Learning to Program with F#

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1 | Preface

This book has been written as an introduction to programming for novice programmers. It is used in the first programming course at the University of Copenhagen's bachelor in computer science program. It has been typeset in LaTeX, and all programs have been developed and tested in Mono version 5.10.1.57.

This book started as a few chapters in 2016 and was to a large extent completed in 2017. This book was developed alongside the course Programmering og Problemløsning (programming and problem solving) and I am very thankful for the positive feedback and suggestions numerous people have given me. I would particularly like to thank Malthe Sporring for his insightful and detailed comments to every (!) page of this book. I also would like to acknowledge the invaluable feedback from my co-teachers: Torben Mogensen, Martin Elsmann, Christina Lioma; my teaching assistants: Sune Hellfritzsch, Emil Bak, Jesper Erno, Rasmus Johannesson, Jan Rolandsen, Peter Pedersen, Joachim Tilsted Kristensen, Lukas Svarre Engedal, Matthias Brix, Kristian Fogh Nissen, Emil Petersen, Jens Larsen, Emil Bak, Lasse Grønborg, Mads Obitsø, Maurits Pallesen, Tor Skovsgaard, Baldar Ivarsen, Alexander Christensen, Lars-Bo Nielsen, Frederik Schmidt, Lukas Engedal, Jan Rolandsen. And finally, thanks to all the students of our course who have had the patience and endurance to labor and enjoy learning to program using F#.

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2 | Introduction

Programming is a creative process in which exciting problems may be solved and new tools and applications may be created. With programming skills, you can create high-level applications to run on a mobile device that interact with other users, databases, and artificial intelligence; you may create programs that run on supercomputers for simulating weather systems on alien planets or social phenomena in the internet economy; and you may create programs that run on small custom-made hardware for controlling your home appliances.

2.1. How to Learn to Solve Problems by Programming

In order to learn how to program, there are a couple of steps that are useful:

1. Choose a programming language: A programming language, such as F#, is a vocabulary and a set of grammatical rules for instructing a computer to perform a certain task. It is possible to program without a concrete language, but your ideas and thoughts must still be expressed in some fairly rigorous way. Theoretical computer scientists typically do not rely on computers nor programming languages but uses mathematics to prove properties of algorithms. However, most computer scientists program using a computer, and with a real language you have the added benefit of checking your algorithm, and hence your thoughts, rigorously on a real computer. This book teaches a subset of F#. The purpose is not to be a reference guide to this language but to use it as a vessel to teach you, the reader, how to convert your ideas into programs.

2. Learn the language: A computer language is a structure for thought, and it influences which thoughts you choose to express as a program, and how you choose to do it. Any conversion requires you to acquire a sufficient level of fluency in order for you to be able to make programs. You do not need to be a master in F# nor to know every corner of the language, and you will expand your knowledge as you expose yourself to solving problems in the language, but you must invest an initial amount of time and energy in order to learn the basics of the language. This book aims at getting you started quickly, which is why we intentionally teach just a small subset of F#. On the internet and through other works you will be able to learn much more.

3. Practice: In order to be a good programmer, the most essential step is: practice, practice, practice! It has been estimated that to master anything, then you have to have spent at least 10000 hours practicing, so get started logging hours! It of course matters, how you practice. This book teaches a number of different programming themes. The point is that programming is thinking, and the scaffold you use shapes

your thoughts. It is therefore important to recognize this scaffold and to have the ability to choose one which suits your ideas and your goals best. The best way to expand your abilities is to sharpen your present abilities, push yourself into new territory, and try something new. Do not be afraid to make errors or be frustrated at first. These are the experiences that make you grow.

4. Solve real problems: I have found that using my programming skills in real situations with customers demanding specific solutions, has forced me to put the programming tools and techniques that I use into perspective. Sometimes a task requires a cheap and fast solution, other times customers want a long-perspective solution with bug fixes, upgrades, and new features. Practicing solving real problems helps you strike a balance between the two when programming. It also allows makes you a more practical programmer, by allowing you to recognize its applications in your everyday experiences. Regardless, real problems create real programmers.

2.2. How to Solve Problems

Programming is the act of solving a problem by writing a program to be executed on a computer. A general method for solving problems, given by George Pólya [9] and adapted to programming, is:

Understand the problem: To solve any problem it is crucial that the problem formulation is understood. What is to be solved? Do you understand everything in the description of the problem? Is all information for finding the solution available or is something missing?

Design a plan: Good designs lead to programs are faster to implement, easier to find errors in, and easier to update in the future. Before you start typing a program consider things like: What are the requirements and constraints for the program? Which components should the program have? How are these components supposed to work together? Designing often involves drawing a diagram of the program and writing program sketches on paper.

Implement the plan: Implementation is the act of transforming a program design into code. A crucial part of any implementation is choosing which programming language to use. Furthermore, the solution to many problems will have a number of implementations which vary in how much code they require, to which degree they rely on external libraries, which programming style they are best suited for, what machine resources they require, and how long time they take to run on a computer. With a good design, the coding is usually easy, since the design will have uncovered the major issues and found solutions for these, but sometimes implementation reveals new problems, which require rethinking the design. Most often the implementation step also require a careful documentation of key aspects of the code, e.g., a user manual for the user, and internal notes for fellow programmers that are to maintain and update the code in the future.

Reflect on the result: A crucial part of any programming task is ensuring that the program solves the problem sufficiently. Ask yourself questions such as: What are the program's errors, is the documentation of the code sufficient and relevant for its intended use? Is the code easily maintainable and extendable by other programmers? Which parts of your method would you avoid or replicate in future programming sessions? Can you reuse some of the code you developed in other programs?

Programming is a very complicated process, and Pólya's list is a useful guide but not a fail-safe approach. Always approach problem-solving with an open mind.

2.3. Approaches to Programming

This book focuses on several fundamentally different approaches to programming:

· imperative programming

Imperative programming emphasizes *how a program shall accomplish a solution* and focusses less on *what the solution is*. A cooking recipe is an example of the spirit of imperative programming, where the recipe emphasizes what should be done in each step rather than describing the result. For example, a recipe for bread might tell you to first mix yeast and water, then add flour, etc. In imperative programming what should be done are called *statements* and in the recipe analogy, the steps are the statements. Statements influence the computer's *states*, in the same way that adding flour changes the state of our dough. Almost all computer hardware is designed to execute low-level programs written in imperative style. Imperative programming builds on the Turing machine [10]. As a historical note, the first major language was FORTRAN [6] which emphasized an imperative style of programming.

· statement
· state

· declarative programming

Declarative programming emphasizes *what a program shall accomplish* but not *how*. We will consider Functional programming as an example of declarative programming. A *functional programming* language evaluates *functions* and avoids state changes. The program consists of *expressions* instead of statements. As an example, the function $f(x) = x^2$ takes a number x, evaluates the expression x^2, and returns the resulting number. Everything about the function may be characterized by the relation between the input and output values. Functional programming has its roots in lambda calculus [1]. The first language emphasizing functional programming was Lisp [7].

· functional programming
· function
· expression

· structured programming

Structured programming emphasizes organization of programs in units of code and data. For example, a traffic light may consist of a state (red, yellow, green), and code for updating the state, i.e., switching from one color to the next. We will focus on Object-oriented programming as the example of structured programming. *Object-oriented programming* is a type of programming, where the code and data are structured into *objects*. E.g., a traffic light may be an object in a traffic-routing program. The first object-oriented programming language was Simula 67 developed by Dahl and Nygaard at the Norwegian Computing Center in Oslo [2].

· Object-oriented programming
· object

· event-driven programming

Event-driven programming, which is often used when dynamically interacting with the real world. This is useful, for example, when programming graphical user interfaces, where programs will often need to react to a user clicking on the mouse or to text arriving from a web-server to be displayed on the screen. Event-driven programs are often programmed using *call-back functions*, which are small programs that are ready to run when events occur.

