Inflation and GDP Dynamics in Production Networks: A Sufficient Statistics Approach*

Hassan Afrouzi[†] Saroj Bhattarai[‡]

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Abstract

We provide closed-form solutions for inflation and GDP dynamics in multi-sector New Keynesian economies with arbitrary input-output linkages. The sufficient statistic for dynamics of sectoral prices is the principal square root of the Leontief matrix, appropriately adjusted for the duration of price spells across sectors. By applying this statistic to the U.S. economy, we evaluate how the interplay between sticky prices and production linkages governs the effects of aggregate and sectoral shocks. We rank U.S. sectors according to their contributions to CPI inflation and the GDP gap. Notably, sectoral inflation in the Oil and Gas Extraction industry causes an immediate increase in CPI inflation that rapidly dissipates. In contrast, inflation in the Semiconductor Manufacturing industry leads to a persistent rise in CPI inflation. Finally, we show that production linkages increase monetary non-neutrality by four times due to strategic complementarities introduced by the network.

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Key Words: Production networks; Multi-sector model; Sufficient statistics; Inflation dynamics; Real effects of monetary policy; Sectoral shocks

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[†]Columbia University. hassan.afrouzi@columbia.edu. 1105A IAB, 420 West 118th St, New York, NY 10027.
[‡]UT Austin. saroj.bhattarai@austin.utexas.edu. 2225 Speedway, Stop C3100, UT Austin, Austin, TX 78712.

1 Introduction

Understanding how production linkages impact the *dynamics* of sectoral prices, inflation, and GDP is a key objective in macroeconomics, which has become even more pressing in light of the recent cascade of supply chain disruptions, oil price fluctuations, and monetary policy responses. In particular, a key question is: In an economy with production networks, what determines each sector's contribution to the dynamics of sectoral prices, aggregate inflation, and GDP?

This paper analytically solves a dynamic model with unrestricted input-output linkages across multiple sectors, where firms engage in forward-looking and staggered Calvo-type pricing decisions. We provide closed-form solutions for dynamic responses of prices, inflation, and GDP to aggregate and sectoral shocks, and derive sufficient statistics that can be measured using data. We then explore, analytically and quantitatively, how the interaction of sticky prices and production networks yields new insights into the transmission of shocks.

Model Setup and Theoretical Results. Our core objective in setting up the model is to analytically characterize inflation and GDP dynamics with arbitrary production networks and heterogeneous price stickiness across sectors in general equilibrium. To do so, we simplify the setup in other dimensions, most notably by taking a first-order approximation around an efficient steady-state.

Our first theoretical result is that the equilibrium path of *all* sectoral prices in response to *any* path of shocks is uniquely determined by a system of differential equations that relates the evolution of sectoral prices to the deviations of these prices from their flexible price equilibrium counterparts. Using this representation, we show that *all* model parameters affect the dynamics of sectoral prices exclusively through the Leontief matrix, adjusted appropriately for the duration of price spells across sectors.

Deriving the explicit solution to this system under the appropriate boundary conditions, we find that the sufficient statistic for the dynamic responses of all sectoral prices to aggregate and sectoral shocks is the principal square root of the duration-adjusted Leontief (PRDL) matrix. This general solution enables us to derive the impulse response functions (IRFs) of sectoral prices to monetary and sectoral TFP/wedge shocks in closed form. Notably, we find that all

IRFs decay exponentially at the rate of the PRDL matrix, indicating that it fully captures the persistence of sectoral price dynamics. Expanding on this finding, we investigate the effects of monetary shocks as well as the spillover effects of shocks that emanate from one sector and propagate to others through the network over time.

First, we show that the cumulative impulse response of GDP to a monetary shock is determined by the PRDL matrix and household expenditure shares. Second, production networks, via the PRDL matrix, create strategic complementarities that amplify inflation persistence in response to monetary shocks. These effects are asymmetric across sectors, as monetary shocks distort relative prices, with price stickiness and production linkages jointly determining these responses. All else equal, sectors that spend more on stickier suppliers have more persistent responses and disproportionally affect the tail response of inflation to monetary shocks.

Next, we study the aggregate effects of persistent but transitory sectoral shocks in this economy. The PRDL matrix also governs the dynamics of aggregate inflation and GDP to these shocks, once they have propagated through the *inverse* Leontief matrix as in static models without nominal rigidities. Our analytical solutions, therefore, shed light on two separate roles of the Leontief matrix in the propagation of sectoral shocks. While the inverse Leontief matrix governs the propagation of sectoral shocks through the network on impact—as in static models—the PRDL matrix governs their endogenous propagation across sectors *through* time and thereby, affects the persistence of inflation and GDP responses to these shocks.

With our analytical solutions at hand, we also study how our sufficient statistics relate to the conventional measure of non-neutrality and inflation persistence in New Keynesian models, i.e., the slope of the Phillips curve. It is well-known that in one-sector economies the slope of the Phillips curve is the elasticity of inflation to demand shocks and theoretically captures the inflationary pressures in such models. We confirm this link in a one-sector version of our economy but show that in the multi-sector version, the slope of the Phillips curve is no longer sufficient either for the magnitude or the direction of non-neutrality and inflation persistence. The key behind this observation is that in multi-sector economies the Phillips curve is also affected by relative price distortions whose effects are not captured by its slope. To illustrate this point, we offer a counterexample with two multi-sector economies where the economy with the *steeper* Phillips curve also exhibits *higher* monetary non-neutrality. Later in our quantitative analysis, we find this result to be relevant for our calibration of the U.S. economy as well.[1]

Finally, to uncover the economic forces encoded by our sufficient statistic, we use eigenvalue perturbation theory to approximate the eigenvalues of the PRDL matrix based on the model's primitives. This approach allows us to analytically prove exactly how production linkages at the micro-level amplify (1) monetary non-neutrality and inflation persistence in response to monetary shocks and (2) create spillover effects from sectoral shocks to aggregate inflation. We later show that this perturbation is remarkably accurate for the measured production network of the U.S. economy. This enables us to match each eigenvalue to a specific sector and provide a comprehensive ranking of sectors in terms of their contributions to inflation and GDP dynamics.

Quantitative Results. Using data on input-output tables, price adjustment frequencies, and consumption shares, we construct our sufficient statistic for the U.S. and quantify the importance of production networks for the propagation of shocks. We find that strategic complementarities introduced by production linkages quadruple the cumulative response of GDP to a monetary shock and double the half-life of the consumer price index (CPI) inflation response.

Underneath this aggregate inflation response is a rich distribution of sectoral inflation responses. The study of these sectoral responses allows us to identify industries that disproportionately affect aggregate monetary non-neutrality. For instance, we use our approximate eigenvalues to match the top three dominant eigenvalues of the PRDL matrix to specific sectors that contribute the most to the tail response of inflation. In a quantitative exercise, we show that although the combined consumption share of these three sectors is essentially zero, dropping them from the analysis reduces monetary non-neutrality by 16 percent.

We then quantitatively study the sectoral origins of aggregate fluctuations, with a specific focus on the pass-through of sectoral shocks to aggregate inflation. This is motivated by recent

[1] For a more detailed discussion of this issue, we refer the reader to Hazell, Herreno, Nakamura, and Steinsson (2022) who carefully layout assumptions under which a two-sector economy admits an aggregate Phillips curve whose slope is still sufficient for the elasticity of inflation to demand shocks. As they note, more generally, such a result does not hold in multi-sector economies.

increases in U.S. aggregate inflation that have underlying sources in sectoral shocks, such as supply chain disruptions, oil price increases, and semiconductor machinery price increases. Specifically, we consider sectoral shocks that lead to a one percent increase in sectoral inflation and then study the pass-through of such sectoral inflation increases on aggregate inflation.[2] We first identify which sectors are the main sources of high impact response of aggregate inflation, while removing the mechanical effect coming from the size of the sector.[3] We find that the Oil and Gas Extraction industry is among the top sectors in causing a high initial impact on aggregate inflation, driven by its role as an input to many sectors in the economy.

We next comprehensively rank sectors based on their contribution to the persistence of aggregate inflation dynamics. Relying on our perturbed eigenvalues, we show that the key quantitative determinant of these effects is an input-output adjusted duration of price spells within these sectors. To provide concrete examples, this adjusted duration in Oil and Gas Extraction industry is relatively small due to the high price flexibility in this sector. Thus, a shock to this sector does not lead to persistent aggregate inflation effects *even though* it is an input to many sectors. The Semiconductor Manufacturing Machinery industry, in contrast, has very persistent aggregate inflation effects because its adjusted duration is relatively larger.

Finally, we show that our ranking of sectors based on their contribution to persistent aggregate inflation dynamics is also important for the real effects of sectoral shocks because those persistent effects translate to larger aggregate GDP (gap) effects. This implies that it is precisely the shocks to sectors that are sources of persistent aggregate inflation dynamics that will have a bigger impact on the real macroeconomy. As such, a sectoral inflation increase of one percent in the Semiconductor Manufacturing Machinery industry is much more distortionary than a sectoral inflation increase of one percent in the Oil and Gas Extraction industry.

Related Literature. This paper contributes to the vast literature on the propagation of shocks in multi-sector New Keynesian models with heterogeneous price stickiness and production

[2]We interpret these sectoral shocks as negative supply shocks.

[3]This metric provides an evaluation of the spillover of sectoral inflation to aggregate inflation due to input-output linkages for in the absence of such linkages, this pass-through metric would be zero for all sectors. We are thus capturing what are sometimes called second-round effects of sectoral inflation increases.

linkages.

First, our results on the response of inflation and GDP to monetary shocks are connected to the work of Carvalho (2006) and Nakamura and Steinsson (2010) who showed how heterogeneous price stickiness amplifies monetary non-neutrality in time- and state-dependent models respectively. Furthermore, our results on how production linkages amplify these responses are connected to the insights of Blanchard (1983) and, in particular, Basu (1995) who showed this amplification result stems from strategic complementarities in a setting where the final good in the economy is also used as input for production. More recently, Carvalho, Lee, and Park (2021); Rubbo (2020); La'O and Tahbaz-Salehi (2022); Woodford (2021); Pasten, Schoenle, and Weber (2020) study input-output linkages in models with heterogeneous price stickiness across sectors.

We contribute to this literature by providing an analytical characterization of sectoral inflation dynamics together with sufficient statistics for the dynamics of inflation and GDP to both aggregate and sectoral shocks. Using perturbation theory, we also unpack the economic determinants of this sufficient statistic. Moreover, in addition to studying the effects of an aggregate monetary policy shock, we also study the effects of a sectoral supply shock, which is motivated by supply chain issues during the pandemic as well as recent commodity price increases. These analytical characterizations then inform our quantitative results, as they show how the persistence of aggregate inflation gets affected by production networks, which has important implications for effects on aggregate GDP.

In deriving sufficient statistics for real effects of monetary policy shocks in sticky-price models with strategic complementarities, our paper connects to recent work by Wang and Werning (2021) and Alvarez, Lippi, and Souganidis (2022).[4] Our main contribution to this strand of the literature is that we consider a multi-sector New Keynesian model with input-output linkages. Our environment is, however, simpler on other dimensions: it does not model oligopolistic behavior within a sector (as in Wang and Werning, 2021) or feature menu costs (as in Alvarez, Lippi, and Souganidis, 2022). Our sufficient statistic, which is the principal square

[4]Our analytical results are also related to previous work by Alvarez, Le Bihan, and Lippi (2016); Baley and Blanco (2021) who do not explicitly model strategic complementarities but provide analytical results in settings with idiosyncratic shocks and menu costs.

root of the frequency-adjusted Leontief matrix, is in close correspondence to, and complements, the ones in Wang and Werning (2021); Alvarez, Lippi, and Souganidis (2022), as they all share an underlying transmission mechanism based on strategic complementarities in pricing decisions.

Finally, there is by now a rich literature in static settings that considers various formulations of exogenous production networks in macroeconomic models. For example, Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), Baqaee and Farhi (2020), and Bigio and La'O (2020) are important contributions and Carvalho (2014); Carvalho and Tahbaz-Salehi (2019) are comprehensive surveys of the literature. These papers study how sectoral shocks propagate to the aggregate economy to cause business cycles and how (if at all) they affect aggregate total factor productivity or the labor wedge.[5] Our sectoral shock results are related to these ideas, but we focus on how they affect the dynamics of aggregate inflation and, thereby, the response of GDP in an economy with nominal pricing frictions. Yet other papers, such as Taschereau-Dumouchel (2020), consider endogenous production networks and study phenomena such as cascades. We use exogenous production networks, thereby using a simpler setting, but we study a dynamic model with sticky prices.[6]

Outline. Section 2 presents the general framework and the environment of our model. Section 3 derives sufficient statistics results for the responses of sectoral and aggregate prices and aggregate GDP. Section 4 unpacks the economic properties of our sufficient statistics, first, in a minimal example, and second, in a more general setting using perturbation theory. Section 5 constructs our sufficient statistics using U.S. data and presents quantitative results on inflation and GDP responses to monetary and sectoral shocks. Section 6 presents theoretical and quantitative extensions. Section 7 concludes.

2 Model

2.1. Environment. Time is continuous and is indexed by $t \in \mathbb{R}_+$. The economy consists of a representative household, monetary and fiscal authorities, and n sectors with input-output

[5] See, also, Guerrieri, Lorenzoni, Straub, and Werning (2020) which focuses on the plausibility and spillover effects of Keynesian supply shocks.

[6] Our work is related to Liu and Tsyvinski (2021), which analyzes the dynamics of real variables in a model with adjustment costs in inputs. In contrast, we consider an economy with nominal rigidities but no adjustment costs.

linkages. In each sector $i \in [n] \equiv \{1, 2, \dots, n\}$, a unit measure of monopolistically competitive firms use labor and goods from all sectors to produce and supply to a competitive final good producer within the same industry. These final goods are sold to the household and other industries.

2.1.1. Household. The representative household demands the final goods produced by each industry, supplies labor in a competitive market, and holds nominal bonds with nominal yield i_t. Household's preferences over consumption C and labor supply L is $U(C) - V(L)$, where U and V are strictly increasing with Inada conditions, and $U''(.) < 0$, $V''(.) > 0$. Household solves:

$$\max_{\{(C_{i,t})_{i\in[n]}, L_t, B_t, M_t\}_{t\geq 0}} \int_0^\infty e^{-\rho t}[U(C_t) - V(L_t)]\mathrm{d}t \tag{1}$$

$$s.t. \quad \sum_{i\in[n]} P_{i,t}C_{i,t} + \dot{B}_t \leq W_t L_t + i_t B_t + \text{Profits}_t + T_t, \qquad C_t \equiv \Phi(C_{1,t}, \dots, C_{n,t}) \tag{2}$$

Here, $\Phi(.)$ defines the consumption index C_t over the household's consumption from sectors $(C_{i,t})_{i\in[n]}$. It is degree one homogenous, strictly increasing in each $C_{i,t}$, satisfying Inada conditions. L_t is labor supply at wage W_t, $P_{i,t}$ is sector i's final good price, B_t is demand for nominal bonds, Profits_t denote all firms' profits rebated to the household, and T_t is a lump-sum tax.

2.1.2. Monetary and Fiscal Policy. For our baseline, we assume monetary authority directly controls the path of nominal GDP, $\{M_t \equiv P_t C_t\}_{t\geq 0}$, where P_t is the consumer price index (CPI).[7] A Taylor rule extension is in Section 6.2. The fiscal authority taxes or subsidizes intermediate firms' sales in each sector i at a possibly time-varying rate $\tau_{i,t}$, lump-sum transferred back to the household. A *wedge shock* to sector i is an *unexpected* disturbance in that sector's taxes.

2.1.3. Final Good Producers. A competitive final good producer in each industry i buys from a continuum of intermediate firms in its sector, indexed by $ij : j \in [0,1]$, and produces a final sectoral good using a CES production function. The profit maximization problem of this firm is:

$$\max_{(Y^d_{ij,t})_{j\in[0,1]}} P_{i,t}Y_{i,t} - \int_0^1 P_{ij,t}Y^d_{ij,t}\mathrm{d}j \quad s.t. \quad Y_{i,t} = \left[\int_0^1 (Y^d_{ij,t})^{1-\sigma_i^{-1}}\mathrm{d}j\right]^{\frac{1}{1-\sigma_i^{-1}}} \tag{3}$$

where $Y^d_{ij,t}$ is the producer's demand for variety ij at price $P_{ij,t}$, $Y_{i,t}$ is its production at price $P_{i,t}$, and $\sigma_i > 1$ is the substitution elasticity across varieties in i. Thus, demand for variety ij is:

$$Y^d_{ij,t} = \mathscr{D}(P_{ij,t}/P_{i,t}; Y_{i,t}) \equiv Y_{i,t}\left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\sigma_i} \quad \text{where} \quad P_{i,t} = \left[\int_0^1 P_{ij,t}^{1-\sigma_i}\mathrm{d}j\right]^{\frac{1}{1-\sigma_i}} \tag{4}$$

[7]Such policy can be implemented by a cash-in-advance constraint (e.g. La'O and Tahbaz-Salehi, 2022), money in utility (e.g. Golosov and Lucas, 2007) or nominal GDP growth targeting (e.g. Afrouzi and Yang, 2019).

