

Learning to program with F#

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A | The console in Windows, MacOS X, and Linux

Almost all popular operating systems are accessed through a user-friendly *graphical user interface* (*GUI*) that is designed to make typical tasks easy to learn to solve. As a computer programmer, you often need to access some of the functionalities of the computer, which, unfortunately, are sometimes complicated by this particular graphical user interface. The *console*, also called the *terminal* and the *Windows command line*, is the right hand of a programmer. The console is a simple program that allows you to complete text commands. Almost all the tasks that can be done with the graphical user interface can be done in the terminal and vice versa. Using the console, you will benefit from the terminal's direct control of the programs we write, and in your education, you will benefit from the fast and raw information you get through the console.

- graphical user interface
- GUI
- console
- terminal
- Windows command line

A.1 The basics

When you open a *directory* or *folder* in your preferred operating system, the directory will have a location in the file system, whether from the terminal or through the operating system's graphical user interface. The console will almost always be associated with a particular directory or folder in the file system, and it is said that it is the directory that the console is in. The exact structure of file systems varies between Linux, MacOS X and Windows, but common is that it is a hierarchical structure. This is illustrated in Figure A.1.

- directory
- folder

There are many predefined console commands, available in the console, and you can make your own. In the following, we will review the most important commands in the three different operating systems. These are summarized in Table A.1.

A.2 Windows

In this section we will discuss the commands summarized in Table A.1. Windows 7 and earlier versions: To open the console, press **Start->Run** in the lower left corner, and then type **cmd** in the box. Alternatively you can type **Windows-key + R**. Now you should open a terminal window with a prompt showing something like Listing A.1.

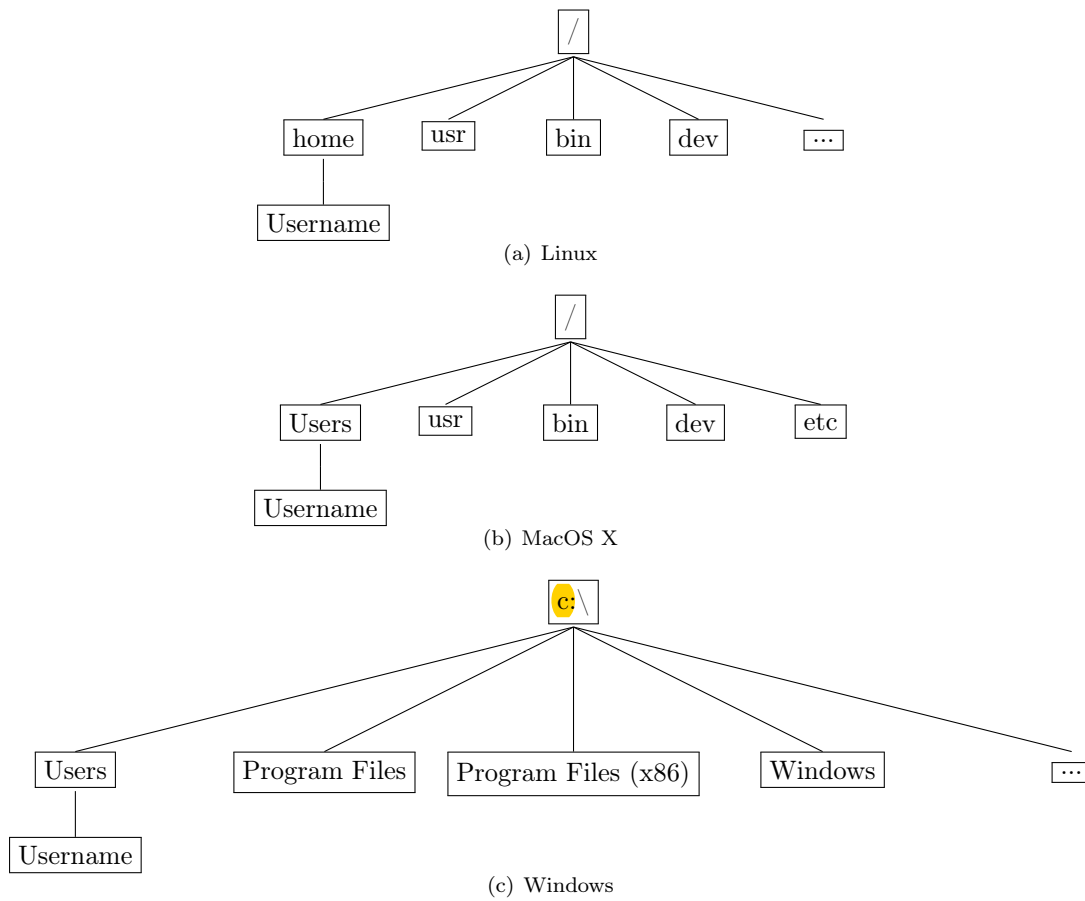


Figure A.1: The top file hierarchy levels of common operating systems.

