Learning to program with F#

Jon Sporring

Department of Computer Science, University of Copenhagen

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Contents

1	Pre	face	6
2	Intr	roduction	7
	2.1	Learning how to solve problems by programming	7
	2.2	How to solve problems	8
	2.3	Approaches to programming	8
	2.4	Why use F#	9
	2.5	How to read this book	10
	Б		
3	Exe	${ m ecuting}\; { m F} \# \; { m code}$	11
	3.1	Source code	11
	3.2	Executing programs	12
4	Qui	ck-start guide	14
5	Usi	${ m rg}~{ m F}\#~{ m as}~{ m a}~{ m calculator}$	19
	5.1	Literals and basic types	19
	5.2	Operators on basic types	24
	5.3	Boolean arithmetic	27
	5.4	Integer arithmetic	28
	5.5	Floating point arithmetic	30
	5.6	Char and string arithmetic	31
	5.7	Programming intermezzo: Hand conversion between decimal and binary numbers	33
6	Val	ues and functions	35
		Value bindings	38

	6.2	Function bindings	43
	6.3	Operators	49
	6.4	Do bindings	50
	6.5	The Printf function	51
	6.6	Variables	54
	6.7	Reference cells	56
	6.8	Tuples	60
7	In-c	ode documentation	64
8	Con	trolling program flow	7 0
	8.1	While and for loops	70
	8.2	Conditional expressions	75
	8.3	Programming intermezzo: Automatic conversion of decimal to binary numbers \dots	77
9	Orga	anising code in libraries and application programs	80
	9.1	Modules	80
	9.2	Namespaces	83
	9.3	Compiled libraries	85
10	Test	ing programs	89
	10.1	White-box testing	91
	10.2	Black-box testing	94
	10.3	Debugging by tracing	97
11	Ord	ered series of data	.06
	11.1	Strings	106
	11.2	Lists	107
	11.3	Arrays	114
	11.4	Multidimensional Arrays	114
	11.5	Comparision	118
12	The	imperative programming paradigm 1	20
	12.1	Imperative design	121

13 Recursion	122
13.1 Recursive functions	122
13.2 The call stack and tail recursion	123
13.3 Mutual recursive functions	128
13.4 To Do	129
14 Programming with types	130
14.1 Type abbreviations	
14.2 Enumerations	
14.2 Enumerations	
14.4 Records	
14.5 Structures	
14.6 Variable types	138
15 Patterns	141
15.1 Wildcard pattern	143
15.2 Constant and literal patterns	144
15.3 Variable patterns	145
15.4 Guards	145
15.5 List patterns	146
15.6 Array, record, and discriminated union patterns	147
15.7 Disjunctive and conjunctive patterns	148
15.8 Active Pattern	149
15.9 Static and dynamic type pattern	152
16 Higher order functions	154
16.1 Function composition	
16.2 Currying	156
17 The functional programming paradigm	158
17.1 Functional design	159
18 Handling Errors and Exceptions	161

10 1 E	Presentions	161
18.2 C	Option types	167
Input	and Output	17 0
19.1 In	nteracting with the console	170
19.2 S	toring and retrieving data from a file	172
19.3 V	Vorking with files and directories	175
19.4 R	Reading from the internet	176
19.5 P	Programming intermezzo: Ask user for existing file	177
Ob :		100
-		180
20.1 C	Constructors and members	180
20.2 A	Accessors	182
20.3 C	Objects are reference types	185
20.4 S	Static classes	186
20.5 N	Mutual recursive classes	186
20.6 F	Function and operator overloading	187
20.7 A	Additional constructors	189
20.8 In	nterfacing with printf family	190
20.9 P	Programming intermezzo	192
20.10In	nheritance	195
20.11A	Abstract class	197
20.12In	nterfaces	199
20.13P	Programming intermezzo: Chess	200
20.14T	Things to remember	208
The o	object-oriented programming paradigm	209
21.1 Io	dentification of objects, behaviors, and interactions by nouns-and-verbs	210
21.2 C	Class diagrams in the Unified Modelling Language	210
21.3 P	Programming intermezzo: designing a racing game	213
21.4 to	odo	216
	18.2 C Input 19.1 I 19.2 S 19.3 V 19.4 F 19.5 F Object 20.1 C 20.2 A 20.3 C 20.4 S 20.5 M 20.6 F 20.7 A 20.8 I 20.9 F 20.10I 20.11A 20.12I 20.13F 20.14T The c 21.1 II 21.2 C 21.3 F	19.1 Interacting with the console 19.2 Storing and retrieving data from a file 19.3 Working with files and directories. 19.4 Reading from the internet 19.5 Programming intermezzo: Ask user for existing file Object-oriented programming 20.1 Constructors and members 20.2 Accessors 20.3 Objects are reference types 20.4 Static classes 20.5 Mutual recursive classes 20.6 Function and operator overloading 20.7 Additional constructors 20.8 Interfacing with printf family 20.9 Programming intermezzo 20.10Inheritance 20.11Abstract class 20.12Interfaces 20.13Programming intermezzo: Chess 20.14Things to remember

	22.1 Drawing primitives in Windows	217
	22.2 Programming intermezzo: Hilbert Curve	227
	22.3 Events, Controls, and Panels	231
23	3 The Event-driven programming paradigm	259
	23.1 Event-driven design	259
\mathbf{A}	The console in Windows, MacOS X, and Linux	260
	A.1 The basics	260
	A.2 Windows	260
	A.3 MacOS X and Linux	264
В	Number systems on the computer	268
	B.1 Binary numbers	268
	B.2 IEEE 754 floating point standard	268
\mathbf{C}	Commonly used character sets	271
	C.1 ASCII	27
	C.2 ISO/IEC 8859	271
	C.3 Unicode	272
D	Common Language Infrastructure	275
\mathbf{E}	Language Details	277
	E.1 Arithmetic operators on basic types	277
	E.2 Basic arithmetic functions	280
	E.3 Precedence and associativity	281
	E.4 Lightweight Syntax	282
\mathbf{F}	To Dos	283
Bi	ibliography	28 4
In	ndex	285

1 | Preface

This book has been written as an introduction to programming for novice programmers. It is used on the first programming course at the University of Copenhagen's bachelor in computer science program. It has been typeset in LaTeX, and all programs have been developed and tested in Mono version 5.2.0.

Jon Sporring Associate Professor, Ph.d. Department of Computer Science, University of Copenhagen September 3, 2017

2 | Introduction

Programming is a creative process in which exciting problems may be solved and new tools and applications may be created. With programming skills, you can create high-level applications to run on a mobile device that interacts with other users, databases, and artificial intelligences; you may create programs that run on super computers for simulating weather systems on alien planets or social phenomena in the internet economy; and you may create programs that run on small custom-made hardware for controlling your home appliances.

2.1 Learning how to solve problems by programming

In order to learn how to program, there are a couple of steps that are useful to follow:

- 1. Choose a programming language: A programming language such as F# is a vocabulary and a set of gramatical rules for instructing a computer to perform a certain task. It is possible to program without a concrete language, but your ideas and thoughts must be expressed in some fairly rigorous way. Actually, theoretical computer science typically does not rely on computers nor programming languages, but uses mathematics to prove properties of algorithms. However, most computer scientists program, and with a real language, you have the added benefit of checking your algorithm, and hence your thoughts, rigorously on a real computer. This book teaches a subset of F#. The purpose is not to be a reference guide to this language, but to use it as a vessel to teach you, the reader, how to convert your ideas into programs.
- 2. Learn the language: A computer language is a structure for thought, and it influences which thoughts you choose to implement as a program, and how you choose to do it. Any conversion requires you to acquire a sufficient level of fluency, for you to be able to make programs. You do not need to be a master in F# nor to know every corner of the language, and you will expand your knowledge as you expose yourself to solving problems in the language, but you must invest an initial amount of time and energy in order to learn the basics of the language. This book aims at getting you started quickly, which is why we intentionally teach just a small subset of F#. On the net and through other works, you will be able to learn much more.
- 3. Practice: If you want to be a good programmer, then there is only one way: practice, practice, practice! It has been estimated that to master anything, then you have to have spent at least 10000 hours of practice, so get started logging hours! It of course matters, what you practice. This book teaches 3 different programming themes. The point is that programming is thinking, and the scaffold that you use, shapes your thoughts. It is therefore important to recognise this scaffold, and to have the ability to choose that which suits your ideas and your goals best. And the best way to expand your abilities is to sharpen your present abilities, push yourself into new territory, and trying something new. Do not be afraid to make errors or be frustrated at first. These are the experiences that make you grow.

4. Solve real problems: I have found that using my programming skills in real situations with customers demanding solutions, that work for them, has allowed me to put into perspective the programming tools and techniques that I use. Often customers want solutions that work, are secure, cheap, and delivered fast, which has pulled me as a programmer in the direction of "if it works, then sell it". On the other hand, in the longer perspective customers also want bug fixes, upgrades, and new features, which require carefully designed code, well written test-suites, and good documentation. And as always, the right solution is somewhere in between. Regardless, real problems create real programmers.

2.2 How to solve problems

Programming is the act of solving a problem by writing a program to be executed on a computer. A general method for solving problems was given by George Pólya [9] and adapted to programming is:

- **Understand the problem:** To solve any problem it is crucial that the problem formulation is understood: What is to be solved? Do you understand everything in the description of the problem? Is all information for finding the solution available or is something missing?
- **Design a plan:** Good designs mean that programs are faster to program, easier to find errors in and update in the future. So, before you start typing a program consider things like: What are the requirements and constraints for the program? Which components should the program have? How are these components supposed to work together? Designing often involves drawing a diagram of the program, and writing program sketches on paper.
- Implement the plan: Implementation is the act of transforming a program design into code. A crucial part of any implementation is choosing which programming language to use. Also, the solution to many problems will have a number of implementations which vary in how much code they require, to which degree they rely on external libraries, which programming style they are best suited for, what machine resources they require, and what their running times are. With a good design, the coding is usually easy, since the design will have uncovered the major issues and found solutions for these, but sometimes implementation reveals new problems, which requires rethinking the design. Most implementations also include writing documentation of the code.
- Reflect on the result: A crucial part in any programming task is ensuring that the program solves the problem sufficiently. E.g., what are the program's errors, is the documentation of the code sufficient and relevant for its intended use? Is the code easily maintainable and extendable by other programmers? Are there any general lessons to be learned from or general code developed by the programming experience, which may be used for future programming sessions?

Programming is a very complicated process, and Pólya's list is a useful guide, but not a fail-safe approach. Always approach problem solving with an open mind.

2.3 Approaches to programming

This book focuses on 3 fundamentally different approaches to programming:

Imperative programming, emphasises how a program shall accomplish a solution and less on what the solution is. A cooking recipe is an example of the spirit of imperative programming, where the recipe emphasises what should be done in each step rather than describing the result. E.g., for making bread, you first mix yeast and water, then add flour, etc. In imperative programming

· imperative programming

 \cdot statement

 \cdot state

what should be done are called *statements* and they influence the computer's *states*, like adding flour changes the state of our dough. Almost all computer hardware is designed to execute low-level programs written in imperative style. Imperative programming builds on the Turing machine [10]. The first major language was FORTRAN [6] which emphasized an imperative style of programming.

- **Declarative programming,** which emphasises what a program shall accomplish but not how. We will consider Functional programming as an example of declarative programming. A functional programming language evaluates functions and avoids state changes. The program consists of expressions instead of statements. As an example the function $f(x) = x^2$ takes a number x and evaluates the expression x^2 , and returns the result. Functional programming has its roots in lambda calculus [1], and the first language emphasizing functional programming was Lisp [7].
- Structured programming, which emphasises organisation of code in units with well-defined interfaces and isolation of internal states and code from other parts of the program. We will focus on Object-oriented programming as the example of structured programming. Object-oriented programming is a type of programming, where the states and programs are structured into objects. A typical object-oriented design takes a problem formulation and identifies key nouns as potential objects and verbs as potential actions to be taken on objects. The first object-oriented programming language was Simula 67 developed by Dahl and Nygaard at the Norwegian Computing Center in Oslo [2].
- **Event-driven programming,** is often used when dynamically interacting with the real world. E.g., when programming graphical user interfaces, programs will often need to react to a user clicking on the mouse or when text arrives from a web-server to be displayed on the screen. Event-driven programs are often programmed using *call-back functions*, which are small programs that are ready to run, when events occur.

