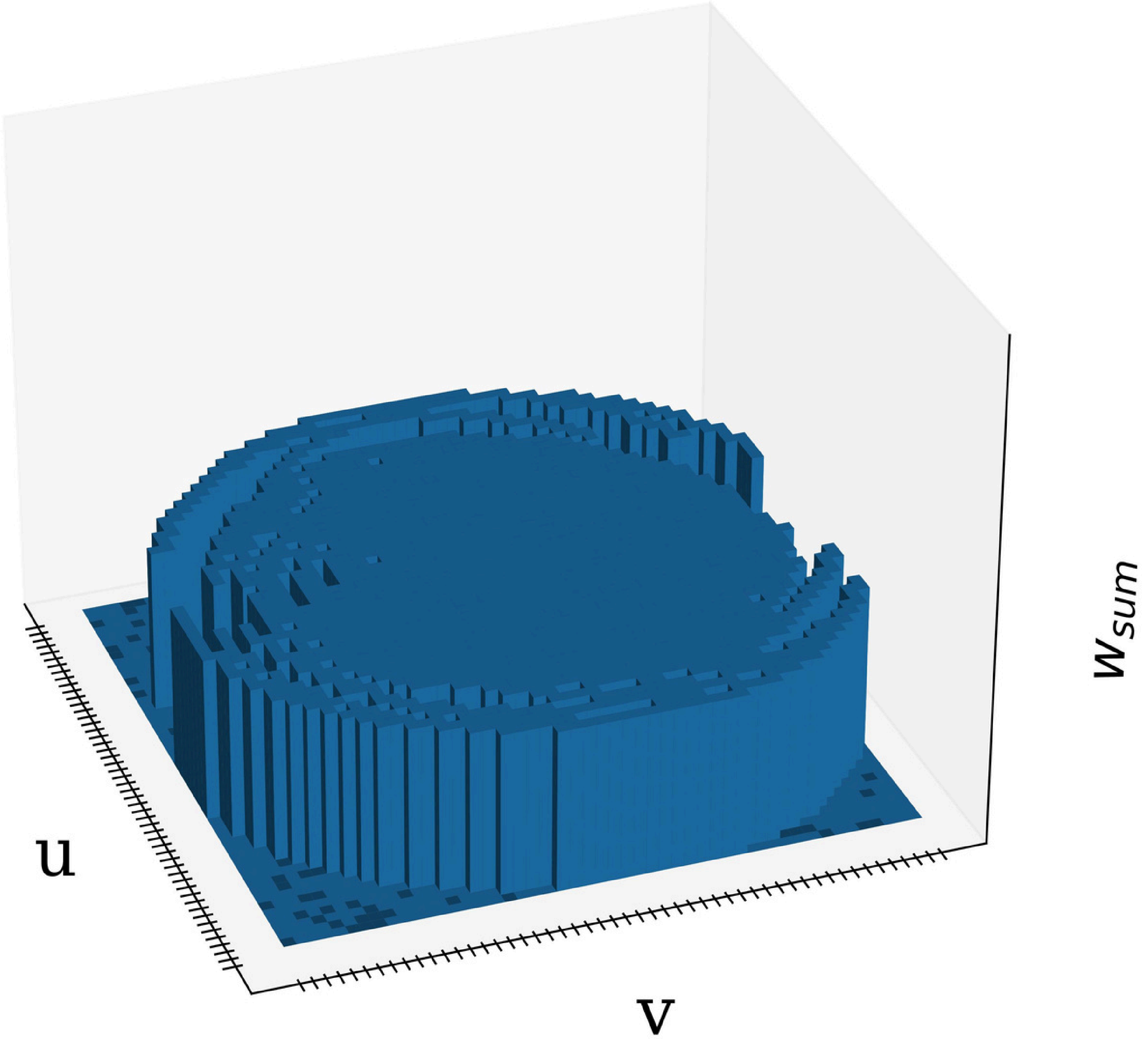
NATURAL WEIGHTING



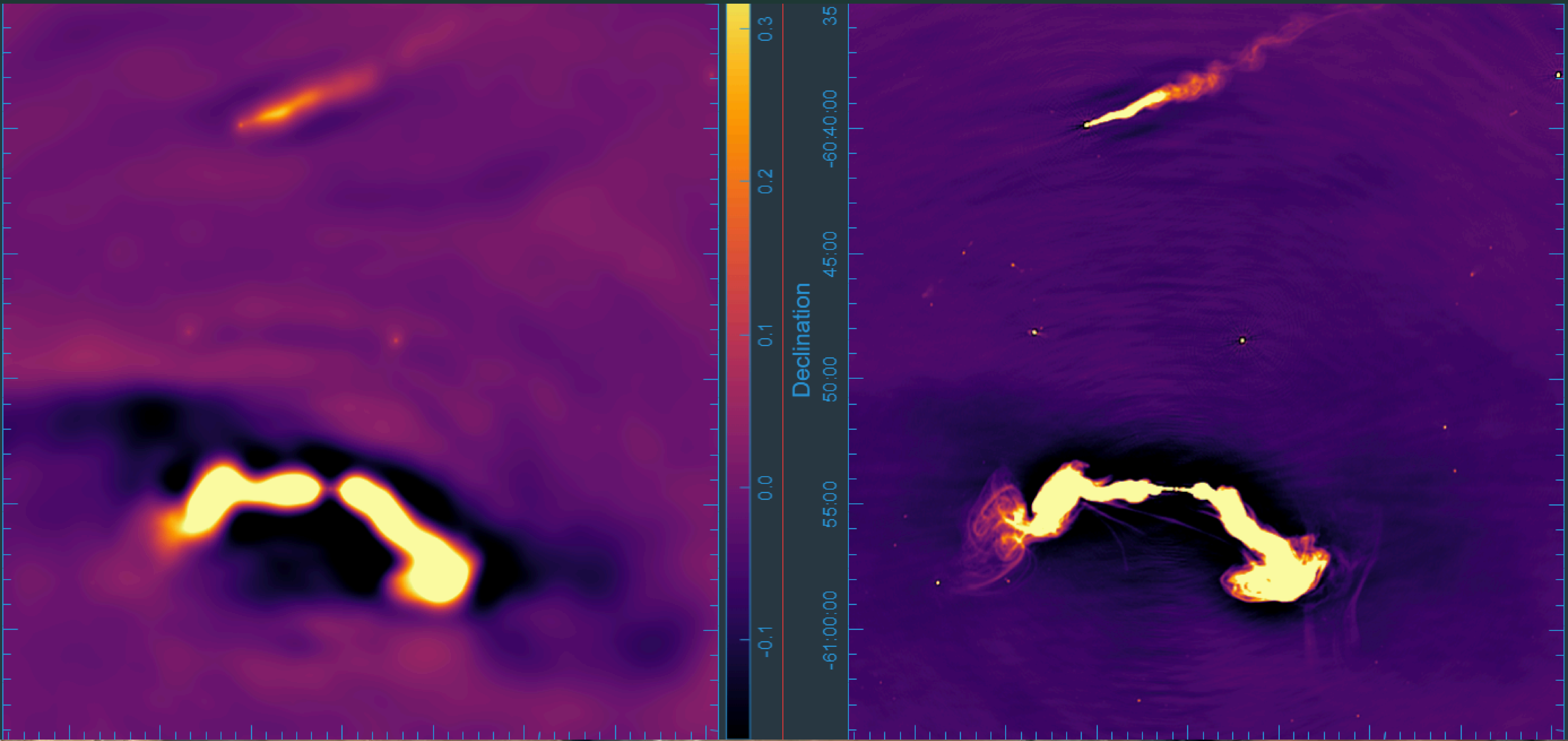
