Inflation and GDP Dynamics in Production Networks: A Sufficient Statistics Approach*

Hassan Afrouzi† Saroj Bhattarai‡

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Abstract

We provide an analytical characterization of inflation and GDP dynamics in multi-sector New Keynesian economies with arbitrary input-output linkages. We find that the sufficient statistic for these dynamics is the principal square root of the Leontief matrix, appropriately adjusted for the duration of price spells across sectors. By applying this statistic to the U.S. economy, we evaluate how the interplay between sticky prices and production linkages governs the effects of aggregate and sectoral shocks, and rank U.S. sectors according to their contributions to inflation and the GDP gap. We show how production linkages (1) amplify the persistence of inflation and GDP responses to monetary shocks, (2) increase the pass-through of sectoral shocks to aggregate inflation, and (3) determine the propagation of sectoral shocks under different monetary rules. Inflation targeting is more contractionary relative to a policy that keeps interest rates fixed when inflation is driven by TFP shocks to sectors with lower adjusted durations (e.g., the Oil and Gas Extraction industry).

JEL Codes: E32, E52, C67
Key Words: Production networks; Multi-sector model; Sufficient statistics; Inflation dynamics; Real effects of monetary policy; Sectoral shocks

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†Columbia University. hassan.afrouzi@columbia.edu. 1105A IAB, 420 West 118th St, New York, NY 10027.

‡UT Austin. saroj.bhattarai@austin.utexas.edu. 2225 Speedway, Stop C3100, UT Austin, Austin, TX 78712.

1 Introduction

Understanding how production linkages impact the *dynamics* of sectoral prices, inflation, and gross domestic product (GDP) is a key objective in macroeconomics, which has become even more pressing in light of the recent supply chain disruptions, oil price fluctuations, and monetary policy responses. This paper aims to shed light on a central question in this area: In an economy with sticky prices and production networks, what determines each sector's contribution to the dynamics of sectoral prices, aggregate inflation, and GDP?

We address this question in a dynamic model with input-output linkages across multiple sectors and staggered forward-looking sticky prices. In this environment, we provide closed-form solutions and derive sufficient statistics for the dynamic responses of prices, inflation, and GDP to aggregate and sectoral shocks around an efficient steady state. By applying perturbation theory to arbitrary networks, we identify how nominal rigidities and production linkages jointly amplify (1) the persistence of inflation and GDP responses to monetary shocks, and (2) the pass-through of sectoral shocks to aggregate inflation. Lastly, we quantify inflation and GDP responses to both aggregate and sectoral shocks and provide a comprehensive ranking of U.S. sectors based on their contribution to the persistence of these responses.

We consider a framework with arbitrary production networks and heterogeneous price stickiness across sectors. In each sector, a continuum of monopolistically competitive intermediate goods firms use labor from a competitive market and goods from other sectors to produce with sector-specific production functions and total factor productivity (TFP) shocks. These firms also make staggered forward-looking pricing decisions, where i.i.d. price change opportunities arrive at sector-specific Poisson rates. A competitive final goods producer in each sector aggregates the intermediate products and sells them for final household consumption or as input for firms in other sectors. In our benchmark, monetary policy controls nominal GDP,[1] while fiscal policy is composed of wedge shocks across sectors, funded by lump-sum transfers. To solve the model, we characterize its efficient steady state and then derive closed form solutions for the local dynamics of the model around this steady state.

Our first theoretical result is that the equilibrium path of *all* sectoral prices in response to *any* path of shocks is uniquely determined by a system of differential equations that relates the evolution of sectoral prices to the deviations of these prices from their flexible price equilibrium counterparts. Using this representation, we show that *all* model parameters affect the dynamics

[1] In extensions, we also consider inflation targeting and a Taylor rule policy.

of sectoral prices exclusively through the Leontief matrix, adjusted appropriately for the duration of price spells across sectors. The explicit solution to this system, which holds for any joint path of monetary policy and sectoral shocks, reveals that the sufficient statistic for the dynamic responses of all sectoral prices to aggregate and sectoral shocks is the principal square root of the duration-adjusted Leontief (PRDL) matrix. In particular, it enables us to derive the impulse response functions (IRFs) of sectoral prices to monetary and sectoral shocks in closed form. It follows that all IRFs, including those of aggregate inflation and GDP, decay exponentially at the rate of the PRDL matrix, reflecting its sufficiency in determining sectoral price dynamics.

Expanding on this finding, we then investigate the effects of monetary shocks as well as the spillover effects of shocks that emanate from one sector and propagate to others through the network over time. First, the PRDL matrix encodes how production networks amplify the cumulative response of GDP as well as inflation persistence in response to monetary shocks. It also captures the effects of monetary shocks on sectoral prices, which are asymmetric across sectors. Monetary shocks distort relative prices due to heterogeneities in production linkages and price stickiness parameters. All else equal, sectors that spend more on stickier suppliers have more persistent responses and disproportionally affect the tail response of inflation.

Next, we study the aggregate effects of persistent but transitory sectoral shocks in this economy. The PRDL matrix also governs the endogenous dynamics of sectoral prices, aggregate inflation, and GDP to these shocks, after they have propagated through the *inverse* Leontief matrix as in static models. Therefore, our analytical solutions shed light on two separate roles of the Leontief matrix in the propagation of sectoral shocks. While the inverse Leontief matrix governs the propagation of sectoral shocks through the network on impact—as in static models—the PRDL matrix governs their endogenous propagation across sectors *through* time and thereby, affects the persistence of inflation and GDP responses to these shocks. We show that these two effects accumulate: more input-output linkages amplify static propagation through the inverse Leontief and create dynamic effects that last longer through the PRDL matrix.

To uncover the economic forces encoded by the PRDL matrix, we use perturbation theory to approximate the eigenvalues of the PRDL matrix based on the model's primitives. This approach allows us to analytically prove that more production linkages at the micro-level amplify (1) monetary non-neutrality and inflation persistence in response to monetary shocks and (2) create spillover effects from sectoral shocks to aggregate inflation. We later show that this perturbation is remarkably accurate for the measured production network of the U.S. economy. This enables us to match each eigenvalue to a specific sector and provide a comprehensive ranking of sectors in terms of their contributions to inflation and GDP dynamics.

Using data on input-output tables, price adjustment frequencies, and consumption shares, we construct our sufficient statistics for the U.S. and quantify the importance of production networks for the propagation of shocks. First, we find that by introducing strategic complementarities in pricing, these production linkages quadruple the cumulative response of GDP and double the half-life of the consumer price index (CPI) inflation response to a monetary shock. Second, we show that underneath these aggregate responses is a rich distribution of sectoral inflation responses. The study of these sectoral responses allows us to identify industries that disproportionately affect aggregate monetary non-neutrality and inflation persistence. These disproportionate effects are quantitatively large. To show this, we identify the top three sectors that contribute the most to the tail response of inflation. In a counterfactual exercise, we show that although the combined consumption share of these three sectors is essentially zero, dropping them from the network reduces monetary non-neutrality by 16 percent.

We then quantitatively study the sectoral origins of aggregate fluctuations, with a particular focus on the pass-through of sectoral shocks to aggregate inflation. Specifically, we consider sectoral idiosyncratic TFP/wedge shocks that lead to a one percent increase in the inflation of their corresponding sector. We then quantify the pass-through of this sectoral inflation to aggregate inflation through the network—i.e., after removing the direct effect coming from the expenditure share of each sector. For instance, we find that the Oil and Gas Extraction industry is among the top sectors in causing a high initial impact on aggregate inflation, driven by its role as an input to many sectors in the economy.

We next rank sectors based on their contribution to the persistence of aggregate inflation dynamics. Relying on our perturbed eigenvalues, we show that the key quantitative determinant of these effects is an input-output adjusted duration of price spells within these sectors. To provide concrete examples, this adjusted duration in Oil and Gas Extraction industry is relatively small due to the high price flexibility in this sector. Thus, a shock to this sector does not lead to persistent aggregate inflation effects *even though* it is an input to many sectors. The Semiconductor Manufacturing Machinery industry, in contrast, has very persistent aggregate inflation effects because its adjusted duration is relatively larger. To understand how these persistent responses connect with the real effects of sectoral shocks, we also show that sectoral shocks that cause higher persistent inflation responses also lead to greater GDP gap effects.

Finally, having established these analytical and quantitative results on the *separate* roles of monetary and sectoral shocks, we study the propagation of sectoral shocks when monetary policy endogenously responds to neutralize their inflationary effects. Conventionally, in benchmark New Keynesian (NK) models, inflationary pressures are determined by the slope of the *aggregate* Phillips curve. In those models, this slope is the elasticity of inflation to demand shocks (output gap). We use our theoretical results to show that in multi-sector models with production networks, the slope of the aggregate Phillips curve is neither sufficient for the magnitude nor the direction of non-neutrality and inflation persistence. The key behind this observation is that in multi-sector economies, the Phillips curve is also affected by relative price distortions that are not captured by its slope. To illustrate this point, we offer a counterexample with two multi-sector economies where the economy with the *steeper* Phillips curve also exhibits *higher* monetary non-neutrality.[2]

We conclude that these relative price distortions are theoretically and quantitatively relevant for the inflationary effects of sectoral shocks, especially when monetary policy stabilizes aggregate inflation conditional on shocks to sectors with higher input-output adjusted price flexibility. For instance, a sectoral shock to Oil and Gas Extraction industry that raises inflation in that sector has a large pass-through to aggregate inflation when monetary policy keeps interest rates fixed.[3] In contrast, if monetary policy responds to this shock by stabilizing aggregate inflation, it generates a substantially negative GDP gap response, in contrast to benchmark NK models.

To illustrate this last point, inflationary TFP shocks in benchmark NK models are *expansionary* to the output gap because sticky prices do not increase as much as they would if they were flexible. In the extreme case when monetary policy fully stabilizes TFP-driven inflation in those models, divine coincidence arises and the output gap is also stabilized. Yet, we find that in the production network of the U.S. economy, stabilizing TFP-driven inflation due to Oil shocks *contracts* the GDP gap of the economy as the TFP shock to the Oil industry trickles to all the downstream sectors that use Oil as an input and have higher price stickiness.

It is important to highlight the interaction of production linkages and price stickiness for this last result. For instance, in contrast to the example above, stabilizing aggregate inflation conditional on an inflationary TFP shock to the Semiconductor Manufacturing Machinery industry is not very costly in terms of GDP gap. This industry is also an input to many sectors, similar to the Oil and Gas Extraction industry, but it has a much higher price stickiness relative

[2]For a more detailed discussion of this issue, we refer the reader to Hazell, Herreno, Nakamura, and Steinsson (2022) who carefully layout assumptions under which a two-sector economy admits an aggregate Phillips curve whose slope is still sufficient for the elasticity of inflation to demand shocks. As they note, more generally, such a result does not hold in multi-sector economies.

[3]In the background, monetary policy achieves determinacy by controlling the nominal GDP, which, under the preferences that we consider, corresponds to fixed interest rates in equilibrium.

to sectors that are downstream to it. Thus, the contractionary effects of stabilizing aggregate inflation are also much smaller because sectoral inflation in that sector does not distort relative prices as much.

Related Literature. This paper contributes to the literature on the propagation of shocks in multi-sector New Keynesian economies with production linkages and heterogeneous price stickiness. Within this literature, the closest recent work to ours is La'O and Tahbaz-Salehi (2022) which studies optimal monetary policy in a static model with production networks and information frictions, as well as Rubbo (2023) which studies optimal monetary policy in a dynamic model with production networks and heterogeneous price stickiness. We contribute to this literature by analyzing the propagation of sectoral shocks and deriving analytical results that shed light on the determinants of this propagation. We also provide results for the propagation of monetary shocks, especially focusing on how monetary policy affects the transmission of sectoral shocks. Our analytical results also shed light on how sectoral prices respond to monetary shocks and how few sectors have disproportionate effects on the tail response of aggregate inflation.

More broadly, our analytical results on the real effects of monetary shocks are related to three strands of the literature. First, they connect to results in Carvalho (2006) and Nakamura and Steinsson (2010) which showed how heterogeneous price stickiness amplifies monetary non-neutrality in time- and state-dependent models respectively. Second, our findings on how production linkages amplify real effects of monetary shocks are connected to the insights of Blanchard (1983); Basu (1995) and more recently La'O and Tahbaz-Salehi (2022) which showed that such amplification stems from strategic complementarities introduced by production networks. Third, in more recent work, Carvalho, Lee, and Park (2021); Pasten, Schoenle, and Weber (2020) and Ghassibe (2021) study the role of production networks in the transmission of monetary shocks.[4] Our main contribution to these strands of the literature is that we consider a multi-sector New Keynesian model with unrestricted input-output linkages.

Moreover, our results on the propagation of sectoral shocks to the aggregate economy in models with production networks build on a rich literature in static settings that considers various formulations of exogenous production networks. Long and Plosser (1983), Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), Jones (2013) are important contributions and Carvalho

[4]In deriving sufficient statistics for real effects of monetary policy shocks in sticky-price models with strategic complementarities, our paper relates to recent work by Wang and Werning (2021) and Alvarez, Lippi, and Souganidis (2022), which derive similar statistics in settings with oligopolies and menu costs, but not production networks. These results are also related to Alvarez, Le Bihan, and Lippi (2016); Baley and Blanco (2021) who provide analytical results in settings with idiosyncratic shocks and menu costs but no strategic complementarities.

(2014); Carvalho and Tahbaz-Salehi (2019) provide comprehensive surveys of the literature. More recently, Bigio and La'O (2020) and Baqaee and Farhi (2020) study misallocation and aggregate productivity in settings with inefficiencies and wedges. Our focus on dynamics as well as our analytical methods also connects with the work in Liu and Tsyvinski (2021), which analyzes the dynamics of real variables in a model with adjustment costs in inputs. While this body of work focuses on how sectoral shocks propagate to the aggregate economy in settings without nominal rigidities, we focus on how the propagation of sectoral shocks interacts with nominal rigidities in dynamic settings with production networks.[5]

Outline. Section 2 presents our model. Section 3 derives our analytical results as well as sufficient statistics, and presents quantitative results on inflation and GDP responses to shocks in the U.S. production network. Section 4 discusses how endogenous monetary policy affects the propagation of shocks. Section 5 presents several extensions. Section 6 concludes.

2 Model

2.1. Environment

Time is continuous and is indexed by $t \in \mathbb{R}_+$. The economy consists of a representative household, monetary and fiscal authorities, and n sectors with input-output linkages. In each sector $i \in [n] \equiv \{1, 2, \ldots, n\}$, a unit measure of monopolistically competitive firms use labor and goods from all sectors to produce and supply to a competitive final good producer within the same industry. These final goods are sold to the household and other industries.

Household. The representative household demands the final goods produced by each industry, supplies labor in a competitive market, and holds nominal bonds with nominal yield i_t. Household's preferences over consumption C and labor supply L is $U(C) - V(L)$, where U and V are strictly increasing with Inada conditions, and $U''(.) < 0$, $V''(.) > 0$. Household solves:

$$\max_{\{(C_{i,t})_{i\in[n]}, L_t, B_t, M_t\}_{t\geq 0}} \int_0^\infty e^{-\rho t}[U(C_t) - V(L_t)]\mathrm{d}t \tag{1}$$

$$s.t. \quad \sum_{i\in[n]} P_{i,t}C_{i,t} + \dot{B}_t \leq W_t L_t + i_t B_t + \text{Profits}_t + T_t, \qquad C_t \equiv \Phi(C_{1,t}, \ldots, C_{n,t}) \tag{2}$$

Here, $\Phi(.)$ defines the consumption index C_t over the household's consumption from sectors $(C_{i,t})_{i\in[n]}$. It is degree one homogenous, strictly increasing in each $C_{i,t}$, satisfying Inada conditions. L_t is labor supply at wage W_t, $P_{i,t}$ is sector i's final good price, B_t is demand for nominal bonds,

[5] Other papers, such as Taschereau-Dumouchel (2020), consider endogenous production networks in real models and study phenomena such as cascades. We use exogenous production networks, thereby using a simpler setting, but we study a dynamic model with sticky prices.

Profits_t denote all firms' profits rebated to the household, and T_t is a lump-sum tax.

Monetary and Fiscal Policy. For our baseline, we assume monetary authority directly controls the path of nominal GDP, $\{M_t \equiv P_t C_t\}_{t\geq 0}$, where P_t is the consumer price index (CPI).[6] A Taylor rule extension is in Section 5.2. The fiscal authority taxes or subsidizes intermediate firms' sales in each sector i at a possibly time-varying rate $\tau_{i,t}$, lump-sum transferred back to the household. A *wedge shock* to sector i is an *unexpected* disturbance in that sector's taxes.

Final Good Producers. A competitive final good producer in each industry i buys from a continuum of intermediate firms in its sector, indexed by $ij : j \in [0,1]$, and produces a final sectoral good using a CES production function. The profit maximization problem of this firm is:

$$\max_{(Y^d_{ij,t})_{j\in[0,1]}} P_{i,t}Y_{i,t} - \int_0^1 P_{ij,t}Y^d_{ij,t}\mathrm{d}j \quad s.t. \quad Y_{i,t} = \left[\int_0^1 (Y^d_{ij,t})^{1-\sigma_i^{-1}}\mathrm{d}j\right]^{\frac{1}{1-\sigma_i^{-1}}} \tag{3}$$

where $Y^d_{ij,t}$ is the producer's demand for variety ij at price $P_{ij,t}$, $Y_{i,t}$ is its production at price $P_{i,t}$, and $\sigma_i > 1$ is the substitution elasticity across varieties in i. Thus, demand for variety ij is:

$$Y^d_{ij,t} = \mathcal{D}(P_{ij,t}/P_{i,t}; Y_{i,t}) \equiv Y_{i,t}\left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\sigma_i} \quad \text{where} \quad P_{i,t} = \left[\int_0^1 P_{ij,t}^{1-\sigma_i}\mathrm{d}j\right]^{\frac{1}{1-\sigma_i}} \tag{4}$$

Final good producers define a unified good for each industry and have zero value added due to being competitive and constant returns to scale (CRS) production.

Intermediate Goods Producers. The intermediate good producer ij uses labor as well as the sectoral goods as inputs and produces with the following CRS production function:

$$Y^s_{ij,t} = Z_{i,t}F_i(L_{ij,t}, X_{ij,1,t}, \ldots, X_{ij,n,t}) \tag{5}$$

where $Z_{i,t}$ is sector i's Hicks-neutral productivity, $L_{ij,t}$ is firm ij's labor demand, and $X_{ij,k,t}$ is its demand for sector k's final good. The function F_i is strictly increasing in all arguments with Inada conditions. The firm's total cost for producing output Y, given $\mathbf{P}_t \equiv (W_t, P_{i,t})_{i\in[n]}$, is:

$$\mathcal{C}_i(Y; \mathbf{P}_t, Z_{i,t}) \equiv \min_{L_{ij,t}, X_{ij,k,t}} W_t L_{ij,t} + \sum_{k\in[n]} P_{k,t}X_{ij,k,t} \quad s.t. \quad Z_{i,t}F_i(L_{ij,t}, X_{ij,1,t}, \ldots, X_{ij,n,t}) \geq Y \tag{6}$$

In each sector i, firms set their prices under a Calvo friction, where i.i.d. price change opportunities arrive at Poisson rates θ_i. Given its cost in Equation (6) and its demand in Equation (4), a firm ij that has the opportunity to change its price at time t chooses its *reset price*, denoted

[6] Such policy can be implemented by a cash-in-advance constraint (e.g. La'O and Tahbaz-Salehi, 2022), money in utility (e.g. Golosov and Lucas, 2007) or nominal GDP growth targeting (e.g. Afrouzi and Yang, 2019).

by $P^{\#}_{ij,t}$, to maximize the expected net present value of its profits until the next price change:

$$P^{\#}_{ij,t} \equiv \arg\max_{P_{ij,t}} \int_0^\infty \theta_i e^{-(\theta_i h + \int_0^h i_{t+s} \mathrm{d}s)} \left[(1-\tau_{i,t}) P_{ij,t} \mathcal{D}(P_{ij,t}/P_{i,t+h}; Y_{i,t+h}) - \mathcal{C}_i(Y^s_{ij,t+h}; \mathbf{P}_{t+h}, Z_{i,t+h})\right] \mathrm{d}h$$

$$s.t. \quad Y^s_{ij,t+h} \geq \mathcal{D}(P_{ij,t}/P_{i,t+h}; Y_{i,t+h}), \quad \forall h \geq 0 \tag{7}$$

where $\theta_i e^{-\theta_i h}$ is the duration density of the next price change, $e^{-\int_0^h i_{t+h} \mathrm{d}s}$ is the discount rate based on nominal rates, and $\tau_{i,t}$ is the tax/subsidy rate on sales. Were prices flexible, maximizing net present value of profits would be equivalent to choosing *desired* prices, denoted by $P^*_{ij,t}$, that maximized firms' static profits within every instant. Desired prices solve:

$$P^*_{ij,t} \equiv \arg\max_{P_{ij,t}} \ (1-\tau_{i,t}) P_{ij,t} \mathcal{D}(P_{ij,t}/P_{i,t}; Y_{i,t}) - \mathcal{C}_i(Y^s_{ij,t}; \mathbf{P}_t, Z_{i,t}) \quad s.t. \quad Y^s_{ij,t} \geq \mathcal{D}(P_{ij,t}/P_{i,t}; Y_{i,t}) \tag{8}$$

Equilibrium Definition. An equilibrium is a set of allocations for households and firms, monetary and fiscal policies, and prices such that: (1) given prices and policies, the allocations are optimal for households and firms, and (2) markets clear. A precise definition is in Appendix B.

2.2. Log-Linearized Approximation

We log-linearize this economy around an efficient steady-state, derivations of which are in Appendix C. For our baseline analysis, we use Golosov and Lucas (2007)'s preferences, $U(C) - V(L) = \log(C) - L$, which simplifies the analytical expressions. In Section 5.1, we consider a more general specification. Going forward, small letters denote the log deviations of their corresponding variables from their steady-state values.

Sectoral Prices. While prices are staggered within sectors, the Calvo assumption implies that we can fully characterize aggregate sectoral prices by desired and reset prices.

First, desired prices are equal to firms' marginal costs, $(mc_{i,t})_{i\in[n]}$, up to a wedge that captures markups or other distortions, $(\omega_{i,t})_{i\in[n]}$. With input-output linkages, $mc_{i,t}$ depends on the aggregate wage, w_t, sectoral prices, $(p_{k,t})_{k\in[n]}$, and the sectoral productivity, $z_{i,t}$:

$$p^*_{i,t} \equiv \omega_{i,t} + mc_{i,t}, \quad mc_{i,t} \equiv \alpha_i w_t + \textstyle\sum_{k\in[n]} a_{ik} p_{k,t} - z_{i,t}, \quad \omega_{i,t} \equiv \log\left(\frac{\sigma_i}{\sigma_i - 1} \times \frac{1}{1-\tau_{i,t}}\right) \tag{9}$$

where α_i and $a_{i,k}$ are sector i's firms' labor share and expenditure share on sector k's final good in the steady-state, respectively. Thus, the steady-state *input-output matrix* is $\mathbf{A} \equiv [a_{ik}] \in \mathbb{R}^{n\times n}$.[7]

Second, the reset price in sector i is the average of all *future* desired prices, discounted at

[7] Baqaee and Farhi (2020) emphasize the distinction between cost-based and sales-based input-output matrices and Domar weights. In an efficient equilibrium, like the one we linearize around, the two are the same.

rate ρ and the probability density of the time between price changes, $e^{-(\rho+\theta_i)h}$:

$$p^{\#}_{i,t} = (\rho + \theta_i) \int_0^\infty e^{-(\rho+\theta_i)h} p^*_{i,t+h} \mathrm{d}h \tag{10}$$

Finally, given sector i's initial aggregate price at $t = 0$, $p_{i,0^-}$, the *aggregate* sectoral price $p_{i,t}$ is an average of the *past* reset prices, weighted by the density of time between price changes:

$$p_{i,t} = \theta_i \int_0^t e^{-\theta_i h} p^{\#}_{i,t-h} \mathrm{d}h + e^{-\theta_i t} p_{i,0^-} \tag{11}$$

Aggregate Price and GDP. The household's demand for goods defines the aggregate Consumer Price Index (CPI) as the expenditure share weighted average of sectoral prices:

$$p_t = \sum_{i\in[n]} \beta_i p_{i,t}, \quad \text{with} \quad \sum_{i\in[n]} \beta_i = 1 \tag{12}$$

where $\boldsymbol{\beta} = (\beta_i)_{i\in[n]}$ is the vector of the household's expenditure shares in the efficient steady-state.

The aggregate GDP, y_t, is equal to aggregate consumption and is given by the difference between the nominal GDP, m_t, and the CPI, p_t: $y_t \equiv m_t - p_t$. Fully elastic labor supply implies that the wage is equal to nominal demand:[8]

$$w_t = p_t + y_t = m_t \quad \text{(fully elastic labor supply)} \tag{13}$$

Equilibrium in the Approximated Economy. Given a path for $(\boldsymbol{\omega}_t, \boldsymbol{z}_t, m_t)_{t\geq 0}$, an equilibrium is a path for GDP, wage and prices, $\vartheta \equiv \{y_t, w_t, p_t, (p^*_{i,t}, p^{\#}_{i,t}, p_{i,t})_{i\in[n]}\}_{t\geq 0}$, such that given a vector of initial sectoral prices, $\mathbf{p}_{0^-} = (p_{i,0^-})_{i\in[n]}$, ϑ solves Equations (9) to (13).

Flexible Prices and GDP. Consider a counterfactual economy where all prices are flexible. By Equation (9), we can derive *flexible prices* of this economy, denoted by $\mathbf{p}^f_t \in \mathbb{R}^n$, as:

$$\mathbf{p}^f_t = w_t \boldsymbol{\alpha} + \mathbf{A}\mathbf{p}^f_t + \boldsymbol{\omega}_t - \boldsymbol{z}_t \quad \Rightarrow \quad \mathbf{p}^f_t = m_t \mathbf{1} + \boldsymbol{\Psi}(\boldsymbol{\omega}_t - \boldsymbol{z}_t) \tag{14}$$

where $\boldsymbol{\alpha} \equiv (\alpha_i)_{i\in[n]}$ contains labor shares, $\mathbf{1}$ is the vector of ones, and $\boldsymbol{\Psi} \equiv (\mathbf{I} - \mathbf{A})^{-1}$ is the inverse Leontief matrix. A key observation is that $\mathbf{p}^f_t$ is only a function of exogenous shocks and model parameters. We can also derive the *flexible price GDP*, y^f_t, in this counterfactual economy as:

$$y^f_t = m_t - \boldsymbol{\beta}^\intercal \mathbf{p}^f_t = \underbrace{\boldsymbol{\lambda}^\intercal \boldsymbol{z}_t}_{\text{aggregate TFP}} - \underbrace{\boldsymbol{\lambda}^\intercal \boldsymbol{\omega}_t}_{\text{labor wedge}}, \qquad \boldsymbol{\lambda} \equiv \left(\tfrac{P_i Y_i}{PC}\right)_{i\in[n]} = \boldsymbol{\Psi}^\intercal \boldsymbol{\beta} \tag{15}$$

where $\boldsymbol{\lambda}$ is the vector of Domar weights in the steady state.[9] Equation (15) shows that two terms determine flexible GDP around the efficient steady-state up to first order: (1) the aggregate TFP, which is the Domar-weighted sectoral productivities (Hulten, 1978), (2) the labor wedge

[8]See Section 5.1 for an extension to the case with partially elastic labor supply.

[9]The Domar weight of a sector i, λ_i, is the ratio of its total sales to the household's total nominal expenditures.

due to distortions, which is the Domar-weighted wedges across sectors (Bigio and La'O, 2020).

3 Sufficient Statistics

Here, we solve sectoral price dynamics in closed form and derive our sufficient statistics results. We then measure these sufficient statistics for the U.S. economy and provide quantitative results on aggregate and sectoral shocks. All the proofs are in Appendix A.

3.1. Dynamics of Prices

Let $\mathbf{p}_t \equiv (p_{i,t})_{i\in[n]}$, $\mathbf{p}_t^{\#} \equiv (p_{i,t}^{\#})_{i\in[n]}$ and $\mathbf{p}_t^{*} \equiv (p_{i,t}^{*})_{i\in[n]}$ be the vectors of sectoral aggregate, reset and desired prices, respectively. Using Equations (9) and (14):[10]

$$\mathbf{p}_t^{*} = (\mathbf{I}-\mathbf{A})\mathbf{p}_t^{f} + \mathbf{A}\mathbf{p}_t \tag{16}$$

where $\mathbf{p}_t^{f}$ is the vector of flexible equilibrium prices in Equation (14). Equation (16) shows that firms' desired prices across sectors is a convex combination of *exogenous* flexible equilibrium prices and *endogenous* sectoral prices in the sticky price economy, with the input-output matrix $\mathbf{A}$ fully capturing the *strategic complementarities* induced by production linkages across the economy (Blanchard, 1983; Basu, 1995; La'O and Tahbaz-Salehi, 2022).

Accordingly, reset and sectoral prices in Equations (10) and (11) solve:

$$\boldsymbol{\pi}_t^{\#} \equiv \dot{\mathbf{p}}_t^{\#} = (\rho\mathbf{I}+\boldsymbol{\Theta})(\mathbf{p}_t^{\#}-\mathbf{p}_t^{*}), \qquad \text{forward-looking with} \qquad \lim_{t\to\infty} e^{-(\rho\mathbf{I}+\boldsymbol{\Theta})t}\mathbf{p}_t^{\#} = 0, \tag{17}$$

$$\boldsymbol{\pi}_t \equiv \dot{\mathbf{p}}_t = \boldsymbol{\Theta}(\mathbf{p}_t^{\#}-\mathbf{p}_t), \qquad \text{backward-looking with} \qquad \mathbf{p}_0 = \mathbf{p}_{0^-} \tag{18}$$

Here, $\boldsymbol{\pi}_t^{\#}$ and $\boldsymbol{\pi}_t$ are the *inflation rates* in reset and aggregate prices across sectors, respectively. $\boldsymbol{\Theta} = \mathrm{diag}(\theta_i) \in \mathbb{R}^{n\times n}$ is a diagonal matrix, with its i'th diagonal entry representing the frequency of price adjustments in sector i.[11] The memorylessness of the Poisson price adjustments (Calvo assumption) allows us to represent this system only in terms of sectoral prices, $\mathbf{p}_t$:

Proposition 1. Sectoral prices evolve according to the following set of differential equations:

$$\dot{\boldsymbol{\pi}}_t = \rho\boldsymbol{\pi}_t + \boldsymbol{\Theta}(\rho\mathbf{I}+\boldsymbol{\Theta})(\mathbf{I}-\mathbf{A})(\mathbf{p}_t-\mathbf{p}_t^{f}), \qquad \text{with } \mathbf{p}_0 = \mathbf{p}_{0^-} \text{ given.} \tag{19}$$

We discuss the main implications of Proposition 1 in the following four remarks.

Remark 1. *Equation (19) represents the* sectoral Phillips curves *of this economy in vector form, linking changes in inflation to the* gap *between prices and their counterparts in a flexible*

[10] Using $\boldsymbol{\alpha} = (\mathbf{I}-\mathbf{A})\mathbf{1}$, the vector form of Equation (9) is $\mathbf{p}_t^{*} = (\mathbf{I}-\mathbf{A})(\mathbf{1}w_t + \boldsymbol{\Psi}(\boldsymbol{\omega}_t - \boldsymbol{z}_t)) + \mathbf{A}\mathbf{p}_t$.

[11] In this draft, we frequently use the exponential function of square matrices, defined by its corresponding power series: $\forall \mathbf{X}\in\mathbb{R}^{n\times n}$, $e^{\mathbf{X}} \equiv \sum_{k=0}^{\infty}\mathbf{X}^k/k!$, which is well-defined because these series always converge.

economy. The matrix $\mathbf{\Gamma} \equiv \mathbf{\Theta}(\rho\mathbf{I} + \mathbf{\Theta})(\mathbf{I} - \mathbf{A})$*—the Leontief matrix,* $\mathbf{I} - \mathbf{A}$*, adjusted by a quadratic form of price adjustment frequencies,* $\mathbf{\Theta}(\rho\mathbf{I} + \mathbf{\Theta})$*—encodes the slopes of these Phillips curves.*

Equation (19) differs from the usual representations of Phillips curves featuring output gap. Such an equivalent representation exists for Equation (19), which we discuss in detail in Section 4. However, we start with the representation above because it is the most straightforward way to demonstrate the following remarks and derive our analytical results.