Most programs do not follow a single programming paradigm as, e.g., one of the above, but are a mix. Nevertheless, this book will treat each paradigm separately to emphasize their advantages and disadvantages.

2.4 Why use F#

This book uses F# also known as Fsharp, which is a functional first programming language that also supports imperative and object-oriented programming. It was originally developed for Microsoft's .Net platform, but is available as open source for many operating systems through Mono. As an introduction to programming, F# is a young programming language still under development, with syntax that at times is a bit complex, but it offers a number of advantages:

- Interactive and compile mode: F# has an interactive and a compile mode of operation: In interactive mode you can write code that is executed immediately in a manner similarly to working with a calculator, while in compile mode, you combine many lines of code possibly in many files into a single application, which is easier to distribute to non F# experts and is faster to execute.
- **Indentation for scope:** F# uses indentation to indicate scope: Some lines of code belong together, e.g., should be executed in a certain order and may share data, and indentation helps in specifying this relationship.
- **Strongly typed:** F# is strongly typed, reducing the number of runtime errors. That is, F# is picky, and will not allow the programmer to mix up types such as numbers and text. This is a great advantage for large programs.
- **Multi-platform:** F# is available on Linux, Mac OS X, Android, iOS, Windows, GPUs, and browsers via the Mono platform.

- · declarative programming
- · functional programming
- \cdot function
- \cdot expression
- $\begin{array}{c} \cdot \, \text{structured} \\ \text{programming} \end{array}$
- \cdot Object-oriented programming
- \cdot object
- \cdot event-driven programming
- \cdot call-back functions

Free to use and open source: F# is supported by the Fsharp foundation (http://fsharp.org) and sponsored by Microsoft.

Assemblies: F# is designed to be able to easily communicate with other .Net and Mono programs through the language-independent, platform-independent bytecode called Common Intermediate Language (CIL) organised as assemblies. Thus, if you find that certain parts of a program are easy to express in F# and others in C++, then you will be able to combine these parts later into a single program.

Modern computing: F# supports all aspects of modern computing including Graphical User Interfaces, Web programming, Information rich programming, Parallel algorithms, . . .

Integrated development environments (IDE): F# is supported by major IDEs such as Visual Studio (https://www.visualstudio.com) and Xamarin Studio (https://www.xamarin.com).

2.5 How to read this book

Learning to program requires mastering a programming language, however most programming languages contains details that are rarely used or used in contexts far from a specific programming topic. Hence, this book only includes a subset of F#, but focuses on language structures necessary to understand 4 common programming paradigms: Imperative programming mainly covered in Chapters 6 to 11, functional programming mainly covered in Chapters 13 to 16, object oriented programming in Chapters 20 and 21, and event driven programming in Chapter 22. A number of general topics are given in the appendix for reference. The disadvantage of this approach is that no single part contains a reference guide to F#, and F# topics are revisited and expanded across the book. For further reading please consult http://fsharp.org.

$3 \mid \text{ Executing F} \# \text{ code}$

3.1 Source code

F# is a functional first programming language, meaning that it has strong support for functional programming, but F# also supports imperative and object-oriented programming. It also has strong support for parallel programming and information rich programs. It was originally developed for Microsoft's .Net platform, but is available as open source for many operating systems through Mono. In this text, we consider F# 4.1 and its Mono implementation, which is different from .Net mainly in terms of the number of libraries accessible. The complete language specification is described in http://fsharp.org/specs/language-spec/.

F# has 2 modes of execution, *interactive* and *compiled*. Interactive mode is well suited for small experiments or back-of-an-envelope calculations, but not for programming in general. Both modes can be accessed via the *console*, see Appendix A for more information on the console. The interactive system is started by calling fsharpi at the command prompt in the console, while compilation is performed with fsharpc, and execution of the compiled code is performed using the mono command.

· interactive mode · compile mode

· console

ram · source code

F# programs comes in many forms, which are identified by suffixes. The *source code* is an F# program written in human readable form using an editor. F# recognises the following types of source code files:

.fs An implementation file, e.g., myModule.fs

 \cdot implementation file

.fsi A signature file, e.g., myModule.fsi

 \cdot signature file

.fsx A script file, e.g., gettingStartedStump.fsx

· script file

.fsscript Same as .fsx, e.g., gettingStartedStump.fsscript

Compiled code is source code translated into a machine readable language, which can be executed by a machine. Compiled F# code is either:

.dll A library file, e.g., myModule.dll

· library file

.exe A stand-alone executable file, e.g., gettingStartedStump.exe

· executable file

The implementation, signature, and script files are all typically compiled to produce an executable file, in which case they are called scripts, but can also be entered into the interactive system, in which case these are called script-fragments. The implementation and signature files are special kinds of script files used for building libraries. Libraries in F# are called modules, and they are collections of smaller programs used by other programs, which will be discussed in detail in Chapter 9.

 \cdot scripts

 $\cdot \ script\text{-}fragment$

3.2 Executing programs

Programs may either be executed by the interpreter or by compiling and executing the compiled code. In Mono the interpreter is called fsharpi and can be used in two ways: interactively, where a user enters one or more script-fragments separated by the ";;" characters, or to execute a script file treated as a single script-fragment. ¹

To illustrate the difference between interactive and compile mode, consider the program in Listing 3.1.

```
Listing 3.1 gettingStartedStump.fsx:
A simple demonstration script.

1 let a = 3.0
2 do printfn "%g" a
```

The code declares a value a to be the decimal value 3.0 and finally prints it to the console. The do printfn is a statement for displaying the content of a value to the screen, and "%g" is a special notation to control how the value is printed. In this case, it is printed as a decimal number. This and more will be discussed at length in the following chapters. For now we will concentrate on how to interact with the F# interpreter and compiler.

An interactive session is obtained by starting the console, typing the fsharpi command, typing the lines of the program, and ending the script-fragment with ";;". The dialogue in Listing 3.2 demonstrates the workflow. What the user types has been highlighted by a box,

We see that after typing fsharpi, then the program starts by stating details about itself followed by > indicating that it is ready to receive commands. The user then types let a = 3.0 and presses enter, to which the interpreter responds with -. This indicates that the line has been received, that the script-fragment is not yet completed, and that it is ready to receive more input. When the user types do printfn "%g" a;; followed by enter, then by ";;" the interpreter knows that the script-fragment is completed, it interprets the script-fragment, responds with 3 and extra type information about the entered code, and with > to indicate, that it is ready for more script-fragments. The interpreter is

¹Jon: Too early to introduce lexeme: "F# uses many characters which at times are given special meanings, e.g., the characters ";;" is compound character denoting end of a script-fragment. Such possibly compound characters are called lexemes."

stopped, when the user types #quit;;. It is also possible to stop the interpreter by typing ctrl-d.

Instead of running fsharpi interactively, we can write the script-fragment from Listing 3.1 into a file, here called gettingStartedStump.fsx. This file can be interpreted directly by fsharpi as shown in Listing 3.3.

```
Listing 3.3: Using the interpreter to execute a script.

1  $ (fsharpi gettingStartedStump.fsx)
2  3
```

Notice that in the file, ";;" is optional. We see that the interpreter executes the code and prints the result on screen without the extra type information.

Finally, the file containing Listing 3.1 may be compiled into an executable file with the program fsharpc, and run using the program mono from the console. This is demonstrated in Listing 3.4.

The compiler takes gettingStartedStump.fsx and produces gettingStarted.exe, which can be run using mono.

Both the interpreter and the compiler translates the source code into a format, which can be executed by the computer. While the compiler performs this translation once and stores the result in the executable file, the interpreter translates the code every time the code is executed. Thus, to run the program again with the interpreter, it must be retranslated as "\$fsharpi gettingStartedStump.fsx". In contrast, compiled code does not need to be recompiled to be run again, only re-executed using "\$ mono gettingStartedStump.exe". On a MacBook Pro, with a 2.9 Ghz Intel Core i5, the time the various stages take for this script are:

Command	Time
fsharpi gettingStartedStump.fsx	1.88s
fsharpc gettingStartedStump.fsx	1.90s
mono gettingStartedStump.exe	0.05s

I.e., executing the script with fsharpi is slightly faster than by first compiling it with fsharpc and then executing the result with mono, 1.88s < 0.05s + 1.90s, if the script were to be executed only once, but every future execution of the script using the compiled version requires only the use of mono, which is much faster than fsharpi, $1.88s \gg 0.05s$.

The interactive session results in extra output on the *type inference* performed, which is very useful for *debugging* and development of code-fragments, but both executing programs with the interpreted directly from a file and compiling and executing the program is much preferred for programming complete programs, since the starting state is well defined, and since this better supports *unit-testing*, which is a method for debugging programs. Thus, **prefer compiling over interpretation.**

- · type inference
- \cdot debugging
- · unit-testing Advice

4 | Quick-start guide

Programming is the art of solving problems by writing a program to be executed by a computer. For example, to solve the following problem,

Problem 4.1 What is the sum of 357 and 864?

we have written the program in F# shown in Listing 4.1.

```
Listing 4.1 quickStartSum.fsx:
A script to add 2 numbers and print the result to the console.

1 let a = 357
2 let b = 864
3 let c = a + b
4 do printfn "%A" c

1 $ fsharpc --nologo quickStartSum.fsx && mono quickStartSum.exe
2 1221
```

In box the above, we see our program was saved as a script in a file called quickStartSum.fsx, and in the console we executed the program by typing the command fsharpc --nologo quickStartSum.fsx && mono quickStartSum.exe. The result is then printed in the console to be 1221. Here, as in the rest of this book, we have used the optional flag --nologo, which informs fsharpc not to print information about its version etc., thus making the output shorter. The && notation tells the console to first run the command on the left, and if that did not report any errors, then run that on the right. This could as well have been performed as two separate commands to the console, and throughout this book, we will use the above shorthand, when convenient.

To solve the problem, we made program consisting of several lines, where each line was a expressions. The first expression let a = 357 in line 1 used the let keyword to bind the value 357 to the name a. This is called a let-binding, and a let-binding makes the name synonymous with the value. Another point to be noted is that F# identifies 357 as an integer number, which is F#'s preferred number type, since computations on integers are very efficient, and since integers are very easy to communicate to other programs. In line 2 we bound the value 864 to the name b, and to the name c, we bound the result of evaluating the sum a + b in line 3. Line 4 is a do-binding, as noted by the keyword do. Do-bindings are also sometimes called statements, and the do keyword is optional in F#. Here the value of c was printed to the console followed by a newline (LF possibly preceded by CR, see Appendix C.1) with the printfn function. A function in F# is an entity that takes zero or more arguments and returns a value. The function printfn is very special, since it can take any number of arguments. It need not return any value, but F# insists that every function must return a value, wherefore printfn

 \cdot expression

16+

· keyword

· binding

· let-binding

· integer number

· do-binding

· do

 $\cdot \, statements$

·printfn

 \cdot function

returns a special type of value called unit and written as "()". The do tells F# to ignore this value. · unit Here printfn has been used with 2 arguments: "%A" and c. Notice that in contrast to many other languages, F# does not use parentheses to frame the list of arguments, nor does it use commas to separate them. In general, the printfn function always has 1 or more arguments, and the first is a format string. A string is a sequence of characters starting and ending with double quotation marks. E.g., let s = "this is a string of characters" binds the string "this is..." to the name s. For the printfn function, the format string may be any string, but if it contains format character sequences, such as %A, then the values following the format string are substituted. The format string must match the value type, that is, here c is of type integer, whereas the format string %A matches many types.

Types are a central concept in F#. In the script 4.1 we bound values of integer type to names. There are several different integer types in F#, here we used the one called int. The values were not declared to have these types, instead the types were inferred by F#. Typing these bindings line by line in an interactive session, then we see the inferred types as shown in Listing 4.2.

· format string

 \cdot string

 \cdot type

· type declaration

 \cdot type inference

```
Listing 4.2: Inferred types are given as part of the response from the interpreter.
> let a = 357;;
val a : int = 357
> let b = 864;;
val b : int = 864
  let c = a + b;;
val c : int = 1221
> do printfn "%A" c;;
1221
val it : unit = ()
```

The interactive session displays the type using the val keyword followed by the name used in the val binding, its type, and its value. Since the value is also responded, then the last printfn statement is superfluous. However, it is ill advised to design programs to be run in an interactive session, since the scripts needs to be manually copied every time it is to be run, and since the starting state may be unclear. Notice that printfn is automatically bound to the name it of it type unit and value "()". F# insists on binding all statements to values, and in lack of an explicit () name, then it will use it. Rumor has it that it is an abbreviation for "irrelevant".

