

1 What are Natural Numbers?

Natural Numbers are the first type of numbers that were used historically and are the numbers that are used for counting, which is why they are sometimes called Counting Numbers. The first known use of written numbers is from a carving in bones from approximately 150,000 years ago, which is thought to be used to count and can be seen in the following picture:



Figure 1: Bone carvings used for counting

Numbers evolved and number systems were defined in order to be able to use them more easily and the number system that we use now, which has its origin in India, became the standard system. We also use other numerical systems, for example when counting time.

1.1 Place Value

We use a decimal number system, which means that when counting, when we reach 10 we start using two digits, when we reach $100 = 10 \times 10$ we use 3 digits, when we get to $1000 = 10 \times 10 \times 10$ we use four digits and so on. This allows us to compare and analyse the size of a number easily. First, let's look at how we can decompose a number into its digits and analyse the value of each one. This value is called the place value of a digit. For example, the number 352 can be decomposed as follows:

$$352 = 300 + 50 + 2 = 3 \times 100 + 50 \times 10 + 2 \times 1$$

So, 352 is composed of 3 hundreds, 5 tens and 2 units. When comparing to the number 2,407, which is composed of 2 thousands, 4 hundreds, 0 tens and 7 units, we can see that 2,704 is larger as it has more

thousands than 352 as it has 0 thousands and the leading place value is larger. If we compare 352 with 391, which is composed of 3 hundreds, 9 tens and 1 unit, we see that the leading place value is the same, in this case 3, but the second place value is larger in 391. Despite the units being larger in 352, 391 is larger as the first different place value is larger in 391. We can see this easily in the following table:

		Hundreds	Tens	Units
352	=	3	5	2
391	=	3	9	1

The first different place value is in the tens category, so this will tell us which number is larger.

This decomposition of numbers into their place values also allows us to write numbers in words quite easily and this is very important in formal documents such as cheques or contracts. From the table we can see that 352 can be written in words as three hundred and fifty-two. Similarly, 391 would be three hundred and ninety-one. In the case of 2,407 we can decompose it in an extended table:

		Thousands	Hundreds	Tens	Units
2,407	=	2	4	0	7

We first note that there are 0 tens, so we would not mention the tens in the written version, which is two thousand four hundred and seven.

For larger numbers we consider extended categories: tens of thousands, hundred thousands, millions, tens of millions, hundred millions, billions, tens of billions, hundred billions, trillions, etc.. We can also extend the table as required for the number of digits each number has. Imagine for example the Kenyan National Budget for 2018 (this is the amount of money the government had to spend in the year 2018) of approximately KSH 3 trillion.

In numbers, this would be 3,000,000,000,000 and can be represented in the table as follows:

		Trillions	Hundred Billions	Tens of Billions	Billions	Hundred Millions	Tens of Millions	Millions	Hundred Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units
3,000,000,000,000	=	3	0	0	0	0	0	0	0	0	0	0	0	0

Let's now consider the number 5,470,082,903,450. In the table, this number would be

		Trillions	Hundred Billions	Tens of Billions	Billions	Hundred Millions	Tens of Millions	Millions	Hundred Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units
5,470,082,903,450	=	5	4	7	0	0	8	2	9	0	3	4	5	0

When writing this number in words recall that we ignore the digits for which their place value is 0. The result in this case would be five trillion four hundred seventy billion eighty-two million nine hundred and three thousand four hundred and fifty. The Billions, Hundred Millions, Tens of Thousands and Units place values are 0 and so they are not mentioned in the word version. All other place values are not 0 and so they have to be mentioned.

If we compare this large number with the number 5,470,082,926,450, we can place them together in the table and find the first place value that is different:

		Trillions	Hundred Billions	Tens of Billions	Billions	Hundred Millions	Tens of Millions	Millions	Hundred Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units
5,470,082,903,450	=	5	4	7	0	0	8	2	9	0	3	4	5	0
5,470,082,926,450	=	5	4	7	0	0	8	2	9	2	6	4	5	0

The numbers are the same until the Tens of Thousands, in which the latter has a place value of 2 and the former a place value of 0, which means that the latter, i.e. 5,470,082,926,450, is the larger number.

Important note - The Actual Value: The actual value of a digit depends on its place value. In the first example we looked at, 352, the Tens place value is 5, but the actual value of this digit is $5 \times 10 = 50$. Note that we multiply the place value by the category it is in. Similarly, in 391, the actual value of the Hundreds digit is $3 \times 100 = 300$. In the last two examples, the actual value of the Tens of Billions digit is $7 \times 10,000,000,000 = 70,000,000,000$.

Let's now look at some examples:

Example 1.1.

To order the following numbers from largest to smallest we decompose them into their place values using the table: 8,974, 8,990, 932 and 10,451

			Tens of Thousands	Thousands	Hundreds	Tens	Units
8,974	=			8	9	7	4
8,990	=			8	9	9	0
932	=				9	3	2
10,451	=	1	0	4	5	1	

The only number that has a non-zero value in the Tens of Thousands is 10,451 and none of the numbers has a larger place value. So the largest number is 10,451. We now compare the Thousands place values and note that for 932 we have 0 (or no Thousands place value), so we are left with 8,974 and 8,990. The first place value that is different is in the Tens, which is larger in 8,990, so 8,990 is larger than 8,974. Finally, the leading place value of 932 is 9 and in the Hundreds, which means that is the smallest number as the rest have place values in larger categories. So the order from largest to smallest is 10,451, 8,990, 8,974, 932.

Example 1.2.

To write the four numbers in the previous example above we guide ourselves with the table:

- 8,947 is composed of 8 Thousands, 9 Hundreds, 7 Tens and 4 Units. So in words 8,947 is eight thousand nine hundred and forty-seven.
- 8,990 is composed of 8 Thousands, 9 Hundreds, 9 Tens and 0 Units (which means we don't need to mention units). So in words 8,990 is eight thousand nine hundred and ninety.
- 932 is composed of 9 Hundreds, 3 Tens and 2 Units. So in words 932 is nine hundred and thirty-two.
- 10,451 is composed of 1 Tens of Thousands, 0 Thousands, 4 Hundreds, 5 Tens and 1 Unit. So in words 10,451 is Ten thousand four hundred and fifty-one.

Example 1.3.

A large forestal company is selling a large amount of wood for KSH 3,406,720 to a woodworking company. They are going to receive a cheque and need to make sure the number is written correctly in words so that the cheque is valid. They can use the place value table to translate it into words as follows:

			Millions	Hundred Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units
3,406,720	=	3	4	0	6	7	2	0	

From the table we can directly write 3,406,720 as three million four hundred and six thousand seven hundred and twenty. Note that we group the three categories of thousands so that we use the word thousand only once and we ignore the tens of thousands and units as they have a place value of 0.

Example 1.4.

What is the actual value of the tens of thousands place value of the number 56,782,309? To find this we find the Tens of Thousands place value, which is 8 and multiply it by the category value, which is 10,000. The actual value then would be $8 \times 10,000 = 80,000$.

1.2 Exercises

1. Order the following numbers from largest to smallest: 75,405, 70,545, 7,545 and 75,045.
2. Write the numbers in the previous exercise in words.
3. What is the actual value of the hundreds in the largest and smallest numbers in the first exercise?
4. A family is hoping to buy a flat in the Livingston area of Nairobi and found one for KSH 20,500,000. They want to offer the asking price and the offer has to be submitted in writing, with the number in words. Write the price of the property in words as it should appear in the offer.