Windows	MacOS X/Linux	Description
<code>dir</code>	<code>ls</code>	Show content of present directory.
<code>cd <dir></code>	<code>cd <dir></code>	Change present directory to <code><dir></code> .
<code>mkdir <dir></code>	<code>mkdir <dir></code>	create directory <code><dir></code> .
<code>rmdir <dir></code>	<code>rmdir <dir></code>	delete <code><dir></code> (Warning: cannot be reverted).
<code>move <file> <file or dir></code>	<code>mv <file> <file or dir></code>	Move <code><fil></code> to <code><file or dir></code> .
<code>copy <file> <file></code>	<code>cp <file> <file></code>	Create a new file called <code><file></code> as a copy of <code><file></code> .
<code>del <file></code>	<code>rm <file></code>	delete <code><file></code> (Warning: cannot be reverted).
<code>echo <string or variable></code>	<code>echo <string or variable></code>	Write a string or content of a variable to screen.

Table A.1: The most important console commands for Windows, MacOS X, and Linux.

Listing A.1: The Windows console.

```

1 Microsoft Windows [Version 6.1.7601]
2 Copyright (c) 2009 Microsoft Corporation. All rights reserved.
3
4 C:\Users\sporrington>

```

To see, which files are in the directory use *dir* as shown in Listing A.2.

· *dir*

Listing A.2: Directory listing with *dir*.

```

1 C:\Users\sporrington>dir
2 Volume in drive C has no label.
3 Volume Serial Number is 94F0-31BD
4
5 Directory of C:\Users\sporrington
6
7 30-07-2015 15:23 <DIR> .
8 30-07-2015 15:23 <DIR> ..
9 30-07-2015 14:27 <DIR> Contacts
10 30-07-2015 14:27 <DIR> Desktop
11 30-07-2015 17:40 <DIR> Documents
12 30-07-2015 15:11 <DIR> Downloads
13 30-07-2015 14:28 <DIR> Favorites
14 30-07-2015 14:27 <DIR> Links
15 30-07-2015 14:27 <DIR> Music
16 30-07-2015 14:27 <DIR> Pictures
17 30-07-2015 14:27 <DIR> Saved Games
18 30-07-2015 17:27 <DIR> Searches
19 30-07-2015 14:27 <DIR> Videos
20          0 File(s)          0 bytes
21        13 Dir(s) 95.004.622.848 bytes free
22
23 C:\Users\sporrington>

```

We see that there are no files and thirteen directories (DIR), the date and time of their creation in the two left columns, their size or if it is a folder, as well as their names in the right-hand column. The first two folders *.* and *..* are found in each folder and refers to this folder as well as the one above in the hierarchy. In this case, the folder *. An* alias for *C:\Users\tracking* and *..* for *C:\Users*.

Use *cd* to change directory, e.g., to *Documents* as in Listing A.3.

· *cd*

Listing A.3: Change directory with *cd*.

```

1 C:\Users\sporrington>cd Documents
2
3 C:\Users\sporrington\Documents>

```

The directory *Documents* is that which Windows Explorer selects, when you press the *Documents* short-cut in Explorer. Note that some systems translate default filenames so that their names may be given different names in different languages in the graphical user interface as compared to the terminal. On a new computer is the *Documents* folder is typically empty, as shown in Listing A.4.

Listing A.4: A new directory is always empty.

```

1 C:\Users\sporrington\Documents>dir
2   Volume in drive C has no label.
3   Volume Serial Number is 94F0-31BD
4
5   Directory of C:\Users\sporrington\Documents
6
7 30-07-2015  19:16    <DIR>          .
8 30-07-2015  19:16    <DIR>          ..
9                0 File(s)                0 bytes
10               2 Dir(s)  94.656.716.800 bytes free
11
12 C:\Users\sporrington\Documents>

```

You can use *mkdir* to create a new directory called, e.g., *myFolder* as illustrated in Listing A.5.