· call-back functions

Most programs do not follow a single programming paradigm as, e.g., one of the above, but are a mix. Nevertheless, this book will treat each paradigm separately to emphasize its advantages and disadvantages.

2.4. Why Use F#

This book uses F#, also known as Fsharp, which is a functional first programming language, meaning that it is designed as a functional programming language that also supports imperative and object-oriented programming. It was originally developed for Microsoft's .Net platform but is available as open source for many operating systems through Mono. As an introduction to programming, F# is a young programming language still under development, with syntax that at times is a bit complex. Still, it offers a number of advantages:

Interactive and compile mode: F# has an interactive and a compile mode of operation. In interactive mode you can write code that is executed immediately in a manner similar to working with a calculator, while in compile mode you combine many lines of code possibly in many files into a single application, which is easier to distribute to people who are not F# experts and is faster to execute.

Indentation for scope: F# uses indentation to indicate scope. Some lines of code belong together and should be executed in a certain order and may share data. Indentation helps in specifying this relationship.

Strongly typed: F# is strongly typed, reducing the number of runtime errors. That is, F# is picky, and will not allow the programmer to mix up types such as numbers and text. This is a great advantage for large programs.

Multi-platform: F# is available on Linux, Mac OS X, Android, iOS, Windows, GPUs, and browsers via the Mono platform.

Free to use and open source: F# is supported by the Fsharp foundation (`http://fsharp.org`) and sponsored by Microsoft.

Assemblies: F# is designed to be able to easily communicate with other .Net and Mono programs through the language-independent, platform-independent bytecode called Common Intermediate Language (CIL) organized as assemblies. Thus, if you find that certain parts of a program are easy to express in F# and others in C++, then you will be able to combine these parts later into a single program.

Modern computing: F# supports all aspects of modern computing including Graphical User Interfaces, Web programming, Information rich programming, Parallel algorithms, ...

Integrated development environments (IDE): F# is supported by major IDEs such as Visual Studio (`https://www.visualstudio.com`) and Xamarin Studio (`https://www.xamarin.com`).

2.5. How to Read This Book

Learning to program requires mastering a programming language, however, most programming languages contain details that are rarely used or used in contexts far from a specific programming topic. Hence, this book only includes a subset of F# but focuses on language structures necessary to understand several common programming paradigms: Imperative programming mainly covered in Chapters 6 to 11, functional programming mainly covered in Chapters 13 to 16, object-oriented programming in Chapters 20 and 22, and event-driven programming in Chapter 23. A number of general topics are given in the appendix for reference. For further reading please consult `http://fsharp.org`.

3 | Executing F# Code

3.1. Source Code

F# is a functional first programming language, meaning that it has strong support for functional programming, but also supports imperative and object-oriented programming.

F# has two modes of execution, *interactive* and *compiled*. Interactive mode allows the user to interact with F# as a dialogue: The user writes statements, and F# responds immediately. Interactive mode is well suited for small experiments or back-of-an-envelope calculations, but not for programming in general. In compile mode, the user writes a complete program, which is translated or compiled using the F# compiler into a compiled file. The compiled file can be run or executed as a stand-alone program using a virtual machine called `mono`. In both the interactive and compile mode, F# statements are translated into something that can be executed on the computer. A major difference is that in interactive mode, the translation is performed everytime the program is executed, while in compiled mode the translation is performed only once. · interactive mode · compile mode

Both interactive and compile modes can be accessed via the *console*, see Appendix A for more information on the console. The interactive system is started by calling `fsharpi` at the command prompt in the console, while compilation is performed with `fsharpc`, and execution of the compiled code is performed using the `mono` command. · console

F# programs come in many forms, which are identified by suffixes. The *source code* is an F# program written in human-readable form using an editor. F# recognizes the following types of source code files: · source code

`.fs` An *implementation file*, e.g., `myModule.fs` · implementation file

`.fsi` A *signature file*, e.g., `myModule.fsi` · signature file

`.fsx` A *script file*, e.g., `gettingStartedStump.fsx` · script file

`.fsscript` Same as `.fsx`, e.g., `gettingStartedStump.fsscript`

Compiled code is source code translated into a machine-readable language, which can be executed by a machine. Compiled F# code is either:

`.dll` A *library file*, e.g., `myModule.dll` · library file

`.exe` A stand-alone *executable file*, e.g., `gettingStartedStump.exe` · executable file

The implementation, signature, and script files are all typically compiled to produce an executable file, in which case they are called *scripts*, but can also be entered into the · scripts interactive system, in which case these are called *script-fragments*. The implementation and signature files are special kinds of script files used for building libraries. Libraries in F# are called modules, and they are collections of smaller programs used by other programs, which will be discussed in detail in Chapter 9.

· script-fragment

3.2. Executing Programs

Programs may either be executed by the interpreter or by compiling and executing the compiled code. In Mono the interpreter is called `fsharpi` and can be used in two ways: interactively, where a user enters one or more script-fragments separated by the "`;;`" characters, or to execute a script file treated as a single script-fragment. [1]

To illustrate the difference between interactive and compile mode, consider the program in Listing 3.1.

**Listing 3.1 gettingStartedStump.fsx:
A simple demonstration script.**

```
let a = 3.0
do printfn "%g" a
```

The code declares a value `a` to be the decimal value 3.0 and finally prints it to the console. The `do printfn` is a statement for displaying the content of a value to the screen, and `"%g"` is a special notation to control how the value is printed. In this case, it is printed as a decimal number. This and more will be discussed at length in the following chapters. For now, we will concentrate on how to interact with the F# interpreter and compiler.

An interactive session is obtained by starting the console, typing the `fsharpi` command, typing the lines of the program, and ending the script-fragment with "`;;`". The dialogue in Listing 3.2 demonstrates the workflow. What the user types has been highlighted by a box.

[1] Jon: **Too early to introduce lexeme: "F# uses many characters which at times are given special meanings, e.g., the characters "`;;`" is compound character denoting the end of a script-fragment. Such possibly compound characters are called lexemes."**

Listing 3.2: An interactive session.

```
$ fsharpi

F# Interactive for F# 4.1 (Open Source Edition)
Freely distributed under the Apache 2.0 Open Source License

For help type #help;;

> let a = 3.0
- do printfn "%g" a;;
3

val a : float = 3.0
val it : unit = ()

> #quit;;
```

We see that after typing `fsharpi`, the program starts by stating details about itself. Then F# writes `>` indicating that it is ready to receive commands. The user types `let a = 3.0` and presses `enter`, to which the interpreter responds with `-`. This indicates that the line has been received, that the script-fragment is not yet completed, and that it is ready to receive more input. When the user types `do printfn "%g" a;;` followed by `enter`, then by "`;;`" the interpreter knows that the script-fragment is completed, it interprets the script-fragment, responds with `3` and some extra information about the entered code, and with `>` to indicate that it is ready for more script-fragments. The interpreter is stopped when the user types `#quit;;`. It is also possible to stop the interpreter by typing `ctrl-d`.

The interactive session results in extra output on the *type inference* performed. In Listing 3.2 F# states that the name `a` has *type* `float` and the value `3.0`. Likewise, the `do` statement F# refers to by the name `it`, and it has the type `unit` and value `()`. Types are very important to F# since they define how different program pieces fit together like lego bricks. They are a key ingrediens for finding errors in programs, also known as *debugging*, and much of the rest of this book is concerned with types.

· type inference
· type
· debugging

Instead of running `fsharpi` interactively, we can write the script-fragment from Listing 3.1 into a file, here called `gettingStartedStump.fsx`. This file can be interpreted directly by `fsharpi` as shown in Listing 3.3.

Listing 3.3: Using the interpreter to execute a script.

```
$ fsharpi gettingStartedStump.fsx
3
```

Notice that in the file, "`;;`" is optional. We see that the interpreter executes the code and prints the result on screen without the extra type information as compared to Listing 3.2.