Were we to solve a slightly different problem,

Problem 4.2

What is the sum of 357.6 and 863.4?

where the only difference is that the numbers now use a decimal point. These are called floating point · decimal point numbers, and the internal representation is quite difference to integer numbers used previously, and the algorithms used to perform arithmetic are also quite different from integers. Now the program would look like Listing 4.3.

- · floating point numbers

Listing 4.3 quickStartSumFloat.fsx: Floating point types and arithmetic. let a = 357.6let b = 863.4let c = a + bdo printfn "%A" c \$ fsharpc --nologo quickStartSumFloat.fsx && mono quickStartSumFloat.exe 1221.0

On the surface, this could appear as an almost negligible change, but the set of integers and the set of real numbers (floats) require quite different representations, in order to be effective on a computer, and as a consequence, the implementation of their operations such as addition are very different. Thus, although the response is an integer, it has type float, which is indicated by 1221.0, and which is not the same as 1221. F# is very picky about types, and generally does not allow types to be mixed, as demonstrated in the interactive session in Listing 4.4.

```
Listing 4.4: Mixing types is often not allowed.
> let a = 357;;
val a : int = 357
> let b = 863.4;;
val b : float = 863.4
> let c = a + b;;
  let c = a + b;;
stdin(4,13): error FS0001: The type 'float' does not match the type 'int'
```

we see that binding a name to a number without a decimal point is inferred to be integer, while when binding to a number with a decimal point, then the type is inferred to be a float, and when trying to add values of integer and floating point, we get an error. The error message contains much information. First it states that the error is in stdin(4,13), which means that the error was found on standard-input at line 4 and column 13. Since the program was executed using fsharpi quickStartSumFloat.fsx, then here standard input means the file quickStartSumFloat.fsx shown in Listing 4.3. The corresponding line and column is also shown in Listing 4.4. After the file, line, and column number, F# informs us of the error number, and a description of the error. Error numbers are an underdeveloped feature in Mono, and should be ignored. However, the verbal description often contains useful information for debugging. In the example we are informed that there is a type mismatch in the expression, i.e., since a is an integer, then F# had expected b to be one too. Debugging is the process of solving errors in programs, and here we can solve the error by either making a into a float or b into an int. The right solution depends on the application.

F# is a functional first programming language, and one implication of this is that names have a lexical scope. A scope are the lines in a program, where a binding is valid, and lexical scope means that to lexical scope find the value of a name F# looks for the value in the above lines. Further, at the outer most level, rebinding is not allowed. If attempted, then F# will return an error as shown in Listing 4.5.

· error message

· debugging

Listing 4.5 quickStartRebindError.fsx: A name cannot be rebound. let a = 357let a = 864\$ fsharpc --nologo -a quickStartRebindError.fsx quickStartRebindError.fsx(2,5): error FS0037: Duplicate definition of value 'a'

However, if the same is performed in an interactive session, then rebinding does not cause an error as shown in Listing 4.6.

```
Listing 4.6: Names may be reused when separated by the lexeme ";;".
> let a = 357;;
val a : int = 357
> let a = 864;;
val a : int = 864
```

The difference is that the ";;" lexeme is used to specifies the end of a script-fragment. A lexeme is a :;; letter or a word, which the F# considers as an atomic unit. Script-fragments may be defined both in ·lexeme scripts and in interactive mode, and rebinding is not allowed at the outermost level in script-fragments. · script-fragment Even with the ";;" lexeme, rebinding is not allowed in compile-mode. In general, avoid rebinding Advice of names.

In F#, functions are also values, and we may define a function sum as part of the solution to the above • function program as shown in Listing 4.7.

```
Listing 4.7 quickStartSumFct.fsx:
A script to add 2 numbers using a user defined function.
let sum x y = x + y
let c = sum 357 864
do printfn "%A" c
$ fsharpc --nologo quickStartSumFct.fsx && mono quickStartSumFct.exe
```

Functions are useful to encapsulate code, such that we can focus on the transformation of data by a encapsulate function while ignore the details on how this is done. Functions are also useful for code reuse, i.e., instead of repeating a piece of code in several places, such code can be encapsulated in a function and replaced with function calls. This makes debugging and maintenance considerably simpler. Entering the function into an interactive session will illustrate the inferred type, the function sum has: val sum: x:int -> y:int -> int. The "->" is the mapping operator in the sense that functions are mappings between sets. The type of the function sum, should be read as val sum: x:int -> (y:int -> int), that is, sum takes an integer and returns a function, which takes an integer and returns an integer. This is an example of a higher-order function.

Type inference in F# may cause problems, since the type of a function is inferred in the context, in which it is defined. E.g., in an interactive session, defining the sum in one scope on a single line will default the types to integers, F#'s favorite type. Thus, if the next script-fragment uses the function with floats, then we will get an error message as shown in Listing 4.8.

```
Listing 4.8: Types are inferred in blocks, and F\# tends to prefer integers.
val sum : x:int -> y:int -> int
> let c = sum 357.6 863.4;;
  let c = sum 357.6 863.4;;
stdin(3,13): error FS0001: This expression was expected to have type
but here has type
     'float'
```

A remedy is to define the function in the same script-fragment as it is used such as shown in Listing 4.9.

```
Listing 4.9: Type inference is per script-fragment.
> let sum x y = x + y
- let c = sum 357.6 863.4;;
val sum : x:float -> y:float -> float
val c : float = 1221.0
```

Alternatively, the types may be explicitly stated as shown in Listing 4.10.

```
Listing 4.10: Function argument and return types may be stated explicitly.
> let sum (x : float) (y : float) : float = x + y;;
val sum : x:float -> y:float -> float
> let c = sum 357.6 863.4;;
val c : float = 1221.0
```

The function sum has two arguments and a return type, and in Listing 4.10 we have specified all three. This is done using the ":" lexeme, and to resolve confusion, we must use parentheses around the arguments such as (y: float), otherwise F# would not be able to understand, whether the type annotation was for the argument or the return value. Often it is sufficient to specify some of the types, since type inference will enforce the remaining types. E.g., in this example, the "+" operator is defined for identical types, so specifying the return value of sum to be a float, implies that the result of the "+" operator is a float, and therefore its arguments must be floats, and finally then the arguments for sum must be floats. However, in this book we advocate the following advice: specify types unless Advice explicitly working with generic functions.

In this chapter, we have scratched the surface of learning how to program by concentrating on a number of key programming concepts and how they are expressed in the F# language. In the following chapters, we will expand the description of F# with features used in all programming approaches.

5 | Using F# as a calculator

In this chapter, we will exclusively use the interactive mode to illustrate basic types and operations in F#.

5.1 Literals and basic types

All programs rely on processing of data, and an essential property of data is its type. A literal is a fixed \cdot type value like the number 3, and if we type the number 3 in an interactive session at the input prompt, \cdot literal then F# responds as shown in Listing 5.1.

```
Listing 5.1: Typing the number 3.

1 > 3;;
2 val it : int = 3
```

What this means is that F# has inferred the type to be int and bound it to the identifier it. For more on binding and identifiers see Chapter 6. Types matter, since the operations that can be performed on integers are quite different from those that can be performed on, e.g., strings. E.g., the number 3 has many different representations as shown in Listing 5.2.

Each literal represents the number 3, but their types are different, and hence they are quite different values. The types int for integer numbers, float for floating point numbers, bool for Boolean values, char for characters, and string for strings of characters are the most common types of literals. A table of all $basic\ types$ predefined in F# is given in Table 5.1. Besides these built-in types, F# is designed such that it is easy to define new types.

·float ·bool ·char ·string

· basic types

Metatype	Type name	Description	
Boolean <u>bool</u>		Boolean values true or false	
Integer	<u>int</u>	Integer values from -2,147,483,648 to 2,147,483,647	
	byte	Integer values from 0 to 255	
	sbyte	Integer values from -128 to 127	
	int8	Synonymous with byte	
	uint8	Synonymous with sbyte	
	int16	Integer values from -32768 to 32767	
	uint16	Integer values from 0 to 65535	
	int32	Synonymous with int	
	uint32	Integer values from 0 to 4,294,967,295	
	int64	Integer values from -9,223,372,036,854,775,808 to	
		9,223,372,036,854,775,807	
	uint64	Integer values from 0 to 18,446,744,073,709,551,615	
Real <u>float</u> 64		64-bit IEEE 754 floating point value from $-\infty$ to ∞	
	double	Synonymous with float	
	single	A 32-bit floating point type	
	float32	Synonymous with single	
	decimal	A floating point data type that has at least 28 significant digits	
Character <u>char</u> Unicode character		Unicode character	
	string	Unicode sequence of characters	
None	<u>unit</u>	The value ()	
Object	obj	An object	
Exception	exn	An exception	

Table 5.1: List of some of the basic types. The most commonly used types are underlined. For at description of integer see Appendix B.1, for floating point numbers see Appendix B.2, for ASCII and Unicode characters see Appendix C, for objects see Chapter 20, and for exceptions see Chapter 18.

Humans like to use the decimal number system for representing numbers. Decimal numbers are base 10, which means that a value is represented as two sequences of decimal digits separated by a decimal point, where each digit d has a position and a value $d \in \{0,1,2,\ldots,9\}$. The part before the decimal point is called the whole part and the part after is called the fractional part of the number. The whole part with neither a decimal point nor a fractional part is called an integer. As an example 35.7 is a decimal number, whose value is $3 \cdot 10^1 + 5 \cdot 10^0 + 7 \cdot 10^{-1}$, and 128 is an integer, whose value is $1 \cdot 10^2 + 2 \cdot 10^1 + 8 \cdot 10^0$. In F# a decimal number is called a floating point number. Floating point numbers may alternatively be given using scientific notation, such as 3.5e-4 and 4e2, where the e-notation is translated to a value as $3.5e-4=3.5 \cdot 10^{-4}=0.00035$, and $4e2=4 \cdot 10^2=400$.

The basic unit of information in almost all computers is the binary digit or *bit* for short. Internally, programs and data is all represented as bits, hence F# has a strong support for binary numbers. A binary number consists of a sequence of binary digits separated by a decimal point, where each digit can have values $b \in \{0,1\}$, and the base is 2. E.g., the binary number $101.01_2 = 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 + 0 \cdot 2^{-1} + 1 \cdot 2^{-2} = 5.25$. Subscripts are often used to indicate the base of a number, e.g., 101.01_2 and 101.01_{10} are different numbers. Since base 10 is so common, the subscript for base 10 numbers is often omitted.

Binary numbers are closely related to *octal* and *hexadecimal numbers*, where octals uses 8 as basis, and where each octal digit can be represented by exactly 3 bits, while hexadecimal numbers uses 16 as basis, and where each hexadecimal digit can be written in binary using exactly 4 bits. The hexadecimal digits uses 0–9 to represent the values 0–9 and a–f in lower or alternatively upper case to represent the values 10-15. Thus, Octals and hexadecimals conveniently serve as shorthand for the much longer binary representation. As examples, the octal number 37_8 is $3 \cdot 8^1 + 7 \cdot 8^0 = 31$, and the hexadecimal number f_{31_6} is $15 \cdot 16^1 + 3 \cdot 16^0 = 243$.

- · decimal number
- \cdot base
- · decimal point
- · digit
- · whole part
- · fractional part
- $\cdot \, \text{integer}$
- · floating point number
- \cdot scientific notation
- · bit
- · binary number

 $\cdot \ octal \ number$

 $\cdot \ hexadecimal \ number$

Character	Escape sequence	Description
BS	\b	Backspace
LF	\n	Line feed
CR	\r	Carriage return
HT	\t	Horizontal tabulation
\	\\	Backslash
"	\"	Quotation mark
,	\'	Apostrophe
BEL	\a	Bell
FF	\f	Form feed
VT	\v	Vertical tabulation
	\uXXXX, \UXXXXXXXX, \DDD	Unicode character

Table 5.2: Escape characters. For the unicode characters 'X' are hexadecimal digits, while for tricode characters 'D' is a decimal character.

To denote integers on bases different than 10, F# uses the prefix '0b' for binary, '0o' for octal, and '0x' for hexadecimal numbers. For example, the value 367_{10} may be written as an integer 367, as a binary number 0b101101111, as a octal number 0o557, and as a hexadecimal number 0x16f. The character sequences 0b12 and ff are not numbers.