Final good producers define a unified good for each industry and have zero value added due to being competitive and constant returns to scale (CRS) production.

2.1.4. Intermediate Goods Producers. The intermediate good producer ij uses labor as well as the sectoral goods as inputs and produces with the following CRS production function:

$$Y^s_{ij,t} = Z_{i,t} F_i(L_{ij,t}, X_{ij,1,t}, \ldots, X_{ij,n,t}) \tag{5}$$

where $Z_{i,t}$ is sector i's Hicks-neutral productivity, $L_{ij,t}$ is firm ij's labor demand, and $X_{ij,k,t}$ is its demand for sector k's final good. The function F_i is strictly increasing in all arguments with Inada conditions. The firm's total cost for producing output Y, given prices $\mathbf{P}_t \equiv (W_t, P_{i,t})_{i\in[n]}$, is:

$$\mathscr{C}_i(Y; \mathbf{P}_t, Z_{i,t}) \equiv \min_{(L_{ij,t}, X_{ij,k,t})_{k\in[n]}} W_t L_{ij,t} + \sum_{k\in[n]} P_{k,t} X_{ij,k,t} \quad s.t. \quad Z_{i,t} F_i(L_{ij,t}, X_{ij,1,t}, \ldots, X_{ij,n,t}) \geq Y \tag{6}$$

In each sector i, firms set their prices under a Calvo friction, where i.i.d. price change opportunities arrive at Poisson rates θ_i. Given its cost in Equation (6) and its demand in Equation (4), a firm ij that has the opportunity to change its price at time t chooses its *reset price*, denoted by $P^{\#}_{ij,t}$, to maximize the expected net present value of its profits until the next price change:

$$P^{\#}_{ij,t} \equiv \arg\max_{P_{ij,t}} \int_0^\infty \theta_i e^{-(\theta_i h + \int_0^h i_{t+s} \mathrm{d}s)} \left[(1-\tau_{i,t}) P_{ij,t} \mathscr{D}(P_{ij,t}/P_{i,t+h}; Y_{i,t+h}) - \mathscr{C}_i(Y^s_{ij,t+h}; \mathbf{P}_{t+h}, Z_{i,t+h}) \right] \mathrm{d}h$$
$$s.t. \quad Y^s_{ij,t+h} \geq \mathscr{D}(P_{ij,t}/P_{i,t+h}; Y_{i,t+h}), \quad \forall h \geq 0 \tag{7}$$

where $\theta_i e^{-\theta_i h}$ is the duration density of the next price change, $e^{-\int_0^h i_{t+h} \mathrm{d}s}$ is the discount rate based on nominal rates, and $\tau_{i,t}$ is the tax/subsidy rate on sales. Were prices flexible, maximizing net present value of profits would be equivalent to choosing *desired* prices, denoted by $P^*_{ij,t}$, that maximized firms' static profits within every instant. Desired prices solve:

$$P^*_{ij,t} \equiv \arg\max_{P_{ij,t}} (1-\tau_{i,t}) P_{ij,t} \mathscr{D}(P_{ij,t}/P_{i,t}; Y_{i,t}) - \mathscr{C}_i(Y^s_{ij,t}; \mathbf{P}_t, Z_{i,t}) \quad s.t. \quad Y^s_{ij,t} \geq \mathscr{D}(P_{ij,t}/P_{i,t}; Y_{i,t}) \tag{8}$$

2.1.5. Equilibrium Definition. An equilibrium is a set of allocations for households and firms, monetary and fiscal policies, and prices such that: (1) given prices and policies, the allocations are optimal for households and firms, and (2) markets clear. A precise definition is in Appendix B.

2.2. Log-Linearized Approximation. We log-linearize this economy around an efficient steady-state, derivations of which are in Appendix C. For our baseline analysis, we use Golosov and Lucas (2007)'s preferences, $U(C) - V(L) = \log(C) - L$, which simplifies the analytical expressions. In Section 6.1, we consider a more general specification. Going forward, small letters denote the log deviations of their corresponding variables from their steady-state values.

2.2.1. Sectoral Prices. While prices are staggered within sectors, the Calvo assumption implies that we can fully characterize aggregate sectoral prices by desired and reset prices.

First, desired prices are equal to firms' marginal costs, $(mc_{i,t})_{i\in[n]}$, up to a wedge that captures markups or other distortions, $(\omega_{i,t})_{i\in[n]}$. With input-output linkages, $mc_{i,t}$ depends on the aggregate wage, w_t, sectoral prices, $(p_{k,t})_{k\in[n]}$, and the sectoral productivity, $z_{i,t}$:

$$p^*_{i,t} \equiv \omega_{i,t} + mc_{i,t}, \quad mc_{i,t} \equiv \alpha_i w_t + \textstyle\sum_{k\in[n]} a_{ik} p_{k,t} - z_{i,t}, \quad \omega_{i,t} \equiv \log\left(\frac{\sigma_i}{\sigma_i - 1} \times \frac{1}{1-\tau_{i,t}}\right) \tag{9}$$

where α_i and $a_{i,k}$ are sector i's firms' labor share and expenditure share on sector k's final good in the steady-state, respectively. Thus, the steady-state *input-output matrix* is $\mathbf{A} \equiv [a_{ik}] \in \mathbb{R}^{n\times n}$.[8]

Second, the reset price in sector i is the average of all *future* desired prices, discounted at rate ρ and the probability density of the time between price changes, $e^{-(\rho+\theta_i)h}$:

$$p^{\#}_{i,t} = (\rho + \theta_i) \int_0^\infty e^{-(\rho+\theta_i)h} p^*_{i,t+h} \mathrm{d}h \tag{10}$$

Finally, given sector i's initial aggregate price at $t = 0$, $p_{i,0^-}$, the *aggregate* sectoral price $p_{i,t}$ is an average of the *past* reset prices, weighted by the density of time between price changes:

$$p_{i,t} = \theta_i \int_0^t e^{-\theta_i h} p^{\#}_{i,t-h} \mathrm{d}h + e^{-\theta_i t} p_{i,0^-} \tag{11}$$

2.2.2. Aggregate Price and GDP. The household's demand for goods defines the aggregate Consumer Price Index (CPI) as the expenditure share weighted average of sectoral prices:

$$p_t = \textstyle\sum_{i\in[n]} \beta_i p_{i,t}, \quad \text{with} \quad \sum_{i\in[n]} \beta_i = 1 \tag{12}$$

where $\boldsymbol{\beta} = (\beta_i)_{i\in[n]}$ is the vector of the household's expenditure shares in the efficient steady-state.

The aggregate GDP, y_t, is equal to aggregate consumption and is given by the difference between the nominal GDP, m_t, and the CPI, p_t: $y_t \equiv m_t - p_t$. Fully elastic labor supply implies that the wage is equal to nominal demand:[9]

$$w_t = p_t + y_t = m_t \quad \text{(fully elastic labor supply)} \tag{13}$$

[8] Baqaee and Farhi (2020) emphasize the important distinction between cost-based and sales-based input-output matrices and Domar weights. In an efficient equilibrium, like the one we linearize around, the two are the same.

[9] See Section 6.1 for an extension to the case with partially elastic labor supply.

2.2.3. Equilibrium in the Approximated Economy. Given a path for $(\boldsymbol{\omega}_t, \boldsymbol{z}_t, m_t)_{t\geq 0}$, an equilibrium is a path for GDP, wage and prices, $\vartheta \equiv \{y_t, w_t, p_t, (p^*_{i,t}, p^\#_{i,t}, p_{i,t})_{i\in[n]}\}_{t\geq 0}$, such that given a vector of initial sectoral prices, $\mathbf{p}_{0^-} = (p_{i,0^-})_{i\in[n]}$, ϑ solves Equations (9) to (13).

2.2.4. Flexible Prices and GDP. Consider a counterfactual economy where all prices are flexible. By Equation (9), we can derive *flexible prices* of this economy, denoted by $\mathbf{p}^f_t \in \mathbb{R}^n$, as:

$$\mathbf{p}^f_t = w_t \boldsymbol{\alpha} + \mathbf{A}\mathbf{p}^f_t + \boldsymbol{\omega}_t - \boldsymbol{z}_t \quad \Rightarrow \quad \mathbf{p}^f_t = m_t \mathbf{1} + \boldsymbol{\Psi}(\boldsymbol{\omega}_t - \boldsymbol{z}_t) \tag{14}$$

where $\boldsymbol{\alpha} \equiv (\alpha_i)_{i\in[n]}$ contains labor shares, $\mathbf{1}$ is the vector of ones, and $\boldsymbol{\Psi} \equiv (\mathbf{I} - \mathbf{A})^{-1}$ is the inverse Leontief matrix. A key observation is that $\mathbf{p}^f_t$ is only a function of exogenous shocks and model parameters. We can also derive the *flexible price GDP*, y^f_t, in this counterfactual economy as:

$$y^f_t = m_t - \boldsymbol{\beta}^\top \mathbf{p}^f_t = \underbrace{\boldsymbol{\lambda}^\top \boldsymbol{z}_t}_{\text{aggregate TFP}} - \underbrace{\boldsymbol{\lambda}^\top \boldsymbol{\omega}_t}_{\text{labor wedge}}, \qquad \boldsymbol{\lambda} \equiv \left(\tfrac{P_i Y_i}{PC}\right)_{i\in[n]} = \boldsymbol{\Psi}^\top \boldsymbol{\beta} \tag{15}$$

where $\boldsymbol{\lambda}$ is the vector of Domar weights in the steady state.[10] Equation (15) shows that two terms determine flexible GDP around the efficient steady-state up to first order: (1) the aggregate TFP, which is the Domar-weighted sectoral productivities (Hulten, 1978), (2) the labor wedge caused by distortions, which is the Domar-weighted wedges across sectors (Bigio and La'O, 2020).

3 Sufficient Statistics

Here, we solve sectoral price dynamics in closed form and derive our sufficient statistics results. All the proofs are in Appendix A.

3.1. Dynamics of Prices. Let $\mathbf{p}_t \equiv (p_{i,t})_{i\in[n]}$, $\mathbf{p}^\#_t \equiv (p^\#_{i,t})_{i\in[n]}$ and $\mathbf{p}^*_t \equiv (p^*_{i,t})_{i\in[n]}$ be the vectors of sectoral aggregate, reset and desired prices, respectively. Using Equations (9) and (14):[11]

$$\mathbf{p}^*_t = (\mathbf{I} - \mathbf{A})\mathbf{p}^f_t + \mathbf{A}\mathbf{p}_t \tag{16}$$

where $\mathbf{p}^f_t$ is the vector of flexible equilibrium prices in Equation (14). Equation (16) shows that firms' desired prices across sectors is a convex combination of *exogenous* flexible equilibrium prices and *endogenous* sectoral prices in the sticky price economy, with the input-output matrix $\mathbf{A}$ fully capturing the *strategic complementarities* induced by production linkages across the

[10] The Domar weight of a sector i, λ_i, is the ratio of its total sales to the household's total nominal expenditures.

[11] Using $\boldsymbol{\alpha} = (\mathbf{I} - \mathbf{A})\mathbf{1}$, the vector form of Equation (9) is $\mathbf{p}^*_t = (\mathbf{I} - \mathbf{A})(\mathbf{1}w_t + \boldsymbol{\Psi}(\boldsymbol{\omega}_t - \boldsymbol{z}_t)) + \mathbf{A}\mathbf{p}_t$.

economy (Blanchard, 1983; Basu, 1995; La'O and Tahbaz-Salehi, 2022).

Accordingly, reset and sectoral prices in Equations (10) and (11) solve:

$$\vec{\boldsymbol{\pi}}_t^\# \equiv \tfrac{\mathrm{d}}{\mathrm{d}t}\mathbf{p}_t^\# = (\rho\mathbf{I}+\boldsymbol{\Theta})(\mathbf{p}_t^\# - \mathbf{p}_t^*), \qquad \text{forward-looking with} \qquad \lim_{t\to\infty} e^{-(\rho\mathbf{I}+\boldsymbol{\Theta})t}\mathbf{p}_t^\# = 0, \tag{17}$$

$$\vec{\boldsymbol{\pi}}_t \equiv \tfrac{\mathrm{d}}{\mathrm{d}t}\mathbf{p}_t = \boldsymbol{\Theta}(\mathbf{p}_t^\# - \mathbf{p}_t), \qquad \text{backward-looking with} \qquad \mathbf{p}_0 = \mathbf{p}_{0^-} \tag{18}$$

Here, $\vec{\boldsymbol{\pi}}_t^\#$ and $\vec{\boldsymbol{\pi}}_t$ are the *inflation rates* in reset and aggregate prices across sectors, respectively. $\boldsymbol{\Theta} = \mathrm{diag}(\theta_i) \in \mathbb{R}^{n\times n}$ is a diagonal matrix, with its i'th diagonal entry representing the frequency of price adjustments in sector i.[12] The memorylessness of the Poisson price adjustments (Calvo assumption) allows us to represent this system only in terms of sectoral prices, $\mathbf{p}_t$:

Proposition 1. Sectoral prices evolve according to the following set of differential equations:

$$\tfrac{\mathrm{d}}{\mathrm{d}t}\vec{\boldsymbol{\pi}}_t = \rho\vec{\boldsymbol{\pi}}_t + \boldsymbol{\Theta}(\rho\mathbf{I}+\boldsymbol{\Theta})(\mathbf{I}-\mathbf{A})(\mathbf{p}_t - \mathbf{p}_t^f), \qquad \text{with } \mathbf{p}_0 = \mathbf{p}_{0^-} \text{ given.} \tag{19}$$

We discuss the main implications of Proposition 1 in the following four remarks.

Remark 1. Equation (19) represents the *sectoral Phillips curves* of this economy in vector form, linking changes in inflation to the *gap* between prices and their counterparts in a flexible economy. The matrix $\boldsymbol{\Gamma} \equiv \boldsymbol{\Theta}(\rho\mathbf{I}+\boldsymbol{\Theta})(\mathbf{I}-\mathbf{A})$—the Leontief matrix, $\mathbf{I}-\mathbf{A}$, adjusted by a quadratic form of price adjustment frequencies, $\boldsymbol{\Theta}(\rho\mathbf{I}+\boldsymbol{\Theta})$—encodes the slopes of these Phillips curves.

Equation (19) differs from the usual representations of Phillips curves in terms of an output gap term. Such an equivalent representation exists for Equation (19) as well, which we discuss in detail in Section 3.3. However, we first present these Phillips curves in terms of nominal price gaps as it is the most straightforward way to demonstrate the following point and derive our analytical results.

Remark 2. Sectoral Phillips curves, with boundary conditions $\mathbf{p}_0 = \mathbf{p}_{0^-}$ and non-explosive prices, uniquely pin down the path of sectoral prices for a given path of flexible prices $(\mathbf{p}_t^f)_{t\geq 0}$.

The key to this observation is that the only endogenous variables in the system of second-order differential equations in Equation (19) are nominal prices and their inflation rates, $\mathbf{p}_t$ and $\vec{\boldsymbol{\pi}}_t$, with $\mathbf{p}_t^f$ acting as an *exogenous* forcing term. Intuitively, nominal prices in the sticky price

[12] Throughout this draft, we frequently use the exponential function of square matrices, defined by its corresponding power series: $\forall \mathbf{X} \in \mathbb{R}^{n\times n}$, $e^{\mathbf{X}} \equiv \sum_{k=0}^{\infty} \mathbf{X}^k/k!$, which is well-defined because these series always converge.

economy should adjust towards their flexible levels, $\mathbf{p}_t^f$. This is formalized in Equation (19), where inflation in sectoral prices depends solely on the time series of nominal price gaps, $\mathbf{p}_t - \mathbf{p}_t^f$.

Remark 3. All shocks $(\boldsymbol{\omega}_t, \boldsymbol{z}_t, m_t^s)_{t\geq 0}$ affect price dynamics *only* through flexible prices, $(\mathbf{p}_t^f)_{t\geq 0}$.

The observation in Remark 3 demonstrates the power of expressing inflation dynamics in terms of *nominal price gaps*. It implies that solving for the dynamics of prices for a given path of $\mathbf{p}_t^f$ is equivalent to having characterized impulse response functions of *all* the prices in the economy to all three types shocks–TFP, markup/wedge, and monetary–in a unified framework.

Remark 4. All parameters affect the dynamics of sectoral prices *only* through the *duration-adjusted Leontief matrix*, $\mathbf{\Gamma}$, and the household's discount rate, ρ.

Intuitively, the dynamics of prices in a production network depend on the frequency of price adjustments (captured by $\mathbf{\Theta}$) and how these shocks propagate through input-output linkages (captured by the Leontief matrix). Proposition 1 formalizes this intuition by showing that the interaction of these two mechanisms is captured by $\mathbf{\Gamma}$ and ρ. Importantly, note that substitution elasticities across different inputs have no impact on price dynamics at the first order. This is due to the flatness of the marginal cost function with respect to inputs at the optimum by Shephard's Lemma (see, e.g., Baqaee and Farhi, 2020).