· *mkdir*

Listing A.5: Creating a directory with *mkdir*.

```

1 C:\Users\sporrington\Documents>mkdir minMappe
2
3 C:\Users\sporrington\Documents>dir
4   Volume in drive C has no label.
5   Volume Serial Number is 94F0-31BD
6
7   Directory of C:\Users\sporrington\Documents
8
9 30-07-2015  19:17    <DIR>          .
10 30-07-2015  19:17    <DIR>          ..
11 30-07-2015  19:17    <DIR>          myFolder
12                0 File(s)                0 bytes
13               3 Dir(s)  94.656.638.976 bytes free
14
15 C:\Users\sporrington\Documents>

```

By using *dir* we inspect the result.

Files can be *created*, e.g., *echo* and *redirection* as demonstrated in Listing A.6.

· *echo*
· *redirection*

Listing A.6: The Windows console.

```

1 C:\Users\sporrington\Documents>echo "Hi" > hi.txt
2
3 C:\Users\sporrington\Documents>dir
4 Volume in drive C has no label.
5 Volume Serial Number is 94F0-31BD
6
7 Directory of C:\Users\sporrington\Documents
8
9 30-07-2015  19:18    <DIR>          .
10 30-07-2015  19:18    <DIR>          ..
11 30-07-2015  19:17    <DIR>          myFolder
12 30-07-2015  19:18                8 hi.txt
13                1 File(s)                8 bytes
14                3 Dir(s)  94.656.634.880 bytes free
15
16 C:\Users\sporrington\Documents>

```

To move the file `hi.txt` to the directory `myFolder` use `move` as shown in Listing A.7.

· `move`

Listing A.7: Move a file with `move`.

```

1 C:\Users\sporrington\Documents>move hi.txt myFolder
2     1 file(s) moved.
3
4 C:\Users\sporrington\Documents>

```

Finally, use `del` to delete a file and `rmdir` to delete a directory as shown in Listing A.8.

· `del`
· `rmdir`

Listing A.8: Delete files and directories with `del` and `rmdir`.

```

1 C:\Users\sporrington\Documents>cd myFolder
2
3 C:\Users\sporrington\Documents\minMappe>del hi.txt
4
5 C:\Users\sporrington\Documents\minMappe>cd ..
6
7 C:\Users\sporrington\Documents>rmdir myFolder
8
9 C:\Users\sporrington\Documents>dir
10 Volume in drive C has no label.
11 Volume Serial Number is 94F0-31BD
12
13 Directory of C:\Users\sporrington\Documents
14
15 30-07-2015  19:20    <DIR>          .
16 30-07-2015  19:20    <DIR>          ..
17                0 File(s)                0 bytes
18                2 Dir(s)  94.651.142.144 bytes free
19
20 C:\Users\sporrington\Documents>

```

The commands available from the console must be in its *search path*. The search path can be seen · `search path`

using `echo` as shown in Listing A.9.

Listing A.9: The Windows console.

```
1 C:\Users\sporrington\Documents>echo %Path%
2 C:\Windows\system32;C:\Windows;C:\Windows\System32\Wbem;
  C:\Windows\System32\WindowsPowerShell\v1.0\;"\Program
  Files\emacs-24.5\bin\"
3
4 C:\Users\sporrington\Documents>
```

The path can be changed using the Control panel in the graphical user interface. In the Control panel, choose System and Security → System → Advanced system settings → Environment Variables, and in the System variables box, double-click on Path and add or remove a path from the list. The search path is a list of paths separated by “;”. Beware, Windows uses the search path for many different tasks, so remove only paths that you are certain are not used for anything.

A useful feature of the console is that you can use the `tab`-key to cycle through filenames. E.g., if you write `cd` followed by a space and `tab` a couple of times, then the terminal will suggest you the available directories.

A.3 MacOS X and Linux

MacOS X (OSX) and Linux are very similar, and both have the option of using `bash` as console. It is in the standard console on MacOS X and on many Linux distributions. A summary of the most important `bash` commands are shown in Table A.1. In MacOS X, you find the console by opening Finder and navigate to Applications → Utilities → Terminal. In Linux, the console can be started by typing `Ctrl + Alt + T`. Some Linux distributions have other key-combinations such as `Super + T`.

Once opened, the console is shown in a window with content as shown in Listing A.10.

Listing A.10: The Windows console.

```
1 Last login: Thu Jul 30 11:52:07 on ttys000
2 FN11194:~ sporring$
```

“FN11194” is the name of the computer, the character `~` is used as an alias for the user’s home directory, and “sporrington” is the username for the user presently logged onto the system. Use `ls` to see which files are present, as shown in Listing A.11.