A character is a Unicode code point, and character literals are enclosed in single quotation marks. Appendix C.3 contains more details on code points. The character type in F# is denoted char. Examples of characters are 'a', 'D', '3'. However '23' and 'abc' are not characters. Some characters do not have a visual representation such as the tabulation character. These can still be represented as a character using escape sequences. A character escape sequence starts with "\" followed by either a letter for simple escapes such as \t for tabulation and \n for newline. Escape sequences can also be a numerical representation of a code point, and three versions exist: The trigraph \DDD, where D is a decimal digit, is used to specify the first 256 code points, the hexadecimal escape codes \uXXXX, where X is a hexadecimal digit, is used to specify the first 65536 code points, and \uXXXXXXXX is used to specify any of the approximately 4.3 · 10⁹ possible code points. All escape sequences are shown in Table 5.2. Examples of char representations of the letter 'a' are: 'a', '\097', '\u00061', '\u000000061'.

A string is a sequence of characters enclosed in double quotation marks. Examples are "a", "this · string is a string", and "-&#\@". Note that the string "a" and the character 'a' are not the same. Some strings are so common that they are given special names: One or more spaces, " " is called whitespace, · whitespace and both "\n" and "\r\n" are called newline. The escape-character "\" may be used to break a line · newline in two. This and other examples are shown in Listing 5.3.

 \cdot character

 \cdot Unicode

· code point

 \cdot char

 \cdot escape sequences

Type	syntax	Examples	Value
int, int32	<int hex="" or=""></int>	3, 0x3	3
	<int hex="" or="">l</int>	31, 0x31	
uint32	<int hex="" or="">u</int>	3u	3
	<int hex="" or="">ul</int>	3ul	
byte, uint8	<int hex="" or="">uy</int>	97uy	97
	' <char>'B</char>	'a'B	
byte[]	" <string>"B</string>	"a\n"B	[97uy; 10uy]
	@" <string>"B</string>	@"a\n"B	[97uy; 92uy; 110uy]
sbyte, int8	<int hex="" or="">y</int>	Зу	3
int16	<int hex="" or="">s</int>	3s	3
uint16	<int hex="" or="">us</int>	3us	3
int64	<int hex="" or="">L</int>	3L	3
uint64	<int hex="" or="">UL</int>	3UL	3
	<int hex="" or="">uL</int>	3uL	
float, double	<float></float>	3.0	3.0
	<hex>LF</hex>	0x013fLF	9.387247271e-323
single, float32	<float>F</float>	3.0F	3.0
	<float>f</float>	3.0f	3.0
	<hex>lf</hex>	0x013flf	4.4701421e-43f
decimal	<float int="" or="">M</float>	3.0M,3M	3.0
	<float int="" or="">m</float>	3.0m,3m	
string	" <string>"</string>	"a \"quote\".\n"	a "quote". <newline></newline>
	@" <string>"</string>	@"a ""quote"".\n"	a "quote". \n .
	"" <string>""</string>	"""a "quote".\n"""	a "quote".\n

Table 5.3: List of literal type. Syntax notation is used such that, e.g., <> means that the programmer replaces the brackets and content with a value on appropriate form. The [||] notation means that the value is an array, see Section 11.3 for details.

Note that the response from fsharpi is shown in double quotation marks, but this is not part of the string.

F# supports literal types, where the type of a literal is indicated as a prefix og suffix as shown in the · literal type Table 5.3. The table uses a simple syntax notation such that <integer or hexadecimal>UL means that the user supplies an integer or a hexadecimal number followed by the characters 'UL'.

The literal type is closely connected to how the values are represented internally. E.g., a value of type int32 use 32 bits and can be both positive and negative, while a uint32 value also use 32 bits, but is unsigned. A byte is an 8-bit number, and sbyte is a signed 8-bit number. Values of type float uses 64 bits, while float32 only uses 32 bits. The number of bits used to represent numbers directly relates to the range and precession these types can represent. This is summarized in Table 5.1 and discussed in more detail in Appendix B. String literals may be verbatim by the @-notation or triple · verbatim double quotation marks, meaning that the escape sequences are not converted to their code point. The two types of string verbatim treat quotation marks differently as illustrated in the table. Further examples are shown in Listing 5.4.

```
Listing 5.4: Named and implied literals.
> 3;;
val it : int = 3
> 4u;;
val it : uint32 = 4u
> 5.6;;
val it : float = 5.6
> 7.9f;;
val it : float32 = 7.9000001f
> 'A';;
val it : char = 'A'
> 'B'B;;
val it : byte = 66uy
> "ABC";;
val it : string = "ABC"
> @"abc\nde";;
val it : string = "abc\nde"
```

Many basic types are compatible, and the type of a literal may be changed by typecasting. An example • typecasting of casting to a float is shown in Listing 5.5.

```
Listing 5.5: Casting an integer to a floating point number.
> float 3;;
 val it : float = 3.0
```

which is a float, since when float is given an argument, then it acts as a function rather than a type, and for the integer 3 it returns the floating point number 3.0. For more on functions see Chapter 6. Boolean values are often treated as the integer values 0 and 1, but no short-hand function names exists for their conversions. Instead use functions from the System. Convert family of functions, as demonstrated in Listing 5.6.

Listing 5.6: Casting booleans. > System.Convert.ToBoolean 1;; val it : bool = true > System.Convert.ToBoolean 0;; val it : bool = false > System.Convert.ToInt32 true;; val it : int = 1> System.Convert.ToInt32 false;; val it : int = 0

Here System. Convert. ToBoolean is the identifier of a function ToBoolean, which is a member of the ... member class Convert that is included in the namespace System. Namespaces, classes, and members will be · class · namespace discussed in Chapter 9.

Typecasting is often a destructive operation, e.g., typecasting a float to int removes the fractional part without rounding as shown in Listing 5.7.

```
Listing 5.7: Fractional part is removed by downcasting.
> int 357.6;;
val it : int = 357
```

Here we typecasted to a lesser type, in the sense that the set of integers is a subset of floating point numbers, and this is called *downcasting*. The opposite is called *upcasting* and is often non-destructive, as Listing 5.5 showed, where an integer was casted to a float while retaining its value. As a side note, rounding a number y.x, where y is the whole part and x is the fractional part, is the operation of mapping numbers in the interval $y.x \in [y.0, y.5)$ to y and $y.x \in [y.5, y+1)$ to y+1. This can be performed by downcasting as shown in Listing 5.8.

```
· downcasting
```

·upcasting

 \cdot rounding

· whole part

· fractional part

```
Listing 5.8: Fractional part is removed by downcasting.
> int (357.6 + 0.5);;
val it : int = 358
```

As the example shows, for floating points whose fractional part is equal to or larger than y.5 adding 0.5 will make them above (y+1).0, and downcasting will thus downcase to (y+1).0. Conversely fractional pars below will downcast to y.0. Thus, rounding is achieved by downcasting.

5.2Operators on basic types

Listing 5.8 is an example of an arithmetic expression using an binary operator written using infix \cdot expression notation, since the operator appears in between the operands. The "+" operator is binary, since it takes two arguments, and since it is written between its arguments, then it uses infix notation. Expressions is the basic building block of all F# programs and this section will discuss operator expressions on basic types.

- · binary operator
- \cdot infix notation
- \cdot operands

The syntax of basic binary operators is shown in Listing 5.9.

```
Listing 5.9 Syntax for a binary expression.
<expr><op><expr>
```

Here <expr> is any expression supplied by the programmer, and <op> is a binary, infix operator. F# supports a range of arithmetic binary infix operators on its built-in types such as addition, subtraction, multiplication, division, and exponentiation using the "+", "-", "*", "/", "**" lexemes. Not all operators are defined for all types, e.g., addition is defined for integer and float types as well as for characters and strings, but multiplication is only defined for integer and floating-point types. A complete list of built-in operators on basic types is shown in Table E.1 and E.2 and a range of mathematical functions shown in Table E.3. An example is 3+4. Note that expressions can themselves be arguments to expressions, and thus, 4+5+6 is also a legal statement. This is called recursion, which means that a rule or a function is used by the rule or function itself in its definition. See Chapter 13 for more on recursive functions.

 \cdot recursion

Unary operators takes only one argument and have the syntax shown in Listing 5.10

```
Listing 5.10 A unary expressions.
<op><expr>
```

An example of a unary operator is -3, where - here is used to negate a positive integer. Since the operator appears before the operand it is a prefix operator.

· prefix operator

The concept of precedence is an important concept in arithmetic expressions. If parentheses are omitted in Listing 5.8, then F# will interpret the expression as (int 357.6) + 0.5, which is erroneous, since addition of an integer with a float is undefined. This is an example of precedence, i.e., function evaluation takes precedence over addition meaning that it is performed before addition. Consider the arithmetic expression,

· precedence

```
Listing 5.11: A simple arithmetic expression.
> 3 + 4 * 5;;
val it : int = 23
```

Here, the addition and multiplication functions are shown in infix notation with the operator lexemes · operator "+" and "*". To arrive at the resulting value 23, F# has to decide in which order to perform the calculation. There are 2 possible orders, 3 + (4 * 5) or (3 + 4) * 5, which gives different results. For integer arithmetic, the correct order is of course to multiply before addition, and we say that multiplication takes precedence over addition. Every atomic operation that F# can perform is ordered in terms of its precedence, and for some common built-in operators shown in Table 5.4, the precedence is shown by the order they are given in the table.

· precedence

Associativity implies the order in which calculations are performed for operators of same precedence. For some operators and type combinations association matters little, e.g., multiplication associates to the left and exponentiation associates to the right, as demonstrated in Listing 5.12.

¹Jon: minor comment on indexing and slice-ranges.

Operator	Associativity	Description
+ <expr>, -<expr>,</expr></expr>	Left	Unary identity, negation, and bitwise negation operator
~~~ <expr></expr>		
f <expr></expr>	Left	Function application
<expr> ** <expr></expr></expr>	Right	Exponent
<pre><expr> * <expr>,</expr></expr></pre>	Left	Multiplication, division and remainder
<expr> / <expr>,</expr></expr>		
<expr> % <expr></expr></expr>		
<expr> + <expr>,</expr></expr>	Left	Addition and subtraction binary operators
<expr> - <expr></expr></expr>		
<expr> ^^^ <expr></expr></expr>	Right	bitwise exclusive or
<expr> &lt; <expr>,</expr></expr>	Left	Comparison operators, bitwise shift, and bitwise 'and'
<expr> &lt;= <expr>,</expr></expr>		and 'or'.
<expr> &gt; <expr>,</expr></expr>		
<expr> &gt;= <expr>,</expr></expr>		
<expr> = <expr>,</expr></expr>		
<expr> &lt;&gt; <expr>,</expr></expr>		
<expr> &lt;&lt;&lt; <expr>,</expr></expr>		
<expr> &gt;&gt;&gt; <expr>,</expr></expr>		
<expr> &amp;&amp;&amp; <expr>,</expr></expr>		
<pre><expr>     <expr> ,</expr></expr></pre>		
<expr> &amp;&amp; <expr></expr></expr>	Left	Boolean and
<expr>    <expr></expr></expr>	Left	Boolean or

Table 5.4: Some common operators, their precedence, and their associativity. Rows are ordered from highest to lowest precedences, such that <*expr*> * <*expr*> has higher precedence than <*expr*> + <*expr*>. Operators in the same row has same precedence. Full table is given in Table E.5.

the expression for 3.0 * 4.0 * 5.0 associates to the left, and thus is interpreted as (3.0 * 4.0) * 5.0, but gives the same results as 3.0 * (4.0 * 5.0), since association does not matter for multiplication of numbers. However, the expression for 4.0 ** 3.0 ** 2.0 associates to the right, and thus is interpreted as 4.0 ** (3.0 ** 2.0), which is quite different from (4.0 ** 3.0) ** 2.0. Whenever in Advice doubt of association or any other basic semantic rules, it is a good idea to use parentheses as here. It is also a good idea to test your understanding of the syntax and semantic

 $\cdot$  and

 $\cdot$  not

· truth table

a	b	a && b	a    b	not a
false	false	false	false	true
false	true	false	true	true
true	false	false	true	false
true	true	true	true	false

Table 5.5: Truth table for boolean 'and', 'or', and 'not' operators. Value 0 is false and 1 is true.

rules by making a simple script.

#### 5.3 Boolean arithmetic

Boolean arithmetic is the basis of almost all computers and particularly important for controlling program flow, which will be discussed in Chapter 8. Boolean values are one of 2 possible values, true or false, which is also sometimes written as 1 and 0. Basic operations on Boolean values are 'and', 'or', and 'not', which in F# are written respectively as the binary operators &&, ||, and the function not. Since the domain of Boolean values is so small, all possible combination of input on these values can be written on tabular form, known as a truth table, and the truth tables for the basic Boolean operators and functions are shown in Table 5.5. A good mnemonic for remembering the result of the 'and' and 'or' operators is to use 1 for true, 0 for false, multiplication for the Boolean 'and' operator, and addition for Boolean 'or' operator, e.g., true and false in this mnemonic translates to  $1 \cdot 0 = 0$ , and the results translates back to the Boolean value false. In F# the truth table for the basic Boolean operators can be produced by a program as shown in Listing 5.13.

