Strategic Inattention, Inflation Dynamics, and the Non-Neutrality of Money*

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Abstract

This paper studies how competition affects firms' expectations in a new dynamic general equilibrium model with *rational inattention* and *oligopolistic competition* where firms acquire information about their competitors' beliefs. In the model, firms with fewer competitors are less attentive to aggregate variables—a novel prediction supported by survey evidence. A calibrated version of the model matches the relationship between a firm's number of competitors and its uncertainty about aggregate inflation as a *non-targeted moment*. A quantitative exercise reveals that firms' strategic inattention to aggregates significantly amplifies monetary non-neutrality and shifts output response disproportionately towards less competitive oligopolies by distorting relative prices.

JEL Codes: E31; E32; E71
Key Words: rational inattention, inflation expectations, oligopolistic competition, inflation dynamics, monetary non-neutrality.

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1 Introduction

Almost every modern monetary model relates aggregate price changes to firms' expectations about aggregate inflation.[1] However, recent literature documents that firms' inflation expectations are inaccurate and disconnected from aggregate inflation (Candia, Coibion, and Gorodnichenko, 2021).[2] Furthermore, the accuracy of firms' expectations about aggregate variables correlates with the number of their competitors (Coibion, Gorodnichenko, and Kumar, 2018). These facts are inconsistent with our standard models and raise the following two questions: (1) How does competition affect firms' expectations? (2) What are the macroeconomic implications of the interaction between competition and expectation formation?

This papers develops a new dynamic general equilibrium model with *rational inattention* and *oligopolistic competition* to study these questions. The interaction of these two model ingredients generates an endogenous relationship between the number of firms' competitors and their expectations about aggregate variables. While both rational inattention and oligopolistic competition are necessary for this relationship—hereafter, referred to as *strategic inattention*—neither one is sufficient on its own. To examine the quantitative fit of the model, I calibrate it to firm-level survey data and find that it matches the relationship between firms' beliefs and the number of their competitors as a *non-targeted* moment. Finally, I find that strategic inattention has quantitatively significant implications for output and inflation responses to monetary policy shocks. It amplifies monetary non-neutrality by up to 77% and shifts the output response disproportionately towards less competitive firms.

The basic model of this paper in Section 2 provides a closed-form characterization of oligopolistic firms' optimal beliefs under rational inattention. Rationally inattentive firms make mistakes in perceiving fundamental shocks. Thus, with a *finite* number of competitors, the average price of a firm's competitors exhibits non-fundamental volatility, which is costly to the firms themselves and to their competitors through strategic complementarities in pricing. Accordingly, when information acquisition is endogenous, oligopolistic firms are *strategically inattentive*: they have an incentive to pay direct attention to the mistakes of their competitors, even at the expense of paying less attention to the fundamental shocks. The model connects higher strategic inattention motives to (1) fewer competitors *and* (2) higher degrees of strategic complementarities in pricing. Thus, the model predicts an endogenous relationship between competition and firms' beliefs about aggregate variables: firms with fewer competitors are less informed about aggregate variables and have more uncertain beliefs.

[1] In New Keynesian models, inflation is increasing in expected aggregate inflation in the future (Woodford, 2003b). In models of information rigidity, it is increasing in past expectation of current inflation (Lucas, 1972; Mankiw and Reis, 2002; Reis, 2006).

[2] See, also, Kumar, Afrouzi, Coibion, and Gorodnichenko (2015) who document that managers in New Zealand make average errors of 2 to 3 percentage points in perceiving current as well as forecasting future inflation.

Strategic inattention also sheds light on why firms' price changes are disconnected from their inflation expectations. Firms that compete with only a few others do not optimize over their prices relative to an aggregate price index but rather relative to the prices of their direct competitors, a feature that is reflected in their beliefs under rational inattention. As firms pay direct attention to the beliefs of their competitors, prices are, on average, closer to firms' expectations of their competitors' prices than to the aggregate price. Accordingly, expectations about aggregates are no longer the relevant index for firms' pricing decisions. Instead, as shown analytically in the static model and later in the context of the closed-form Phillips curve in the dynamic model, a more appropriate index for aggregate prices is firms' aggregated expectations of their own competitors' prices. Importantly, strategic inattention creates a wedge between the relevant expectations for prices and aggregate inflation expectations.

Direct motivating evidence from firm-level survey data in Section 3 supports the presence of strategic inattention among firms. First, to assess whether the conditions required by the basic model hold in the data, a novel question is included in a survey of New Zealand firms that measures significant strategic complementarities in pricing. Furthermore, when asked how many direct competitors they face in their main product market, firms report an average of 8 competitors, with 45% percent of them reporting a number less than 6. Second, as predicted by the model, firms with fewer competitors are more uncertain about aggregate prices.[3] Third, firms are more aware of their own industry prices than aggregate prices, which is also consistent with the model's prediction that firms should pay direct attention to the beliefs of their competitors.

To study the quantitative implications of strategic inattention, Section 4 extends the basic static model of the paper to a dynamic general equilibrium model that endogenizes strategic complementarities and the total amount of information processed by firms. In the model, oligopolistic competition arises from the representative household's preferences over different varieties. On the firm side, this demand structure generates many small oligopolies with heterogeneity in the number of firms operating within them. Firms are rationally inattentive and acquire information about their competitors' beliefs and fundamental shocks over time. On the methodological front, the model requires solving for the equilibrium strategy of a dynamic rational inattention game within every oligopoly, which, to the best of my knowledge, is novel to this paper. I solve for these equilibrium strategies by extending recent methods for solving single-agent dynamic rational inattention models.[4]

To validate the model, I calibrate it to the firm-level survey data and find that the model

[3] Coibion, Gorodnichenko, and Kumar (2018) document a similar result for the size of forecast and nowcast errors. The model in this paper provides support for this prediction, but also delivers a precise prediction in terms of the *variance* of beliefs which is tested in Section 3.

[4] In particular, I use the method developed by Afrouzi and Yang (2019) which builds on and generalizes the first-order condition methods developed in Maćkowiak, Matějka, and Wiederholt (2018).

matches firms' strategic inattention to inflation—i.e., the relationship between firms' beliefs about aggregate inflation and the number of their competitors—as *non-targeted* moments. In the calibrated model, firms in more competitive oligopolies acquire more information in total and allocate a larger amount of attention towards aggregate shocks. This prediction of the calibrated model matches the empirical evidence that firms with a larger number of competitors are more informed about aggregate variables (Coibion, Gorodnichenko, and Kumar, 2018, as well as my own analysis in Section 3).

The remainder of the paper in Section 5 studies the *aggregate* and *reallocative* implications of strategic inattention for the propagation of monetary policy shocks to output and inflation. Since firms in less competitive oligopolies acquire less information about the aggregate shocks, the response of their prices to these shocks are smaller and more persistent, both of which amplify monetary non-neutrality. In a set of counterfactual exercises, I find that strategic inattention has quantitatively significant *aggregate* effects: it increases the volatility of output due to monetary shocks by up to 77% and increases the half-life of output by up to 30% (1 quarter). Moreover, it lowers the volatility of inflation caused by monetary shocks by up to 13% and increases its half-life by up to 17% (2 months). The fact that inflation responds more persistently to shocks among firms with fewer competitors is consistent with evidence documented by Schoenle (2018).

In addition to affecting the response of aggregate prices and output, strategic inattention also distorts the response of *relative* prices and *concentrates* output response towards oligopolies with fewer firms. Since such oligopolies are more strategically inattentive, their prices respond more sluggishly to monetary shocks, attracting demand towards more concentrated oligopolies. Thus, more oligopolistic firms contribute *more* to the output response of the economy relative to their steady-state market share. To examine these effects, I define the *concentration multiplier* of oligopolies with K competitors as the ratio of the cumulative response of outputs in those oligopolies relative to the aggregate output response. These multipliers are defined such that they are equal to one in a model without heterogeneity in output response. However, with the heterogeneity caused by strategic inattention, more concentrated oligopolies drive a higher share of the output response. For instance, the concentration multiplier of duopolies in the calibrated model is 1.26; i.e., their cumulative output response is 26% larger than the average cumulative output response in the model.

The final step in Section 5 is a conceptual decomposition that inspects the mechanisms that are at work in the quantitative model. It is well-known that real rigidities significantly amplify monetary non-neutrality (Woodford, 2003a). Since strategic complementarities in the dynamic model are endogenous to the environment of firms and vary with competition, it may as well be that all the quantitative results are driven by differences in real rigidities across oligopolies than by strategic inattention. But this is not the case. In fact, real rigidities work against strategic inattention in the calibrated model because firms with more competitors have higher strategic complementarities.

Therefore, the net effects of oligopolistic competition on monetary non-neutrality result from two opposing forces. On the one hand, firms with fewer competitors pay less attention to monetary policy shocks due to strategic incentives, a force that *amplifies* monetary non-neutrality through larger information frictions (strategic inattention mechanism). On the other hand, firms in oligopolies with fewer competitors have lower degrees of strategic complementarities in the calibrated model, a force that increases the weight of these firms' higher-order beliefs on their prices and *attenuates* the degree of monetary non-neutrality (real rigidities mechanism). A decomposition of the net effects of these two forces shows that while they are both quantitatively significant, strategic inattention dominates, and the resultant effect is such that monetary non-neutrality is amplified when the number of competitors within oligopolies are smaller. Therefore, the effect of strategic inattention is not only independent of the conventional real rigidities channel in amplifying monetary non-neutrality, but it is also strong enough to overcome the latter even when the two work in opposite directions.