Listing A.11: Display a directory content with `ls`.

```
1 FN11194:~ sporring$ ls
2 Applications      Documents          Library           Music             Public
3 Desktop           Downloads         Movies            Pictures
4 FN11194:~ sporring$
```

More details about the files are available by using flags to `ls` as demonstrated in Listing A.12.

Listing A.12: Display extra information about files using flags to `ls`.

```

1 FN11194:~ sporring$ ls -l
2 drwx----- 6 sporring staff 204 Jul 30 14:07 Applications
3 drwx-----+ 32 sporring staff 1088 Jul 30 14:34 Desktop
4 drwx-----+ 76 sporring staff 2584 Jul 2 15:53 Documents
5 drwx-----+ 4 sporring staff 136 Jul 30 14:35 Downloads
6 drwx-----@ 63 sporring staff 2142 Jul 30 14:07 Library
7 drwx-----+ 3 sporring staff 102 Jun 29 21:48 Movies
8 drwx-----+ 4 sporring staff 136 Jul 4 17:40 Music
9 drwx-----+ 3 sporring staff 102 Jun 29 21:48 Pictures
10 drwxr-xr-x+ 5 sporring staff 170 Jun 29 21:48 Public
11 FN11194:~ sporring$

```

The flag `-l` means long, and many other flags can be found by querying the built-in manual with `man ls`. The output is divided into columns, where the left column shows a number of codes: “d” means they are directory followed by three sets of optional “rwx” denoting whether the owner, the associated group of users and anyone can “r” - read, “w” - write, or “x” - execute the files. In this case, only the owner **can** any of the three. For directories, “x” means permission to enter. The second column can often be ignored, but shows how many links there are to the file or directory. Then follows the username of the owner, which in this case is `sporring`. The files are also associated with a group of users, and in this case, they all are **associate** with the group called `staff`. Then follows the file or directory size, the date of last change, and the file or directory name. There are always two hidden directories: `.` and `..`, where `.` is an alias for the present directory, and `..` for the directory above. Hidden files will be shown with the `-a` flag.

Use `cd` to change to the directory `Documents` as shown in Listing A.13.

· `cd`

Listing A.13: Change directory with `cd`.

```

1 FN11194:~ sporring$ cd Documents/
2 FN11194:Documents sporring$

```

Note that some graphical user interfaces translate standard filenames and directories to the local language, such that navigating using the graphical user interface will reveal other files and directories, which, however, are aliases. On a new computer many directories are predefined but empty, e.g., `Documents` as shown in Listing A.14.

Listing A.14: New directories are empty.

```

1 FN11194:Documents sporring$ ls
2 FN11194:Documents sporring$

```

You can yourself create a new directory using `mkdir` as demonstrated in Listing A.15.

· `mkdir`

Listing A.15: Creating a directory using `mkdir`.

```

1 FN11194:Documents sporring$ mkdir myFolder
2 FN11194:Documents sporring$ ls
3 myFolder
4 FN11194:tmp sporring$

```

A file can be created using *echo* and with *redirection* as shown in Listing A.16.

· *echo*
· *redirection*

Listing A.16: Creating a directory with *echo*.

```
1 FN11194:Documents sporring$ echo "hi" > hi.txt
2 FN11194:Documents sporring$ ls
3 hi.txt                myFolder
```

To move the file *hi.txt* into *myFolder* use *mv*. This is demonstrated in Listing A.17.

· *mv*

Listing A.17: Moving files with *mv*.

```
1 FN11194:Documents sporring$ echo mv hi.txt myFolder/
2 FN11194:Documents sporring$
```

To delete the file and the directory, use *rm* and *rmdir* as shown in Listing A.18.

· *rm*
· *rmdir*

Listing A.18: The Windows console.

```
1 FN11194:Documents sporring$ cd myFolder/
2 FN11194:myFolder sporring$ rm hi.txt
3 FN11194:myFolder sporring$ cd ..
4 FN11194:Documents sporring$ rmdir myFolder/
5 FN11194:Documents sporring$ ls
6 FN11194:Documents sporring$
```

Only commands found on the *search-path* are available in the console. The content of the *search-path* is seen using the *echo* command as demonstrated in Listing A.19.