```
Listing 5.13: Boolean operators and truth tables.
> printfn "a b a*b a+b not a"
  printfn "%A %A %A %A %A"
    false false (false && false) (false || false) (not false)
  printfn "%A %A %A %A %A"
    false true (false && true) (false || true) (not false)
  printfn "%A %A %A %A %A"
    true false (true && false) (true || false) (not true)
  printfn "%A %A %A %A %A"
    true true (true && true) (true || true) (not true);;
    a*b a+b not a
false false false true
false true false true true
true false false true false
true true true false
val it : unit = ()
```

Here, we used the printfn function, to present the results of many expressions on something that resembles a tabular form. The spacing produced using the printfn function is not elegant, and in Section 6.5 we will discuss better options for producing more beautiful output. Notice that the arguments for printfn was given on the next line with indentation. The indentation is an important part of telling F# which part of what you write belongs together. This is an example of the so-called lightweight syntax. Generally, F# ignores newlines and whitespaces except when using the lightweight syntax, and the examples of the difference between regular and lightweight syntax is discussed in Chapter 6.

### 5.4 Integer arithmetic

The set of integers is infinitely large, but since all computers have limited resources, it is not possible to represent it in its entirety. The various integer types listed in Table 5.1 are finite subsets reduced by limiting their ranges. An in-depth description of integer implementation can be found in Appendix B. The type int is the most common type.

Table E.1, E.2, and E.3 give examples operators and functions pre-defined for integer types. Notice that fewer functions are available for integers than for floating point numbers. For most addition, subtraction, multiplication, and negation, the result is straight forward. However, performing arithmetic operations on integers requires extra care, since the result may cause *overflow* and *underflow*. E.g., the range of the integer type sbyte is [-128...127], which causes problems in the example in Listing 5.14.

· overflow · underflow

```
Listing 5.14: Adding integers may cause overflow.

1 > 100y;;
2 val it : sbyte = 100y
3
4 > 30y;;
5 val it : sbyte = 30y
6
7 > 100y + 30y;;
8 val it : sbyte = -126y
```

Here 100 + 30 = 130, which is larger than the biggest sbyte, and the result is an overflow. Similarly, we get an underflow, when the arithmetic result falls below the smallest value storable in an sbyte as demonstrated in Listing 5.15.

```
Listing 5.15: Subtracting integers may cause underflow.

> -100y - 30y;;
val it : sbyte = 126y
```

I.e., we were expecting a negative number, but got a positive number instead.

The overflow error in Listing 5.14 can be understood in terms of the binary representation of integers: In binary,  $130 = 10000010_2$ , and this binary pattern is interpreted differently as byte and sbyte, see Listing 5.16.

```
Listing 5.16: The left most bit is interpreted differently for signed and unsigned integers, which gives rise to potential overflow errors.
```

```
1  > 0b10000010uy;;
2  val it : byte = 130uy
3
4  > 0b10000010y;;
5  val it : sbyte = -126y
```

That is, for signed bytes, the left-most bit is used to represent the sign, and since the addition of  $100 = 01100100_2$  and  $30 = 00011110_b$  is  $130 = 10000010_2$  causes the left-most bit to be used, then this is wrongly interpreted as a negative number, when stored in an sbyte. Similar arguments can be

made explaining underflows.

The division and remainder operators, which discards the fractional part after division, and the re- integer division mainder operator calculates the remainder after integer division, as demonstrated in Listing 5.17.

```
Listing 5.17: Integer division and remainder operators.

1 > 7 / 3;;
2 val it : int = 2
3
4 > 7 % 3;;
5 val it : int = 1
```

Together integer division and remainder is a lossless representation of the original number, see Listing 5.18.

And we see that integer division of 7 by 3 followed by multiplication by 3 is less that 7, and the difference is 7 % 3.

Notice that neither overflow nor underflow error gave rise to an error message, which is why such bugs are difficult to find. Dividing any non-zero number with 0 is infinite, which is also outside the domain of any of the integer types, but in this case, F# casts an *exception* as shown in Listing 5.19.

 $\cdot$  exception

The output looks daunting at first sight, but the first and last line of the error message are the most important parts, which tells us what exception was cast and why the program stopped. The middle are technical details concerning which part of the program caused this, and can be ignored for the time being. Exceptions are a type of *runtime error*, and are treated in Chapter 18

 $\cdot$  runtime error

Integer exponentiation is not defined as an operator, but this is available the built-in function pown.

a	b	a ~~~ b
false	false	false
false	true	true
true	false	true
true	true	false

Table 5.6: Boolean exclusive or truth table.

This function is demonstrated in Listing 5.20.

```
Listing 5.20: Integer exponent function.

1 > pown 2 5;;
2 val it : int = 32
```

which is equal to  $2^5$ .

For binary arithmetic on integers, the following operators are available: <leftExpr> <<< <ri>which shifts the bit pattern of <leftExpr> <rightExpr> positions to the left while inserting 0's to right; <leftExpr> >>> <rightExpr>, which shifts the bit pattern of <leftExpr> <rightExpr> positions to the right while inserting 0's to left; <expr> &&& <expr>, bitwise 'and', returns the result of taking the Boolean 'and' operator position-wise; <expr> ||| <expr>, bitwise 'or', as 'and' but using the Boolean 'or' operator; and <expr> ~~~ <expr>, bitwise xor, which is returns the result of the Boolean 'xor' operator defined by the truth table in Table 5.6.

 $\cdot xor$ 

· exclusive or

### 5.5 Floating point arithmetic

Like integers, the set of reals is also infinitely large, hence, floating point types are finite subsets reduced by sampling the space of reals. An in-depth description of floating point implementations can be found in Appendix B. The type float is the most common type.

Table E.1, E.2, and E.3 give examples operators and functions pre-defined for floating point types. Note that the remainder operator for floats calculates the remainder after division and discarding the fractional part, see Listing 5.21.

```
Listing 5.21: Floating point division and remainder operators.

1 > 7.0 / 2.5;;
2 val it : float = 2.8

3 4 > 7.0 % 2.5;;
5 val it : float = 2.0
```

The remainder for floating point numbers can be fractional, but division, discarding fractional part, and remainder is still a lossless representation of the original number as demonstrated in Listing 5.22.

#### Listing 5.22: Floating point division, truncation, and remainder is a lossless representation of a number.

```
> float (int (7.0 / 2.5));;
val it : float = 2.0
 (float (int (7.0 / 2.5))) * 2.5;;
val it : float = 5.0
> (float (int (7.0 / 2.5))) * 2.5 + 7.0 % 2.5;;
val it : float = 7.0
```

Arithmetic using float will not cause over- and underflow problems, since the IEEE 754 standard includes the special numbers  $\pm \infty$  and NaN. As shown in Listing 5.23, no exception is thrown.

#### Listing 5.23: Floating point numbers include infinity and Not-a-Number.

```
> 1.0/0.0;;
val it : float = infinity
> 0.0/0.0;;
val it : float = nan
```

However, the float type has limited precision, since there is only a finite number of numbers that can be stored in a float. E.g., addition and subtraction can give surprising results as demonstrated in Listing 5.24.

```
Listing 5.24: Floating point arithmetic has finite precision.
> 357.8 + 0.1 - 357.9;;
val it : float = 5.684341886e-14
```

That is, addition and subtraction associates to the left, hence the expression is interpreted as (357.8 + 0.1) - 357.9, and we see that we do not get the expected 0. The reason is that the calculation is done stepwise, and in the process, the numbers are represented using the imprecise floating point standard. Thus, 357.8 + 0.1 is represented as number close to but not identical to what 357.9 is represented as, and thus, when subtracting these two representations, we get a very small number but not 0. Such errors tend to accumulate and comparing the result of expressions of floating point values should therefore be treated with care. Thus, equivalence of two floating point expressions should only be considered up to sufficient precision, e.g., comparing 357.8 + 0.1 and 357.9 up to 1e-10 precision should be tested as, abs ((357.8 + 0.1) - 357.9) < 1e-10.

#### 5.6 Char and string arithmetic

Addition is the only operator defined for characters. Nevertheless, character arithmetic is often done by casting to integer. A typical example is conversion of case, e.g., to convert the lowercase character 'z' to uppercase, we use the ASCIIbetical order and add the difference between any Basic Latin Block · ASCIIbetical order letters in upper- and lowercase as integers and cast back to char, see Listing 5.25.

· . []

#### Listing 5.25: Converting case by casting and integer arithmetic.

```
1 > char (int 'z' - int 'a' + int 'A');;
2 val it : char = 'Z'
```

I.e., the code point difference between upper and lower case for any alphabetical character 'a' to 'z' is constant, hence we can change case by adding or subtracting the difference between any corresponding character. Unfortunately, this does not generalize to characters from other languages.

A large collection of operators and functions exist for **string**. The simplest is concatenation using the "+" operator as demonstrated in Listing 5.26.

```
Listing 5.26: Example of string concatenation.

1 > "hello" + " " + "world";;
2 val it : string = "hello world"
```

Characters and strings cannot be concatenated, which is why the above example used the string of a space " " instead of the space character ' '. The characters of a string may be indexed as using the . [] notation. This is demonstrated in Listing 5.27.

```
Listing 5.27: String indexing using square brackets.
```

```
1  > "abcdefg".[0];;
2  val it : char = 'a'
3
4  > "abcdefg".[3];;
5  val it : char = 'd'
6
7  > "abcdefg".[3..];;
8  val it : string = "defg"
9
10  > "abcdefg".[..3];;
11  val it : string = "abcd"
12
13  > "abcdefg".[1..3];;
14  val it : string = "bcd"
15
16  > "abcdefg".[*];
17  val it : string = "abcdefg"
```

Notice that the first character has index 0, and to get the last character in a string, we use the string's length property. This is done as shown in Listing 5.28.

