Learning to program with F#

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# Contents

1	Preface						
2	Intr	roduction	6				
	2.1	How to learn to program	6				
	2.2	How to solve problems	7				
	2.3	Approaches to programming	7				
	2.4	Why use F#	8				
	2.5	How to read this book	9				
Ι	$\mathbf{F}$	# basics	10				
	Б						
3	Exe	${ m ecuting}\; { m F\#\;code}$	11				
	3.1	Source code	11				
	3.2	Executing programs	11				
4	Qui	ck-start guide	14				
5	Usi	${ m rg}~{ m F}\#~{ m as}~{ m a}~{ m calculator}$	19				
	5.1	Literals and basic types	19				
	5.2	Operators on basic types	24				
	5.3	Boolean arithmetic	26				
	5.4	Integer arithmetic	27				
	5.5	Floating point arithmetic	30				
	5.6	Char and string arithmetic	31				

6	Con	stants, functions, and variables	34
	6.1	Values	36
	6.2	Non-recursive functions	40
	6.3	User-defined operators	43
	6.4	The Printf function	45
	6.5	Variables	47
7	In-c	code documentation	<b>52</b>
8	Con	trolling program flow	57
	8.1	For and while loops	57
	8.2	Conditional expressions	61
	8.3	Programming intermezzo	
	8.4	Recursive functions	63
9	Ord	ered series of data	65
	9.1	Tuples	66
	9.2	Lists	68
	9.3	Arrays	71
10		ting programs	<b>7</b> 6
	10.1	White-box testing	78
	10.2	Back-box testing	80
	10.3	Debugging by tracing	83
11	Exc	eptions	91
12	Inp	ut and Output	97
	12.1	Interacting with the console	97
	12.2	Storing and retrieving data from a file	98
	12.3	Working with files and directories	102
	12.4	Programming intermezzo	103

II	Imperative programming	105
13	Graphical User Interfaces	107
14	Imperative programming	108
	14.1 Introduction	108
	14.2 Generating random texts	
	14.2.1 0'th order statistics	110
	14.2.2 1'th order statistics	
III	I Declarative programming	113
<b>15</b>	Sequences and computation expressions	114
	15.1 Sequences	
16	Patterns	119
	16.1 Pattern matching	119
17	Types and measures	121
	17.1 Unit of Measure	121
18	Functional programming	124
IV	Structured programming	127
19	Namespaces and Modules	128
20	Object-oriented programming	129
V	Appendix	130
A	Number systems on the computer	131
	A.1 Binary numbers	131
	A.2 IEEE 754 floating point standard	

В	Commonly used character sets	135
	B.1 ASCII	. 135
	B.2 ISO/IEC 8859	. 136
	B.3 Unicode	. 136
$\mathbf{C}$	A brief introduction to Extended Backus-Naur Form	140
D	${f F} b$	143
$\mathbf{E}$	Language Details	148
	E.1 Arithmetic operators on basic types	. 148
	E.2 Basic arithmetic functions	. 151
	E.3 Precedence and associativity	. 152
	E.4 Lightweight Syntax	. 154
$\mathbf{F}$	The Some Basic Libraries	149
	F.1 System.String	. 149
	F.2 List, arrays, and sequences	. 154
	F.3 Mutable Collections	. 156
	F.3.1 Mutable lists	. 156
	F.3.2 Stacks	. 156
	F.3.3 Queues	. 157
	F.3.4 Sets and dictionaries	. 157
Bi	ibliography	155
In	ndex	156

# Chapter 5

# Using F# as a calculator

### 5.1 Literals and basic types

All programs rely on processing of data, and an essential property of data is its type. A literal is a  $\cdot$  type fixed value such as the number 3, and if we type the number 3 in an interactive session at the input  $\cdot$  literal prompt, then F# responds as follows,

```
Listing 5.1, firstType.fsx:
Typing the number 3.

> 3;;
val it : int = 3
```

What this means is that F# has inferred the type to be int and bound it to the identifier it. Rumor has it, that the identifier it is an abbreviation for 'irrelevant'. For more on binding and identifiers see Chapter 6. Types matter, since the operations that can be performed on integers are quite different from those that can be performed on, e.g., strings. I.e.,

```
Listing 5.2, typeMatters.fsx:

Many representations of the number 3 but using different types.

> 3;;

val it : int = 3
> 3.0;;

val it : float = 3.0
> '3';;

val it : char = '3'
> "3";;

val it : string = "3"
```

Each literal represent the number 3, but their types are different, and hence they are quite different values. The types int for integer numbers, float for floating point numbers, bool for boolean values, char for characters, and string for strings of characters are the most common types of literals. A table of all  $basic\ types$  predefined in F# is given in Table 5.1. Besides these built-in types, F# is designed such that it is easy to define new types.

Humans like to use the decimal number system for representing numbers. Decimal numbers are base 10,

· float

 $\cdot$  int

·it

- $\cdot$  bool
- ·char
- ·string
- · basic types
- $\cdot$  decimal number
- $\cdot$  base

Metatype	Type name	Description
Boolean	bool	Boolean values true or false
Integer	int	Integer values from -2,147,483,648 to 2,147,483,647
	byte	Integer values from 0 to 255
	sbyte	Integer values from -128 to 127
	int32	Synonymous with int
	uint32	Integer values from 0 to 4,294,967,295
Real	float	64-bit IEEE 754 floating point value from $-\infty$ to $\infty$
	double	Synonymous with float
Character	char	Unicode character
	string	Unicode sequence of characters
None unit No value denoted		No value denoted
Object obj An object		An object
Exception	exn	An exception

Table 5.1: List of some of the basic types. The most commonly used types are highlighted in bold. For at description of integer see Appendix A.1, for floating point numbers see Appendix A.2, for ASCII and Unicode characters see Appendix B, for objects see Chapter 20, and for exceptions see Chapter 11.