· *search-path*

Listing A.19: The content of the *search-path*.

```
1 FN11194:Documents sporring$ echo $PATH
2 /Applications/Maple 17//Applications/PackageManager.app/Contents/MacOS/:
  /Applications/MATLAB_R2014b.app/bin/:/opt/local/bin:/opt/local/sbin:
  /usr/local/bin:/usr/bin:/bin:/usr/sbin:/sbin:/opt/X11/bin:
  /Library/TeX/texbin
3 FN11194:Documents sporring$
```

The *search-path* can be changed by editing the setup file for Bash. On MacOS X it is called *~/.profile*, and on Linux it is either *~/.bash_profile* or *~/.bashrc*. Here new paths can be added by adding the following *line* `export PATH="<new path>:<another new path>:$PATH"`.

A useful feature of Bash is that the console can help you write commands. E.g., if you write *fs* followed by pressing the *tab*-key, and if Mono is in the *search-path*, then Bash will typically respond by completing the line as *fsharp*, and by further pressing the *tab*-key sometimes, Bash will show the list of options, typically *fshpari* and *fsharp*. Also, most commands have an extensive manual, which can be accessed using the *man* command. E.g., the manual for *rm* is retrieved by *man rm*.

B | Number systems on the computer

B.1 Binary numbers

Humans like to use the *decimal number* system for representing numbers. Decimal numbers are *base 10* means that for a number consisting of a sequence of digits separated by a *decimal point*, where each *digit* can have values $d \in \{0, 1, 2, \dots, 9\}$ and the weight of each digit is proportional to its place in the sequence of digits **w.r.t.** the decimal point, i.e., the number $357.6 = 3 \cdot 10^2 + 5 \cdot 10^1 + 7 \cdot 10^0 + 6 \cdot 10^{-1}$ or in general:

$$v = \sum_{i=-m}^n d_i 10^i \quad (\text{B.1})$$

The basic unit of information in almost all computers is the binary digit or *bit* for short. A *binary number* consists of a sequence of binary digits separated by a decimal point, where each digit can have values $b \in \{0, 1\}$, and the base is 2. The general equation is,

$$v = \sum_{i=-m}^n b_i 2^i \quad (\text{B.2})$$

and examples are $1011.1_2 = 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 + 1 \cdot 2^{-1} = 11.5$. Notice that we use subscript 2 to denote a binary number, while no subscript is used for decimal numbers. The left-most bit is called the *most significant bit*, and the right-most bit is called the *least significant bit*. Due to typical organization of computer memory, 8 binary digits is called a *byte*, and 32 digits a *word*.

Other number systems are often used, e.g., *octal numbers*, which are base 8 numbers, where each digit is $o \in \{0, 1, \dots, 7\}$. Octals are useful short-hand for binary, since 3 binary digits maps to the set of octal digits. Likewise, *hexadecimal numbers* are base 16 with digits $h \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, a, b, c, d, e, f\}$, such that $a_{16} = 10$, $b_{16} = 11$ and so on. Hexadecimals are convenient since 4 binary digits map directly to the set of octal digits. Thus $367 = 101101111_2 = 557_8 = 16f_{16}$. A list of the integers 0–63 in various bases is given **in Table B.1**.

B.2 IEEE 754 floating point standard

The set of real numbers **also called reals** includes all fractions and irrational numbers. It is infinite in size both in the sense that there is no largest nor smallest **number and between** any 2 given numbers there are infinitely many numbers. Reals are widely used for calculation, but since any computer only has finite memory, it is impossible to represent **all possible reals**. Hence, any computation performed on a computer with reals must rely on approximations. *IEEE 754 double precision floating-point format (binary64)*, known as a *double*, is a standard for representing an approximation of reals using 64 bits.

Dec	Bin	Oct	Hex	Dec	Bin	Oct	Hex
0	0	0	0	32	100000	40	20
1	1	1	1	33	100001	41	21
2	10	2	2	34	100010	42	22
3	11	3	3	35	100011	43	23
4	100	4	4	36	100100	44	24
5	101	5	5	37	100101	45	25
6	110	6	6	38	100110	46	26
7	111	7	7	39	100111	47	27
8	1000	10	8	40	101000	50	28
9	1001	11	9	41	101001	51	29
10	1010	12	a	42	101010	52	2a
11	1011	13	b	43	101011	53	2b
12	1100	14	c	44	101100	54	2c
13	1101	15	d	45	101101	55	2d
14	1110	16	e	46	101110	56	2e
15	1111	17	f	47	101111	57	2f
16	10000	20	10	48	110000	60	30
17	10001	21	11	49	110001	61	31
18	10010	22	12	50	110010	62	32
19	10011	23	13	51	110011	63	33
20	10100	24	14	52	110100	64	34
21	10101	25	15	53	110101	65	35
22	10110	26	16	54	110110	66	36
23	10111	27	17	55	110111	67	37
24	11000	30	18	56	111000	70	38
25	11001	31	19	57	111001	71	39
26	11010	32	1a	58	111010	72	3a
27	11011	33	1b	59	111011	73	3b
28	11100	34	1c	60	111100	74	3c
29	11101	35	1d	61	111101	75	3d
30	11110	36	1e	62	111110	76	3e
31	11111	37	1f	63	111111	77	3f