Finally, the file containing Listing 3.1 may be compiled into an executable file with the program `fsharpc`, and run using the program `mono` from the console. This is demonstrated in Listing 3.4.

Listing 3.4: Compiling and executing a script.

```
$ fsharpc gettingStartedStump.fsx
F# Compiler for F# 4.1 (Open Source Edition)
Freely distributed under the Apache 2.0 Open Source License
$ mono gettingStartedStump.exe
3
```

The compiler takes `gettingStartedStump.fsx` and produces `gettingStarted.exe`, which can be run using `mono`.

Both the interpreter and the compiler translates the source code into a format which can be executed by the computer. While the compiler performs this translation once and stores the result in the executable file, the interpreter translates the code every time the code is executed. Thus, to run the program again with the interpreter, it must be retranslated as "`$fsharpi gettingStartedStump.fsx`". In contrast, compiled code does not need to be recompiled to be run again, only re-executed using "`$ mono gettingStartedStump.exe`". On a MacBook Pro, with a 2.9 GHz Intel Core i5, the time the various stages take for this script are:

Command	Time
`fsharpi gettingStartedStump.fsx`	1.88s
`fsharpc gettingStartedStump.fsx`	1.90s
`mono gettingStartedStump.exe`	0.05s

I.e., executing the script with `fsharpi` is slightly faster than by first compiling it with `fsharpc` and then executing the result with `mono` ($1.88s < 0.05s + 1.90s$), if the script were to be executed only once, but every future execution of the script using the compiled version requires only the use of `mono`, which is much faster than `fsharpi` ($1.88s \gg 0.05s$).

Executing programs with the interpreted directly from a file and compiling and executing the program is much preferred for programming complete programs, since the starting state is well defined, and since this better supports *unit-testing*, which is a method for debugging programs. Thus, **prefer compiling over interpretation.**

· unit-testing
Advice

4 | Quick-start Guide

Programming is the art of solving problems by writing a program to be executed by a computer. For example, to solve the following problem,

Problem 4.1

What is the sum of 357 and 864?

we have written the programshown in Listing 4.1.

Listing 4.1 quickStartSum.fsx:
A script to add 2 numbers and print the result to the console.

```
1 let a = 357
2 let b = 864
3 let c = a + b
4 do printfn "%A" c
```

```
1 $ fsharpc --nologo quickStartSum.fsx && mono quickStartSum.exe
2 1221
```

In the box the above, we see our program was saved as a script in a file called `quickStartSum.fsx`, and in the console we executed the program by typing the command `fsharpc --nologo quickStartSum.fsx && mono quickStartSum.exe`. The result is then printed in the console to be `1221`. Here, as in the rest of this book, we have used the optional flag `--nologo`, which informs `fsharpc` not to print information about its version etc., thus making the output shorter. The `&&` notation tells the console to first run the command on the left, and if that did not report any errors, then run the command on the right. This could also have been performed as two separate commands to the console, and throughout this book we will use the above shorthand when convenient.

To solve the problem, we made program consisting of several lines, where each line was an *expressions*. The first expression, `let a = 357`, in line 1 used the *`let` keyword* to *bind* the value 357 to the name `a`. This is called a *let-binding* and makes the name synonymous with the value. Another notable point is that F# identifies 357 as an *integer number*, which is F#'s preferred number type, since computations on integers are very efficient, and since integers are very easy to communicate to other programs. In line 2 we bound the value 864 to the name `b`, and to the name `c` we bound the result of evaluating the sum `a + b` in line 3. Line 4 is a *do-binding*, as noted by the keyword *`do`*. Do-bindings are also sometimes called *statements*, and the `do` keyword is optional in F#. Here the value of `c` was printed to the console followed by a newline with the *`printfn` function*. A function

· expression
· `let`
· keyword
· binding
· let-binding
· integer number
· do-binding
· `do`
· statements
· `printfn`
· function

in F# is an entity that takes zero or more arguments and returns a value. The function `printfn` is very special, since it can take any number of arguments and returns the special value *"()"* which has type ***unit***. The `do` tells F# to ignore this value. Here `printfn` has been used with 2 arguments: `"%A"` and `c`. Notice that in contrast to many other languages, F# does not use parentheses to frame the list of arguments, nor does it use commas to separate them. In general, the `printfn` function always has 1 or more arguments, and the first is a *format string*. A *string* is a sequence of characters starting and ending with double quotation marks. E.g., `let s = "this is a string of characters"` binds the string `"this is..."` to the name `s`. For the `printfn` function, the format string may be any string, but if it contains format character sequences, such as `%A`, then format character sequence are replaced by the arguments to `printfn` which follows the format string. The format string must match the value *type*, that is, here `c` is of type integer, whereas the format string `%A` matches many types.

· `()`
· `unit`
· format string
· string
· type

Types are a central concept in F#. In the script 4.1 we bound values of integer type to names. There are several different integer types in F#, here we used the one called `int`. The values were not *declared* to have these types, instead the types were *inferred* by F#. Typing these bindings line by line in an interactive session, we see the inferred types as shown in Listing 4.2.

· type declaration
· type inference

Listing 4.2: Inferred types are given as part of the response from the interpreter.

```
> let a = 357;;
val a : int = 357

> let b = 864;;
val b : int = 864

> let c = a + b;;
val c : int = 1221

> do printfn "%A" c;;
1221
val it : unit = ()
```

The interactive session displays the type using the ***val*** keyword followed by the name used in the binding, its type, and its value. Since the value is also responded, the last `printfn` statement is superfluous. However, **it is ill-advised to design programs to be run in an interactive session, since the scripts need to be manually copied every time it is to be run, and since the starting state may be unclear.** Notice that `printfn` is automatically bound to the name ***it*** of type `unit` and value *"()"*. F# insists on binding all statements to values, and in lack of an explicit name, it will use `it`. Rumor has it that `it` is an abbreviation for "irrelevant".

· `val`
Advice
· `it`
· `()`

The following problem,

Problem 4.2

What is the sum of 357.6 and 863.4?

uses *decimal point* numbers instead of integers. These numbers are called *floating point* numbers, and their internal representation is quite different to integer numbers used previously. Likewise, algorithms used to perform arithmetic are quite different from integers.

· decimal point
· floating point

Now the program would look like Listing 4.3.

**Listing 4.3 quickStartSumFloat.fsx:
Floating point types and arithmetic.**

```
1 let a = 357.6
2 let b = 863.4
3 let c = a + b
4 do printfn "%A" c
```

```
1 $ fsharpc --nologo quickStartSumFloat.fsx && mono
    quickStartSumFloat.exe
2 1221.0
```

On the surface, this could appear as an almost negligible change, but the set of integers and the set of real numbers (floats) require quite different representations in order to be effective on a computer, and as a consequence, the implementation of their operations, such as addition, are very different. Thus, although the response is an integer, it has type `float` which is indicated by `1221.0` and which is not the same as `1221`. F# is very picky about types, and generally does not allow types to be mixed, as demonstrated in the interactive session in Listing 4.4.

Listing 4.4: Mixing types is often not allowed.

```
> let a = 357;;
val a : int = 357

> let b = 863.4;;
val b : float = 863.4

> let c = a + b;;
  let c = a + b;;
  ------------^

stdin(4,13): error FS0001: The type 'float' does not match the
  type 'int'
```

We see that binding a name to a number without a decimal point is inferred to be an integer, while when binding to a number with a decimal point the type is inferred to be a float, and that our attempt of adding an integer and floating point value gives an error. The *error message* contains much information. First, it states that the error is in `stdin(4,13)`, which means that the error was found on standard-input at line 4 and column 13. Since the program was executed using `fsharpi quickStartSumFloat.fsx`, here standard input means the file `quickStartSumFloat.fsx` shown in Listing 4.3. The corresponding line and column are also shown in Listing 4.4. After the file, line, and column number, F# informs us of the error number and a description of the error. Error numbers are an underdeveloped feature in Mono and should be ignored. However, the verbal description often contains useful information for *debugging*. In the example we are informed that there is a type mismatch in the expression, i.e., since `a` is an integer, F# expected `b` to be one too. Debugging is the process of solving errors in programs, and here we can solve the error by either making `a` into a float or `b` into an int. The right solution depends on the application.