```
Listing 5.28: String length attribute and string indexing.
```

```
1 > "abcdefg".Length;;
2 val it : int = 7
3
4 > "abcdefg".[7-1];;
5 val it : char = 'g'
```

Since index counting starts at 0, and the string length is 7, then the index of the last character is 6. The is a long list of built-in functions in System.String for working with strings, some of which will be discussed in Section 11.1.

The dot notation is an example of Structured programming, where technically speaking, the string "abcdefg" is an immutable object of class string, [] is an object method, and Length is a property. For more on object, classes, and methods see Chapter 20.

· dot notation · object · class · method

Strings are compared letter by letter. For two strings to be equal, they must have the same length and all the letters must be identical. E.g., "abs" = "absalon" is false, while "abs" = "abs" is true. The "<>" operator is the boolean negation of the "=" operator, e.g., "abs" <> "absalon" is true, while "abs" <> "abs" is false. For the "<", "<=", ">", and ">=" operators, the strings are ordered alphabetically, such that "abs" < "absalon" && "absalon" < "milk" is true, that is, the "<" operator on two strings is true, if the left operand should come before the right, when sorting alphabetically. The algorithm for deciding the boolean value of leftOp < rightOp is as follows: we start by examining the first character, and if leftOp.[0] and rightOp.[0] are different, then the leftOp < rightOp is equal to leftOp.[0] < rightOp.[0]. E.g., "milk" < "abs" is the same as 'm' < 'a', which is false, since the letter 'm' does not come before the letter 'a' in the alphabet, or more precisely, the codepoint of 'm' is not less than the codepoint of 'a'. If leftOp.[0] and rightOp.[0] are equal, then we move onto the next letter and repeat the investigation, e.g., "abe" < "abs" is true, since "ab" = "ab" is true and 'e' < 's' is true. If we reach the end of either of the two strings, then the short is smaller than the larger, e.g., "abs" < "absalon" is true, while "abs" < "absalon" is true, while

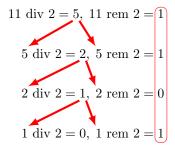
# 5.7 Programming intermezzo: Hand conversion between decimal and binary numbers

Conversion of integers between decimal and binary form is a key concept in order to understand some of the basic properties of calculations on the computer. From binary to decimal is straight forward using the power-of-two algorithm, i.e., given a sequence of n+1 bits that represent an integer  $b_n b_{n-1} \dots b_0$ , where  $b_n$  and  $b_0$  are the most and least significant bits, then the decimal value is calculated as,

$$v = \sum_{i=0}^{n} b_i 2^i \tag{5.1}$$

For example,  $10011_2 = 1 + 2 + 16 = 19$ . From decimal to binary is a little more complex, but a simple divide-by-two algorithm exists. The key to understanding the divide-by-two algorithm is to realize that when you divide a number by two, then that is equivalent to shifting its binary representation 1 to the right. E.g.,  $10 = 1010_2$  and  $10/2 = 5 = 101_2$ . Odd numbers have  $b_0 = 1$ , e.g.,  $11_{10} = 1011_2$  and  $11_{10}/2 = 5.5 = 101.1_2$ . Hence, if we divide any number by two and get a non-integer number, then its least significant bit was 1. Another way to express this is that the least significant bit is the remainder after integer division by two. Sequential application of this idea leads directly to the divide-by-two algorithm. E.g., if we were to convert the number  $11_{10}$  on decimal form to binary form we would

perform the following steps:



Here we used div and rem to signify the integer division and remainder operators. The algorithm stops, when the result of integer division is zero. Reading off the remainder from below and up we find the sequence  $1011_2$ , which is the binary form of the decimal number  $11_{10}$ . Using interactive mode, we can calculate the same as shown in Listing 5.29.

Thus, but reading the second integer-response from printfn from below and up, we again obtain the binary form of  $11_{10}$  to be  $1011_2$ . For integers with a fractional part, the divide-by-two may be used on the whole part, while multiply may be used in a similar manner on the fractional part.

# The console in Windows, MacOS X, and Linux

Almost all popular operating systems are accessed through a user-friendly graphical user interface (GUI) that is designed to make typical tasks easy to learn to solve. As a computer programmer, you often need to access some of the functionalities of the computer, which, unfortunately, are sometimes complicated by this particular graphical user interface. The console, also called the terminal and the Windows command line, is the right hand of a programmer. The console is a simple program that allows you to complete text commands. Almost all the tasks that can be done with the graphical user interface can be done in the console and vice versa. Using the console, you will benefit from its direct control of the programs we write, and in your education, you will benefit from the fast and raw information you get through the console.

- · graphical user interface
- $\cdot GUI$
- $\cdot$  console
- $\cdot$  terminal
- $\cdot$  Windows command line

#### The basics A.1

When you open a directory or folder in your preferred operating system, the directory will have a directory location in the file system, whether from the console or through the operating system's graphical user interface. The console will almost always be associated with a particular directory or folder in the file system, and it is said that it is the directory that the console is in. The exact structure of file systems varies between Linux, MaxOS X and Windows, but common is that it is a hierarchical structure. This is illustrated in Figure A.1.

There are many predefined console commands, available in the console, and you can make your own. In the following, we will review the most important commands in the three different operating systems. These are summarized in Table A.1.

#### ${f Windows}$

In this section we will discuss the commands summarized in Table A.1. Windows 7 and earlier versions: To open the console, press Start->Run in the lower left corner, and then type cmd In the box. In Windows 8 and 10, you right-click on the windows icon, choose Run or equivalent in your local language, and type cmd. Alternatively you can type Windows-key + R. Now you should open a console window with a prompt showing something like Listing A.1.

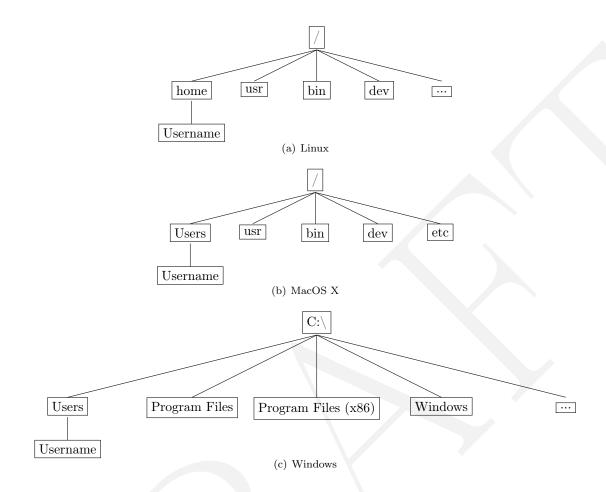


Figure A.1: The top file hierarchy levels of common operating systems.

Windows	MacOS X/Linux	Description
dir	ls	Show content of present direc-
		tory.
cd <dir></dir>	cd <dir></dir>	Change present directory to
		<dir>.</dir>
mkdir <dir></dir>	mkdir <dir></dir>	create directory <dir>.</dir>
rmdir <dir></dir>	rmdir <dir></dir>	delete <dir> (Warning: can-</dir>
		not be reverted).
move <file> <file dir="" or=""></file></file>	mv <file> <file dir="" or=""></file></file>	Move <fil> to <file or<="" td=""></file></fil>
		dir>.
copy <file> <file></file></file>	cp <file> <file></file></file>	Create a new file called <file></file>
		as a copy of <file>.</file>
del <file></file>	rm <file></file>	delete <file> (Warning: can-</file>
		not be reverted).
echo <string or="" variable=""></string>	echo <string or="" variable=""></string>	Write a string or content of a
		variable to screen.

Table A.1: The most important console commands for Windows, MacOS X, and Linux.

·dir

# Listing A.1: The Windows console. Microsoft Windows [Version 6.1.7601] Copyright (c) 2009 Microsoft Corporation. All rights reserved. C:\Users\sporring>

To see which files are in the directory, use *dir* as shown in Listing A.2.

```
Listing A.2: Directory listing with dir.
C:\Users\sporring>dir
 Volume in drive C has no label.
 Volume Serial Number is 94F0-31BD
 Directory of C:\Users\sporring
30-07-2015 15:23
                      <DIR>
30-07-2015
            15:23
                      <DIR>
30-07-2015
            14:27
                      <DIR>
                                     Contacts
30-07-2015 14:27
                      <DIR>
                                     Desktop
30-07-2015 17:40
                      <DIR>
                                     Documents
30-07-2015 15:11
                      <DIR>
                                      Downloads
30-07-2015 14:28
                      <DIR>
                                     Favorites
30-07-2015 14:27
                      <DIR>
                                     Links
30-07-2015 14:27
                      <DIR>
                                     Music
30-07-2015
            14:27
                      <DIR>
                                     Pictures
30-07-2015
            14:27
                      <DIR>
                                      Saved Games
30-07-2015
            17:27
                      <DIR>
                                      Searches
30-07-2015
            14:27
                      <DIR>
                                      Videos
                                       0 bytes
                0 File(s)
               13 Dir(s) 95.004.622.848 bytes free
C:\Users\sporring>
```

We see that there are no files and thirteen directories (DIR). The columns tell from left to right, the date and time of their creation, the file size or if it is a folder, and the name file or directory name. The first two folders "." and ".." are found in each folder and refers to this folder as well as the one above in the hierarchy. In this case, the folder "." is an alias for C:\Users\tracking and ".." for C:\Users.

Use *cd* to change directory, e.g., to Documents as in Listing A.3.

```
Listing A.3: Change directory with cd.

C:\Users\sporring>cd Documents

C:\Users\sporring\Documents>
```

The directory Documents is that Windows Explorer selects, when you press the Documents short-cut in Explorer. Note that some systems translate default filenames so their names may be given different names in different languages in the graphical user interface as compared to the console.

You can use *mkdir* to create a new directory called, e.g., myFolder as illustrated in Listing A.4.

 $\cdot$  mkdir

 $\cdot \, cd$ 

#### Listing A.4: Creating a directory with mkdir. C:\Users\sporring\Documents>mkdir myFolder C:\Users\sporring\Documents>dir Volume in drive C has no label. Volume Serial Number is 94F0-31BD Directory of C:\Users\sporring\Documents 30-07-2015 19:17 <DIR> 30-07-2015 19:17 <DIR> 30-07-2015 19:17 <DIR> myFolder 0 bytes 0 File(s) 3 Dir(s) 94.656.638.976 bytes free C:\Users\sporring\Documents>

By using dir we inspect the result.

Files can be created by, e.g., *echo* and *redirection* as demonstrated in Listing A.5.

· echo · redirection

```
Listing A.5: Creating a file with echo and redirection.
C:\Users\sporring\Documents>echo "Hi" > hi.txt
C:\Users\sporring\Documents>dir
 Volume in drive C has no label.
 Volume Serial Number is 94F0-31BD
 Directory of C:\Users\sporring\Documents
30-07-2015 19:18
                     <DIR>
30-07-2015 19:18
                     <DIR>
30-07-2015 19:17
                     <DIR>
                                   myFolder
30-07-2015 19:18
                                  8 hi.txt
               1 File(s)
                                      8 bytes
               3 Dir(s) 94.656.634.880 bytes free
C:\Users\sporring\Documents>
```

To move the file hi.txt to the directory myFolder use move as shown in Listing A.6.

```
Listing A.6: Move a file with move.

1 C:\Users\sporring\Documents>move hi.txt myFolder
2 1 file(s) moved.

3 4 C:\Users\sporring\Documents>
```

Finally, use *del* to delete a file and *rmdir* to delete a directory as shown in Listing A.7.

·del ·rmdir

 $\cdot$  move

#### Listing A.7: Delete files and directories with del and rmdir. C:\Users\sporring\Documents>cd myFolder C:\Users\sporring\Documents\myFolder>del hi.txt C:\Users\sporring\Documents\myFolder>cd .. C:\Users\sporring\Documents>rmdir myFolder C:\Users\sporring\Documents>dir Volume in drive C has no label. Volume Serial Number is 94F0-31BD Directory of C:\Users\sporring\Documents 30-07-2015 19:20 <DIR> 30-07-2015 19:20 <DIR> 0 bytes 0 File(s) 2 Dir(s) 94.651.142.144 bytes free C:\Users\sporring\Documents>

The commands available from the console must be in its search path. The search path can be seen · search path using echo as shown in Listing A.8.

The path can be changed using the Control panel in the graphical user interface. In Windows 7, choose the Control panel, choose System and Security  $\rightarrow$  System  $\rightarrow$  Advanced system settings  $\rightarrow$  Environment Variables. In Windows 10 you can find this window by searching for "Environment" in the Control panel. In the window's System variables box, double-click on Path and add or remove a path from the list. The search path is a list of paths separated by ";". Beware, Windows uses the search path for many different tasks, so remove only paths that you are certain are not used for anything.

A useful feature of the console is that you can use the tab-key to cycle through filenames. E.g., if you write cd followed by a space and tab a couple of times, then the console will suggest you the available directories.

#### A.3 MacOS X and Linux

MacOS X (OSX) and Linux are very similar, and both have the option of using *bash* as console. It ·bash is in the standard console on MacOS X and on many Linux distributions. A summary of the most important bash commands are shown in Table A.1. In MacOS X, you find the console by opening

Finder and navigating to Applications  $\rightarrow$  Utilities -> Terminal. In Linux, the console can be started by typing Ctrl + Alt + T. Some Linux distributions have other key-combinations such as Super + T.

Once opened, the console is shown in a window with content as shown in Listing A.9.

```
Listing A.9: The Windows console.

Last login: Thu Jul 30 11:52:07 on ttys000
FN11194:~ sporring$
```

"FN11194" is the name of the computer, the character  $\sim$  is used as an alias for the user's home directory, and "sporring" is the username for the user presently logged onto the system. Use ls to see  $\cdot ls$  which files are present, as shown in Listing A.10.

```
Listing A.10: Display a directory content with 1s.

1 FN11194:~ sporring$ 1s
2 Applications Documents Library Music Public
3 Desktop Downloads Movies Pictures
4 FN11194:~ sporring$
```

More details about the files are available by using flags to 1s as demonstrated in Listing A.11.