Related Literature. This paper is motivated by the recent literature that investigates how firms' expectations are related to their environment. The most related work in this area is Coibion, Gorodnichenko, and Kumar (2018) which provides direct evidence on the relationship between firms' number of competitors and their expectations. To the best of my knowledge, the model in this paper is the first to provide an explanation for this relationship and to investigate its implications. Most notably, in the model, inflation responds more persistently to shocks among firms with fewer competitors. Schoenle (2018) documents a similar relationship in the U.S. PPI data and provides evidence for this mechanism.

The model proposed in this paper is mainly related to the vast literature on rational inattention (Sims, 1998, 2003) and, especially, its applications to pricing models and business cycles dynamics (most notably, Maćkowiak and Wiederholt, 2009, 2015; Matějka, 2015).[5] The previous work in this literature has mainly focused on monopolistic competition models. The main contribution of this paper is to study the consequences of rational inattention in *oligopolistic* competition models, which is essential to the main objective of this study that aims to understand the effects of competition on firms' expectations, and, through that, on inflation dynamics and monetary non-neutrality.

The oligopolistic structure of competition studied here is related to the literature that has

[5] See, also, Pasten and Schoenle (2016); Stevens (2020); Yang (2019) for recent discussions and Mackowiak, Matějka, and Wiederholt (2021) for a detailed review of this literature. More broadly, the paper is also related to the literature on the effects of information rigidities and monetary policy (e.g., Lucas, 1972; Mankiw and Reis, 2002; Woodford, 2003b; Reis, 2006; Nimark, 2008; Angeletos and La'O, 2009; Angeletos and Lian, 2016; Melosi, 2016; Baley and Blanco, 2019).

focused on its macroeconomic implications (Rotemberg and Saloner, 1986; Rotemberg and Woodford, 1992; Atkeson and Burstein, 2008). While this paper's main focus is to understand the interaction of oligopolistic competition with rational inattention, the implications of the model for monetary non-neutrality complement concurrent work by Mongey (2021) and Wang and Werning (2021), which focus on the interactions of nominal rigidities with oligopolistic competition. These three models provide a unified view of how competition affects output and inflation dynamics, but under different mechanisms.[6] In particular, the mechanism of interest here is strategic inattention, which affects aggregate dynamics through firms' *expectations* in a micro-founded model with endogenous information acquisition.

The model's implications for inflation dynamics and monetary non-neutrality is also of particular interest given the recent evidence on the rise of concentration (see, for instance, Autor, Dorn, Katz, Patterson, and Van Reenen, 2020; Covarrubias, Gutiérrez, and Philippon, 2020; Kwon, Ma, and Zimmermann, 2021). My results suggest that these trends are also changing the landscape of monetary policy by affecting the propagation of these shocks to real and nominal variables.

Finally, this paper is also related to the literature that formalizes the incentives to learn about others' beliefs in strategic environments (Hellwig and Veldkamp, 2009; Myatt and Wallace, 2012), which focus on a continuum of agents and impose certain restrictions on the set of available information. I depart from this literature by focusing on an unrestricted set of available information to unmask how the number of players affects information acquisition incentives in a dynamic general equilibrium model. The main finding that relates to this body of work is that players' beliefs under rational inattention models—where the cost of information is defined in terms of Shannon's mutual information function—exhibit correlated non-fundamental volatility as long as the number of players is finite. In that sense, the static model of this paper is related to Denti (2020), which studies unrestricted information acquisition with a finite set of actions and states but under more general preferences. In more recent work, Hébert and La'O (2021) study large static games under a much broader class of information cost functions and, among other results, characterize what type of cost functions would generate non-fundamental volatility even with an infinite number of players.

2 A Static Model with Analytical Solution

This section studies how expectations are affected by oligopolistic competition in a static model with an analytical solution. The model predicts that oligopolistic firms *directly pay attention*

[6] Studying monetary non-neutrality with monopolistic competition under each of these frictions has a long history. For information friction models, see Lucas (1972); Woodford (2003a). For random price adjustments in New Keynesian models, see Woodford (2003b)'s review of that literature. For price adjustment under menu costs see, for instance, Caplin and Spulber (1987); Golosov and Lucas (2007); Nakamura and Steinsson (2010).

to the beliefs of their competitors. Consequently, equilibrium beliefs are correlated beyond what is implied by fundamental shocks. This *excess* correlation depends on two key parameters: the number of a firm's competitors and the degree of strategic complementarities in pricing. Moreover, excess correlation in beliefs disappears in the monopolistic competition limit.

The rest of the section discusses these results in detail. While I focus on the economics of the forces at work in the main text, Appendix C provides a rigorous treatment of the model. In particular, proofs of main propositions are included in Appendix C.7.

2.1 The Environment

There are a large number of sectors in the economy indexed by $j \in \{1, \dots, J\}$. There are K firms in every sector, where K represents the number of firms in an industry that *directly* compete with one another.[7] Firms are price-setters, and their profits are affected by a normally distributed fundamental shock that I denote by $q \sim \mathcal{N}(0,1)$. For any realization of the fundamental q and a set of prices chosen by firms $(p_{l,m})_{(l,m)\in J\times K}$, firm j,k experiences the following quadratic losses in profits by selecting the price $p_{j,k}$:

$$L_{j,k}((q, p_{j,k})_{(j,k)\in J\times K}) = (p_{j,k} - (1-\alpha)q - \alpha\frac{1}{K-1}\sum_{l\neq k} p_{j,l})^2,$$

where $\alpha \in [0,1)$ denotes the degree of *within* sector strategic complementarity.[8] In Section 4, I show that minimizing this loss is equivalent to maximizing a second-order approximation of a properly defined profit function and discuss how α depends on the certain properties of demand.

2.2 Firms' Problem

Firms are rationally inattentive and acquire information subject to a finite attention capacity. Firms also choose a pricing strategy that maps their information set to a price.

To unmask firms' incentives in information acquisition, I model the information choice set such that arbitrarily precise information about shocks and the beliefs of others are available—i.e., firms can directly choose the joint distribution of their price with q and others' prices. While this is a well-known feature of single-agent rational inattention problems, the feasibility of such strategies is not obvious in game-theoretic settings. For instance, suppose a firm considers a deviation in which it acquires information about the beliefs of other firms beyond what is revealed by the fundamental q. Since prices are realized simultaneously across firms *after* information is acquired, how can firms pay attention to each other's beliefs before actions are

[7] When asked how many direct competitors they face in their main product market, firms in New Zealand report an average of 8 (See Figure 1).

[8] Here the fundamental q, and prices, $(p_{j,k})_{j\in J,k\in K}$, can be interpreted as log-deviations from a steady-state symmetric equilibrium, which allows us to normalize their mean to zero.

realized? What type of a choice set makes such information acquisition strategies possible? More importantly, does such a choice set exist, and if so, what does it look like?

Appendix C.1 formalizes the answer to these questions. It starts with a definition of a *rich* choice set of available signals as a set that allows for any arbitrary deviation among Gaussian distributions.[9] It continues to show the existence of such a set by constructing one as the vector space generated by the fundamental q and a set of countably infinite independent normal random variables (Lemma C.1). Henceforth, I refer to this vector space as $\mathscr{S}$. Finally, Corollary C.2 concludes Appendix C.1 by showing that any deviation in joint Gaussian distributions can be generated by a random variable in this vector space.

Therefore, a pure strategy for firm j,k is to choose a set of signals from the information choice set, $S_{j,k} \subseteq \mathscr{S}$, and a pricing strategy that is measurable with respect to the σ-algebra generated by its signals $S_{j,k}$, denoted by $p_{j,k} : S_{j,k} \to \mathbb{R}$. Given a strategy profile for others, $(S_{l,m} \subseteq \mathscr{S})_{(l,m)\neq(j,k)}$, firm j,k's problem is

$$\min_{S_{j,k}\subseteq\mathscr{S}} \mathbb{E}\left[\min_{p_{j,k}:S_{j,k}\to\mathbb{R}} \mathbb{E}\left[\left(p_{j,k}(S_{j,k}) - (1-\alpha)q - \alpha\frac{1}{K-1}\sum_{l\neq k} p_{j,l}(S_{j,l})\right)^2 |S_{j,k}\right]\right] \tag{1}$$
$$s.t. \quad \mathscr{I}(S_{j,k};(q,p_{l,m}(S_{l,m}))_{(l,m)\neq(j,k)}) \leq \kappa$$

Here $\mathscr{I}(S_{j,k};(q,p_{l,m}(S_{l,m}))_{(l,m)\neq(j,k)})$ is Shannon's mutual information function and measures the amount of information that the firm's signal reveals about the fundamental shock and the prices of other firms.[10] This constraint requires that a firm cannot acquire more than κ bits of information, which is exogenous in this section but endogenous in Section 4. We are now ready to define the equilibrium.

Definition 1. *A pure strategy Gaussian equilibrium for this economy is a strategy profile* $(S_{j,k} \subseteq \mathscr{S}, p_{j,k} : S_{j,k} \to \mathbb{R})_{(j,k)\in J\times K}$ *from which no firm has an incentive to deviate—i.e.,* $\forall (j,k) \in J \times K$, $(S_{j,k}, p_{j,k})$ *solves* j,k*'s problem as stated in Equation (1), given* $(S_{l,m}, p_{l,m})_{(l,m)\neq(j,k)}$.