· error message

· debugging

F# is a functional first programming language, and one implication of this is that names have a *lexical scope*. A scope is the lines in a program where a binding is valid, and lexical scope means that to find the value of a name, F# looks for the value in the above lines. Furthermore, at the outermost level, rebinding is not allowed. If attempted, then F# will return an error as shown in Listing 4.5.

· lexical scope

Listing 4.5 quickStartRebindError.fsx:
A name cannot be rebound.

```
let a = 357
let a = 864
```

```
$ fsharpc --nologo -a quickStartRebindError.fsx

quickStartRebindError.fsx(2,5): error FS0037: Duplicate
  definition of value 'a'
```

However, if the same code is executed in an interactive session, then rebinding does not cause an error, as shown in Listing 4.6.

Listing 4.6: Names may be reused when separated by the lexeme "`;;`".

```
> let a = 357;;
val a : int = 357

> let a = 864;;
val a : int = 864
```

The difference is that the *"`;;`" lexeme* is used to specify the end of a *script-fragment*. A lexeme is a letter or word, which F# considers as an atomic unit. Script-fragments may be defined both in scripts and in interactive mode, and rebinding is not allowed at the outermost level in script-fragments. Even with the "`;;`" lexeme, rebinding is not allowed in compile-mode. In general, **avoid rebinding of names.**

· `;;`
· lexeme
· script-fragment

Advice

In F#, *functions* are also values, and we may define a function `sum` as part of the solution to the above program, as shown in Listing 4.7.

· function

Listing 4.7 quickStartSumFct.fsx:
A script to add 2 numbers using a user-defined function.

```
let sum x y = x + y
let c = sum 357 864
do printfn "%A" c
```

```
$ fsharpc --nologo quickStartSumFct.fsx && mono
  quickStartSumFct.exe
1221
```

Functions are useful to *encapsulate* code, such that we can focus on the transformation of data by a function while ignoring the details on how this is done. Functions are also

· encapsulate

useful for code reuse, i.e., instead of repeating a piece of code in several places, such code can be encapsulated in a function and replaced with function calls. This makes debugging and maintenance considerably simpler. Entering the function into an interactive session will illustrate the inferred type the function `sum` has: `val sum : x:int -> y:int -> int`. The "`->`" is the mapping operator in the sense that functions are mappings between sets. The type of the function `sum`, should be read as `val sum : x:int -> (y:int -> int)`, that is, `sum` takes an integer and returns a function, which takes an integer and returns an integer. This is an example of a higher-order function.

Type inference in F# may cause problems, since the type of a function is inferred based on the context in which it is defined. E.g., in an interactive session, defining the sum in one scope on a single line will default the types to integers, F#'s favorite type. Thus, if the next script-fragment uses the function with floats, then we will get an error message as shown in Listing 4.8.

Listing 4.8: Types are inferred in blocks, and F# tends to prefer integers.

```
val sum : x:int -> y:int -> int

> let c = sum 357.6 863.4;;
  let c = sum 357.6 863.4;;
  ------------^^^^^

stdin(3,13): error FS0001: This expression was expected to have type
    'int'
but here has type
    'float'
```

A remedy is to define the function in the same script-fragment as it is used, such as shown in Listing 4.9.

Listing 4.9: Type inference is per script-fragment.

```
> let sum x y = x + y
- let c = sum 357.6 863.4;;
val sum : x:float -> y:float -> float
val c : float = 1221.0
```

Alternatively, the types may be explicitly stated as shown in Listing 4.10.

Listing 4.10: Function argument and return types may be stated explicitly.

```
> let sum (x : float) (y : float) : float = x + y;;
val sum : x:float -> y:float -> float

> let c = sum 357.6 863.4;;
val c : float = 1221.0
```

The function `sum` has two arguments and a return type, and in Listing 4.10 we have specified all three. This is done using the "`:`" lexeme, and to resolve confusion, we must use parentheses around the arguments, such as `(y : float)`, otherwise F# would not be able to understand whether the type annotation was for the argument or the return value. Often it is sufficient to specify just some of the types, since type inference will enforce the remaining types. E.g., in this example, the "+" operator is defined for identical types, so specifying the return value of `sum` to be a float implies that the result of the "+" operator is a float, and therefore that its arguments must be floats, and finally then that the arguments for `sum` must be floats. However, in this book we advocate the following advice: **specify types unless explicitly working with generic functions.** Advice

In this chapter, we have scratched the surface of learning how to program by concentrating on a number of key programming concepts and how they are expressed in the F# language. In the following chapters, we will expand the description of F# with features used in all programming approaches.

A | The Console in Windows, MacOS X, and Linux

Almost all popular operating systems are accessed through a user-friendly *graphical user interface* (*GUI*) that is designed to make typical tasks easy to learn to solve. As a computer programmer, you often need to access some of the functionalities of the computer, which, unfortunately, are sometimes complicated by this particular graphical user interface. The *console*, also called the *terminal* and the *Windows command line*, is the right hand of a programmer. The console is a simple program that allows you to complete text commands. Almost all the tasks that can be done with the graphical user interface can be done in the console and vice versa. Using the console, you will benefit from its direct control of the programs we write, and in your education, you will benefit from the fast and raw information you get through the console.

· graphical user interface
· GUI
· console
· terminal
· Windows command line

A.1. The Basics

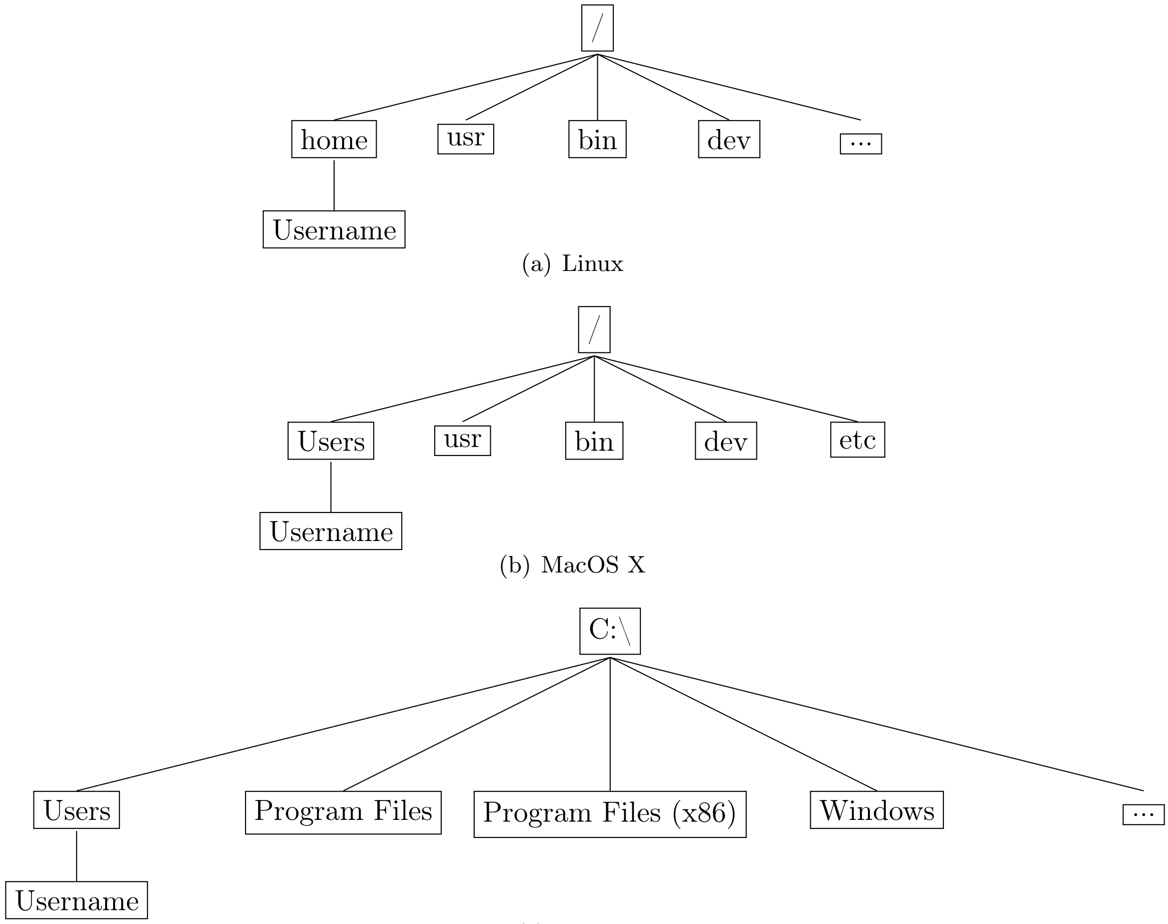
When you open a *directory* or *folder* in your preferred operating system, the directory will have a location in the file system, whether from the console or through the operating system's graphical user interface. The console will almost always be associated with a particular directory or folder in the file system, and it is said that it is the directory that the console is in. The exact structure of file systems varies between Linux, MacOS X, and Windows, but common is that it is a hierarchical structure. This is illustrated in Figure A.1.

