CSGames Qualifications 2020

Theoretical Computing

Use this Latex template to answer directly

Question 1

Name the data structure that fits the description:

- 1. Element search is in $O(\log n)$
- 2. Element insertion is in O(1)
- 3. Element search is in O(1)

Question 2

Name a sorting algorithm that can complete in O(n)

Question 3

Can a sorting algorithm finish in O(log n) everytime without preprocessing the data?

Question 4

Explain what is a regular language?

Question 5

If we have $v = \{a, b\}$, give $v \star$.

Question 6

Given the regular expression R = (ab*)*, find an equivalent regular expression that only have a star height of 1 (no nested Kleene star. The allowed operations are union, concatenation and Kleene star.

Question 7

Is this equation true or false? justify.

$$(\neg r \Rightarrow q) \lor \neg (q \lor s) \lor s \land q \land r \land \neg (t \land s) \Rightarrow (q \land \neg t) \lor r \land \neg s)$$

Question 8

If a language L is regular, it's complement is also a regular language. true or false? justify.

Question 9

Explain in details the impact of a black box that could factorize products of prime numbers in polinomial time on the rsa cryptosystem.

Question 10

Explain the following regular expression:

$$\epsilon \cup 0(0 \cup 1) \star \cup 1((0((1(0 \cup 1)(0 \cup 1)\star) \cup 0(0 \cup 1)\star)\star) \cup (1(0 \cup 1)(0 \cup 1)\star))\star$$

Question 11

Evaluate $\Omega = (\lambda x.xx)(\lambda x.xx)$

Question 12

Using Church encoding, evaluate $\lambda x.\lambda y.x(xy)$

Question 13

Explain, in your own words, Turing's proof of undecidability of the halting problem.

Question 14

Prove that $L = \{xx : x \in \sum^{\star}\}$ is an irregular language.

Question 15

Prove P = NP