Table B.1: A list of the integers 0–63 in decimal, binary, octal, and hexadecimal.

These bits are divided into 3 parts: sign, exponent and fraction,

$$s e_1 e_2 \dots e_{11} m_1 m_2 \dots m_{52}.$$

where s , e_i , and m_j are binary digits. The bits are converted to a number using the equation by first calculating the exponent e and the mantissa m ,

$$e = \sum_{i=1}^{11} e_i 2^{11-i}, \quad (\text{B.3})$$

$$m = \sum_{j=1}^{52} m_j 2^{-j}. \quad (\text{B.4})$$

I.e., the exponent is an integer, where $0 \leq e < 2^{11}$, and the mantissa is a rational, where $0 \leq m < 1$. For most combinations of e and m the real number v is calculated as,

$$v = (-1)^s (1 + m) 2^{e-1023} \quad (\text{B.5})$$

with the exception that

	$m = 0$	$m \neq 0$
$e = 0$	$v = (-1)^s 0$ (signed zero)	$v = (-1)^s m 2^{1-1023}$ (subnormals)
$e = 2^{11} - 1$	$v = (-1)^s \infty$	$v = (-1)^s \text{NaN}$ (not a number)

· subnormals
· NaN
· not a number

where $e = 2^{11} - 1 = 1111111111_2 = 2047$. The largest and smallest number that is not infinity is thus

$$e = 2^{11} - 2 = 2046 \quad (\text{B.6})$$

$$m = \sum_{j=1}^{52} 2^{-j} = 1 - 2^{-52} \simeq 1. \quad (\text{B.7})$$

$$v_{\max} = \pm (2 - 2^{-52}) 2^{1023} \simeq \pm 2^{1024} \simeq \pm 10^{308} \quad (\text{B.8})$$

The density of numbers varies in such a way that when $e - 1023 = 52$, then

$$v = (-1)^s \left(1 + \sum_{j=1}^{52} m_j 2^{-j} \right) 2^{52} \quad (\text{B.9})$$

$$= \pm \left(2^{52} + \sum_{j=1}^{52} m_j 2^{-j} 2^{52} \right) \quad (\text{B.10})$$

$$= \pm \left(2^{52} + \sum_{j=1}^{52} m_j 2^{52-j} \right) \quad (\text{B.11})$$

$$\stackrel{k=52-j}{=} \pm \left(2^{52} + \sum_{k=51}^0 m_{52-k} 2^k \right) \quad (\text{B.12})$$

which are all integers in the range $2^{52} \leq |v| < 2^{53}$. When $e - 1023 = 53$, then the same calculation gives

$$v \stackrel{k=53-j}{=} \pm \left(2^{53} + \sum_{k=52}^1 m_{53-k} 2^k \right) \quad (\text{B.13})$$

which are every second integer in the range $2^{53} \leq |v| < 2^{54}$, and so on for **larger e** . When $e - 1023 = 51$, **then** the same calculation gives,

$$v \stackrel{k=51-j}{=} \pm \left(2^{51} + \sum_{k=50}^{-1} m_{51-k} 2^k \right) \quad (\text{B.14})$$

which **gives** a distance between numbers of $1/2$ in the range $2^{51} \leq |v| < 2^{52}$, and so on for **smaller e** . Thus we may conclude that the distance between numbers in the interval $2^n \leq |v| < 2^{n+1}$ is 2^{n-52} , for $-1022 = 1 - 1023 \leq n < 2046 - 1023 = 1023$. For subnormals, the distance between numbers are

$$v = (-1)^s \left(\sum_{j=1}^{52} m_j 2^{-j} \right) 2^{-1022} \quad (\text{B.15})$$

$$= \pm \left(\sum_{j=1}^{52} m_j 2^{-j} 2^{-1022} \right) \quad (\text{B.16})$$

$$= \pm \left(\sum_{j=1}^{52} m_j 2^{-j-1022} \right) \quad (\text{B.17})$$

$$\stackrel{k=-j-1022}{=} \pm \left(\sum_{j=-1023}^{-1074} m_{-k-1022} 2^k \right) \quad (\text{B.18})$$

which gives a distance between numbers of $2^{-1074} \simeq 10^{-323}$ in the range $0 < |v| < 2^{-1022} \simeq 10^{-308}$.