NATURAL WEIGHTING

UNIFORM WEIGHTING



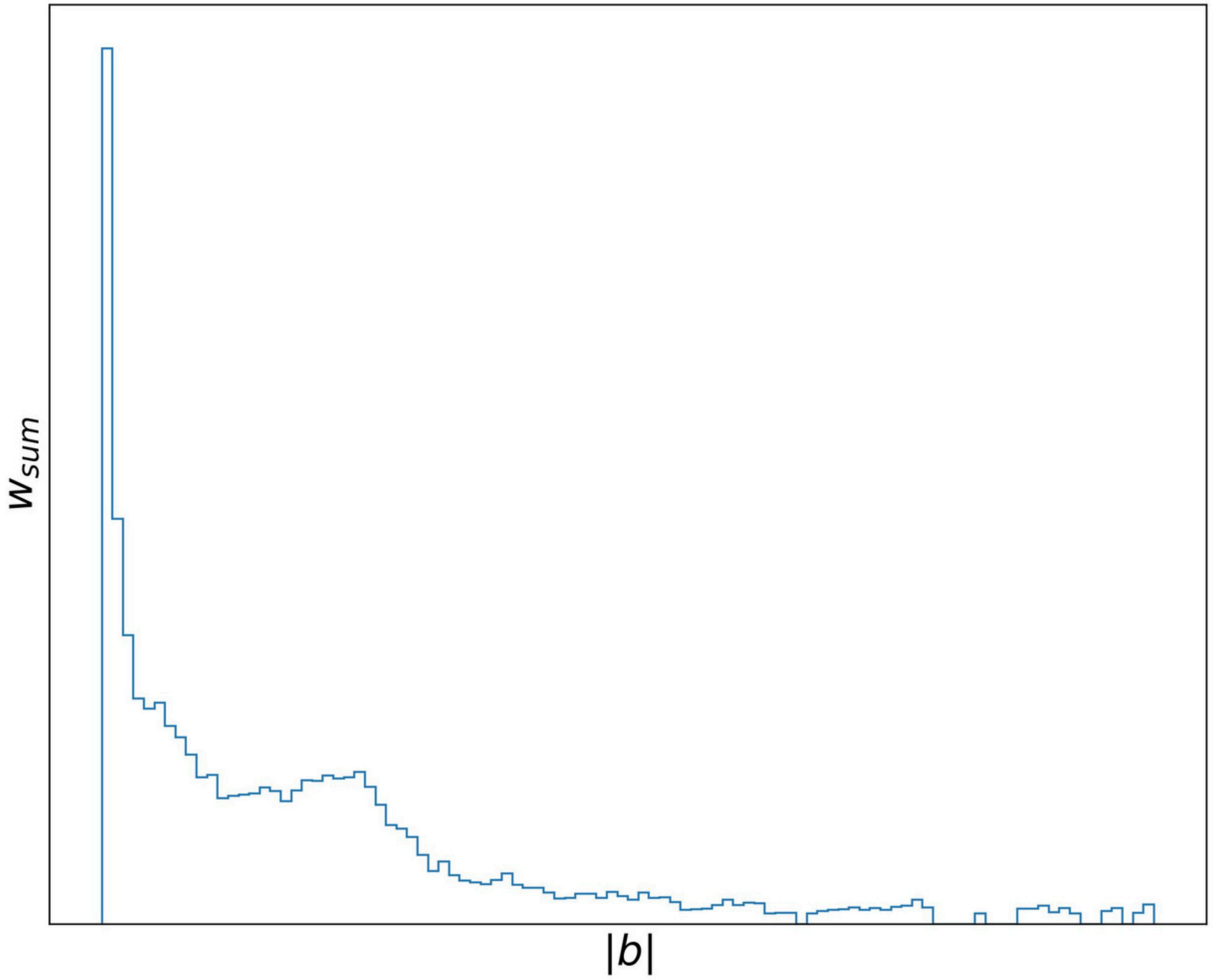