Given that ρ is usually calibrated close to zero, we will assume $\rho = 0$ going forward.[13] This makes $\mathbf{\Gamma}$ the sole object through which model parameters affect prices, allowing us to fully focus on the economic intuition behind its effects. We now state the main result of this section.

Proposition 2. Suppose $\mathbf{p}_t^f$ is piece-wise continuous and is bounded,[14] and let $\rho = 0$. Then, given $\mathbf{p}_t^f$ and a vector of initial prices $\mathbf{p}_{0^-}$, the *principal square root of the duration-adjusted*

[13] With an annual interest rate of 0.04, $\rho \approx \ln(1.04)/12 \approx 0.003$ at a monthly frequency. However, there is a literature that reinterprets a larger ρ as a parameter for disciplining how myopic firms are in price-setting (see, e.g., Gabaix, 2020). See Minton and Wheaton (2022) for a discussion of myopia in production networks.

[14] In our setting with perfect foresight, piece-wise continuity ensures that $\mathbf{p}_t^f$ is Riemann integrable with unexpected shocks introducing at most countable jumps in flexible prices. The boundedness assumption is not restrictive with zero trend inflation. With trend inflation, boundedness should be replaced with exponential order.

Leontief (PRDL) matrix, $\sqrt{\boldsymbol{\Gamma}}$, exists and is a sufficient statistic for dynamics of sectoral prices:[15]

$$\mathbf{p}_t = \underbrace{e^{-\sqrt{\boldsymbol{\Gamma}}t}\mathbf{p}_{0^-} + \sqrt{\boldsymbol{\Gamma}}e^{-\sqrt{\boldsymbol{\Gamma}}t}\int_0^t \sinh(\sqrt{\boldsymbol{\Gamma}}h)\mathbf{p}_h^f \mathrm{d}h}_{\text{inertial effect of past prices due to stickiness}} + \underbrace{\sqrt{\boldsymbol{\Gamma}}\sinh(\sqrt{\boldsymbol{\Gamma}}t)\int_t^\infty e^{-\sqrt{\boldsymbol{\Gamma}}h}\mathbf{p}_h^f \mathrm{d}h}_{\text{forward looking effect of future prices}} \tag{20}$$

Drawing on Remarks 1 to 4, Proposition 2 presents the analytical solution for dynamics of *all* sectoral prices. This solution specifically highlights the interplay between the forward-looking nature of pricing decisions and the backward-looking nature of aggregation Equations (17) and (18). While firms take the future path of $\mathbf{p}_t^f$ into account when setting prices, aggregate prices also depend on the past path of $\mathbf{p}_t^f$ due to the persistence of stickiness over time.

Furthermore, Proposition 2 illustrates that it is not $\boldsymbol{\Gamma}$ itself that is crucial for price dynamics, but rather its *principal square root*, which is the square root of $\boldsymbol{\Gamma}$ all of whose eigenvalues have positive real parts. From an economic standpoint, this square root emerges as a result of the system's dual forward-looking and backward-looking nature. Firms take the future and past paths of flexible prices into account when adjusting prices so that these paths affect dynamics partially insofar as such changes were not incorporated at the time of adjustment. Additionally, the principal square root is the relevant square root because it is the one that adheres to stability boundary conditions. Proving the existence of $\sqrt{\boldsymbol{\Gamma}}$ mainly relies on the economic assumptions that all sectors have strictly positive labor shares and price adjustment frequencies.[16]

Next, we explore the analytical solution presented in Proposition 2 by examining the IRFs of sectoral prices, CPI inflation, and GDP (gap) to monetary, sectoral TFP, and wedge shocks.

3.2. Impulse Response Functions. Using Proposition 2, we can obtain IRFs to specific shocks by plugging in the appropriate path of such shocks. Suppose the economy is in its steady state at $t = 0^-$ (left limit at $t = 0$), meaning that exogenous variables $(\boldsymbol{z}_t, \boldsymbol{\omega}_t, m_t) = (\boldsymbol{z}_{0^-}, \boldsymbol{\omega}_{0^-}, m_{0^-})$ for all $t < 0$ and all prices are at their flexible level: $\mathbf{p}_{0^-} - \mathbf{p}_{0^-}^f = 0$.

[15] The hyperbolic sine of a square matrix $\mathbf{X}$ is defined as $\sinh(\mathbf{X}) \equiv (e^{\mathbf{X}} - e^{-\mathbf{X}})/2$.

[16] This ensures that the inverse Leontief matrix exists and has positive real entries (see, Carvalho and Tahbaz-Salehi, 2019, p. 639). We can then show $\boldsymbol{\Gamma}$ is a M-matrix: By Theorem 2.3 in (Berman and Plemmons, 1994, p. 134, condition N_{38}), this is true if $\boldsymbol{\Gamma}$ is inverse-positive; i.e., $\boldsymbol{\Gamma}^{-1} \geq 0$ elementwise. Since $\boldsymbol{\Theta}(\rho\mathbf{I} + \boldsymbol{\Theta})$ is invertible because $\theta_i > 0, \forall i$, and $\mathbf{I} - \mathbf{A}$ is invertible because inverse Leontief exists, $\boldsymbol{\Gamma}^{-1}$ exists and is the infinite sum of positive matrices: $\boldsymbol{\Gamma}^{-1} = \sum_{n=0}^{\infty}\mathbf{A}^n(\rho\mathbf{I} + \boldsymbol{\Theta})^{-1}\boldsymbol{\Theta}^{-1} \geq 0$. Finally, having shown that $\boldsymbol{\Gamma}$ is a non-singular M-matrix, we can apply Theorem 5 in Alefeld and Schneider (1982) which shows that every non-singular M-matrix has exactly one M-matrix as its square root, which is also its principal square root by properties of M-matrices.

3.2.1. Monetary Shocks. An expansionary monetary shock is a one-time unexpected but permanent increase in nominal GDP: $m_t = m_{0^-} + \delta_m, \forall t \geq 0$ where δ_m denotes the shock size. The implied path for $\mathbf{p}_t^f$ is $\mathbf{p}_t^f = \mathbf{p}_{0^-}^f + \delta_m \mathbf{1}$, where $\mathbf{1}$ is a vector of ones.

Proposition 3. The impulse response functions of sectoral prices, $\mathbf{p}_t$; CPI inflation, $\pi_t = \boldsymbol{\beta}^\intercal \vec{\boldsymbol{\pi}}_t$; GDP, y_t; and GDP gap, $\tilde{y}_t \equiv y_t - y_t^f$ to an expansionary monetary shock are given by:

$$\tfrac{\partial}{\partial \delta_m} \mathbf{p}_t = (\mathbf{I} - e^{-\sqrt{\boldsymbol{\Gamma}} t})\mathbf{1}, \qquad \tfrac{\partial}{\partial \delta_m} \pi_t = \boldsymbol{\beta}^\intercal \sqrt{\boldsymbol{\Gamma}} e^{-\sqrt{\boldsymbol{\Gamma}} t} \mathbf{1}, \qquad \tfrac{\partial}{\partial \delta_m} y_t = \tfrac{\partial}{\partial \delta_m} \tilde{y}_t = \boldsymbol{\beta}^\intercal e^{-\sqrt{\boldsymbol{\Gamma}} t} \mathbf{1} \tag{21}$$

Proposition 3 shows: (1) The only relevant objects for the sectoral price, inflation, and GDP dynamics are $\sqrt{\boldsymbol{\Gamma}}$ and expenditure shares $\boldsymbol{\beta}$. Thus, we can compute these IRFs for the input-output structure of the U.S. economy once we construct $\sqrt{\boldsymbol{\Gamma}}$ and the expenditure shares $\boldsymbol{\beta}$ from the data. (2) Although *relative* sectoral prices converge back to the steady-state in the long run, the aggregate monetary shock distorts these relative prices on the transition path. These distortions are also captured by $\sqrt{\boldsymbol{\Gamma}}$ and thus are measurable. (3) $\sqrt{\boldsymbol{\Gamma}}$ also captures the degree of monetary non-neutrality in the economy since GDP response to a monetary shock is zero in the flexible economy. We see this in the cumulative impulse response (CIR) of GDP, obtained by integrating the area under its impulse response function:

$$\mathrm{CIR}_{\tilde{y},m} \equiv \int_0^\infty \tfrac{\partial}{\partial \delta_m} \tilde{y}_t \mathrm{d}t = \boldsymbol{\beta}^\intercal \sqrt{\boldsymbol{\Gamma}}^{-1} \mathbf{1} \tag{22}$$

3.2.2. TFP and Wedge Shocks. How do sectoral prices, CPI and GDP respond to a TFP/wedge shock in sector $i \in [n]$? To answer this question, we consider the following shock to sector i:

$$\omega_{i,t} - z_{i,t} = \omega_{i,0^-} - z_{i,0^-} + e^{-\phi_i t} \delta_z^i, \quad \forall t \geq 0 \tag{23}$$

Here, a positive δ_z^i captures a negative TFP or a positive wedge shock to sector i that decays at the rate $\phi_i \geq 0$. We note that $\phi_i = 0$ would correspond to a permanent TFP/Wedge shock while a positive ϕ_i denotes a temporary disturbance that disappears at rate ϕ_i. The implied path for $\mathbf{p}_t^f$, given such as shock, is $\mathbf{p}_t^f = \mathbf{p}_{0^-}^f + e^{-\phi_i t} \delta_z^i \boldsymbol{\Psi} \mathbf{e}_i$, where $\boldsymbol{\Psi}$ is the inverse Leonteif matrix and $\mathbf{e}_i$ is the i'th standard basis vector. Economically, $\boldsymbol{\Psi} \mathbf{e}_i$ is a measure of sector i's *upstreamness* as it measures how much sector i, directly and indirectly, supplies to other sectors.

Proposition 4. Suppose $\phi_i \notin \text{eig}(\sqrt{\mathbf{\Gamma}})$.[17] Then, the IRFs of sectoral prices, $\mathbf{p}_t$; CPI inflation, $\pi_t = \boldsymbol{\beta}'\vec{\boldsymbol{\pi}}_t$; GDP, y_t; and GDP gap, $\tilde{y}_t = y_t - y_t^f$, to a TFP/wedge shock in sector i are given by:

$$\frac{\partial}{\partial \delta_z^i}\mathbf{p}_t = (e^{-\phi_i t}\mathbf{I} - e^{-\sqrt{\mathbf{\Gamma}}t})(\mathbf{I} - \phi_i^2\mathbf{\Gamma}^{-1})^{-1}\mathbf{\Psi}\mathbf{e}_i, \qquad \frac{\partial}{\partial \delta_z^i}\pi_t = \boldsymbol{\beta}^\top(\sqrt{\mathbf{\Gamma}}e^{-\sqrt{\mathbf{\Gamma}}t} - \phi_i e^{-\phi_i t}\mathbf{I})(\mathbf{I} - \phi_i^2\mathbf{\Gamma}^{-1})^{-1}\mathbf{\Psi}\mathbf{e}_i \quad (24)$$

$$\frac{\partial}{\partial \delta_z^i}y_t = \boldsymbol{\beta}^\top(e^{-\sqrt{\mathbf{\Gamma}}t} - e^{-\phi_i t}\mathbf{I})(\mathbf{I} - \phi_i^2\mathbf{\Gamma}^{-1})^{-1}\mathbf{\Psi}\mathbf{e}_i, \quad \frac{\partial}{\partial \delta_z^i}\tilde{y}_t = \boldsymbol{\beta}^\top(e^{-\sqrt{\mathbf{\Gamma}}t} - \phi_i^2\mathbf{\Gamma}^{-1}e^{-\phi_i t})(\mathbf{I} - \phi_i^2\mathbf{\Gamma}^{-1})^{-1}\mathbf{\Psi}\mathbf{e}_i \quad (25)$$

The most important observation from Proposition 4 is that, aside from the exogenous dynamics introduced by the shock ($e^{-\phi_i t}$), all endogenous dynamics are captured by $e^{-\sqrt{\mathbf{\Gamma}}}$. This is best illustrated in the limiting case when the shock is almost permanent $\phi_i \downarrow 0$:

$$\frac{\partial}{\partial \delta_z^i}\mathbf{p}_t|_{\phi_i\downarrow 0} = (\mathbf{I} - e^{-\sqrt{\mathbf{\Gamma}}t})\mathbf{\Psi}\mathbf{e}_i, \quad \frac{\partial}{\partial \delta_z^i}\pi_t|_{\phi_i\downarrow 0} = \boldsymbol{\beta}^\top\sqrt{\mathbf{\Gamma}}e^{-\sqrt{\mathbf{\Gamma}}t}\mathbf{\Psi}\mathbf{e}_i, \quad \frac{\partial}{\partial \delta_z^i}\tilde{y}_t|_{\phi_i\downarrow 0} = \boldsymbol{\beta}^\top e^{-\sqrt{\mathbf{\Gamma}}t}\mathbf{\Psi}\mathbf{e}_i \quad (26)$$

This observation uncovers *two* separate roles of the Leontief matrix in the dynamic economy.

Remark 5. The inverse Leontief matrix, $\mathbf{\Psi}$, determines the *static* propagation of TFP/wedge shocks by passing them through the network ($\mathbf{e}_i \to \mathbf{\Psi}\mathbf{e}_i$). The principal square root, $\sqrt{\mathbf{\Gamma}}$, determines the *dynamic* propagation of these shocks over time ($\mathbf{\Psi}\mathbf{e}_i \to e^{-\sqrt{\mathbf{\Gamma}}t}\mathbf{\Psi}\mathbf{e}_i$).

Moreover, in response to TFP/wedge shocks, the GDP response combines both the response under flexible prices and the response of the GDP gap under sticky prices. To separate these, we define the *GDP gap* as $\tilde{y}_t \equiv y_t - y_t^f$ and decompose the CIR of GDP to its two components:

$$\text{CIR}_{y,z_i} \equiv \int_0^\infty \frac{\partial}{\partial \delta_z^i} y_t \mathrm{d}t = \underbrace{-\phi_i^{-1}\lambda_i}_{\substack{\text{CIR}_{y^f,z_i} \equiv \text{Flexible GDP Response} \\ \text{(Domar-weighted cumulative TFP)}}} + \underbrace{\boldsymbol{\beta}^\top(\phi_i\mathbf{I} + \sqrt{\mathbf{\Gamma}})^{-1}\mathbf{\Psi}\mathbf{e}_i}_{\text{CIR}_{x,z_i} \equiv \text{Cumulative GDP Gap Response}} \quad (27)$$

This decomposition provides intuition for the limiting case when $\phi_i \to 0$. Note that in this case, the flexible GDP CIR explodes because, with a permanent shock to TFP, the economy diverges from the initial steady-state (which is why we are only considering the case when $\phi_i \to 0$ and not $\phi_i = 0$). However, the GDP gap CIR is not explosive in this limit as the effects of sticky prices are only temporary deviations from the flexible price response:

$$\text{CIR}_{x,z^i}|_{\phi_i\to 0} = \boldsymbol{\beta}^\top\sqrt{\mathbf{\Gamma}}^{-1}\mathbf{\Psi}\mathbf{e}_i \quad (28)$$

Equations (22) and (28) have a similar interpretation: Both show that with permanent shocks, the CIR of the GDP gap is the inner product of the expenditure weighted $\sqrt{\mathbf{\Gamma}}^{-1}$ and the instan-

[17] In words, assume that the persistence parameter ϕ_i is not an eigenvalue of the $\sqrt{\mathbf{\Gamma}}$ matrix. This is a technical assumption that simplifies analytical derivations. A limit of IRFs can be taken and are valid when $\phi_i \in \text{eig}(\mathbf{\Gamma})$.

taneous pass-through of a shock to firms' flexible prices ($\mathbf{1}$ for monetary shocks and $\boldsymbol{\Psi}\mathbf{e}_i$ for TFP/wedge shocks). We next turn to unpacking the economic interpretation of $\sqrt{\boldsymbol{\Gamma}}$.

3.3. Discussion of Aggregate and Sectoral Phillips Curves. In Proposition 1, we derived sectoral Phillips curves in terms of inflation and nominal price gaps, after which we discussed how this representation allows us to derive analytical results. In this section, we relate our results to more conventional representations of Phillips curves in New Keynesian (NK) economies, which involve output gaps, combined with real wage gaps in sticky-price and sticky-wage models (e.g., Woodford, 2003; Galí, 2008), or relative price gaps in multi-sector economies (e.g., Aoki, 2001; Benigno, 2004).

As the Phillips curve is an endogenous relationship between inflation and nominal price changes, there is no unique representation of it. But, there are two fundamental reasons why the literature has converged on one specific representation, involving output gaps. To discuss these reasons and relate our results to them, we first derive a representation involving output gaps in our model.