It can be shown that all equilibria strategies are among a specific type of strategies, in which every firm observes only one signal. This is connected to a well-known result in single-agent rational inattention problems, which states that the actions of rationally inattentive decision-makers are sufficient statistics for the underlying signals that generate those actions.[11] In this

[9] My definition of a rich information set corresponds to the concept of flexibility in information acquisition in Denti (2018).

[10] $\mathscr{I}(.;.)$ is Shannon's mutual information function. In this paper, I focus on Gaussian random variables, in which case $\mathscr{I}(X;Y) = \frac{1}{2}\log_2(\det(var(X))) - \frac{1}{2}\log_2(\det(var(X|Y)))$. The Gaussian nature of the information structure is self-consistent in the equilibrium. When a firm's opponents choose Gaussian signals, it is also optimal to choose a Gaussian signal under the quadratic loss. See, e.g. Maćkowiak and Wiederholt (2009) or Afrouzi and Yang (2019), for optimality of Gaussian signals under quadratic objectives with Gaussian fundamentals.

[11] See, e.g., Steiner, Stewart, and Matějka (2017); Maćkowiak, Matějka, and Wiederholt (2018); Afrouzi and Yang (2019) for proofs of similar lemmas in single decision making problems.

game-theoretic setting, denote strategies that satisfy this property as *recommendation strategies*—more specifically, strategies in which the signal recommends the optimal price generated by its σ-algebra: $(S_{j,k} \in \mathscr{S}, p_{j,k} = S_{j,k})$. We can then show that any strategy for any firm is at least weakly dominated by a recommendation strategy, the proof of which requires the following two results. (1) *Feasibility of recommendation strategies*: since $\mathscr{S}$ contains only Gaussian signals, we need to show that the optimal prices generated by any strategy profile are also in $\mathscr{S}$ (Lemma C.3 in Appendix C.2). (2) *Optimality of recommendation strategies*: Lemma C.5 in Appendix C.3 proves that for any arbitrary strategy, there exists a recommendation strategy that generates a weakly higher payoff for the firm. Thus, without loss of generality, we can assume that firms always choose to observe only one signal that is equal to their price in the equilibrium.

It then follows that all equilibria are unique in the joint distribution that they imply for firms' prices and q, which is done in Appendix C.5. The optimality of recommendation strategies combined with the uniqueness of the equilibrium in the joint distribution of prices and q allows us to directly focus on how firms' prices are related to one another. Let $p_{j,k} = S_{j,k}$ be the price that firm j,k charges in the equilibrium. Appendix C.4 characterizes these prices as:

$$p_{j,k} = \lambda\left((1-\alpha)q + \alpha\frac{1}{K-1}\sum_{l\neq k} p_{j,l}\right) + z_{j,k}, \quad z_{j,k} \perp (q, S_{m,l})_{(m,l)\neq(j,k)}$$

$$\mathbb{E}[z_{j,k}] = 0, \quad \mathbb{V}\mathrm{ar}(z_{j,k}) = \lambda(1-\lambda)\mathbb{V}\mathrm{ar}\left((1-\alpha)q + \alpha\frac{1}{K-1}\sum_{l\neq k} p_{j,l}\right)$$

Where $\lambda \equiv 1-2^{-2\kappa}$ and $z_{j,k}$ is noise in prices introduced by rational inattention. Two observations immediately follow: larger capacity, κ, increases the covariance of prices with the fundamental and decreases the variance of the rational inattention noise. In particular, when $\kappa \to \infty$, λ approaches one, the noise disappears and $p_{j,k} = q, \forall j,k$.

The rest of this section unpacks the properties of this solution and studies its economic implications for given levels of κ, α, and K.

2.3 Economics of Attention Allocation

Rationally inattentive firms make mistakes in observing the fundamental q—captured by the positive variance of $z_{j,k}$ in the equation above—which contaminate their prices and affect the profits of their competitors. While the previous section mathematically characterized the unique equilibrium, this section recasts the attention problem of the firms to show that mistakes of oligopolistic firms are correlated in the equilibrium. Moreover, this correlation increases with the degree of strategic complementarity α and decreases with the number of competitors $K \geq 2$, vanishing to zero as $K \to \infty$.[12]

[12] In more recent work, Hébert and La'O (2021) study correlated beliefs in large games (corresponding to $K \to \infty$) but for a broader class of payoff and information cost functions.

I start by precisely defining what I mean by *mistakes*: a mistake is a part of a firm's price that is unpredictable by the fundamentals of the economy (i.e., are independent of q). Formally, any firm's price can be decomposed into its projection on the fundamental and a part that is orthogonal to it:

$$p_{j,k} = \delta q + v_{j,k},\ v_{j,k} \perp q,\ \delta \in \mathbb{R}.$$

The vector $(v_{j,k})_{j,k \in J\times K}$, therefore, contains the *mistakes* of all firms in pricing, with their joint distribution endogenously determined in the equilibrium.

It is important to note that these mistakes are not necessarily independent across firms. In fact, by endogenizing the information choices of firms, one of the objectives here is to understand how the mistakes of different firms relate to one another in the equilibrium, or intuitively how much managers of competing firms *attend* to the mistakes of their rivals and incorporate them in their own prices. Moreover, the coefficient δ, which determines the degree to which prices covary with the fundamental of the economy, is also an equilibrium object. Our goal is to understand how δ and the joint distribution of mistakes rely on the underlying parameters of the model: α, K, and κ. But, first, we need to define what we mean by attention precisely.

Definition 2. *The* amount of attention *that a firm pays to a random variable is the mutual information between their set of signals and that random variable. Moreover, we say a firm* knows more about X than Y *if it pays more attention to X than Y.*

It is straightforward to verify that a firm's attention to a shock increases with the absolute correlation of the firm's price with that shock.[13] Building on this notion, Appendix C.6 shows that when others play a strategy in which $\frac{1}{K-1}\sum_{l\neq k} p_{j,l} = \delta q + v_{j,-k}$, the attention problem of firm j,k can be recast into choosing two separate degrees of correlation:

$$\begin{aligned} &\max_{\rho_q \geq 0, \rho_v \geq 0}\ \rho_q + \frac{\alpha\sigma_v}{1-\alpha(1-\delta)}\rho_v \quad , \\ s.t. \qquad & \rho_q^2 + \rho_v^2 \leq \lambda \equiv 1 - 2^{-2\kappa} \end{aligned}$$

Here $\sigma_v \equiv \sqrt{\mathbb{V}\text{ar}(v_{j,-k})}$ is the standard deviation of the average mistakes of j,k's competitors, ρ_q is the correlation of the firm's signal with the fundamental, and ρ_v is its correlation with the average mistake of its competitors. The following proposition states the properties of the equilibrium. Its proof in Appendix C.7 also derives closed-form solutions for these correlations.

Proposition 1. In equilibrium,

1. Firms pay strictly positive attention to the mistakes of their competitors ($\rho_v^* > 0$) if $\alpha > 0$ and K is finite.

[13] Formally, for two normal random variables X and Y, let $\mathscr{I}(X,Y)$ denote Shannon's mutual information between the two. Then $\mathscr{I}(X,Y) = -\frac{1}{2}\log_2(1-\rho_{X,Y}^2)$ where $\rho_{X,Y}$ is the correlation between X and Y. Notice that $\mathscr{I}(X,Y)$ is increasing in $\rho_{X,Y}^2$.

2. Firms' knowledge of the fundamental increases in the number of their competitors and decreases in the degree of strategic complementarity:

$$\frac{\partial}{\partial K}\rho_q^* > 0,\ \frac{\partial}{\partial \alpha}\rho_q^* < 0.$$

3. Firms do not pay attention to mistakes of those in other industries: $\forall (j,k),(l,m)$, if $j \neq l$, $p_{j,k} \perp p_{l,m} | q$.

The first part of Proposition 1 follows from the fact that firms are affected by the mistakes of their competitors and find it optimal to pay strictly positive attention to them. However, since mistakes are orthogonal to the fundamental, any attention to others' mistakes has to be traded off with attention to the fundamental. This leads to the second part of Proposition 1: firms with a smaller degree of strategic complementarity α or a larger number of competitors K are less affected by others mistakes and pay more attention to the fundamental.

The degree of strategic complementarity, α, is the parameter that relates the payoff of a firm to the mistakes of its competitors. The larger is α, the more the profits of a firm depends on the mistakes of its competitors. Accordingly, the firm finds it more in their interest to track those mistakes. This result illustrates the importance of micro-founding these strategic complementarities, which is one of the main objectives of the model in Section 4.

Moreover, the effect of K is captured by the presence of σ_v in the objective of the firms. Intuitively, firms are only affected by the average of their rivals' mistakes, which gets smaller as the number of a firm's competitors increases and goes to zero as $K \to \infty$ (which is an equilibrium outcome).[14] Therefore, the larger the number of a firm's competitors, the more their mistakes "wash out." This allows a more competitive firm to substitute away from paying attention to others' mistakes and pay more attention to the fundamental shock.

Finally, Equation (C.5) in Appendix C.5 shows that, in the equilibrium, the covariance of aggregate price and the fundamental shock is given by

$$\delta = \frac{\lambda - \alpha\lambda}{1 - \alpha\lambda}$$

Therefore, the degree to which prices covary with the fundamental q increases with the capacity of processing information, κ, and decreases with the degree of strategic complementarity, α. Moreover, δ is independent of the number of firms in the static model, but this is not a robust feature of a more general model and goes away in the dynamic setting of Section 4, where beliefs are dynamic and strategic complementarities are micro-founded.