C | Commonly used character sets

Letters, digits, symbols and space are the core of how we store data, write programs, and communicate with computers and each other. These symbols are in short called characters, and **represents** a mapping between numbers, also known as codes, and a pictorial representation of the character. E.g., the ASCII code for the letter 'A' is 65. These mappings are for short called character sets, and due to differences in natural languages and symbols used across the globe, many different character sets are in use. E.g., the English alphabet contains the letters 'a' to 'z', which is shared by many other European languages, but which have other symbols and **accents** for example, Danish has further the letters 'æ', 'ø', and 'å'. Many non-European languages have completely different symbols, where **Chinese** character set is probably the most extreme, **where** some definitions contains 106,230 different characters albeit only 2,600 are included in the official Chinese language test at highest level.

Presently, the most common character set used is Unicode Transformation Format (UTF), whose most popular encoding schemes are 8-bit (UTF-8) and 16-bit (UTF-16). Many other character sets exist, and many of the later **builds** on the American Standard Code for Information Interchange (ASCII). The ISO-8859 codes were an intermediate set of character sets that are still in use, but which is greatly inferior to UTF. Here we will briefly give an overview of ASCII, ISO-8859-1 (Latin1), and UTF.

C.1 ASCII

The *American Standard Code for Information Interchange* (ASCII) [8], is a 7 bit code tuned for the letters of the english language, numbers, punctuation symbols, control codes and space, see **Tables C.1 and C.2**. The first 32 codes are reserved for non-printable control characters to control printers and similar devices or to provide meta-information. The meaning of each control **characters** is not universally agreed upon.

- American Standard Code for Information Interchange
- ASCII

The code order is known as *ASCIIbetical order*, and it is sometimes used to perform arithmetic on codes, e.g., an upper case letter with code c may be converted to lower case by adding 32 to its code. The ASCIIbetical order also has consequence for sorting, i.e., when sorting characters according to their ASCII code, **then** 'A' comes before 'a', which comes before the symbol '{'.

- ASCIIbetical order

C.2 ISO/IEC 8859

The ISO/IEC 8859 report http://www.iso.org/iso/catalogue_detail?csnumber=28245 defines 10 sets of codes specifying up to 191 codes and graphic characters using 8 bits. Set 1 **also known as ISO/IEC 8859-1, Latin alphabet No. 1, or Latin1** covers many European languages and is designed to be compatible with ASCII, such that code for the printable characters in ASCII are the same in ISO 8859-1. **In Table C.3 is shown** the characters above 7e. Codes 00-1f and 7f-9f are undefined in ISO 8859-1.

- Latin1

x0+0x	00	10	20	30	40	50	60	70
00	NUL	DLE	SP	0	@	P	'	p
01	SOH	DC1	!	1	A	Q	a	q
02	STX	DC2	"	2	B	R	b	r
03	ETX	DC3	#	3	C	S	c	s
04	EOT	DC4	\$	4	D	T	d	t
05	ENQ	NAK	%	5	E	U	e	u
06	ACK	SYN	&	6	F	V	f	v
07	BEL	ETB	'	7	G	W	g	w
08	BS	CAN	(8	H	X	h	x
09	HT	EM)	9	I	Y	i	y
0A	LF	SUB	*	:	J	Z	j	z
0B	VT	ESC	+	;	K	[k	{
0C	FF	FS	,	<	L	\	l	
0D	CR	GS	-	=	M]	m	}
0E	SO	RS	.	>	N	^	n	~
0F	SI	US	/	?	O	_	o	DEL

Table C.1: ASCII

C.3 Unicode

Unicode is a character standard defined by the Unicode Consortium, <http://unicode.org> as the *Unicode Standard*. Unicode allows for 1,114,112 different codes. Each code is called a *code point*, which represents an abstract character. However, not all abstract characters require a unit of several code points to be specified. Code points are divided into 17 planes each with $2^{16} = 65,536$ code points. Planes are further subdivided into named *blocks*. The first plane is called the *Basic Multilingual plane* and it are the first 128 code points is called the *Basic Latin block* and are identical to ASCII, see Table C.1, and code points 128-255 is called the *Latin-1 Supplement block*, and are identical to the upper range of ISO 8859-1, see Table C.3. Each code-point has a number of attributes such as the *Unicode general category*. Presently more than 128,000 code points covering 135 modern and historic writing systems, and obtained at <http://www.unicode.org/Public/UNIDATA/UnicodeData.txt>, which includes the code point, name, and general category.