Remark 2. *Sectoral Phillips curves, with boundary conditions* $\mathbf{p}_0 = \mathbf{p}_{0^-}$ *and non-explosive prices, uniquely pin down the path of sectoral prices for a given path of flexible prices* $(\mathbf{p}_t^f)_{t\geq 0}$*.*

The key to this observation is that the only endogenous variables in the system of second-order differential equations in Equation (19) are nominal prices and their inflation rates, $\mathbf{p}_t$ and $\boldsymbol{\pi}_t$, with $\mathbf{p}_t^f$ acting as an *exogenous* forcing term. Intuitively, nominal prices in the sticky price economy should adjust towards their flexible levels, $\mathbf{p}_t^f$. This is formalized in Equation (19), where inflation in sectoral prices depends solely on the time series of nominal price gaps, $\mathbf{p}_t - \mathbf{p}_t^f$.

Remark 3. *All shocks* $(\boldsymbol{\omega}_t, \boldsymbol{z}_t, m_t)_{t\geq 0}$ *affect price dynamics* only *through flexible prices,* $(\mathbf{p}_t^f)_{t\geq 0}$*.*

The observation in Remark 3 demonstrates the power of expressing inflation dynamics in terms of *nominal price gaps*. It implies that solving for the dynamics of prices for a given path of $\mathbf{p}_t^f$ is equivalent to having characterized impulse response functions of *all* the prices in the economy to all three types shocks–TFP, markup/wedge, and monetary–in a unified framework.

Remark 4. *All parameters affect the dynamics of sectoral prices* only *through the* duration-adjusted Leontief matrix*,* $\mathbf{\Gamma}$*, and the household's discount rate,* ρ*.*

Intuitively, the dynamics of prices in a production network depend on the frequency of price adjustments ($\mathbf{\Theta}$) and how these shocks propagate through input-output linkages (the Leontief matrix). Proposition 1 formally shows how these two mechanisms interact through $\mathbf{\Gamma}$ and ρ. Moreover, note that substitution elasticities across different inputs have no impact on price dynamics at the first order. This is due to the flatness of the marginal cost function with respect to inputs at the optimum by Shephard's Lemma (see, e.g., Baqaee and Farhi, 2020).

Given that ρ is usually calibrated close to zero, we will assume $\rho = 0$ going forward.[12] This makes $\mathbf{\Gamma}$ the sole object through which model parameters affect prices, allowing us to fully focus on the economic intuition behind its effects. We now state the main result of this section.

[12] With an annual interest rate of 0.04, $\rho \approx \ln(1.04)/12 \approx 0.003$ at a monthly frequency. However, there is a literature that reinterprets a larger ρ as a parameter for disciplining how myopic firms are in price-setting (see, e.g., Gabaix, 2020). See Minton and Wheaton (2022) for a discussion of myopia in production networks.

Proposition 2. Suppose $\mathbf{p}_t^f$ is piece-wise continuous and is bounded,[13] and let $\rho = 0$. Then, given $\mathbf{p}_t^f$ and a vector of initial prices $\mathbf{p}_{0^-}$, the *principal square root of the duration-adjusted Leontief (PRDL) matrix*, $\sqrt{\boldsymbol{\Gamma}}$, exists and is a sufficient statistic for dynamics of sectoral prices:[14]

$$\mathbf{p}_t = \underbrace{e^{-\sqrt{\boldsymbol{\Gamma}}t}\mathbf{p}_{0^-} + \sqrt{\boldsymbol{\Gamma}}e^{-\sqrt{\boldsymbol{\Gamma}}t}\int_0^t \sinh(\sqrt{\boldsymbol{\Gamma}}h)\mathbf{p}_h^f \mathrm{d}h}_{\text{inertial effect of past prices due to stickiness}} + \underbrace{\sqrt{\boldsymbol{\Gamma}}\sinh(\sqrt{\boldsymbol{\Gamma}}t)\int_t^\infty e^{-\sqrt{\boldsymbol{\Gamma}}h}\mathbf{p}_h^f \mathrm{d}h}_{\text{forward looking effect of future prices}} \tag{20}$$

Drawing on Remarks 1 to 4, Proposition 2 presents the analytical solution for dynamics of *all* sectoral prices. This solution specifically highlights the interplay between the forward-looking nature of pricing decisions and the backward-looking nature of aggregation Equations (17) and (18). While firms take the future path of $\mathbf{p}_t^f$ into account when setting prices, aggregate prices also depend on the past path of $\mathbf{p}_t^f$ due to the persistence of stickiness over time.

Furthermore, Proposition 2 illustrates that it is not $\boldsymbol{\Gamma}$ itself that is crucial for price dynamics, but rather its *principal square root*, which is the square root of $\boldsymbol{\Gamma}$ all of whose eigenvalues have positive real parts. From an economic standpoint, this square root emerges as a result of the system's dual forward-looking and backward-looking nature. Firms take the future and past paths of flexible prices into account when adjusting prices so that these paths affect dynamics partially insofar as such changes were not incorporated at the time of adjustment. Additionally, the principal square root is the relevant square root because it is the one that adheres to stability boundary conditions. Proving the existence of $\sqrt{\boldsymbol{\Gamma}}$ mainly relies on the economic assumptions that all sectors have strictly positive labor shares and price adjustment frequencies.[15]

Next, we explore the analytical solution presented in Proposition 2 by examining the IRFs of sectoral prices, CPI inflation, and GDP (gap) to monetary, sectoral TFP, and wedge shocks.

3.2. Impulse Response Functions

Using Proposition 2, we can obtain IRFs by plugging in specific paths for $\mathbf{p}_t^f$ implied by shocks. Consider the economy in its steady state at $t = 0^-$ (left limit at $t = 0$), so that exogenous variables $(\boldsymbol{z}_t, \boldsymbol{\omega}_t, m_t) = (\boldsymbol{z}_{0^-}, \boldsymbol{\omega}_{0^-}, m_{0^-})$ for $t \uparrow 0$ and all prices are at their flexible level: $\mathbf{p}_{0^-} - \mathbf{p}_{0^-}^f = 0$.

[13] In our setting with perfect foresight, piece-wise continuity ensures that $\mathbf{p}_t^f$ is Riemann integrable with unexpected shocks introducing at most countable jumps in flexible prices. The boundedness assumption is not restrictive with zero trend inflation. With trend inflation, boundedness is replaced with exponential order.

[14] The hyperbolic sine of a square matrix $\mathbf{X}$ is defined as $\sinh(\mathbf{X}) \equiv (e^{\mathbf{X}} - e^{-\mathbf{X}})/2$.

[15] This ensures that the inverse Leontief matrix exists and has positive real entries (see, Carvalho and Tahbaz-Salehi, 2019, p. 639). We can then show $\boldsymbol{\Gamma}$ is a M-matrix: By Theorem 2.3 in (Berman and Plemmons, 1994, p. 134, condition N_{38}), this is true if $\boldsymbol{\Gamma}$ is inverse-positive; i.e., $\boldsymbol{\Gamma}^{-1} \geq 0$ elementwise. Since $\boldsymbol{\Theta}(\rho\mathbf{I} + \boldsymbol{\Theta})$ is invertible because $\theta_i > 0, \forall i$, and $\mathbf{I} - \mathbf{A}$ is invertible because inverse Leontief exists, $\boldsymbol{\Gamma}^{-1}$ exists and is the infinite sum of positive matrices: $\boldsymbol{\Gamma}^{-1} = \sum_{n=0}^{\infty} \mathbf{A}^n(\rho\mathbf{I} + \boldsymbol{\Theta})^{-1}\boldsymbol{\Theta}^{-1} \geq 0$. Finally, having shown that $\boldsymbol{\Gamma}$ is a non-singular M-matrix, we can apply Theorem 5 in Alefeld and Schneider (1982) which shows that every non-singular M-matrix has exactly one M-matrix as its square root, which is also its principal square root by properties of M-matrices.

3.2.1. Monetary Shocks. An expansionary monetary shock is a one-time unexpected but permanent increase in nominal GDP: $m_t = m_{0-} + \delta_m, \forall t \geq 0$ where δ_m denotes the shock size. The implied path for $\mathbf{p}_t^f$ is $\mathbf{p}_t^f = \mathbf{p}_{0-}^f + \delta_m \mathbf{1}$, where $\mathbf{1}$ is a vector of ones.

Proposition 3. The impulse response functions of sectoral prices, $\mathbf{p}_t$; CPI inflation, $\pi_t = \boldsymbol{\beta}^\intercal \boldsymbol{\pi}_t$; GDP, y_t; and GDP gap, $\tilde{y}_t \equiv y_t - y_t^f$ to an expansionary monetary shock are given by:

$$\tfrac{\partial}{\partial \delta_m} \mathbf{p}_t = (\mathbf{I} - e^{-\sqrt{\boldsymbol{\Gamma}} t}) \mathbf{1}, \qquad \tfrac{\partial}{\partial \delta_m} \pi_t = \boldsymbol{\beta}^\intercal \sqrt{\boldsymbol{\Gamma}} e^{-\sqrt{\boldsymbol{\Gamma}} t} \mathbf{1}, \qquad \tfrac{\partial}{\partial \delta_m} y_t = \tfrac{\partial}{\partial \delta_m} \tilde{y}_t = \boldsymbol{\beta}^\intercal e^{-\sqrt{\boldsymbol{\Gamma}} t} \mathbf{1} \tag{21}$$

Proposition 3 shows: (1) The only relevant objects for the sectoral price, inflation, and GDP dynamics are $\sqrt{\boldsymbol{\Gamma}}$ and expenditure shares $\boldsymbol{\beta}$. Thus, we can compute these IRFs for the input-output structure of the U.S. economy once we construct $\sqrt{\boldsymbol{\Gamma}}$ and the expenditure shares $\boldsymbol{\beta}$ from the data. (2) Although *relative* sectoral prices converge back to the steady-state in the long run, the aggregate monetary shock distorts these relative prices on the transition path. These distortions are also captured by $\sqrt{\boldsymbol{\Gamma}}$ and thus are measurable. (3) $\sqrt{\boldsymbol{\Gamma}}$ also captures the degree of monetary non-neutrality in the economy since GDP response to a monetary shock is zero in the flexible economy. We see this in the cumulative impulse response (CIR) of GDP, obtained by integrating the area under its impulse response function:

$$\mathrm{CIR}_{\tilde{y},m} \equiv \int_0^\infty \tfrac{\partial}{\partial \delta_m} \tilde{y}_t \mathrm{d}t = \boldsymbol{\beta}^\intercal \sqrt{\boldsymbol{\Gamma}}^{-1} \mathbf{1} \tag{22}$$

3.2.2. TFP and Wedge Shocks. How do sectoral prices, CPI and GDP respond to sectoral TFP/wedge shocks? To answer this question, we consider the following shock to any sector i:

$$\omega_{i,t} - z_{i,t} = \omega_{i,0-} - z_{i,0-} + e^{-\phi_i t} \delta_z^i, \quad \forall t \geq 0 \tag{23}$$

Here, a positive δ_z^i captures a negative TFP or a positive wedge shock to sector i that decays at the rate $\phi_i \geq 0$. We note that $\phi_i = 0$ would correspond to a permanent TFP/Wedge shock while a positive ϕ_i denotes a temporary disturbance that disappears at rate ϕ_i. The implied path for $\mathbf{p}_t^f$, given such as shock, is $\mathbf{p}_t^f = \mathbf{p}_{0-}^f + e^{-\phi_i t} \delta_z^i \boldsymbol{\Psi} \mathbf{e}_i$, where $\boldsymbol{\Psi}$ is the inverse Leonteif matrix and $\mathbf{e}_i$ is the i'th standard basis vector. Economically, $\boldsymbol{\Psi} \mathbf{e}_i$ is a measure of sector i's *upstreamness* as it measures how much sector i, directly and indirectly, supplies to other sectors.

Proposition 4. Suppose $\phi_i \notin \mathrm{eig}(\sqrt{\boldsymbol{\Gamma}})$.[16] Then, the IRFs of sectoral prices, $\mathbf{p}_t$; CPI inflation, $\pi_t = \boldsymbol{\beta}^\intercal \boldsymbol{\pi}_t$; GDP, y_t; and GDP gap, $\tilde{y}_t = y_t - y_t^f$, to a TFP/wedge shock in sector i are given by:

$$\tfrac{\partial}{\partial \delta_z^i} \mathbf{p}_t = (e^{-\phi_i t} \mathbf{I} - e^{-\sqrt{\boldsymbol{\Gamma}} t})(\mathbf{I} - \phi_i^2 \boldsymbol{\Gamma}^{-1})^{-1} \boldsymbol{\Psi} \mathbf{e}_i, \qquad \tfrac{\partial}{\partial \delta_z^i} \pi_t = \boldsymbol{\beta}^\intercal (\sqrt{\boldsymbol{\Gamma}} e^{-\sqrt{\boldsymbol{\Gamma}} t} - \phi_i e^{-\phi_i t} \mathbf{I})(\mathbf{I} - \phi_i^2 \boldsymbol{\Gamma}^{-1})^{-1} \boldsymbol{\Psi} \mathbf{e}_i$$

[16] I.e., assume ϕ_i is not an eigenvalue of the $\sqrt{\boldsymbol{\Gamma}}$ matrix. This is a technical assumption that simplifies analytical derivations, but it is not restrictive: A limit of IRFs can be taken and is valid when $\phi_i \to x \in \mathrm{eig}(\sqrt{\boldsymbol{\Gamma}})$.

$$\frac{\partial}{\partial \delta_z^i} y_t = \boldsymbol{\beta}^\intercal (e^{-\sqrt{\boldsymbol{\Gamma}}t} - e^{-\phi_i t}\mathbf{I})(\mathbf{I} - \phi_i^2 \boldsymbol{\Gamma}^{-1})^{-1} \boldsymbol{\Psi}\mathbf{e}_i, \quad \frac{\partial}{\partial \delta_z^i} \tilde{y}_t = \boldsymbol{\beta}^\intercal (e^{-\sqrt{\boldsymbol{\Gamma}}t} - \phi_i^2 \boldsymbol{\Gamma}^{-1} e^{-\phi_i t})(\mathbf{I} - \phi_i^2 \boldsymbol{\Gamma}^{-1})^{-1} \boldsymbol{\Psi}\mathbf{e}_i$$

The most important observation from Proposition 4 is that, aside from the exogenous dynamics introduced by the shock ($e^{-\phi_i t}$), all endogenous dynamics are captured by $e^{-\sqrt{\boldsymbol{\Gamma}}}$. This is best illustrated in the limiting case when the shock is almost permanent $\phi_i \downarrow 0$:

$$\frac{\partial}{\partial \delta_z^i} \mathbf{p}_t|_{\phi_i \downarrow 0} = (\mathbf{I} - e^{-\sqrt{\boldsymbol{\Gamma}}t}) \boldsymbol{\Psi}\mathbf{e}_i, \quad \frac{\partial}{\partial \delta_z^i} \pi_t|_{\phi_i \downarrow 0} = \boldsymbol{\beta}^\intercal \sqrt{\boldsymbol{\Gamma}} e^{-\sqrt{\boldsymbol{\Gamma}}t} \boldsymbol{\Psi}\mathbf{e}_i, \quad \frac{\partial}{\partial \delta_z^i} \tilde{y}_t|_{\phi_i \downarrow 0} = \boldsymbol{\beta}^\intercal e^{-\sqrt{\boldsymbol{\Gamma}}t} \boldsymbol{\Psi}\mathbf{e}_i \tag{24}$$

This observation uncovers *two* separate roles of the Leontief matrix in the dynamic economy.

Remark 5. *The inverse Leontief matrix, $\boldsymbol{\Psi}$, determines the* static *propagation of TFP/wedge shocks by passing them through the network ($\mathbf{e}_i \to \boldsymbol{\Psi}\mathbf{e}_i$). The principal square root, $\sqrt{\boldsymbol{\Gamma}}$, determines the* dynamic *propagation of these shocks over time ($\boldsymbol{\Psi}\mathbf{e}_i \to e^{-\sqrt{\boldsymbol{\Gamma}}t}\boldsymbol{\Psi}\mathbf{e}_i$).*

Moreover, in response to TFP/wedge shocks, the GDP response combines both the response under flexible prices and the response of the GDP gap under sticky prices. To separate these, we define the *GDP gap* as $\tilde{y}_t \equiv y_t - y_t^f$ and decompose the CIR of GDP to its two components:

$$\mathrm{CIR}_{y,z_i} \equiv \int_0^\infty \frac{\partial}{\partial \delta_z^i} y_t \mathrm{d}t = \underbrace{-\phi_i^{-1}\lambda_i}_{\substack{\mathrm{CIR}_{y^f,z_i} \equiv \text{Flexible GDP Response} \\ \text{(Domar-weighted cumulative TFP)}}} + \underbrace{\boldsymbol{\beta}^\intercal (\phi_i \mathbf{I} + \sqrt{\boldsymbol{\Gamma}})^{-1} \boldsymbol{\Psi}\mathbf{e}_i}_{\mathrm{CIR}_{\tilde{y},z_i} \equiv \text{Cumulative GDP Gap Response}} \tag{25}$$

This decomposition provides intuition for the limiting case when $\phi_i \to 0$. Note that in this case, the flexible GDP CIR explodes because, with a permanent shock to TFP, the economy diverges from the initial steady-state (which is why we are only considering the case when $\phi_i \to 0$ and not $\phi_i = 0$). However, the GDP gap CIR is not explosive in this limit as the effects of sticky prices are only temporary deviations from the flexible price response:

$$\mathrm{CIR}_{x,z^i}|_{\phi_i \to 0} = \boldsymbol{\beta}^\intercal \sqrt{\boldsymbol{\Gamma}}^{-1} \boldsymbol{\Psi}\mathbf{e}_i \tag{26}$$

Equations (22) and (26) have a similar interpretation: Both show that with permanent shocks, the CIR of the GDP gap is the inner product of the expenditure weighted $\sqrt{\boldsymbol{\Gamma}}^{-1}$ and the instantaneous pass-through of a shock to firms' flexible prices ($\mathbf{1}$ for monetary shocks and $\boldsymbol{\Psi}\mathbf{e}_i$ for TFP/wedge shocks). We next turn to unpacking the economic interpretation of $\sqrt{\boldsymbol{\Gamma}}$.

3.3. Perturbation Around Diagonal Economies

We have shown $\sqrt{\boldsymbol{\Gamma}}$ encodes all the economic forces of the model in shaping the endogenous dynamics of prices and GDP. But what is its economic interpretation? In principle, we could use the Jordan decomposition of $\sqrt{\boldsymbol{\Gamma}}$ to perform a spectral analysis, but this approach does not take us far in terms of economic intuition. For instance, suppose $\sqrt{\boldsymbol{\Gamma}}$ is diagonalizable so there exists a diagonal $\mathbf{D} = \mathrm{diag}(d_1, \ldots, d_n)$, and an invertible matrix $\mathbf{P}$ such that $\sqrt{\mathbf{\Gamma}} = \mathbf{PDP}^{-1}$, which for instance would imply GDP and inflation responses to a monetary shock are

$$\tfrac{\partial}{\partial \delta_m} \tilde{y}_t = \boldsymbol{\beta}^\intercal e^{-\sqrt{\mathbf{\Gamma}} t} \mathbf{1} = \sum_{i=1}^{n} w_i e^{-d_i t}, \tag{27}$$

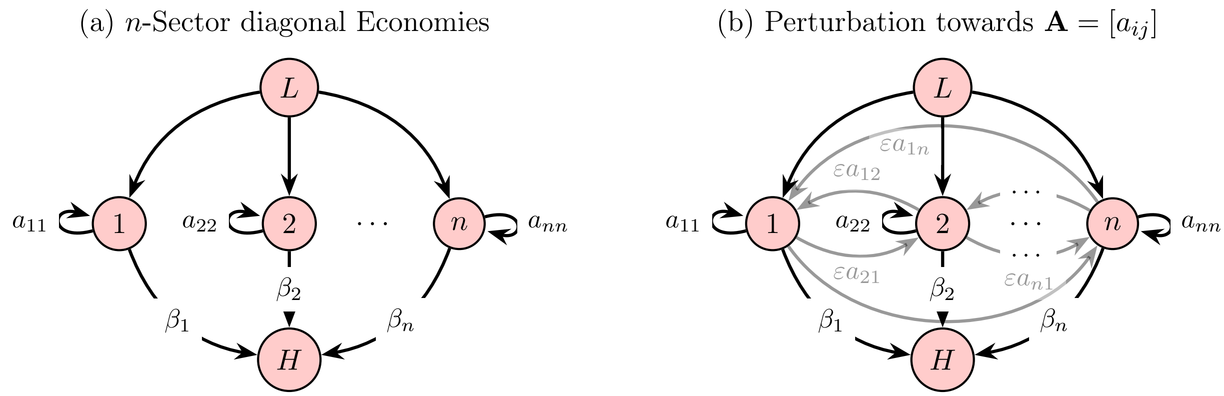
$$\tfrac{\partial}{\partial \delta_m} \pi_t = \boldsymbol{\beta}^\intercal \sqrt{\mathbf{\Gamma}} e^{-\sqrt{\mathbf{\Gamma}} t} \mathbf{1} = \sum_{i=1}^{n} d_i w_i e^{-d_i t}, \quad w_i \equiv \boldsymbol{\beta}^\intercal \mathbf{P} \mathbf{e}_i \mathbf{e}_i^\intercal \mathbf{P}^{-1} \mathbf{1} \tag{28}$$

The problem is it is unclear how the structure of the economy is reflected in the eigenvalues $\{d_i\}$ and coefficients $\{w_i\}$. The key idea here is to approximate an *arbitrary* input-output economy around "diagonal" economies, whose eigendecomposition has a clear economic interpretation.

We do not use this approximation in the quantitative results presented in the sections below but derive it here to provide intuition. We start by defining a diagonal economy as follows:

Definition 1. A **diagonal** economy is characterized by a diagonal input-output matrix.

Figure 1: Perturbation around diagonal Economies



Notes: Figure 1a draws the structure of diagonal economies where sectors operate independently but are allowed to use their own output in roundabout production. Figure 1b shows our parameterized perturbation of an arbitrary input-output matrix $\mathbf{A}$ around its diagonal structure: the perturbation is given by keeping sectors' own input shares from their output fixed, but only adds their input from other sectors proportional to an $\varepsilon > 0$.

Figure 1a depicts diagonal economies. These are multi-sector economies with heterogeneous price stickiness where sectors only use their own output in roundabout production. Diagonal economies are useful benchmarks because for each sector i, the corresponding eigenvalue is its frequency adjusted by the square root of their labor share, $d_i = \theta_i \sqrt{1 - a_{ii}}$, and the corresponding weight in Equation (27) is the household's expenditure share for that sector:

$$\tfrac{\partial}{\partial \delta_m} \tilde{y}_t = \sum_{i=1}^{n} \beta_i e^{-\theta_i \sqrt{1-a_{ii}}\, t}, \qquad \tfrac{\partial}{\partial \delta_m} \pi_t = \sum_{i=1}^{n} \beta_i \theta_i \sqrt{1 - a_{ii}} e^{-\theta_i \sqrt{1-a_{ii}}\, t} \tag{29}$$

These expressions are now interpretable; e.g., GDP response is the expenditure-weighted average of exponential functions, each decaying at the rate of the sector's adjusted frequency.

Now, consider an *arbitrary* n-sector economy with frequency matrix $\mathbf{\Theta} = \text{diag}(\theta_1, \ldots, \theta_n)$ and input-output matrix $\mathbf{A} = [a_{ij}]$, and define the corresponding diagonal economy as $\mathbf{A}_D \equiv \text{diag}(a_{11}, \ldots, a_{nn})$. Thus, we can write the duration-adjusted Leontief matrix $\mathbf{\Gamma} = \mathbf{\Theta}^2(\mathbf{I} - \mathbf{A})$ as the sum of the one in the diagonal economy $\mathbf{\Gamma}_D = \mathbf{\Theta}^2(\mathbf{I} - \mathbf{A}_D)$ and the off-diagonal matrix $\mathbf{\Gamma}_R$:

$$\mathbf{\Gamma} = \mathbf{\Gamma}_D + \mathbf{\Gamma}_R, \quad \text{with} \quad \mathbf{\Gamma}_R \equiv \mathbf{\Theta}^2(\mathbf{A}_D - \mathbf{A}) \tag{30}$$

This is a classic exercise in perturbation theory where we replace $\mathbf{\Gamma}$ with $\mathbf{\Gamma}(\varepsilon) = \mathbf{\Gamma}_D + \varepsilon \mathbf{\Gamma}_R$ for some $\varepsilon > 0$ and express the eigenvalues and eigenvectors as power series in ε (see, e.g., Bender and Orszag, 1999, p. 350). The economic interpretation is that we move from the diagonal economy, $\mathbf{A}_D$, towards the arbitrary economy, $\mathbf{A}$, in proportional to ε, as shown in Figure 1b. Notably, $\varepsilon = 0$ corresponds to the diagonal economy, and $\varepsilon = 1$ corresponds to the arbitrary economy, $\mathbf{A}$. Generally, eigenvalues and eigenvectors do not need to be differentiable in ε, especially for non-symmetric matrices as in our case. However, assuming that eigenvalues of $\mathbf{\Gamma}_D$ are distinct (i.e., sectors of the diagonal economy have distinct adjusted frequencies),[17] we obtain the following Lemma from Theorems 1 and 2 in Greenbaum, Li, and Overton (2020).

Lemma 1. Let $\xi_i \equiv \theta_i \sqrt{1 - a_{ii}}$ and assume ξ_i's are distinct. Let $(d_i(\varepsilon), \mathbf{v}_i(\varepsilon))$ be an eigenvalue/eigenvector pair for the principal square root of the perturbed economy, $\sqrt{\mathbf{\Gamma}(\varepsilon)}$. Then,

$$d_i(\varepsilon) = \xi_i + \mathcal{O}(\|\varepsilon\|^2) \qquad\qquad \mathbf{v}_i(\varepsilon) = \mathbf{e}_i + \varepsilon \Big[\frac{\theta_j^2 a_{ji}}{\xi_j^2 - \xi_i^2} \mathbf{1}_{\{j \neq i\}}\Big] + \mathcal{O}(\|\varepsilon\|^2) \tag{31}$$

Lemma 1 is useful because it links the mathematical properties of $\sqrt{\mathbf{\Gamma}}$ to its economic properties. It shows that up to first-order in ε, the eigenvalues of $\sqrt{\mathbf{\Gamma}}$ are the same as the diagonal economy; i.e. $\frac{\partial}{\partial \varepsilon} d_i(\varepsilon)|_{\varepsilon=0} = 0$. Importantly, note that in theory, this perturbation does not have to be accurate for $\varepsilon = 1$. But as we plot in Figure E.1 in the Appendix, it is a *remarkably accurate approximation* for the eigenvalues of the measured $\sqrt{\mathbf{\Gamma}}$ for the U.S. economy.

3.3.1. Aggregate and Sectoral Effects of Monetary Shocks. We now discuss how monetary shocks propagate in our approximate economy. We first present the results for sectoral inflation and then aggregate these responses to obtain the effects on GDP and CPI inflation.

Proposition 5 (Sectoral Inflation Responses)**.** Suppose $\{\xi_i \equiv \theta_i \sqrt{1 - a_{ii}}\}_{i \in [n]}$ are distinct. The

[17] This is a fairly weak assumption because ξ_i's are almost surely distinct if the distributions of $\mathbf{\Theta}$ and $\mathbf{A}$ in the data are drawn from distributions with densities with respect to the Lebesgue measure. In other words, the event that two sectors have the same adjusted frequencies in the data has zero probability.

impulse response of inflation in sector $i \in [n]$ to a monetary shock is:

$$\tfrac{\partial}{\partial \delta_m}\pi_{i,t} = \underbrace{e^{-\xi_i t}}_{\text{diagonal baseline}} \underbrace{+\,\varepsilon \sum_{j \neq i} \frac{\xi_i a_{ij}}{1-a_{ii}} \times \frac{\xi_i}{\xi_i+\xi_j} \times \frac{\xi_j e^{-\xi_j t} - \xi_i e^{-\xi_i t}}{\xi_i - \xi_j}}_{\text{first order effect of the network}} + \mathcal{O}(\|\varepsilon\|^2) \tag{32}$$

Equation (32) shows that introducing production linkages creates spillover effects on the inflation of sector i through all of its suppliers, captured by the term labeled the "first order effect of the network." Since i's suppliers have sticky prices, increasing production linkages (higher ϵ) leads to an initial dampening of the inflation response in sector i to a monetary shock. However, since money is neutral in the long-run, this dampened response has to be compensated for in terms of inflation in the long run, which implies that inflation in sector i is more persistent with higher ϵ. The following corollary shows how these sectoral effects translate into the response of aggregate inflation to monetary shocks.

Proposition 6 (Impact and Asymptotic Inflation Response)**.** Input-output linkages dampen CPI inflation response to a monetary shock on impact but amplify its persistence.

$$\underbrace{\tfrac{\partial}{\partial \varepsilon}\big[\tfrac{\partial}{\partial \delta_m}\pi_0\big]\big|_{\varepsilon=0}}_{\partial \text{impact response}/\partial\varepsilon} = -\sum_{i=1}^{n} \beta_i \sum_{j\neq i} \frac{\xi_i a_{ij}}{1-a_{ii}} \times \frac{\xi_i}{\xi_i+\xi_j} < 0 \tag{33}$$

$$\iota \equiv \arg\min_i\{\xi_i\} \Rightarrow \quad \underbrace{\tfrac{\partial}{\partial \varepsilon}\big[\tfrac{\partial}{\partial \delta_m}\pi_t|_{t\to\infty}\big]\big|_{\varepsilon=0}}_{\partial \text{asymptotic response}/\partial\varepsilon} \sim \sum_{j\neq\iota} \beta_j \Big(\frac{\xi_j^2 a_{j\iota}}{1-a_{jj}} + \frac{\xi_\iota^2 a_{\iota j}}{1-a_\iota}\Big) \frac{\xi_\iota e^{-\xi_\iota t}}{|\xi_i^2 - \xi_\iota^2|} > 0 \tag{34}$$

Finally, we can show that this increase in the persistence of inflationary responses corresponds closely with an increase in monetary non-neutrality, as shown in the next proposition.

Proposition 7 (Monetary Non-Neutrality)**.** Input-output linkages amplify monetary non-neutrality measured by the CIR of GDP to a monetary shock.

$$\tfrac{\partial}{\partial \varepsilon}\mathrm{CIR}_{\tilde{y},\delta_m}\big|_{\varepsilon=0} = \sum_{i=1}^{n}\sum_{j\neq i} \beta_j \times \frac{a_{ji}}{1-a_{jj}} \times \xi_i^{-1} \times \frac{\xi_i^{-1}}{\xi_i^{-1}+\xi_j^{-1}} > 0 \tag{35}$$

3.3.2. Aggregate Effects of Sectoral Shocks. We now characterize the pass-through of sectoral inflation to aggregate CPI inflation. The experiment is to consider a negative sectoral TFP shock to sector i that raises the inflation rate in that sector by 1 percent on impact. Our goal is to characterize how much aggregate CPI inflation rises in response to this sectoral shock, and how this pass-through is affected by the network. The following proposition presents this pass-through for impact response of inflation. The full expression for the dynamic response of inflation is also attainable but more complicated. It is included in the proof of the proposition.

Proposition 8 (Pass-through of Sectoral to Aggregate Inflation)**.** Input-output linkages amplify the pass-through of sectoral inflation rates to aggregate CPI inflation. Formally,

$$\frac{\partial \pi_0}{\partial \pi_{i,0}}\Big|_{\delta_z^i} = \underbrace{\beta_i}_{\text{direct pass-through}} \underbrace{+\varepsilon \sum_{j\neq i} \overbrace{a_{ji}}^{①} \times \overbrace{\frac{\beta_j}{1-a_{jj}}}^{②} \times \overbrace{\frac{\phi_i^{-1}}{\xi_j^{-1}+\phi_i^{-1}}}^{③} \times \overbrace{\frac{\xi_i^{-1}}{\xi_i^{-1}+\xi_j^{-1}}}^{④}}_{\text{first-order indirect pass-through via network}} + \underbrace{\mathcal{O}(\|\varepsilon\|^2)}_{\text{higher-order effects}} \tag{36}$$

Equation (36) relates the pass-through of sectoral inflation rate in sector i to aggregate inflation *conditional* on a negative TFP shock to sector i. The first term on the right-hand side is the direct pass-through of sectoral inflation to aggregate inflation: a one percent inflation in sector i directly feeds to inflation proportional to the expenditure share of the sector, denoted by β_i. The second term, which itself consists of four components, labeled by ① – ④, captures the first-order *indirect* pass-through of sectoral inflation to aggregate inflation through the network.