NATURAL VS UNIFORM

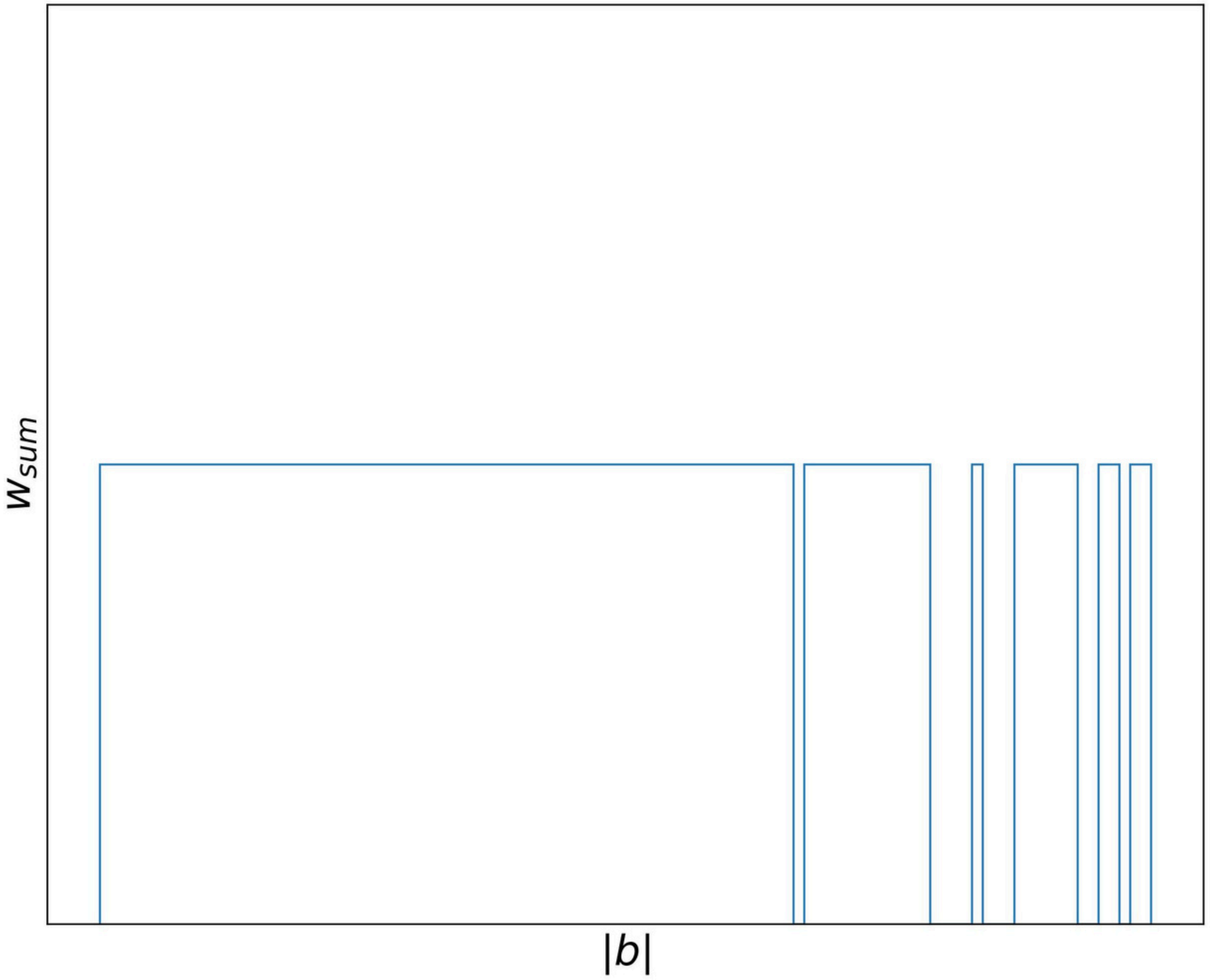


WHY DO WE LOSE SENSITIVITY?