· directory
· folder

There are many predefined console commands, available in the console, and you can also make your own. In the following sections, we will review the most important commands in the three different operating systems. These are summarized in Table A.1.

A.2. Windows

In this section we will discuss the commands summarized in Table A.1. Windows 7 and earlier versions: To open the console, press `Start->Run` in the lower left corner, and then type `cmd` in the box. In Windows 8 and 10, you right-click on the windows icon, choose `Run` or equivalent in your local language, and type `cmd`. Alternatively, you can type `Windows-key + R`. Now you should open a console window with a prompt showing something like Listing A.1.



(a) Linux

(b) MacOS X

(c) Windows

Figure A.1.: The top file hierarchy levels of common operating systems.

Windows	MacOS X/Linux	Description
`dir`	`ls`	Show content of present directory.
`cd <dir>`	`cd <dir>`	Change present directory to `<dir>`.
`mkdir <dir>`	`mkdir <dir>`	Create directory `<dir>`.
`rmdir <dir>`	`rmdir <dir>`	Delete `<dir>` (Warning: cannot be reverted).
`move <file> <file or dir>`	`mv <file> <file or dir>`	Move `<fil>` to `<file or dir>`.
`copy <file1> <file2>`	`cp <file1> <file2>`	Create a new file called `<file2>` as a copy of `<file1>`.
`del <file>`	`rm <file>`	delete `<file>` (Warning: cannot be reverted).
`echo <string or variable>`	`echo <string or variable>`	Write a string or content of a variable to screen.

Table A.1.: The most important console commands for Windows, MacOS X, and Linux.

Listing A.1: The Windows console.

```
Microsoft Windows [Version 6.1.7601]
Copyright (c) 2009 Microsoft Corporation.  All rights reserved.

C:\Users\sporring>
```

To see which files are in the directory, use ***dir***, as shown in Listing A.2. · `dir`

Listing A.2: Directory listing with `dir`.

```
C:\Users\sporring>dir
 Volume in drive C has no label.
 Volume Serial Number is 94F0-31BD

 Directory of C:\Users\sporring

30-07-2015  15:23    <DIR>          .
30-07-2015  15:23    <DIR>          ..
30-07-2015  14:27    <DIR>          Contacts
30-07-2015  14:27    <DIR>          Desktop
30-07-2015  17:40    <DIR>          Documents
30-07-2015  15:11    <DIR>          Downloads
30-07-2015  14:28    <DIR>          Favorites
30-07-2015  14:27    <DIR>          Links
30-07-2015  14:27    <DIR>          Music
30-07-2015  14:27    <DIR>          Pictures
30-07-2015  14:27    <DIR>          Saved Games
30-07-2015  17:27    <DIR>          Searches
30-07-2015  14:27    <DIR>          Videos
               0 File(s)              0 bytes
              13 Dir(s)  95.004.622.848 bytes free

C:\Users\sporring>
```

We see that there are no files and thirteen directories (DIR). The columns tell from left to right: the date and time of their creation, the file size or if it is a folder, and the name file or directory name. The first two folders "`.`" and "`..`" are found in each folder and refer to this folder as well as the one above in the hierarchy. In this case, the folder "`.`" is an alias for `C:\Users\sporring` and "`..`" for `C:\Users`.

Use ***cd*** to change directory, e.g., to `Documents`, as in Listing A.3. · `cd`

Listing A.3: Change directory with `cd`.

```
C:\Users\sporring>cd Documents

C:\Users\sporring\Documents>
```

Note that some systems translate default filenames, so their names may be given different names in different languages in the graphical user interface as compared to the console.

You can use ***mkdir*** to create a new directory called, e.g., `myFolder`, as illustrated in List- · `mkdir` ing A.4.

Listing A.4: Creating a directory with `mkdir`.

```
C:\Users\sporring\Documents>mkdir myFolder

C:\Users\sporring\Documents>dir
 Volume in drive C has no label.
 Volume Serial Number is 94F0-31BD

 Directory of C:\Users\sporring\Documents

30-07-2015  19:17    <DIR>          .
30-07-2015  19:17    <DIR>          ..
30-07-2015  19:17    <DIR>          myFolder
               0 File(s)              0 bytes
               3 Dir(s)  94.656.638.976 bytes free

C:\Users\sporring\Documents>
```

By using `dir` we inspect the result.

Files can be created by, e.g., *`echo`* and *redirection*, as demonstrated in Listing A.5.

· `echo`
· redirection

Listing A.5: Creating a file with `echo` and redirection.

```
C:\Users\sporring\Documents>echo "Hi" > hi.txt

C:\Users\sporring\Documents>dir
 Volume in drive C has no label.
 Volume Serial Number is 94F0-31BD

 Directory of C:\Users\sporring\Documents

30-07-2015  19:18    <DIR>          .
30-07-2015  19:18    <DIR>          ..
30-07-2015  19:17    <DIR>          myFolder
30-07-2015  19:18                 8 hi.txt
               1 File(s)              8 bytes
               3 Dir(s)  94.656.634.880 bytes free

C:\Users\sporring\Documents>
```

To move the file `hi.txt` to the directory `myFolder`, use *`move`*, as shown in Listing A.6.

· `move`

Listing A.6: Move a file with `move`.

```
C:\Users\sporring\Documents>move hi.txt myFolder
        1 file(s) moved.

C:\Users\sporring\Documents>
```

Finally, use *`del`* to delete a file and *`rmdir`* to delete a directory, as shown in Listing A.7.