The indirect effect can be understood as follows: an inflationary shock in sector i, up to first-order, propagates through its buyers. Thus, we need to sum over all the other sectors that purchase from i. When considering a buyer $j \neq i$, the impact of i's inflationary shock on the economy through j is proportional to j's expenditure share on i, ①, and j's own Domar weight in the baseline economy, ②. These two components jointly determine the potency of i's shock on j and resemble what is known from static models. The next two terms, however, capture dynamic considerations. The term labeled ③ accounts for the fact that if the duration of the shock to i, ϕ_i^{-1}, is small compared to the adjusted duration of price spells in the downstream sector j, ξ_j^{-1}, then the shock's pass-through via j is weakened. This occurs because stickier downstream sectors, measured by their adjusted duration ξ_j^{-1}, are less responsive to a transient shock because they anticipate it will dissipate relatively faster than prices in their sector will adjust. The term under ④ captures a similar effect, but relative to the adjusted duration of price spells in the upstream sector i itself. When the adjusted duration of price spells in the upstream sector i is relatively small compared to that of the downstream sector j, then firms in j are not very responsive to the price changes of supplier i since they anticipate those prices will readjust faster than their own prices.

3.4. Measurement and Quantitative Implications

We now measure our sufficient statistics for the U.S. and study the dynamic responses of inflation and GDP under these statistics.

3.4.1. Sufficient Statistics Construction From Data. Proposition 2 shows that the sufficient statistics for inflation and GDP dynamics are the PRDL matrix, $\sqrt{\mathbf{\Gamma}}$, and the consumption expenditure shares vector, $\boldsymbol{\beta}$. We use the make and use input-output (IO) tables from 2012, made available by the BEA, to construct the input-output matrix $\mathbf{A}$; the consumption expenditure share vector $\boldsymbol{\beta}$; and the sectoral labor shares vector $\boldsymbol{\alpha}$. We construct them at the detailed-level disaggregation, excluding the government sectors, which leads to 393 sectors. Figure E.2 presents the matrix $\mathbf{A}$ we construct from the data, in a heat-map version. Moreover, we construct the diagonal matrix $\boldsymbol{\Theta}^2$, whose diagonal elements are the squared frequency of price adjustment in each sector, using data on 341 sectors from Pasten, Schoenle, and Weber (2020).

3.4.2. Dynamic Aggregate Responses to a Monetary Policy Shock. Panel A of Figure 2 shows impulse responses of aggregate inflation and GDP to an expansionary monetary policy shock in our calibrated economy. The size of this shock is noramlized so that inflation responds by 1 percent on impact, after which it slowly goes back to its steady state level at zero. The persistence of this convergence is governed by our measured $\sqrt{\boldsymbol{\Gamma}}$, with a half-life of around 6 months. Moreover, the shock has substantial real effects. GDP rises by around 10 percent on impact and decays slowly back to zero. The cumulated response of GDP is about 131 percent.

To illustrate the roles of various model ingredients that lead to such substantial real effects, we consider several counterfactual experiments. In these counterfactuals, we keep the initial impact on inflation the same at 1 percent.[18] In Panel B of Figure 2, we compare our calibrated economy to a counterfactual horizontal economy, which does not feature any input-output linkages and where labor is the only input in production. The cumulated impulse response of GDP is 4.1 times larger in our baseline economy. Strategic complementarity in price setting that arises through input-output linkages, as we pointed out while discussing the analytical results in the previous Section, is the driving force for this result. This in turn leads to a more persistent inflation response, which amplifies GDP response both on impact and over time.

In addition to input-output linkages, another source that amplifies the real effects of monetary policy is heterogenous price stickiness across sectors, as we pointed out and formalized in the previous Section. To investigate the role of this channel, in Panel C of Figure 2, we compare our calibrated baseline economy to a counterfactual economy that has homogenous price stickiness across sectors. We calibrate the frequency of price changes in this economy to be the same as the weighted average of the frequency of price changes across sectors in our baseline economy. This economy still features input-output linkages, and through that, strategic complementarity in

[18]The monetary policy shock size is therefore different across the baseline and the counterfactual cases. Recall that the cumulated impulse response of aggregate inflation corresponds to the monetary policy shock size in our model. Keeping the initial impact on aggregate inflation the same across various model specifications brings out the crucial role played by the persistence of inflation.

price setting. The cumulated impulse response of GDP is 2.4 times larger in our baseline economy, which shows that heterogeneity in price stickiness across sectors does play a quantitatively important role in magnifying monetary non-neutrality. The quantitative importance of this channel, however, is not as high as that of input-output linkages.

Finally, shutting down both channels, in Panel D of Figure 2, we compare our calibrated baseline economy to a counterfactual horizontal economy that also has homogenous price stickiness across sectors and find that the cumulated impulse response of GDP is 6.9 times larger in our baseline economy.[19] This total effect is approximately equal to the sum of the two separate counterfactual effects we showed above.

3.4.3. Heterogeneous Sectoral Inflation Responses to a Monetary Policy Shock. Underlying the aggregate inflation response to the monetary policy shock we discussed above is a distribution of sectoral inflation responses. In Figure 3, we show impulse responses of some selected sectors' inflation to an expansionary monetary policy shock. Sectoral inflation responses differ significantly both in terms of the impact response and the persistence and moreover, sectors where inflation responds by a larger amount initially have more short-lived responses. In particular, Figure 3 shows that sectoral inflation in the Oil and Gas Extraction industry is high in the initial periods but dissipates fast, while sectoral inflation in the Semiconductor Manufacturing Machinery industry responds by a small amount initially but is persistently positive over time. For completeness, Table E.1 provides a ranking of the top twenty sectors by their initial sectoral inflation response while Table E.2 provides a ranking of the top twenty sectors by the half-life of their sectoral inflation response.

For interpretation, we turn to the approximation in Section 3.3, where we showed that inflation in sectors with more flexible prices and less input-output linkages respond more strongly initially. Specifically, Equation (32) showed that the relevant statistic for impact sectoral inflation response (evaluated at $t = 0$) is $\xi_i - \varepsilon \sum_{j \neq i} \frac{\xi_i a_{ij}}{1-a_{ii}} \frac{\xi_i}{\xi_i + \xi_j}$. Panel A of Figure 4 shows the correlation between the actual ranks of sectors and the ranks predicted from this statistic. As is clear, the approximated statistic accounts extremely well for the exact numerical results. Moreover, as mentioned above, sectors where inflation responds more initially tend to have short-lived responses. Panel B of Figure 4 shows the correlation between actual ranks of sectors

[19]Note that even in this textbook type multisector New Keynesian model, inflation effects are persistent because our modeling of monetary policy preserves an endogenous state variable in the model. This is a standard approach in the literature on sufficient statistics of monetary policy shocks, but is a different approach than assuming a Taylor rule where the interest rate feedback coefficient is on inflation. We show results from this case later.

given by half-life of sectoral inflation response and the ranks predicted from this statistic for impact response. As is clear, the correlation is strongly negative.

3.4.4. Sectoral Origins of Aggregate Inflation and GDP Dynamics. Motivated by differential sectoral inflation dynamics, supply chain issues and commodity price increases, and persistent aggregate inflation in the U.S. recently, we now study aggregate implications of sectoral shocks. Specifically, we compute sectoral shocks that lead to a 1 percent increase in sectoral inflation and then study the pass-through of such sectoral inflation increases on aggregate inflation.[20] The average duration of the sectoral shocks is 6 months.[21]

We first identify sectors that lead to a high on-impact response of aggregate inflation in Table 1. We provide a ranking of the top twenty sectors by their initial effect on aggregate inflation, where we remove the effect coming from the size of the sector. This metric, therefore, provides an evaluation of the spillover of sectoral inflation to aggregate inflation due to input-output linkages for in the absence of such linkages, this pass-through metric would be zero for all sectors.[22] As one example, the Oil and Gas Extraction industry ranks very high in Table 1. As we showed analytically using an approximation in Section 3.3, sectors that serve as input to other sectors and have more sticky prices cause greater spillover to aggregate inflation. Specifically, in Equation (36) we showed that the relevant statistic for this impact pass-through on aggregate inflation is $\sum_{j\neq i}\beta_j \frac{a_{ji}}{1-a_{jj}} \frac{\phi_i^{-1}}{\phi_i^{-1}+\xi_j^{-1}} \frac{\xi_i^{-1}}{\xi_i^{-1}+\xi_j^{-1}}$. Panel A of Figure 5 shows the correlation between the actual ranks of sectors and the ranks predicted from this statistic. As is clear, the approximated statistic accounts well for the exact numerical results, thereby providing an economic interpretation to the rankings.

We next identify sectors that lead to persistent aggregate inflation dynamics when sectoral inflation increases by 1 percent. Table 2 provides a ranking of the top twenty sectors by the half-life of the aggregate inflation response. One clear pattern emerges: Sectors with more sticky prices lead to persistent aggregate inflation dynamics when sectoral shocks cause a rise in sectoral inflation. Semiconductor Manufacturing Machinery industry is one sector that ranks high in Table 2. Identifying which sectors are the main sources of persistent aggregate inflation dynamics is critical because those persistent effects translate to larger aggregate GDP gap effects. To make this clear, in Panel B of Figure 5, we show that the cumulated impulse response of

[20]We interpret these sectoral shocks as negative supply shocks.

[21]Note that while the average duration of the sectoral shock is the same across all sectors, the size of the sectoral shock is different as we calibrate the size such that sectoral inflation increases by 1 percent across all sectors.

[22]We are thus capturing what are sometimes called second-round effects of sectoral inflation increases.

aggregate GDP gap is very tightly correlated with the half-life of aggregate inflation.[23] This implies that it is precisely the shocks to sectors that are the sources of persistent aggregate inflation dynamics that will have a bigger impact on the real macroeconomy.

3.4.5. A Spectral Analysis of Aggregate Inflation Persistence. So far, we have highlighted the critical role played by the persistence of aggregate inflation in driving macroeconomic dynamics. In particular, for the monetary shock, we showed in Section 3.4.2 that model features which increase the persistence of aggregate inflation leads to higher monetary non-neutrality. We now investigate further the origins of aggregate inflation persistence by identifying which sectors play a key role in propagating monetary policy shock in the longer run. In terms of long-run dynamics, given our analytical solution, the smallest eigenvalues of $\mathbf{\Gamma} \equiv \mathbf{\Theta}^2(\mathbf{I}-\mathbf{A})$ play the dominant role. Of course, eigenvalues as such do not have an economic meaning and cannot be assigned to particular sectors. In the diagonal economy considered in Section 3.3 however, eigenvalues are given by $\theta_i\sqrt{1-a_{ii}}$. So, in Table 3, we sort the eigenvalues and present those of $\mathbf{\Gamma} \equiv \mathbf{\Theta}^2(\mathbf{I}-\mathbf{A})$ together with $\theta_i\sqrt{1-a_{ii}}$ for several industries. As is clear, the eigenvalues are extremely close across these two cases, thus helping us provide economic meaning in terms of sectors that lead to the smallest eigenvalues. Figure E.1 shows that this extremely close association holds across the full range of eigenvalues and sectors.

To show the aggregate implications of these sectors with the lowest eigenvalues, we do a counterfactual exercise by dropping the three sectors with the smallest eigenvalues and recomputing the impulse responses of inflation and GDP.[24] Just dropping these three sectors leads to a noticeable change in the cumulative IRF of GDP, with the cumulative IRF of real GDP in calibrated economy higher by around 16 percent.[25] These results show that a few sectors play a very influential role in driving monetary non-neutrality in the economy as they determine the persistence of aggregate inflation. To show this clearly, in Figure E.3 we plot the impulse responses of inflation and GDP to a monetary shock for both our calibrated and counterfactual economies. They depict that over the longer horizon, inflation response is lower in the counterfactual economy and this difference in dynamics gets reflected in a lower response

[23] We compute the ratio of the cumulated impulse respone of GDP to the cumulated impulse response of GDP under flexible prices for a unit sectoral shock.

[24] In this exercise, we recompute the counterfactual input-output matrix by moving the share of these dropped sectors (as inputs) to the labor share. Moreover, these sectors correspond closely to sectors that have the highest half-life of sectoral inflation to a monetary shock.

[25] Two of these sectors have a zero sectoral share in aggregate real GDP while the third one has an extremely small sectoral share in aggregate GDP of 0.0015 percent. As such, in a horizontal economy, dropping them would not have affected the response of aggregate GDP at all.

of real GDP throughout.

4 Propagation with Endogenous Monetary Policy Responses

So far, we have focused on the economy's responses to monetary policy and sectoral TFP/wedge shocks separately. In this section, we investigate the response of inflation and GDP to sectoral shocks, when monetary policy endogenously responds to these shocks; specifically, when monetary policy aims to stabilize CPI inflation. We begin by outlining the aggregate Phillips curve and identify novel forces resulting from multi-sector production linkages. Additionally, we discuss how monetary policy responses to sectoral shocks can nontrivially impact their transmission.

4.1. Phillips Curves: Revisited

In Proposition 1, we derived sectoral Phillips curves in terms of inflation and nominal price gaps and discussed how this representation delivers analytical results for general paths of money, productivities, and wedges. To study endogenous monetary policy responses, however, it is useful for us to relate our results to more conventional representations of Phillips curves in New Keynesian (NK) economies, which involve output gaps, combined with real wage gaps in sticky-price and sticky-wage models (e.g., Woodford, 2003a; Galí, 2008), or relative price gaps in multi-sector economies (e.g., Aoki, 2001; Benigno, 2004).

To this end, consider the sectoral Phillips curves in Proposition 1 and define the relative sectoral prices, $\mathbf{q}_t \equiv \mathbf{p}_t - p_t\mathbf{1}$, as the vector of sectoral prices relative to the CPI price index in log form. Similarly, let $\mathbf{q}_t^f \equiv \mathbf{p}_t^f - p_t^f\mathbf{1}$ denote the same object in the flexible price economy, and $\tilde{y}_t \equiv y_t - y_t^f$ denote the GDP gap. Then, we can re-write Equation (19) as:

$$\dot{\boldsymbol{\pi}}_t = \rho\boldsymbol{\pi}_t + \boldsymbol{\Gamma}(\mathbf{q}_t - \mathbf{q}_t^f) - \boldsymbol{\Gamma}\mathbf{1}\tilde{y}_t \tag{37}$$

which shows that the nominal price gaps can be decomposed into *relative* price gaps and a term that involves the aggregate GDP gap.[26] Thus, in a network economy, relative price distortions affect inflation dynamics independent of the GDP gap. For instance, even if monetary policy fully stabilized the GDP gap ($\tilde{y}_t = 0$), inflation rates across sectors would still move until relative prices are at their flexible levels. More importantly, $\boldsymbol{\Gamma} = \boldsymbol{\Theta}(\rho\mathbf{I} + \boldsymbol{\Theta})(\mathbf{I} - \mathbf{A})$ remains the sufficient statistic that governs the effect of these relative price distortions on inflation dynamics.

[26]This equation is perhaps more familiar in its discrete-time form. To see this, note that $\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\pi}_t = \lim_{\mathrm{d}t\to 0}(\boldsymbol{\pi}_{t+\mathrm{d}t} - \boldsymbol{\pi}_t)/\mathrm{d}t$. For small $\mathrm{d}t$ one can substitute this to get the familiar discrete time version under perfect foresight:

$$\boldsymbol{\pi}_t = (1 - \rho\mathrm{d}t)\boldsymbol{\pi}_{t+\mathrm{d}t} - \boldsymbol{\Gamma}(\mathbf{q}_t - \mathbf{q}_t^f)\mathrm{d}t + \boldsymbol{\Gamma}\mathbf{1}\tilde{y}_t\mathrm{d}t \tag{38}$$

Therefore, we obtain the equivalence result that any two economies with the same $\mathbf{\Gamma}$ matrix deliver the same inflation dynamics, whether it reflects heterogeneous price stickiness as in Aoki (2001); Benigno (2004) or production linkages across sectors. This result also reflects the new forces that are introduced through networks. Multisector economies without production linkages restrict $\mathbf{\Gamma}$ to be diagonal, implicitly putting the most sticky sectors as the drivers of inflation persistence. With production linkages, $\mathbf{\Gamma}$ can be non-diagonal, implicitly capturing the indirect effects of price stickiness across sectors on inflation persistence through production linkages.

Moreover, the independent role of relative price distortions in these Phillips curves undermines the conventional wisdom on the sufficiency of the slope of the aggregate Phillips curve in determining inflation dynamics. To elaborate on this point, let us consider the aggregate Phillips curve for the CPI inflation, which can be derived by multiplying this with $\boldsymbol{\beta}^\intercal$ from the left:

$$\dot{\pi}_t = \rho\pi_t + \boldsymbol{\beta}^\intercal\mathbf{\Gamma}(\mathbf{q}_t - \mathbf{q}_t^f) - \boldsymbol{\beta}^\intercal\mathbf{\Gamma}\mathbf{1}\tilde{y}_t \tag{39}$$

The coefficient on the GDP gap in Equation (39), $\boldsymbol{\beta}^\intercal\mathbf{\Gamma}\mathbf{1}$, is usually referred to as the slope of the Phillips curve and it holds significance in the literature for the following two reasons.

First, in one-sector economies, the term involving relative price gaps is zero.[27] So the slope $\boldsymbol{\beta}^\intercal\mathbf{\Gamma}\mathbf{1} > 0$ uniquely captures all inflationary forces, determining the degree to which changes in GDP gap translate into changes in inflation. With multiple sectors and production linkages, however, this slope is insufficient for characterizing inflation dynamics and can even be misleading if the role of relative prices is not taken into account, as we discuss further below.

The second reason is its empirical interpretation. Conditional on the term involving relative price gaps being zero—e.g., the one-sector model—the slope of the Phillips curve is the *elasticity* of inflation to *demand* shocks. Note that the Phillips curve is an endogenous relationship and does not imply a causal relationship on its own. However, once one writes the Phillips curve so that the only term on the right-hand side is the GDP gap (*if* such a representation is possible), then by estimating its slope, one can identify the inflationary effects of demand shocks.

Nonetheless, such a representation does not necessarily exist in multisector economies. For instance, Hazell, Herreno, Nakamura, and Steinsson (2022) demonstrate this in a two-sector model with GHH preferences, which eliminates the term involving relative price gaps. As they observe, without such assumptions, multi-sector economies do not necessarily admit aggregate Phillips curves with only inflation and output gap terms. We can also observe this in

[27] To see this, note that in a one-sector economy the relative price $\mathbf{q}_t = p_t - p_t = 0$ because the CPI is equal to the final price of the economy's single sector. Similarly, $\mathbf{q}_t^f = 0$. Thus, for a one-sector economy $\dot{\pi}_t = -\boldsymbol{\beta}^\intercal\mathbf{\Gamma}\mathbf{1}\tilde{y}_t$.

Equation (39): since $\mathbf{\Gamma}$ is invertible under the assumptions of the model, the only model that admits a representation of the aggregate Phillips curve with only inflation and output gap is one where the expenditure share $\boldsymbol{\beta}$ is a left eigenvector of the $\mathbf{\Gamma}$ matrix, which is a very strict restriction on price stickiness and production linkages across sectors. Two obvious examples of such economies are the one-sector economy and multi-sector economies with no production linkages ($\mathbf{A} = 0$) and homogenous price stickiness ($\mathbf{\Theta} = \theta\mathbf{I}$).

More generally, Rubbo (2023) addresses this issue from an alternative perspective and shows that while the aggregate Phillips curve in multi-sector economies with production networks involves terms other than the GDP gap, there always exists a composite price index whose corresponding Phillips curve only includes inflation in that price index and the GDP gap. Rubbo (2023) refers to this price as the "divine coincidence index" because a policy that stabilizes this index in an economy with only TFP shocks also automatically stabilizes GDP gap.[28]

But what can the slope of the aggregate Phillips curve for CPI inflation, $\boldsymbol{\beta}^\intercal\mathbf{\Gamma}\mathbf{1}$, tell us about inflation and GDP dynamics in economies with production networks? More specifically, can it still predict the inflationary pressures of demand shocks, at least qualitatively? The answer is no. To illustrate this, we construct a counterexample where the slope is not only insufficient in predicting the magnitudes of these dynamics but is also misleading in predicting the direction of the effects of monetary shocks (the demand shock in our model) on inflation and GDP.

To this end, consider a counterexample with the following two economies: (1) A *horizontal economy* where price change frequencies across $n \geq 1$ sectors are heterogeneous, with no input-output linkages ($\mathbf{A} = \mathbf{0}$). It follows that $\mathbf{\Gamma}_1 = \mathrm{diag}(\theta_1^2, \ldots, \theta_n^2)$, where θ_i is the frequency of price changes in sector i, with at least two sectors having distinct frequencies when $n > 1$. (2) A *homogenuous economy*, also with no input-output linkages, where price change frequencies are homogeneous across sectors given by the expenditure-weighted average of frequencies in the horizontal economy, i.e., $\bar{\theta} = \sum_i \beta_i\theta_i$. We then obtain the following result.

Proposition 9. Consider the horizontal and homogeneous economies described above. Then, (1) When $n = 1$, in both economies, monetary non-neutrality is higher when the aggregate Phillips curve is flatter. (2) When $n > 1$, with at least two sectors having distinct frequencies of

[28] Rubbo (2023) proves this result generally for all production network economies. Special cases of this result in two sector economies with heterogeneous price stickiness as well as models with sticky prices and sticky wages are also discussed in (Woodford, 2003b, page 442) and (Galí, 2008, Equation (33) and the discussion on page 137), respectively. To see Rubbo (2023)'s point in our framework, define the divine coincidence price index as $p_t^{DC} \equiv \boldsymbol{\beta}^\intercal\mathbf{\Gamma}^{-1}\mathbf{p}_t/(\boldsymbol{\beta}^\intercal\mathbf{\Gamma}^{-1}\mathbf{1})$. Using Equation (37), inflation in this price index evolves according to

$$\dot{\pi}_t^{DC} = \rho\pi_t^{DC} - \tfrac{1}{\boldsymbol{\beta}^\intercal\mathbf{\Gamma}^{-1}\mathbf{1}}\tilde{y}_t, \qquad \boldsymbol{\beta}^\intercal\mathbf{\Gamma}^{-1}\mathbf{1} = \textstyle\sum_{i\in[n]}\lambda_i\theta_i^{-2} \tag{40}$$

price changes, the horizontal economy experiences strictly higher monetary non-neutrality *even though* that it has a strictly steeper aggregate Phillips curve than the homogeneous economy.

4.2. Quantitative Results

We now present quantitative results that are counterparts to our theoretical discussions above. We compute the slope of the aggregate Phillips curve in various economies and also show how different monetary policy responses can alter the transmission of sectoral shocks.

In Section 3.4.2 we presented the extent of monetary non-neutrality in our baseline economy and in various counterfactual economies. We next assess whether the slopes of the aggregate Phillips curves, $\boldsymbol{\beta}^{\intercal}\boldsymbol{\Gamma}\mathbf{1}$, as given in Equation (39), align with the effects of the monetary policy shock in these economies. The slopes of the aggregate Phillips curve are as follows: In the calibrated economy, it is 0.0187; in the horizontal economy, it is 0.1135; in the homogeneous price stickiness economy, it is 0.0190; and in the horizontal and homogeneous price stickiness economy, it is 0.0526. As is clear, the slopes of the aggregate Phillips curves do not serve as sufficient statistics for ranking of monetary non-neutrality. For instance, the slope is steeper in the horizontal economy compared to the economy that is both horizontal and has homogeneous price stickiness across sectors. As we showed in Section 3.4.2 however, monetary non-neutrality is much lower in the economy that is both horizontal and has homogeneous price stickiness across sectors. This failure of $\boldsymbol{\beta}^{\intercal}\boldsymbol{\Gamma}\mathbf{1}$ to predict the GDP effects of monetary policy shocks is due to the effects of relative price gaps that are present in the aggregate Phillips curves in our model, as given in Equation (39), and which we discussed in detail above in Section 4. Our quantitative results here are thus consistent with the counterexample we presented analytically in Proposition 9.

We next show how different monetary policy responses can alter the tranmission of sectoral shocks. In Figure 6, we plot for two sectors the impulses responses for sectoral shocks under our baseline monetary policy and under a strict aggregate inflation targeting poliy.[29] The sectoral shocks are calibrated to lead to a 1 percent increase in sectoral inflation under the baseline monetary policy specification. As we emphasized in Section 3.4.4, sectoral inflation in the Oil and Gas Extraction industry passess through substantially on imapct to aggregate inflation, but the effects are very transient. The key result we want to highlight is that if monetary policy responds by stabilizing aggregate inflation when a shock to the Oil and Gas Extraction industry increases sectoral inflation, it creates a large negative GDP gap. In fact, this

[29] Note that as our baseline monetary policy controls nominal GDP, with our preferences, the policy can be thought of equivalently as one that leads to a constant nominal interest rate in equilibrium.

policy is so contractionary that it leads the GDP in the economy to fall below the GDP under flexible prices.[30] In contrast, stabilizing aggregate inflation when a shock to the Semiconductor Manufacturing Machinery industry increases sectoral inflation is not similarly contractionary in terms of aggregate GDP. The reason for this sharp differences across sectors is that the Oil and Gas Extraction industry is a relatively flexible price sector, even in input-output adjusted price duration terms, and as such, rises in sectoral inflation in that sector do not cause large dispersion in relative prices if policy does not respond to the sectoral shock. When monetary policy does respond by stabilizing aggregate inflation however, it creates large relative price gaps that lead to a negative aggregate GDP gap. For completeness, Figure E.4 shows results from a counterfactual economy that has homogeneous frequency of price adjustment across sectors and where it is clear that responding to inflation originating in the Oil and Gas Extraction industry does not lead to negative GDP gap effects.

5 Extensions

We now present several extensions of our theoretical and quantitative results.

5.1. General Labor Supply Elasticity

So far, we used preferences that imply an infinite Frisch elasticity of labor supply. Our solution techniques, analytical results, and quantitative insights do not, however, depend on this simplification. In Appendix A.10, we present the details and present here the counterpart of Proposition 1 with $\rho = 0$:

$$\dot{\boldsymbol{\pi}}_t = \boldsymbol{\Gamma}(\mathbf{I} + \psi \mathbf{1}\boldsymbol{\beta}^\intercal)(\mathbf{p}_t - \mathbf{p}_t^f), \qquad \mathbf{p}_t^f \equiv m_t \mathbf{1} - \boldsymbol{\Psi} \boldsymbol{z}_t + (\boldsymbol{\Psi} - \frac{\psi}{1+\psi}\mathbf{1}\boldsymbol{\lambda}^\intercal)\boldsymbol{\omega}_t \tag{41}$$

where ψ is the inverse Frisch elasticity of labor supply. We can then extend Propositions 3 and 4 to this case by replacing $\boldsymbol{\Gamma}$ with $\boldsymbol{\Gamma}_\psi \equiv \boldsymbol{\Gamma}(\mathbf{I} + \psi \mathbf{1}\boldsymbol{\beta}^\intercal)$ and adjusting for $\mathbf{p}_t^f$ as above. In particular, the impulse responses for monetary and sectoral productivity shocks only change through $\boldsymbol{\Gamma}_\psi$. The impulse responses for sectoral wedge shocks, however, also need to be adjusted through $\mathbf{p}_t^f$.

In Figure E.5 we show impulse responses of aggregate inflation and GDP to an expansionary monetary policy shock when the Frisch elasticity is calibrated at 2. For comparison, we also present the results from our baseline calibration. As is clear, since a finite Frisch elasticity introduces aggregate strategic substitutability, it reduces the persistence of inflation and thereby, the extent of monetary non-neutrality. More importantly, this calibration does not alter our

[30]Generally, for TFP shocks, sticky prices lead to a GDP response that is lower than the GDP response under flexible prices.

quantitative results on the various forces that drive monetary non-neutrality, as shown in Figure E.6 - Figure E.8 . Finally, Figure E.9 shows that the distribution of sectoral inflation response after an aggregate monetary policy shock depicts the same patterns as in Section 3.4.2.

5.2. Taylor Rule as Monetary Policy Rule

So far, we used a monetary policy rule as determining a path of nominal GDP which kept the analysis similar to the theoretical literature on monetary non-neutrality and highlighted the role of endogenous persistence in the model. We now model monetary policy as following a rule in which the nominal interest rate responds to aggregate inflation. Our model derivations generalize to using such a Taylor rule and the details are in Appendix A.11. We need to impose boundary conditions that ensure that inflation and relative sectoral prices are stationary and for solving the resulting set of equilibrium system of equations, we use a Schur decomposition.

Here we discuss some key aspects of the model equilibrium. First, the counterpart of Proposition 1 with $\rho = 0$ and a Taylor rule with a monetary shock v_t, $i_t = \phi_\pi \boldsymbol{\beta}^\intercal \boldsymbol{\pi}_t + v_t$, is:

$$\ddot{\boldsymbol{\pi}}_t = \boldsymbol{\Gamma}(\mathbf{I} - \phi_\pi \mathbf{1}\boldsymbol{\beta}^\intercal)(\boldsymbol{\pi}_t - \boldsymbol{\pi}_t^f), \qquad \boldsymbol{\pi}_t^f \equiv (\mathbf{I} - \phi_\pi \mathbf{1}\boldsymbol{\beta}^\intercal)^{-1}(\mathbf{1}v_t - \boldsymbol{\Psi}(\dot{\boldsymbol{z}}_t - \dot{\boldsymbol{\omega}}_t)) \tag{42}$$

Here, $\boldsymbol{\pi}_t^f$ is the sectoral inflation rate that would have prevailed in a flexible price economy with the same Taylor rule and is exogenous to the system of differential equations. We can see that this equation differs from our Proposition 1 in two aspects. First, it is now a second-order differential equation in $\boldsymbol{\pi}_t$ rather than in prices. This is because, with an inflation-targeting Taylor rule, the economy is no longer price stationary, similat to the case in one-sector New Keynesian models. Second, we see that the dynamics of the second-order differential equations are still governed by $\boldsymbol{\Gamma}$, but now, it is adjusted for the endogenous response of monetary policy through the Taylor rule: $\boldsymbol{\Gamma}_{\phi,\pi} \equiv \boldsymbol{\Gamma}(\mathbf{I} - \phi_\pi \mathbf{1}\boldsymbol{\beta}^\intercal)$.

A Taylor rule in terms of inflation makes sticky price models forward-looking and thus the source of persistence is exogenous.[31] In our baseline calibration, fixing the Taylor rule coefficient at the standard value of $\phi_\pi = 1.5$, we introduce persistent shocks to the Taylor rule. We then calibrate the size and persistence of the shocks to generate a response of aggregate inflation that matches as closely as possible the aggregate inflation response in our nominal GDP rule economy of Section 3.4.2.[32] Figure E.10 shows the impulse responses of aggregate inflation and GDP to an expansionary monetary policy shock. The monetary non-neutrality, by design, is essentially the same as in Section 3.4.2.

[31] In the standard three equation sticky price model with a Taylor rule, the economy is fully forward-looking.
[32] We match exactly the initial response and the half-life of aggregate inflation in these two economies.

Given this baseline calibrated Taylor rule economy, we investigate the various forces that drive monetary non-neutrality using counterfactual exercises, which are presented in Figure E.11 - Figure E.13.[33] Overall, these results are consistent with our main conclusion that both production networks and heterogenous price stickiness play a quantitatively important role in amplifying monetary non-neutrality. We note that the precise extent of amplification coming from them jointly, compared to the horizontal economy with homogenous price stickiness across sectors, is a bit smaller than in Section 3.4.2. The reason that in this economy, persistent dynamics in inflation come about through persistence in the monetary policy shock itself, which increases the monetary non-neutrality even in the basic multi-sector economy.[34] Additionally, Figure E.14 shows that the distribution of sectoral inflation response after an aggregate monetary policy shock depicts the same patterns as in Section 3.4.2.

6 Conclusion

We provide sufficient statistics for inflation and GDP *dynamics* in multisector dynamic New Keynesian economies with input-output linkages. We show that the sufficient statistic for these dynamic responses is the *principal square root* of the Leontief matrix appropriately adjusted for the sectoral frequencies of price adjustments.

We construct this sufficient statistic using data from input-output tables and frequencies of price adjustments across sectors in the U.S. In quantitative experiments on this calibrated economy, we find a significant role for production networks in the propagation of aggregate monetary and sectoral TFP shocks. First, monetary shocks lead to effects on GDP that are four times as large, relative to a baseline multisector economy with a horizontal production network. Second, sectoral shocks that increase sectoral inflation can lead to substantial effects on aggregate inflation through spillovers that come about through production networks.

In future work, we plan to extend our framework and analysis in several directions. For instance, it will be interesting to study welfare and optimal policy implications in our model. A model with state-dependent pricing, due to fixed costs of changing nominal prices, is likely to lead to new insights. We also plan to extend the model to capture another important source

[33]In these counterfactual exercises, we keep the monetary policy shock and persistence the same as the baseline calibration. The reason is that with the Taylor rule as a monetary policy rule, inflation becomes forward-looking in the model and as such, differences in model features show up as affecting the level response of inflation, and not the persistence. We thus will not fix the impact response of inflation across various counterfactual exercises. For intuition, in the one-sector model with the Taylor rule, the slope of the Phillips curve that incorporates strategic complementarity only shows up as affecting the impact response of inflation.