which that a value is represented as two sequences of decimal digits separated by a decimal point, where each digit can have values  $d \in \{0, 1, 2, ..., 9\}$ , and the value, which each digit represents is proportional to its position. The part before the decimal point is called the whole part and the part after is called the fractional part of the number. The whole part without a decimal point and a fractional part is called an integer. As an example 35.7 is a decimal number, whose value is  $3 \cdot 10^1 + 5 \cdot 10^0 + 7 \cdot 10^{-1}$ , and 128 is an integer, whose value is  $1 \cdot 10^2 + 2 \cdot 10^1 + 8 \cdot 10^0$ . In F# a decimal number is called a floating point number and in this text we use Extended Backus-Naur Form (EBNF) to describe the grammar of F#. In EBNF, the grammar describing a decimal number is,

```
Listing 5.3: Decimal numbers.

dDigit = "0" | "1" | "2" | "3" | "4" | "5" | "6" | "7" | "8" | "9";

dInt = dDigit {dDigit}; (*no spaces*)

dFloat = dInt "." {dDigit}; (*no spaces*)
```

In EBNF dDigit, dInt, and dFloat are names of tokens, while "0", "1", ..., "9", and "." are terminals. Tokens and terminals together with formatting rules describe possible sequences, which are valid. E.g., a dDigit is defined by the = notation to be either 0 or 1 or ... or 9, as signified by the | syntax. The definition of a token is ended by a ;. The "{ }" in EBNF signfies zero or more repetitions of its content, such that a dInt is, e.g., dDigit, dDigit dDigit, dDigit dDigit dDigit dDigit dDigit and so on. Since a dDigit is any decimal digit, we conclude that 3, 45, and 0124972930485738 are examples of dInt. A dFloat is the concatenation of one or more digits, a dot, and zero or more digits, such as 0.4235, 3., but not .5 nor .. Sometimes EBNF implicitly allows for spaces between tokens and terminals, so here we have used the comments notation (\*\*) to explicitly remind ourselves, that no spaces are allowed between the whole part, decimal point, and the fractional part. A complete description of EBNF is given in Appendix C.

Floating point numbers may alternatively be given using *scientific notation*, such as 3.5e-4 and 4e2, where the e-notation is translated to a value as  $3.5e-4 = 3.5 \cdot 10^{-4} = 0.00035$ , and  $4e2 = 4 \cdot 10^2 = 400$ . To describe this in EBNF we write

- · decimal point
- $\cdot \operatorname{digit}$
- · whole part
- · fractional part
- $\cdot \, {\rm integer} \,$
- · floating point number
- · Extended Backus-Naur Form
- $\cdot$  EBNF

 $\cdot$  scientific notation

```
Listing 5.4: Scientific notation.

sFloat = (dInt | dFloat) ("e" | "E" ) ["+" | "-"] dInt; (*no spaces*)
float = dFloat | sFloat;
```

Note that the number before the lexeme e may be an dInt or a dFloat, but the exponent value must be an dInt.

The basic unit of information in almost all computers is the binary digit or bit for short. Internally, programs and data is all represented as bits, hence F# has a strong support for binary numbers. A binary number consists of a sequence of binary digits separated by a decimal point, where each digit can have values  $b \in \{0,1\}$ , and the base is 2. E.g., the binary number  $101.01_2 = 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 + 0 \cdot 2^{-1} + 1 \cdot 2^{-2} = 5.25$ . Binary numbers are closely related to octal and hexadecimal numbers, where octals uses 8 as basis, and where each octal digit can be represented by exactly 3 bits, while hexadecimal numbers uses 16 as basis, and where each hexadecimal digit can be written in binary using exactly 4 bits. The hexadecimal digits uses 0–9 to represent the values 0–9 and a–f in lower or alternatively upper case to represent the values 10-15. Octals and hexadecimals thus conveniently serve as shorthand for the much longer binary representation. F# has a syntax for writing integers on binary, octal, decimal, and hexadecimal numbers as,

```
\cdot bit
```

- · binary number
- · octal number
- · hexadecimal number

```
Listing 5.5: Binary, hexadecimal, and octal numbers.

bDigit = "0" | "1";
oDigit = "0" | "1" | "2" | "3" | "4" | "5" | "6" | "7";
xDigit =
    "0" | "1" | "2" | "3" | "4" | "5" | "6" | "7" | "8" | "9"
    | "A" | "B" | "C" | "D" | "E" | "F" | "a" | "b" | "c" | "d" | "e" | "f";
bitInt = "0" ("b" | "B") bDigit {bDigit}; (*no spaces*)
octInt = "0" ("o" | "0") oDigit {oDigit}; (*no spaces*)
hexInt = "0" ("x" | "X") xDigit {xDigit}; (*no spaces*)
xInt = bitInt | octInt | hexInt;
int = dInt | xInt;
```

For example the value 367 in base 10 may be written as a dInt integer as 367, as a bitInt binary number as 0b101101111, as a octInt octal number as 0o557, and as a hexInt hexadecimal number as 0x16f. In contrast, 0b12 and ff are neither an dInt nor an xInt.

A character is a Unicode code point, and character literals are enclosed in single quotation marks, see Appendix B.3 for a description of code points. The EBNF for characters is,

- ·character
- $\cdot$  Unicode
- · code point

```
Listing 5.6: Character escape sequences.

codePoint = ?Any unicode codepoint?;
escapeChar =

"\" ("b" | "n" | "r" | "t" | "\" | """ | "a" | "f" | "v")

| "\u" xDigit xDigit xDigit xDigit

| "\U" xDigit xDigit xDigit xDigit xDigit xDigit xDigit xDigit

| "\" dDigit dDigit dDigit; (*no spaces*)

char = "'" codePoint | escapeChar "'"; (*no spaces*)
```

where codePoint is a UTF8 encoding of a char. The escape characters escapeChar are special sequences that are interpreted as a single code point shown in Table 5.2. The trigraph \DDD uses decimal specification for the first 256 code points, and the hexadecimal escape codes \uXXXX, \UXXXXXXX allow for the full specification of any code point. Examples of a char are 'a', '\_', '\n', and '\065'.

Character	Escape sequence	Description
BS	\b	Backspace
LF	\n	Line feed
CR	\r	Carriage return
HT	\t	Horizontal tabulation
\	\\	Backslash
"	\"	Quotation mark
,	\ '	Apostrophe
BEL	\a	Bell
FF	\f	Form feed
VT	\v	Vertical tabulation
	\uXXXX, \UXXXXXXXX, \DDD	Unicode character

Table 5.2: Escape characters. For the unicode characters 'X' are hexadecimal digits, while for tricode characters 'D' is a decimal character.