[14] σ_v is determined by the endogenous choices of firms. To see how this quantity declines with the number of competitors in the equilibrium, notice that if $K \to \infty$, a firm has no incentive to pay attention to others' mistakes if they are independent as the law of large numbers would imply $\sigma_v = 0$. Since incentives are symmetric, in the equilibrium, all firms prefer to have independent mistakes, implying that $\sigma_v = 0$.

2.4 Equilibrium Prices and Expectations

Conventional models relate firms' prices to their expectations of aggregate prices (inflation in dynamic models). However, empirical evidence on firms' expectations about aggregate inflation suggests that this link is not present in the data, and there is a disconnect between firms' prices and their expectations of aggregate inflation (Coibion, Gorodnichenko, and Kumar, 2018). The static model in this section, however, predicts a different relationship between prices and expectations. Here, firms' prices are related to their expectations of their *competitors' prices*, and the aggregate price is related to an average of those expectations:

$$p = (1-\alpha)\overline{\mathbb{E}^{j,k}[q]} + \alpha\overline{\mathbb{E}^{j,k}[p_{j,-k}]}.$$

The key notion here is that the average of expectations is not the same as the expectation of the average; more precisely, firms' average expectations of their competitors' prices are not the same as their expectations of average prices in the economy. While the conventional models predict these two objects are the same, the following proposition shows that rational inattention with oligopolistic competition creates a wedge between prices and aggregate expectations of firms about the aggregate price.

Proposition 2. In equilibrium, the aggregate price co-moves more with the average expectations from own-industry prices than average expectations of the aggregate price itself. Formally,

$$cov(p, \overline{\mathbb{E}^{j,k}[p_{j,-k}]}) > cov(p, \overline{\mathbb{E}^{j,k}[p]}).$$

Moreover, the two converge to each other as $K \to \infty$.

Therefore, what matters for the determination of the aggregate price is what firms know about the prices of their competitors, not what they know about the aggregate price. It follows as a corollary that the realized price is also closer to the average own-industry price expectations than the average expectation of the aggregate price.

Corollary 3. *In equilibrium, the realized price is closer in absolute value to the average expectations from own-industry prices than the average expectation of the aggregate price itself.*

$$|p - \overline{\mathbb{E}^{j,k}[p_{j,-k}]}| < |p - \overline{\mathbb{E}^{j,k}[p]}|$$

The intuition behind these results relies on firms' incentives in paying attention to the mistakes of their competitors. In equilibrium, the signals that firms observe are more informative of their own sector's prices than the aggregate economy:

$$S_{j,k} = \underbrace{\overbrace{p}^{\text{covaries with aggregate price}} + u_j}_{\text{covaries with industry prices}} + e_{j,k}$$

Here, we have decomposed the mistake of firm j,k as $v_{j,k} = u_j + e_{j,k}$, where $u_j \perp p$ is the common mistake in sector j and $e_{j,k}$ is the independent part of firm j,k's mistake. The fact that $\mathbb{V}\text{ar}(u_j) \neq 0$, by Proposition 1, implies that the firm is more informed in predicting its own-industry price changes than the aggregate price, and the two would become the same only if there was no coordination within industries in information acquisition, which happens when $K \to \infty$. This result, along with its counterpart in the dynamic model, shows how stable inflation can be an equilibrium outcome even when firms' expectations of that inflation are ill-informed.

3 Motivating Facts from Survey Data

The last section showed that two parameters are key for the formation of firms' expectations: the number of direct competitors and the degree of strategic complementarity. How important are these mechanisms? The goal of this section is to answer this question in two steps: first, I provide measurements of these two parameters and, second, I test the main predictions of the simple model in the data. To do so, I use a quantitative survey of firms' expectations from New Zealand, which is comprehensively discussed in Kumar, Afrouzi, Coibion, and Gorodnichenko (2015) and Coibion, Gorodnichenko, and Kumar (2018), to assess the predictions of the model in the previous section. The survey was conducted in multiple waves among a random sample of firms in New Zealand with broad sectoral coverage.

The new empirical contributions in this paper relative to the previous work that has used this data is that I (1) implement and utilize a new question in the survey to back-out the degree of strategic complementarity for firms, and (2) document that firms with more competitors have more certain posteriors about the aggregate inflation.

3.1 Number of Competitors and Strategic Complementarity

The bsaic model in the previous section links the strength of strategic inattention to two key parameters: the number of a firm's direct competitors and the degree of micro-level strategic complementarities. Both of these objects are hard to measure. While it is straightforward to measure the number of firms within a particular industry, it is not clear which subsets of those firms *directly* compete with one another because product similarity is only a necessary element of competition but not sufficient. Similarly, measuring the degree of strategic complementarity is challenging due to endogenous responses of firms to common shocks: strategic complementarity is defined as the elasticity of a firm's price to the average price of its competitors. But finding exogenous movements in firms' prices that are not responses to some common shock is cumbersome.[15] Data from surveys can circumvent these challenges by explicitly asking firms

[15] Recently, there has been some further progress in this area: Amiti, Itskhoki, and Konings (2019) use international shocks as instruments for shocks that only move competitors' prices and provide estimates of strategic complementarities for Belgian manufacturing firms.

about these objects. In particular, two questions in the survey from New Zealand firms directly measure these within a representative sample.

The first question asks firms *"How many direct competitors does this firm face in its main product line?"* Figure 1 shows the distribution of firms' responses to this question. Columns (1) and (2) in Table 1 show that the average firm in the sample faces only eight competitors, with 45% of firms reporting that they face six or fewer competitors. A breakdown of firms' answers from different industries shows that this average is fairly uniform across them.

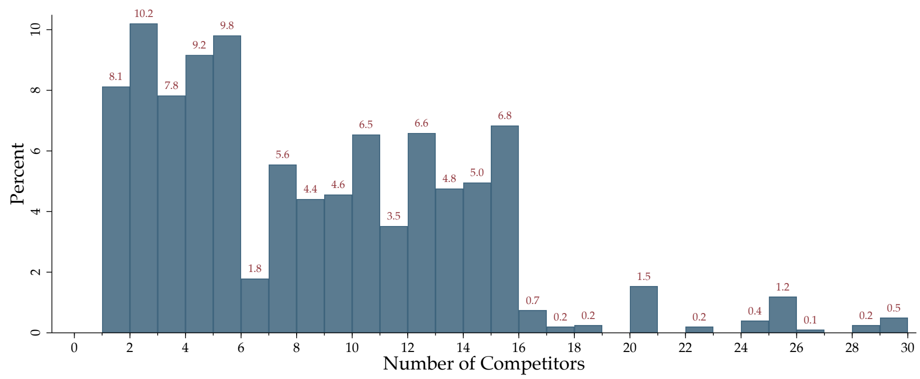

Figure 1: Distribution of the Number of Competitors

Notes: the figure presents the distribution of the number of competitors that firms report they face in their direct product market in the survey data from New Zealand. The numbers over bars denote the percentage of firms within the corresponding bin. Firms with more than 30 competitors are dropped (only less than 1 percent of firms report they have more than 30 competitors, with a max of 42).

We also implemented a question in the survey to measure the degree of micro-level strategic complementarity across firms. This has been a challenging parameter to estimate in the literature due to major endogeneity concerns: it is rarely possible to find exogenous variations in the prices of a firms' competitors that are not correlated with aggregates or the firm's own costs. To bypass this issue, I rely on the following hypothetical question to measure the degree of strategic complementarity:

"Suppose that you get news that the general level of prices went up by 10% in the economy:

(a) By what percentage do you think your competitors would raise their prices on average?

(b) By what percentage would your firm raise its price on average?

(c) By what percentage would your firm raise its price if your competitors did not change their price at all in response to this news?"

Table 1: Number of Competitors and Degree of Strategic Complementarity

	Number of Competitors				*Strategic Complementarity*			
	(1)		*(2)*		*(3)*		*(4)*	
Constant	8.449	(0.113)	8.367	(0.198)	0.817	(0.008)	0.795	(0.015)
Manufacturing			-	-			-	-
Construction			-1.285	(0.425)			0.063	(0.032)
Trade			0.183	(0.334)			0.010	(0.026)
Services			0.319	(0.287)			0.031	(0.021)
Observations	3072		2667		2824		2445	

Standard errors in parentheses

Notes: the table presents statistics for the number of competitors and the degree of strategic complementarity in the survey data from New Zealand. Columns (1) and (2) report the average number of competitors that firms report they face in their main product market. Columns (3) and (4) show the coefficient for the degree of strategic complementarity from Equation (2).

The question proposes a change in the firms' environment that is coming through aggregate variables, which affects both their costs and those of their competitors. The question then measures three different quantities that allow me to disentangle the degree of strategic complementarity:

$$p_{j,k} = \overbrace{\underbrace{(1-\alpha)\mathbb{E}^{j,k}[q]}_{\text{Answer to c.}} + \underbrace{\alpha\mathbb{E}^{j,k}[p_{j,-k}]}_{\text{Answer to a.}}}^{\text{Answer to b.}}. \tag{2}$$

The average α implied by the responses of firms to this question is 0.82 and uniform across different industries, as reported in columns (3) and (4) of Table 1.[16] More recently, Coibion, Gorodnichenko, Kumar, and Ryngaert (2021) follow my approach here and use this method to estimate strategic complementarities.

3.2 Uncertainty about Inflation versus Number of Competitors

A novel prediction of the model is that firms with more competitors should be more aware of aggregate prices. We can directly test this prediction using the following question in the survey.