- Unicode Standard
- code point
- blocks
- Basic Multilingual plane
- Basic Latin block
- Latin-1 Supplement block
- Unicode general category

A Unicode code point is an abstraction from the encoding and the graphical representation of a character. A code point is written as “U+” followed by its hexadecimal number, and for the Basic Multilingual plane 4 digits are used, e.g., the code point with the unique name LATIN CAPITAL LETTER A has the Unicode code point is “U+0041”, and is in this text it is visualized as ‘A’. More digits are used for code points of the remaining planes.

The general category is used in **grammars** to specify valid characters, e.g., in naming identifiers in F#. Some categories and their letters in the first 256 code points are shown in Table C.5.

To store and retrieve code points, they must be encoded and decoded. A common encoding is *UTF-8*, which encodes code points as 1 to 4 bytes, and which is backward-compatible with ASCII and ISO 8859-1. Hence, in all 3 coding systems the character with code 65 represents the character ‘A’. Another popular encoding scheme is *UTF-16*, which encodes characters as 2 or 4 bytes, but which is not backward-compatible with ASCII or ISO 8859-1. UTF-16 is used internally in many **compiles**, interpreters and operating systems.

- UTF-8
- UTF-16

Code	Description
NUL	Null
SOH	Start of heading
STX	Start of text
ETX	End of text
EOT	End of transmission
ENQ	Enquiry
ACK	Acknowledge
BEL	Bell
BS	Backspace
HT	Horizontal tabulation
LF	Line feed
VT	Vertical tabulation
FF	Form feed
CR	Carriage return
SO	Shift out
SI	Shift in
DLE	Data link escape
DC1	Device control one
DC2	Device control two
DC3	Device control three
DC4	Device control four
NAK	Negative acknowledge
SYN	Synchronous idle
ETB	End of transmission block
CAN	Cancel
EM	End of medium
SUB	Substitute
ESC	Escape
FS	File separator
GS	Group separator
RS	Record separator
US	Unit separator
SP	Space
DEL	Delete

Table C.2: ASCII symbols.

x0+0x	80	90	A0	B0	C0	D0	E0	F0
00			NBSP	°	À	Ð	à	ð
01			¡	±	Á	Ñ	á	ñ
02			¢	²	Â	Ò	â	ò
03			£	³	Ã	Ó	ã	ó
04			¤	´	Ä	Ö	ä	ö
05			¥	µ	Å	Õ	å	õ
06			¦	¶	Æ	Ö	æ	ö
07			§	·	Ç	×	ç	÷
08			¨	¸	È	Ø	è	ø
09			©	¹	É	Ù	é	ù
0a			ª	º	Ê	Ú	ê	ú
0b			«	»	Ë	Û	ë	û
0c			¬	$\frac{1}{4}$	Ì	Ü	ì	ü
0d			SHY	$\frac{1}{2}$	Í	Ý	í	ý
0e			®	$\frac{3}{4}$	Î	Þ	î	þ
0f			¯	¸	Ï	ß	ï	ÿ

Table C.3: ISO-8859-1 (latin1) non-ASCII part. Note that the codes 7f – 9f are undefined.

Code	Description
NBSP	Non-breakable space
SHY	Soft hyphen

Table C.4: ISO-8859-1 special symbols.

General category	Code points	Name
Lu	U+0041–U+005A, U+00C0–U+00D6, U+00D8–U+00DE	Upper case letters
Ll	U+0061–U+007A, U+00B5, U+00DF–U+00F6, U+00F8–U+00FF	Lower case letter
Lt	None	Digraphic letter, with first part uppercase
Lm	None	Modifier letter
Lo	U+00AA, U+00BA	Gender ordinal indicator
Nl	None	Letterlike numeric character
Pc	U+005F	Low line
Mn	None	Nonspacing combining mark
Mc	None	Spacing combining mark
Cf	U+00AD	Soft Hyphen

Table C.5: Some general categories for the first 256 code points.

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