[34]In addition, compared to the results in Section 3.4.2, production networks and heterogenous price stickiness play a similar role quantitatively.

of dynamics, through endogenous capital accumulation, to further develop the framework for business cycle analysis.

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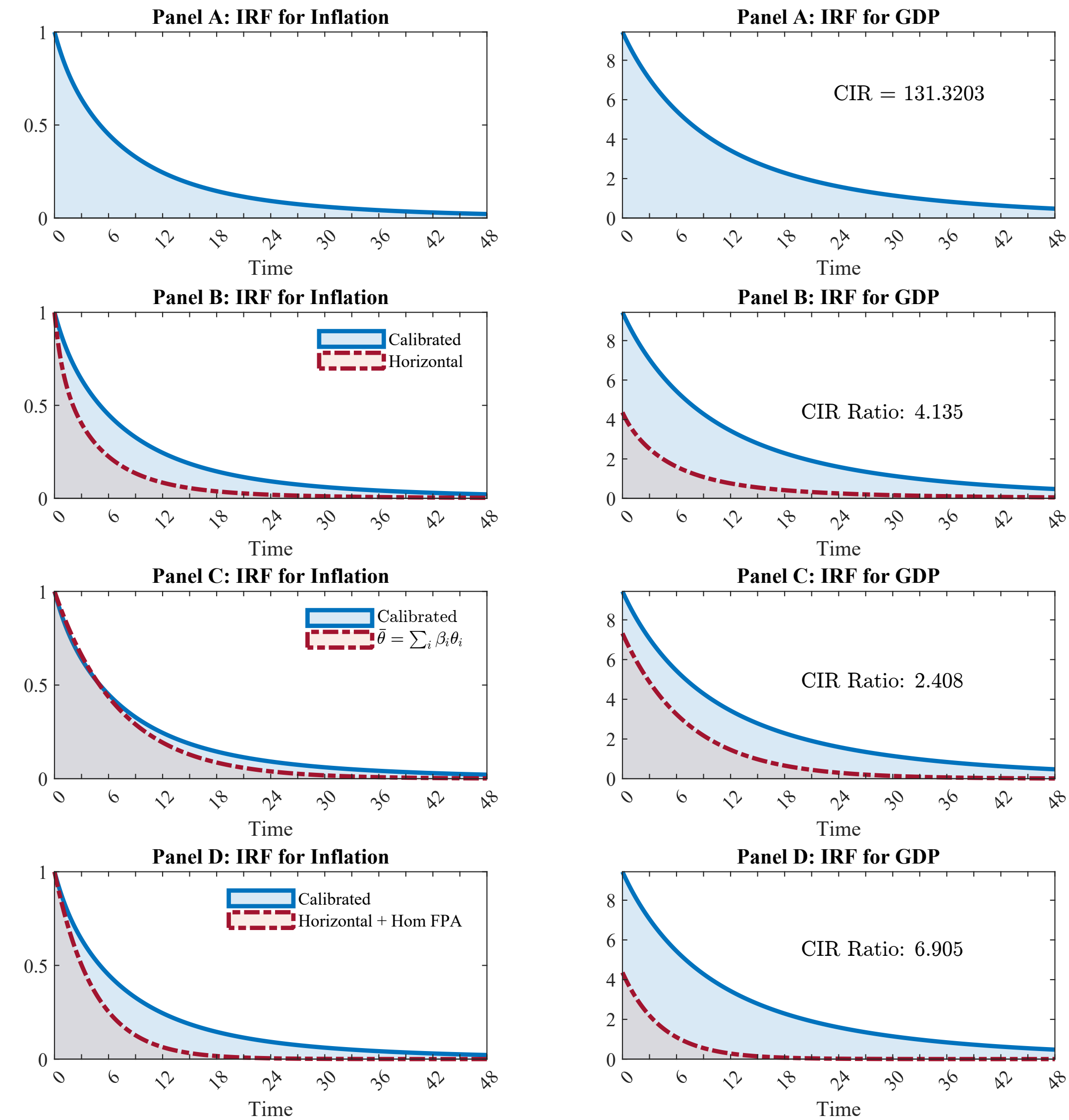
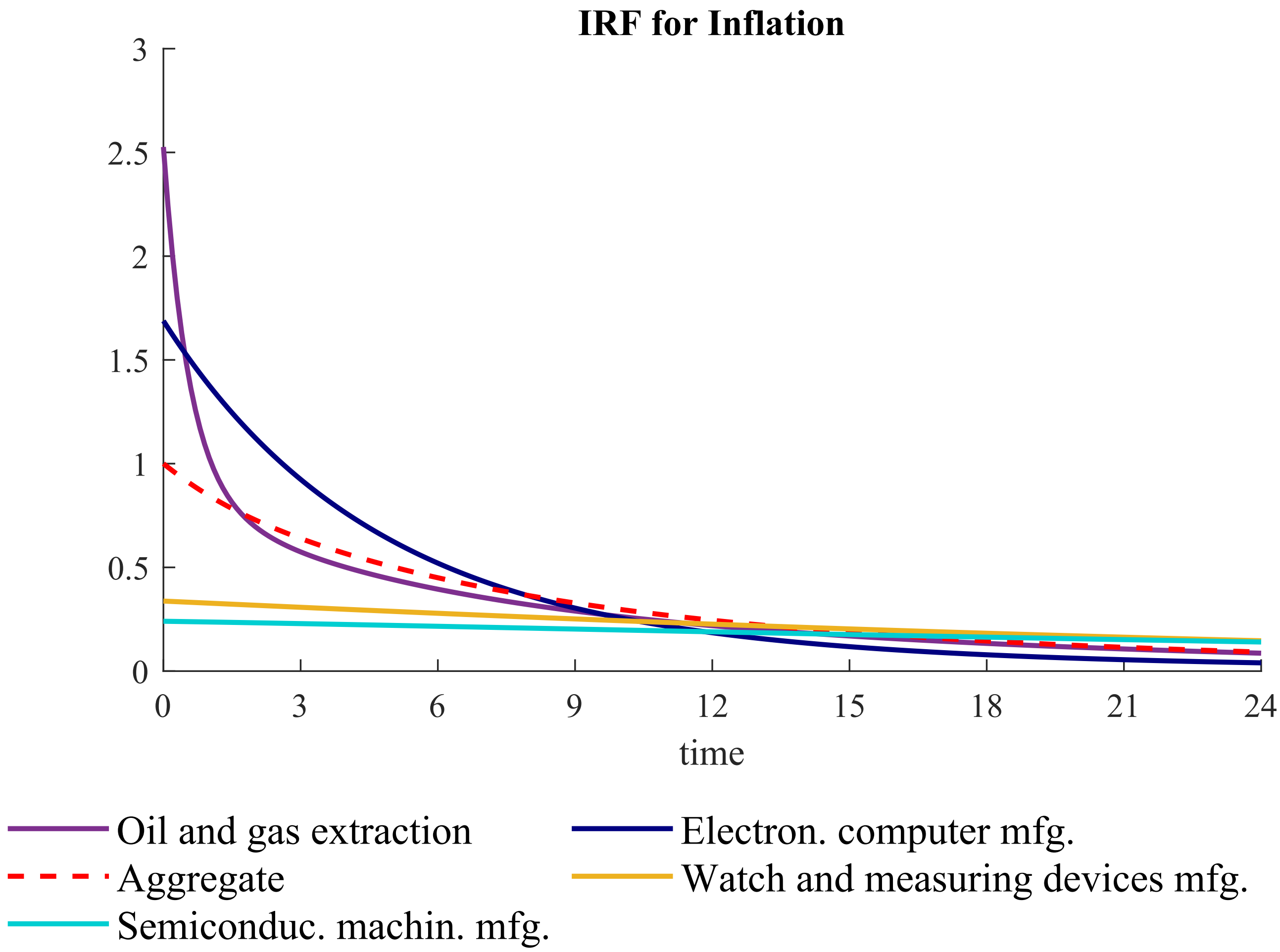
7 Figures

Figure 2: IRFs to a monetary policy shock



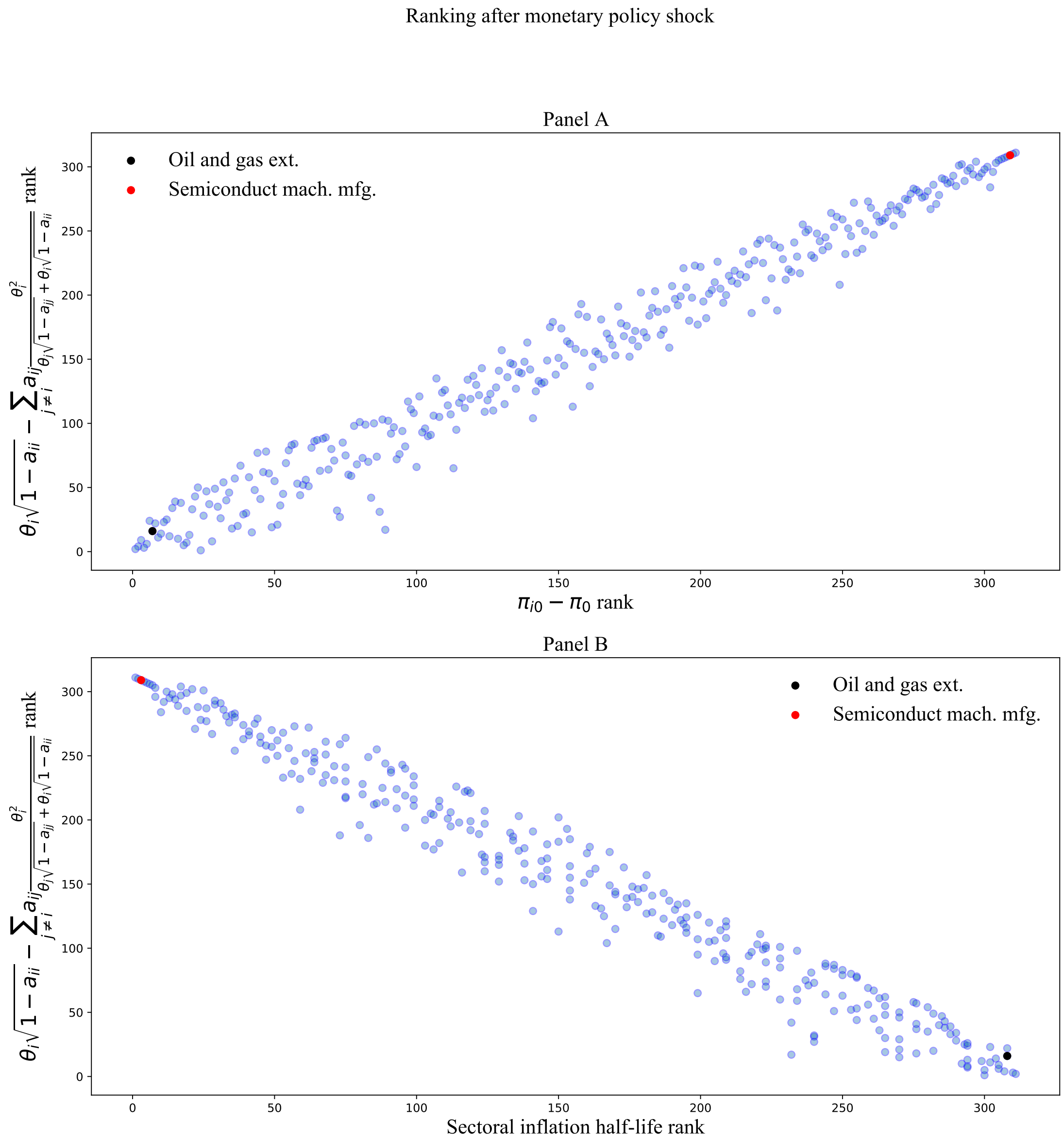
Notes: This figure plots the impulse response functions for inflation and GDP to a monetary shock that generates a one percentage increase in inflation on impact. The calibration of the model is at a monthly frequency. The different panels show the results from the baseline calibrated economy (Panel A) as well as various counterfactual economies (Panels B, C, and D). CIR denotes the cumulative impulse response. CIR Ratio denotes the ratio of CIR of the baseline economy to the counterfactual economy.

Figure 3: Sectoral inflation response to a monetary policy shock



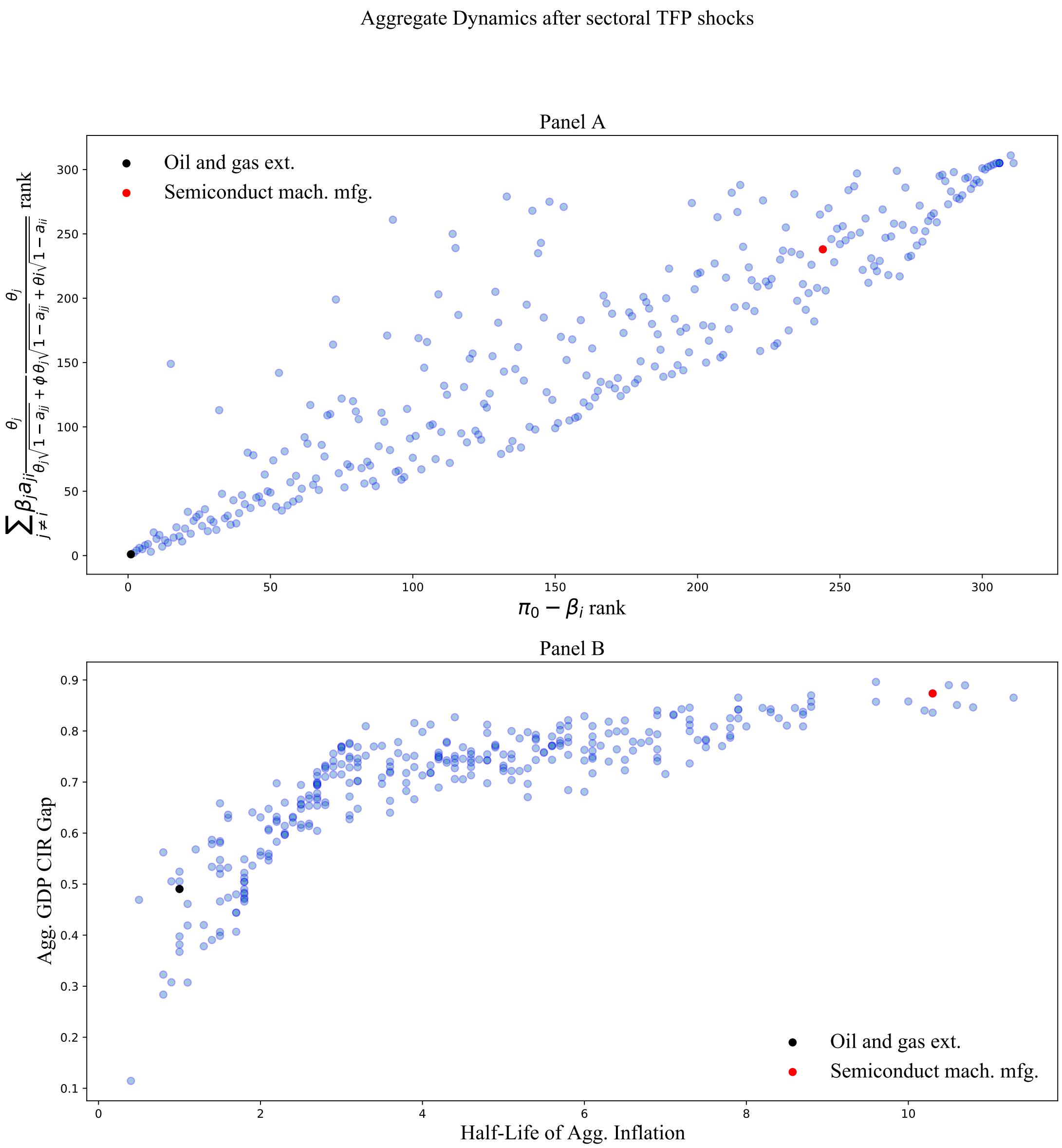
Notes: This figure plots the impulse response functions for aggregate inflation and sectoral inflation to a monetary shock that generates a one percentage increase in aggregate inflation on impact. The calibration of the model is at a monthly frequency. The aggregate inflation response is shown in dashed lines.

Figure 4: Correlation of actual ranks of sectors and ranks using an approximated statistic for sectoral inflation response to a monetary policy shock



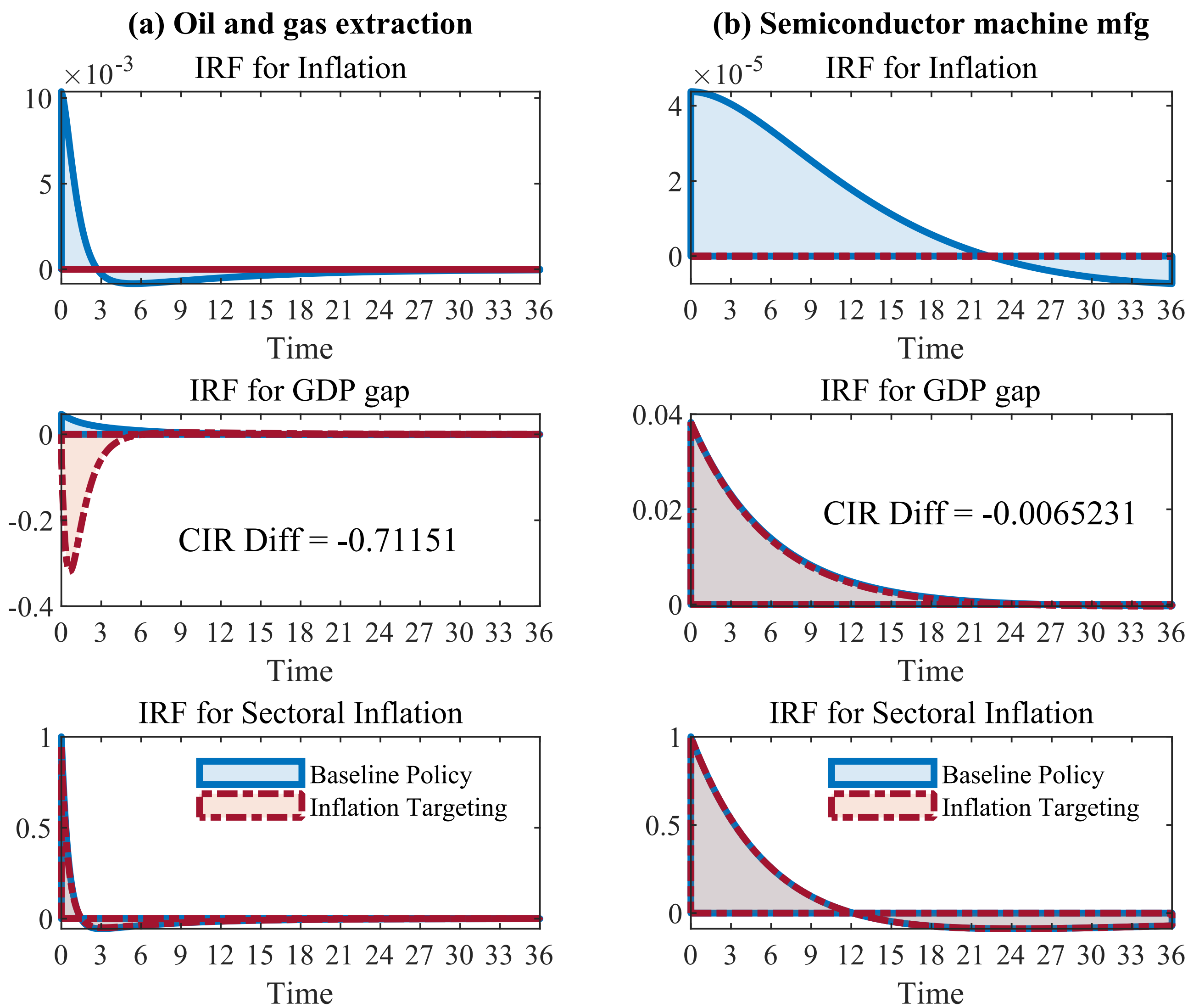
Notes: This figure plots the actual ranks and ranks using an approximated statistic for sectoral inflation response to a monetary policy shock. Panel A plots sectoral inflation impact response while Panel B plots the sectoral inflation half-life. Each dot in the figure represents a sector.

Figure 5: Aggregate inflation and GDP dynamics following sectoral shocks



Notes: Panel A of the figure plots actual ranks of sectors and ranks using an approximated statistic for aggregate inflation impact response after a sectoral shock increases sectoral inflation by one percentage on impact. Panel B of the figure plots how aggregate GDP gap and half-life of aggregate inflation are correlated when a sectoral shock increases sectoral inflation by one percentage on impact. Each dot in the figure represents a sector.

Figure 6: Dynamics following sectoral shocks under different policies



Notes: This figure plots the impulse response functions for inflation, gdp gap, and sectoral inflation to a sectoral shock that increases sectoral inflation by one percent on impact in the baseline policy economy. It compares the baseline policy economy with an economy where monetary policy stabilizes aggregate inflation. Panel A: Oil and gas extraction. Panel B: Semiconductor machine manufacturing.

8 Tables

Table 1: Ranking of industries by pass-through to aggregate inflation after a sectoral shock

Industry	Agg. Inflation Impact Resp.
Oil and gas extraction	0.009543
Insurance agencies, brokerages, and related act...	0.008415
Employment services	0.006016
Legal services	0.005696
Management consulting services	0.005642
Advertising, public relations, and related serv...	0.005026
Accounting, tax preparation, bookkeeping, and p...	0.004993
Warehousing and storage	0.004981
Architectural, engineering, and related services	0.004981
Electric power generation, transmission, and di...	0.003828
Services to buildings and dwellings	0.003702
Monetary authorities and depository credit inte...	0.003621
Scenic and sightseeing transportation and suppo...	0.003418
Securities and commodity contracts intermediati...	0.003354
Other support activities for mining	0.003241
Truck transportation	0.003186
Commercial and industrial machinery and equipme...	0.003146
Wired telecommunications carriers	0.003121
Other financial investment activities	0.003025
Other nondurable goods merchant wholesalers	0.002608

Notes: Ranking of industries by aggregate inflation impact response when a sectoral shock leads to an increase in 1% in the shocked sector's inflation on impact. Average duration of the sectoral shock is 6 months.

Table 2: Ranking of industries by half-life of aggregate inflation repsonse after a sectoral shock

Industry	Half Life Agg. Inflation
Packaging machinery manufacturing	11.3
Miscellaneous nonmetallic mineral products	10.8
Coating, engraving, heat treating and allied ac...	10.7
All other forging, stamping, and sintering	10.6
Industrial process furnace and oven manufacturing	10.5
Semiconductor machinery manufacturing	10.3
Printing ink manufacturing	10.3
Speed changer, industrial high-speed drive, and...	10.2
Machine shops	10.0
Insurance agencies, brokerages, and related act...	9.6
Turned product and screw, nut, and bolt manufac...	9.6
Electricity and signal testing instruments manu...	8.8
Other communications equipment manufacturing	8.8
Fluid power process machinery	8.8
Support activities for printing	8.7
Relay and industrial control manufacturing	8.7
Industrial and commercial fan and blower and ai...	8.7
Optical instrument and lens manufacturing	8.6
In-vitro diagnostic substance manufacturing	8.5
Other electronic component manufacturing	8.4

Notes: Ranking of industries by half-life of aggregate inflation response when a sectoral shock that leads to an increase in 1% in the shocked sector's inflation on impact. Average duration of the sectoral shock is 6 months.

Table 3: Comparison of eigenvalues of the calibrated economy with eigenvalues of the diagonal economy associated with specific industries

Industry	θ_i	$\theta_i\sqrt{1-a_{ii}}$	Eigenvalue $\sqrt{\Gamma}$
Insurance agencies, brokerages, and related act...	0.035586	0.022688	0.022439
Coating, engraving, heat treating and allied ac...	0.027804	0.02744	0.027327
Warehousing and storage	0.032407	0.030659	0.030562
Semiconductor machinery manufacturing	0.034003	0.032861	0.032858
Flavoring syrup and concentrate manufacturing	0.038897	0.038458	0.038413
Showcase, partition, shelving, and locker manuf...	0.039775	0.039335	0.039325
Packaging machinery manufacturing	0.040667	0.039349	0.039346
Machine shops	0.044323	0.043501	0.042797
Watch, clock, and other measuring and controlli...	0.043928	0.043682	0.043607
Other communications equipment manufacturing	0.044149	0.043945	0.043919
Turned product and screw, nut, and bolt manufac...	0.044987	0.044227	0.044319
Electricity and signal testing instruments manu...	0.048076	0.044627	0.044622
Broadcast and wireless communications equipment	0.053673	0.045249	0.045218
Fluid power process machinery	0.047158	0.045863	0.045821
Optical instrument and lens manufacturing	0.048201	0.04615	0.046098
All other miscellaneous manufacturing	0.047515	0.046339	0.046138
Miscellaneous nonmetallic mineral products	0.049119	0.046373	0.04629
Other aircraft parts and auxiliary equipment ma...	0.051709	0.046385	0.046363
Cutlery and handtool manufacturing	0.047783	0.047746	0.047703
Analytical laboratory instrument manufacturing	0.04835	0.048093	0.048118
Other industrial machinery manufacturing	0.049155	0.048275	0.048118
Breakfast cereal manufacturing	0.048738	0.048585	0.048335
Cut stone and stone product manufacturing	0.063157	0.048644	0.048573
Advertising, public relations, and related serv...	0.049135	0.048695	0.048643
Metal crown, closure, and other metal stamping ...	0.048895	0.048722	0.048708
Toilet preparation manufacturing	0.050453	0.050085	0.05007
Doll, toy, and game manufacturing	0.050442	0.050401	0.050399
Offices of physicians	0.050503	0.050503	0.050503
Waste management and remediation services	0.054119	0.050815	0.050563
Motorcycle, bicycle, and parts manufacturing	0.057306	0.050978	0.050979

Notes: The actual eigenvalues of the calibrated economy are compared with eigenvalues of the counterfactual diagonal economy. In the diagonal economy, the eigenvalues are associated with specific industries, which are given in the first column.

APPENDIX
(FOR ONLINE PUBLICATION)

A Proofs

A.1. Proof of Proposition 1

Differentiating Equation (18) with respect to time and substituting Equation (17) we arrive at

$$\begin{aligned}\dot{\boldsymbol{\pi}}_t = \ddot{\mathbf{p}}_t = \boldsymbol{\Theta}(\boldsymbol{\pi}_t^{\#} - \boldsymbol{\pi}_t) &= \boldsymbol{\Theta}(\rho\mathbf{I} + \boldsymbol{\Theta})(\mathbf{p}_t^{\#} - \mathbf{p}_t^*) - \boldsymbol{\Theta}\boldsymbol{\pi}_t \\ &= \boldsymbol{\Theta}(\rho\mathbf{I} + \boldsymbol{\Theta})(\mathbf{p}_t - \mathbf{p}_t^*) + \underbrace{\boldsymbol{\Theta}(\rho\mathbf{I} + \boldsymbol{\Theta})(\mathbf{p}_t^{\#} - \mathbf{p}_t) - \boldsymbol{\Theta}\boldsymbol{\pi}_t}_{=\rho\boldsymbol{\pi}_t \text{ by Equation (18)}} \end{aligned} \tag{A.1}$$

Now using the definition of $\mathbf{p}_t^*$ from Equation (9) observe that:

$$\mathbf{p}_t - \mathbf{p}_t^* = \mathbf{p}_t - \boldsymbol{\omega}_t + \boldsymbol{z}_t - m_t^s\boldsymbol{\alpha} + \mathbf{A}\mathbf{p}_t = -(\mathbf{I} - \mathbf{A})(\underbrace{m_t^s\mathbf{1} + \boldsymbol{\Psi}(\boldsymbol{\omega}_t - \boldsymbol{z}_t)}_{=\mathbf{p}_t^f \text{ by Equation (14)}} - \mathbf{p}_t) \tag{A.2}$$

Combining Equations (A.1) and (A.2) gives us the desired result. ■

A.2. Proof of Proposition 2

For $\rho = 0$, the differential equation in Equation (19) is

$$\dot{\boldsymbol{\pi}}_t = \ddot{\mathbf{p}}_t = \boldsymbol{\Gamma}(\mathbf{p}_t - \mathbf{p}_t^f) \tag{A.3}$$

Since $\mathbf{p}_t^f$ is piece-wise continuous and bounded, it has a Laplace transform for any $s \geq 0$. Let $\mathbf{P}^f(s) = \mathcal{L}_s(\mathbf{p}_t^f) \equiv \int_0^\infty e^{-st}\mathbf{p}_t^f \mathrm{d}t$ denote the Laplace transform of $\mathbf{p}_t^f$. Similarly, let $\mathbf{P}(s) = \mathcal{L}_s(\mathbf{p}_t)$ denote the Laplace transform of $\mathbf{p}_t$. Then, applying the Laplace transform to the differential equation above, we have:

$$\mathbf{P}(s) = (s^2\mathbf{I} - \boldsymbol{\Gamma})^{-1}(s\mathbf{p}_{0+} + \boldsymbol{\pi}_{0+}) - (s^2\mathbf{I} - \boldsymbol{\Gamma})^{-1}\boldsymbol{\Gamma}\mathbf{P}^f(s) \tag{A.4}$$

Now, let $\sqrt{\boldsymbol{\Gamma}}$ denote the principal square root of $\boldsymbol{\Gamma}$; i.e., the square root of $\boldsymbol{\Gamma}$ all of whose eigenvalues have non-negative real parts. This matrix exists and is a non-singular M-matrix by Theorem 5 in Alefeld and Schneider (1982). Thus, we have:

$$\mathbf{p}_t = \sqrt{\boldsymbol{\Gamma}}^{-1}\sinh(\sqrt{\boldsymbol{\Gamma}}t)\boldsymbol{\pi}_{0+} + \cosh(\sqrt{\boldsymbol{\Gamma}}t)\mathbf{p}_{0+} - \mathcal{L}_t^{-1}\left[(s^2\mathbf{I} - \boldsymbol{\Gamma})^{-1}\boldsymbol{\Gamma}\mathbf{P}^f(s)\right] \tag{A.5}$$

where $\mathbf{c}_0$ and $\mathbf{c}_1$ are vectors in $\mathbf{R}^n$ and are appropriate linear transformations of $\mathbf{p}_{0+}$ and $\boldsymbol{\pi}_{0+}$. Moreover, the last terms is the inverse Laplace transform of the product of $(s^2\mathbf{I} - \boldsymbol{\Gamma})^{-1}\boldsymbol{\Gamma}$ and $\mathbf{P}^f(s)$. Since the inverse Laplace transform of a product is the convolution of inverse Laplace of individual functions, we have:

$$\begin{aligned}\mathcal{L}_t^{-1}\left[(s^2\mathbf{I} - \boldsymbol{\Gamma})^{-1}\boldsymbol{\Gamma}\mathbf{P}^f(s)\right] &= \int_0^t \mathcal{L}_{t-h}^{-1}\left[(s^2\mathbf{I} - \boldsymbol{\Gamma})^{-1}\boldsymbol{\Gamma}\right]\mathbf{p}_h^f \mathrm{d}h \\ &= \sqrt{\boldsymbol{\Gamma}}\int_0^t \sinh(\sqrt{\boldsymbol{\Gamma}}(t-h))\mathbf{p}_h^f \mathrm{d}h \end{aligned} \tag{A.6}$$

Combining Equations (A.5) and (A.6) and using the definitions of sinh(.) and cosh(.) we arrive at

$$\begin{aligned}\mathbf{p}_t = &\frac{1}{2}e^{\sqrt{\mathbf{\Gamma}}t}\left[\sqrt{\mathbf{\Gamma}}^{-1}\boldsymbol{\pi}_{0+} + \mathbf{p}_{0+} - \sqrt{\mathbf{\Gamma}}\int_0^t e^{-\sqrt{\mathbf{\Gamma}}h}\mathbf{p}_h^f \mathrm{d}h\right] \\ &- \frac{1}{2}e^{-\sqrt{\mathbf{\Gamma}}t}\left[\sqrt{\mathbf{\Gamma}}^{-1}\boldsymbol{\pi}_{0+} - \mathbf{p}_{0+} - \sqrt{\mathbf{\Gamma}}\int_0^t e^{\sqrt{\mathbf{\Gamma}}h}\mathbf{p}_h^f \mathrm{d}h\right] \end{aligned} \tag{A.7}$$

Now, in terms of boundary conditions $\mathbf{p}_t$ satisfies the following two: (1) it is continuous at $t = 0$, since the probability of price change opportunities arriving at a short interval around any point is arbitrarily small—i.e., $\mathbf{p}_{0+} = \mathbf{p}_{0-}$ because no firm changes their price exactly at $t = 0$ as it is a measure zero event, (2) we are looking for the solution in which prices are non-explosive; in fact bounded because $\mathbf{p}_t^f$ is bounded. So the term multiplying $e^{\sqrt{\mathbf{\Gamma}}t}$ has to be zero as $t \to \infty$ and we have:

$$\sqrt{\mathbf{\Gamma}}^{-1}\boldsymbol{\pi}_{0+} + \mathbf{p}_{0-} = \sqrt{\mathbf{\Gamma}}\int_0^\infty e^{-\sqrt{\mathbf{\Gamma}}h}\mathbf{p}_h^f \mathrm{d}h \tag{A.8}$$

Plugging these boundary conditions into the solution we have:

$$\begin{aligned}\mathbf{p}_t &= e^{-\sqrt{\mathbf{\Gamma}}t}\mathbf{p}_{0-} + \frac{\sqrt{\mathbf{\Gamma}}}{2}e^{\sqrt{\mathbf{\Gamma}}t}\int_t^\infty e^{-\sqrt{\mathbf{\Gamma}}h}\mathbf{p}_h^f \mathrm{d}h - \frac{\sqrt{\mathbf{\Gamma}}}{2}e^{-\sqrt{\mathbf{\Gamma}}t}\int_0^\infty e^{-\sqrt{\mathbf{\Gamma}}h}\mathbf{p}_h^f \mathrm{d}h + \frac{\sqrt{\mathbf{\Gamma}}}{2}e^{-\sqrt{\mathbf{\Gamma}}t}\int_0^t e^{\sqrt{\mathbf{\Gamma}}h}\mathbf{p}_h^f \mathrm{d}h \\ &= e^{-\sqrt{\mathbf{\Gamma}}t}\mathbf{p}_{0-} + \sqrt{\mathbf{\Gamma}}e^{-\sqrt{\mathbf{\Gamma}}t}\int_0^t \sinh(\sqrt{\mathbf{\Gamma}}h)\mathbf{p}_h^f \mathrm{d}h + \sqrt{\mathbf{\Gamma}}\sinh(\sqrt{\mathbf{\Gamma}}t)\int_t^\infty e^{-\sqrt{\mathbf{\Gamma}}h}\mathbf{p}_h^f \mathrm{d}h \end{aligned} \tag{A.9}$$

■

A.3. Proof of Propositions 3 and 4

First, note that we can combine all the shocks in both propositions into a single path for $\mathbf{p}_t^f$ as:

$$\mathbf{p}_t^f = \mathbf{p}_{0-} + \delta_m \mathbf{1} + \mathbf{\Psi}e^{-\mathbf{\Phi}t}\boldsymbol{\delta}_z, \qquad \mathbf{\Phi} \equiv \mathrm{diag}(\phi_1, \ldots, \phi_n), \qquad \boldsymbol{\delta}_z \equiv \sum_{i=1}^n \delta_z^i \mathbf{e}_i \tag{A.10}$$

where $\mathbf{p}_{0-}$ are the steady-state prices before shocks, δ_m is the monetary shock, and δ_z^i is the TFP/wedge shock to sector i. We can then plug this path into Proposition 2 to derive the response of the economy to all of these shocks jointly. Since we have log-linearized the model the response of the economy to this aggregated path is simply the sum of the impulse responses to individual shocks. So we can solve the model for the joint path in Equation (A.10) and then decompose it to individual IRFs.