NATURAL



UNIFORM



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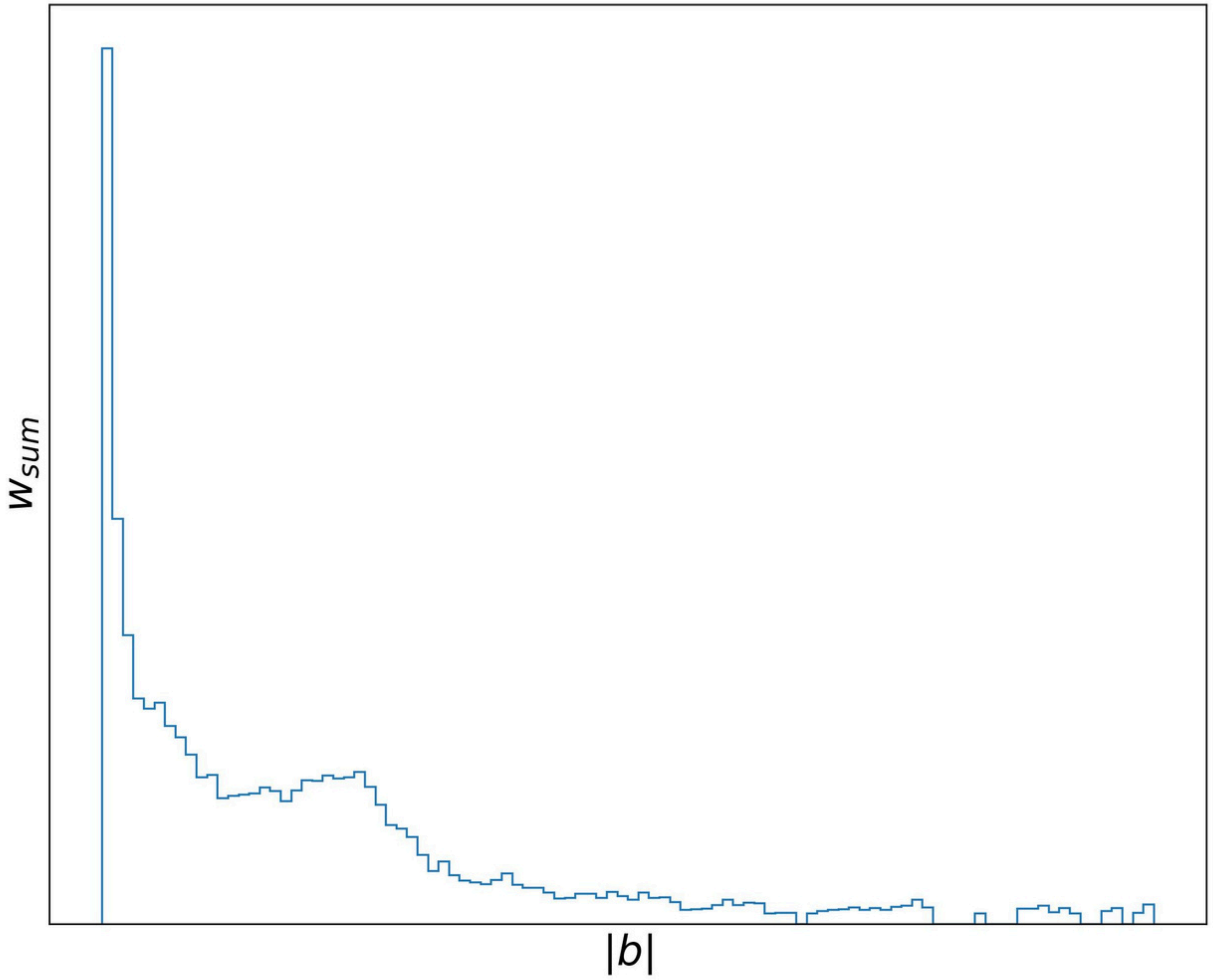
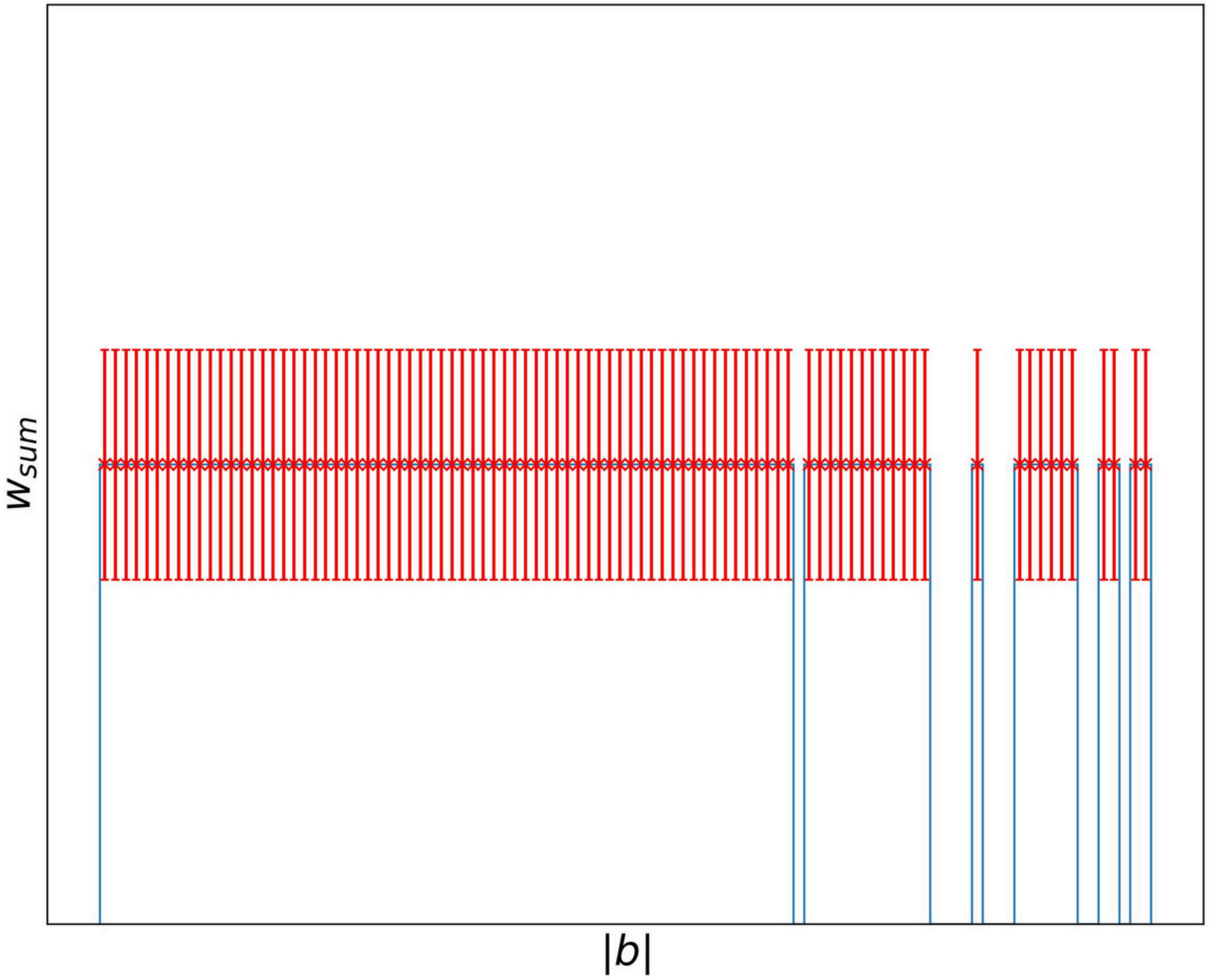