A string is a sequence of characters enclosed in double quotation marks,

 $\cdot$  string

```
Listing 5.7: Strings.

stringChar = char - '"';
string = '"' { stringChar } '"';
verbatimString = '@"' { char - ('"' | '\"' ) | '""' } '"';
```

Examples are "a", "this is a string", and "-&#\@". Newlines and following whitespaces,

· newline · whitespace

```
Listing 5.8: Whitespace and newline.
whitespace = " " {" "};
newline = "\n" | "\r" "\n";
```

are taken literally, but may be ignored by a preceding \character. Further examples of strings are,

```
Listing 5.9, stringLiterals.fsx:

Examples of string literals.

> "abcde";;
val it : string = "abcde"

- de";;
val it : string = "abc
de"

> "abc\
- de";;
val it : string = "abcde"

> "abc\nde";;
val it : string = "abcde"

> "abc\nde";;
val it : string = "abc
de"
```

The response is shown in double quotation marks, which are not part of the string.

F# supports literal types, where the type of a literal is indicated as a prefix og suffix as shown in the - literal type Table 5.3. Examples are,

type	EBNF	Examples
int, int32	(dInt   xInt)["l"]	3
uint32	(dInt   xInt)("u"  "ul")	3u
byte, uint8	((dInt   xInt) "uy")   (char "B")	Зuy
byte[]	["@"] string "B"	"abc"B and "@http:\\"B
sbyte, int8	(dInt   xInt)"y"	Зу
float, double	float   (xInt "LF")	3.0
string	simpleString	"a \"quote\".\n"
	'@"'{(char - ('"'  '\"'))  '""'} '"'	@"a ""quote"".\n"

Table 5.3: List of literal type. No spacing is allowed between the literal and the prefix or suffix.

```
Listing 5.10, namedLiterals.fsx:
Named and implied literals.
> 3;;
val it : int = 3
> 4u;;
val it : uint32 = 4u
> 5.6;;
val it : float = 5.6
> 7.9f;;
val it : float32 = 7.9000001f
 'A';;
val it : char = 'A'
> 'B'B;;
val it : byte = 66uy
> "ABC";;
val it : string = "ABC"
```

Strings literals may be verbatim by the @-notation meaning that the escape sequences are not converted to their code point., e.g.,

 $\cdot$  verbatim

```
Listing 5.11, string Verbatim.fsx:

Examples of a string literal.

> @"abc\nde";;
val it : string = "abc\nde"
```

Many basic types are compatible, and the type of a literal may be changed by typecasting. E.g.,

 $\cdot$  typecasting

```
Listing 5.12, upcasting.fsx:

Casting an integer to a floating point number.

> float 3;;
val it : float = 3.0
```

which is a float, since when float is given an argument, then it acts as a function rather than a type, and for the integer 3 it returns the floating point number 3.0. For more on functions see Chapter 6. Boolean values are often treated as the integer values 0 and 1, but no short-hand function names exists for their conversions. Instead use,

```
Listing 5.13, castingBooleans.fsx:
Casting booleans.

> System.Convert.ToBoolean 1;;
val it: bool = true
> System.Convert.ToBoolean 0;;
val it: bool = false
> System.Convert.ToInt32 true;;
val it: int = 1
> System.Convert.ToInt32 false;;
val it: int = 0
```

Here System.Convert.ToBoolean is the identifier of a function ToBoolean, which is a *member* of the *class* Convert that is included in the *namespace* System. Namespaces, classes, and members are all part of Structured programming to be discussed in Part IV.

 $\cdot$  member

 $\begin{array}{c} \cdot \ class \\ \cdot \ namespace \end{array}$ 

Typecasting is often a destructive operation, e.g., typecasting a float to int removes the fractional part without rounding,

```
Listing 5.14, downcasting.fsx:
Fractional part is removed by downcasting.

> int 357.6;;
val it : int = 357
```

Here we typecasted to a lesser type, in the sense that the set of integers is a subset of floating point numbers, and this is called *downcasting*. The opposite is called *upcasting* and is often non-destructive, as Listing 5.12 showed, where an integer was casted to a float while retaining its value. As a side note, *rounding* a number y.x, where y is the *whole part* and x is the *fractional part*, is the operation of mapping numbers in the interval  $y.x \in [y.0, y.5)$  to y and  $y.x \in [y.5, y+1)$  to y+1. This can be performed by downcasting as follows,

```
· downcasting
```

- ·upcasting
- · rounding
- $\cdot$  whole part
- · fractional part

```
Listing 5.15, rounding.fsx:
Fractional part is removed by downcasting.

> int (357.6 + 0.5);;
val it : int = 358
```

since if  $y.x \in [y.0, y.5)$ , then  $y.x + 0.5 \in [y.5, y+1)$ , from which downcasting removes the fractional part resulting in y. And if  $y.x \in [y.5, y+1)$ , then  $y.x + 0.5 \in [y+1, y+1.5)$ , from which downcasting removes the fractional part resulting in y+1. Hence, the result is rounding.

### 5.2 Operators on basic types

Listing 5.15 is an example of an arithmetic expression using an infix operator. Expressions is the basic building block of all F# programs, and its grammar has many possible options. In the example, + is the operator, and it is an infix operator, since it takes values on its left and right side. The grammar for expressions are defined recursively, and some of it is given by,

- $\cdot$  expression
- · infix operator

```
Listing 5.16: Expressions.

const = byte | sbyte | int32 | uint32 | int | ieee64 | char | string | verbatimString | "false" | "true" | "()";
sliceRange = expr | expr ".." (*no space between expr and ".."*) | ".." expr (*no space between expr and ".."*) | expr ".." expr (*no space between expr and ".."*) | vexpr ".." expr (*no space between expr and ".."*) | vexpr ".." expr (*no space between expr and ".."*) | vexpr = ... | const (*a const value*) | "(" expr ")" (*block*) | expr expr (*application*) | expr infixOp expr (*infix application*) | prefixOp expr (*prefix application*) | expr ".[" expr "]" (*index lookup, no space before "."*) | expr ".[" sliceRange "]" (*index lookup, no space before "."*)
```

Recursion means that a rule or a function is used by the rule or function itself in its definition, e.g., in the definition of expression, the token expression occurs both on the left and the right side of the = symbol. See Part III for more on recursion. Infix notation means that the *operator* op appears between the two *operands*, and since there are 2 operands, it is a *binary operator*. As the grammar shows, the operands themselves can be expressions. Examples are 3+4 and 4+5+6. Some operators only takes one operand, e.g., -3, where - here is used to negate a postive integer. Since the operator appears before the operand it is a *prefix operator*, and since it only takes one argument it is also a *unary operator*. Finally, some expressions are function names, which can be applied to expressions. F# supports a range of arithmetic infix and prefix operators on its built-in types such as addition, subtraction, multiplication, division, and exponentiation using the +, -, \*, /, \*\* binary operators respectively. Not all operators are defined for all types, e.g., addition is defined for integer and float types as well as for characters and strings, but multiplication is only defined for integer and floating point types. A complete list of built-in operators on basic types is shown in Table E.1 and E.2 and a range of mathematical functions shown in Table E.3.