While the joint response can be derived from Proposition 2 by solving explicitly for the integrals in Equation (20), it is more convenient to guess a particular solution for the differential equation in Equation (19) for the particular path of flexible prices specified in Equation (A.10): with $\rho = 0$, a path of non-explosive prices, $\mathbf{p}_t$, is uniquely characterized by

$$\dot{\boldsymbol{\pi}}_t = \ddot{\mathbf{p}}_t = \mathbf{\Gamma}(\mathbf{p}_t - \mathbf{p}_t^f) = \mathbf{\Gamma}(\mathbf{p}_t - \mathbf{p}_{0-} - \delta_m\mathbf{1} - \mathbf{\Psi}e^{-\mathbf{\Phi}t}\boldsymbol{\delta}_z) \qquad \text{with} \qquad \mathbf{p}_0 = \mathbf{p}_{0-}^f \tag{A.11}$$

Noting that this is a system of non-homogenous differential equations, the *general* solution to this system can be written as $\mathbf{p}_t = \mathbf{p}_t^p + \mathbf{p}_t^g$, where $\mathbf{p}_t^p$ is a particular solution to the non-homogenous system of differential equations above and $\mathbf{p}_t^g$ is the general solution to the homogenous system, $\ddot{\mathbf{p}}_t^g = \mathbf{\Gamma}\mathbf{p}_t^g$. To obtain the solution we start with the guess that a candidate for the particular solution is

$$\mathbf{p}_t^p = \mathbf{p}_{0^-}^f + \delta_m \mathbf{1} + \mathbf{X}e^{-\mathbf{\Phi}t}\boldsymbol{\delta}_z \tag{A.12}$$

for some $\mathbf{X} \in \mathbb{R}^{n\times n}$. Plugging this into Equation (A.11) we obtain $(\mathbf{\Gamma}\mathbf{X} - \mathbf{X}\mathbf{\Phi}^2 - \mathbf{\Gamma}\mathbf{\Psi})e^{-\mathbf{\Phi}t}\boldsymbol{\delta}_z = 0$. Since we want this equation to hold for any $t \geq 0$ and any $\boldsymbol{\delta}_z$, it follows that our guess is verified when $\mathbf{X}$ is the solution to the Sylvester equation

$$\mathbf{\Gamma}\mathbf{X} - \mathbf{X}\mathbf{\Phi}^2 = \mathbf{\Gamma}\mathbf{\Psi} \tag{A.13}$$

which is unique because we assumed that $\mathbf{\Gamma}$ and $\mathbf{\Phi}^2$ do not have any common eigenvalues (see, e.g., Horn and Johnson, 2012, Theorem 2.4.4.1).

As for the general solution, $\mathbf{p}_t^g$, one can solve this differential equation by the method of undetermined coefficients for second-order matrix differential equations (see Apostol, 1975). In particular, one can easily confirm that such a solution has the form:

$$\mathbf{p}_t^g = \sum_{k=0}^{\infty} \frac{\mathbf{\Gamma}^k t^{2k}}{(2k)!}\mathbf{c}_0 + \sum_{k=0}^{\infty} \frac{\mathbf{\Gamma}^k t^{2k+1}}{(2k+1)!}\mathbf{c}_1 \tag{A.14}$$

whose domain of convergence in t includes our time domain $[0,\infty)$ and $\mathbf{c}_0, \mathbf{c}_1$ are constant vectors in $\mathbb{R}^n$. Now, letting $\sqrt{\mathbf{\Gamma}}$ denote the principal square root of $\mathbf{\Gamma}$, which exists and is a non-singular M-matrix by Theorem 5 in Alefeld and Schneider (1982), we can write the equation above as

$$\mathbf{p}_t^g = \underbrace{\sum_{k=0}^{\infty} \frac{(\sqrt{\mathbf{\Gamma}}t)^k}{k!}}_{=e^{\sqrt{\mathbf{\Gamma}}t}}(\underbrace{\frac{\mathbf{c}_0 + \sqrt{\mathbf{\Gamma}}^{-1}\mathbf{c}_1}{2}}_{\equiv\tilde{\mathbf{c}}_0}) + \underbrace{\sum_{k=0}^{\infty} \frac{(-\sqrt{\mathbf{\Gamma}}t)^k}{k!}}_{=e^{-\sqrt{\mathbf{\Gamma}}t}}(\underbrace{\frac{\mathbf{c}_0 - \sqrt{\mathbf{\Gamma}}^{-1}\mathbf{c}_1}{2}}_{\equiv\tilde{\mathbf{c}}_1}) \tag{A.15}$$

Thus, the general solution to the non-homogenous system is given by

$$\mathbf{p}_t = \mathbf{p}_t^p + \mathbf{p}_t^g = \mathbf{p}_{0^-}^f + \delta_m\mathbf{1} + \mathbf{X}e^{-\mathbf{\Phi}t}\boldsymbol{\delta}_z + e^{\sqrt{\mathbf{\Gamma}}t}\tilde{\mathbf{c}}_0 + e^{-\sqrt{\mathbf{\Gamma}}t}\tilde{\mathbf{c}}_1 \tag{A.16}$$

Now, to determine the constant vectors $\tilde{\mathbf{c}}_0, \tilde{\mathbf{c}}_1$, we have the two sets of boundary conditions. (1) $\mathbf{p}_0 = \mathbf{p}_{0^-}^f$ (notice with positive and finite frequencies of price changes, no firm gets an opportunity to change their prices at instant zero so the left and right limits are the same). (2) With zero trend inflation (which is the assumption here), prices converge to a steady-state level as $t \to \infty$—i.e., the price function is non-explosive over time. The second set of boundary conditions immediately imply $\tilde{\mathbf{c}}_0 = 0$ because all of the eigenvalues of $\mathbf{\Gamma}$ have strictly positive real parts because it is an M-matrix. The first set of boundary conditions imply: $\tilde{\mathbf{c}}_1 = -\delta_m\mathbf{1} - \mathbf{X}\boldsymbol{\delta}_z$. Thus,

$$\mathbf{p}_t = \mathbf{p}_{0^-}^f + \delta_m(\mathbf{I} - e^{-\sqrt{\mathbf{\Gamma}}t})\mathbf{1} + \mathbf{X}e^{-\mathbf{\Phi}t}\boldsymbol{\delta}_z - e^{-\sqrt{\mathbf{\Gamma}}t}\mathbf{X}\boldsymbol{\delta}_z \tag{A.17}$$

Now to calculate the terms that involve $\mathbf{X}$ note that

$$\begin{aligned} \mathbf{X}e^{-\mathbf{\Phi}t}\boldsymbol{\delta}_z - e^{-\sqrt{\mathbf{\Gamma}}t}\mathbf{X}\boldsymbol{\delta}_z &= \sum_{i=1}^{n} \delta_z^i [\mathbf{X}e^{-\mathbf{\Phi}t} - e^{-\sqrt{\mathbf{\Gamma}}t}\mathbf{X}]\mathbf{e}_i \\ &= \sum_{i=1}^{n} \delta_z^i [e^{-\phi_i t}\mathbf{I} - e^{-\sqrt{\mathbf{\Gamma}}t}]\mathbf{X}\mathbf{e}_i \end{aligned} \tag{A.18}$$

where the second line follows from the fact that $e^{-\mathbf{\Phi}t}\mathbf{e}_i = e^{-\phi_i t}\mathbf{I}$. Now, we need to calculate $\mathbf{X}\mathbf{e}_i$. Using the Sylvester equation that characterizes $\mathbf{X}$ in Equation (A.18), we have:

$$\begin{aligned} \mathbf{\Gamma}\mathbf{X}\mathbf{e}_i - \mathbf{X}\mathbf{\Phi}^2\mathbf{e}_i = \mathbf{\Gamma}\mathbf{\Psi}\mathbf{e}_i &\Rightarrow (\mathbf{\Gamma} - \phi_i^2\mathbf{I})\mathbf{X}\mathbf{e}_i = \mathbf{\Gamma}\mathbf{\Psi} \\ &\Rightarrow \mathbf{X}\mathbf{e}_i = (\mathbf{I} - \phi_i^2\mathbf{\Gamma}^{-1})^{-1}\mathbf{\Psi} \end{aligned} \tag{A.19}$$

where the second line follows from the fact that both $\mathbf{\Gamma}$ and $\mathbf{\Gamma} - \phi_i^2\mathbf{I}$ are invertible.[35] Thus,

$$\begin{aligned} \mathbf{p}_t &= \mathbf{p}^f_{0^-} + \delta_m(\mathbf{I} - e^{-\sqrt{\mathbf{\Gamma}}t})\mathbf{1} + \mathbf{X}e^{-\mathbf{\Phi}t}\boldsymbol{\delta}_z - e^{-\sqrt{\mathbf{\Gamma}}t}\mathbf{X}\boldsymbol{\delta}_z \\ &= \mathbf{p}^f_{0^-} + \delta_m(\mathbf{I} - e^{-\sqrt{\mathbf{\Gamma}}t})\mathbf{1} + \sum_{i=1}^{n} \delta_z^i [e^{-\phi_i t}\mathbf{I} - e^{-\sqrt{\mathbf{\Gamma}}t}](\mathbf{I} - \phi_i^2\mathbf{\Gamma}^{-1})^{-1}\mathbf{\Psi}\mathbf{e}_i \end{aligned} \tag{A.20}$$

To get the IRFs in Proposition 3 note that

$$\frac{\mathrm{d}}{\mathrm{d}\delta_m}\mathbf{p}_t = (\mathbf{I} - e^{-\sqrt{\mathbf{\Gamma}}t})\mathbf{1} \tag{A.21}$$

$$\frac{\mathrm{d}}{\mathrm{d}\delta_m}\pi_t = \frac{\mathrm{d}}{\mathrm{d}\delta_m}\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\beta}^\intercal\mathbf{p}_t = \boldsymbol{\beta}^\intercal\sqrt{\mathbf{\Gamma}}e^{-\sqrt{\mathbf{\Gamma}}t}\mathbf{1} \tag{A.22}$$

$$\frac{\mathrm{d}}{\mathrm{d}\delta_m}y_t = \frac{\mathrm{d}}{\mathrm{d}\delta_m}(m_t - \boldsymbol{\beta}^\intercal\mathbf{p}_t) = \boldsymbol{\beta}^\intercal e^{-\sqrt{\mathbf{\Gamma}}t}\mathbf{1} \tag{A.23}$$

$$\frac{\mathrm{d}}{\mathrm{d}\delta_m}\tilde{y}_t = \frac{\mathrm{d}}{\mathrm{d}\delta_m}(y_t - y_t^f) = \frac{\mathrm{d}}{\mathrm{d}\delta_m}\boldsymbol{\beta}^\intercal(\mathbf{p}_t^f - \mathbf{p}_t) = \boldsymbol{\beta}^\intercal e^{-\sqrt{\mathbf{\Gamma}}t}\mathbf{1} \tag{A.24}$$

To get the IRFs in Proposition 4 note that

$$\frac{\mathrm{d}}{\mathrm{d}\delta_z^i}\mathbf{p}_t = (e^{-\phi_i t}\mathbf{I} - e^{-\sqrt{\mathbf{\Gamma}}t})(\mathbf{I} - \phi_i^2\mathbf{\Gamma}^{-1})^{-1}\mathbf{\Psi}\mathbf{e}_i \tag{A.25}$$

$$\frac{\mathrm{d}}{\mathrm{d}\delta_z^i}\pi_t = \frac{\mathrm{d}}{\mathrm{d}\delta_z^i}\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\beta}^\intercal\mathbf{p}_t = \boldsymbol{\beta}^\intercal(\sqrt{\mathbf{\Gamma}}e^{-\sqrt{\mathbf{\Gamma}}t} - \phi_i e^{-\phi_i t}\mathbf{I})(\mathbf{I} - \phi_i^2\mathbf{\Gamma}^{-1})^{-1}\mathbf{\Psi}\mathbf{e}_i \tag{A.26}$$

$$\frac{\mathrm{d}}{\mathrm{d}\delta_z^i}y_t = \frac{\mathrm{d}}{\mathrm{d}\delta_z^i}(m_t - \boldsymbol{\beta}^\intercal\mathbf{p}_t) = \boldsymbol{\beta}^\intercal(e^{-\sqrt{\mathbf{\Gamma}}t} - e^{-\phi_i t}\mathbf{I})(\mathbf{I} - \phi_i^2\mathbf{\Gamma}^{-1})^{-1}\mathbf{\Psi}\mathbf{e}_i \tag{A.27}$$

$$\frac{\mathrm{d}}{\mathrm{d}\delta_m}\tilde{y}_t = \frac{\mathrm{d}}{\mathrm{d}\delta_m}(y_t - y_t^f) = \boldsymbol{\beta}^\intercal(e^{-\sqrt{\mathbf{\Gamma}}t} - \phi_i^2 e^{-\phi_i t}\mathbf{\Gamma}^{-1})(\mathbf{I} - \phi_i^2\mathbf{\Gamma}^{-1})^{-1}\mathbf{\Psi}\mathbf{e}_i \tag{A.28}$$

■

A.4. Proof of Proposition 5

TBA.

A.5. Proof of Proposition 6

TBA.

[35] $\mathbf{\Gamma}$ is invertible because both $\mathbf{\Theta}$ and $\mathbf{I} - \mathbf{A}$ are invertible by assumptions $\theta_i > 0$ and $\alpha_i > 0$, $\forall i$. To see why $\mathbf{\Gamma} - \phi_i^2\mathbf{I}$ is invertible, take the Jordan form of $\mathbf{\Gamma}$ as $\mathbf{\Gamma} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$ and note that $\mathbf{\Gamma} - \phi_i^2\mathbf{I} = \mathbf{P}(\mathbf{D} - \phi_i^2\mathbf{I})\mp^{-1}$. Note that each block of $\mathbf{D} - \phi_i^2$ has non-zero diagonal because $\phi_i^2 \notin \operatorname{eig}(\mathbf{\Gamma})$ so $(\mathbf{D} - \phi_i^2\mathbf{I})$ is invertible and $(\mathbf{\Gamma} - \phi_i^2\mathbf{I})^{-1} = \mathbf{P}(\mathbf{D} - \phi_i^2\mathbf{I})^{-1}\mathbf{P}^{-1}$.

A.6. Proof of Proposition 7

TBA.

A.7. Proof of Proposition 8

TBA.

A.8. Proof of Proposition 9

Let $\mathbf{\Gamma}_1 = \mathrm{diag}(\theta_1^2, \ldots, \theta_n^2)$ and $\mathbf{\Gamma}_2 = (\sum_{i\in[n]} \beta_i\theta_i)^2\mathbf{I}$ denote the duration-adjusted Leontief matrices in the horizontal and homogeneous economies, respectively. It follows the PRDLs of these economies are given by $\sqrt{\mathbf{\Gamma}_1} = \mathrm{diag}(\theta_1, \ldots, \theta_2)$ and $\sqrt{\mathbf{\Gamma}_2} = (\sum_{i\in[n]} \beta_i\theta_i)\mathbf{I}$ (since PRDL is the square root that has eigenvalues with positive parts, for both $\mathbf{\Gamma}_1$ and $\mathbf{\Gamma}_2$ we pick the positive eigenvalues when constructing $\sqrt{\mathbf{\Gamma}_1}$ and $\sqrt{\mathbf{\Gamma}_2}$.)

Now, let us construct the slope of the aggregate Phillips curve in these two economies. Letting $\kappa_1 = \boldsymbol{\beta}^\intercal\mathbf{\Gamma}_1\mathbf{1}$ and $\kappa_2 = \boldsymbol{\beta}^\intercal\mathbf{\Gamma}_2\mathbf{1}$ denote the slope of the aggregate Phillips curve in the horizontal and homogeneous economies, respectively, we have

$$\kappa_1 = \sum_{i\in[n]} \beta_i\theta_i^2, \qquad \kappa_2 = \sum_{i\in[n]} (\beta_i\theta_i)^2 \tag{A.29}$$

indicating that the difference between these two slopes is the expenditure-weighted variance of price adjustment frequencies across sectors:

$$\kappa_1 - \kappa_2 = \sum_{i\in[n]} \beta_i(\theta_i - \sum_{i\in[n]} \beta_i\theta_i)^2 = \mathrm{var}_\beta(\theta_i) \geq 0 \tag{A.30}$$

Note that with $n = 1$, this variance is zero and $\kappa_1 = \kappa_2$. However, with $n \geq 2$, since we have assumed that frequencies are distinct in at least two sectors, $\kappa_1 > \kappa_2$ and the horizontal economy has strictly a steeper aggregate Phillips curve than the homogeneous economy.

Now, recall that the CIR of output (gap) to a monetary shock in a given economy is given by $\boldsymbol{\beta}^\intercal\sqrt{\mathbf{\Gamma}}^{-1}\mathbf{1}$. Letting CIR_1 and CIR_2 denote the CIRs of output (gap) to a monetary shock in the horizontal and homogeneous economies, respectively, we have

$$\mathrm{CIR}_1 = \sum_{i\in[n]} \frac{\beta_i}{\theta_i}, \qquad \mathrm{CIR}_2 = \frac{1}{\sum_{i\in[n]} \beta_i\theta_i} \tag{A.31}$$

Since the function $f(X) \equiv 1/X$ is a strictly convex function, we can apply Jensen's inequality to conclude that

$$\mathrm{CIR}_1 = \mathbb{E}_\beta[f(\theta_i)] \geq f(\mathbb{E}_\beta[\theta_i]) = \mathrm{CIR}_2 \tag{A.32}$$

where the inequality holds with equality if $n = 1$, and holds strictly with $n > 1$ as we have assumed that at least two sectors have distinct frequencies. Thus, we have the following implications for Parts 1 and 2:

Part 1. With $n = 1$, $\kappa_1 = \kappa_2 = \theta_1^2$ and $\text{CIR}_1 = \text{CIR}_2 = 1/\theta_1$. Thus, monetary non-neutrality in both economies is larger if θ_1 is smaller which is equivalent to $\kappa_1 = \kappa_2 = \theta_1^2$ being smaller (or, equivalently, the Phillips curve being flatter).

Part 2. With $n > 1$, we established above that $\kappa_1 > \kappa_2$ (i.e., the horizontal economy has a strictly steeper aggregate Phillips curve), but also $\text{CIR}_1 > \text{CIR}_2$ (i.e. the horizontal economy experiences strictly higher monetary non-neutrality). Thus, the horizontal economy has a steeper aggregate Phillips curve and higher monetary non-neutrality than the homogeneous economy.

A.9. Proof of Lemma 1

Consider the matrix

$$\mathbf{\Gamma}(\varepsilon) = \mathbf{\Gamma}_D + \varepsilon \mathbf{\Gamma}_R \tag{A.33}$$

as defined in the main text, where $\mathbf{\Gamma}_D = \mathbf{\Theta}^2(\mathbf{I} - \mathbf{A})$ is the diagonal and $\mathbf{\Gamma}_R = \mathbf{\Theta}^2(\mathbf{A}_D - \mathbf{A})$, with $\mathbf{A}_D \equiv \text{diag}(\mathbf{A})$ defined as the diagonal of the input-output matrix $\mathbf{A}$. Note that $\varepsilon = 1$ gives us the $\mathbf{\Gamma}$ for the original model and $\varepsilon = 0$ gives us the $\mathbf{\Gamma}_D$ of the diagonal model. Now, we want to perturb the eigendecomposition of the matrix $\sqrt{\mathbf{\Gamma}(\varepsilon)}$ up to first order in ε. To this end, let

$$\sqrt{\mathbf{\Gamma}(\varepsilon)} = \mathbf{P}(\varepsilon)\mathbf{D}(\varepsilon)\mathbf{P}(\varepsilon)^{-1} \tag{A.34}$$

denote the Jordan decomposition of the principal square root $\sqrt{\mathbf{\Gamma}(\varepsilon)}$. It then follows that

$$\mathbf{\Gamma}(\varepsilon) = \mathbf{P}(\varepsilon)\mathbf{D}(\varepsilon)^2\mathbf{P}(\varepsilon)^{-1} \tag{A.35}$$

Since $\mathbf{\Gamma}(0) = \mathbf{\Gamma}_D$ is already diagonal we have $\mathbf{D}(0) = \sqrt{\mathbf{\Gamma}_D}$ and $\mathbf{P}(0) = \mathbf{I}$. So letting $\mathbf{V}(\varepsilon) \equiv \mathbf{P}(\varepsilon) - \mathbf{P}(0) = \mathbf{P}(\varepsilon) - \mathbf{I}$, and $\mathbf{\Delta}(\varepsilon) \equiv \mathbf{D}(\varepsilon) - \mathbf{D}(0) = \mathbf{D}(\varepsilon) - \sqrt{\mathbf{\Gamma}_D}$, we have

$$\mathbf{\Gamma}(\varepsilon) = (\mathbf{I} + \mathbf{V}(\varepsilon))(\sqrt{\mathbf{\Gamma}_D} + \mathbf{\Delta}(\varepsilon))^2(\mathbf{I} + \mathbf{V}(\varepsilon))^{-1} \tag{A.36}$$

At this point we can apply Theorems 1 and 2 from Greenbaum, Li, and Overton (2020), which also shows differentiability of $\mathbf{V}(\varepsilon)$ and $\mathbf{\Delta}(\varepsilon)$ to obtain the results presented in the lemma.

For completeness, however, let us take differentiability as given and calculate these derivatives in our case. Assuming differentiability at $\varepsilon = 0$, we can now do a first order Taylor expansion of $\mathbf{V}(\varepsilon)$ and $\mathbf{\Delta}(\varepsilon)$ in a small neighborhood around $\varepsilon = 0$ to get

$$\mathbf{V}(\varepsilon) = \varepsilon \mathbf{V}'(0) + \mathcal{O}(\|\varepsilon\|^2) \tag{A.37}$$

$$\mathbf{\Delta}(\varepsilon) = \varepsilon \mathbf{\Delta}'(0) + \mathcal{O}(\|\varepsilon\|^2) \tag{A.38}$$

for some matrices $\mathbf{V}'(0)$ and $\mathbf{\Delta}'(0)$ that we still need to characterize. Note that since eigenvalues of $\mathbf{\Gamma}(0) = \mathbf{\Gamma}_D$ are distinct by assumption, for small enough ε, eigenvalues of $\mathbf{\Gamma}(\varepsilon)$ are also distinct by continuity, meaning that $\mathbf{\Gamma}(\varepsilon)$ is diagonalizable and $\Delta(\varepsilon) = \varepsilon \mathbf{\Delta}'(0)$ is diagonal in a neighborhood around $\varepsilon = 0$.

Now, to characterize $\mathbf{V}'(0)$ and $\mathbf{\Delta}'(0)$, we the above Taylor expansions into Equation (A.36) and use the identity that $(\mathbf{I}+\varepsilon\mathbf{V}'(0))^{-1}=\mathbf{I}-\varepsilon\mathbf{V}'(0)+\mathcal{O}(\|\varepsilon\|^2)$, we get

$$\mathbf{\Gamma}(\varepsilon)=\mathbf{\Gamma}_D+\varepsilon\mathbf{\Gamma}_R=\mathbf{\Gamma}_D+\varepsilon\mathbf{V}'(0)\mathbf{\Gamma}_D-\varepsilon\mathbf{\Gamma}_D\mathbf{V}'(0)+\varepsilon\sqrt{\mathbf{\Gamma}_D}\mathbf{\Delta}'(0)+\varepsilon\mathbf{\Delta}'(0)\sqrt{\mathbf{\Gamma}_D}+\mathcal{O}(\|\varepsilon\|^2) \quad (A.39)$$

Canceling out the $\mathbf{\Gamma}_D$ terms, dividing by ε and taking the limit as $\varepsilon\to 0$ we get

$$\mathbf{\Gamma}_R=\mathbf{V}'(0)\mathbf{\Gamma}_D-\mathbf{\Gamma}_D\mathbf{V}'(0)+\sqrt{\mathbf{\Gamma}_D}\mathbf{\Delta}'(0)+\mathbf{\Delta}'(0)\sqrt{\mathbf{\Gamma}_D} \quad (A.40)$$

We know the left hand side of this equation $\mathbf{\Gamma}_R=\mathbf{\Theta}^2(\mathbf{A}_D-\mathbf{A})$ as well as $\mathbf{\Gamma}_D=\mathbf{\Theta}^2(\mathbf{I}-\mathbf{A}_D)$ on the right hand side. Moreover, we know that $\mathbf{\Delta}'(0)$ is diagonal. Now multiplying by standard basis vectors $\mathbf{e}_i$ and $\mathbf{e}_j$ from both sides, we get

$$[\mathbf{\Gamma}_R]_{ii}=[\mathbf{\Delta}'(0)]_{ii}\Rightarrow 2[\sqrt{\mathbf{\Gamma}_D}]_{ii}[\mathbf{\Delta}]_{ii}=0 \qquad \forall i=j \quad (A.41)$$

$$[\mathbf{\Gamma}_R]_{ji}=([\mathbf{\Gamma}_D]_{ii}-[\mathbf{\Gamma}_D]_{jj})[\mathbf{V}'(0)]_{ji}\Rightarrow[\mathbf{V}'(0)]_{ji}=\frac{\theta_j^2a_{ji}}{\xi_j^2-\xi_i^2} \qquad \forall i\neq j \quad (A.42)$$

Thus, we obtain that

$$\mathbf{D}(\varepsilon)=\mathbf{D}(0)+\varepsilon\mathbf{\Delta}'(0)+\mathcal{O}(\|\varepsilon\|^2)=\sqrt{\mathbf{\Gamma}_D}+\varepsilon\mathbf{\Delta}'(0)+\mathcal{O}(\|\varepsilon\|^2)=\operatorname{diag}(\xi_i)+\mathcal{O}(\|\varepsilon\|^2) \quad (A.43)$$

$$\mathbf{P}(\varepsilon)=\mathbf{I}+\varepsilon\mathbf{V}'(0)+\mathcal{O}(\|\varepsilon\|^2)=\mathbf{I}+\varepsilon\mathbf{V}'(0)+\mathcal{O}(\|\varepsilon\|^2)=\mathbf{I}+\varepsilon\Big[\frac{\theta_j^2a_{ji}}{\xi_j^2-\xi_i^2}\Big]+\mathcal{O}(\|\varepsilon\|^2) \quad (A.44)$$

■

A.10. Derivations for Finite Frisch Elasticity

TBA.

A.11. Taylor Rule with Schur Decomposition

TBA.

B Equilibrium Definition

Here, we precisely define the equilibrium concept used in this paper.

Definition 2. A **sticky price equilibrium** for this economy is

(a) an allocation for the household, $\mathcal{A}_h=\{(C_{i,t})_{i\in[n]},C_t,L_t,B_t\}_{t\geq 0}\cup\{B_{0-}\}$,

(b) an allocation for all firms $\mathcal{A}_f=\{(Y_{i,t},Y^d_{ij,t},Y^s_{ij,t},L_{ij,t},X_{ij,k,t})_{i\in[n],j\in[0,1]}\}_{t\geq 0}$,

(c) a set of monetary and fiscal policies $\mathcal{A}_g=\{(M_t,T_t,\tau_{1,t},\ldots,\tau_{n,t})_{t\geq 0}\}$,

(d) and a set of prices $\mathcal{P}=\{(P_{i,t},P_{ij,t})_{i\in[n],j\in[0,1]},W_t,P_t,i_t\}_{t\geq 0}\cup\{(P_{ij,0-})_{i\in[n],j\in[0,1]}\}$

such that

1. given $\mathcal{P}$ and $\mathcal{A}_g$, $\mathcal{A}_h$ solves the household's problem in Equation (1),
2. given $\mathcal{P}$ and $\mathcal{A}_g$, $\mathcal{A}_f$ solves the final goods producers problems in Equation (3), intermediate goods producers' cost minimization in Equation (6) and their pricing problem in Equation (7),

3. labor, money, bonds and final sectoral goods markets clear and government budget constraint is satisfied:

$$M_t = M_t^s, \quad B_t = 0, \quad L_t = \sum_{i\in[n]} \int_0^1 L_{ij,t}\mathrm{d}j, \quad \sum_{i\in[n]} \int_0^1 (1-\tau_{i,t})P_{ij,t}Y_{ij,t}\mathrm{d}j = T_t \quad \forall t \geq 0 \tag{B.1}$$

$$Y_{k,t} = C_{k,t} + \sum_{i\in[n]} \int_0^1 X_{ij,k,t}\mathrm{d}j \quad \forall k \in [n], \quad \forall t \geq 0 \tag{B.2}$$

Furthermore, to understand how the stickiness of prices will affect and distort the equilibrium allocations, we will make comparisons between the equilibrium defined above and its *flexible-price* analog, formally defined below.

Definition 3. A **flexible price equilibrium** is an equilibrium defined similar to Definition 2 with the only difference that intermediate goods producers' prices solve the flexible price problems specified in Equation (8) instead of the sticky price problem in Equation (7).

Finally, since we have defined our economy without any aggregate or sectoral shocks, we will pay specific attention to *stationary* equilibria, which we define below.

Definition 4. A **stationary equilibrium** for this economy is an equilibrium as in Definition 2 or Definition 3 with the additional requirement that all the allocative variables in the household's allocation in $\mathcal{A}_h$ and the sectoral production of final good producers $(Y_{i,t})_{i\in[n]}$ as well as the distributions of the allocative variables for intermediate good producers in $\mathcal{A}_i$ are constant over time.[36]

C Derivations of Optimality Conditions in the Model

Here, we characterize the flexible- and sticky-price stationary equilibria of this economy.