UNIFORM



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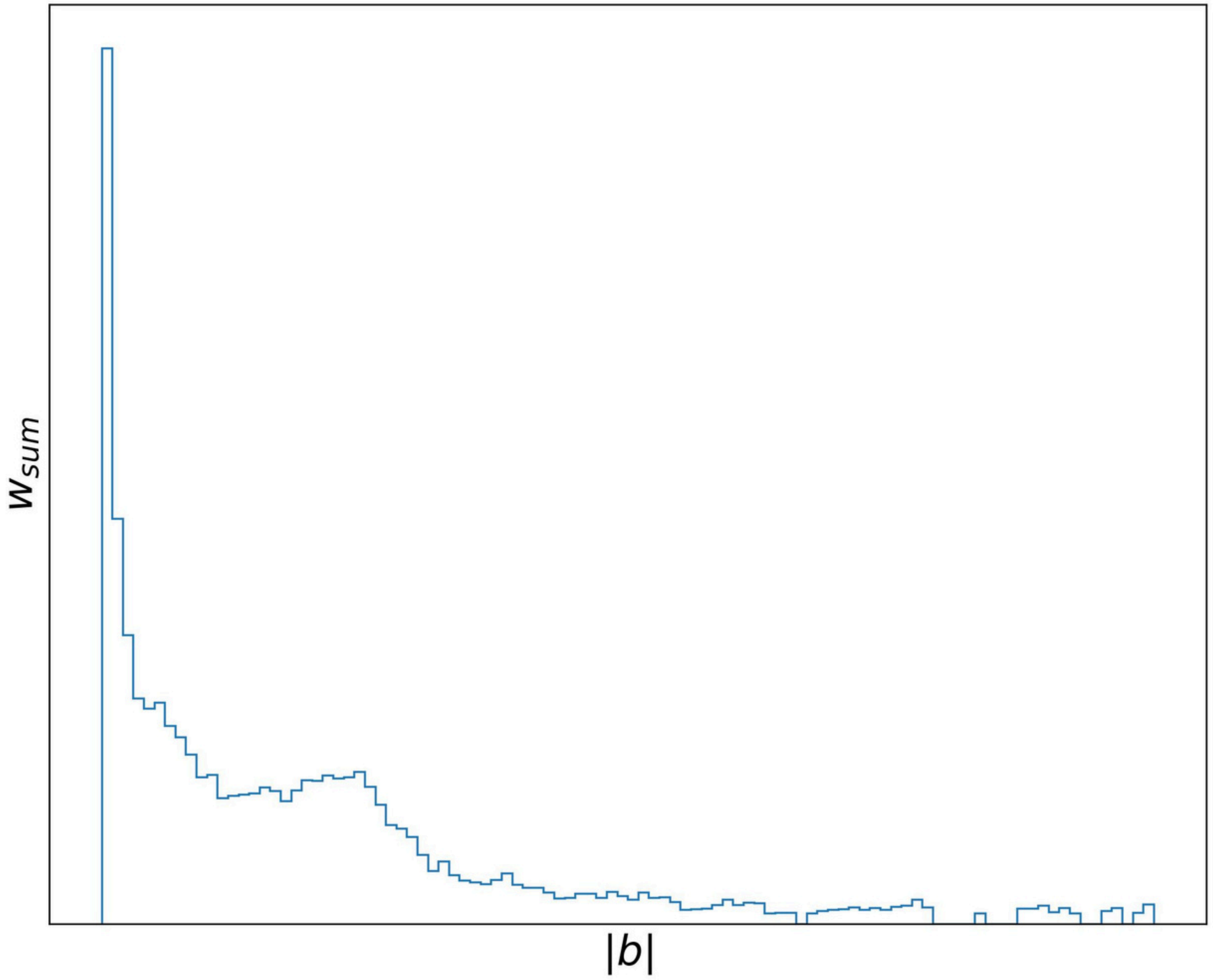