In the sixth wave of the survey, conducted in 2016, firms were asked to report the distribution of their beliefs about both aggregate and their industry inflation through the following two similarly worded questions: *"Please assign probabilities (from 0-100) to the following ranges of overall price changes in [the economy] [your industry] over the next 12 months for New Zealand."* For both questions, firms were provided with an identical set of bins to which they assigned their subjective probabilities.[17]

[16]For reference, the usual calibration for the strategic complementarity in the U.S. in monopolistic competition models is around 0.9 (see, e.g., Mankiw and Reis, 2002; Woodford, 2003b) which is slightly larger than what I estimate here for micro-level complementarities.

[17]These were two separate questions in the survey that I have combined here in order to avoid repetition. The

Proposition 1 predicts that knowledge about the aggregate price should be increasing in the number of a firm's competitors. This is a unique testable prediction of the oligopolistic rational inattention model. To test this prediction, I run the following regression:

$$\log(\sigma_i^{\pi}) = \beta_0 + \beta_1 \log(K_i) + \epsilon_i, \tag{3}$$

where σ_i^{π} is firm i's subjective uncertainty about the aggregate inflation—defined as the standard deviation of their self-reported distribution for inflation—and K_i is the number of competitors that they report in their main product market. The model's prediction translates to the null hypothesis that $\beta_1 < 0$. Table 2 reports the result of this regression and shows that this is indeed the case. This result is also robust to including firm controls such as firms' age and size (measured by employment) as well as industry fixed effects.

Table 2: Subjective Uncertainty of Firms and the Number of Competitors.

	$\log(\sigma_i^{\pi})$	
	(1)	*(2)*
log(#competitors)	-0.116	-0.113
	(0.012)	(0.013)
Firm controls and FEs	No	Yes
Observations	1,662	1,552

Robust standard errors in parentheses

Notes: the table reports the result of regressing the log standard deviation of firms' reported distribution for their forecast of aggregate inflation on the log of their number of competitors as well as a set of firm controls (age, size measured by employment, and fixed effects for construction, manufacturing, financial services and trade industries).

The significance of this coefficient in explaining firms' uncertainty about aggregates is an observation that is not reconcilable with full information rational expectation models or, to the best of my knowledge, any other macroeconomic model of information rigidity prior to this paper, and indicates the importance of strategic incentives in how much firms pay attention to aggregate variables in the economy.

3.3 Knowledge about Industry versus Aggregate Inflation

Another main prediction of the model is that firms are more aware of their competitors' price changes than the aggregate price.

In the fourth wave of the survey, conducted in the last quarter of 2014, firms were asked to provide their nowcasts of both industry and aggregate yearly inflation.

assigned bins varied from -25 percent to 25 percent with 5 percent increments. The wide range is provided because firms are highly uncertain about inflation and assign positive probabilities to high inflation rates. The large negative magnitudes were also provided to avoid priming concerns.

Table 3 reports the size of firms' nowcast errors in perceiving these two inflation rates.[18] The average absolute nowcast error across firms about their own industry inflation is 1.2 percentage points, a magnitude that is considerably lower than the average absolute nowcast error about aggregate inflation, 3.1 percentage points.

Furthermore, Figure 2 shows that in addition to this striking difference in the averages, the distributions of these nowcast errors are skewed in opposite directions: for nearly two-thirds of firms, their nowcast error of the aggregate inflation is larger than the mean error, while the reverse is true in the case of industry inflation.

Table 3: Size of Firms' Nowcast Errors

	Observations	*Industry inflation*		*Aggregate inflation*	
		mean	*std*	*mean*	*std*
Industry	*(1)*	*(2)*	*(3)*	*(4)*	*(5)*
Construction	52	0.75	0.54	3.95	1.95
Manufacturing	363	1.43	1.72	2.55	2.04
Financial Services	352	1.51	1.51	4.23	1.73
Trade	302	0.63	0.90	2.31	1.93
Total	1,069	1.20	1.49	3.11	2.09

Notes: the table reports the size of firms' nowcast errors in perceiving aggregate inflation versus industry inflation for the 12 months ending in December 2014.

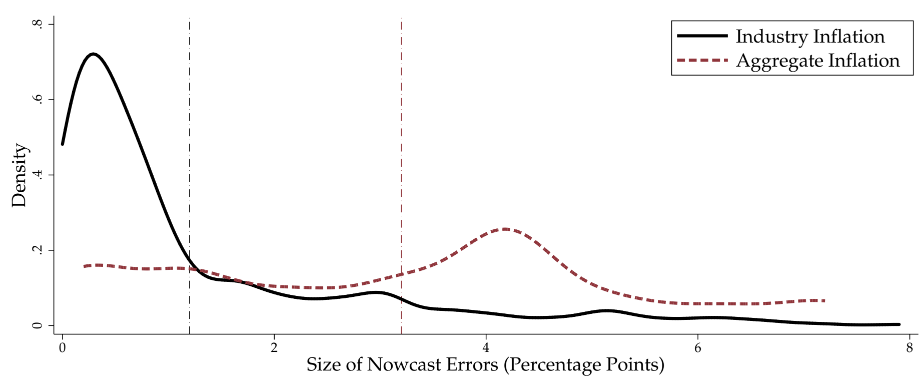

Figure 2: Distributions of the Size of Firms' Nowcast Errors

Notes: the figure presents the distribution of the size of firms' errors in perceiving the aggregate and their industry inflation in New Zealand. The dashed vertical lines denote the means of these distributions.

4 A Micro-founded Dynamic Model

This section aims to extend the simple static model of Section 2 to a dynamic general equilibrium model to quantitatively analyze the effects of firms' strategic incentives in information

[18]Nowcast errors for industry inflation are measured as the distance between firms' nowcast and the realized inflation in their industry.

acquisition for the propagation of monetary policy shocks to aggregate output and inflation.

In particular, the model in this section improves on the static model by (1) micro-founding the loss function and micro-level strategic complementarities as a function of a representative household's demand for different varieties of goods (2) endogenizing the choice of information processing capacity on the part of firms, and (3) considering the dynamic incentives of firms in information acquisition in addition to their strategic incentives (dynamically inattentive firms realize that information has a continuation value and choose their information accordingly).

All the derivations and proofs for the propositions in this section are in Appendix G.

4.1 Environment

Households. The economy consists of a large number of sectors, $j \in J \equiv \{1,\ldots,J\}$; and each sector j consists of $K_j \geq 2$ firms that produce weakly substitutable goods. Here, K_j is drawn from an exogenous distribution $\mathcal{K}$ (that I will, later on, calibrate to the Distribution of the number of competitors in the data). The representative household takes the nominal prices of these goods as given and forms a demand over each firm's product in the economy. In particular, the aggregate time t consumption of the household is

$$C_t \equiv \prod_{j\in J} C_{j,t}^{J^{-1}}, \quad C_{j,t} \equiv \left(K_j^{-1} \sum_{k\in K_j} C_{j,k,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{4}$$

where $C_{j,t}$ is the composite demand of the household for the goods produced in sector j and is determined by a CES aggregation of within sector goods with an elasticity of substitution $\eta > 1$.[19] Equation (4) denotes that the aggregate consumption of the household is Cobb-Douglas in the composite goods of sectors. Therefore, the representative household's problem is

$$\max_{((C_{j,k,t})_{(j,k)\in J\times K}, C_t, L_t, B_t)_{t=0}^{\infty}} \quad \mathbb{E}_0^f \sum_{t=0}^{\infty} \beta^t [\log(C_t) - L_t] \tag{5}$$
$$s.t. \quad \sum_{j,k} P_{j,k,t} C_{j,k,t} + B_t \leq W_t L_t + (1+i_{t-1})B_{t-1} + \sum_{j,k} \Pi_{j,k,t} - T$$

where $\mathbb{E}_t^f[.]$ is the full information rational expectations operator at time t,[20] C_t is the aggregate consumption bundle defined in Equation (4), L_t is the labor supply of the household, B_t is their demand for nominal bonds, W_t is the nominal wage, i_t is the net nominal interest rate, $\Pi_{j,k,t}$ denotes the profit of firm j,k at time t, and T is a constant lump sum tax that is used by the government to finance a hiring subsidy for firms in order to eliminate any long-run inefficiencies

[19] A more general aggregator can be considered here—see, e.g., Rotemberg and Woodford (1992). I derive the implied demand under a general form for this aggregator function in Appendix E. Another specific case is the Kimball aggregator, which I discuss in Appendix F.

[20] Since the main purpose of this paper is to study the effects of rational inattention under imperfect competition among firms, I assume that households are fully informed about prices and wages. This is a common assumption in the literature—see, e.g., Melosi (2016)—and simplifies the household side of the economy as a natural first step in separating the implications of rational inattention for households versus firms.

of imperfect competition.

The CES aggregator within sector goods leads to the following demand function for the product of firm j,k:

$$C_{j,k,t} = Q_t \mathscr{D}(P_{j,k,t}; P_{j,-k,t}), \quad \mathscr{D}(P_{j,k,t}; P_{j,-k,t}) \equiv J^{-1} \frac{P_{j,k,t}^{-\eta}}{\sum_{l \in K_j} P_{j,l,t}^{1-\eta}} \tag{6}$$

where $Q_t \equiv P_t C_t$ is the nominal aggregate demand (P_t is the price of the aggregate consumption bundle C_t), $P_{j,k,t}$ is firm j,k's price at t, and $P_{j,-k,t}$ is the vector of other firms' prices in sector j. Furthermore, the household's intertemporal Euler and labor supply equations are given by:

$$W_t = Q_t, \quad 1 = \beta(1+i_t)\mathbb{E}_t^f [\frac{Q_t}{Q_{t+1}}]$$

where the linear disutility of labor implies that nominal wage is proportional to the nominal demand.[21]

Firms. Firms are rationally inattentive. At each period t, firms take their information set from the previous period as given and choose an arbitrary number of signals from a rich set of *available signals*, $\mathscr{S}^t$, subject to an information processing constraint.[22] It is important to note that this choice happens *before* these signals are realized and, similar to conventional rational inattention models, depends on the joint distribution of these signals with the payoff relevant variables of firms.