C.1. Households' Optimality Conditions

We can decompose the household's consumption problem into two stages, where for a *given* level of C_t the household minimizes her expenditure on sectoral goods (compensated demand) and then decides on the optimal level of C_t as a function of life-time income (uncompensated demand). The compensated demand of the household for sectoral goods given the vector of sectoral prices $\mathbf{P}_t = (P_{1,t}, \dots, P_{n,t})$ gives us the expenditure function:

$$\begin{aligned}\mathcal{E}(C_t; \mathbf{P}_t) &\equiv \min_{C_{1,t},\dots,C_{n,t}} \sum_{i\in[n]} P_{i,t}C_{i,t} \quad \text{subject to} \quad \Phi(C_{1,t},\dots,C_{n,t}) \geq C_t \\ &= P_tC_t, \quad P_t \equiv \mathcal{E}(1, \mathbf{P}_t)\end{aligned} \tag{C.1}$$

where the second line follows from the first degree homogeneity of the function $\Phi(.)$ and P_t is the cost of a *unit* of C_t and, or in short, the price of C_t. Note that due to first degree homogeneity

[36]Note that the production and input demands of individual intermediate goods producers do not need to be time-invariant in the stationary equilibrium, but their distributions do.

of $\Phi(.)$, P_t does not depend on household's choices and is *just* a function of the sectoral prices, $\mathbf{P}_t$. Applying Shephard's lemma, we obtain that the household's expenditure share of sectoral good i is proportional to the elasticity of the expenditure fuction with respect to the price of i:

$$P_{i,t}C^*_{i,t} = \beta_i(\mathbf{P}_t) \times P_t C_t \quad \text{where} \quad \beta_i(\mathbf{P}_t) \equiv \frac{\partial \log(\mathcal{E}(C_t, \mathbf{P}_t))}{\partial \log(P_{i,t})} \tag{C.2}$$

It is important to note that due to the first-degree homogeneity of the expenditure function, these elasticities are independent of aggregate consumption C_t and only depend on sectoral prices, $\mathbf{P}_t$. Moreover, it is easy to verify that they are also a homogeneous of degree zero in these prices so that the vector of household's expenditure shares, denoted by $\boldsymbol{\beta}_t \in \mathbb{R}^n$, can be written as a function of sectoral prices relative to wage:

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}(\mathbf{P}_t/W_t) \tag{C.3}$$

Notably, a vector of constant expenditure shares corresponds to $\Phi(.)$ being a Cobb-Douglas aggregator where sectoral goods are neither complements nor substitutes.

Given the household's expenditure function and the aggregate price index P_t in Equation (C.1), it is straightforward to derive the labor supply and Euler equations for bonds:

$$\underbrace{\gamma(C_t)\frac{\dot{C}_t}{C_t}}_{\text{marginal loss from saving}} = \underbrace{i_t - \rho - \frac{\dot{P}_t}{P_t}}_{\text{marginal gain from saving}} \quad \text{where} \quad \underbrace{\gamma(C_t) \equiv -\frac{U''(C_t)C_t}{U'(C_t)}}_{\text{inverse elasticity of intertemporal substitution}} \tag{C.4}$$

$$\underbrace{\frac{V'(L_t)}{U'(C_t)}}_{\text{MRS}_{LC}} = \underbrace{\frac{W_t}{P_t}}_{\text{real wage}} \Rightarrow \psi(L_t)\frac{\dot{L}_t}{L_t} + \gamma(C_t)\frac{\dot{C}_t}{C_t} = \frac{\dot{W}_t}{W_t} - \frac{\dot{P}_t}{P_t} \quad \text{where} \quad \underbrace{\psi(L_t) \equiv \frac{V''(L_t)L_t}{V'(L_t)}}_{\text{inverse Frisch elasticity of labor supply}} \tag{C.5}$$

Moreover, as long as the interest rate $i_t > 0$, which we will confirm is the case in the stationary equilibria as well as in small enough neighborhoods around them, the cash-in-advance constraint binds. Thus, if the central bank targeted a constant nominal GDP growth of μ,

$$\frac{\dot{P}_t}{P_t} + \frac{\dot{C}_t}{C_t} = \frac{\dot{M}_t}{M_t} = \mu \tag{C.6}$$

Note that by combining Equations (C.4) to (C.6) we can write the growth rate of wages as well as the nominal interest rates as a function of consumption and labor supply growths:

$$\frac{\dot{W}_t}{W_t} = \mu + \psi(L_t)\frac{\dot{L}_t}{L_t} + (\gamma(C_t) - 1)\frac{\dot{C}_t}{C_t}, \qquad i_t = \rho + \mu + (\gamma(C_t) - 1)\frac{\dot{C}_t}{C_t} \tag{C.7}$$

As shown and utilized by Golosov and Lucas (2007) and more recenty by Wang and Werning (2021), a convenient set of preferences that simplify these conditions tremendously are $U(C_t) = \log(C_t)$ and $V(L_t) = L_t$ which imply $\gamma(C_t) = 1$ and $\psi(L_t) = 0$. Plugging these elasticities into Equation (C.7), we can see how these preferences simplifiy aggregate dynamics by setting wage growth to the constant rate of μ and interest rates to a constant rate at $\rho + \mu$.

C.2. Firms' Cost Minimization and Input-Output Matrices

We start by characterizing firms' expenditure shares on inputs by first solving their expenditure minimization problems. Since expenditure minimization is a static decision within every period, our characterization of these expenditure shares closely follow Bigio and La'O (2020); Baqaee and Farhi (2020), and we refer the reader to these papers for more detailed treatments.

Let us start with the observation that the firms' cost function in Equation (6), given the wage and sectoral prices $\mathbf{P}_t = (W_t, P_{i,t})_{i\in[n]}$, is homogenous of degree one in production:

$$\begin{aligned}\mathcal{C}_i(Y^s_{ij,t}; \mathbf{P}_t, Z_{i,t}) &= \min_{L_{jk,t},(X_{ij,k,t})_{k\in[n]}} W_t L_{ij,t} + \sum_{k\in[n]} P_{k,t} X_{ij,k,t} \quad \text{subject to} \quad Z_{i,t} F_i(L_{ij,t}, (X_{ij,k,t})_{k\in[n]}) \geq Y^s_{ij,t} \\ &= \mathsf{MC}_i(\mathbf{P}_t, Z_{i,t}) \times Y^s_{ij,t}, \quad \mathsf{MC}_i(\mathbf{P}_t, Z_{i,t}) \equiv \mathcal{C}_i(1; \mathbf{P}_t, 1)/Z_{i,t} \end{aligned} \tag{C.8}$$

where the second line follows from the first degree homogeneity of the production function $Z_i F_i(.)$ and $\mathsf{MC}_i(\mathbf{P}_t, Z_{i,t})$ is the cost of producing a *unit* of output, or in short, the firm's marginal cost of production. Note that due to the first degree homogeneity of the production function, marginal costs are independent of level of production and depend only on the sector's production function and input prices. Applying Shephard's lemma and re-arranging firms' optimal demand for inputs gives us the result that firms' expenditure share of any input is the elasticity of the cost function with respect to that input:

$$W_t L^*_{ij,t} = \alpha_i(\mathbf{P}_t) \times \mathsf{MC}_i(\mathbf{P}_t, Z_{i,t}) Y^s_{ij,t}, \quad P_{k,t} X^*_{ij,k,t} = a_{ik}(\mathbf{P}_t) \times \mathsf{MC}_i(\mathbf{P}_t, Z_{i,t}) Y^s_{ij,t}, \quad \forall k \in [n] \tag{C.9}$$

where $\alpha_i(\mathbf{P}_t)$ and $a_{ik}(\mathbf{P}_t)$ are the elasticities of the sector i's cost function with respect to labor and sector k's final good respectively:

$$\alpha_i(\mathbf{P}_t) \equiv \frac{\partial \log(\mathcal{C}_i(Y; \mathbf{P}_t, 1)/Z_{i,t})}{\partial \log(W_t)}, \qquad a_{ik}(\mathbf{P}_t) \equiv \frac{\partial \log(\mathcal{C}_i(Y; \mathbf{P}_t, 1)/Z_{i,t})}{\partial \log(P_{k,t})} \quad \forall k \in [n] \tag{C.10}$$

with the property that $\alpha_i(\mathbf{P}_t) + \sum_{k\in[n]} a_{ik}(\mathbf{P}_t) = 1$. It is important to note that the first degree homogeneity of the cost function in Equation (6) also implies that these elasticities are only functions of the aggregate wage and sectoral prices. It is also well-known that these elasticities are directly related to the *cost-based* input-output matrix, denoted by $\mathbf{A}_t \in \mathbb{R}^{n\times n}$, and the labor share vector, denoted by $\boldsymbol{\alpha}_t \in \mathbb{R}^n$:

$$[\mathbf{A}_t]_{i,k} \equiv \frac{\text{total expenditure of sector } i \text{ on sector } k}{\text{total expenditure on inputs in sector } i} = a_{ik}(\mathbf{P}_t), \quad \forall (i,k) \in [n]^2 \tag{C.11}$$

$$[\boldsymbol{\alpha}_t]_i \equiv \frac{\text{total expenditure of sector } i \text{ on labor}}{\text{total expenditure on inputs in sector } i} = \alpha_i(\mathbf{P}_t), \quad \forall i \in [n] \tag{C.12}$$

where the second equality holds *only* under firms' optimal expenditure shares and follows from integrating Equation (C.9). Since these elasticities are also homogenous of degree zero in the price vector $\mathbf{P}_t$, Equations (C.11) and (C.12) imply that in *any equilibrium*, the cost-based input-output matrix and the vector of sectoral labor shares are only a function of the sectoral prices relative to the nominal wage; i.e.,

$$\mathbf{A}_t = \mathbf{A}(\mathbf{P}_t/W_t) = [a_{ik}(\mathbf{P}_t/W_t)], \qquad \boldsymbol{\alpha}_t = \boldsymbol{\alpha}(\mathbf{P}_t/W_t) = [\alpha_i(\mathbf{P}_t/W_t)] \tag{C.13}$$

A notable example is Cobb-Douglas production functions, which imply constant elasticities for the cost function—because inputs are neither substitutes nor complements—and lead to a constant input-output matrix and constant vector of labor shares over time.

C.3. Firms' Optimal Prices

Having characterized firms' cost functions, we now derive the optimal *desired prices*, $P^*_{ij,t}$, in Equation (8) and *reset prices*, $P^{\#}_{ij,t}$ in Equation (C.15). It follows that the optimal desired price is a markup over the marginal cost of production and proportional to the wedge introduced through taxes/subsidies:

$$P^*_{ij,t} = P^*_{i,t} \equiv \underbrace{\frac{1}{1-\tau_i}}_{\text{tax/subsidy wedge}} \times \underbrace{\frac{\sigma_i}{\sigma_i - 1}}_{\text{markup}} \times \underbrace{\frac{\mathsf{MC}_i(\mathbf{P}_t, 1)}{Z_{i,t}}}_{\text{marginal cost}} \tag{C.14}$$

It is then straightforward to show that the firms' optimal reset prices are a weighted average of all future desired prices in industry i:

$$P^{\#}_{ij,t} = P^{\#}_{i,t} \equiv \underbrace{\int_0^\infty \overbrace{\frac{e^{-(\theta_i h + \int_0^h i_{t+s}\mathrm{d}s)} Y_{i,t+h} P^{\sigma_i}_{i,t+h}}{\int_0^\infty e^{-(\theta_i h + \int_0^h i_{t+s}\mathrm{d}s)} Y_{i,t+h} P^{\sigma_i}_{i,t+h}\mathrm{d}h}}^{\text{weight (density) on } P^*_{i,t+h}} \times P^*_{i,t+h}\mathrm{d}h}_{\text{weighted average of all future desired prices}} \tag{C.15}$$

Given this reset price, we can then calculate the aggregate price of sector i from Equation (4) as:

$$P^{1-\sigma_i}_{i,t} = \int_0^1 P^{1-\sigma_i}_{ij,t}\mathrm{d}j = \theta_i \int_0^t e^{-\theta_i h}(P^{\#}_{i,t-h})^{1-\sigma_i}\mathrm{d}h + e^{-\theta_i t}\int_0^1 P^{1-\sigma_i}_{ij,0^-}\mathrm{d}j \tag{C.16}$$

where the second equality follows from the observation that at time t the density of firms that reset their prices h periods ago to $P^{\#}_{i,t}$ is governed by the exponential distribution of time between price changes and is equal to $\theta_i e^{-\theta_i h}$.

C.4. Market Clearing and Total Value Added

Define the sales-based Domar weight of sector $i \in [n]$ at time t as the ratio of the final producer's sales relative to the households total expenditure on consumption:

$$\lambda_{i,t} \equiv P_{i,t}Y_{i,t}/(P_t C_t) \tag{C.17}$$

Now, substituting optimal consumption of the household from sector $k \in [n]$ in Equation (C.2) and optimal demand of firms for the final good of sector $k \in [n]$ in Equation (C.9) into the market clearing condition for final good of sector k and dividing by household's total expenditure, we get

$$\lambda_{k,t} = \beta_i(\mathbf{P}_t/W_t) + \sum_{i\in[n]} a_{ik}(1, \mathbf{P}_t/W_t)\lambda_{i,t}\Delta_{i,t}/\mu_{i,t} \tag{C.18}$$

where $\mu_{i,t} \equiv P_{i,t}/\mathsf{MC}_i(\mathbf{P}_t, W_t)$ is the markup of sector i and $\Delta_{i,t}$ is the well-known measure of price dispersion in the New Keynesian literature defined as

$$\Delta_{i,t} = \int_0^1 (P_{ij,t}/P_{i,t})^{-\sigma_i} \mathrm{d}j \geq 1 \tag{C.19}$$

Where the inequality follows from applying Jensen's inequality to the definion of the aggregate price index $P_{i,t}$.[37] Thus, letting $\boldsymbol{\lambda}_t \equiv (\lambda_{i,t})_{i\in[n]}$ denote the vector of sales-based domar weights at time t across sectors and $\boldsymbol{\mathcal{M}}_t \equiv \mathrm{diag}(\mu_{i,t}/\Delta_{i,t})$ as the diagonal matrix whose i'th diagonal entry is the price dispersion adjusted markup wedge of sector i, we can write Equation (C.18) in the following matrix form:

$$\boldsymbol{\lambda}_t = (\mathbf{I} - \mathbf{A}_t^\intercal \boldsymbol{\mathcal{M}}_t^{-1})^{-1}\boldsymbol{\beta}_t \tag{C.20}$$

Finally, substituting firms labor demand into the labor market clearing condition, we arrive at the following expression for the labor share:

$$\frac{W_t L_t}{P_t C_t} = \boldsymbol{\alpha}_t^\intercal \boldsymbol{\mathcal{M}}_t^{-1} \boldsymbol{\lambda}_t \tag{C.21}$$

C.5. Efficient Flexible Price Stationary Equilibrium

We log-linearize the model around an efficient flexible price stationary equilibrium. Before that, we first discuss the allocation in this steady state. To implement the efficient allocation in the steady-state, the government sets taxes to undo distortions arising from monopolistic competition. That is, by setting $\tau_i = -\frac{1}{\sigma_i}, \forall i \in [n]$

$$P_{ij}^* = \mathsf{MC}_i, \forall j \in [0,1], i \in [n] \tag{C.22}$$

where $\mathsf{MC}_i \equiv \mathcal{C}_i(1; \mathbf{P}, Z_i)$ is the marginal cost of sector $i \in [n]$ at the efficient stationary equilibrium. The marginal cost is given by (C.8) evaluated at $(\mathbf{P}, Z_i)$. From the firm's cost minimization problem at the stationary equilibrium, we also get the demand for labor and intermediate inputs

$$WL_{ij} = \alpha_i(\mathbf{P}) \times \mathsf{MC}_i(\mathbf{P}, Z_i) Y_{ij}^s \tag{C.23}$$

$$P_k X_{ij,k} = a_{ik}(\mathbf{P}) \times \mathsf{MC}_i(\mathbf{P}, Z_i) Y_{ij}^s, \ \forall k \in [n] \tag{C.24}$$

where $\alpha_i(\mathbf{P})$ and $a_{ik}(\mathbf{P})$ are the elasticities of the sector i's cost function with respect to labor and sector k's final good at the stationary equilibrium, respectively:

$$\alpha_i(\mathbf{P}) \equiv \frac{\partial \log(\mathcal{C}_i(Y; \mathbf{P}, Z_i))}{\partial \log(W)}, \quad a_{ik}(\mathbf{P}) \equiv \frac{\partial \log(\mathcal{C}_i(Y; \mathbf{P}, Z_i))}{\partial \log(P_k)}, \ \forall k \in [n] \tag{C.25}$$

[37]Note that $1 = [\int_0^1 (P_{ij,t}/P_{i,t})^{1-\sigma_i} \mathrm{d}j]^{\frac{\sigma_i}{\sigma_i - 1}} \mathrm{d}j = [\int_0^1 ((P_{ij,t}/P_i, t)^{-\sigma_i})^{\frac{\sigma_i - 1}{\sigma_i}} \mathrm{d}j]^{\frac{\sigma_i}{\sigma_i - 1}} \mathrm{d}j \leq \int_0^1 (P_{i,t}/P_t)^{-\sigma_i} \mathrm{d}j$.

Then, the cost-based input-output matrix and the sectoral labor shares at the efficient stationary equilibrium are given by

$$\mathbf{A} = \mathbf{A}(\mathbf{P}/W) = [a_{ik}(1, \mathbf{P}/W)], \qquad \boldsymbol{\alpha} = \boldsymbol{\alpha}(\mathbf{P}/W) = [\alpha_i(1, \mathbf{P}/W)] \tag{C.26}$$

where we used the observation that the cost-based input-output matrix and the vector of sectoral labor shares are only a function of the sectoral prices relative to the nominal wage. From the representative retailer's optimality conditions and the monopolistically competitive firm's optimal price, the aggregate sectoral price is

$$P_i = \left(\int_0^1 \mathsf{MC}_i(\mathbf{P}, Z_i)^{1-\sigma_i} dj\right)^{\frac{1}{1-\sigma_i}} = \mathsf{MC}_i(\mathbf{P}, Z_i) = P^*_{ij}, \ \forall i \in [n] \ \forall j \in [0,1] \tag{C.27}$$

and the demand for the variety ij given by

$$Y^d_{ij} = Y_i, \ \forall i \in [n], \ \forall j \in [0,1] \tag{C.28}$$

Since monopolistically competitive firms produce to meet demand,

$$Y^s_{ij} = Y^d_{ij} = Y_i, \ \forall i \in [n], j \in [0,1] \tag{C.29}$$

and all firms $j \in [0,1]$ within a sector $i \in [n]$ produce the same amount of output at the efficient stationary equilibrium. Turning to the household optimality conditions, the expenditure minimization of the household gives

$$P_i C_i = \beta_i(\mathbf{P}) \times PC, \ \beta_i(\mathbf{P}) \equiv \frac{\partial \log(\mathcal{E}(C; \mathbf{P}))}{\partial \log(P_i)} \tag{C.30}$$

where $\mathcal{E}(C; \mathbf{P})$ is the expenditure function in the stationary equilibrium, and $P \equiv \mathcal{E}(1; \mathbf{P})$. In an efficient stationary equilibrium, the Euler equation becomes

$$i = \rho \tag{C.31}$$

The intra-temporal first-order condition for the household gives

$$PC = W = M \tag{C.32}$$

where the second equality comes from the assumption that the monetary authority directly controls the path for nominal GDP. Finally, market clearing implies in

$$B = 0 \tag{C.33}$$

$$L = \sum_{i \in [n]} L_i \tag{C.34}$$

$$\sum_{i \in [n]} \tau_i P_i Y_i = T \tag{C.35}$$

$$Y_k = C_k + \sum_{i\in[n]} \int_0^1 X_{ij,k}\mathrm{d}j,\ \forall k \in [n] \tag{C.36}$$

The sales-based Domar weight of sector $i \in [n]$ is given by

$$\lambda_i = {}^{P_iY_i}/_{PC} \tag{C.37}$$

Then, multiplying the market clearing for the sector's k good by P_k and dividing by PC

$$P_kY_k = P_kC_k + \sum_{i\in[n]} \int_0^1 P_kX_{ij,k}\mathrm{d}j,\ \forall k \in [n] \tag{C.38}$$

$$\frac{P_kY_k}{PC} = \frac{P_kC_k}{PC} + \frac{1}{PC}\sum_{i\in[n]} \int_0^1 P_kX_{ij,k}\mathrm{d}j \tag{C.39}$$

$$\lambda_k = \beta_k(\mathbf{P}/W) + \frac{1}{PC}\sum_{i\in[n]} a_{ik}(1, \mathbf{P}/W) \times \mathsf{MC}_i(\mathbf{P}, Z_i)Y_i \tag{C.40}$$

$$\lambda_k = \beta_k(\mathbf{P}/W) + \sum_{i\in[n]} a_{ik}(1, \mathbf{P}/W) \times \frac{P_iY_i}{PC} \tag{C.41}$$

$$\lambda_k = \beta_k(\mathbf{P}/W) + \sum_{i\in[n]} a_{ik}(1, \mathbf{P}/W) \times \lambda_i \tag{C.42}$$

Rearranging the terms and stacking all equations you get

$$(\mathbf{I} - \mathbf{A}^\intercal)\boldsymbol{\lambda} = \boldsymbol{\beta} \tag{C.43}$$

$$\boldsymbol{\lambda} = (\mathbf{I} - \mathbf{A}^\intercal)^{-1}\boldsymbol{\beta} \tag{C.44}$$

where $\boldsymbol{\lambda} = (\lambda_i)_{i\in[n]}$ denotes the vector of sales-based Domar weights in the stationary equilibrium. Then, multiplying the labor market clearing condition by W, dividing by PC and substituting firms' labor demand into it, we arrive at the following expression for the labor share:

$$\frac{WL}{PC} = \sum_{i\in[n]} \frac{WL_i}{PC} \tag{C.45}$$

$$= \sum_{i\in[n]} \alpha_i(1, \mathsf{P}/W) \times \frac{P_iY_i}{PC} \tag{C.46}$$

$$= \sum_{i\in[n]} \alpha_i(1, \mathsf{P}/W) \times \lambda_i \tag{C.47}$$

$$= \boldsymbol{\alpha}^\intercal \boldsymbol{\lambda} \tag{C.48}$$

Recall that $\mathbf{A}$, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ are objects that depend on $({}^{\mathbf{P}}/_{W})$. While $W = M$ is set exogenously by the Central Bank, $\mathbf{P}$ is an endogenous object. To solve for the sectoral prices, we plug each sector firm's labor and intermediate input demand into their production function:

$$1 = Z_iF_i\left(\frac{\alpha_i(\mathbf{P}) \times \mathsf{MC}_i(\mathbf{P}, Z_i)Y_i}{W}, \left(\frac{a_{ik}(\mathbf{P}) \times \mathsf{MC}_i(\mathbf{P}, Z_i)Y_i}{P_k}\right)_{i\in[n]}\right) \tag{C.49}$$

$$=Z_i F_i\left(\frac{\alpha_i(M,\mathbf{P})\times \mathsf{MC}_i(M,\mathbf{P},Z_i)}{M},\left(\frac{a_{ik}(M,\mathbf{P})\times \mathsf{MC}_i(M,\mathbf{P},Z_i)}{P_k}\right)_{i\in[n]}\right),\ \forall i\in[n] \tag{C.50}$$

where in the second line we use $W=M$ and the fact that $F_i(.)$ is homogenous of degree one in its inputs. These form a system of n nonlinear equations with n unknowns, $(P_i)_{i\in[n]}$. Once we solve for the sectoral prices as a function of $(Z_i)_{i\in[n]}$ and M, we can solve for the price level $P\equiv\mathcal{E}(1;\mathbf{P})$. With the price level P, the aggregate GDP is given by $C={}^{M}/_{\mathcal{E}(1;\mathbf{P})}$, and sectoral consumption $C_i=\beta_i(\mathbf{P})^{M}/_{P_i}$.

C.6. Log-linearization

TBA.

D Data Appendix

Proposition 2 shows that the sufficient statistics for inflation and output dynamics in response to shocks in our model are the duration-adjusted Leontief matrix, $\Gamma = \Theta^2(\mathbf{I} - \mathbf{A})$., and the consumption expenditure shares across sectors, given by β. We now describe in detail how we construct Γ and β using detailed sectoral US data.

First, we use the input-output (IO) tables from the BEA to construct the input-output linkages across sectors[38], given by the matrix A; the consumption expenditure shares across sectors, given by the vector β; and the sectoral labor shares, given by the vector α. In particular, to construct A, we use both the "make" and "use" IO tables.[39] The "use" IO table also provides data on the compensation of employees, which is used to construct the sectoral labor shares α. Moreover, the "use" IO table also provides data on personal consumption expenditure, which is used to construct the consumption expenditure shares across sectors, β. Figure F.1 in Appendix F presents the matrix A we construct from the data, in a heat-map version.

Next, we construct the diagonal matrix Θ^2, whose diagonal elements are the squared frequency of price adjustment in each sector, using data on 341 sectors from Pasten et al. (2020). First, we match data from Pasten et al. (2020) on the frequency of price changes with the 2002 concordance table between IO industry codes and the 2002 NAICS codes. Then, we match these codes with the 2012 concordance table between IO industry codes and 2012 NAICS codes. The last step is performed in order to get frequency of price adjustment data for sectors in the 2012 IO table.

D.1. Constructing the Input-Output Matrix

In this subsection, we describe how we use the "Make" and "Use" matrices to get the cost-based industry-by-industry input-output table. Specifically, we use the 2012 "Make" table after redefinitions and the 2012 "Use" table after redefinitions in producers' value.

Recall that the "Make" table is a matrix of Industry-by-Commodity. Given a row, each column shows the values of each commodity produced by the industry in the row. The "Use" table is a matrix of Commodity-by-Industry. Given a column, each row shows the value of each commodity used by the industry (or final use) in the column. In order to create an industry-by-industry IO table, we combine both. We follow the Handbook of Input-Output Table Compilation and Analysis from the UN and Concepts and Methods of the United States Input-Output Accounts from the BEA, adjusting the matrices accordingly in order to calculate the cost-based IO matrix. We exclude the government sector, Scraps, Used and secondhand

[38]We construct industry-by-industry IO tables. We use industry and sector interchangeably.

[39]The "make" table is a matrix of industries on the rows and commodities on the columns that gives the value of each commodity on the column produced by the industry on the rows. The "use" table is a matrix of commodities on the rows and industries on the columns that gives the value of each commodity on the row that was used by each industry in the column. We combine both matrices to give an industry-by-industry IO matrix.

goods, Noncomparable imports, and Rest of the world adjustment[40]

Input-Output matrix (A). From the "Use" table from the BEA, a given column j gives:

$$\begin{aligned}\text{Total Industry Output}_j =& \text{Total Intermediate}_j \\ &+\text{Compensation of Employees}_j \\ &+\text{Taxes on production and imports, less subsidies}_j \\ &+\text{Gross operating surplus}_j\end{aligned}$$

where Total Intermediate$_j$ is the sum of the dollar amount of each commodity used by industry j. The total cost is given by

$$\text{Total Industry Cost}_j = \text{Total Intermediate}_j + \text{Compensation of Employees}_j$$

Therefore,

$$\underbrace{P_j Y_j}_{\text{Total Industry Output}} = \underbrace{(1+\omega_j)}_{\text{Wedge}} \underbrace{\left(\sum_i P_i X_{ji} + W L_j\right)}_{\text{Total Industry Cost}}$$

where we implicitly assume that the wedge is attributed to taxes and gross operating surplus. That is

$$(1+\omega_j) \equiv \frac{\text{Total Intermediate}_j + \text{Compensation of Employees}_j + \text{Taxes}_j + \text{Gross Operating Surplus}_j}{\text{Total Intermediate}_j + \text{Compensation of Employees}_j} \tag{D.1}$$

Let $\text{diag}(1+\omega)$ be the diagonal matrix in which each j-th diagonal is the the industry j wedge. We calculate the cost-based IO matrix by first calculating the revenue-based IO matrix and then, using these wedges, recovering the cost-based IO matrix. First, we calculate the revenue-based IO matrix. Let $\mathbf{U}_{(N_C+1)\times N_I}$ be the "Use" matrix (commodity-by-industry) that gives for each cell u_{ij} the dollar value of commodity i used in the production of industry j and in the last row the compensation of employees. Let $\mathbf{M}_{N_I \times N_C}$ be the "Make" matrix (industry-by-commodity) that gives for each cell m_{ij} the dollar value of commodity j produced by i. Let $\mathbf{g}_{N_I\times 1}$ be the vector of industry total output and $\mathbf{q}_{N_C\times 1}$ be the vector of commodity output, where N_C is the number of commodities and N_I is the number of industries. Then, define the following matrices

$$\mathbf{B} = \mathbf{U} \times \text{diag}(\mathbf{g})^{-1} \tag{D.2}$$

$$\mathbf{D} = \mathbf{M} \times \text{diag}(\mathbf{q})^{-1} \tag{D.3}$$

where $\text{diag}(\mathbf{g})$ is the diagonal matrix of vector $\mathbf{g}$ and $\text{diag}(\mathbf{q})$ is the diagonal matrix of vector $\mathbf{q}$. The matrix $\mathbf{D}$ is a market share matrix. Its entry d_{ij} gives the market share of industry i in the

[40] Baqaee and Fahri (2020) also exclude these sectors. Besides them, we exclude Customs duties, which is an industry with zero commodity use and zero compensation of employees.

production of commodity j. The matrix $\mathbf{B}$ is a direct input matrix. Its entry b_{ij} gives the dollar amount share of commodity i in the output of industry j. Let

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_I^{N_C \times N_I} \\ \tilde{\boldsymbol{\alpha}}'_{1 \times N_I} \end{bmatrix} \tag{D.4}$$

where $\mathbf{B}_I$ is the part of $\mathbf{B}$ that includes all intermediate inputs and industries and $\tilde{\boldsymbol{\alpha}}'$ the vector with compensation of employees for each industry. Then, the revenue-based industry-by-industry IO matrix is given by

$$\tilde{\mathbf{A}} = (\mathbf{D}\mathbf{B}_I)^T \tag{D.5}$$

To go from the revenue-based IO matrix to the cost-based IO matrix, first recall that

$$\tilde{A}_{ij} = \frac{P_j X_{ij}}{P_i Y_i} \tag{D.6}$$

where $P_j X_{ij}$ is the expenditure of industry i on industry j, $P_i Y_i$ is the revenue of the industry i. The cost-based IO matrix is given by

$$\mathbf{A} = [A_{ij}]_{i \in [n], j \in [n]}, \; A_{ij} = \frac{P_j X_{ij}}{\mathcal{C}_i} \tag{D.7}$$

where $\mathcal{C}_i \equiv \sum_k P_k X_{ik} + W L_i$ is the total cost of industry i. Since $P_i Y_i = (1+\omega_i)\mathcal{C}_i$, we have that

$$A_{ij} = \frac{P_j X_{ij}}{\mathcal{C}_i} = \frac{P_i Y_i}{\mathcal{C}_i} \frac{\mathcal{C}_i}{P_i Y_i} \frac{P_j X_{ij}}{\mathcal{C}_i} = (1+\omega_i)\tilde{A}_{ij} \tag{D.8}$$

Hence

$$\mathbf{A} = \text{diag}(1+\omega)\tilde{\mathbf{A}} \tag{D.9}$$

Labor Share ($\boldsymbol{\alpha}$). Once we calculate the IO matrix, the labor share is given by

$$\boldsymbol{\alpha} = \text{diag}(1+\omega)\tilde{\boldsymbol{\alpha}} \tag{D.10}$$

Consumption Share ($\boldsymbol{\beta}$). The "Use" table gives the Personal Consumption Expenditures on each commodity. Since, we are working with an industry-by-industry IO matrix, we need to calculate an industry consumption share vector. In order to do that, let C_i be the consumption dollar amount of commodity i, and $\mathbf{c}$ be the vector of consumption dollar amount of all commodities in the economy. Then, the vector of consumption dollar amount of each industry is given by $\mathbf{Dc}$. To see this,

$$\mathbf{Dc} = \begin{bmatrix} \sum_j d_{1j} c_j \\ \sum_j d_{2j} c_j \\ \vdots \\ \sum_j d_{nj} c_j \end{bmatrix} \tag{D.11}$$

Recall that d_{ij} gives the market share of industry i in the production of commodity j. Therefore, $d_{ij} c_j$ is the amount in dollars spent by households on commodity j produced by i. Then, the sum over j gives the total expenditure in dollars of households on commodities produced by industry i. That is, the total expenditure in dollars of households on industry i. Then,

$$\boldsymbol{\beta} = \frac{\mathbf{Dc}}{\mathbf{1}'\mathbf{Dc}} \tag{D.12}$$

D.2. Constructing the Frequency of Price Adjustment Matrix

To get the 2012 detail-level industry frequency of price adjustments from the 2002 detail-level industry frequency of price adjustments, we had to manually match them. There were five cases in which industries could fall:

1. Industries with exact matching: the 2002 detail-level industry exactly correspond to the 2012 detail-level industry. In these cases, we use the 2002 detail-level industry frequency of price adjustment as the 2012 detail-level industry frequency of price adjustment. E.g.: Poultry and egg production (2002 IO Code: 112300, 2002 NAICS Code: 1123; 2012 IO Code: 112300; 2012 NAICS Code: 1123).
2. Industries with close matching: the 2002 detail-level industry closely correspond to the 2012 detail-level industry. In these cases, we use the 2002 detail-level industry frequency of price adjustment as the 2012 detail-level industry frequency of price adjustment. E.g.: In 2012 there is Metal crown, closure, and other metal stamping (except automotive) (2012 IO Code: 332119, 2012 NAICS Code: 332119). In 2002, there is Crown and closure manufacturing and metal stamping (2002 IO Code: 33211B, 2002 NAICS Code: 332115-6).
3. Industry present in 2002, but not in 2012: these are detail-level industries that were present in 2002, but not in 2012. These are 2002 industries that seem to be put into a coarser industry in 2012. We match the 2002 industries with the coarser 2012 industry. If there are more than one 2002 industry that are associated with the coarser industry in 2012 with frequency of price adjustment data, we use their average frequency of price adjustment as the 2012 industry frequency of price adjustment. E.g.: Other crop farming (2012 IO Code: 111900, 2012 NAICS Code: 1119). In 2002, there were three industries for which we have data on frequency of price adjustment, that seem to belong to that industry: All other crop farming (2002 IO Code: 1119B0; 2002 NAICS Code: 11194, 111992, 111998), Tobacco farming (2002 IO Code: 111910, 2002 NAICS Code: 11191), Cotton farming (2002 IO Code: 111920, 2002 NAICS Code: 11192). We take the average of these industries' frequency of price adjustment and use as the Other crop farming frequency of price adjustment.
4. Industry present in 2012, but not in 2002: these are detail-level industries that were present in 2012, but not in 2002. These are industries in 2012 that seem to be put into a coarser industry in 2002. In these cases, we use the 2002 coarser industry frequency of price adjustment to impute the 2012 finer industry frequency of price adjustment. E.g.: In 2002, retail trade was a single industry (2002 IO Code: 4A0000; 2002 NAICS Code:

44, 45). In 2012, within retail trade there were Motor vehicle and parts dealers (2012 IO Code: 441000, 2012 NAICS Code: 441), Food and beverage stores (2012 IO Code: 445000, 2012 NAICS Code: 445), General merchandise stores (2012 IO Code: 452000, 2012 NAICS Code: 452), Building material and garden equipment and supplies dealers (2012 IO Code: 444000, 2012 NAICS Code: 444), Health and personal care stores (2012 IO Code: 446000, 2012 NAICS Code: 446), Gasoline stations (2012 IO Code: 447000, 2012 NAICS Code: 447), Clothing and clothing accessories stores (2012 IO Code: 448000, 2012 NAICS Code: 448), Nonstore retailers (2012 IO Code: 454000, 2012 NAICS Code: 454), All other retail (2012 IO Code: 4B0000, 2012 NAICS Code: 442, 443, 451, 453). For all these 2012 industries, we impute their frequency of price adjustment with the 2002 Retail Trade value.