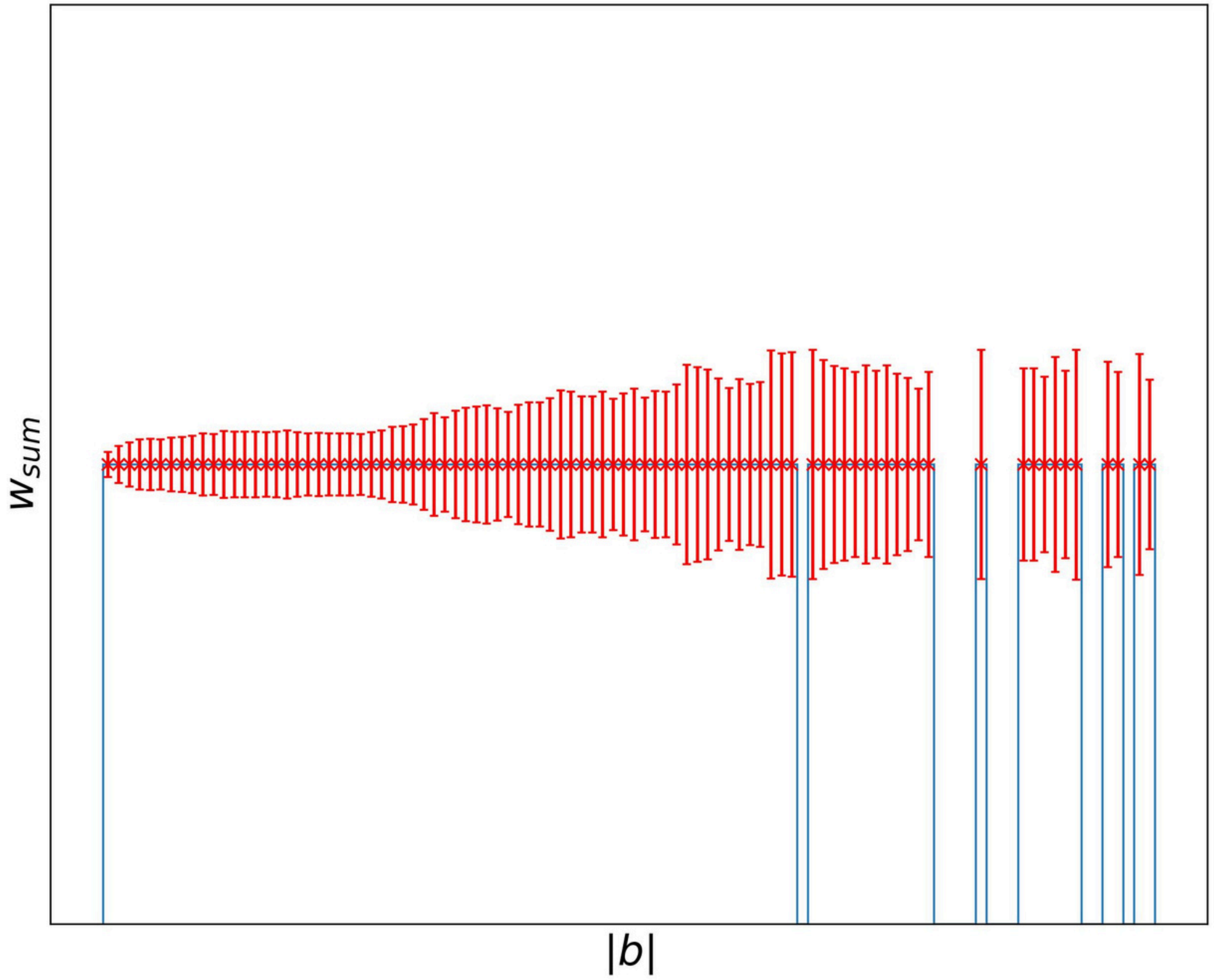

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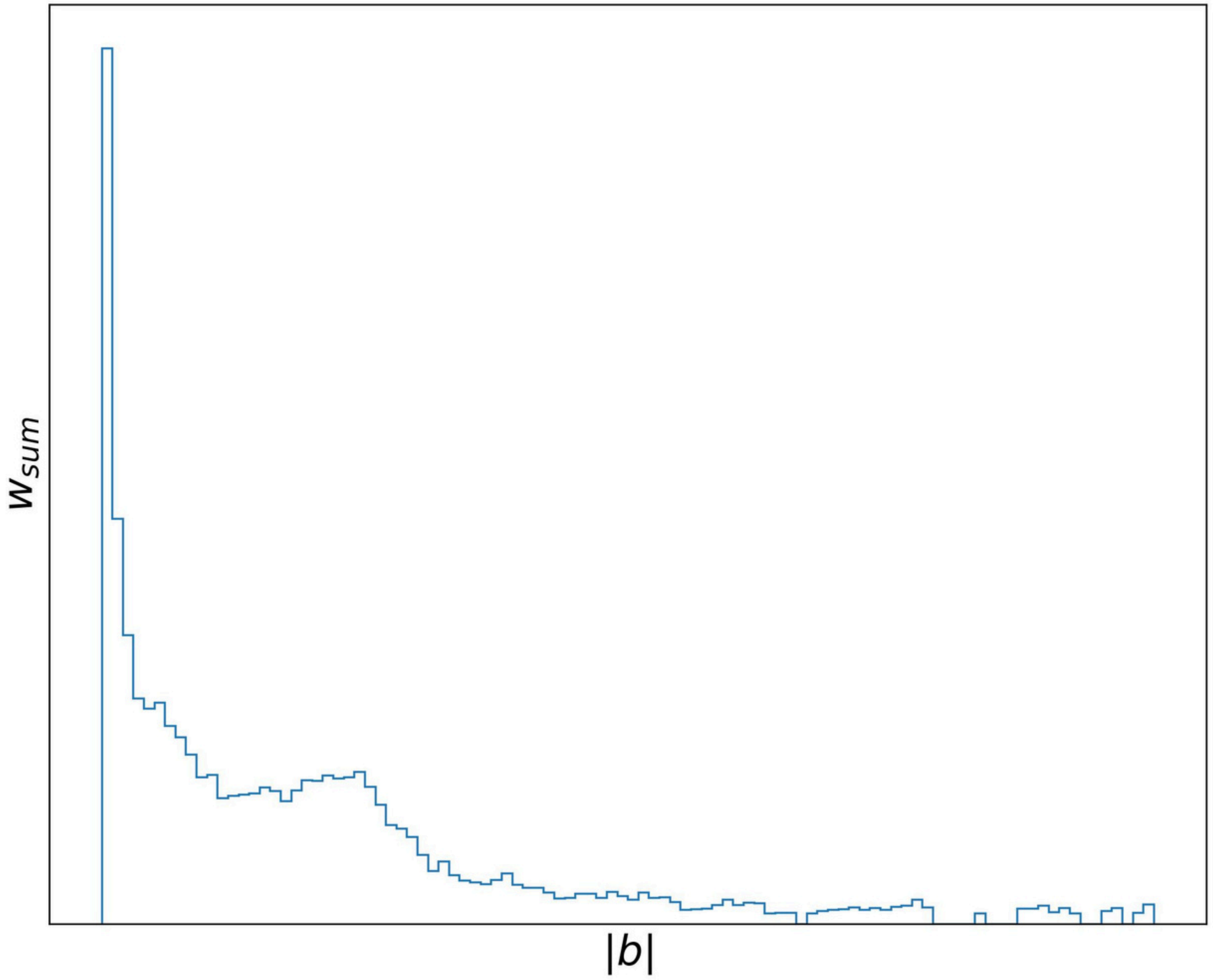