Consider the sectoral Phillips curves in Proposition 1 and define the relative prices $\mathbf{q}_t \equiv \mathbf{p}_t - p_t\mathbf{1}$ as the vector of sectoral prices relative to the CPI price index in log form. Moreover, define the GDP gap $\tilde{y}_t \equiv y_t - y_t^f$ as the gap between the sticky price equilibrium GDP and the flexible price equilibrium GDP. Then, it is straightforward to re-write Equation (19) as:

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{\boldsymbol{\pi}}_t = \rho\vec{\boldsymbol{\pi}}_t + \boldsymbol{\Gamma}(\mathbf{q}_t - \mathbf{q}_t^f) - \boldsymbol{\Gamma}\mathbf{1}\tilde{y}_t \tag{29}$$

which shows that the nominal price gaps can be decomposed into *relative* price gaps and a term that involves the aggregate GDP gap of this economy.[18] Finally, the aggregate Phillips curve for the CPI inflation can be derived by multiplying this with $\boldsymbol{\beta}^\intercal$ from the left:

$$\frac{\mathrm{d}}{\mathrm{d}t}\pi_t = \rho\pi_t + \boldsymbol{\beta}^\intercal\boldsymbol{\Gamma}(\mathbf{q}_t - \mathbf{q}_t^f) - \boldsymbol{\beta}^\intercal\boldsymbol{\Gamma}\mathbf{1}\tilde{y}_t \tag{31}$$

The coefficient on the GDP gap in Equation (31), $\boldsymbol{\beta}^\intercal\boldsymbol{\Gamma}\mathbf{1}$, is usually referred to as the slope of

[18]This equation is perhaps more familiar in its discrete-time form. To see this, note that $\frac{\mathrm{d}}{\mathrm{d}t}\vec{\boldsymbol{\pi}}_t = \lim_{\mathrm{d}t\to 0}(\vec{\boldsymbol{\pi}}_{t+\mathrm{d}t} - \vec{\boldsymbol{\pi}}_t)/\mathrm{d}t$. For small $\mathrm{d}t$ one can substitute this to get the familiar discrete time version under perfect foresight:

$$\vec{\boldsymbol{\pi}}_t = (1-\rho\mathrm{d}t)\vec{\boldsymbol{\pi}}_{t+\mathrm{d}t} - \boldsymbol{\Gamma}(\mathbf{q}_t - \mathbf{q}_t^f)\mathrm{d}t + \boldsymbol{\Gamma}\mathbf{1}\tilde{y}_t\mathrm{d}t \tag{30}$$

the Phillips curve and it holds considerable significance in the literature for the following two reasons.

First, in one-sector economies, where the term involving relative price gaps is zero,[19] the slope uniquely captures all inflationary forces of the model: in order for inflation to increase in the economy, *output gap* must rise, with $\boldsymbol{\beta}^\intercal \boldsymbol{\Gamma}\mathbf{1} > 0$ determining the degree to which changes in output gap translate into changes in inflation. This gives an important theoretical role to the slope of the Phillips curve as it becomes a *sufficient statistic* for inflation and output dynamics in one-sector NK models. We show below that this is also the case in our economy when there is only one sector, but also discuss how with multiple sectors, the slope of the aggregate Phillips curve not only is insufficient for characterizing inflation dynamics, but can also be misleading if the role of relative prices is not taken into account.

The second reason the slope of the Phillips curve holds such significance in the literature is its empirical interpretation. In certain economies, such as one sector NK model or particular multi-sector economies such as the one in Hazell, Herreno, Nakamura, and Steinsson (2022) where the term involving relative price gaps is zero, the slope of the Phillips curve is the *elasticity* of inflation to *demand* shocks. In other words, if one wants to estimate the inflationary effects of a pure demand shock, in these classes of models, the slope of the Phillips curve is the only statistic that one needs to estimate. This is the underlying motivation for the vast empirical literature that estimates the slope of the Phillips curve.

To fully appreciate this view, it is useful to think about the Phillips curve as the supply curve of the economy relating nominal price changes to the GDP gap, given the optimal behavior of price-setters. Such an interpretation raises the simultaneity problem in the estimation of the price-output relationship as a second equation from the demand side, usually, the Euler equation from the household side also relates inflation to the output gap. Note that such simultaneous equations can be written in multiple ways as they represent endogenous relationships between prices and quantities, but only one representation corresponds to how demand shocks affect

[19]To see this, note that in a one-sector economy the relative price $\mathbf{q}_t = p_t - p_t = 0$ because the CPI is equal to the final price of the economy's single sector. Similarly, $\mathbf{q}_t^f = 0$. Thus, for a one-sector economy $\frac{\mathrm{d}}{\mathrm{d}t}\pi_t = -\boldsymbol{\beta}^\intercal \boldsymbol{\Gamma}\mathbf{1}\tilde{y}_t$.

inflation and it does so uniquely only when the term involving relative price gaps is zero. For instance, Hazell, Herreno, Nakamura, and Steinsson (2022) achieve this by assuming GHH preferences which eliminate the term involving relative price gaps in their aggregate Phillips curve and ensures that the slope of the regional Phillips curves is the same as the slope of the aggregate Phillips curve. As they observe, without such assumptions, multi-sector economies do not necessarily admit aggregate Phillips curves with only inflation and output gap terms.

Rubbo (2020) addresses this issue from an alternative perspective and shows that while the aggregate Phillips curve in multi-sector economies with production networks involves terms other than the GDP gap, there always exists a composite price index whose corresponding Phillips curve only includes inflation in that price index and the GDP gap. Rubbo (2020) refers to this price as the "divine coincidence index" because a policy that stabilizes this index in an economy with only TFP shocks also automatically stabilizes GDP gap.[20] Note however that beyond a policy that targets a zero GDP gap, stabilizing the divine coincidence price index neither stabilizes the aggregate price level defined by the household's consumption basket nor does it necessarily relate to an optimal price index that a central bank would target once it considers the household's welfare.[21]

But what can the slope of the aggregate Phillips curve for CPI inflation, $\boldsymbol{\beta}^\top \boldsymbol{\Gamma} \mathbf{1}$, as given in Equation (31) tell us about inflation and GDP dynamics in multi-sector economies with production networks? More specifically, does it still hold significance in predicting the inflationary pressures of demand shocks or does it fail to capture the confounding effects of relative price distortions? Our analytical results allow us to answer this question by comparing the slope of the aggregate Phillips curve with the closed-form solutions for inflation and output dynamics. In particular, we construct an example where the slope is not only insufficient in predicting the

[20] Rubbo (2020) proves this result generally for all production network economies. A special case of this result in sticky price and sticky wage models is also discussed in Galí (2008) (see Equation (33) and the discussion on p. 137). To see Rubbo (2020)'s point in our framework, define the divine coincidence price index as $p_t^{DC} \equiv \boldsymbol{\beta}^\top \boldsymbol{\Gamma}^{-1} \mathbf{p}_t / (\boldsymbol{\beta}^\top \boldsymbol{\Gamma}^{-1} \mathbf{1})$. Using Equation (29), one can show that inflation in this price index evolves according to

$$\frac{\mathrm{d}}{\mathrm{d}t} \pi_t^{DC} = \rho \pi_t^{DC} - \frac{1}{\boldsymbol{\beta}^\top \boldsymbol{\Gamma}^{-1} \mathbf{1}} \tilde{y}_t \tag{32}$$

[21] We refer the reader to La'O and Tahbaz-Salehi (2022); Rubbo (2020) for a more detailed discussion of optimal price indices for monetary policy in production network economies.

magnitudes of these dynamics, but is also misleading in predicting the direction of the effects of monetary shocks (the demand shock in our model) on inflation and GDP.

To this end, consider a minimal counterexample with the following two economies: (1) A *horizontal economy* where price change frequencies across $n \geq 1$ sectors are heterogeneous, with no input-output linkages ($\mathbf{A} = \mathbf{0}$). It follows that $\mathbf{\Gamma}_1 = \text{diag}(\theta_1^2, \ldots, \theta_n^2)$, where θ_i is the frequency of price changes in sector i. It also immediately follows that $\sqrt{\mathbf{\Gamma}_1} = \mathbf{\Theta}$.[22] (2) A *homogenuous economy*, also with no input-output linkages, where price change frequencies are homogeneous across sectors given by the expenditure-weighted average of frequencies in the horizontal economy, i.e., $\bar{\theta} = \sum_i \beta_i \theta_i$. We then obtain the following result.

Proposition 5. Consider the horizontal and homogeneous economies above. Then, 1. When $n = 1$, in both horizontal and homogenous economies, monetary non-neutrality is higher when the Phillips curve is flatter. 2. With $n > 1$, the horizontal economy experiences higher monetary non-neutrality *even though* that it has a steeper Phillips curve than the homogeneous economy.

4 Unbundling the Sufficient Statistic $\sqrt{\mathbf{\Gamma}}$

We have shown that one can recover the full dynamics of sectoral prices by measuring $\sqrt{\mathbf{\Gamma}}$. In this section, we study the economic forces encoded by $\sqrt{\mathbf{\Gamma}}$ through two complementary approaches. First, we present a two-sector model that embeds the key mechanisms in the simplest possible way. Second, we perturb an *arbitrary* n-sector economy around benchmarks that give economic interpretation to eigenvalues and eigenvectors of $\sqrt{\mathbf{\Gamma}}$.

4.1. Minimal Two Sector Example. We start with a minimal example, constructed with two constraints in mind: (1) capture the main forces in the simplest possible economy, and (2) imbed simple versions of Basu (1995) and Carvalho (2006) to identify the new mechanisms at work.[23]

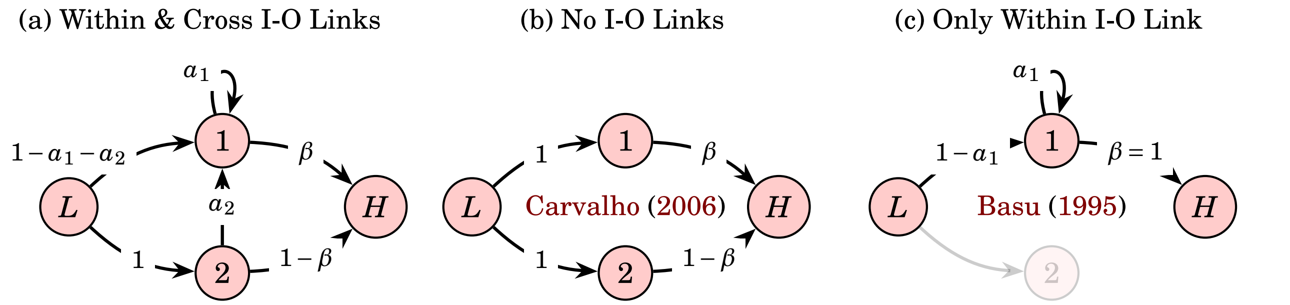
Sectors 1 and 2 have price adjustment frequencies θ_1 and θ_2. Sector 2 only uses labor, but sector 1 uses its own and Sector 2's goods, with expenditure shares a_1 and a_2. β is the

[22] Recall $\sqrt{\mathbf{\Gamma}}$ is the square root of $\mathbf{\Gamma}$ whose eigenvalues have positive real parts. So, for any i, we pick $\sqrt{\theta_i^2} = \theta_i$.

[23] Nakamura and Steinsson (2010) go further in investigating the role of heterogeneity across sectors with menu costs and also have an extension with Basu (1995) roundabout production. We take a step back from their analysis by modeling price rigidity through Calvo frictions. Instead, we focus on unrestricted input-output linkages across sectors.

household's expenditure share on sector 1 with the other $1-\beta$ going to sector 2. Figure 1a shows the network structure of this economy. Figures 1b and 1c shows how economies of Basu (1995) and Carvalho (2006) are represented within this economy.

Figure 1: Example of Two Sector Economies



Notes: Figure 1a draws our minimalistic example of two sector economies. Figures 1b and 1c shows how this class of two sector economies nest two prominant examples from the literature.

The input-output matrix, $\mathbf{A}$, the inverse Leontief matrix, $\mathbf{\Psi} = (\mathbf{I}-\mathbf{A})^{-1}$, the duration-adjusted Leontief matrix, $\mathbf{\Gamma} = \mathbf{\Theta}^2(\mathbf{I}-\mathbf{A})$, and its principal square root, $\sqrt{\mathbf{\Gamma}}$, of this economy are:

$$\mathbf{A} = \begin{bmatrix} a_1 & a_2 \\ 0 & 0 \end{bmatrix}, \mathbf{\Psi} = \begin{bmatrix} \frac{1}{1-a_1} & \frac{a_2}{1-a_1} \\ 0 & 1 \end{bmatrix}, \mathbf{\Gamma} = \begin{bmatrix} \theta_1^2(1-a_1) & -\theta_1^2 a_2 \\ 0 & \theta_2^2 \end{bmatrix}, \sqrt{\mathbf{\Gamma}} = \begin{bmatrix} \theta_1\sqrt{1-a_1} & -\frac{\theta_1^2 a_2}{\theta_1\sqrt{1-a_1}+\theta_2} \\ 0 & \theta_2 \end{bmatrix} \quad (33)$$

To simplify expressions, here we also assume that TFP/wedge shocks are almost permanent ($\phi \downarrow 0$). The $\phi > 0$ case, closed form expression for $e^{-\sqrt{\mathbf{\Gamma}}t}$, and other derivations are in Appendix D.

Step 1: One Sector with No Input-Output Linkages. Suppose $a_1 = a_2 = 0$ (so that there are no input-output linkages) and $\beta = 1$ (so that only sector 1 sells to the household). Thus, $\mathbf{\Psi} = 1$ and $\sqrt{\mathbf{\Gamma}} = \theta_1$. Using Propositions 3 and 4, the IRFs to all shocks are exponential with decay rate θ_1:

$$\frac{\partial}{\partial \delta_m}\pi_t = \frac{\partial}{\partial \delta_z}\pi_t\Big|_{\phi\downarrow 0} = \theta_1 e^{-\theta_1 t}, \qquad \frac{\partial}{\partial \delta_m}\tilde{y}_t = \frac{\partial}{\partial \delta_z}\tilde{y}_t\Big|_{\phi\downarrow 0} = e^{-\theta_1 t} \quad (34)$$

The key observation is that inflation and GDP are more persistent with more sticky prices.

Step 2: One Sector with Roundabout Production (Basu, 1995). Consider the same economy as in Step 1, but now allow $a_1 > 0$ so that sector 1 uses its output in roundabout production. Thus, $\mathbf{\Psi} = \frac{1}{1-a_1}$ and $\sqrt{\mathbf{\Gamma}} = \theta_1\sqrt{1-a_1}$. Using Propositions 3 and 4, GDP gap and CPI inflation IRFs are:

$$\frac{\partial}{\partial \delta_m}\pi_t = \theta_1\sqrt{1-a_1}e^{-\theta_1\sqrt{1-a_1}t}, \qquad \frac{\partial}{\partial \delta_m}\tilde{y}_t = e^{-\theta_1\sqrt{1-a_1}t} \quad (35)$$

$$\tfrac{\partial}{\partial \delta_z}\pi_t\big|_{\phi\downarrow 0} = \frac{1}{1-a_1}\times \theta_1\sqrt{1-a_1}\,e^{-\theta_1\sqrt{1-a_1}\,t}, \qquad \tfrac{\partial}{\partial \delta_z}\tilde{y}_t\big|_{\phi\downarrow 0} = \frac{1}{1-a_1}\times e^{-\theta_1\sqrt{1-a_1}\,t} \tag{36}$$

All responses are still exponential functions of time, but with *higher* persistence ($\theta_1\sqrt{1-a_1} < \theta_1$). This propagation happens because resetting firms do not fully adjust to their flexible prices due to strategic complementarities. We also see the *static* role of inverse Leontief, $\frac{1}{1-a_1}$, for TFP/wedge shocks as it scales the IRFs for *all* horizons. We can decompose the dynamic and static amplifications effects of roundabout production through the CIR of GDP gap:

$$\mathrm{CIR}_{x,\delta_m} = \underbrace{\frac{1}{\theta_1}}_{\substack{\text{duration of}\\\text{price spells}}} \times \underbrace{\frac{1}{\sqrt{1-a_1}}}_{\substack{\text{dynamic}\\\text{amplification}}}, \quad \mathrm{CIR}_{x,\delta_z} = \underbrace{\frac{1}{1-a_1}}_{\substack{\text{inverse Leontief}\\\text{(static amplification)}}} \times \underbrace{\frac{1}{\theta_1}}_{\substack{\text{duration of}\\\text{price spells}}} \times \underbrace{\frac{1}{\sqrt{1-a_1}}}_{\text{dynamic amplification}} \tag{37}$$

Step 3: Two Sectors with No Input-Output Linkages (Carvalho, 2006). Suppose $a_1 = a_2 = 0$. In this case, $\boldsymbol{\Psi} = \mathbf{I}$ and $e^{-\sqrt{\boldsymbol{\Gamma}}t} = \mathrm{diag}(e^{-\theta_1 t}, e^{-\theta_2 t})$. Using Propositions 3 and 4, IRFs of GDP and CPI inflation to monetary shocks are weighted sums of two exponentials decaying at rates θ_1 and θ_2:

$$\tfrac{\partial}{\partial \delta_m}\tilde{y}_t = \beta e^{-\theta_1 t} + (1-\beta)e^{-\theta_2 t} \Rightarrow \mathrm{CIR}_{\tilde{y},\delta_m} = \beta\theta^{-1} + (1-\beta)\theta_2^{-1}, \quad \tfrac{\partial}{\partial \delta_m}\pi_t = \beta\theta_1 e^{-\theta_1 t} + (1-\beta)\theta_2 e^{-\theta_2 t} \tag{38}$$

Key observations are: (1) heterogeneity amplifies monetary non-neutrality (apply Jensen's inequality to the convex IRFs); (2) CIR is expenditure-weighted duration of price stickiness; and (3) the more sticky sector is the "dominant" sector, meaning that the asymptotic behavior of inflation is determined by the sector with the lower price adjustment frequency. As for TFP/wedge shocks, since the two sectors are independent, there is no contagion across sectors.