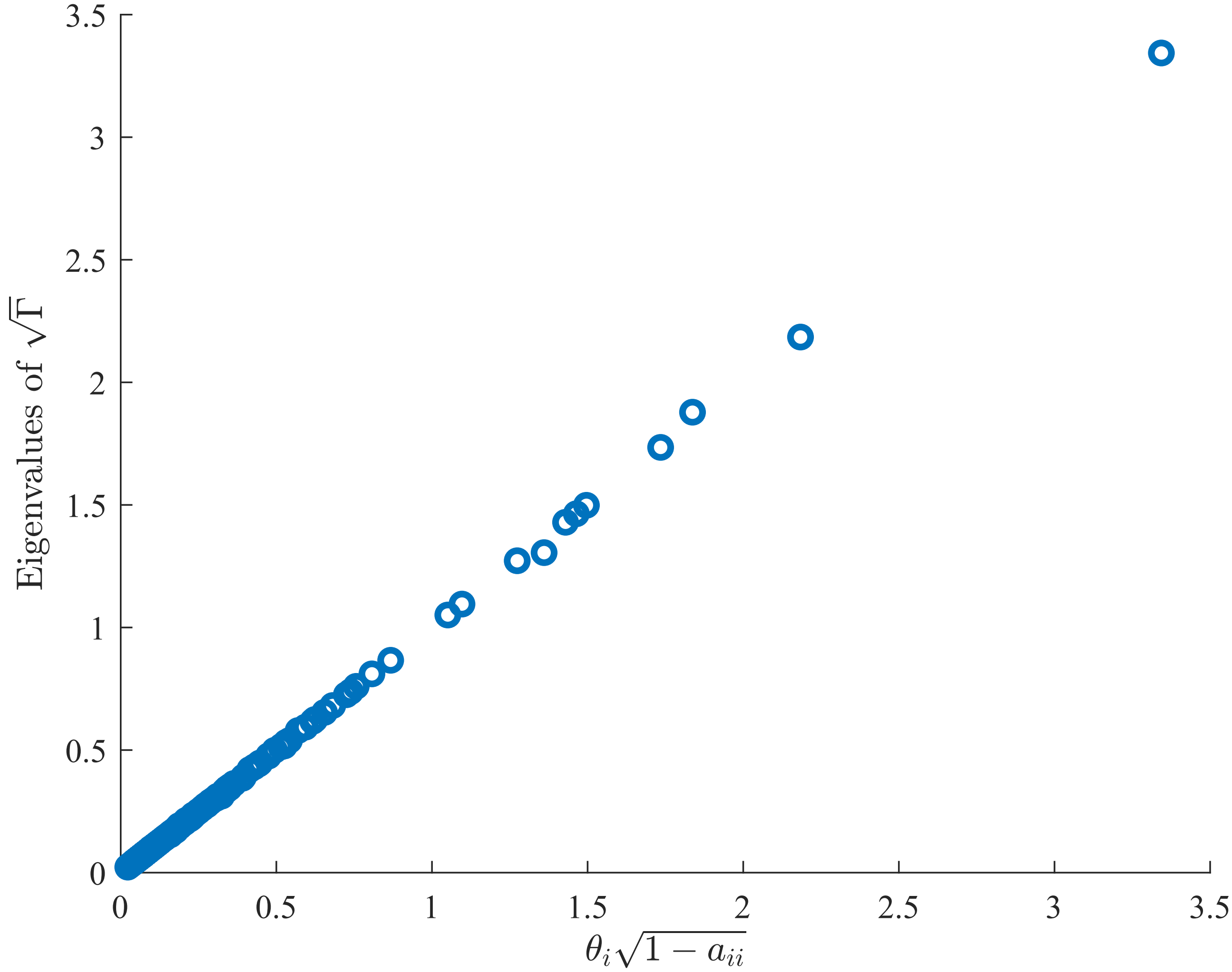
5. Industry present in 2012, but not in 2002 without correspondence: these are 2012 detail-level industries for which there was no correspondent 2002 detail-level industry. In these cases, we impute their frequency of price adjustment with the average frequency of price adjustment among industries with data. E.g.: Motion picture and video industries (2012 IO Code: 512100; 2012 NAICS Code: 5121).

For the industries in cases three, four and five, a concordance table is available upon request. The average frequency of price adjustment among sectors with data is given by 0.171. Its continuous counterpart is 0.1875. This is the value that is used to impute the sectors that are present in 2012, but not in 2002 without any correspondence in the simulations [41]. Finally, the consumption weighted average frequency of price adjustment is given by $\bar{\theta} = \sum_i \beta_i \theta_i$, where β_i is sector's i consumption share, θ_i its frequency of price adjustment. This is the value that is used for the counterfactual economy in which we set an homogenous frequency of price adjustment.

[41] Its value is given by $-\ln(1-0.171)$

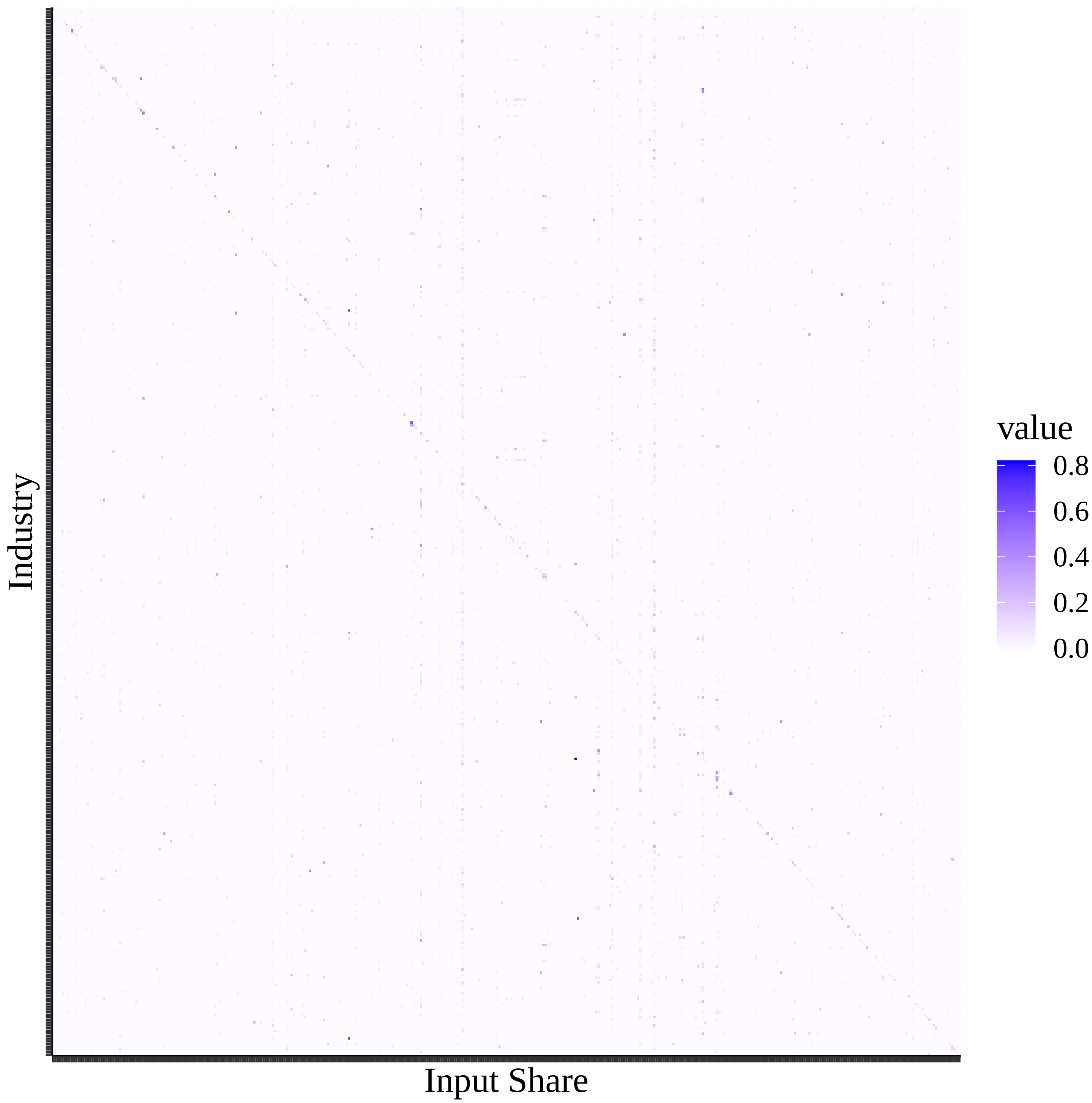
E Additional Figures and Tables

Figure E.1: Relationship between eigenvalues in the diagonal economy and the baseline economy



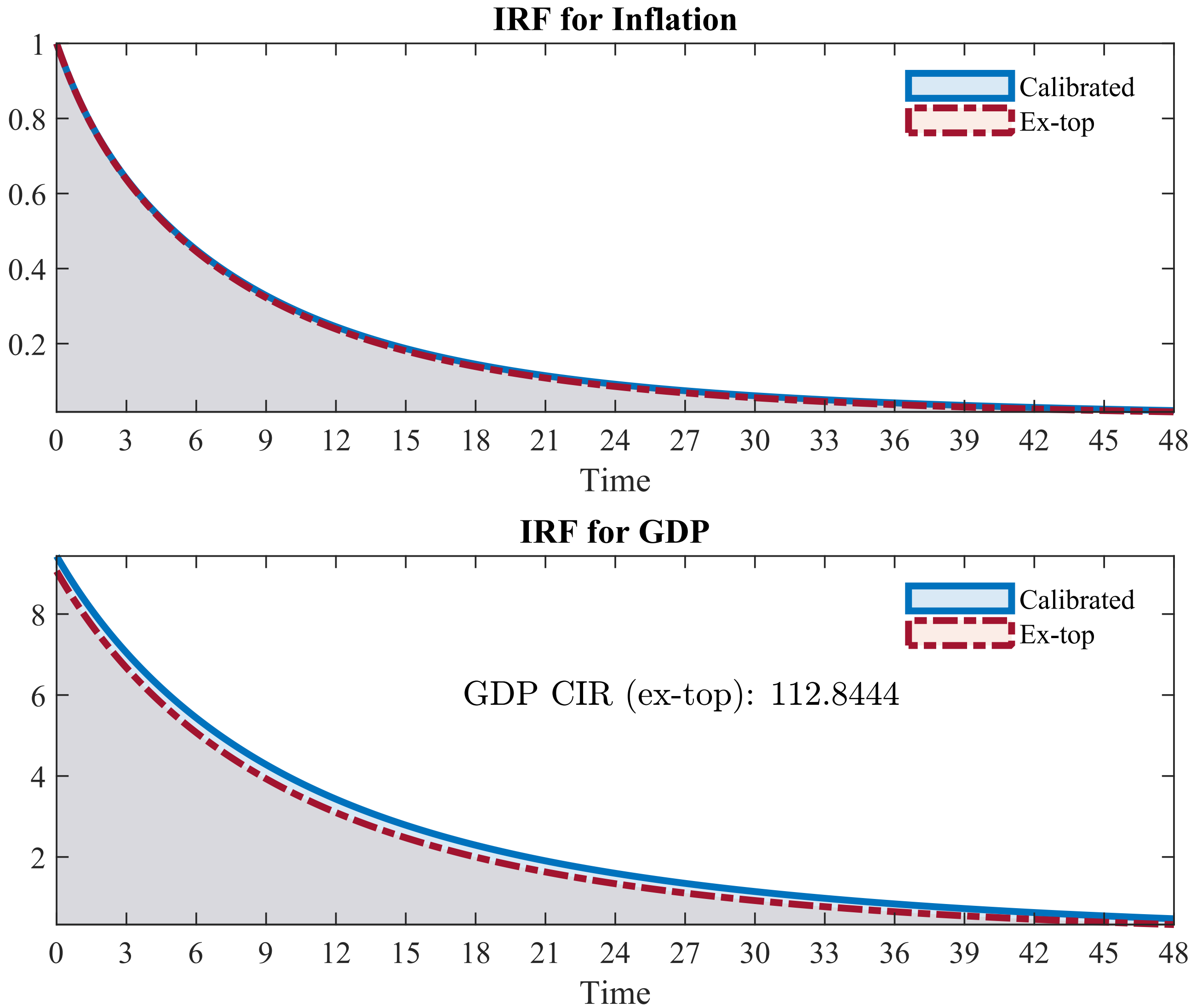
Notes: This figure plots the relationship between the eigenvalues in the diagonal economy and the eigenvalues in the baseline calibrated economy

Figure E.2: U.S. sectoral input-output matrix (heat map) in 2012



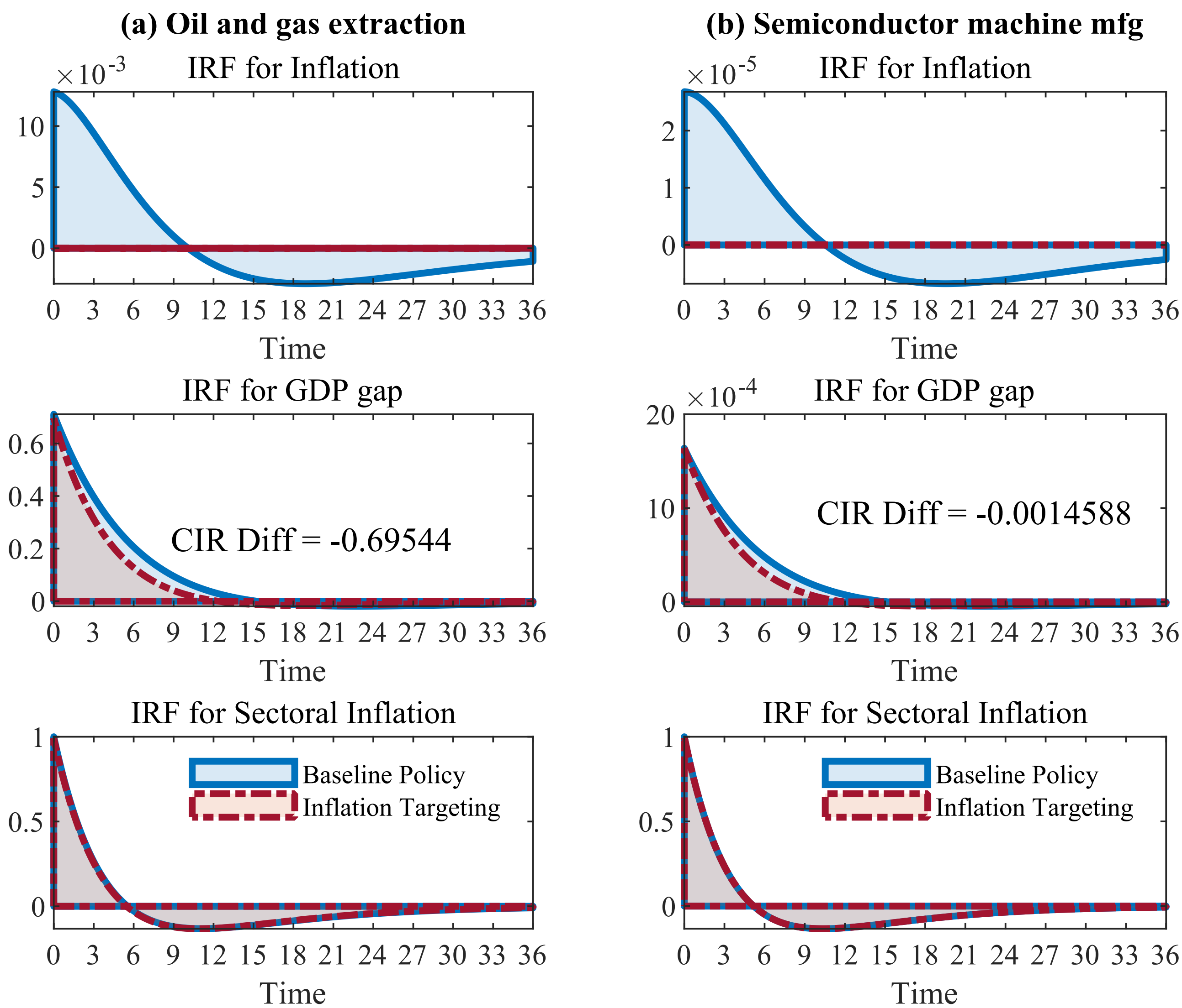
Notes: This figure presents the sectoral input-output matrix in a heat map version, using data from the make and use input-otput tables produced by the BEA in 2012. The industry classification is at the detail-level disaggregation, for a total of 393 sectors.

Figure E.3: Impulse response functions to a monetary policy shock in two economies



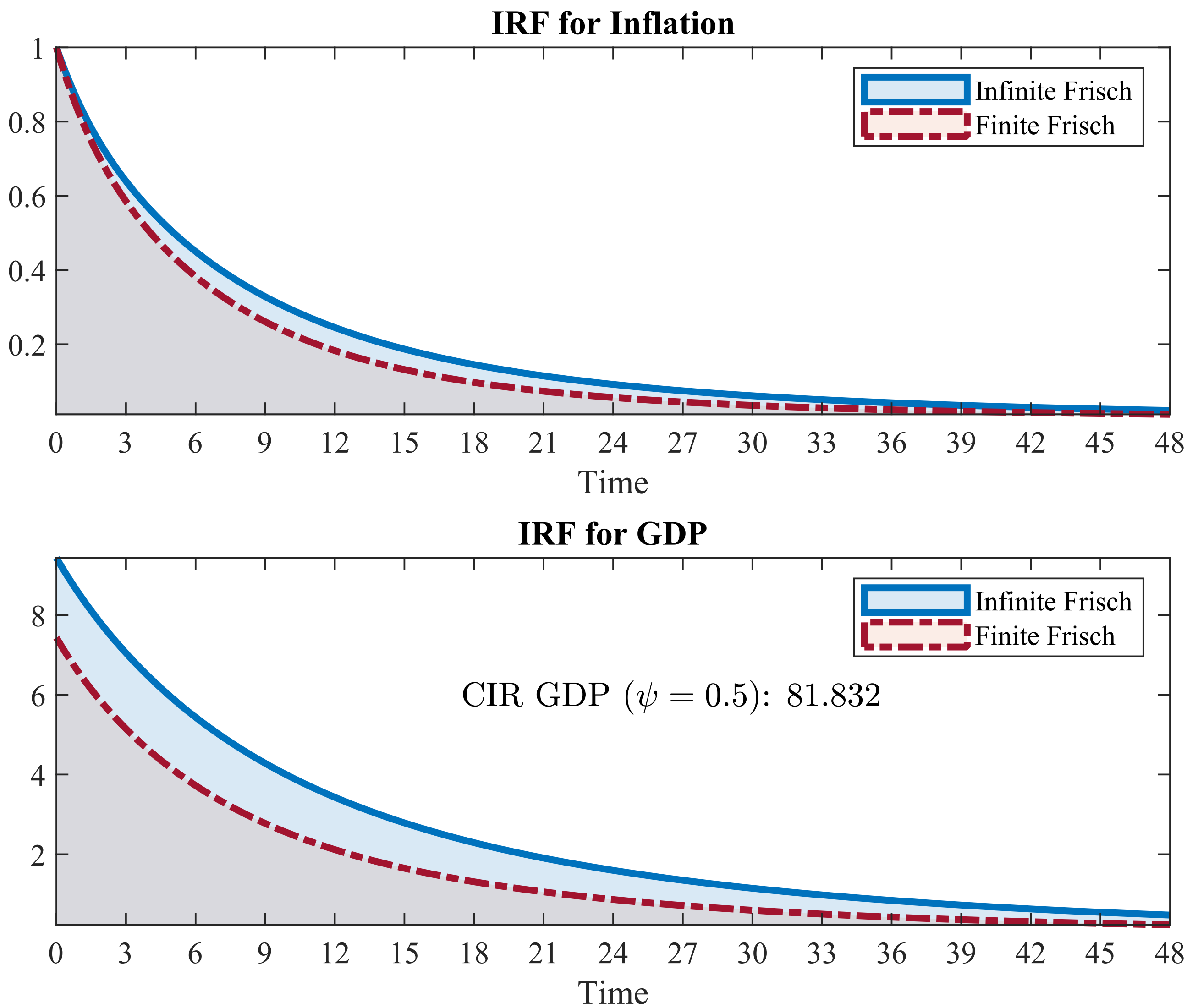
Notes: This figure plots the inflation and GDP responses after a monetary policy shock that leads to a one percent increase in inflation on impact in the baseline economy and excluding the top-3 sectors by eigenvalues in the diagonal economy.

Figure E.4: Dynamics following sectoral shocks in a homogeneous frequency of price adjustment economy



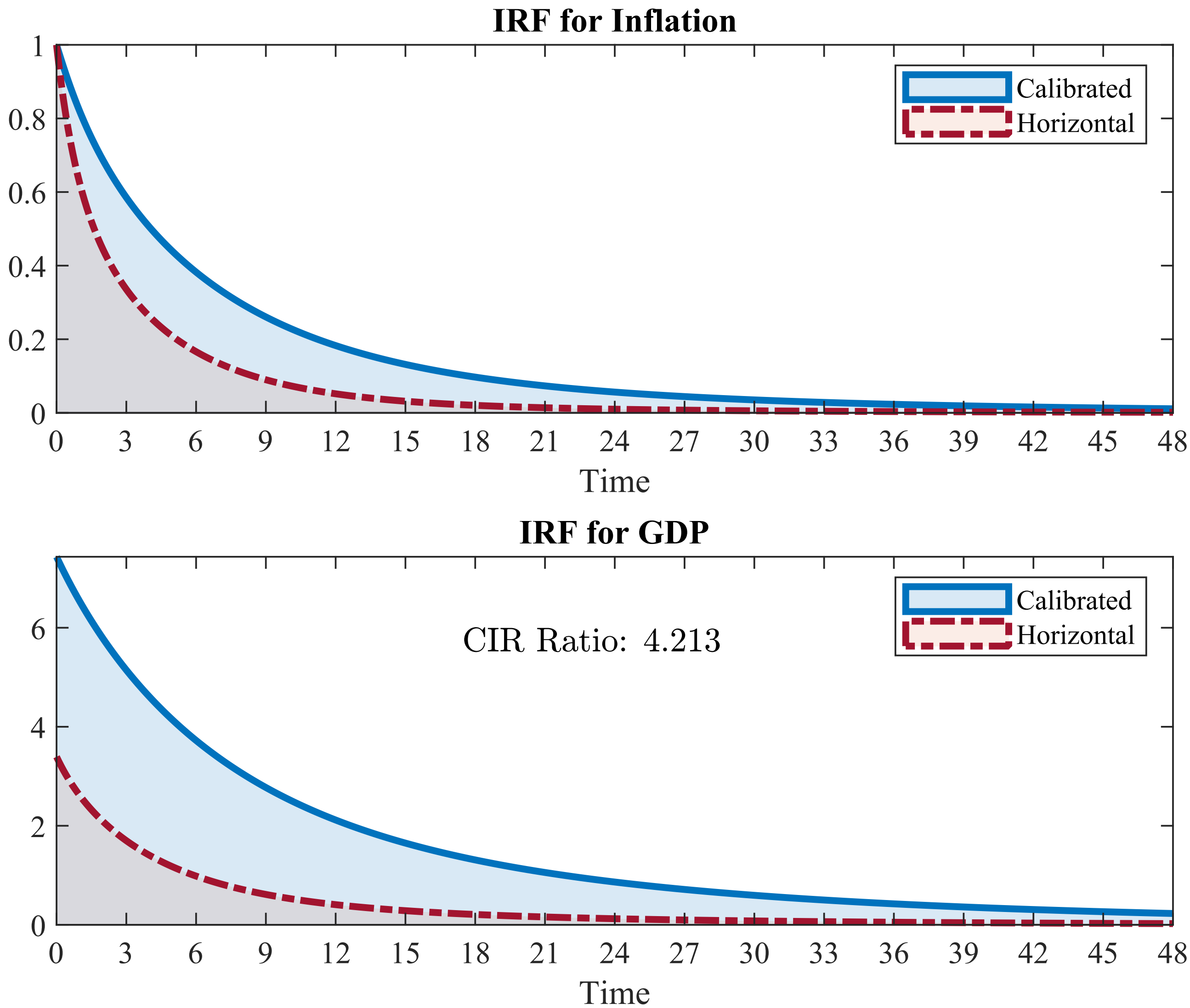
Notes: This figure plots the impulse response functions for inflation, gdp gap, and sectoral inflation to a sectoral shock that increases sectoral inflation by one percent on impact in the baseline policy economy. It compares the baseline policy economy with an economy where monetary policy stabilizes aggregate inflation. Panel A: Oil and gas extraction. Panel B: Semiconductor machine manufacturing. This calibration imposes a homogenoues frequency of price adjustment across sectors.

Figure E.5: Impulse response functions to a monetary policy shock in two economies



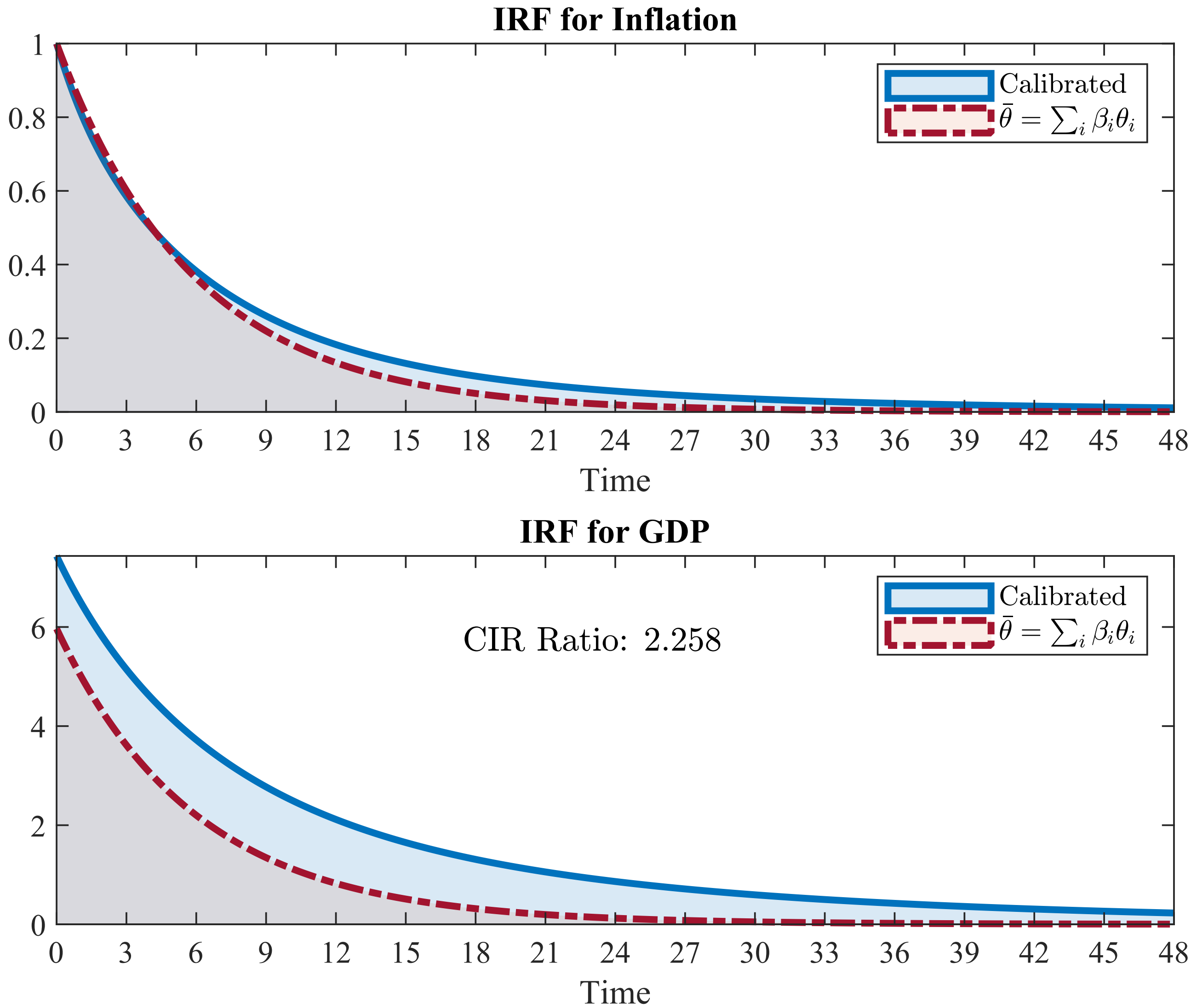
Notes: This figure plots the impulse response functions for inflation and GDP to a monetary shock that generates a one percentage increase in inflation on impact. The calibration of the model is at a monthly frequency. CIR denotes the cumulative impulse response. The calibration uses a (finite) Frisch elasticity of 2.

