E3)

$$X = [x_v\ y_v\ \theta_v\ x_1\ y_1 \cdots x_M\ y_M]^T$$

$$f = \begin{bmatrix} x_v + (\delta_d + v_d)\cos\theta_v \\ y_v + (\delta_d + v_d)\sin\theta_v \\ \theta_v + \delta_\theta + v_\theta \\ x_1 \\ y_1 \\ \vdots \\ x_M \\ y_M \end{bmatrix}$$

$(x_v, y_v, \theta_v) \rightarrow$ vehicle pose

$x_1 y_1 \cdots x_M y_M$ are land marks.

$$F_x = \frac{\partial f}{\partial x} = \begin{bmatrix} \frac{d(x_v + (\delta_d + v_d)\cos\theta_v)}{d x_v} & \frac{d(x_v (\delta_d + v_d)\cos\theta_v)}{d y_v} & \frac{d(x_v + (\delta_d + v_d)\cos\theta_v)}{\partial \theta_v} & \cdots & \frac{d(x_v + (\delta_d + v_d)\cos\theta_v)}{d y_M} \\ \frac{\partial (y_v + (\delta_d + v_d)\sin\theta_v)}{d x_v} & \frac{d(y_v + (\delta_d + v_d)\sin\theta_v)}{d y_v} & \frac{d(y_v + (\delta_d + v_d)\sin\theta_v)}{d\theta_v} & \cdots & \frac{d(y_v + (\delta_d + v_d)\sin\theta_v)}{d y_M} \\ \frac{d(\theta_v + \delta_\theta + v_\theta)}{d x_v} & \frac{d(\theta_v + \delta_\theta + v_\theta)}{d y_v} & \frac{d(\theta_v + \delta_\theta + v_\theta)}{d\theta_v} & \cdots & \frac{d(\theta_v + \delta_\theta + v_\theta)}{d y_M} \\ \frac{d x_1}{d x_v} & \frac{d x_1}{d y_v} & \cdots & & \frac{d x_1}{d y_M} \\ \vdots & & & & \vdots \\ \frac{d y_M}{d x_v} & \cdots & & & \frac{d y_M}{d y_M} \end{bmatrix}$$

Scanned with CamScanner

$$F_x = \frac{\partial f}{\partial x} = \begin{bmatrix} 1 & 0 & -\delta_d \sin\theta_v & 0 \cdots 0 \cdots & 0 \\ 0 & 1 & \delta_d \cos\theta_v & 0 \cdots 0 \cdots & 0 \\ 0 & 0 & 1 & 0 \cdots 0 \cdots & 0 \\ \vdots & \cdots & & \ddots & 0 \\ & & & 1 \cdots & 0 \\ & & & & 1 \\ 0 & 0 & 0 & 0 \quad 0 & \end{bmatrix}$$

$$F_v = \frac{\partial f}{\partial v} = \begin{bmatrix} \frac{d(x_v + (\delta_d + v_d)\cos\theta_v)}{d(\delta_d)} & \frac{d(x_v + (\delta_d + v_d)\cos\theta_v)}{d\delta_\theta} \\ \frac{d(y_v + (\delta_d + v_d)\sin\theta_v)}{d\delta_d} & \frac{d(y_v + (\delta_d + v_d)\sin\theta_v)}{d\delta_\theta} \\ \frac{d(\theta_v + \delta_\theta + v_\theta)}{d\delta_d} & \frac{d(\theta_v + \delta_\theta + v_\theta)}{d\delta_\theta} \\ \frac{dx_1}{d\delta_d} & \frac{dx_1}{d\delta_\theta} \\ \vdots & \vdots \\ \frac{dy_M}{d\delta_d} & \frac{dy_M}{d\delta_\theta} \end{bmatrix}$$

$v = (\delta_d, \delta_\theta)$

$$F_v = \begin{bmatrix} \cos\theta_v & 0 \\ \sin\theta_v & 0 \\ 0 & 1 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix} \qquad h = \begin{bmatrix} \sqrt{(y_i-y_v)^2+(x_i-x_v)^2} + \omega_r \\ \tan^{-1}\left(\frac{y_i-y_v}{x_i-x_v}\right) - \theta + \omega_\beta \end{bmatrix}$$

$$\frac{\partial h}{\partial p_i} = \begin{bmatrix} \frac{d\left(\sqrt{(y_i-y_v)^2+(x_i-x_v)^2}+\omega_r\right)}{dx_i} & \frac{d\left(\sqrt{(y_i-y_v)^2+(x_i-x_v)^2}+\omega_r\right)}{dy_i} \\ \frac{d\left(\tan^{-1}(y_i-y_v)/(x_i-x_v)-\theta+\omega_\beta\right)}{dx_i} & \frac{d\left(\tan^{-1}(y_i-y_v)/(x_i-x_v)-\theta+\omega_\beta\right)}{dy_i} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}\left[(y_i-y_v)^2+(x_i-x_v)^2\right]^{1/2-1} \cdot 2(x_i-x_v) & \frac{1}{2}\left[(y_i-y_v)^2+(x_i-x_v)^2\right]^{1/2-1} \cdot 2(y_i-y_v) \\ \frac{1}{1+\left(\frac{y_i-y_v}{x_i-x_v}\right)^2}(-1)\,(y_i-y_v)\cdot(x_i-x_v)^{-2} & \frac{1}{1+\left(\frac{y_i-y_v}{x_i-x_v}\right)^2}\cdot\frac{1}{x_i-x_v} \end{bmatrix}$$