```
Listing A.11: Display extra information about files using flags to 1s.
FN11194:~ sporring$ ls -l
drwx ----
             6 sporring staff
                                 204 Jul 30 14:07 Applications
drwx----+ 32 sporring
                         staff
                                1088 Jul 30 14:34 Desktop
drwx----+ 76 sporring
                         staff
                                2584 Jul
                                          2 15:53 Documents
drwx----+ 4 sporring
                         staff
                                 136 Jul 30 14:35 Downloads
drwx----@ 63 sporring
                         staff
                                 2142 Jul 30 14:07 Library
drwx----+ 3 sporring
                         staff
                                 102 Jun 29 21:48 Movies
drwx----+ 4 sporring
                         staff
                                 136 Jul
                                          4 17:40 Music
drwx ----+
             3 sporring
                                  102 Jun 29 21:48 Pictures
                         staff
drwxr-xr-x+
             5 sporring
                         staff
                                  170 Jun 29 21:48 Public
FN11194: sporring$
```

The flag -1 means long, and many other flags can be found by querying the built-in manual with man ls. The output is divided into columns, where the left column shows a number of codes: "d" means they are directory followed by three sets of optional "rwx" denoting whether the owner, the associated group of users and anyone can "r" - read, "w" - write, or "x" - execute the files. In this case, only the owner can do any of the three. For directories, "x" means permission to enter. The second column can often be ignored, but shows how many links there are to the file or directory. Then follows the username of the owner, which in this case is sporring. The files are also associated with a group of users, and in this case, they all are associated with the group called staff. Then follows the file or directory size, the date of last change, and the file or directory name. There are always two hidden directories: "." and "..", where "." is an alias for the present directory, and ".." for the directory above. Hidden files will be shown with the -a flag.

Use cd to change to the directory Documents as shown in Listing A.12.

```
Listing A.12: Change directory with cd.

1 FN11194:~ sporring$ cd Documents/
2 FN11194:Documents sporring$
```

Note that some graphical user interfaces translate standard filenames and directories to the local language, such that navigating using the graphical user interface will reveal other files and directories, which, however, are aliases.

You can yourself create a new directory using mkdir as demonstrated in Listing A.13.

·mkdir

```
Listing A.13: Creating a directory using mkdir.

1 FN11194:Documents sporring$ mkdir myFolder
2 FN11194:Documents sporring$ ls
3 myFolder
4 FN11194:tmp sporring$
```

A file can be created using echo and with redirection as shown in Listing A.14.

 $\cdot$  echo  $\cdot$  redirection

• mv

 $\cdot rm$ 

```
Listing A.14: Creating a file with echo and redirection.

FN11194:Documents sporring$ echo "hi" > hi.txt
FN11194:Documents sporring$ ls
hi.txt myFolder
```

To move the file hi.txt into myFolder, use mv. This is demonstrated in Listing A.15.

```
Listing A.15: Moving files with mv.

FN11194:Documents sporring$ echo mv hi.txt myFolder/
FN11194:Documents sporring$
```

To delete the file and the directory, use rm and rmdir as shown in Listing A.16.

```
Listing A.16: The Windows console.