The concept of *precedence* is an important concept in arithmetic expressions.<sup>1</sup> If parentheses are omitted in Listing 5.15, then F# will interpret the expression as (int 357.6) + 0.5, which is erroneous, since addition of an integer with a float is undefined. This is an example of precedence, i.e., function evaluation takes precedence over addition meaning that it is performed before addition. Consider the arithmetic expression,

```
· precedence
```

 $\cdot$  operator

 $\cdot$  operands

· binary operator

· prefix operator

· unary operator

```
Listing 5.17, simpleArithmetic.fsx:
A simple arithmetic expression.

> 3 + 4 * 5;;
val it : int = 23
```

Here, the addition and multiplication functions are shown in *infix notation* with the *operator* lexemes + and \*. To arrive at the resulting value 23, F# has to decide in which order to perform the calculation. There are 2 possible orders, 3 + (4 \* 5) or (3 + 4) \* 5, which gives different results. For integer arithmetic, the correct order is of course to multiply before addition, and we say that multiplication takes *precedence* over addition. Every atomic operation that F# can perform is ordered in terms of its precedences, and for some common built-in operators shown in Table E.5, the precedence is shown by the order they are given in the table.

 $\cdot$  infix notation

 $\cdot$  operator

 $\cdot$  precedence

<sup>&</sup>lt;sup>1</sup>Todo: minor comment on indexing and slice-ranges.

a	b	a && b	a    b	not a
false	false	false	false	true
false	true	false	true	true
true	false	false	true	false
true	true	true	true	false

Table 5.4: Truth table for boolean 'and', 'or', and 'not' operators. Value 0 is false and 1 is true.

Associativity implies the order in which calculations are performed for operators of same precedence. For some operators and type combinations association matters little, e.g., multiplication associates to the left and exponentiation associates to the right, e.g., in

```
Listing 5.18, precedence.fsx:

Precedences rules define implicite parentheses.

> 3.0*4.0*5.0;;
val it : float = 60.0
> (3.0*4.0)*5.0;;
val it : float = 60.0
> 3.0*(4.0*5.0);;
val it : float = 60.0
> 4.0 ** 3.0 ** 2.0;;
val it : float = 262144.0
> (4.0 ** 3.0) ** 2.0;;
val it : float = 4096.0
> 4.0 ** (3.0 ** 2.0);;
val it : float = 262144.0
```

the expression for 3.0 \* 4.0 \* 5.0 associates to the left, and thus is interpreted as (3.0 \* 4.0)\* 5.0, but gives the same results as 3.0 \* (4.0 \* 5.0), since association does not matter for multiplication of numbers. However, the expression for 4.0 \*\* 3.0 \*\* 2.0 associates to the right, and thus is interpreted as 4.0 \*\* (3.0 \*\* 2.0), which is quite different from (4.0 \*\* 3.0)\*\* 2.0. Whenever in doubt of association or any other basic semantic rules, it is a good idea to use parentheses as here. It is also a good idea to test your understanding of the syntax and semantic rules by making a simple scripts.

Advice

### 5.3 Boolean arithmetic

Boolean arithmetic is the basis of almost all computers and particularly important for controlling program flow, which will be discussed in Chapter 8. Boolean values are one of 2 possible values, true or false, which is also sometimes written as 1 and 0. Basic operations on boolean values are 'and', 'or', and 'not', which in F# is written as the binary operators &&, ||, and the function not. Since the domain of boolean values is so small, then all possible combination of input on these values can be written on tabular form, known as a truth table, and the truth tables for the basic boolean operators and functions is shown in Table 5.4. A good mnemonics for remembering the result of the 'and' and 'or' operators is to use 1 for true, 0 for false, multiplication for the boolean 'and' operator, and addition for boolean 'or' operator, e.g., true and false in this mnemonic translates to  $1 \cdot 0 = 0$ , and the results translates back to the boolean value false. In F# the truth table for the basic boolean operators is reproduced by,

 $\cdot \text{ and }$ 

 $\cdot \text{ or }$ 

 $\cdot$  not

· truth table

### Listing 5.19, truthTable.fsx: Boolean operators and truth tables. printfn "a b a\*b a+b not a" printfn "%A %A %A %A %A" false false (false && false) (false || false) (not false) printfn "%A %A %A %A %A" false true (false && true) (false || true) (not false) printfn "%A %A %A %A %A" true false (true && false) (true || false) (not true) printfn "%A %A %A %A %A" true true (true && true) (true || true) (not true);; a\*b a+b not a false false false true false true false true true true false false true false true true true false val it : unit = ()

In Listing 5.19 we used the printfn function, to present the results of many expressions on something that resembles a tabular form. The spacing produced using the printfn function is not elegant, and in Section 6.4 we will discuss better options for producing more beautiful output. Notice, that the arguments for printfn was given on the next line with indentation. The indentation is an important part of telling F#, which part of what you write belongs together. This is an example of the so-called lightweight syntax. Generally, F# ignores newlines and whitespaces except when using the lightweight syntax, and the examples of the difference between regular and lightweight syntax is discussed in Chapter 6.

### 5.4 Integer arithmetic

The set of integers is infinitely large, but since all computers have limited resources, it is not possible to represent it in their entirety. The various integer types listed in Table 5.1 are finite subsets reduced by limiting their ranges. An in-depth description of integer implementation can be found in Appendix A. The type int is the most common type.