· `del`
· `rmdir`

Listing A.7: Delete files and directories with `del` and `rmdir`.

```
C:\Users\sporring\Documents>cd myFolder

C:\Users\sporring\Documents\myFolder>del hi.txt

C:\Users\sporring\Documents\myFolder>cd ..

C:\Users\sporring\Documents>rmdir myFolder

C:\Users\sporring\Documents>dir
 Volume in drive C has no label.
 Volume Serial Number is 94F0-31BD

 Directory of C:\Users\sporring\Documents

30-07-2015  19:20    <DIR>          .
30-07-2015  19:20    <DIR>          ..
               0 File(s)              0 bytes
               2 Dir(s)  94.651.142.144 bytes free

C:\Users\sporring\Documents>
```

The commands available from the console must be in its *search path*. The search path can be seen using `echo`, as shown in Listing A.8. · search path

Listing A.8: Displaying the search path.

```
C:\Users\sporring\Documents>echo %Path%
C:\Windows\system32;C:\Windows;C:\Windows\System32\Wbem;
  C:\Windows\System32\WindowsPowerShell\v1.0\;"\Program
  Files\emacs-24.5\bin\"

C:\Users\sporring\Documents>
```

The path can be changed using the Control panel in the graphical user interface. In Windows 7, choose the Control panel, choose `System and Security` → `System` → `Advanced system settings` → `Environment Variables`. In Windows 10, you can find this window by searching for “Environment” in the Control panel. In the window’s `System variables` box, double-click on `Path` and add or remove a path from the list. The search path is a list of paths separated by “;”. Beware, Windows uses the search path for many different tasks, so remove only paths that you are certain are not used for anything.

A useful feature of the console is that you can use the `tab`-key to cycle through filenames. E.g., if you write `cd` followed by a space and `tab` a couple of times, then the console will suggest to you the available directories.

A.3. MacOS X and Linux

MacOS X (OSX) and Linux are very similar, and both have the option of using ***bash*** as console. It is in the standard console on MacOS X and on many Linux distributions. A summary of the most important `bash` commands is shown in Table A.1. In MacOS X, · `bash` you find the console by opening `Finder` and navigating to `Applications` → `Utilities` -> `Terminal`. In Linux, the console can be started by typing `Ctrl + Alt + T`. Some Linux distributions have other key-combinations such as `Super + T`.

Once opened, the console is shown in a window with content, as shown in Listing A.9.

Listing A.9: The MacOS console.

```
Last login: Thu Jul 30 11:52:07 on ttys000
FN11194:~ sporring$
```

"FN11194" is the name of the computer, the character ∼ is used as an alias for the user's home directory, and "sporring" is the username for the user presently logged onto the system. Use *ls* to see which files are present, as shown in Listing A.10. · `ls`

Listing A.10: Display a directory content with `ls`.

```
FN11194:~ sporring$ ls
Applications     Documents     Library         Music          Public
Desktop          Downloads     Movies          Pictures
FN11194:~ sporring$
```

More details about the files are available by using flags to `ls` as demonstrated in Listing A.11.

Listing A.11: Display extra information about files using flags to `ls`.

```
FN11194:~ sporring$ ls -l
drwx------    6 sporring  staff   204 Jul 30 14:07 Applications
drwx------+  32 sporring  staff  1088 Jul 30 14:34 Desktop
drwx------+  76 sporring  staff  2584 Jul  2 15:53 Documents
drwx------+   4 sporring  staff   136 Jul 30 14:35 Downloads
drwx------@  63 sporring  staff  2142 Jul 30 14:07 Library
drwx------+   3 sporring  staff   102 Jun 29 21:48 Movies
drwx------+   4 sporring  staff   136 Jul  4 17:40 Music
drwx------+   3 sporring  staff   102 Jun 29 21:48 Pictures
drwxr-xr-x+   5 sporring  staff   170 Jun 29 21:48 Public
FN11194:~ sporring$
```

The flag `-l` means long, and many other flags can be found by querying the built-in manual with `man ls`. The output is divided into columns, where the left column shows a number of codes: "d" stands for directory, and the set of three of optional "rwx" denote whether respectively the owner, the associated group of users, and anyone can respectively "r" - read, "w" - write, and "x" - execute the file. In all directories but the `Public` directory, only the owner can do any of the three. For directories, "x" means permission to enter. The second column can often be ignored, but shows how many links there are to the file or directory. Then follows the username of the owner, which in this case is `sporring`. The files are also associated with a group of users, and in this case, they all are associated with the group called `staff`. Then follows the file or directory size, the date of last change, and the file or directory name. There are always two hidden directories: "`.`" and "`..`", where "`.`" is an alias for the present directory, and "`..`" for the directory above. Hidden files will be shown with the `-a` flag.

Use *cd* to change to the directory, for example to `Documents` as shown in Listing A.12. · cd

Listing A.12: Change directory with `cd`.

```
FN11194:~ sporring$ cd Documents/
FN11194:Documents sporring$
```

Note that some graphical user interfaces translate standard filenames and directories to the local language, such that navigating using the graphical user interface will reveal other files and directories, which, however, are aliases.

You can create a new directory using *mkdir*, as demonstrated in Listing A.13. · mkdir

Listing A.13: Creating a directory using `mkdir`.

```
FN11194:Documents sporring$  mkdir myFolder
FN11194:Documents sporring$ ls
myFolder
FN11194:tmp sporring$
```

A file can be created using *echo* and with *redirection*, as shown in Listing A.14. · echo · redirection

Listing A.14: Creating a file with `echo` and redirection.

```
FN11194:Documents sporring$ echo "hi" > hi.txt
FN11194:Documents sporring$ ls
hi.txt        myFolder
```

To move the file `hi.txt` into `myFolder`, use *mv*. This is demonstrated in Listing A.15. · mv

Listing A.15: Moving files with `mv`.

```
FN11194:Documents sporring$ echo mv hi.txt myFolder/
FN11194:Documents sporring$
```

To delete the file and the directory, use *rm* and *rmdir*, as shown in Listing A.16. · rm · rmdir

Listing A.16: Deleting files and directories.

```
FN11194:Documents sporring$ cd myFolder/
FN11194:myFolder sporring$ rm hi.txt
FN11194:myFolder sporring$ cd ..
FN11194:Documents sporring$ rmdir myFolder/
FN11194:Documents sporring$ ls
FN11194:Documents sporring$
```

Only commands found on the *search-path* are available in the console. The content of the search-path is seen using the `echo` command, as demonstrated in Listing A.17. · search-path

Listing A.17: The content of the search-path.

```
FN11194:Documents sporring$ echo $PATH
/Applications/Maple
  17/:/Applications/PackageMaker.app/Contents/MacOS/:
  /Applications/MATLAB_R2014b.app/bin/:/opt/local/bin:
  /opt/local/sbin:/usr/local/bin:/usr/bin:/bin:/usr/sbin:
  /sbin:/opt/X11/bin:/Library/TeX/texbin
FN11194:Documents sporring$
```

The search-path can be changed by editing the setup file for Bash. On MacOS X it is called `~/.profile`, and on Linux it is either `~/.bash_profile` or `~/.bashrc`. Here new paths can be added by adding the following line: `export PATH="<new path>:<another new path>:$PATH"`.

A useful feature of Bash is that the console can help you write commands. E.g., if you write `fs` followed by pressing the `tab`-key, and if Mono is in the search-path, then Bash will typically respond by completing the line as `fsharp`, and by further pressing the `tab`-key some times, Bash will show the list of options, typically `fshpari` and `fsharpc`. Also, most commands have an extensive manual which can be accessed using the `man` command. E.g., the manual for `rm` is retrieved by `man rm`.

B | Number Systems on the Computer

B.1. Binary Numbers

Humans like to use the *decimal number* system for representing numbers. Decimal numbers are *base* 10 meaning that a decimal number consists of a sequence of digits separated by a *decimal point*, where each *digit* can have values $d \in \{0, 1, 2, \ldots, 9\}$ and the weight of each digit is proportional to its place in the sequence of digits with respect to the decimal point, i.e., the number $357.6 = 3 \cdot 10^2 + 5 \cdot 10^1 + 7 \cdot 10^0 + 6 \cdot 10^{-1}$, or in general, for a number consisting of digits d_i with $n + 1$ and m digits to the left and right of the decimal point, the value v is calculated as:

· decimal number
· base
· decimal point
· digit

$$v = \sum_{i=-m}^{n} d_i 10^i. \tag{B.1}$$

The basic unit of information in almost all computers is the binary digit, or *bit* for short. A *binary number* consists of a sequence of binary digits separated by a decimal point, where each digit can have values $b \in \{0, 1\}$, and the base is 2. The general equation is,

· bit
· binary number

$$v = \sum_{i=-m}^{n} b_i 2^i, \tag{B.2}$$

and examples are $1011.1_2 = 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 + 1 \cdot 2^{-1} = 11.5$. Notice that we use subscript 2 to denote a binary number, while no subscript is used for decimal numbers. The left-most bit is called the *most significant bit*, and the right-most bit is called the *least significant bit*. Due to typical organisation of computer memory, 8 binary digits is called a *byte*, and 32 digits a *word*.