FN11194:Documents sporring$ cd myFolder/
FN11194:myFolder sporring$ rm hi.txt
FN11194:myFolder sporring$ cd ..
FN11194:Documents sporring$ rmdir myFolder/
FN11194:Documents sporring$ ls
FN11194:Documents sporring$
```

Only commands found on the *search-path* are available in the console. The content of the search-path · search-path is seen using the echo command as demonstrated in Listing A.17.

## Listing A.17: The content of the search-path. FN11194:Documents sporring\$ echo \$PATH /Applications/Maple 17/:/Applications/PackageMaker.app/Contents/MacOS/: /Applications/MATLAB_R2014b.app/bin/:/opt/local/bin:/opt/local/sbin:

/usr/local/bin:/usr/bin:/usr/sbin:/sbin:/opt/X11/bin:

/Library/TeX/texbin FN11194:Documents sporring\$

The search-path can be changed by editing the setup file for Bash. On MacOS X it is called ~/.profile, and on Linux it is either ~/.bash_profile or ~/.bashrc. Here new paths can be added by adding the following line: export PATH="<new path>:<another new path>:\$PATH".

A useful feature of Bash is that the console can help you write commands. E.g., if you write fs followed by pressing the tab-key, and if Mono is in the search-path, then Bash will typically respond by completing the line as fsharp, and by further pressing the tab-key some times, Bash will show the list of options, typically fshpari and fsharpc. Also, most commands have an extensive manual, which can be accessed using the man command. E.g., the manual for rm is retrieved by man rm.

## Number systems on the computer

#### B.1 Binary numbers

Humans like to use the decimal number system for representing numbers. Decimal numbers are ·decimal number base 10 means that for a number consisting of a sequence of digits separated by a decimal point, where each digit can have values  $d \in \{0, 1, 2, \dots, 9\}$  and the weight of each digit is proportional  $\cdot$  decimal point to its place in the sequence of digits with respect to the decimal point, i.e., the number 357.6 = . digit  $3 \cdot 10^2 + 5 \cdot 10^1 + 7 \cdot 10^0 + 6 \cdot 10^{-1}$  or in general:

$$v = \sum_{i=-m}^{n} d_i 10^i \tag{B.1}$$

The basic unit of information in almost all computers is the binary digit or bit for short. A binary bit number consists of a sequence of binary digits separated by a decimal point, where each digit can have binary number values  $b \in \{0,1\}$ , and the base is 2. The general equation is,

$$v = \sum_{i=-m}^{n} b_i 2^i \tag{B.2}$$

and examples are  $1011.1_2 = 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 + 1 \cdot 2^{-1} = 11.5$ . Notice that we use subscript 2 to denote a binary number, while no subscript is used for decimal numbers. The left-most bit is called the most significant bit, and the right-most bit is called the least significant bit. Due to typical organisation of computer memory, 8 binary digits is called a byte, and 32 digits a word.

· most significant bit

· least significant bit

· byte

 $\cdot$  word

 $\cdot$  octal number

 $\cdot$  hexadecimal number

Other number systems are often used, e.g., octal numbers, which are base 8 numbers, where each digit is  $o \in \{0, 1, \dots, 7\}$ . Octals are useful short-hand for binary, since 3 binary digits maps to the set of octal digits. Likewise, hexadecimal numbers are base 16 with digits  $h \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, a, b, c, d, e, f\}$ , such that  $a_{16} = 10$ ,  $b_{16} = 11$  and so on. Hexadecimals are convenient since 4 binary digits map directly to the set of octal digits. Thus  $367 = 101101111_2 = 557_8 = 16f_{16}$ . A list of the integers 0–63 is various bases is given in Table B.1.

#### B.2IEEE 754 floating point standard

The set of real numbers also called *reals* includes all fractions and irrational numbers. It is infinite in size both in the sense that there is no largest nor smallest number, and between any 2 given numbers there are infinitely many numbers. Reals are widely used for calculation, but since any computer only has finite memory, it is impossible to represent all possible reals on one. Hence, any computation performed on a computer with reals must rely on approximations. IEEE 754 double precision floatingpoint format (binary64), known as a double, is a standard for representing an approximation of reals

· IEEE 754 double precision floating-point format

· binarv64

 $\cdot$  double

Dec	Bin	Oct	Hex	Dec	Bin	Oct	Hex
0	0	0	0	32	100000	40	20
1	1	1	1	33	100001	41	21
2	10	2	2	34	100010	42	22
3	11	3	3	35	100011	43	23
4	100	4	4	36	100100	44	24
5	101	5	5	37	100101	45	25
6	110	6	6	38	100110	46	26
7	111	7	7	39	100111	47	27
8	1000	10	8	40	101000	50	28
9	1001	11	9	41	101001	51	29
10	1010	12	a	42	101010	52	2a
11	1011	13	b	43	101011	53	2b
12	1100	14	c	44	101100	54	2c
13	1101	15	d	45	101101	55	2d
14	1110	16	e	46	101110	56	2e
15	1111	17	f	47	101111	57	2f
16	10000	20	10	48	110000	60	30
17	10001	21	11	49	110001	61	31
18	10010	22	12	50	110010	62	32
19	10011	23	13	51	110011	63	33
20	10100	24	14	52	110100	64	34
21	10101	25	15	53	110101	65	35
22	10110	26	16	54	110110	66	36
23	10111	27	17	55	110111	67	37
24	11000	30	18	56	111000	70	38
25	11001	31	19	57	111001	71	39
26	11010	32	1a	58	111010	72	3a
27	11011	33	1b	59	111011	73	3b
28	11100	34	1c	60	111100	74	3c
29	11101	35	1d	61	111101	75	3d
30	11110	36	1e	62	111110	76	3e
31	11111	37	1f	63	111111	77	3f

Table B.1: A list of the integers 0–63 in decimal, binary, octal, and hexadecimal.

using 64 bits. These bits are divided into 3 parts: sign, exponent and fraction,

$$s e_1 e_2 \dots e_{11} m_1 m_2 \dots m_{52},$$

where s,  $e_i$ , and  $m_j$  are binary digits. The bits are converted to a number using the equation by first calculating the exponent e and the mantissa m,

$$e = \sum_{i=1}^{11} e_i 2^{11-i}, \tag{B.3}$$

$$m = \sum_{j=1}^{52} m_j 2^{-j}.$$
 (B.4)

I.e., the exponent is an integer, where  $0 \le e < 2^{11}$ , and the mantissa is a rational, where  $0 \le m < 1$ . For most combinations of e and m the real number v is calculated as,

$$v = (-1)^{s} (1+m) 2^{e-1023}$$
(B.5)

with the exception that

	m = 0	$m \neq 0$
e = 0	$v = (-1)^s 0 \text{ (signed zero)}$	$v = (-1)^s m 2^{1-1023}$ (subnormals)
$e = 2^{11} - 1$	$v = (-1)^s \infty$	$v = (-1)^s \text{ NaN (not a number)}$

 $\cdot$  subnormals

· NaN

where  $e = 2^{11} - 1 = 111111111111_2 = 2047$ . The largest and smallest number that is not infinity is thus · not a number

$$e = 2^{11} - 2 = 2046 \tag{B.6}$$

$$m = \sum_{j=1}^{52} 2^{-j} = 1 - 2^{-52} \simeq 1.$$
 (B.7)

$$v_{\text{max}} = \pm (2 - 2^{-52}) 2^{1023} \simeq \pm 2^{1024} \simeq \pm 10^{308}$$
 (B.8)

The density of numbers varies in such a way that when e - 1023 = 52, then

$$v = (-1)^{s} \left( 1 + \sum_{j=1}^{52} m_j 2^{-j} \right) 2^{52}$$
 (B.9)

$$= \pm \left(2^{52} + \sum_{j=1}^{52} m_j 2^{-j} 2^{52}\right) \tag{B.10}$$

$$= \pm \left(2^{52} + \sum_{j=1}^{52} m_j 2^{52-j}\right) \tag{B.11}$$

$$\stackrel{k=52-j}{=} \pm \left(2^{52} + \sum_{k=51}^{0} m_{52-k} 2^k\right)$$
 (B.12)

which are all integers in the range  $2^{52} \le |v| < 2^{53}$ . When e - 1023 = 53, then the same calculation gives

$$v \stackrel{k=53-j}{=} \pm \left(2^{53} + \sum_{k=52}^{1} m_{53-k} 2^k\right)$$
 (B.13)

which are every second integer in the range  $2^{53} \le |v| < 2^{54}$ , and so on for larger values of e. When e - 1023 = 51, the same calculation gives,

$$v \stackrel{k=51-j}{=} \pm \left( 2^{51} + \sum_{k=50}^{-1} m_{51-k} 2^k \right)$$
 (B.14)

which is a distance between numbers of 1/2 in the range  $2^{51} \le |v| < 2^{52}$ , and so on for smaller values of e. Thus we may conclude that the distance between numbers in the interval  $2^n \le |v| < 2^{n+1}$  is  $2^{n-52}$ , for  $-1022 = 1 - 1023 \le n < 2046 - 1023 = 1023$ . For subnormals, the distance between numbers are

$$v = (-1)^s \left(\sum_{j=1}^{52} m_j 2^{-j}\right) 2^{-1022}$$
(B.15)

$$= \pm \left(\sum_{j=1}^{52} m_j 2^{-j} 2^{-1022}\right) \tag{B.16}$$

$$= \pm \left(\sum_{j=1}^{52} m_j 2^{-j-1022}\right) \tag{B.17}$$

$$\stackrel{k=-j-1022}{=} \pm \left( \sum_{j=-1023}^{-1074} m_{-k-1022} 2^k \right)$$
 (B.18)

which gives a distance between numbers of  $2^{-1074} \simeq 10^{-323}$  in the range  $0 < |v| < 2^{-1022} \simeq 10^{-308}$ .

### C | Commonly used character sets

Letters, digits, symbols and space are the core of how we store data, write programs, and communicate with computers and each other. These symbols are in short called characters, and represent a mapping between numbers, also known as codes, and a pictorial representation of the character. E.g., the ASCII code for the letter 'A' is 65. These mappings are for short called character sets, and due to differences in natural languages and symbols used across the globe, many different character sets are in use. E.g., the English alphabet contains the letters 'a' to 'z', which is shared by many other European languages, but which have other symbols and accents. For example, Danish has further the letters 'æ', 'ø', and 'å'. Many non-European languages have completely different symbols, where the Chinese character set is probably the most extreme, and some definitions contains 106,230 different characters albeit only 2,600 are included in the official Chinese language test at highest level.

Presently, the most common character set used is Unicode Transformation Format (UTF), whose most popular encoding schemes are 8-bit (UTF-8) and 16-bit (UTF-16). Many other character sets exist, and many of the later build on the American Standard Code for Information Interchange (ASCII). The ISO-8859 codes were an intermediate set of character sets that are still in use, but which is greatly inferior to UTF. Here we will briefly give an overview of ASCII, ISO-8859-1 (Latin1), and UTF.

#### C.1 ASCII

The American Standard Code for Information Interchange (ASCII) [8], is a 7 bit code tuned for the letters of the english language, numbers, punctuation symbols, control codes and space, see Tables C.1 and C.2. The first 32 codes are reserved for non-printable control characters to control printers and similar devices or to provide meta-information. The meaning of each control character is not universally agreed upon.

· American Standard Code for Information Interchange

· ASCII

The code order is known as  $ASCIIbetical\ order$ , and it is sometimes used to perform arithmetic on codes, e.g., an upper case letter with code c may be converted to lower case by adding 32 to its code. The ASCIIbetical order also has consequence for sorting, i.e., when sorting characters according to their ASCII code, 'A' comes before 'a', which comes before the symbol '{'}.

· ASCIIbetical order

#### C.2 ISO/IEC 8859

The ISO/IEC 8859 report http://www.iso.org/iso/catalogue_detail?csnumber=28245 defines 10 sets of codes specifying up to 191 codes and graphic characters using 8 bits. Set 1, also known as ISO/IEC 8859-1, Latin alphabet No. 1 or *Latin1*, covers many European languages and is designed to be compatible with ASCII, such that code for the printable characters in ASCII are the same in ISO 8859-1. Table C.3 shows the characters above 7e. Codes 00-1f and 7f-9f are undefined in ISO 8859-1.

Latin1

x0+0x	00	10	20	30	40	50	60	70
00	NUL	DLE	SP	0	0	P		p
01	SOH	DC1	!	1	A	Q	a	q
02	STX	DC2	"	2	В	R	b	r
03	ETX	DC3	#	3	С	S	С	s
04	EOT	DC4	\$	4	D	Т	d	t
05	ENQ	NAK	%	5	E	U	e	u
06	ACK	SYN	&	6	F	V	f	V
07	BEL	ETB	,	7	G	W	g	W
08	BS	CAN	(	8	Η	X	h	X
09	HT	EM	)	9	I	Y	i	У
0A	$_{ m LF}$	SUB	*	:	J	Z	j	Z
0B	VT	ESC	+	;	K	[	k	{
0C	$\operatorname{FF}$	FS	,	<	L	\	1	
0D	CR	GS	_	=	M	]	m	}
0E	SO	RS		>	N	^	n	~
0F	SI	US	/	?	O	_	О	DEL

Table C.1: ASCII

#### C.3 Unicode

Unicode is a character standard defined by the Unicode Consortium, http://unicode.org as the Unicode Standard. Unicode allows for 1,114,112 different codes. Each code is called a code point, which represents an abstract character. However, not all abstract characters require a unit of several code points to be specified. Code points are divided into 17 planes each with  $2^{16} = 65,536$  code points. Planes are further subdivided into named blocks. The first plane is called the Basic Multilingual plane and its block of the first 128 code points is called the Basic Latin block and are identical to ASCII, see Table C.1, and code points 128-255 is called the Latin-1 Supplement block, and are identical to the upper range of ISO 8859-1, see Table C.3. Each code-point has a number of attributes such as the Unicode general category. Presently more than 128,000 code points are defined covering 135 modern and historic writing systems, and obtained at http://www.unicode.org/Public/UNIDATA/UnicodeData.txt, which includes the code point, name, and general category.

A Unicode code point is an abstraction from the encoding and the graphical representation of a character. A code point is written as "U+" followed by its hexadecimal number, and for the Basic Multilingual plane 4 digits are used, e.g., the code point with the unique name LATIN CAPITAL LETTER A has the Unicode code point "U+0041", and is in this text it is visualized as 'A'. More digits are used for code points of the remaining planes.

The general category is used to specify valid characters that not necessarily have a visual representation but possibly transforms text. Some categories and their letters in the first 256 code points are shown in Table C.5.

To store and retrieve code points, they must be encoded and decoded. A common encoding is UTF-8, which encodes code points as 1 to 4 bytes, and which is backward-compatible with ASCII and ISO 8859-1. Hence, in all 3 coding systems the character with code 65 represents the character 'A'. Another popular encoding scheme is UTF-16, which encodes characters as 2 or 4 bytes, but which is not backward-compatible with ASCII or ISO 8859-1. UTF-16 is used internally in many compilers, interpreters and operating systems.

- · Unicode Standard
- · code point
- · blocks
- · Basic Multilingual plane
- $\cdot$ Basic Latin block
- · Latin-1 Supplement block
- · Unicode general category

UTF-8

UTF-16

Code	Description
NUL	Null
SOH	Start of heading
STX	Start of text
ETX	End of text
EOT	End of transmission
ENQ	Enquiry
ACK	Acknowledge
$\operatorname{BEL}$	Bell
BS	Backspace
HT	Horizontal tabulation
LF	Line feed
VT	Vertical tabulation
FF	Form feed
CR	Carriage return
SO	Shift out
SI	Shift in
DLE	Data link escape
DC1	Device control one
DC2	Device control two
DC3	Device control three
DC4	Device control four
NAK	Negative acknowledge
SYN	Synchronous idle
ETB	End of transmission block
CAN	Cancel
EM	End of medium
SUB	Substitute
ESC	Escape
FS	File separator
GS	Group separator
RS	Record separator
US	Unit separator
SP	Space
DEL	Delete

Table C.2: ASCII symbols.

x0+0x	80	90	A0	В0	C0	D0	E0	F0
00			NBSP	0	À	Đ	à	ð
01			i	土	Á	Ñ	á	ñ
02			¢	2	Â	Ò	â	ò
03			£	3	Ã	Ó	$ ilde{ ext{a}}$	ó
04			¤	,	Ä	Ô	ä	ô
05			¥	$\mu$	Å	Õ	å	õ
06				¶	Æ	Ö	æ	ö
07			§	•	Ç	×	ç	÷
08			••	3	È	Ø	è	Ø
09			©	1	É	Ù	é	ù
0a			a	Q	Ê	Ú	ê	ú
0b			«	<b>»</b>	Ë	Û	ë	û
0c			_	$\frac{1}{4}$	Ì	Ü	ì	ü
0d			SHY	$ \begin{array}{r} \frac{1}{4} \\ \frac{1}{2} \\ \frac{3}{4} \end{array} $	Í	Ý	í	ý
0e			R	$\frac{3}{4}$	Î	Þ	î	þ
Of			-	i	Ϊ	ſß	ï	ÿ

Table C.3: ISO-8859-1 (latin1) non-ASCII part. Note that the codes 7f – 9f are undefined.

Code	Description
NBSP	Non-breakable space
SHY	Soft hypen

Table C.4: ISO-8859-1 special symbols.

General	Code points	Name
category	•	
Lu	$U+0041-U+005A,\ U+00C0-U+00D6,$	Upper case letters
	$\mathrm{U}+00\mathrm{D}8-\mathrm{U}+00\mathrm{D}\mathrm{E}$	
Ll	$U+0061-U+007A,\ U+00B5,$	Lower case letter
	$U+00DF-U+00F6,\ U+00F8-U+00FF$	
$\operatorname{Lt}$	None	Digraphic letter, with first part uppercase
Lm	None	Modifier letter
Lo	$\mathrm{U}{+}00\mathrm{AA},\mathrm{U}{+}00\mathrm{BA}$	Gender ordinal indicator
Nl	None	Letterlike numeric character
Pc	$\mathrm{U}{+}005\mathrm{F}$	Low line
Mn	None	Nonspacing combining mark
Mc	None	Spacing combining mark
Cf	$\mathrm{U}{+}00\mathrm{AD}$	Soft Hyphen

Table C.5: Some general categories for the first 256 code points.

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## Index

(), 15	dot notation, 33
. [], 32	double, 268
;;, 17	double, 22
	downcasting, 24
American Standard Code for Information Inter-	
change, 271	echo, $263$ , $266$
and, 27	encapsulate, 17
ASCII, 271	error message, 16
ASCIIbetical order, 31, 271	escape sequences, 21
	event-driven programming, 9
base, 20, 268	exception, 29
bash, 264	exclusive or, 30
Basic Latin block, 272	executable file, 11
Basic Multilingual plane, 272	exn, 19
basic types, 19	expression, 9, 14, 24
binary number, 20, 268	•
binary operator, 24	float, 19
binary64, 268	float32, 22
binding, 14	floating point number, 20
bit, 20, 268	floating point numbers, 15
blocks, 272	folder, 260
bool, 19	format string, 15
byte, 268	fractional part, 20, 24
byte[], 22	function, 9, 14, 17
byte, 22	functional programming, 9
call-back functions, 9	graphical user interface, 260
cd, 262, 265	GUI, 260
char, 19, 21	GC1, 200
	hexadecimal number, 20, 268
character, 21	nexacconical number, 20, 200
class, 24, 33	IEEE 754 double precision floating-point format,
code point, 21, 272	268
compile mode, 11	imperative programming, 8
console, 11, 260	implementation file, 11
debugging 12 16	infix notation, 24
debugging, 13, 16	int, 19
decimal, 22	int16, 22
decimal number, 20, 268	int32, 22
decimal point, 15, 20, 268	int64, 22
declarative programming, 9	int8, 22
del, 263	integer, 20
digit, 20, 268	9 /
dir, 262	integer division, 29
directory, 260	integer number, 14
do, 14	interactive mode, 11
do-hinding 14	it. 15. 19

INDEX 286

keyword, 14	single, 22
	source code, 11
Latin-1 Supplement block, 272	state, 9
Latin1, 271	statement, 9
least significant bit, 268	statements, 14
let, 14	string, $15$ , $21$
let-binding, 14	string, 19
lexeme, 17	structured programming, 9
lexical scope, 16	subnormals, 270
library file, 11	
literal, 19	terminal, 260
literal type, 22	truth table, 27
ls, 265	type, 15, 19
member, 24	type declaration, 15
method, 33	type inference, 13, 15
	typecasting, 23
mkdir, 262, 266	in+16 22
most significant bit, 268	uint16, 22
move, 266	uint32, 22
mv, 266	uint64, 22
namespace, 24	uint8, 22 underflow, 28
NaN, 270	
newline, 21	Unicode, 21 Unicode, general entergry, 272
not, 27	Unicode general category, 272 Unicode Standard, 272
not a number, 270	
	unit, 15, 19 unit-testing, 13
obj, 19	upcasting, 24
object, 9, 33	- 9
Object-oriented programming, 9	UTF-16, 272 UTF-8, 272
octal number, 20, 268	011-0, 212
operands, 24	val, 15
operator, 25	verbatim, 23
or, 27	, 32 33 33 33 3
overflow, 28	whitespace, 21
	whole part, 20, 24
precedence, 25	Windows command line, 260
prefix operator, 25	word, 268
printfn, 14	
1- 960	xor, 30
reals, 268	
recursion, 25	
redirection, 263, 266	
remainder, 29	
rm, 266	
rmdir, 263, 266	
rounding, 24	
runtime error, 29	
sbyte, 22	
scientific notation, 20	
script file, 11	
script-fragment, 11, 17	
scripts, 11	
search path, 264	
search-path, 266	
signature file, 11	
· · · · · · · · · · · · · · · · · · ·	