Table E.1, E.2, and E.3 gives examples operators and functions pre-defined for integer types. Notice that fewer functions are available for integers than for floating point numbers. For most addition, subtraction, multiplication, and negation the result straight forward. However, performing arithmetic operations on integers requires extra care, since the result may cause *overflow* and *underflow*. E.g., the range of the integer type  $\mathtt{sbyte}$  is [-128...127], which causes problems in the following example,

· overflow · underflow

```
Listing 5.20, overflow.fsx:
Adding integers may cause overflow.

> 100y;;
val it : sbyte = 100y
> 30y;;
val it : sbyte = 30y
> 100y + 30y;;
val it : sbyte = -126y
```

Here 100 + 30 = 130, which is larger than the biggest sbyte, and the result is an overflow. Similarly,

we get an underflow, when the arithmetic result falls below the smallest value storable in an sbyte,

```
Listing 5.21, underflow.fsx:
Subtracting integers may cause underflow.

> -100y - 30y;;
val it : sbyte = 126y
```

I.e., we were expecting a negative number, but got a postive number instead.

The overflow error in Listing 5.20 can be understood in terms of the binary representation of integers: In binary,  $130 = 10000010_2$ , and this binary pattern is interpreted differently as byte and sbyte,

```
Listing 5.22, overflowBits.fsx:
The left most bit is interpreted differently for signed and unsigned integers, which
gives rise to potential overflow errors.

> 0b10000010uy;;
val it : byte = 130uy
> 0b10000010y;;
val it : sbyte = -126y
```

That is, for signed bytes, the left-most bit is used to represent the sign, and since the addition of  $100 = 01100100_2$  and  $30 = 00011110_b$  is  $130 = 10000010_2$  causes the left-most bit to be used, then this is wrongly interpreted as a negative number, when stored in an sbyte. Similar arguments can be made explaining underflows.

The division and remainder operators, which discards the fractional part after division, and the re-mainder operator calculates the remainder after integer division, e.g.,

· integer division · remainder

```
Listing 5.23, integerDivisionRemainder.fsx:
Integer division and remainder operators.

> 7 / 3;;
val it : int = 2
> 7 % 3;;
val it : int = 1
```

Together integer division and remainder is a lossless representation of the original number as,

```
Listing 5.24, integerDivisionRemainderLossless.fsx:
Integer division and remainder is a lossless representation of an integer, compare with Listing 5.23.

> (7 / 3) * 3;;
val it : int = 6
> (7 / 3) * 3 + (7 % 3);;
val it : int = 7
```

And we see that integer division of 7 by 3 followed by multiplication by 3 is less that 7, and the difference is 7 % 3.

Notice that neither overflow nor underflow error gave rise to an error message, which is why such bugs are difficult to find. Dividing any non-zero number with 0 is infinite, which is also outside the domain

a	b	a ~~~ b
false	e false fals	
false	true	true
true	false	true
false	true	false

Table 5.5: Boolean exclusive or truth table.

of any of the integer types, but in this case, F# casts an exception,

 $\cdot$  exception

```
Listing 5.25, integerDivisionByZeroError.fsx:
Integer division by zero causes an exception run-time error.

> 3/0;;
System.DivideByZeroException: Attempted to divide by zero.
at <StartupCode$FSI_0002>.$FSI_0002.main@ () <0x68079f8 + 0x0000e> in < filename unknown>:0
at (wrapper managed-to-native) System.Reflection.MonoMethod:
    InternalInvoke (System.Reflection.MonoMethod,object,object[],System.Exception&)
at System.Reflection.MonoMethod.Invoke (System.Object obj, BindingFlags invokeAttr, System.Reflection.Binder binder, System.Object[]
    parameters, System.Globalization.CultureInfo culture) <0x1a7c270 + 0 x000a1> in <filename unknown>:0
Stopped due to error
```

The output looks daunting at first sight, but the first and last line of the error message are the most important parts, which tells us what exception was cast and why the program stopped. The middle are technical details concerning which part of the program caused this, and can be ignored for the time being. Exceptions are a type of *run-time error*, and are treated in Chapter 11

· run-time error

Integer exponentiation is not defined as an operator, but this is available the built-in function pown, e.g.,

```
Listing 5.26, integerPown.fsx:
Integer exponent function.

> pown 2 5;;
val it : int = 32
```

which is equal to  $2^5$ .

For binary arithmetic on integers, the following operators are available: leftOp <<< rightOp, which shifts the bit pattern of leftOp rightOp positions to the left while inserting 0's to right; leftOp >>> rightOp, which shifts the bit pattern of leftOp rightOp positions to the right while inserting 0's to left; leftOp &&& rightOp, bitwise 'and', returns the result of taking the boolean 'and' operator position-wise; leftOp ||| rightOp, bitwise 'or', as 'and' but using the boolean 'or' operator; and leftOp ~~~ leftOp, bitwise xor, which is returns the result of the boolean 'xor' operator defined by the truth table in Table 5.5.

<sup>·</sup> xor

<sup>·</sup> exclusive or

#### 5.5 Floating point arithmetic

The set of reals is infinitely large, and since all computers have limited resources, it is not possible to represent it in their entirety. Floating point types are finite subsets reduced by sampling the space of reals. An in-depth description of floating point implementations can be found in Appendix A. The type float is the most common type.

Table E.1, E.2, and E.3 gives examples operators and functions pre-defined for floating point types. For most addition, subtraction, multiplication, divisions, and negation the result straight forward. The remainder operator for floats calculates the remainder after division and discarding the fractional part,

```
Listing 5.27, floatDivisionRemainder.fsx:
Floating point division and remainder operators.

> 7.0 / 2.5;;
val it : float = 2.8
> 7.0 % 2.5;;
val it : float = 2.0
```

The remainder for floating point numbers can be fractional, but division, discarding fractional part, and remainder is still a lossless representation of the original number as,

```
Listing 5.28, floatDivisionRemainderLossless.fsx:
Floating point division, truncation, and remainder is a lossless representation of a number.

> float (int (7.0 / 2.5));;
val it : float = 2.0
> (float (int (7.0 / 2.5))) * 2.5;;
val it : float = 5.0
> (float (int (7.0 / 2.5))) * 2.5 + 7.0 % 2.5;;
val it : float = 7.0
```

Arithmetic using float will not cause over- and underflow problems, since the IEEE 754 standard includes the special numbers  $\pm \infty$  and NaN. E.g.,

```
Listing 5.29, floatDivisionByZero.fsx:
Floating point numbers include infinity and Not-a-Number.

> 1.0/0.0;;
val it : float = infinity
> 0.0/0.0;;
val it : float = nan
```

However, the float type has limited precision, since there is only a finite number of numbers that can be stored in a float. E.g.,

```
Listing 5.30, floatImprecission.fsx:
Floating point arithmetic has finite precision.

> 357.8 + 0.1 - 357.9;;
val it : float = 5.684341886e-14
```

That is, addition and subtraction associates to the left, hence the expression is interpreted as (357.8 + 0.1) - 357.9, and we see that we do not get the expected 0, since only a limited number of floating point values are available, and the numbers 357.8 + 0.1 and 357.9 do not result in the same floating point representation. Such errors tend to accumulate and comparing the result of expressions of floating point values should therefore be treated with care. Thus, **equivalence of two floating point expressions should only be considered up to sufficient precision, e.g., comparing** 357.8 + 0.1 and 357.9 up to 1e-10 precision should be tested as, abs ((357.8 + 0.1) - 357.9) < 1e-10.