Step 4: Two Sectors with Input-Output Linkages. Let us now consider the case of $a_2 > 0$; i.e., when there is input-output linkages *across* sectors. To condense expressions, we assume here that $a_1 = 0$. The more general case with $a_1 > 0$ as well as omitted IRFs are presented in Appendix D.

Monetary Shocks. By Proposition 3, IRFs of sectoral inflation rates to monetary shocks are:

$$\tfrac{\partial}{\partial \delta_m}\pi_{1,t} = \underbrace{\theta_1 e^{-\theta_1 t}}_{\text{Step 1 baseline}} \underbrace{+\, a_2\theta_1 \times \frac{\theta_1}{\theta_1+\theta_2} \times \frac{\theta_2 e^{-\theta_2 t} - \theta_1 e^{-\theta_1 t}}{\theta_1 - \theta_2}}_{\text{residual effect of the network}} \qquad \tfrac{\partial}{\partial \delta_m}\pi_{2,t} = \underbrace{\theta_2 e^{-\theta_2 t}}_{\text{Step 1 baseline}} \tag{39}$$

Sector 2's response is the same as Step 1 because it does not rely on the network for production. In contrast, sector 1's response includes an additional term that captures the residual effect of the network. This effect is negative initially and becomes positive after some $t^* > 0$. Intuitively, inflation is more sluggish in sector 1 with a higher a_2 because of sector 2's effect on sector

1's marginal cost, which propagates its stickiness. This behavior is also reflected in the CPI inflation through sector 1. We can formalize these statements by looking at the *impact response* of inflation in sector 1 at $t = 0$ as well as its *asymptotic behavior* as $t \to \infty$.

Corollary 1. An increase in sector 1's expenditure on sector 2's output; i.e., an increase in a_2, dampens the impact response of CPI inflation to monetary shocks and amplifies its persistence.

$$\underbrace{\frac{\partial}{\partial a_2}\left[\frac{\partial}{\partial \delta_m}\pi_t\Big|_{t=0}\right] = -\beta\frac{\theta_1^2}{\theta_1+\theta_2} < 0,}_{\partial\text{impact response}/\partial a_2} \qquad \underbrace{\frac{\partial}{\partial a_2}\left[\frac{\partial}{\partial \delta_m}\pi_t|_{t\to\infty}\right] \sim \beta\frac{\theta_1^2}{|\theta_1^2-\theta_2^2|}\min\{\theta_1,\theta_2\}e^{-\min\{\theta_1,\theta_2\}t}}_{\partial\text{asymptotic response}/\partial a_2} > 0 \tag{40}$$

Equation (40) shows that higher a_2 reduces the impact response of inflation (left-side) and amplifies its persistence captured by its asymptotic expansion as $t \to \infty$ (right-side), proportional to the dominant eigenvalue of the economy, given by $\min\{\theta_1,\theta_2\}e^{-\min\{\theta_1,\theta_2\}t}$.

Next, we examine the CIR of GDP as a measure of monetary non-neutrality.

Corollary 2. An increase in sector 1's expenditure on sector 2's output; i.e. an increase in a_2, amplifies monetary non-neutrality. Formally:

$$\mathrm{CIR}_{\tilde{y},m} = \underbrace{\beta\times\theta_1^{-1}+(1-\beta)\times\theta_2^{-1}}_{\text{Carvalho (2006) benchmark}} + \underbrace{\beta\times a_2\times\theta_2^{-1}\times\frac{\theta_2^{-1}}{\theta_1^{-1}+\theta_2^{-1}}}_{\text{network amplification effect }(\geq 0)} \tag{41}$$

The term under the first bracket is the CIR when $a_2 = 0$, which corresponds to our Carvalho (2006) example in Step 3 and shows that monetary non-neutrality is proportional to the expenditure weighted average of the duration of the price spells (captured by θ_1^{-1} and θ_2^{-1}).

The second term is sector 2's *indirect* effect on CIR *through* sector 1, which depends on three factors: household's indirect expenditure share on sector 2 through sector 1 ($\beta\times a_2$), the average duration of price spells in sector 2 (θ_2^{-1}), and, finally, the *relative* duration of price spells in sector 2 ($\frac{\theta_2^{-1}}{\theta_1^{-1}+\theta_2^{-1}}$). When sector 1's prices are relatively flexible ($\frac{\theta_1}{\theta_2}\to\infty$), sector 2 is the bottleneck for sector 1's price adjustment, which maximizes sector 2's indirect effect on monetary non-neutrality. Conversely, when sector 1's prices are much stickier ($\frac{\theta_1}{\theta_2}\to 0$), sector 1's prices will take a long time to adjust anyways, which minimizes sector 2's indirect effect.

TFP / Wedge Shocks. In our two-sector example, only a TFP/wedge shock to sector 2 would propagate through the network because it is the only upstream sector. Accordingly, here we only discuss a shock to sector 2 (δ_z^2). Appendix D has the details.

By Proposition 4, the IRFs of sectoral and CPI inflation as well as the CIR of GDP gap are:

$$\frac{\partial}{\partial \delta_z^2}\pi_{1,t} = \underbrace{a_2 \frac{\theta_1\theta_2}{\theta_1+\theta_2}\frac{\theta_1 e^{-\theta_2 t} - \theta_2 e^{-\theta_1 t}}{\theta_1-\theta_2}}_{\text{network effect}\geq 0}, \quad \frac{\partial}{\partial \delta_z^2}\pi_t = \underbrace{(1-\beta)\theta_2 e^{-\theta_2 t}}_{\text{Step 1 baseline}} + \underbrace{\beta a_2 \frac{\theta_1\theta_2}{\theta_1+\theta_2}\frac{\theta_1 e^{-\theta_2 t} - \theta_2 e^{-\theta_1 t}}{\theta_1-\theta_2}}_{\text{Excess inflation: network effect}\geq 0} \tag{42}$$

$$\frac{\partial}{\partial \delta_z^2}\pi_{2,t} = \underbrace{\theta_2 e^{-\theta_2 t}}_{\text{Step 1 baseline}}, \quad \text{CIR}_{\tilde{y},\delta_z^2} = \underbrace{(1-\beta)\frac{1}{\theta_2}}_{\text{Step 1 baseline}} + \underbrace{\beta a_2 \times (\frac{1}{\theta_1} + \frac{1}{\theta_2} - \frac{1}{\theta_1+\theta_2})}_{\text{network amplification effect}\geq 0} \tag{43}$$

We make two observations. First, not only a negative TFP or positive wedge shock in sector 2 leads to inflation in that sector (as expected), but it also causes inflation in sector 1 as long as $a_2 > 0$, while increasing the CIR of GDP gap. The intuition is that as sector 2's price rises due to the shock, so does the marginal cost of production for sector 1, which leads to inflation in that sector as well. This observation also carries over to CPI inflation response.

Corollary 3. A higher expenditure share of sector 1 on sector 2's output; i.e. an increase in a_2, amplifies inflation response and the CIR of GDP gap to sectoral shocks.

$$\frac{\partial}{\partial a_2}\frac{\partial}{\partial \delta_z^2}\pi_t = \beta \times \frac{1}{\theta_1^{-1}+\theta_2^{-1}} \times \frac{\theta_1 e^{-\theta_2 t} - \theta_2 e^{-\theta_1 t}}{\theta_1-\theta_2} \geq 0, \quad \frac{\partial}{\partial a_2}\text{CIR}_{\tilde{y},\delta_z^2} = \beta \times (\frac{1}{\theta_1} + \frac{1}{\theta_2} - \frac{1}{\theta_1+\theta_2}) \geq 0 \tag{44}$$

Second, when there are no input-output linkages ($a_2 = 0$), the asymptotic behavior of inflation is driven by sector 2's price stickiness, but as soon as there are some input-output linkages $a_2 > 0$, the asymptotic behavior is driven by the stickier sector, even if it is sector 1.

Corollary 4. The asymptotic behavior of excess inflation is driven by the stickier sector.

$$\frac{\partial}{\partial a_2}[\frac{\partial}{\partial \delta_z^2}\pi_t\big|_{t\to\infty}] \sim \beta \frac{\theta_1\theta_2}{|\theta_2^2-\theta_1^2|}\max\{\theta_1,\theta_2\}e^{-\min\{\theta_1,\theta_2\}t} \tag{45}$$

Corollary 4 is a dominant eigenvalue result, as the eigenvalues of $-\sqrt{\mathbf{\Gamma}}$ are $-\theta_1$ and $-\theta_2$.

4.2. Perturbation Around Diagonal Economies. Corollaries 1 to 4 have given us intuition on how the input-output structure propagate shocks through the sufficient statistic $\sqrt{\mathbf{\Gamma}}$, but in a two-sector example. How general are these corollaries? In this section, we conduct a complementary analysis to understand the economic forces behind $\sqrt{\mathbf{\Gamma}}$.

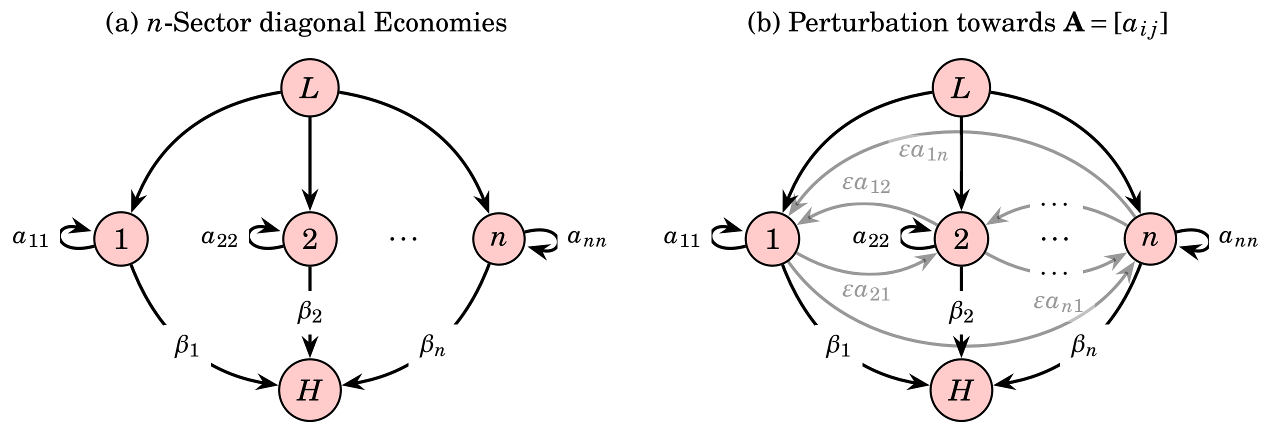
In principle, we could use the Jordan decomposition of $\sqrt{\mathbf{\Gamma}}$ to do this, but this approach does not take us far in terms of economic intuition. For instance, suppose $\sqrt{\mathbf{\Gamma}}$ is diagonalizable so there exists a diagonal $\mathbf{D} = \text{diag}(d_1, \ldots, d_n)$, and an invertible matrix $\mathbf{P}$ such that $\sqrt{\mathbf{\Gamma}} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$, which for instance would imply GDP and inflation responses to a monetary shock are

$$\tfrac{\partial}{\partial \delta_m}\tilde{y}_t = \boldsymbol{\beta}^\top e^{-\sqrt{\boldsymbol{\Gamma}}t}\mathbf{1} = \sum_{i=1}^{n} w_i e^{-d_i t}, \quad \tfrac{\partial}{\partial \delta_m}\pi_t = \boldsymbol{\beta}^\top \sqrt{\boldsymbol{\Gamma}} e^{-\sqrt{\boldsymbol{\Gamma}}t}\mathbf{1} = \sum_{i=1}^{n} d_i w_i e^{-d_i t}, \quad w_i \equiv \boldsymbol{\beta}^\top \mathbf{P}\mathbf{e}_i \mathbf{e}_i^\top \mathbf{P}^{-1}\mathbf{1} \tag{46}$$

The problem is it is unclear how the structure of the economy is reflected in the eigenvalues $\{d_i\}$ and coefficients $\{w_i\}$. The key idea here is to approximate an *arbitrary* input-output economy around "diagonal" economies, whose eigendecomposition has a clear economic interpretation.

Definition 1. A **diagonal** economy is characterized by a diagonal input-output matrix.

Figure 2: Perturbation around diagonal Economies



Notes: Figure 2a draws the structure of diagonal economies where sectors operate independently but are allowed to use their own output in roundabout production. Figure 2b shows our parameterized perturbation of an arbitrary input-output matrix $\mathbf{A}$ around its diagonal structure: the perturbation is given by keeping sectors' own input shares from their output fixed, but only adds their input from other sectors proportional to an $\varepsilon > 0$.

Figure 2a depicts diagonal economies. These are multi-sector economies with heterogeneous price stickiness where sectors only use their own output in roundabout production. Diagonal economies are useful benchmarks because for each sector i, the corresponding eigenvalue is its frequency adjusted by the square root of their labor share, $d_i = \theta_i\sqrt{1-a_{ii}}$, and the corresponding weight in Equation (46) is the household's expenditure share for that sector:

$$\tfrac{\partial}{\partial \delta_m}\tilde{y}_t = \sum_{i=1}^{n} \beta_i e^{-\theta_i\sqrt{1-a_{ii}}\,t}, \qquad \tfrac{\partial}{\partial \delta_m}\pi_t = \sum_{i=1}^{n} \beta_i\theta_i\sqrt{1-a_{ii}}\, e^{-\theta_i\sqrt{1-a_{ii}}\,t} \tag{47}$$

These expressions are now interpretable; e.g., GDP response is the expenditure-weighted average of exponential functions, each decaying at the rate of the sector's adjusted frequency.

Now, consider an *arbitrary* n-sector economy with frequency matrix $\boldsymbol{\Theta} = \text{diag}(\theta_1,\ldots,\theta_n)$ and input-output matrix $\mathbf{A} = [a_{ij}]$, and define the corresponding diagonal economy as $\mathbf{A}_D \equiv$

diag$(a_{11},\dots,a_{nn})$. Thus, we can write the duration-adjusted Leontief matrix $\mathbf{\Gamma} = \mathbf{\Theta}^2(\mathbf{I}-\mathbf{A})$ as the sum of the one in the diagonal economy $\mathbf{\Gamma}_D = \mathbf{\Theta}^2(\mathbf{I}-\mathbf{A}_D)$ and the off-diagonal matrix $\mathbf{\Gamma}_R$:

$$\mathbf{\Gamma} = \mathbf{\Gamma}_D + \mathbf{\Gamma}_R, \quad \text{with} \quad \mathbf{\Gamma}_R \equiv \mathbf{\Theta}^2(\mathbf{A}_D - \mathbf{A}) \tag{48}$$

This is a classic exercise in perturbation theory where we replace $\mathbf{\Gamma}$ with $\mathbf{\Gamma}(\varepsilon) = \mathbf{\Gamma}_D + \varepsilon\mathbf{\Gamma}_R$ for some $\varepsilon > 0$ and express the eigenvalues and eigenvectors as power series in ε (see, e.g., Bender and Orszag, 1999, p. 350). The economic interpretation is that we move from the diagonal economy, $\mathbf{A}_D$, towards the arbitrary economy, $\mathbf{A}$, in proportional to ε, as shown in Figure 2b. Notably, $\varepsilon = 0$ corresponds to the diagonal economy, and $\varepsilon = 1$ corresponds to the arbitrary economy, $\mathbf{A}$. Generally, eigenvalues and eigenvectors do not need to be differentiable in ε, especially for non-symmetric matrices as in our case. However, assuming that eigenvalues of $\mathbf{\Gamma}_D$ are distinct (i.e., sectors of the diagonal economy have distinct adjusted frequencies),[24] we obtain the following Lemma from Theorems 1 and 2 in Greenbaum, Li, and Overton (2020).