$$H_{p_i} = \begin{bmatrix} \frac{x_i-x_v}{\sqrt{(y_i-y_v)^2+(x_i-x_v)^2}} & \frac{(y_i-y_v)}{\sqrt{(y_i-y_v)^2+(x_i-x_v)^2}} \\ \frac{-(y_i-y_v)}{(x_i-x_v)^2+(y_i-y_v)^2} & \frac{x_i-x_v}{(x_i-x_v)^2+(y_i-y_v)^2} \end{bmatrix} = \frac{\partial h}{\partial p_j}$$

$$H_x = (H_v \cdots 0 \cdots H_{P_i} \cdots 0)$$

$$H_v = \begin{bmatrix} \dfrac{d\left(\sqrt{(y_i-y_v)^2+(x_i-x_v)^2}+\omega_r\right)}{d x_v} & \dfrac{d\left(\sqrt{(y_i-y_v)^2+(x_i-x_v)^2}+\omega_r\right)}{d y_v} & \dfrac{d\left(\sqrt{(y_i-y_v)^2+(x_i-x_v)^2}+\omega_r\right)}{d \theta_v} \\ \dfrac{d\left(\tan^{-1}(y_i-y_v)/(x_i-x_v)-\theta+\omega_r\right)}{d x_v} & \dfrac{d\left(\tan^{-1}(y_i-y_v)/(x_i-x_v)-\theta+\omega_r\right)}{d y_v} & \dfrac{d\left(\tan^{-1}\frac{(y_i-y_v)}{(x_i-x_v)}-\theta+\omega_r\right)}{d\theta_v} \end{bmatrix}$$

$$H_v = \begin{bmatrix} \dfrac{-x_i-x_v}{\sqrt{(y_i-y_v)^2+(x_i-x_v)^2}} & \dfrac{-y_i-y_v}{\sqrt{(y_i-y_v)^2+(x_i-x_v)^2}} & 0 \\ \dfrac{y_i-y_v}{(y_i-y_v)^2+(x_i-x_v)^2} & \dfrac{-x_i-x_v}{(y_i-y_v)^2+(x_i-x_v)^2} & -1 \end{bmatrix}$$

Scanned with CamScanner

$$Y_z = \begin{bmatrix} I_{n \times n} & & 0_{n \times 2} \\ G_{x_v} & 0_{2 \times n-3} & G_z \end{bmatrix} \qquad g = \begin{pmatrix} x_v + r\cos(\theta_v + \beta) \\ y_v + r\sin(\theta_v + \beta) \end{pmatrix}$$

$$G_{x_v} = \frac{\partial g}{\partial x_v} = \begin{bmatrix} \frac{d(x_v + r\cos(\theta_v+\beta))}{dx_v} & \frac{d(x_v + r\cos(\theta_v+\beta))}{dy_v} & \frac{d(x_v + r\cos(\theta_v+\beta))}{d\theta_v} \\ \frac{d(y_v + r\sin(\theta_v+\beta))}{dx_v} & \frac{d(y_v + r\sin(\theta_v+\beta))}{dy_v} & \frac{d(y_v + r\sin(\theta_v+\beta))}{d\theta_v} \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0 & -r\sin(\theta_v+\beta)\cdot 1 + 0 \\ 0 & 1+0 & 0 + r\cos(\theta_v+\beta)\cdot 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -r\sin(\theta_v+\beta) \\ 0 & 1 & r\cos(\theta_v+\beta) \end{bmatrix}$$

$$G_z = \frac{\partial g}{\partial z} = \begin{bmatrix} \frac{d(x_v + r\cos(\theta_v+\beta))}{dr} & \frac{d(x_v + r\cos(\theta_v+\beta))}{d\beta} \\ \frac{d(y_v + r\sin(\theta_v+\beta))}{dr} & \frac{d(y_v + r\sin(\theta_v+\beta))}{d\beta} \end{bmatrix} \quad z = (r, \beta)$$

$$= \begin{bmatrix} 0 + \cos(\theta_v+\beta) & -r\sin(\theta_v+\beta) \\ 0 + \sin(\theta_v+\beta) & r\cos(\theta_v+\beta) \end{bmatrix}$$

$$G_2 = \begin{bmatrix} \cos(\theta_V + \beta) & -r\sin(\theta_V + \beta) \\ \sin(\theta_V + \beta) & r\cos(\theta_V + \beta) \end{bmatrix}$$