Advice

### 5.6 Char and string arithmetic

Addition is the only operator defined for characters, nevertheless, character arithmetic is often done by casting to integer. A typical example is conversion of case, e.g., to convert the lowercase character 'z' to uppercase, we use the *ASCIIbetical order* and add the difference between any Basic Latin Block letters in upper- and lowercase as integers and cast back to char, e.g.,

 $\cdot$  ASCIIbetical order

```
Listing 5.31, upcaseChar.fsx:

Converting case by casting and integer arithmetic.

> char (int 'z' - int 'a' + int 'A');;
val it : char = 'Z'
```

I.e., the code point difference between upper and lower case for any alphabetical character 'a' to 'z' is constant, hence we can change case by adding or subtracting the difference between any corresponding character. Unfortunately, this does not generalize to characters from other languages.

A large collection of operators and functions exist for string. The most simple is concatenation using, e.g.,

```
Listing 5.32, stringConcatenation.fsx:
Example of string concatenation.

> "hello" + " " + "world";;
val it : string = "hello world"
```

Characters and strings cannot be concatenated, which is why the above example used the string of a space " " instead of the space character ' '. The characters of a string may be indexed as using the . [] notation,

· . []

# Listing 5.33, stringIndexing.fsx: String indexing using square brackets. > "abcdefg".[0];; val it : char = 'a' > "abcdefg".[3];; val it : char = 'd' > "abcdefg".[3..];; val it : string = "defg" > "abcdefg".[...3];; val it : string = "abcd" > "abcdefg".[1...3];; val it : string = "bcd" > "abcdefg".[\*];; val it : string = "abcdefg"

Notice, that the first character has index 0, and to get the last character in a string, we use the string's length property as,

```
Listing 5.34, stringIndexingLength.fsx:
String length attribute and string indexing.

> "abcdefg".Length;;
val it : int = 7
> "abcdefg".[7-1];;
val it : char = 'g'
```

Since index counting starts at 0, and the string length is 7, then the index of the last character is 6. The is a long list of built-in functions in System.String for working with strings, some of which will be discussed in Chapter F.1.

The dot notation is an example of Structured programming, where technically speaking, the string "abcdefg" is an immutable object of class string, [] is an object method, and Length is a property. For more on object, classes, and methods see Chapter 20.

Strings are compared letter by letter. For two strings to be equal, they must have the same length and all the letters must be identical. E.g., "abs" = "absalon" is false, while "abs" = "abs" is true. The <> operator is the boolean negation of the = operator, e.g., "abs" <> "absalon" is true, while "abs" <> "abs" is false. For the < , <=, >, and >= operators, the strings are ordered alphabetically, such that "abs" < "absalon" && "absalon" < "milk" is true, that is, the < operator on two strings is true, if the left operand should come before the right, when sorting alphabetically. The algorithm for deciding the boolean value of leftOp < rightOp is as follows: we start by examining the first character, and if leftOp.[0] and rightOp.[0] are different, then the leftOp < rightOp is equal to leftOp.[0] < rightOp.[0]. E.g., "milk" < "abs" is the same as 'm' < 'a', which is false, since the letter 'm' does not come before the letter 'a' in the alphabet, or more precisely, the codepoint of 'm' is not less than the codepoint of 'a'. If leftOp.[0] and rightOp.[0] are equal, then we move onto the next letter and repeat the investigation, e.g., "abs" < "abs" is true, since "ab" = "ab" is true and 'e' < 's' is true. If we reach the end of either of the two strings, then the short is smaller than the larger, e.g., "abs" < "absalon" is true, while "abs" < "abs" is false. The <=, >, and >= operators are defined similarly.

- $\cdot \ dot \ notation$
- $\cdot$  object
- $\cdot$  class
- $\cdot$  method

# Part V Appendix

# Appendix E

# Language Details

This appendix lists various language details.