· most significant bit
· least significant bit
· byte
· word

Other number systems are often used, e.g., *octal numbers*, which are base 8 numbers and have digits $o \in \{0, 1, \ldots, 7\}$. Octals are useful short-hand for binary, since 3 binary digits map to the set of octal digits. Likewise, *hexadecimal numbers* are base 16 with digits $h \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, a, b, c, d, e, f\}$, such that $a_{16} = 10$, $b_{16} = 11$ and so on. Hexadecimals are convenient, since 4 binary digits map directly to the set of hexadecimal digits. Thus $367 = 101101111_2 = 557_8 = 16f_{16}$. A list of the integers 0–63 in various bases is given in Table B.1.

· octal number
· hexadecimal number

B.2. IEEE 754 Floating Point Standard

The set of real numbers, also called *reals*, includes all fractions and irrational numbers. It

Dec	Bin	Oct	Hex	Dec	Bin	Oct	Hex
0	0	0	0	32	100000	40	20
1	1	1	1	33	100001	41	21
2	10	2	2	34	100010	42	22
3	11	3	3	35	100011	43	23
4	100	4	4	36	100100	44	24
5	101	5	5	37	100101	45	25
6	110	6	6	38	100110	46	26
7	111	7	7	39	100111	47	27
8	1000	10	8	40	101000	50	28
9	1001	11	9	41	101001	51	29
10	1010	12	a	42	101010	52	2a
11	1011	13	b	43	101011	53	2b
12	1100	14	c	44	101100	54	2c
13	1101	15	d	45	101101	55	2d
14	1110	16	e	46	101110	56	2e
15	1111	17	f	47	101111	57	2f
16	10000	20	10	48	110000	60	30
17	10001	21	11	49	110001	61	31
18	10010	22	12	50	110010	62	32
19	10011	23	13	51	110011	63	33
20	10100	24	14	52	110100	64	34
21	10101	25	15	53	110101	65	35
22	10110	26	16	54	110110	66	36
23	10111	27	17	55	110111	67	37
24	11000	30	18	56	111000	70	38
25	11001	31	19	57	111001	71	39
26	11010	32	1a	58	111010	72	3a
27	11011	33	1b	59	111011	73	3b
28	11100	34	1c	60	111100	74	3c
29	11101	35	1d	61	111101	75	3d
30	11110	36	1e	62	111110	76	3e
31	11111	37	1f	63	111111	77	3f

Table B.1.: A list of the integers 0–63 in decimal, binary, octal, and hexadecimal.

is infinite in size both in the sense that there is no largest nor smallest number, and that between any 2 given numbers there are infinitely many numbers. Reals are widely used for calculation, but since any computer only has finite memory, there are infinitely many numbers which cannot be represent on a computer. Hence, any computation performed on a computer with reals must rely on approximations. *IEEE 754 double precision floating-point format* (*binary64*), known as a *double*, is a standard for representing an approximation of reals using 64 bits. These bits are divided into 3 parts: sign, exponent and fraction,

· IEEE 754 double precision floating-point format
· binary64
· double

$$s\, e_1 e_2 \dots e_{11}\, m_1 m_2 \dots m_{52},$$

where s, e_i, and m_j are binary digits. The bits are converted to a number using the equation by first calculating the exponent e and the mantissa m,

$$e = \sum_{i=1}^{11} e_i 2^{11-i}, \tag{B.3}$$

$$m = \sum_{j=1}^{52} m_j 2^{-j}. \tag{B.4}$$

I.e., the exponent is an integer, where $0 \leq e < 2^{11}$, and the mantissa is a rational, where $0 \leq m < 1$. For most combinations of e and m, the real number v is calculated as,

$$v = (-1)^s (1+m) 2^{e-1023} \tag{B.5}$$

with the exceptions that

	$m = 0$	$m \neq 0$
$e = 0$	$v = (-1)^s\, 0$ (signed zero)	$v = (-1)^s\, m2^{1-1023}$ (subnormals)
$e = 2^{11} - 1$	$v = (-1)^s \infty$	$v = (-1)^s$ NaN (not a number)

· subnormals
· NaN
· not a number

where $e = 2^{11} - 1 = 11111111111_2 = 2047$. The largest and smallest number that is not infinity is thus

$$e = 2^{11} - 2 = 2046, \tag{B.6}$$

$$m = \sum_{j=1}^{52} 2^{-j} = 1 - 2^{-52} \simeq 1, \tag{B.7}$$

$$v_{\max} = \pm \left(2 - 2^{-52}\right) 2^{1023} \simeq \pm 2^{1024} \simeq \pm 10^{308}. \tag{B.8}$$

The density of numbers varies in such a way that when $e - 1023 = 52$, then

$$v = (-1)^s \left(1 + \sum_{j=1}^{52} m_j 2^{-j} \right) 2^{52} \tag{B.9}$$

$$= \pm \left(2^{52} + \sum_{j=1}^{52} m_j 2^{-j} 2^{52} \right) \tag{B.10}$$

$$= \pm \left(2^{52} + \sum_{j=1}^{52} m_j 2^{52-j} \right) \tag{B.11}$$

$$\overset{k=52-j}{=} \pm \left(2^{52} + \sum_{k=51}^{0} m_{52-k} 2^k \right), \tag{B.12}$$

which are all integers in the range $2^{52} \leq |v| < 2^{53}$. When $e - 1023 = 53$, then the same calculation gives

$$v \overset{k=53-j}{=} \pm \left(2^{53} + \sum_{k=52}^{1} m_{53-k} 2^k \right), \tag{B.13}$$

which are every second integer in the range $2^{53} \leq |v| < 2^{54}$, and so on for larger values of e. When $e - 1023 = 51$, the same calculation gives,

$$v \overset{k=51-j}{=} \pm \left(2^{51} + \sum_{k=50}^{-1} m_{51-k} 2^k \right), \tag{B.14}$$

which is a distance between numbers of 1/2 in the range $2^{51} \leq |v| < 2^{52}$, and so on for smaller values of e. Thus we may conclude that the distance between numbers in the interval $2^n \leq |v| < 2^{n+1}$ is 2^{n-52}, for $-1022 = 1 - 1023 \leq n < 2046 - 1023 = 1023$. For subnormals, the distance between numbers is

$$v = (-1)^s \left(\sum_{j=1}^{52} m_j 2^{-j} \right) 2^{-1022} \tag{B.15}$$

$$= \pm \left(\sum_{j=1}^{52} m_j 2^{-j} 2^{-1022} \right) \tag{B.16}$$

$$= \pm \left(\sum_{j=1}^{52} m_j 2^{-j-1022} \right) \tag{B.17}$$

$$\stackrel{k=-j-1022}{=} \pm \left(\sum_{j=-1023}^{-1074} m_{-k-1022} 2^k \right), \tag{B.18}$$

which gives a distance between numbers of $2^{-1074} \simeq 10^{-323}$ in the range $0 < |v| < 2^{-1022} \simeq 10^{-308}$.

C | Commonly Used Character Sets

Letters, digits, symbols, and space are the core of how we store data, write programs, and communicate with computers and each other. These symbols are in short called characters and represent a mapping between numbers, also known as codes, and a pictorial representation of the character. E.g., the ASCII code for the letter 'A' is 65. These mappings are for short called character sets, and due to differences in natural languages and symbols used across the globe, many different character sets are in use. E.g., the English alphabet contains the letters 'a' to 'z'. These letters are common to many other European languages which in addition use even more symbols and accents. For example, Danish has further the letters 'æ', 'ø', and 'å'. Many non-European languages have completely different symbols, where the Chinese character set is probably the most extreme, and some definitions contain 106,230 different characters, albeit only 2,600 are included in the official Chinese language test at the highest level.