In contrast to the static model where I assumed the capacity of processing information was exogenous, here I assume firms can choose this capacity where the cost of every one bit of information is $\omega \texttt{rs}_j$ units of labor. Here, $\omega > 0$ is the parameter that governs the cost of information, and $\texttt{rs}_j = (JK_j)^{-1}$ is the revenue share (or relative size) of the firm in the *full-information* symmetric equilibrium. The assumption that the cost of information is proportional to the firms' relative size in the full-information benchmark is made for three reasons. First, it makes the analysis consistent with the empirical evidence, since all the regressions presented in this paper about strategic inattention and the references to the literature control for firms' relative size. Second, from a theoretical perspective, it makes firms' rational inattention problems size-independent so that as we take the monopolistic competition limit, information acquisition does not become infinitely costly for firms (I will revisit this in more detail later when I derive a second-order approximation to the firms' problem). Finally, in the absence of this assumption, information would be relatively more costly for smaller firms to acquire, which is inconsistent

[21]The linear disutility in labor is a common assumption in the models of monetary non-neutrality—see, for instance, Golosov and Lucas (2007), which eliminates the source of *across* sector strategic complementarity from the household side. I use this assumption to the same end in order to mainly focus on micro-founding *within* sector strategic complementarities.

[22]See Appendix D for the formal specification of $\mathscr{S}^t$.

with the evidence on how firm size correlates with attention—if anything, larger firms are *more* inattentive to aggregate variables (Coibion, Gorodnichenko, and Kumar, 2018; Candia, Coibion, and Gorodnichenko, 2021).[23]

Once firms make their information choice, all new shocks and signals are realized, and each firm observes the realization of their signal. Firms then choose their prices conditional on this information set.[24] After setting their prices, demand for each variety is realized. Firms then hire labor from the competitive labor market to produce with a production function that has decreasing returns in labor; $Y_{j,k,t} = (L^p_{j,k,t})^{\frac{1}{1+\gamma}}$ and meet their demand. Here, $\gamma = 0$ corresponds to constant returns to scale, and positive γ captures the degree of decreasing returns to scale in labor.

Formally, a strategy for firm j,k is to choose a capacity for processing information conditional on their initial information set at any time, $\kappa_{j,k,t} : S^{t-1}_{j,k} \to \mathbb{R}_+$, a set of signals to observe, $S_{j,k,t} \subset \mathscr{S}^t$, and a pricing strategy that maps its information set to their optimal actions, $P_{j,k,t} : S^t_{j,k} \to \mathbb{R}$, where $S^t_{j,k} = \{S_{j,k,\tau}\}^t_{\tau=0}$ is the firm's information set at time t. Firms then hire enough labor for their good production to satisfy demand.[25] Given a strategy for all the other firms in the economy, firm j,k's problem is to maximize the net present value of their lifetime profits given the initial information set that they inherit from the previous period:

$$\max_{\{S_{j,k,t} \subset \mathscr{S}^t, P_{j,k,t}(S^t_{j,k}), \kappa_{j,k,t}(S^{t-1}_{j,k})\}_{t \geq 0}} \tag{7}$$

$$\mathbb{E}[\sum_{t=0}^{\infty} \underbrace{\beta^t Q_t^{-1}}_{\text{discount factor}} (\underbrace{P_{j,k,t} Y^d_{j,k,t}}_{\text{revenue}} - \underbrace{(1-\bar{\mathtt{s}}_j) W_t (Y^d_{j,k,t})^{1+\gamma}}_{\text{production cost}} - \underbrace{(1-\bar{\mathtt{s}}_j) W_t \times \omega \mathtt{rs}_j \times \kappa_{j,k,t}}_{\text{cost of attention}}) | S^{-1}_{j,k}]$$

$$s.t. \quad Y^d_{j,k,t} = Q_t \mathscr{D}(P_{j,k,t}; P_{j,-k,t}) \tag{demand}$$

$$\mathscr{I}(S_{j,k,t}, (Q_\tau, P_{l,m,\tau}(S^\tau_{l,m}))^{(l,m)\neq(j,k)}_{0\leq\tau\leq t} | S^{t-1}_{j,k}) \leq \kappa_{j,k,t} \tag{information processing constraint}$$

$$S^t_{j,k} = S^{t-1}_{j,k} \cup S_{j,k,t}, \quad S^{-1}_{j,k} \text{ given.} \tag{evolution of the information set}$$

where the information processing constraint requires that the amount of information that a firm can add to its information set about the state of the economy at a given time is bounded above by the capacity $\kappa_{j,k,t}$. The function $\mathscr{I}(.,.)$ is Shannon's mutual information function, which measures the reduction in conditional entropy experienced by the firm across two consecutive

[23]Similar assumptions have been made in the menu cost literature. For instance, Gertler and Leahy (2008) assume that menu costs are proportional to firms' size so that pricing decisions are size-invariant.

[24]Since my main objective is to examine the real effects of monetary policy through endogenous information acquisition of these firms, I abstract away from other sources of monetary non-neutrality, and in particular, assume that prices are perfectly flexible.

[25]This is a ubiquitous assumption in the literature with sticky prices, which rules out temporary shutdowns of production by firms due to negative profits induced by suboptimal prices. Formally, this assumption requires that supply *has* to be equal to demand. See e.g. Woodford (2003b); Golosov and Lucas (2007).

periods.[26] Moreover, $\bar{s}_j$ denotes a constant hiring subsidy to firms in sector j that eliminates the steady-state inefficiencies from imperfect competition and implements the optimal level of output in that steady-state (see Galí, 2015, p. 73).

Monetary Policy and General Equilibrium. Following the literature, I assume that monetary policy determines the growth of nominal aggregate demand and model it as an AR(1) process with persistence ρ:[27]

$$\Delta \log(Q_t) = \rho \Delta \log(Q_{t-1}) + u_t. \tag{8}$$

Equilibrium. A general equilibrium for the economy is an allocation for the household, $\Omega^H \equiv \{(C_{j,k,t})_{j\in J,k\in K_j}, L_t^s, B_t\}_{t=0}^{\infty}$, a strategy profile for firms given an initial set of signals

$$\Omega^F \equiv \{(S_{j,k,t} \subset \mathscr{S}^t, P_{j,k,t}, \kappa_{j,k,t}, L^p_{j,k,t}, Y^d_{j,k,t})_{t=0}^{\infty}\}_{j\in J,k\in K_j} \cup \{S^{-1}_{j,k}\}_{j\in J,k\in K_j},$$

and a set of prices $\{i_t, P_t, W_t\}_{t=0}^{\infty}$ such that (a) given prices and Ω^F, the household's allocation solves their problem in Equation (5); (b) given prices and Ω^H, no firm has an incentive to deviate from Ω^F; (c) given prices, Ω^F and Ω^H, $\{Q_t \equiv P_t C_t\}_{t=0}^{\infty}$ satisfies the monetary policy rule in Equation (8); (d) labor and goods markets clear.

4.2 Sources of Strategic Complementarity

Strategic complementarities in pricing are at the core of this paper's focus on understanding how firms allocate their attention across aggregate variables and the beliefs of their competitors. Therefore, a brief discussion of the sources of strategic complementarities in the model is necessary.

There are two sources of strategic complementarity in the model: (1) decreasing returns to scale in labor ($\gamma > 0$) and (2) sensitivity of optimal markups to relative prices. Complementarities due to decreasing returns to scale are not specific to oligopolistic environments and are commonly used in monetary models. They exist because firms' marginal costs are sensitive to their levels of production—which in turn depends on their relative prices. I assume decreasing returns to scale mainly for calibration purposes.[28]

However, the sensitivity of optimal markups to relative prices under CES demand is only a source for strategic complementarity when firms are oligopolistic. Contrary to models of monopolistic competition where a constant elasticity of substitution across varieties implies constant markups, an oligopolistic environment relates markups to firms' relative prices. This is because the granularity of firms in an oligopoly implies that any change in a firm's price

[26] See Appendix B for the formal specification of Shannon's mutual information function.

[27] See, for instance, Mankiw and Reis (2002); Woodford (2003a); Golosov and Lucas (2007); Nakamura and Steinsson (2010).