# E.1 Arithmetic operators on basic types

Operator	left0p	right0p	Expression	Result	Description
leftOp + rightOp	ints	ints	5 + 2	7	Addition
	floats	floats	5.0 + 2.0	7.0	
	chars	chars	'a' + 'b'	'\195'	Addition of
					codes
	strings	strings	"ab" + "cd"	"abcd"	Concatenation
leftOp - rightOp	ints	ints	5 - 2	3	Subtraction
	floats	floats	5.0 - 2.0	3.0	
leftOp * rightOp	ints	ints	5 * 2	10	Multiplication
	floats	floats	5.0 * 2.0	10.0	_
left0p / right0p	ints	ints	5 / 2	2	Integer di- vision
	floats	floats	5.0 / 2.0	2.5	Division
leftOp % rightOp	ints	ints	5 % 2	1	Remainder
	floats	floats	5.0 % 2.0	1.0	
leftOp ** rightOp	floats	floats	5.0 ** 2.0	25.0	Exponentiation
leftOp && rightOp	bool	bool	true && false	false	boolean
1 0 1					and
leftOp    rightOp	bool	bool	true    false	false	boolean or
leftOp &&& rightOp	ints	ints	0b1010 &&& 0b1100	0b1000	bitwise
1 0 1					bool and
leftOp     rightOp	ints	ints	0b1010     0b1100	0b1110	bitwise
1 0 1					boolean or
leftOp ^^^ rightOp	ints	ints	0b1010 ^^^ 0b1101	0b0111	bitwise
1 0 1					boolean
					exclusive
					or
leftOp <<< rightOp	ints	ints	0b00001100uy <<< 2	0b00110000uy	bitwise
1 0 1			v	, and the second	shift left
leftOp >>> rightOp	ints	ints	0b00001100uy >>> 2	0b00000011uy	bitwise
1 0 1			v	v	and
+op	ints		+3	3	identity
•	floats		+3.0	3.0	Ů
-op	ints		-3	-3	negation
1	floats		-3.0	-3.0	o o
not op	bool		not true	false	boolean
· · · · · ·					negation
~~~op	ints		~~~0b00001100uy	0b11110011uy	bitwise
٦٠	11100		Jeogga	Jarring	
					boolean

Table E.1: Arithmetic operators on basic types. Ints, floats, chars, and strings means all built-in integer types etc.. Note that for the bitwise operations, digits 0 and 1 are taken to be true and false.

Operator	left0p	right0p	Expression	Result	Description
leftOp < rightOp	bool	bool	true < false	false	Less than
	ints	ints	5 < 2	false	
	floats	floats	5.0 < 2.0	false	
	chars	chars	'a' < 'b'	true	
	strings	strings	"ab" < "cd"	true	
leftOp > rightOp	bool	bool	true > false	true	Greater than
	ints	ints	5 > 2	true	
	floats	floats	5.0 > 2.0	true	
	chars	chars	'a' > 'b'	false	
	strings	strings	"ab" > "cd"	false	
leftOp = rightOp	bool	bool	true = false	false	Equal
	ints	ints	5 = 2	false	
	floats	floats	5.0 = 2.0	false	
	chars	chars	'a' = 'b'	false	
	strings	strings	"ab" = "cd"	false	
leftOp <= rightOp	bool	bool	true <= false	false	Less than or equal
	ints	ints	5 <= 2	false	
	floats	floats	5.0 <= 2.0	false	
	chars	chars	'a' <= 'b'	true	
	strings	strings	"ab" <= "cd"	true	
leftOp >= rightOp	bool	bool	true >= false	true	Greater than or equal
	ints	ints	5 >= 2	true	
	floats	floats	5.0 >= 2.0	true	
	chars	chars	'a' >= 'b'	false	
	strings	strings	"ab" >= "cd"	false	
leftOp <> rightOp	bool	bool	true <> false	true	Not Equal
	ints	ints	5 <> 2	true	
	floats	floats	5.0 <> 2.0	true	
	chars	chars	'a' <> 'b'	true	
	strings	strings	"ab" <> "cd"	true	

Table E.2: Comparison operators on basic types. Ints, floats, chars, and strings means all built-in integer types etc..

# E.2 Basic arithmetic functions

Type	Function name	Example	Result	Description
Ints and floats	abs	abs -3	3	Absolute value
Floats	acos	acos 0.8	0.6435011088	Inverse cosine
Floats	asin	asin 0.8	0.927295218	Inverse sinus
Floats	atan	atan 0.8	0.6747409422	Inverse tangent
Floats	atan2	atan2 0.8 2.3	0.3347368373	Inverse tangentvariant
Floats	ceil	ceil 0.8	1.0	Ceiling
Floats	cos	cos 0.8	0.6967067093	Cosine
Floats	cosh	cosh 0.8	1.337434946	Hyperbolic cosine
Floats	exp	exp 0.8	2.225540928	Natural exponent
Floats	floor	floor 0.8	0.0	Floor
Floats	log	log 0.8	-0.2231435513	Natural logarithm
Floats	log10	log10 0.8	-0.09691001301	Base-10 logarithm
Ints, floats,	max	max 3.0 4.0	4.0	Maximum
chars, and strings				
Ints, floats,	min	min 3.0 4.0	3.0	Minimum
chars, and strings				_
Ints	pown	pown 3 2	9	Integer exponent
Floats	round	round 0.8	1.0	Rounding
Ints and floats	sign	sign -3	-1	Sign
Floats	sin	sin 0.8	0.7173560909	Sinus
Floats	sinh	sinh 0.8	0.8881059822	Hyperbolic sinus
Floats	sqrt	sqrt 0.8	0.894427191	Square root
Floats	tan	tan 0.8	1.029638557	Tangent
Floats	tanh	tanh 0.8	0.6640367703	Hyperbolic tangent

Table E.3: Predefined functions for arithmetic operations

Name	Example	Description
fst	fst (1, 2)	
snd	snd (1, 2)	
failwith	failwith	
invalidArg	invalidArg	
raise	raise	
reraise	reraise	
ref	ref	
ceil	ceil	

Table E.4: Built-in functions.

## E.3 Precedence and associativity

Operator	Associativity	Description
+op, -op, ~~~op	Left	Unary identity, negation, and bitwise negation operator
f x	Left	Function application
leftOp ** rightOp	Right	Exponent
leftOp * rightOp,	Left	Multiplication, division and remainder
<pre>left0p / right0p,</pre>		
leftOp % rightOp		
leftOp + rightOp,	Left	Addition and subtraction binary operators
leftOp - rightOp		
leftOp ^^^ rightOp	Right	bitwise exclusive or
<pre>left0p &lt; right0p,</pre>	Left	Comparison operators, bitwise shift, and bitwise 'and'
<pre>left0p &lt;= right0p,</pre>		and 'or'.
<pre>left0p &gt; right0p,</pre>		
<pre>left0p &gt;= right0p,</pre>		
<pre>left0p = right0p,</pre>		
<pre>left0p &lt;&gt; right0p,</pre>		
<pre>left0p &lt;&lt;&lt; right0p,</pre>		
<pre>left0p &gt;&gt;&gt; right0p,</pre>		
leftOp &&& rightOp,		
leftOp     rightOp,		
&&	Left	Boolean and
11	Left	Boolean or

Table E.5: Some common operators, their precedence, and their associativity. Rows are ordered from highest to lowest precedences, such that leftOp \* rightOp has higher precedence than leftOp + rightOp. Operators in the same row has same precedence. Full table is given in Table E.6.

 $<sup>\</sup>cdot$  boolean or

 $<sup>\</sup>cdot$  boolean and

Operator	Associativity	Description