Lemma 1. Let $\xi_i \equiv \theta_i\sqrt{1-a_{ii}}$ and assume ξ_i's are distinct. Let $(d_i(\varepsilon), \mathbf{v}_i(\varepsilon))$ be an eigenvalue/eigenvector pair for the principal square root of the perturbed economy, $\sqrt{\mathbf{\Gamma}(\varepsilon)}$. Then,

$$d_i(\varepsilon) = \xi_i + \mathcal{O}(\|\varepsilon\|^2) \qquad \mathbf{v}_i(\varepsilon) = \mathbf{e}_i + \varepsilon\Big[\frac{\theta_j^2 a_{ji}}{\xi_j^2 - \xi_i^2}\mathbf{1}_{\{j\neq i\}}\Big] + \mathcal{O}(\|\varepsilon\|^2) \tag{49}$$

Lemma 1 is useful because it links the mathematical properties of $\sqrt{\mathbf{\Gamma}}$ to its economic properties. It shows that up to first-order in ε, the eigenvalues of $\sqrt{\mathbf{\Gamma}}$ are the same as the diagonal economy; i.e. $\frac{\partial}{\partial\varepsilon} d_i(\varepsilon)|_{\varepsilon=0} = 0$. Importantly, note that in theory, this perturbation does not have to be accurate for $\varepsilon = 1$. But as we plot in Figure E.16 in the Appendix, it is a *remarkably accurate approximation* for the eigenvalues of the measured $\sqrt{\mathbf{\Gamma}}$ for the U.S. economy.

4.2.1. Aggregate and Sectoral Effects of Monetary Shocks. We start by presenting how monetary policy shocks propagate in our approximate economy. We first present the results for sectoral inflation and then aggregate these responses to obtain the effects on GDP and CPI inflation.

Proposition 6 (Sectoral Inflation Responses)**.** Suppose $\{\xi_i \equiv \theta_i\sqrt{1-a_{ii}}\}_{i\in[n]}$ are distinct. The

[24] This is a fairly weak assumption because ξ_i's are almost surely distinct if the distributions of $\mathbf{\Theta}$ and $\mathbf{A}$ in the data are drawn from distributions with densities with respect to the Lebesgue measure. In other words, the event that two sectors have the same adjusted frequencies in the data has zero probability.

impulse response of inflation in sector $i \in [n]$ to a monetary shock is:

$$\frac{\partial}{\partial \delta_m}\pi_{i,t} = \underbrace{e^{-\xi_i t}}_{\text{diagonal baseline}} + \underbrace{\varepsilon \sum_{j\neq i} \frac{\xi_i a_{ij}}{1-a_{ii}} \times \frac{\xi_i}{\xi_i+\xi_j} \times \frac{\xi_j e^{-\xi_j t} - \xi_i e^{-\xi_i t}}{\xi_i - \xi_j}}_{\text{first order effect of the network}} + \mathcal{O}(\|\varepsilon\|^2) \tag{50}$$

Comparing Equation (39) and Equation (50), these first-order effects are *exactly* equal to the equivalent expression in the two-sector economy. The key to this equivalence is that the maximum path length in our two-sector model was one. Thus, the total effects of the network were summarized by its first-order effects, which is what our approximation here is capturing.

Proposition 7 (Impact and Asymptotic Inflation Response)**.** Input-output linkages dampen CPI inflation response to a monetary shock on impact but amplify its persistence.

$$\underbrace{\frac{\partial}{\partial \varepsilon}[\frac{\partial}{\partial \delta_m}\pi_0]\big|_{\varepsilon=0}}_{\partial\text{impact response}/\partial\varepsilon} = -\sum_{i=1}^{n} \beta_i \sum_{j\neq i} \frac{\xi_i a_{ij}}{1-a_{ii}} \times \frac{\xi_i}{\xi_i+\xi_j} < 0 \tag{51}$$

$$\iota \equiv \arg\min_i\{\xi_i\} \Rightarrow \quad \underbrace{\frac{\partial}{\partial \varepsilon}[\frac{\partial}{\partial \delta_m}\pi_t|_{t\to\infty}]\big|_{\varepsilon=0}}_{\partial\text{asymptotic response}/\partial\varepsilon} \sim \sum_{j\neq\iota} \beta_j \Big(\frac{\xi_j^2 a_{j\iota}}{1-a_{jj}} + \frac{\xi_\iota^2 a_{\iota j}}{1-a_\iota}\Big) \frac{\xi_\iota e^{-\xi_\iota t}}{|\xi_i^2 - \xi_\iota^2|} > 0 \tag{52}$$

Again, these approximations are identitcal to the *exact* expressions for the two-sector model.

Proposition 8 (Monetary Non-Neutrality)**.** Input-output linkages amplify monetary non-neutrality measured by the CIR of GDP to a monetary shock.

$$\frac{\partial}{\partial \varepsilon}\mathrm{CIR}_{\tilde{y},\delta_m}\big|_{\varepsilon=0} = \sum_{i=1}^{n}\sum_{j\neq i} \beta_j \times \frac{a_{ji}}{1-a_{jj}} \times \xi_i^{-1} \times \frac{\xi_i^{-1}}{\xi_i^{-1}+\xi_j^{-1}} > 0 \tag{53}$$

4.2.2. Aggregate Effects of Sectoral Shocks. We start by characterizing the pass-through of sectoral inflation to aggregate CPI inflation. The experiment of interest is to consider a sectoral negative TFP shock to sector i that raises the inflation rate in that sector by 1 percent on impact. Our goal is to characterize how much aggregate CPI inflation rises in response to this sectoral shock, and how this pass-through is affected by the network. The following proposition presents this pass-through for impact response of inflation. The full expression for the dynamic response of inflation is also attainable but more complicated. It is included in the proof of the proposition.

Proposition 9 (Pass-through of Sectoral to Aggregate Inflation)**.** Input-output linkages amplify the pass-through of sectoral inflation rates to aggregate CPI inflation. Formally,

$$\frac{\partial \pi_0}{\partial \pi_{i,0}}\Big|_{\delta_z^i} = \underbrace{\beta_i}_{\text{direct pass-through}} \underbrace{+\varepsilon \sum_{j\neq i} \overbrace{a_{ji}}^{①} \times \overbrace{\frac{\beta_j}{1-a_{jj}}}^{②} \times \overbrace{\frac{\phi_i^{-1}}{\xi_j^{-1}+\phi_i^{-1}}}^{③} \times \overbrace{\frac{\xi_i^{-1}}{\xi_i^{-1}+\xi_j^{-1}}}^{④}}_{\text{first-order indirect pass-through via network}} + \underbrace{\mathcal{O}(\|\varepsilon\|^2)}_{\text{higher-order effects}} \tag{54}$$

Equation (54) relates the pass-through of sectoral inflation rate in sector i to aggregate inflation *conditional* on a negative TFP shock to sector i. The first term on the right-hand side is the direct pass-through of sectoral inflation to aggregate inflation: a one percent inflation in sector i directly feeds to inflation proportional to the expenditure share of the sector, denoted by β_i. The second term, which itself consists of four components, labeled by ①–④, captures the first-order *indirect* pass-through of sectoral inflation to aggregate inflation through the network.

The indirect effect can be understood as follows: an inflationary shock in sector i, up to first-order, propagates through its buyers. Thus, we need to sum over all the other sectors that purchase from i. When considering a buyer $j \neq i$, the impact of i's inflationary shock on the economy through j is proportional to j's expenditure share on i, ①, and j's own Domar weight in the baseline economy, ②. These two components jointly determine the potency of i's shock on j and resemble what is known from static models. The next two terms, however, capture dynamic considerations. ③ accounts for the fact that if the duration of the shock to i, ϕ_i^{-1}, is small compared to the adjusted duration of price spells in the downstream sector j, ξ_j^{-1}, then the shock's pass-through via j is weakened. This occurs because stickier downstream sectors, measured by their adjusted duration ξ_j^{-1}, are less responsive to a transient shock because they anticipate it will dissipate relatively faster than prices in their sector will adjust. ④ captures a similar effect, but relative to the adjusted duration of price spells in the upstream sector i itself. When the adjusted duration of price spells in the upstream sector i is relatively small compared to that of the downstream sector j, then firms in j are not very responsive to the price changes of supplier i since they anticipate those prices will readjust faster than their own prices.

5 Quantitative Results

We now present quantitative results on dynamic responses of inflation and GDP to aggregate and sectoral shocks, using U.S. data to construct our sufficient statistics.

5.1. Sufficient Statistics Construction From Data. Proposition 2 shows that the sufficient statistics for inflation and GDP dynamics are the PRDL matrix, $\sqrt{\boldsymbol{\Gamma}}$, and the vector of consumption expenditure shares across sectors, as given by $\boldsymbol{\beta}$. Here, we briefly describe how we construct $\sqrt{\boldsymbol{\Gamma}}$ and $\boldsymbol{\beta}$ from the data.

First, we use the make and use input-output (IO) tables from 2012, made available by the BEA, to construct the input-output matrix $\mathbf{A}$; the consumption expenditure share vector $\boldsymbol{\beta}$; and the sectoral labor shares vector $\boldsymbol{\alpha}$. We construct them at the detailed-level disaggregation, excluding the government sectors, which leads to 393 sectors. Figure E.15 presents the matrix $\mathbf{A}$ we construct from the data, in a heat-map version. Next, we construct the diagonal matrix $\boldsymbol{\Theta}^2$, whose diagonal elements are the squared frequency of price adjustment in each sector, using data on 341 sectors from Pasten, Schoenle, and Weber (2020).[25]

5.2. Dynamic Aggregate Responses to a Monetary Policy Shock. For our calibrated economy, in Panel A of Figure 3, we show impulse responses of aggregate inflation and GDP to an expansionary monetary policy shock. The monetary policy shock size is chosen such that it leads to a 1 percent increase in inflation on impact.[26] After increasing by 1 percent on impact, inflation slowly goes back to steady-state, as endogenous state variables evolve over time and input-output linkages and differential price stickiness across sectors slow down the inflation adjustment. More importantly, there are substantial real effects on GDP of this shock to nominal GDP, as seen by the large initial effect on GDP of around 10 percent. Critically, these effects on GDP are persistent and decay slowly, and the cumulated impulse response of GDP is about 131 percent.

[25]In our continuous time model, the monthly frequency of price adjustment in the data corresponds to one minus the probability that the Poisson variable does not arrive within one month. Let fpa be the frequency of price adjustment in Pasten, Schoenle, and Weber (2020). Then, $\theta = -\log(1 - fpa)$. The consumption share weighted average frequency of price changes across sectors is 0.171 (0.188), before (after) our continuous-time transformation. We use the average frequency of price changes for sectors that are missing the price change data.

[26]Recall that the shock is a permanent shock to the level of nominal GDP.

To illustrate the roles of various model ingredients that lead to such substantial real effects, we now do several counterfactual experiments. In these counterfactuals, we keep the initial impact on inflation the same at 1 percent.[27] In Panel B of Figure 3, we compare our calibrated economy to a counterfactual horizontal economy, which does not feature any input-output linkages and where labor is the only input in production. The cumulated impulse response of GDP is 4.1 times larger in our baseline economy. Strategic complementarity in price setting that arises through input-output linkages, as we pointed out while discussing the analytical results in the previous Section, is the driving force for this result. This in turn leads to a more persistent inflation response, which amplifies GDP response both on impact and over time.

In addition to input-output linkages, another source that amplifies the real effects of monetary policy is heterogenous price stickiness across sectors, as we pointed out and formalized in the previous Section. To investigate the role of this channel, in Panel C of Figure 3, we compare our calibrated baseline economy to a counterfactual economy that has homogenous price stickiness across sectors. We calibrate the frequency of price changes in this economy to be the same as the weighted average of the frequency of price changes across sectors in our baseline economy. This economy still features input-output linkages, and through that, strategic complementarity in price setting. The cumulated impulse response of GDP is 2.4 times larger in our baseline economy, which shows that heterogeneity in price stickiness across sectors does play a quantitatively important role in magnifying monetary non-neutrality. The quantitative importance of this channel, however, is not as high as that of input-output linkages.

Finally, shutting down both channels, in Panel D of Figure 3, we compare our calibrated baseline economy to a counterfactual horizontal economy that also has homogenous price stickiness across sectors and find that the cumulated impulse response of GDP is 6.9 times larger in our baseline economy.[28] This total effect is approximately equal to the sum of the two

[27] The monetary policy shock size is therefore different across the baseline and the counterfactual cases. Recall that the cumulated impulse response of aggregate inflation corresponds to the monetary policy shock size in our model. Keeping the initial impact on aggregate inflation the same across various model specifications brings out the crucial role played by the persistence of inflation.

[28] Note that even in this textbook type multisector New Keynesian model, inflation effects are persistent because our modeling of monetary policy preserves an endogenous state variable in the model. This is a standard approach in the literature on sufficient statistics of monetary policy shocks, but is a different approach than assuming a

separate counterfactual effects we showed above.

Having presented the extent of monetary non-neutrality in these four economies, we now assess whether the slopes of the aggregate Phillips curves, $\boldsymbol{\beta}^{\top}\boldsymbol{\Gamma}\mathbf{1}$, as given in Equation (31), align with the effects of the monetary policy shock in these economies. The slopes of the aggregate Phillips curve are as follows: In the calibrated economy, it is 0.0187; in the horizontal economy, it is 0.1135; in the homogeneous price stickiness economy, it is 0.0190; and in the horizontal and homogeneous price stickiness economy, it is 0.0526. As is clear, the slopes of the aggregate Phillips curves do not serve as sufficient statistics for ranking of monetary non-neutrality. For instance, the slope is steeper in the horizontal economy compared to the economy that is both horizontal and has homogeneous price stickiness across sectors. This failure of $\boldsymbol{\beta}^{\top}\boldsymbol{\Gamma}\mathbf{1}$ to predict the effects of monetary policy shocks is due to the effects of relative price gaps that are present in the aggregate Phillips curves in our model, as given in Equation (31), and which we discussed in detail in Section 3.3. Moreover, our quantitative results here are consistent with the counterexample we presented analytically in Proposition 5.

5.3. Heterogeneous Sectoral Inflation Responses to a Monetary Policy Shock. Underlying the aggregate inflation response to the monetary policy shock we discussed above is a distribution of sectoral inflation responses. In Figure 4, we show impulse responses of some selected sectors' inflation to an expansionary monetary policy shock. As is clear, sectoral inflation responses differ significantly both in terms of the impact response and the persistence of the response. In particular, sectors where inflation responds by a larger amount initially have more short-lived responses. This also implies that sectoral inflation responses cross the aggregate inflation response. In particular, Figure 4 shows that sectoral inflation in the Oil and Gas Extraction industry is high in the initial periods but dissipates fast, while sectoral inflation in the Semiconductor Manufacturing Machinery industry responds by a small amount initially but is persistently positive over time. For completeness, Table E.1 in Appendix E provides a ranking of the top twenty sectors by their initial sectoral inflation response while Table E.2 in Appendix E provides a ranking of the top twenty sectors by the half-life of their sectoral

Taylor rule where the interest rate feedback coefficient is on inflation. We show results from this case later.

inflation response.

As we showed analytically using an approximation in Section 4.2, inflation in sectors with more flexible prices and less input-output linkages respond more strongly initially. Specifically, we showed in Equation (50) that the relevant statistic for impact sectoral inflation response (evaluated at $t=0$) is $\xi_i - \varepsilon \sum_{j\neq i} \frac{\xi_i a_{ij}}{1-a_{ii}} \frac{\xi_i}{\xi_i+\xi_j}$. Panel A of Figure 5 shows the correlation between the actual ranks of sectors and the ranks predicted from this statistic. As is clear, the approximated statistic accounts extremely well for the exact numerical results. Moreover, as mentioned above, sectors where inflation responds more initially tend to have short-lived responses. Panel B of Figure 5 shows the correlation between actual ranks of sectors given by half-life of sectoral inflation response and the ranks predicted from this statistic for impact response. As is clear, the correlation is strongly negative.

5.4. Sectoral Origins of Aggregate Inflation and GDP Dynamics. Motivated by differential sectoral inflation dynamics, supply chain issues and commodity price increases, and persistent aggregate inflation in the U.S. recently, we now consider aggregate implications of sectoral shocks. Specifically, we compute sectoral shocks that lead to a 1 percent increase in sectoral inflation and then study the pass-through of such sectoral inflation increases on aggregate inflation.[29] The average duration of the sectoral shocks is 6 months in this exercise.[30]

We first identify sectors that are the main sources of high on impact response of aggregate inflation. Table 1 provides a ranking of the top twenty sectors by their initial effect on aggregate inflation where we remove the effect coming from the size of the sector. This metric, therefore, provides an evaluation of the spillover of sectoral inflation to aggregate inflation due to input-output linkages for in the absence of such linkages, this pass-through metric would be zero for all sectors.[31] As one example, the Oil and Gas Extraction industry ranks very high in Table 1. As we showed analytically using an approximation in Section 4.2, sectors that serve as input to other sectors and have more sticky prices cause greater spillover to aggregate inflation according

[29]We interpret these sectoral shocks as negative supply shocks.

[30]Note that while the average duration of the sectoral shock is the same across all sectors, the size of the sectoral shock is different as we calibrate the size such that sectoral inflation increases by 1 percent across all sectors.

