

# CSGames Qualifications 2020

## Theoretical Computing

Use this Latex template to answer directly

### Question 1

Name the data structure that fits the description:

1. Element search is in  $O(\log n)$
2. Element insertion is in  $O(1)$
3. Element lookup is in  $O(1)$

### Question 2

Name a sorting algorithm that can complete in  $O(n)$

### Question 3

Can a sorting algorithm finish in  $O(\log n)$  without preprocessing the data?

### Question 4

Explain what is a regular language?

### Question 5

If we have  $v = \{a, b\}$ , give  $v^*$ .

## Question 6

Given the regular expression  $R = (ab^*)^*$ , find an equivalent regular expression that only have a star height of 1 (no nested Kleene star). The allowed operations are union, concatenation and Kleene star.

## Question 7

Is this equation true or false? justify.

$$(\neg r \Rightarrow q) \vee \neg(q \vee s) \vee s \wedge q \wedge r \wedge \neg(t \wedge s) \Rightarrow (q \wedge \neg t) \vee (r \wedge \neg s)$$

## Question 8

If a language  $L$  is regular, it's complement is also a regular language. true or false? justify.

## Question 9

Explain in details the impact of a black box that could factorize products of prime numbers in polynomial time on the rsa cryptosystem.

## Question 10

Explain the following regular expression:

$$\epsilon \cup 0(0 \cup 1)^* \cup 1((0((1(0 \cup 1)(0 \cup 1)^*) \cup 0(0 \cup 1)^*)^*) \cup (1(0 \cup 1)(0 \cup 1)^*))^*$$

## Question 11

Evaluate  $\Omega = (\lambda x.xx)(\lambda x.xx)$

## Question 12

Using Church encoding, evaluate  $\lambda x.\lambda y.x(xy)$

## Question 13

Explain, in your own words, Turing's proof of undecidability of the halting problem.

## Question 14

Prove that  $L = \{xx : x \in \Sigma^*\}$  is an irregular language.

## Question 15

Prove  $P = NP$