[28] See Woodford (2003b) or Galí (2015) for discussions and applications of this channel in generating strategic complementarities.

influences the distribution of demand across its competitors. Accordingly, demand elasticities for firms within an oligopoly depend on the relative prices of all those firms and are no longer constant. A look at the best response of a firm to a particular realization of $P_{j,-k,t}$ and Q_t manifests this relationship:

$$P^*_{j,k,t} = \underbrace{\mu(P^*_{j,k,t}, P_{j,-k,t})}_{\text{optimal markup}} \times \underbrace{(1-\bar{s}_j)(1+\gamma)Q_t^{1+\gamma}\mathscr{D}(P^*_{j,k,t}; P_{j,-k,t})^{\gamma}}_{\text{marginal cost}} \tag{9}$$

where $P^*_{j,k,t}$ is the implied optimal price given Q_t and the vector of others prices $P_{j,-k,t}$ and the optimal markup has the familiar expression in terms of the elasticity of a firm's demand:

$$\mu(P^*_{j,k,t}, P_{j,-k,t}) \equiv \frac{\varepsilon_D(P^*_{j,k,t}, P_{j,-k,t})}{\varepsilon_D(P^*_{j,k,t}, P_{j,-k,t}) - 1}, \quad \varepsilon_D(P_{j,k,t}, P_{j,-k,t}) \equiv -\frac{\partial Y_{j,k,t}}{\partial P_{j,k,t}} \frac{P_{j,k,t}}{Y_{j,k,t}}$$

Here, $\varepsilon_D(P_{j,k,t}, P_{j,-k,t})$ is firm j,k's elasticity of demand with respect to its own price. Following Atkeson and Burstein (2008), it is informative to write these elasticities in terms of a firm's market share within its own sector:

$$\varepsilon_D(P_{j,k,t}, P_{j,-k,t}) = \eta - (\eta-1)m_{j,k,t}, \quad m_{j,k,t} \equiv \frac{P_{j,k,t}Y^d_{j,k,t}}{\sum_{l\in K_j} P_{j,l,t}Y^d_{j,l,t}} \tag{10}$$

An immediate observation is that the *level* of optimal markups increase in a firm's market share and converges to the monopolistic competition markup when this market share goes to zero:

$$\mu(P^*_{j,k,t}, P_{j,-k,t}) = \frac{\eta}{\eta-1} + \frac{1}{\eta-1}\frac{m_{j,k,t}}{1-m_{j,k,t}} \tag{11}$$

Moreover, given the definition of market shares, one can derive the degree of strategic complementarity for a given set of prices by differentiating the firm's best response. For the special case when there are constant returns to scale ($\gamma = 0$) so that sensitivity of markups is the only source of complementarity, this adopts an explicit representation in terms of the market shares:

$$\frac{dP^*_{j,k,t}}{P^*_{j,k,t}}\Big|_{\gamma=0} = \frac{dQ_t}{Q_t} + \underbrace{(1-\eta^{-1})m_{j,k,t}}_{\text{strategic complementarity}} \left(\underbrace{\frac{\sum_{l\neq k} m_{j,l,t}dP_{j,l,t}/P_{j,l,t}}{\sum_{l\neq k} m_{j,l,t}}}_{\text{average price-change of others}} - \underbrace{\frac{dQ_t}{Q_t}}_{\text{change in wage}} \right) \tag{12}$$

An important observation is that strategic complementarity $\alpha^{\gamma=0}_{j,k,t} \equiv (1-\eta^{-1})m_{j,k,t}$ *increases* in the firm's own market share and *decreases* in the total market share of their competitors. This might seem unintuitive at first glance: after all, why should a firm's price be *more* sensitive to the prices of their competitors when those competitors hold *lower* market share? This becomes more puzzling in an extreme case when a single firm holds almost all the market with its market share approaching 1. The expression above implies that such a firm has the maximum strategic complementarity of $1-\eta^{-1}$. But how can that be? Shouldn't a firm that holds almost all the market simply disregard its competitors and act as a monopoly?

The answer relies on the structure of demand implied by CES preferences: these preferences are such that the marginal consumer reduces a higher share of her demand with respect to a one percent change in the prices of a firm's competitors when that firm holds higher market share. Thus, while a monopolistic firm enjoys the sheer lack of competition, the mere existence of small competitors shatters the autonomy of a firm in responding to their marginal costs, especially at higher levels of market share. Therefore, while a monopolistic firm with CES demand would charge a constant markup over its marginal cost, an *almost* monopolistic firm chooses to match the average price change of their competitors with weight $1-\eta^{-1}$.

In the other extreme, when $m_{j,k,t}$ becomes small, the expression above becomes arbitrarily small, and strategic complementarity disappears. This is not consistent with my findings in the empirical section of the paper, where firms with a large number of competitors, and hence potentially lower market share, still report high levels of strategic complementarity. This suggests that the sensitivity of markups is not the sole determinant of complementarities across firms, and other forces might be at work. I capture this in the model by introducing decreasing returns to scale in labor, which is a standard approach in monetary models, especially in the absence of oligopolistic competition (see, e.g., Galí, 2015).

Nonetheless, the intuition outlined for the case of $\gamma = 0$ carries on to the case when $\gamma > 0$. With some tedious algebra in differentiating the best response of the firm with decreasing returns to scale, we can obtain the expression for strategic complementarity as

$$\alpha_{j,k,t}^{\gamma>0} = (1-\eta^{-1})m_{j,k,t} + \left(1-(1-\eta^{-1})m_{j,k,t}\right)\left(1-\frac{1+\gamma}{1+\gamma\eta(1-(1-\eta^{-1})m_{j,k,t})^2}\right) \tag{13}$$

This exposition of the strategic complementarity shows that at high levels of market share, the strategic complementarity is mainly driven by the sensitivity of the markup and gets closer to the strategic complementarity for the case of $\gamma = 0$ as $m_{j,k,t} \to 1$. However, now when $m_{j,k,t}$ becomes small, strategic complementarity remains positive and converges to $\frac{\gamma(\eta-1)}{1+\gamma\eta}$ as a firm's market share goes to zero.

4.3 Solution Method and Incentives in Information Acquisition

An Approximate Problem. I use a second-order approximation to the firms' problem to solve the model. The justification for this approach stems from the issue that in the firms' problem, as stated in the previous section, both the choice and the state variables are joint distributions between prices and the fundamental shocks. This is a known dimensionality curse in decision-making models with rational inattention, which is exacerbated in this model by the game-theoretic nature of firms' decisions: the solution requires solving for an additional fixed point across best response distributions.

Second-order approximations are a common remedy to this problem in the literature.[29] It is a well-known property of rational inattention models that when payoffs are quadratic and priors are Gaussian, optimal distributions are also Gaussian.[30] Since Gaussian distributions are characterized by their first two moments, this approximation reduces the dimensionality of the problem to the squared dimension of the space from which the Gaussian distribution is drawn.[31]

I derive this second-order approximation around the full-information equilibrium of this economy, which can be thought of as the case where information acquisition is free. Since prices are flexible, in the symmetric equilibrium of the full-information economy, all firms within a given sector j have the same market share and charge the same markup μ_j—given by Equation (11)—over their marginal cost, $(1-\bar{\mathtt{s}}_j)Q_t$:

$$P^{\text{full}}_{j,k,t} = \mu_j(1-\bar{\mathtt{s}}_j)Q_t = Q_t, \forall j \in J, k \in K_j, t \geq 0 \tag{14}$$

where the second equality follows from setting $\bar{\mathtt{s}}_j = 1-\mu_j^{-1}$ to eliminate steady-state distortions coming from market power (Galí, 2015).[32] In Appendix E, I derive the implied approximate problem of the firm as

$$\max_{\{\kappa_{j,k,t}, S_{j,k,t}, p_{j,k,t}(S^t_{j,k})\}_{t\geq 0}} -\mathtt{rs}_j\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\left(\underbrace{\frac{1}{2}B_j(p_{j,k,t}(S^t_{j,k})-p^*_{j,k,t})^2}_{\text{loss from mispricing}} + \underbrace{\omega\kappa_{j,k,t}}_{\text{cost of capacity}} \,|S^{-1}_{j,k}\right)\right] \tag{15}$$

$$\begin{aligned} s.t.\quad & p^*_{j,k,t} \equiv (1-\alpha_j)q_t - \alpha_j p_{j,-k,t}(S_{j,-k,t}) && (16)\\ & \mathscr{I}\left(S_{j,k,t}, (q_\tau, p_{l,m,\tau}(S^\tau_{l,m}))^{(l,m)\neq(j,k)}_{0\leq\tau\leq t}\right) \leq \kappa_{j,k,t} \\ & S^t_{j,k} = S^{t-1}_{j,k} \cup S_{j,k,t}, \quad S^{-1}_{j,k} \text{ given.} \end{aligned}$$

where $p_{j,k,t} \equiv \log(P_{j,k,t})$, $p_{j,-k,t} \equiv \frac{1}{K_j-1}\sum_{l\neq k}\log(P_{j,l,t})$ and

$$B_j \equiv \frac{\varepsilon^j_D}{1-\alpha_j} = \frac{\eta+\gamma(\eta-(\eta-1)K_j^{-1})^2}{1+\gamma} \tag{17}$$

captures the curvature of firms' profit functions in sector j around their optimal price.

Notice that firms' losses from mispricing are always proportional to their size (revenue share)—here, captured by $\mathtt{rs}_j$. Thus, by assuming that the marginal cost of attention is also proportional to the revenue share of the firm in the symmetric full-information equilibrium, we can pull out $\mathtt{rs}_j$ from the maximization and the attention problem becomes homogeneous in

[29] See, e.g., Maćkowiak and Wiederholt (2009, 2015); Paciello and Wiederholt (2014); Maćkowiak, Matějka, and Wiederholt (2018).

[30] See, e.g. Sims (2003); Maćkowiak and Wiederholt (2009); Afrouzi and Yang (2019) for proofs of this result in different environments.

[31] See Afrouzi and Yang (2019) for a detailed discussion.

[32] Here, the presence of $\bar{\mathtt{s}}_j$ makes the second order approximation convenient by ensuring that all relative prices are the same in the full-information economy but is not necessary, nor does it alter the economic forces at work.

firms' steady-state relative size, $\texttt{rs}_j$, which multiplies both losses from mispricing and the cost of attention). This allows the model to consistently relate to the empirical evidence on strategic inattention, which controls for firm size.