Figure E.6: Impulse response functions to a monetary policy shock in two economies



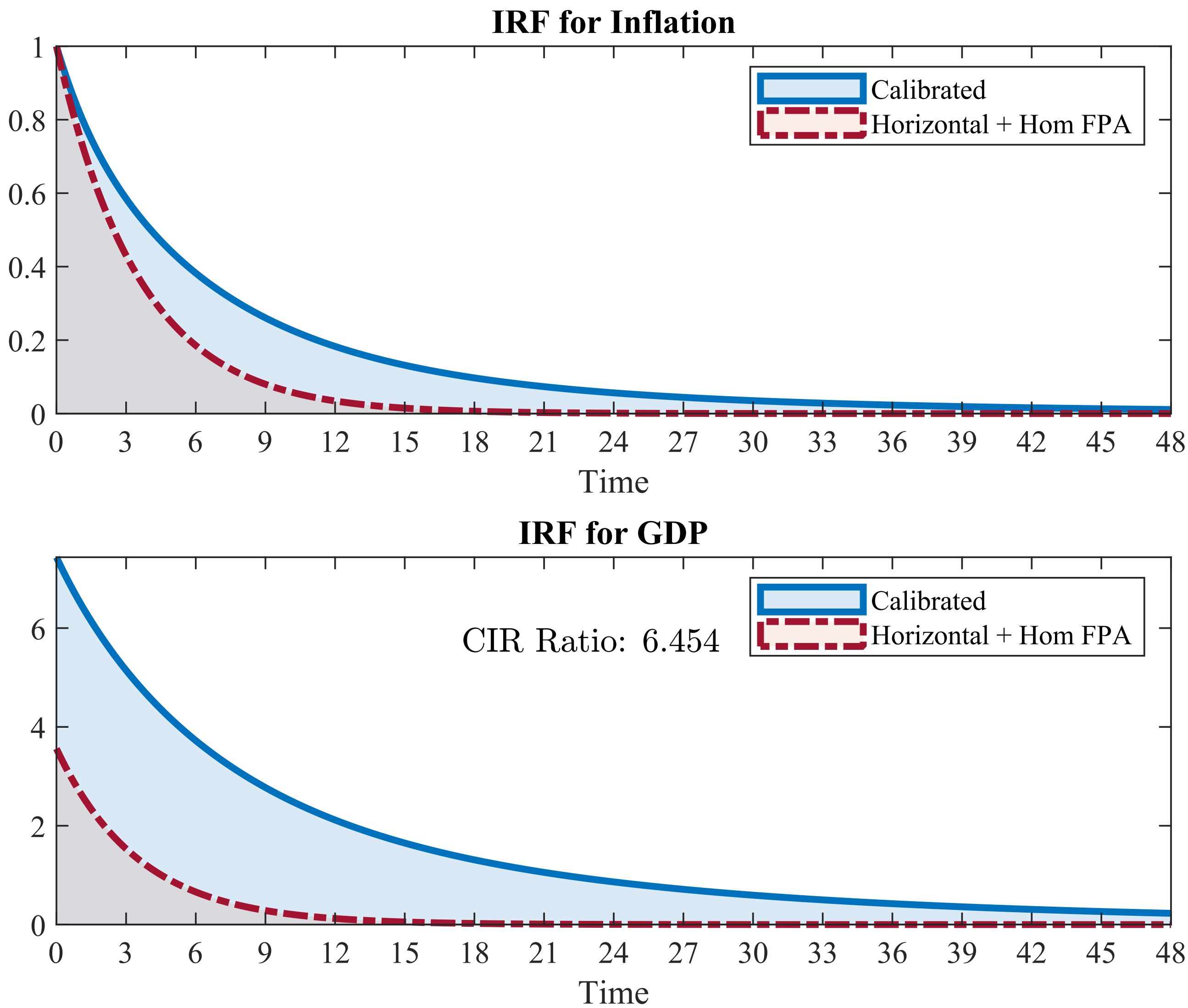
Notes: This figure plots the impulse response functions for inflation and GDP to a monetary shock that generates a one percentage increase in inflation on impact. It compares our baseline economy that has production networks with an economy that has a horizonal production structure where only labor is used as an input for production. The calibration of the model is at a monthly frequency. CIR denotes the cumulative impulse response. The calibration uses a (finite) Frisch elasticity of 2.

Figure E.7: Impulse response functions to a monetary policy shock in two economies



Notes: This figure plots the impulse response functions for inflation and GDP to a monetary shock that generates a one percentage increase in inflation on impact. It compares our baseline economy that has heterogeneous price stickiness across sectors with an economy that has homogeneous price stickiness across sectors. The homogeneous price adjustment frequency is calibrated to be the weighted average of the price adjustment frequencies across sectors. The calibration of the model is at a monthly frequency. CIR denotes the cumulative impulse response. The calibration uses a (finite) Frisch elasticity of 2.

Figure E.8: Impulse response functions to a monetary policy shock in two economies



Notes: This figure plots the impulse response functions for inflation and GDP to a monetary shock that generates a one percentage increase in inflation on impact. It compares our baseline economy that has production networks and heterogeneous price stickiness across sectors with an economy that has both a horizonal production structure where only labor is used as an input for production as well as homogeneous price stickiness across sectors. The homogeneous price adjustment frequency is calibrated to be the weighted average of the price adjustment frequencies across sectors. The calibration of the model is at a monthly frequency. CIR denotes the cumulative impulse response. The calibration uses a (finite) Frisch elasticity of 2.

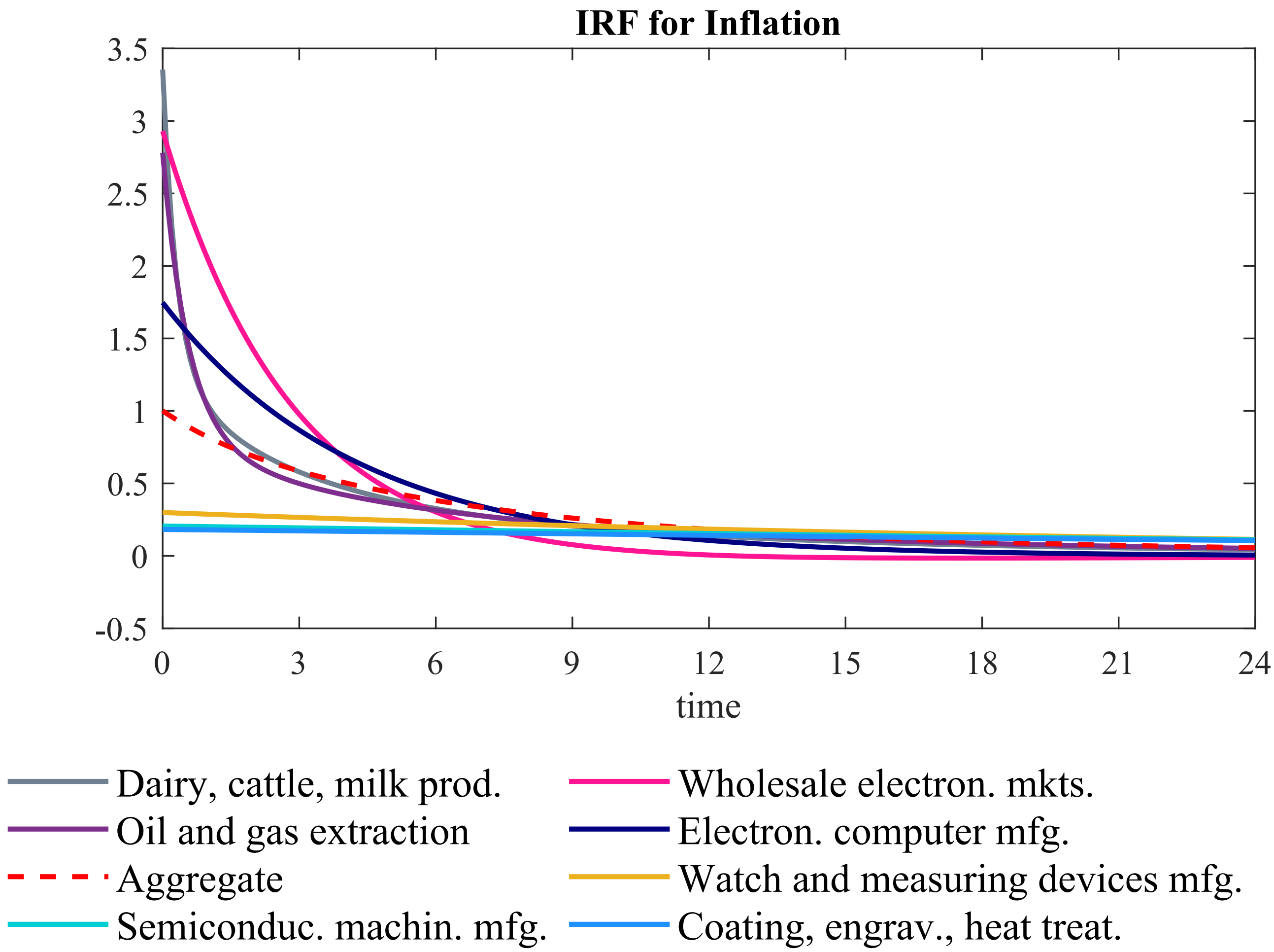
Table E.1: Ranking of industries by inflation impact after a monetary policy shock

Industry	Inflation Impact Resp.
Alumina refining and primary aluminum production	3.671238
Other crop farming	2.586257
Monetary authorities and depository credit inte...	2.525314
Dairy cattle and milk production	2.020186
Animal production, except cattle and poultry an...	1.954667
Wholesale electronic markets and agents and bro...	1.729891
Oil and gas extraction	1.526314
Automobile manufacturing	1.460931
Natural gas distribution	1.283793
Copper, nickel, lead, and zinc mining	1.238399
Fishing, hunting and trapping	1.206477
Rail transportation	1.05083
Nonferrous Metal (except Aluminum) Smelting and...	0.992207
Professional and commercial equipment and supplies	0.963642
Machinery, equipment, and supplies	0.848447
Poultry processing	0.834817
Electric lamp bulb and part manufacturing	0.821602
Poultry and egg production	0.804583
Fluid milk and butter manufacturing	0.801039
Petrochemical manufacturing	0.797405

Table E.2: Ranking of industries by inflation half-life after a monetary policy shock

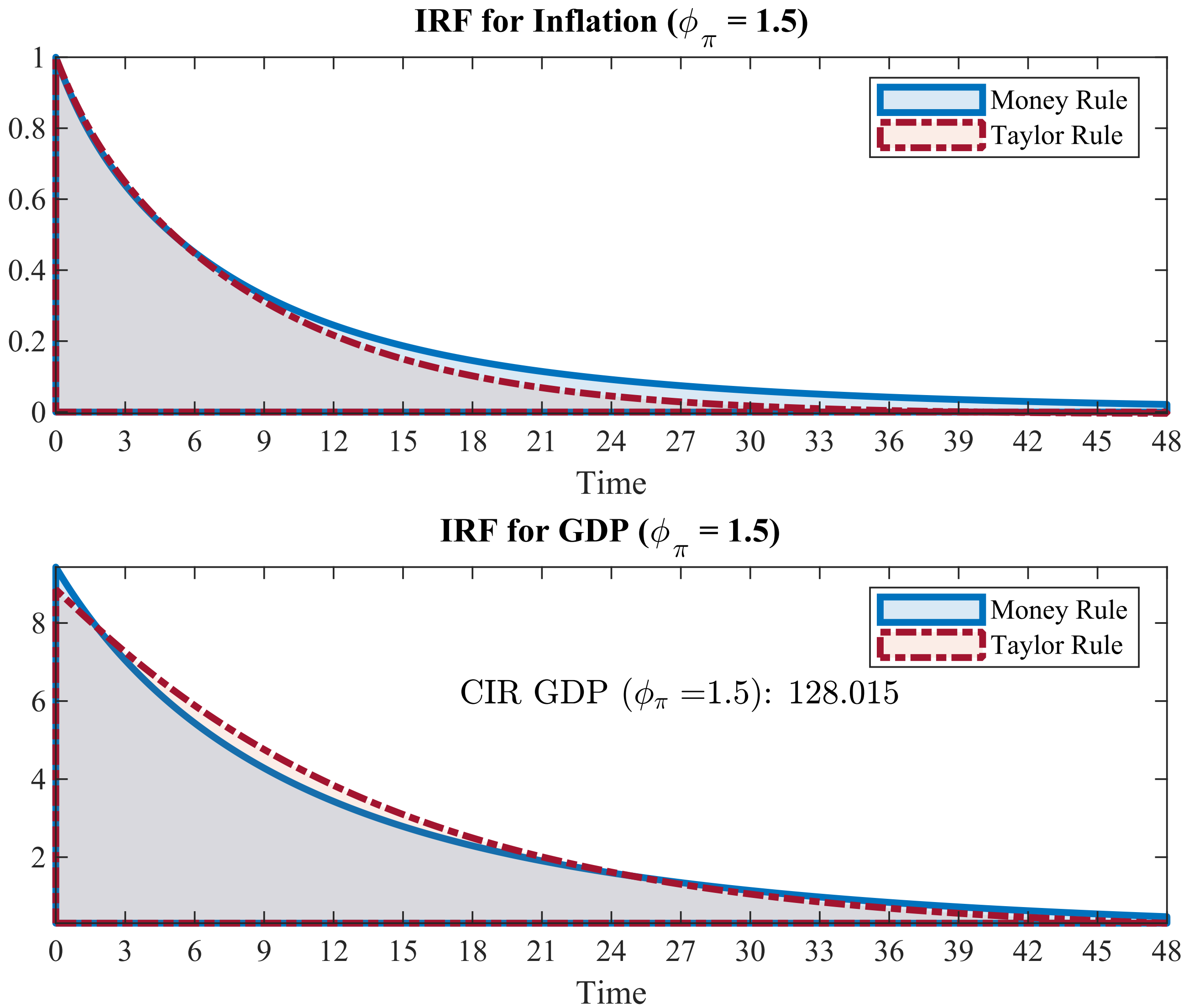
Industry	Inflation Half-Life
Insurance agencies, brokerages, and related act...	33.6
Coating, engraving, heat treating and allied ac...	33.1
Semiconductor machinery manufacturing	29.8
Warehousing and storage	29.0
Packaging machinery manufacturing	25.2
Flavoring syrup and concentrate manufacturing	25.1
Showcase, partition, shelving, and locker manuf...	24.3
Toilet preparation manufacturing	23.7
Turned product and screw, nut, and bolt manufac...	23.7
Breakfast cereal manufacturing	23.0
Other engine equipment manufacturing	22.5
Other industrial machinery manufacturing	22.4
Miscellaneous nonmetallic mineral products	22.2
Fluid power process machinery	21.9
All other miscellaneous manufacturing	21.6
Cut stone and stone product manufacturing	21.5
Other aircraft parts and auxiliary equipment ma...	21.2
Electricity and signal testing instruments manu...	21.2
Metal crown, closure, and other metal stamping ...	20.9
Machine shops	20.9

Figure E.9: Inflation response to a monetary policy shock



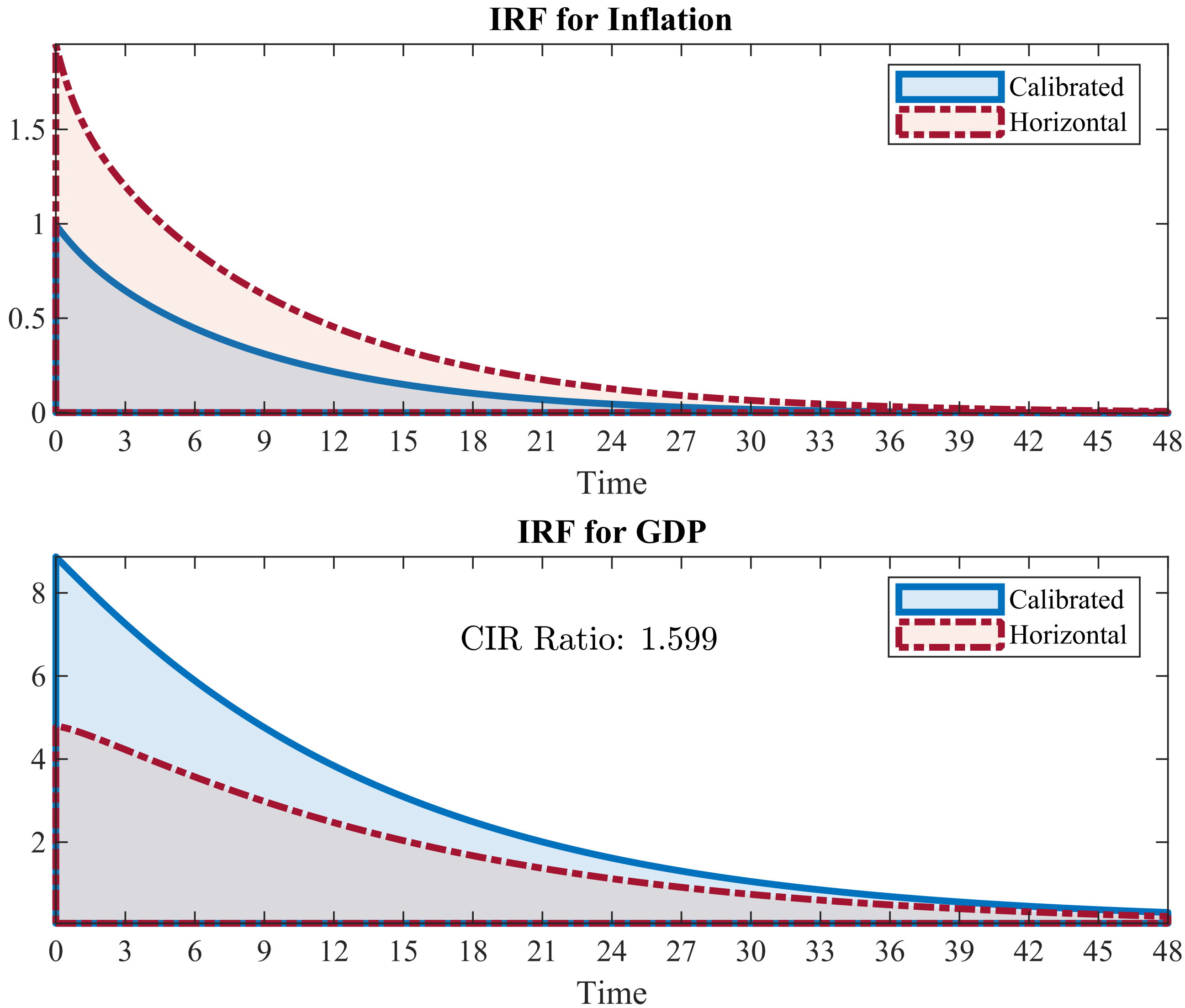
Notes: This figure plots the impulse response functions for aggregate inflation and sectoral inflation to a monetary shock that generates a one percentage increase in aggregate inflation on impact. The calibration of the model is at a monthly frequency. The aggregate inflation response is shown in dashed lines. The calibration uses a (finite) Frisch elasticity of 2.

Figure E.10: Impulse response functions to a monetary policy shock in two economies



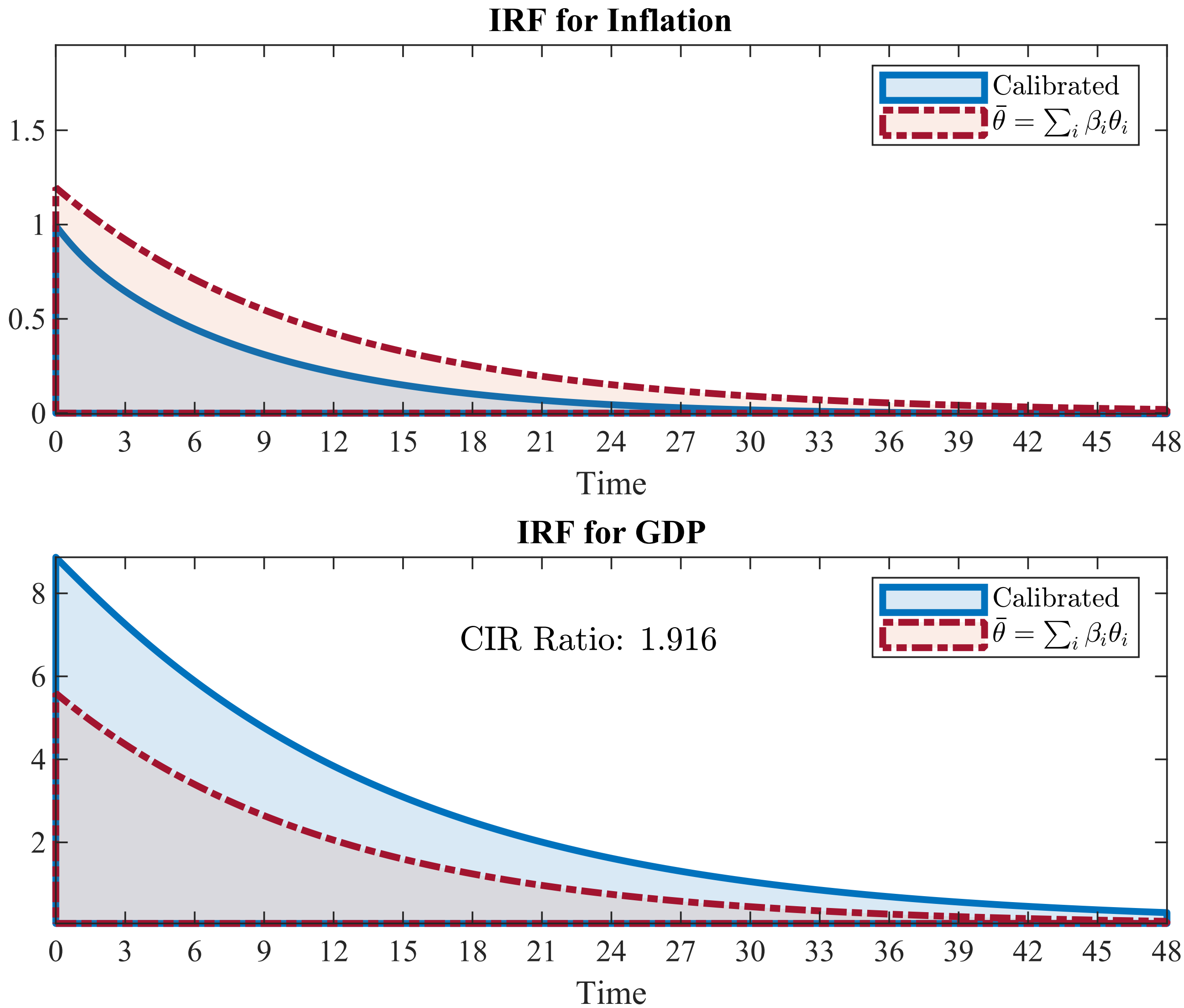
Notes: The figure compares the impulse responses for inflation and GDP to a monetary shock in the nominal GDP rule economy and the Taylor rule economy. The initial shock size and the persistence of the shock in the Taylor rule economy is calibrated to match: 1) aggregate inflation response as one percentage on impact; 2) half-life of aggregate inflation the same as in the nominal GDP rule economy. The calibration of the model is at a monthly frequency. CIR denotes the cumulative impulse response. The calibration fixes the feedback paramter on the Taylor rule to $\phi_\pi = 1.5$.

Figure E.11: Impulse response functions to a monetary policy shock in two economies



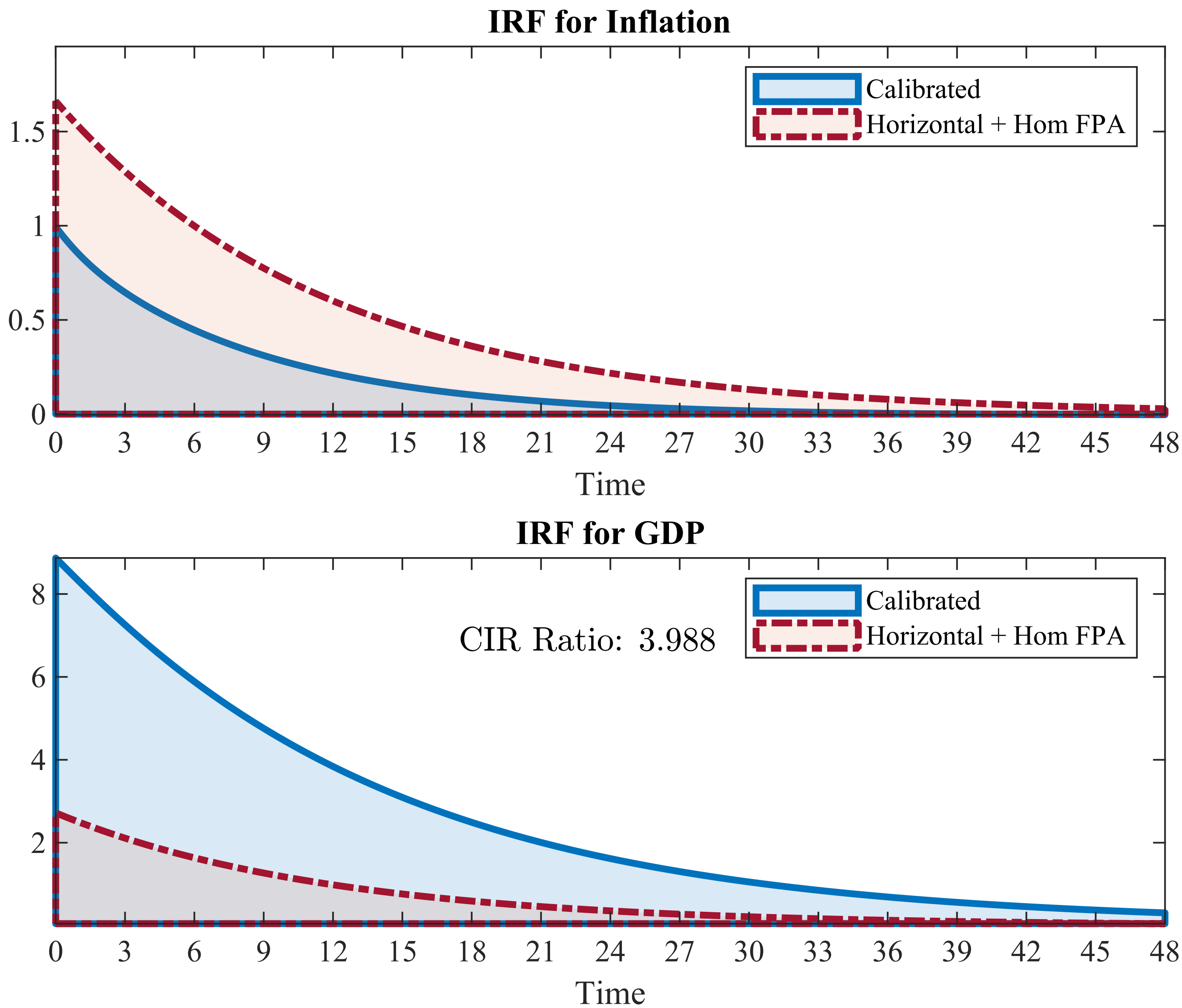
Notes: This figure plots the impulse response functions for inflation and GDP to a monetary shock. It compares our baseline Taylor rule economy that has production networks with an economy that has a horizonal production structure where only labor is used as an input for production. The calibration of the model is at a monthly frequency. CIR denotes the cumulative impulse response. The calibration fixes the feedback paramter on the Taylor rule to $\phi_\pi = 1.5$. The monetary shock size and persistence is the same across the two economies.

Figure E.12: Impulse response functions to a monetary policy shock in two economies



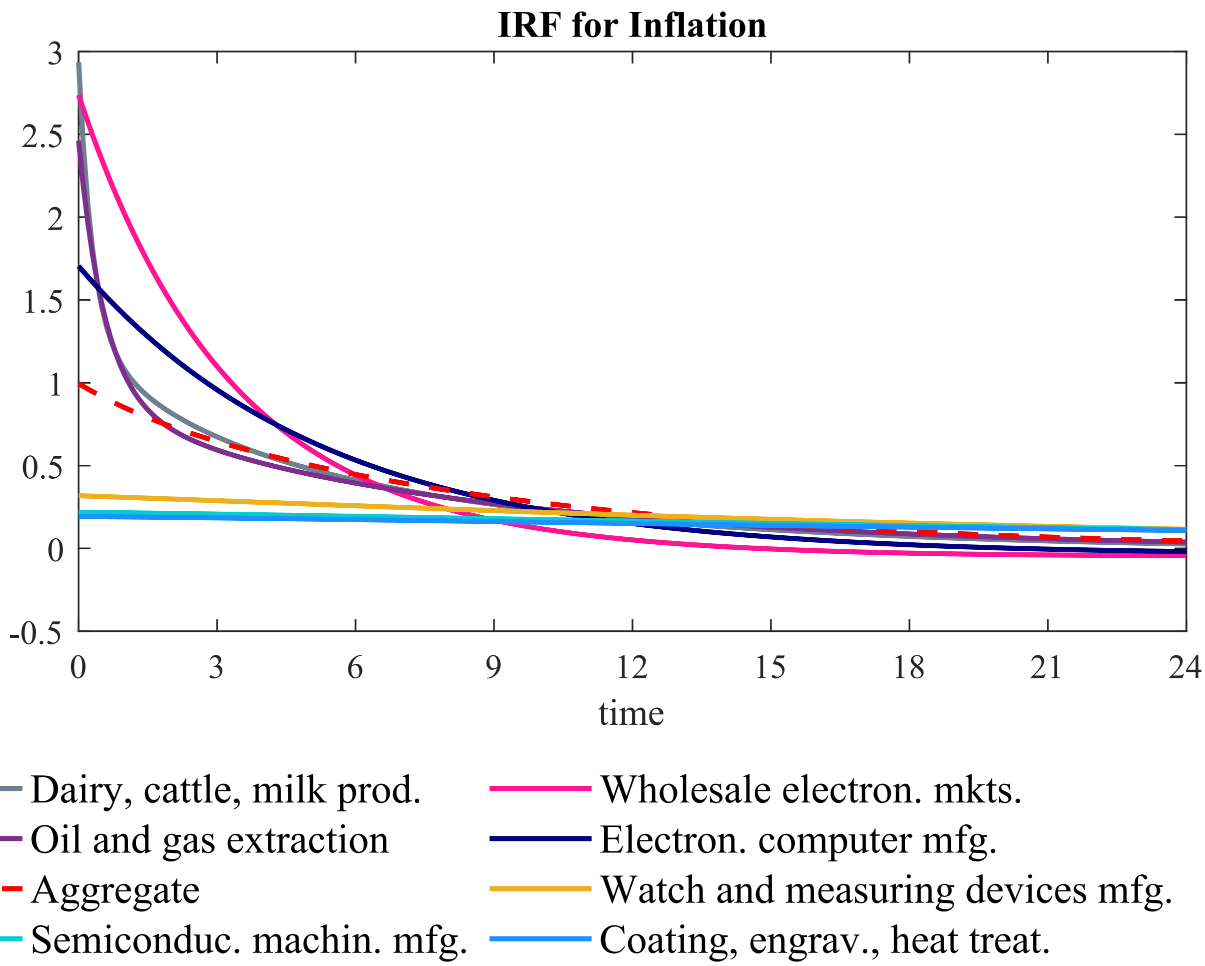
Notes: This figure plots the impulse response functions for inflation and GDP to a monetary shock. It compares our baseline Taylor rule economy that has heterogeneous price stickiness across sectors with an economy that has homogeneous price stickiness across sectors. The homogeneous price adjustment frequency is calibrated to be the weighted average of the price adjustment frequencies across sectors. The calibration of the model is at a monthly frequency. CIR denotes the cumulative impulse response.The calibration fixes the feedback paramter on the Taylor rule to $\phi_\pi = 1.5$. The monetary shock size and persistence is the same across the two economies.

Figure E.13: Impulse response functions to a monetary policy shock in two economies



Notes: This figure plots the impulse response functions for inflation and GDP to a monetary shock that generates a one percentage increase in inflation on impact. It compares our baseline economy that has production networks and heterogeneous price stickiness across sectors with an economy that has both a horizonal production structure where only labor is used as an input for production as well as homogeneous price stickiness across sectors. The homogeneous price adjustment frequency is calibrated to be the weighted average of the price adjustment frequencies across sectors. The calibration of the model is at a monthly frequency. CIR denotes the cumulative impulse response.The calibration fixes the feedback paramter on the Taylor rule to $\phi_\pi = 1.5$. The monetary shock size and persistence is the same across the two economies.

Figure E.14: Inflation response to a monetary policy shock



Notes: This figure plots the impulse response functions for aggrgate inflation and sectoral inflation to a monetary shock that generates a one percentage increase in aggregate inflation on impact. The calibration of the model is at a monthly frequency. The aggregate inflation response is shown in dashed lines. The calibration fixes the feedback paramter on the Taylor rule to $\phi_\pi = 1.5$.