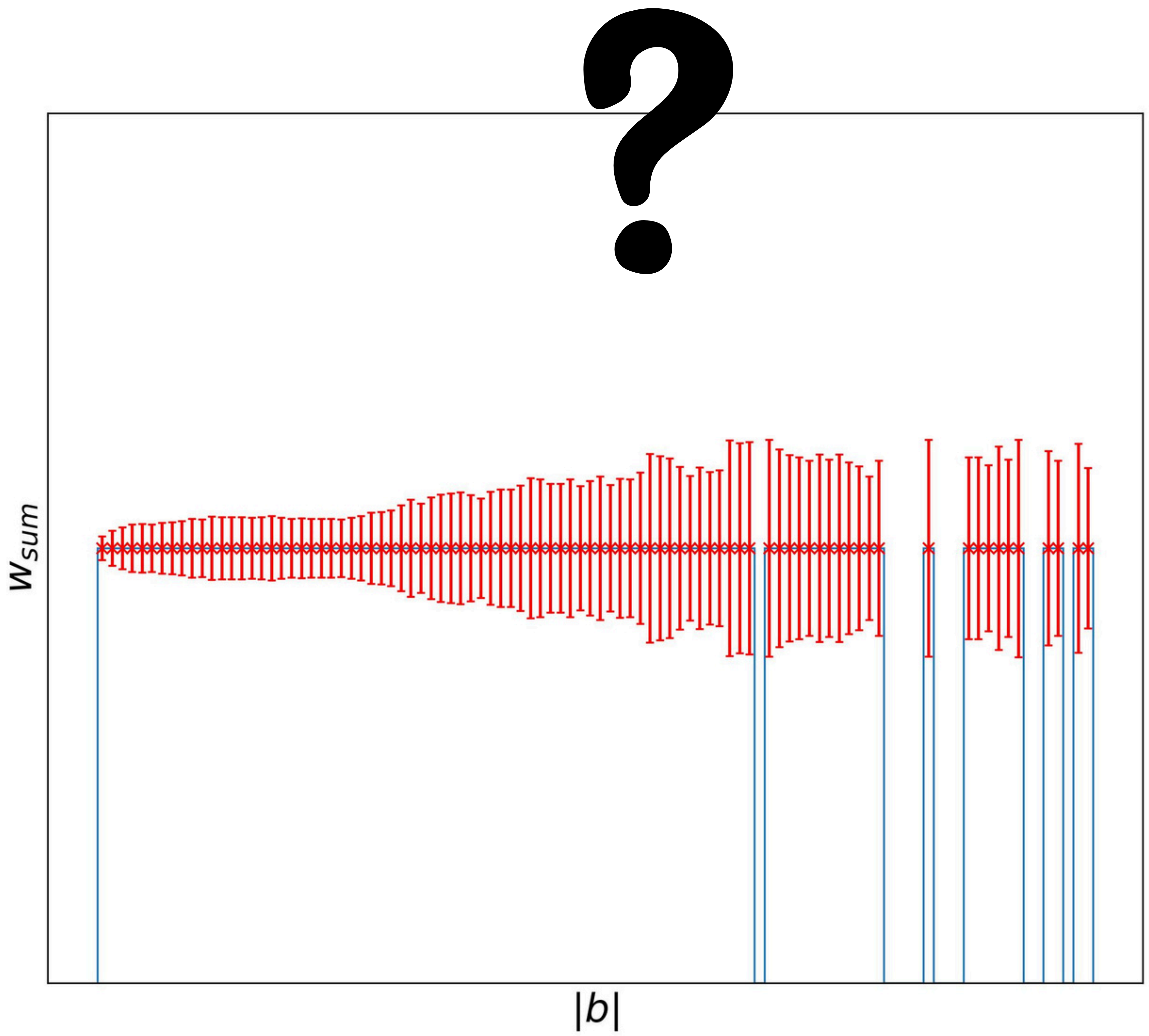

DECONVOLUTION AS A LINEAR MODEL

$$I^{D} = I^{PSF} * x = F^{\dagger}\hat{I}^{PSF}Fx$$

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$$x \approx \left(F^\dagger \hat{I}^{PSF} F \right)^{-1} I^D = F^\dagger (\hat{I}^{PSF})^{-1} F I^D$$

DECONVOLUTION AS A LINEAR MODEL

$$I^D = I^{PSF} * x = F^\dagger \hat{I}^{PS}$$

$$x \approx \left(F^\dagger \hat{I}^{PSF} F\right)^{-1} I^D = F^\dagger (\hat{I}^{PSF})^{-1} \overbrace{F I^D}^{\text{WEIGHTED AND SUMMED DATA}}$$

DECONVOLUTION AS A LINEAR MODEL

$$I^D = I^{PSF} * x = F^\dagger \hat{I}^{PSF} F x$$

$$x \approx \left(F^\dagger \hat{I}^{PSF} F \right)^{-1} I^D = F^\dagger (\underbrace{\hat{I}^{PSF}}_{\text{SUM OF WEIGHTS}})^{-1} F I^D$$

DECONVOLUTION AS A LINEAR MODEL

$$I^D = I^{PSF} * x = F^\dagger \hat{I}^{PSF} F x$$

$$x \approx \left(F^\dagger \hat{I}^{PSF} F\right)^{-1} I^D = F^\dagger (\hat{I}^{PSF})^{-1} F I^D$$

Unfortunately not that simple:

- Ill-posed problem (gaps in uv-coverage)
- Non-periodic boundary conditions
- Want physics informed regularisation
- Not just a deconvolution problem