[31]We are thus capturing what are sometimes called second-round effects of sectoral inflation increases.

to this metric. Specifically, in Equation (54) we showed that the relevant statistic for this impact pass-through on aggregate inflation is $\sum_{j\neq i}\beta_j \frac{\alpha_{ji}}{1-\alpha_{jj}} \frac{\phi_i^{-1}}{\phi_i^{-1}+\xi_j^{-1}} \frac{\xi_i^{-1}}{\xi_i^{-1}+\xi_j^{-1}}$. Panel A of Figure 6 shows the correlation between the actual ranks of sectors and the ranks predicted from this statistic. As is clear, the approximated statistic accounts well for the exact numerical results.

We next identify sectors that are the main sources of persistent aggregate inflation dynamics when sectoral inflation increases by 1 percent. Table 2 provides a ranking of the top twenty sectors by how persistently they affect aggregate inflation, as given by the half-life of the aggregate inflation response. One clear pattern emerges: Sectors with more sticky prices lead to persistent aggregate inflation dynamics when sectoral shocks cause a rise in sectoral inflation. Semiconductor Manufacturing Machinery industry is one sector that ranks high in Table 2.

Identifying which sectors are the main sources of persistent aggregate inflation dynamics is critical because those persistent effects translate to larger aggregate GDP gap effects. To make this clear, in Panel B of Figure 6, we show that the cumulated impulse response of aggregate GDP gap is very tightly correlated with the half-life of aggregate inflation in the face of sectoral shocks.[32] This implies that it is precisely the shocks to sectors that are the sources of persistent aggregate inflation dynamics that will have a bigger impact on the real macroeconomy.

6 Extensions

We now present several extensions of our theoretical and quantitative results.

6.1. General Labor Supply Elasticity. So far, we used log-linear preferences that imply an infinite Frisch elasticity of labor supply. Our solution techniques, analytical results, and quantitative insights do not, however, depend on this simplification. In Appendix A.9, we present the details of the model solution for a finite Frisch elasticity case and present here the counterpart of Proposition 1 with $\rho = 0$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{\boldsymbol{\pi}}_t = \boldsymbol{\Gamma}(\mathbf{I}+\psi\mathbf{1}\boldsymbol{\beta}^\intercal)(\mathbf{p}_t-\mathbf{p}_t^f), \qquad \mathbf{p}_t^f \equiv m_t\mathbf{1}-\boldsymbol{\Psi}\boldsymbol{z}_t+(\boldsymbol{\Psi}-\frac{\psi}{1+\psi}\mathbf{1}\boldsymbol{\lambda}^\intercal)\boldsymbol{\omega}_t \tag{55}$$

[32] We compute the ratio of the cumulated impulse respone of GDP to the cumulated impulse response of GDP under flexible prices.

where ψ is the inverse Frisch elasticity of labor supply. Given these expressions, we can extend Propositions 3 and 4 to this case by replacing $\boldsymbol{\Gamma}$ with $\boldsymbol{\Gamma}_\psi \equiv \boldsymbol{\Gamma}(\mathbf{I}+\psi\mathbf{1}\boldsymbol{\beta}^\intercal)$ and adjusting for $\mathbf{p}_t^f$ as above. In particular, the impulse response for monetary and sectoral productivity shocks only change through $\boldsymbol{\Gamma}_\psi$. The impulse responses for sectoral wedge shocks, however, also need to be adjusted through $\mathbf{p}_t^f$.

To show the quantitative implications, in Figure E.1 we show impulse responses of aggregate inflation and GDP to an expansionary monetary policy shock when Frisch elasticity is calibrated at 2. For comparison, we also present the results from our baseline calibration. As is clear, since a finite Frisch elasticity introduces strategic substitutability from aggregate sources, it reduces the persistence of inflation and thereby, the extent of monetary non-neutrality. More importantly, this calibration does not alter our quantitative results on the various forces that drive monetary non-neutrality, as shown in Figure E.2-Figure E.4 . Finally, Figure E.5 shows that the distribution of sectoral inflation response after an aggregate monetary policy shock depicts the same patterns as in Section 5.2.

6.2. Taylor Rule as Monetary Policy Rule. So far, we used a monetary policy rule as determining a path of nominal GDP which kept the analysis similar to the theoretical literature on monetary non-neutrality and highlighted the role of endogenous persistence in the model. We now consider an extension where monetary policy is modeled as a rule in which the nominal interest rate responds to aggregate inflation. Our model derivations generalize to using such a Taylor rule and the details are in Appendix A.10. We now need to impose boundary conditions that ensure that inflation and relative sectoral prices are stationary. In terms of solving the resulting set of equilibrium system of equations, we use a Schur decomposition-based technique.

Here we discuss some key aspects of the model equilibrium. First, the counterpart of Proposition 1 with $\rho = 0$ and a Taylor rule with a monetary shock v_t, $i_t = \phi_\pi \boldsymbol{\beta}^\intercal \vec{\boldsymbol{\pi}}_t + v_t$, is:

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{\boldsymbol{\pi}}_t = \boldsymbol{\Gamma}(\mathbf{I}-\phi_\pi\mathbf{1}\boldsymbol{\beta}^\intercal)(\vec{\boldsymbol{\pi}}_t - \vec{\boldsymbol{\pi}}_t^f), \qquad \vec{\boldsymbol{\pi}}_t^f \equiv (\mathbf{I}-\phi_\pi\mathbf{1}\boldsymbol{\beta}^\intercal)^{-1}(\mathbf{1}v_t - \boldsymbol{\Psi}(\dot{\boldsymbol{z}}_t - \dot{\boldsymbol{\omega}}_t)) \tag{56}$$

Here, $\vec{\boldsymbol{\pi}}_t^f$ is the sectoral inflation rate that would have prevailed in a flexible price economy with the same Taylor rule and is exogenous to the system of differential equations. We can see that this equation differs from our previous result in two aspects: first, it is now a second-order differential equation in $\vec{\boldsymbol{\pi}}_t$ rather than in prices. This is because, with an inflation-targeting Taylor rule, the economy is no longer price stationary, an observation that holds also in one-sector New Keynesian models. Second, we see that the dynamics of the second-order differential equations are still governed by $\boldsymbol{\Gamma}$, but this time it is adjusted for the endogenous response of monetary policy through the Taylor rule: $\boldsymbol{\Gamma}_{\phi,\pi} \equiv \boldsymbol{\Gamma}(\mathbf{I} - \phi_\pi \mathbf{1}\boldsymbol{\beta}^\top)$.

A Taylor rule in terms of inflation makes sticky price models forward-looking and thus the source of persistence is exogenous.[33] In our baseline calibration, fixing the Taylor rule coefficient at the standard value of $\phi_\pi = 1.5$, we introduce persistent shocks to the Taylor rule. We then calibrate the size and persistence of the shocks to generate a response of aggregate inflation that matches as closely as possible the aggregate inflation response in our nominal GDP rule economy of Section 5.2.[34] Figure E.6 shows the impulse responses of aggregate inflation and GDP to an expansionary monetary policy shock. The monetary non-neutrality, by design, is essentially the same as in Section 5.2.

Given this baseline calibrated Taylor rule economy, we now investigate the various forces that drive monetary non-neutrality in the model using counterfactual exercises, which are presented in Figure E.7-Figure E.9 .[35] Overall, these results are consistent with our main conclusion that both production networks and heterogenous price stickiness play a quantitatively important role in amplifying monetary non-neutrality. We note that the precise extent of amplification coming from them jointly, compared to the horizontal economy that also has homogenous price stickiness across sectors, is a bit smaller than in Section 5.2. The driving force for that result is that in this economy, persistent dynamics in inflation come about through persistence in

[33] In the standard three equation sticky price model with a Taylor rule, the economy is fully forward-looking.

[34] In actual implementation, we match exactly the initial response and the half-life of aggregate inflation in these two economies.

[35] In these counterfactual exercises, we keep the monetary policy shock and persistence the same as the baseline calibration for the Taylor rule economy. The reason is that with the Taylor rule as a monetary policy rule, inflation becomes forward-looking in the model and as such, differences in model features show up as affecting the level response of inflation, and not the persistence. We thus will not fix the impact response of inflation across various counterfactual exercises. For intuition, in the basic one-sector sticky price model with the Taylor rule, the slope of the Phillips curve that incorporates strategic complementarity only shows up as affecting the impact response of inflation.

the monetary policy shock itself, which increases the monetary non-neutrality even in the basic multi-sector economy.[36] Additionally, Figure E.10 shows that the distribution of sectoral inflation response after an aggregate monetary policy shock depicts the same patterns as in Section 5.2.

6.3. Source of Aggregate Inflation Persistence. So far, we have highlighted the critical role played by the persistence of aggregate inflation in driving macroeconomic dynamics. In particular, for the monetary shock, we showed in Section 5.2 that model features which increase the persistence of aggregate inflation leads to higher monetary non-neutrality. We now investigate further the sectoral origins of aggregate inflation persistence by identifying which sectors play a key role in propagating monetary policy shock in the longer run. In terms of long-run dynamics, given our analytical solution, the smallest eigenvalues of $\mathbf{\Gamma} \equiv \mathbf{\Theta}^2(\mathbf{I}-\mathbf{A})$ play the dominant role. Of course, eigenvalues as such do not have an economic meaning and cannot be assigned to particular sectors. In the diagonal economy considered in Section 4 however, eigenvalues are given by $\theta_i\sqrt{1-a_{ii}}$. So, in Table 3 we sort the eigenvalues and present those of $\mathbf{\Gamma} \equiv \mathbf{\Theta}^2(\mathbf{I}-\mathbf{A})$ together with $\theta_i\sqrt{1-a_{ii}}$ for several industries. As is clear, the eigenvalues are extremely close across these two cases, thus helping us provide economic meaning in terms of sectors that lead to the smallest eigenvalues. Figure E.16 shows that this extremely close association holds across the full range of eigenvalues and sectors.

To show the aggregate implications of these sectors with the lowest eigenvalues, we do a counterfactual exercise by dropping the three sectors with the smallest eigenvalues and recompute the impulse responses of inflation and GDP.[37] Just dropping these three sectors leads to a noticeable change in the cumulative IRF of GDP, with the cumulative IRF of real GDP in calibrated economy higher by around 16 percent.[38] These results thus show that a few

[36]In addition, compared to the results in Section 5.2, production networks and heterogenous price stickiness play a similar role quantitatively.

[37]In this exercise, we recompute the counterfactual input-output matrix by moving the share of these dropped sectors (as inputs) to the labor share. Moreover, these sectors correspond closely to sectors that have the highest half-life of sectoral inflation to a monetary shock.

[38]Note that two of these sectors have a zero sectoral share in aggregate real GDP while the third one has an extremely small sectoral share in aggregate GDP of 0.0015 percent. If the economy were horizontal, dropping them would not have affected the response of aggregate GDP at all.

sectors play a very influential role in driving monetary non-neutrality in the economy as they determine the persistence of aggregate inflation. To show this clearly, in Figure E.17 we plot the impulse responses of inflation and GDP to a monetary shock for both our calibrated and counterfactual economies. They depict that over the longer horizon, inflation response is lower in the counterfactual economy and this difference in dynamics gets reflected in a lower response of real GDP throughout the time horizon.

7 Conclusion

We provide sufficient statistics for inflation and GDP *dynamics* in multisector dynamic New Keynesian economies with input-output linkages. We show that the sufficient statistic for these dynamic responses is the *principal square root* of the Leontief matrix appropriately adjusted for the sectoral frequencies of price adjustments.

We construct this sufficient statistic using data from input-output tables and frequencies of price adjustments across sectors in the U.S. In quantitative experiments on this calibrated economy, we find a significant role for production networks in the propagation of aggregate monetary and sectoral TFP shocks. First, monetary shocks lead to effects on GDP that are four times as large, relative to a baseline multisector economy with a horizontal production network. Second, sectoral shocks that increase sectoral inflation can lead to substantial effects on aggregate inflation through spillovers that come about through production networks.

In future work, we plan to extend our framework and analysis in several directions. For instance, it will be interesting to study welfare and optimal policy implications in our model. A model with state-dependent pricing, due to fixed costs of changing nominal prices, is likely to lead to new insights. We also plan to extend the model to capture another important source of dynamics, through endogenous capital accumulation, to further develop the framework for business cycle analysis.

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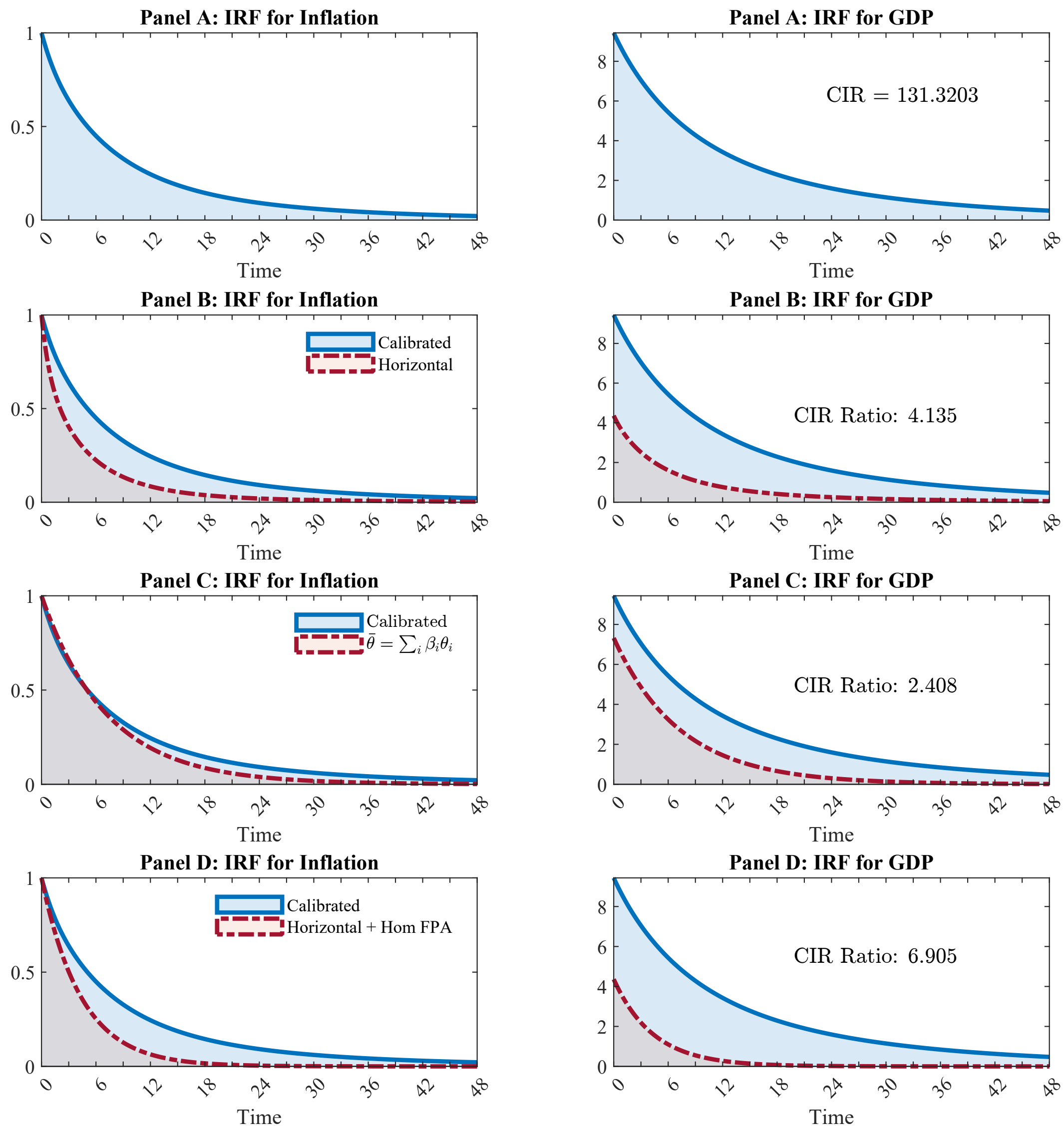
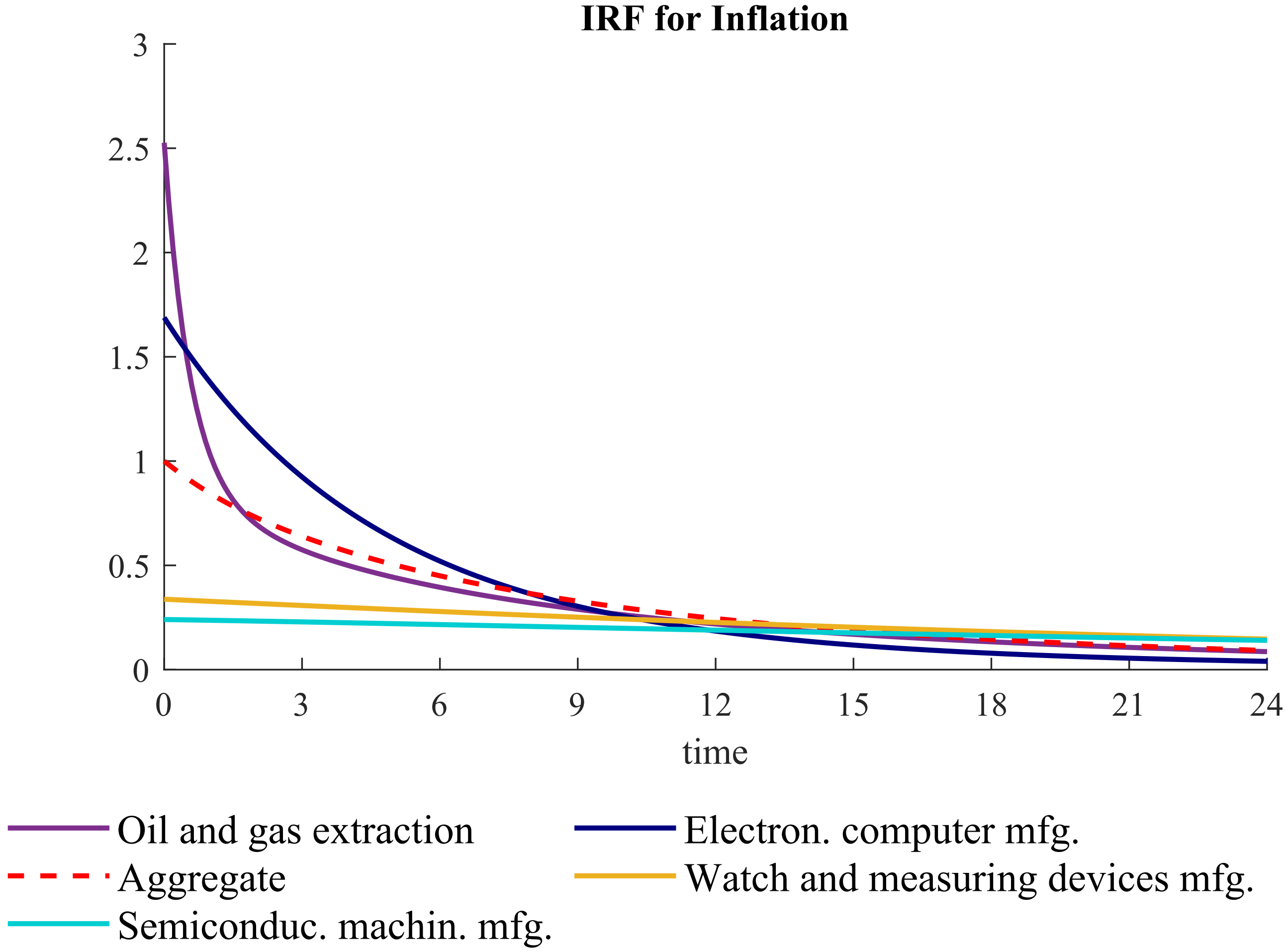
8 Figures