Presently, the most common character set used is Unicode Transformation Format (UTF), whose most popular encoding schemes are 8-bit (UTF-8) and 16-bit (UTF-16). Many other character sets exist, and many of the later build on the American Standard Code for Information Interchange (ASCII). The ISO-8859 codes were an intermediate set of character sets that are still in use, but which is greatly inferior to UTF. Here we will briefly give an overview of ASCII, ISO-8859-1 (Latin1), and UTF.

C.1. ASCII

The *American Standard Code for Information Interchange* (*ASCII*) [8], is a 7 bit code tuned for the letters of the English language, numbers, punctuation symbols, control codes and space, see Tables C.1 and C.2. The first 32 codes are reserved for non-printable control characters to control printers and similar devices or to provide meta-information. The meaning of each control character is not universally agreed upon.

· American Standard Code for Information Interchange
· ASCII

The code order is known as *ASCIIbetical order*, and it is sometimes used to perform arithmetic on codes, e.g., an uppercase letter with code c may be converted to lower case by adding 32 to its code. The ASCIIbetical order also has a consequence for sorting, i.e., when sorting characters according to their ASCII code, 'A' comes before 'a', which comes before the symbol '{'.

· ASCIIbetical order

x0+0x	00	10	20	30	40	50	60	70
00	NUL	DLE	SP	0	@	P	‘	p
01	SOH	DC1	!	1	A	Q	a	q
02	STX	DC2	"	2	B	R	b	r
03	ETX	DC3	#	3	C	S	c	s
04	EOT	DC4	$	4	D	T	d	t
05	ENQ	NAK	%	5	E	U	e	u
06	ACK	SYN	&	6	F	V	f	v
07	BEL	ETB	’	7	G	W	g	w
08	BS	CAN	(	8	H	X	h	x
09	HT	EM	)	9	I	Y	i	y
0A	LF	SUB	*	:	J	Z	j	z
0B	VT	ESC	+	;	K	[	k	{
0C	FF	FS	,	<	L	\	l	|
0D	CR	GS	–	=	M	]	m	}
0E	SO	RS	.	>	N	ˆ	n	˜
0F	SI	US	/	?	O	_	o	DEL

Table C.1.: ASCII

C.2. ISO/IEC 8859

The ISO/IEC 8859 report `http://www.iso.org/iso/catalogue_detail?csnumber=28245` defines 10 sets of codes specifying up to 191 codes and graphics characters using 8 bits. Set 1, also known as ISO/IEC 8859-1, Latin alphabet No. 1, or *Latin1*, covers many European languages and is designed to be compatible with ASCII, such that code for the printable characters in ASCII is the same in ISO 8859-1. Table C.3 shows the characters above 7e. Codes 00-1f and 7f-9f are undefined in ISO 8859-1.

· Latin1

C.3. Unicode

Unicode is a character standard defined by the Unicode Consortium, `http://unicode.org`, as the *Unicode Standard*. Unicode allows for 1,114,112 different codes. Each code is called a *code point* which represents an abstract character. However, not all abstract characters require a unit of several code points to be specified. Code points are divided into 17 planes, each with $2^{16} = 65,536$ code points. Planes are further subdivided into named *blocks*. The first plane is called the *Basic Multilingual plane* and its block of the first 128 code points is called the *Basic Latin block* and is identical to ASCII, see Table C.1, and code points 128-255 are called the *Latin-1 Supplement block*, and are identical to the upper range of ISO 8859-1, see Table C.3. Each code-point has a number of attributes such as the *Unicode general category*. Presently more than 128,000 code points are defined as covering 135 modern and historical writing systems, and obtained at `http://www.unicode.org/Public/UNIDATA/UnicodeData.txt`, which includes the code point, name, and general category.

· Unicode Standard
· code point
· blocks
· Basic Multilingual plane
· Basic Latin block
· Latin-1 Supplement block
· Unicode general category

A Unicode code point is an abstraction from the encoding and the graphical representation of a character. A code point is written as “U+” followed by its hexadecimal number, and for the Basic Multilingual plane, 4 digits are used, e.g., the code point with the unique name LATIN CAPITAL LETTER A has the Unicode code point “U+0041”, and is in this

Code	Description
NUL	Null
SOH	Start of heading
STX	Start of text
ETX	End of text
EOT	End of transmission
ENQ	Enquiry
ACK	Acknowledge
BEL	Bell
BS	Backspace
HT	Horizontal tabulation
LF	Line feed
VT	Vertical tabulation
FF	Form feed
CR	Carriage return
SO	Shift out
SI	Shift in
DLE	Data link escape
DC1	Device control one
DC2	Device control two
DC3	Device control three
DC4	Device control four
NAK	Negative acknowledge
SYN	Synchronous idle
ETB	End of transmission block
CAN	Cancel
EM	End of medium
SUB	Substitute
ESC	Escape
FS	File separator
GS	Group separator
RS	Record separator
US	Unit separator
SP	Space
DEL	Delete

Table C.2.: ASCII symbols.

text visualized as 'A'. More digits are used for code points of the remaining planes.

The general category is used to specify valid characters that do not necessarily have a visual representation but possibly transform text. Some categories and their letters in the first 256 code points are shown in Table C.5.

To store and retrieve code points, they must be encoded and decoded. A common encoding is *UTF-8*, which encodes code points as 1 to 4 bytes, and which is backward-compatible with ASCII and ISO 8859-1. Hence, in all 3 coding systems, the character with code 65 represents the character 'A'. Another popular encoding scheme is *UTF-16*, which encodes characters as 2 or 4 bytes, but which is not backward-compatible with ASCII or ISO 8859-1. UTF-16 is used internally in many compilers, interpreters, and operating systems.

· UTF-8

· UTF-16

x0+0x	80	90	A0	B0	C0	D0	E0	F0
00			NBSP	°	À	Ð	à	ð
01			¡	±	Á	Ñ	á	ñ
02			¢	²	Â	Ò	â	ò
03			£	³	Ã	Ó	ã	ó
04			¤	´	Ä	Ô	ä	ô
05			¥	µ	Å	Õ	å	õ
06			¦	¶	Æ	Ö	æ	ö
07			§	·	Ç	×	ç	÷
08			¨	¸	È	Ø	è	ø
09			©	¹	É	Ù	é	ù
0a			ª	º	Ê	Ú	ê	ú
0b			«	»	Ë	Û	ë	û
0c			¬	¼	Ì	Ü	ì	ü
0d			SHY	½	Í	Ý	í	ý
0e			®	¾	Î	Þ	î	þ
0f			¯	¿	Ï	ß	ï	ÿ

Table C.3.: ISO-8859-1 (latin1) non-ASCII part. Note that the codes 7f – 9f are undefined.

Code	Description
NBSP	Non-breakable space
SHY	Soft hypen

Table C.4.: ISO-8859-1 special symbols.

General category	Code points	Name
Lu	U+0041–U+005A, U+00C0–U+00D6, U+00D8–U+00DE	Upper case letter
Ll	U+0061–U+007A, U+00B5, U+00DF–U+00F6, U+00F8–U+00FF	Lower case letter
Lt	None	Digraphic letter, with first part uppercase
Lm	None	Modifier letter
Lo	U+00AA, U+00BA	Gender ordinal indicator
Nl	None	Letterlike numeric character
Pc	U+005F	Low line
Mn	None	Nonspacing combining mark
Mc	None	Spacing combining mark
Cf	U+00AD	Soft Hyphen

Table C.5.: Some general categories for the first 256 code points.

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