ident "<"types ">"	Left	High-precedence type application
ident "("expr ")"	Left	High-predence application
"."	Left	
prefixOp	Left	All prefix operators
"" rule	Left	Pattern matching rule
ident expr,	Left	
"lazy'' expr,		
"assert'' epxr		
"**"opChar	Right	Exponent like
"*"opChar, "/"opChar,	Left	Infix multiplication like
"%"opChar		
"-"opChar, "+"opChar	Left	Infix addition like
":?''	None	
"::''	Right	
"^'' opChar	Right	
"!="opChar, "<"opChar,	Left	Infix addition like
">"opChar, "=",	\	
" "opChar, "&"opChar,		
"\$"opChar		
":>", ":?>"	Right	The state of the s
"&", "&&"	Left	Boolean and like
"or", "  "	Left	Boolean or like
", "	None	
":="	Right	
"->"	Right	
"if"	None	
"function", "fun",	None	
"match", "try"		
"let"	None	
";"	Right	
"   "	Left	
"when"	Right	
"as"	Right	

Table E.6: Precedence and associativity of operators. Operators in the same row has same precedence. See Listing 6.1 for the definition of prefixOp

# E.4 Lightweight Syntax

To appear later.  $^1$ 

<sup>&</sup>lt;sup>1</sup>Todo: See Lightweight Syntax, Spec-4.0 Chapter 15.1

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# Index

. [], 32	$\min, 23$
ReadKey, 98	nativeint, 20
ReadLine, 98	obj, 17
Read, 98	pown, 23
System.Console.ReadKey, 98	printfn, 47
System.Console.ReadLine, 98	printf, 45, 47
System.Console.Read, 98	round, 23
System.Console.WriteLine, 98	sbyte, 20
System.Console.Write, 98	sign,23
WriteLine, 98	single, 20
Write, 98	$\sinh, 23$
abs, 23	sin, 23
acos, 23	sprintf, 47
asin, 23	sqrt, 23
atan2, 23	stderr, 47, 98
atan, 23	stdin, 98
bignum, 20	stdout, 47, 98
byte[], 20	string, 17
byte, 20	tanh, 23
ceil, 23	tan, 23
char, 17	uint16, 20
cosh, 23	uint32, 20
$\cos, 23$	uint64, 20
decimal, 20	uint8, 20
double, 20	unativeint, 20
eprintfn, 47	unit, 17
eprintf, 47	
exn, 17	American Standard Code for Information Inter-
exp, 23	change, 135
failwithf, 47	and, $28$
float32, 20	anonymous function, 42
float, 17	array sequence expressions, 117
floor, 23	Array.toArray, 73
fprintfn, 47	Array.toList, 73
fprintf, 47	ASCII, 135
ignore, 47	ASCIIbetical order, 32, 135
int16, 20	
int32, 20	base, 17, 131
int64, 20	Basic Latin block, 136
int8, 20	Basic Multilingual plane, 136
int, 17	basic types, 17
it, 17	binary, 131
log10, 23	binary number, 18
$\log, 23$	binary operator, 23
max, 23	binary64, 131
•	binding, 13

bit, 18, 131 generic function, 41 black-box testing, 77 hand tracing, 83 block, 38 Head, 69 blocks, 136 hexadecimal, 131 boolean and, 27 hexadecimal number, 18 boolean or, 27 HTML, 55 branches, 62 Hyper Text Markup Language, 55 branching coverage, 78 bug, 76 IEEE 754 double precision floating-point format, byte, 131 131 Imperativ programming, 108 character, 19 Imperative programming, 6 class, 21, 33 implementation file, 10 code point, 19, 136 infix notation, 23 compiled, 10 infix operator, 22 computation expressions, 68, 71 integer division, 29 conditions, 62 integer number, 17 Cons, 69 interactive, 10 console, 10 IsEmpty, 69 coverage, 78 Item, 69 currying, 43 jagged arrays, 73 debugging, 12, 77, 83 decimal number, 17, 131 keyword, 13 decimal point, 17, 131 Declarative programming, 6 Latin-1 Supplement block, 136 digit, 17, 131 Latin1, 135 dot notation, 33 least significant bit, 131 double, 131 Length, 69 downcasting, 22 length, 66 lexeme, 15 EBNF, 17, 139 lexical scope, 15, 41 efficiency, 76 lexically, 36 encapsulate code, 40 lightweight syntax, 34, 37 encapsulation, 43, 49 list, 68 environment, 83 list sequence expression, 117 exception, 30 List.Empty, 69 exclusive or, 30 List.toArray, 69 executable file, 10 List.toList, 69 expression, 13, 22 literal, 17 expressions, 6 literal type, 20 Extended Backus-Naur Form, 17, 139 Extensible Markup Language, 52 machine code, 108 maintainability, 77 file, 97 member, 21, 66 floating point number, 17 method, 33 format string, 13 mockup code, 83 fractional part, 17, 22 module elements, 128 function, 15 modules, 10 function coverage, 78 most significant bit, 131 Functional programming, 6, 108 Mutable data, 47 functional programming, 6 functionality, 76 namespace, 21 functions, 6 namespace pollution, 122 NaN, 133

nested scope, 38 statement coverage, 78 newline, 20 statements, 6, 108 not, 28 states, 108 not a number, 133 stopping criterium, 64 stream, 97 obfuscation, 68 string, 13, 19 object, 33 Structured programming, 7 Object oriented programming, 108 subnormals, 133 Object-orientered programming, 7 Tail, 69 objects, 7 octal, 131 tail-recursive, 64 octal number, 18 terminal symbols, 139 operand, 40 tracing, 83 truth table, 28 operands, 23 operator, 23, 40 tuple, 66 or, 28 type, 13, 17 overflow, 29 type casting, 21 overshadows, 39 type declaration, 13 type inference, 12, 13 pattern matching, 119, 124 type safety, 40 portability, 77 precedence, 23 unary operator, 23 prefix operator, 23 underflow, 29 Procedural programming, 108 Unicode, 19 procedure, 43 unicode general category, 136 production rules, 139 Unicode Standard, 136 unit of measure, 121 ragged multidimensional list, 71 unit testing, 77 raise an exception, 91 unit-less, 121 range expression, 68 unit-testing, 12 reals, 131 upcasting, 22 recursive function, 63 usability, 76 reference cells, 50 UTF-16, 136 reliability, 76 UTF-8, 136 remainder, 29 rounding, 22 variable, 47 run-time error, 30 verbatim, 21 scientific notation, 18 white-box testing, 77, 78 scope, 38 whitespace, 20 script file, 10 whole part, 17, 22 script-fragment, 15 wild card, 36 word, 131 script-fragments, 10 Seq.initInfinite, 117 XML, 52 Seq.item, 115 xor, 30 Seq.take, 115 Seq.toArray, 117 yield bang, 115 Seq.toList, 117 side-effect, 72 side-effects, 43, 50 signature file, 10 slicing, 72 software testing, 77

state, 6 statement, 13