Figure 3: IRFs to a monetary policy shock



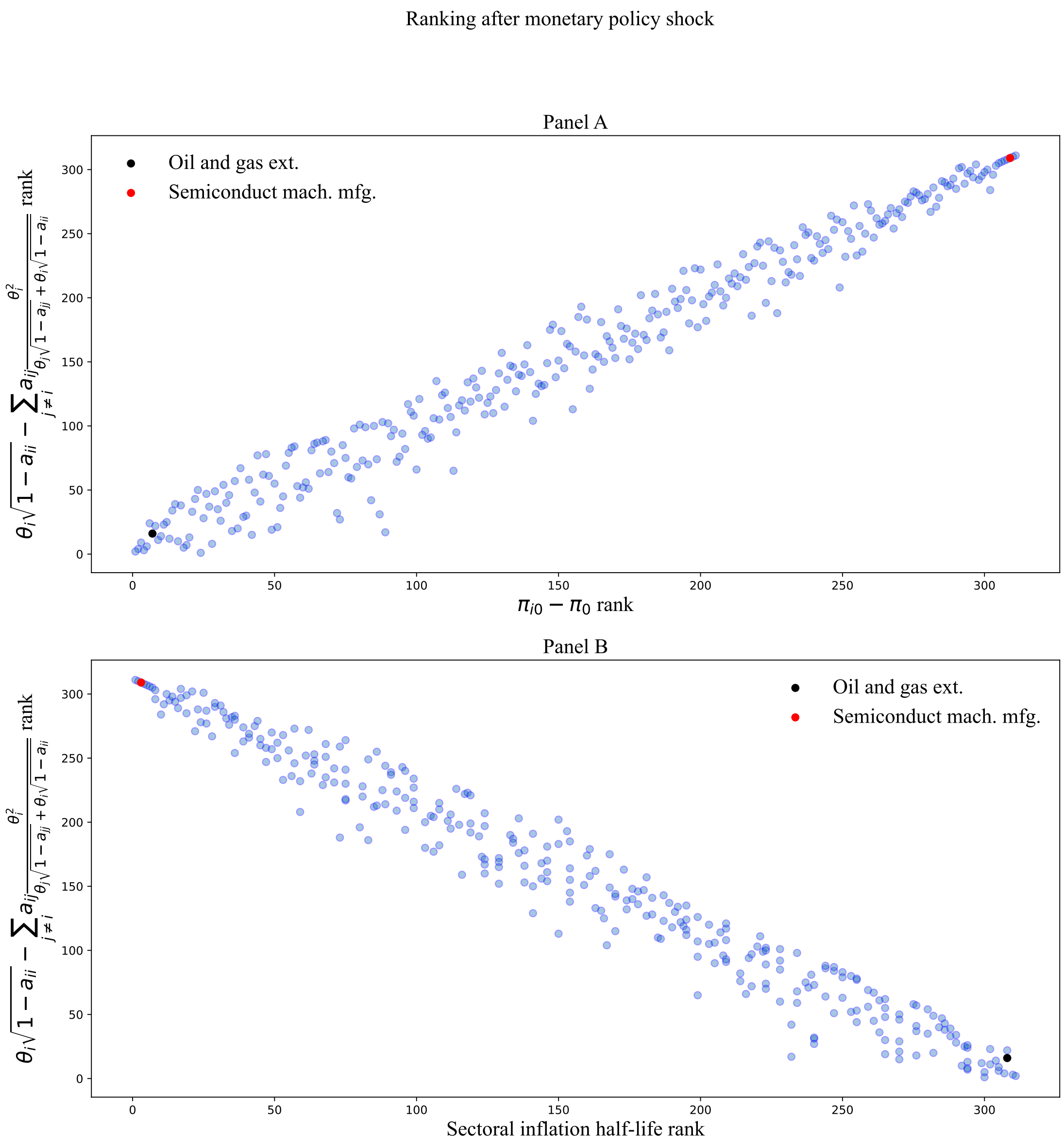
Notes: This figure plots the impulse response functions for inflation and GDP to a monetary shock that generates a one percentage increase in inflation on impact. The calibration of the model is at a monthly frequency. The different panels show the results from the baseline calibrated economy (Panel A) as well as various counterfactual economies (Panels B, C, and D). CIR denotes the cumulative impulse response. CIR Ratio denotes the ratio of CIR of the baseline economy to the counterfactual economy.

Figure 4: Sectoral inflation response to a monetary policy shock



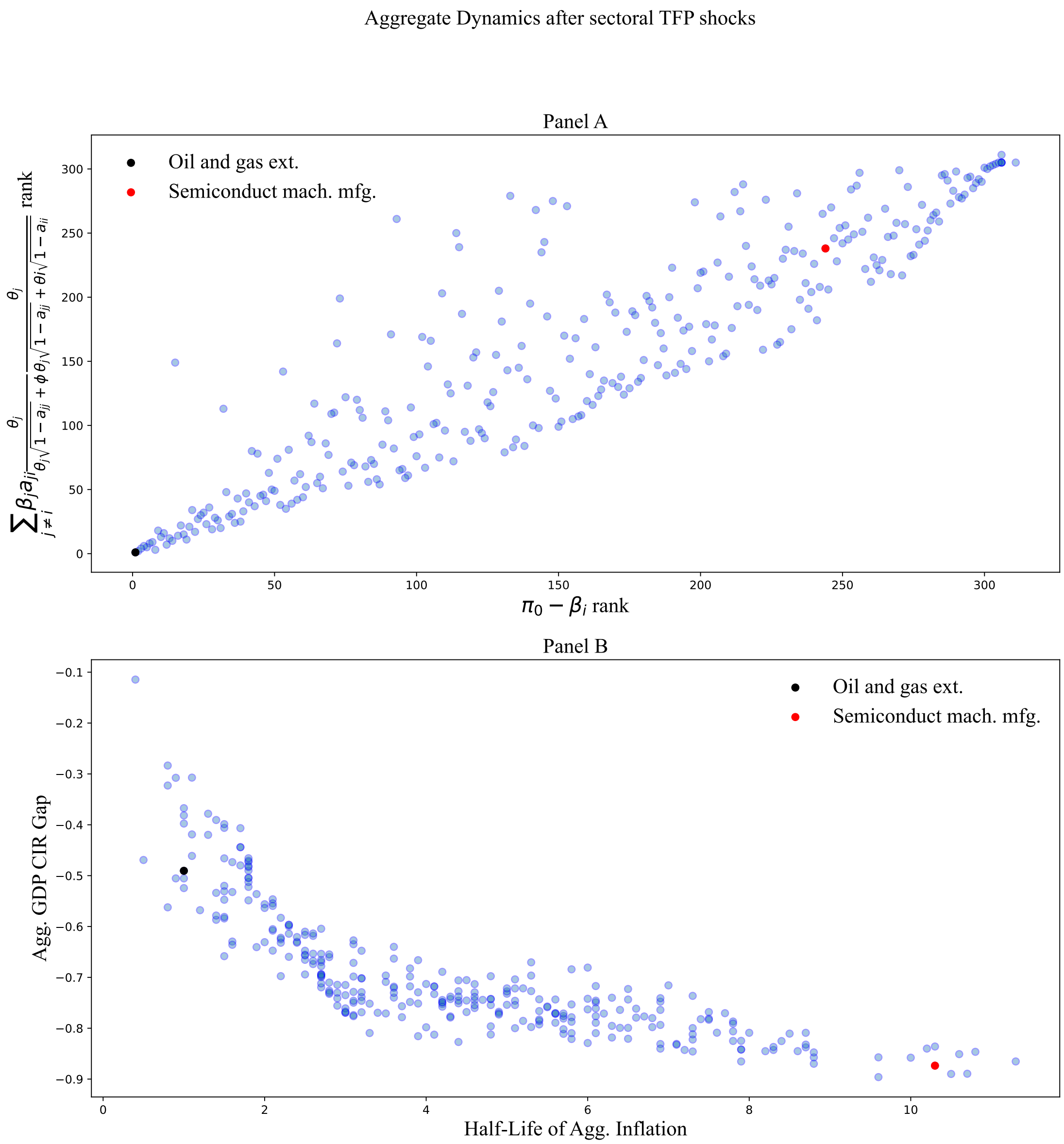
Notes: This figure plots the impulse response functions for aggregate inflation and sectoral inflation to a monetary shock that generates a one percentage increase in aggregate inflation on impact. The calibration of the model is at a monthly frequency. The aggregate inflation response is shown in dashed lines.

Figure 5: Correlation of actual ranks of sectors and ranks using an approximated statistic for sectoral inflation response to a monetary policy shock



Notes: This figure plots the actual ranks and ranks using an approximated statistic for sectoral inflation response to a monetary policy shock. Panel A plots sectoral inflation impact response while Panel B plots the sectoral inflation half-life. Each dot in the figure represents a sector.

Figure 6: Aggregate inflation and GDP dynamics following sectoral shocks



Notes: Panel A of the figure plots actual ranks of sectors and ranks using an approximated statistic for aggregate inflation impact response after a sectoral shock increases sectoral inflation by one percentage on impact. Panel B of the figure plots how aggregate GDP gap and half-life of aggregate inflation are correlated when a sectoral shock increases sectoral inflation by one percentage on impact. Each dot in the figure represents a sector.

9 Tables

Table 1: Ranking of industries by pass-through to aggregate inflation after a sectoral shock

Industry	Agg. Inflation Impact Resp.
Oil and gas extraction	0.009543
Insurance agencies, brokerages, and related act...	0.008415
Employment services	0.006016
Legal services	0.005696
Management consulting services	0.005642
Advertising, public relations, and related serv...	0.005026
Accounting, tax preparation, bookkeeping, and p...	0.004993
Warehousing and storage	0.004981
Architectural, engineering, and related services	0.004981
Electric power generation, transmission, and di...	0.003828
Services to buildings and dwellings	0.003702
Monetary authorities and depository credit inte...	0.003621
Scenic and sightseeing transportation and suppo...	0.003418
Securities and commodity contracts intermediati...	0.003354
Other support activities for mining	0.003241
Truck transportation	0.003186
Commercial and industrial machinery and equipme...	0.003146
Wired telecommunications carriers	0.003121
Other financial investment activities	0.003025
Other nondurable goods merchant wholesalers	0.002608

Notes: Ranking of industries by aggregate inflation impact response when a sectoral shock leads to an increase in 1% in the shocked sector's inflation on impact. Average duration of the sectoral shock is 6 months.

Table 2: Ranking of industries by half-life of aggregate inflation repsonse after a sectoral shock

Industry	Half Life Agg. Inflation
Packaging machinery manufacturing	11.3
Miscellaneous nonmetallic mineral products	10.8
Coating, engraving, heat treating and allied ac...	10.7
All other forging, stamping, and sintering	10.6
Industrial process furnace and oven manufacturing	10.5
Semiconductor machinery manufacturing	10.3
Printing ink manufacturing	10.3
Speed changer, industrial high-speed drive, and...	10.2
Machine shops	10.0
Insurance agencies, brokerages, and related act...	9.6
Turned product and screw, nut, and bolt manufac...	9.6
Electricity and signal testing instruments manu...	8.8
Other communications equipment manufacturing	8.8
Fluid power process machinery	8.8
Support activities for printing	8.7
Relay and industrial control manufacturing	8.7
Industrial and commercial fan and blower and ai...	8.7
Optical instrument and lens manufacturing	8.6
In-vitro diagnostic substance manufacturing	8.5
Other electronic component manufacturing	8.4

Notes: Ranking of industries by half-life of aggregate inflation response when a sectoral shock that leads to an increase in 1% in the shocked sector's inflation on impact. Average duration of the sectoral shock is 6 months.

Table 3: Comparison of eigenvalues of the calibrated economy with eigenvalues of the diagonal economy associated with specific industries

Industry	θ_i	$\theta_i\sqrt{1-a_{ii}}$	Eigenvalue $\sqrt{\Gamma}$
Insurance agencies, brokerages, and related act...	0.035586	0.022688	0.022439
Coating, engraving, heat treating and allied ac...	0.027804	0.02744	0.027327
Warehousing and storage	0.032407	0.030659	0.030562
Semiconductor machinery manufacturing	0.034003	0.032861	0.032858
Flavoring syrup and concentrate manufacturing	0.038897	0.038458	0.038413
Showcase, partition, shelving, and locker manuf...	0.039775	0.039335	0.039325
Packaging machinery manufacturing	0.040667	0.039349	0.039346
Machine shops	0.044323	0.043501	0.042797
Watch, clock, and other measuring and controlli...	0.043928	0.043682	0.043607
Other communications equipment manufacturing	0.044149	0.043945	0.043919
Turned product and screw, nut, and bolt manufac...	0.044987	0.044227	0.044319
Electricity and signal testing instruments manu...	0.048076	0.044627	0.044622
Broadcast and wireless communications equipment	0.053673	0.045249	0.045218
Fluid power process machinery	0.047158	0.045863	0.045821
Optical instrument and lens manufacturing	0.048201	0.04615	0.046098
All other miscellaneous manufacturing	0.047515	0.046339	0.046138
Miscellaneous nonmetallic mineral products	0.049119	0.046373	0.04629
Other aircraft parts and auxiliary equipment ma...	0.051709	0.046385	0.046363
Cutlery and handtool manufacturing	0.047783	0.047746	0.047703
Analytical laboratory instrument manufacturing	0.04835	0.048093	0.048118
Other industrial machinery manufacturing	0.049155	0.048275	0.048118
Breakfast cereal manufacturing	0.048738	0.048585	0.048335
Cut stone and stone product manufacturing	0.063157	0.048644	0.048573
Advertising, public relations, and related serv...	0.049135	0.048695	0.048643
Metal crown, closure, and other metal stamping ...	0.048895	0.048722	0.048708
Toilet preparation manufacturing	0.050453	0.050085	0.05007
Doll, toy, and game manufacturing	0.050442	0.050401	0.050399
Offices of physicians	0.050503	0.050503	0.050503
Waste management and remediation services	0.054119	0.050815	0.050563
Motorcycle, bicycle, and parts manufacturing	0.057306	0.050978	0.050979

Notes: The actual eigenvalues of the calibrated economy are compared with eigenvalues of the counterfactual diagonal economy. In the diagonal economy, the eigenvalues are associated with specific industries, which are given in the first column.

APPENDIX
(FOR ONLINE PUBLICATION)

TO BE COMPLETED.