SOLVING THE INVERSE PROBLEM

- USE BAYESIAN PRINCIPLES TO FORMULATE OPTIMISATION PROBLEM
- NATURAL GRADIENT DETERMINES SCALE EMPHASIS (WEIGHT)
- STRENGTH OF PRIOR RELATIVE TO LIKELIHOOD INCORPORATES SCALE UNCERTAINTY (SETTING HYPERPARAMETERS)
- CAN BE DONE VERY EFFICIENTLY USING THE APPROXIMATION

$$I^D \approx I^{PSF} * x$$

A practical preconditioner for wide-field continuum imaging of radio interferometric data

Hertzog L. Bester,[1,2] Audrey Repetti,[3] Simon Perkins,[1,2] Oleg M. Smirnov,[2,1] and Jonathan S. Kenyon[2,1]

[1]*South African Radio Astronomy Observatory, Cape Town, Western Cape, South Africa;* `lbester@ska.ac.za`

[2]*Rhodes University, Makhanda (Grahamstown), Eastern Cape, South Africa*

[3]*Institute of Sensors, Signals and Systems, Heriot-Watt University, Edinburgh, United Kingdom*

Abstract. The celebrated CLEAN algorithm has been the cornerstone of deconvolution algorithms in radio interferometry almost since its conception in the 1970s. For all its faults, CLEAN is remarkably fast, robust to calibration artefacts and in its ability to model point sources. We demonstrate how the same assumptions that afford CLEAN its speed can be used to accelerate more sophisticated deconvolution algorithms.

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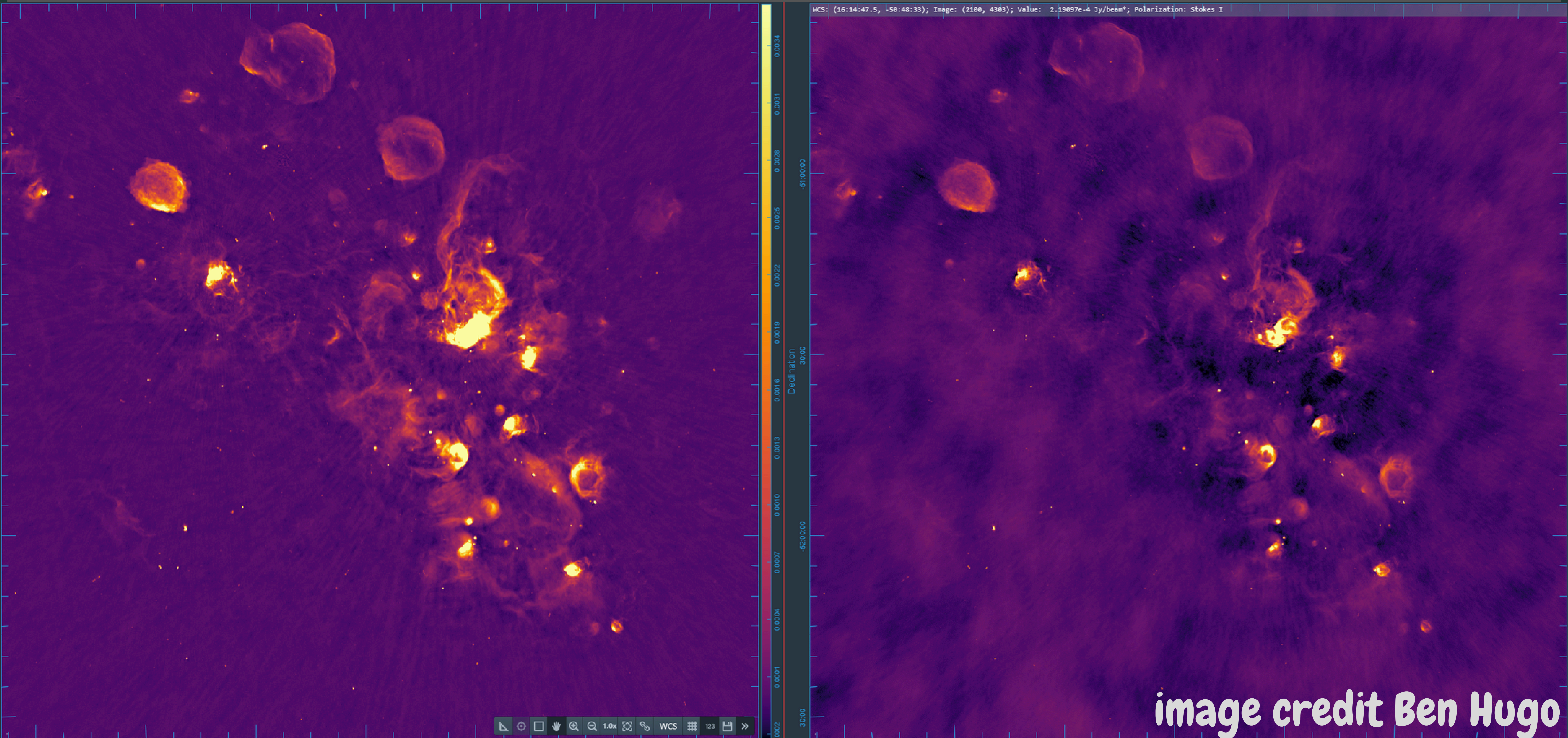
[a]*South African Radio Astronomy Observatory (SARAO), Cape Town, WC, South Africa*
[b]*Centre for Radio Astronomy Techniques & Technologies (RATT), Department of Physics and Electronics, Rhodes University, Makhanda, EC, South Africa*
[c]*Institute for Radioastronomy, National Institute of Astrophysics (INAF IRA), Bologna, Italy*

SPARSITY PRIOR VS MULTI-SCALE CLEAN



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