Moreover, notice from Equation (17) that how this problem depends on oligopolistic demand structure through the demand elasticity and strategic complementarity. Since all the firms in the oligopoly solve the same problem, it is implied that the equilibrium information structure depends on the oligopolistic nature of firms *only* through these two objects. This remark parallels Wang and Werning (2021)'s findings that these objects are sufficient statistics for how prices responds in oligopolies under nominal rigidities for small shocks.[33] My results here show that, up to a second-order approximation, these objects are also sufficient statistics for the *optimal information structure of oligopolistic firms.*

Information Acquisition Incentives. This approximate problem brings out the trade-offs that a firm faces in information acquisition. First, it formulates the profits of a firm as a function of two negative terms. The first term is the firm's losses from mispricing, which captures the fact that under imperfect information, the firm might choose a price that is not optimal under full information. This incorporates all the benefits of information acquisition: more information allows the firm to choose a price that is closer to the optimum, on average. The second term is the cost of producing information processing capacity, which increases with more information acquisition. The cost-benefit analysis between these two determines, first, the optimal capacity that the firm chooses for its information processing and, second, the signals that provide the firm with the best possible information given that capacity.

An important observation is that there is heterogeneity in the relative importance of losses from mispricing and the cost of producing information capacity. Firms with more competitors have more concave profit functions (larger B_j), which motivates them to produce more capacity. This creates a level effect in information acquisition that was absent in the static model where capacity was assumed to be constant. I will revisit this effect in more detail in Section 5.[34]

Furthermore, the problem formulates the losses from mispricing in terms of a quadratic distance from an "ideal" price ($p^*_{j,k,t}$) that is a weighted average between the log of nominal demand and the average price of the firm's competitors. In the expression for this ideal price, the degree of strategic complementarity α_j, as derived in Equation (13) with symmetric market shares $m_{j,k} = K_j^{-1}$, determines the strength of strategic incentives in information acquisition.

[33] Wang and Werning (2021)'s sufficient statistics are in terms of elasticities and super-elasticities of demand. I have derived my approximation in terms of demand elasticity and strategic complementarity, which can be written as a function of the other two. See the section on "General form of α" in Appendix E for a derivation.

[34] There is evidence that supports this level effect. Coibion, Gorodnichenko, and Kumar (2018) document that firms with a higher slope in their profit function around their optimal price have more accurate expectations about inflation.

The symmetric market shares assumption simplifies the solution to the model by allowing us to solve for the equilibrium among symmetric strategies. I discuss this assumption further in Section 6.

Finally, in addition to the strategic incentives discussed in the static model, the dynamic evolution of the information set indicates how firms' information sets become the source of a new *dynamic* trade-off. Since q_t has a persistent process, firms understand that the signals they choose to observe at any given time will not only inform them about their contemporaneous ideal prices but also about their future values.[35] A byproduct of this dynamic environment is that it leads to endogenously *persistent mistakes*: firms' mistakes affect their beliefs and prices persistently and motivate their competitors to pay attention to the time-series of their mistakes over time. Thus, in contrast to the static model where the solution was a joint distribution for prices, in the dynamic model, the solution is an endogenous stochastic process for prices of firms, the persistence of which is endogenously determined by firms' optimal information acquisition strategy.

Broadly, these dynamic incentives have two potential effects on information acquisition. (1) They affect the level of capacity production as a function of the volatility and persistence of "ideal" prices: a more patient firm that faces a persistent process assigns a larger continuation value to the knowledge that contemporaneous signals generate about the future values of said process. (2) Dynamic incentives also affect the allocation of attention between monetary policy shocks and others' history of mistakes. If mistakes are endogenously less persistent than fundamental q_t, then more patient firms will assign a higher continuation value to learning about q and allocate a higher portion of their attention to that.

Solving Firms' Problem. In this section, I briefly discuss the outline of the algorithm for solving the model. A detailed explanation of this algorithm is included in Appendix I.

The model's solution is a joint stochastic process for all firms' prices and the nominal demand that satisfies the equilibrium conditions. In general, the covariance structure of this stochastic process is time-varying depending on the initial information set of firms but eventually converges to a covariance stationary process, which can be interpreted as the information structure that emerges once firms have been acquiring information for a long enough time and is the information structure that I characterize as the solution.[36] I solve for this covariance stationary process using the following iterative procedure. At each iteration, given a guess for the joint

[35] See, e.g., Steiner, Stewart, and Matějka (2017); Maćkowiak, Matějka, and Wiederholt (2018); Miao, Wu, and Young (2020); Afrouzi and Yang (2019) for an extensive discussion of dynamic incentives of a rationally inattentive agent. Moreover, the exposition of dynamic rational inattention problems varies across different applications. For the formulation that is closest to this paper, see Afrouzi and Yang (2019).

[36] This is a common approach in the literature (see, e.g., Maćkowiak and Wiederholt, 2009, 2015, 2020) because transitional dynamics from an arbitrary prior lead to time-varying impulse responses that depend on that initial prior.

stochastic process of prices and nominal demand, I derive the implied strategy for a firm's competitors in a given sector. This strategy then implies a stochastic process for the "ideal" price of a firm specified in Equation (16) and defines the rational inattention problem of that firm.

Strategic inattention implies that at each iteration, the process for a firm's ideal price depends on the history of monetary policy shocks as well as the history of non-fundamental shocks (mistakes) in the prices of a firm's competitors. The state-space of the shocks that a firm desires to learn about is then given by:

$$p_{j,-k,t}(S^t_{j,-k}) = \underbrace{p_{j,-k,t}(S_{j,-k,t})|_q}_{\text{projection on realizations of all } q_{t-\tau}\text{'s}} + \underbrace{v_{j,-k,t}}_{\text{orthogonal to all realizations of } q_{t-\tau}\text{'s}} \tag{18}$$

The first term captures the projection of the firm's competitors' prices on the history of monetary policy shocks. The second term captures the endogenous mistakes of firm j,k's competitors orthogonal to monetary policy shocks. This Equation also highlights a major difference between this model and a model in which there is a continuum of firms with orthogonal mistakes. In the latter, there is no non-fundamental volatility ($v_{j,-k,t} = 0$). The presence of these non-fundamental shocks in the oligopolistic model motivates firms to learn about them. Hence, these endogenous shocks need to be included in the state-space of the processes that firms track, where both their volatility and auto-correlations are *endogenously* determined. Similar to the static model, the richness of the information structure allows firms to acquire information about *both* of these components. This makes the solution of the model complicated and the conventional methods for solving monopolistic competition rational inattention models need to be extended to allow for both the fundamental and the endogenous non-fundamental shocks.

Therefore, to solve the firm's rational inattention problem, I need to write both of these terms as a function of a shock vector with a Markov state-space representation. Moreover, since both these processes are endogenous to the equilibrium, this Markov representation needs to allow for enough flexibility that does not ex-ante rule out the equilibrium processes. Following Wold's representation theorem, one can approximate the stochastic component of any covariance-stationary process with an MA(T) process arbitrarily closely as $T \to \infty$. With that in mind, I approximate the mistakes of the firm's competitors with an MA(T) process. Similarly, I approximate the projection of firms' prices on monetary policy shocks with an integrated MA(T) process (integration is to account for the unit root in prices). In my solution algorithm, I set $T = 40$, which assumes that the IRFs of all variables converge back to the steady-state after 40 quarters (which I then confirm in the calibrated model). I then write the firms' rational inattention problem in terms of the concatenated vector of these shocks (of length $2T$).

Given the guess for the Markov representation that emerges from the procedure above, a firm's problem in any iteration reduces to a single agent dynamic rational inattention problem in a linear quadratic Gaussian setup, which I solve using the method developed in Afrouzi and Yang (2019) and derive the firm's optimal signal structure under its "steady-state" information structure.[37] Given this signal structure, I use standard Kalman-filtering to solve for the dynamics of the firm's price. Doing this for all values of K for which there is a positive mass of sectors, I then derive the new guess for the joint stochastic process of firms' prices and iterate until convergence.

The method in Afrouzi and Yang (2019) solves a rational inattention problem by iterating over the analytically derived policy functions (first-order conditions of the rational inattention problem) and is faster than the value function methods by several orders of magnitude. This speed in obtaining the solution is crucial for the feasibility of solving and calibrating the model in this paper. For any given value of ω (the cost of attention), I need to solve the model for different numbers of competitors ranging from $K = 2$ to $K = 44$ and $K \to \infty$ in a model where the state-space representation includes not only the monetary policy shocks but also an endogenous persistent process for the mistakes of firms. I then need to repeat this procedure by iterating over different ω's to find the value that fits its targeted moment.

4.4 A Special Case with a Closed-Form Phillips Curve

In general, the equilibrium signal structure of firms does not admit a closed-form representation. However, we can go further in characterizing the representation of optimal signals when firms are completely myopic in their information acquisition ($\beta = 0$) which is useful for intuition.

Proposition 4. Given a strategy profile for all other firms in the economy, every firm prefers to see only one signal at any given time. Moreover, if $\beta = 0$, the optimal signal of firm j,k at time t is

$$S_{j,k,t} = (1-\alpha_j)q_t + \alpha_j p_{j,-k,t}(S^t_{j,-k}) + e_{j,k,t}$$

The expression for the optimal signals, in this case, shows how firms incorporate the mistakes of their competitors into their information sets. In particular, by plugging in the decomposition in Equation (18) we can re-write the optimal signal of the firm as

$$S_{j,k,t} = \overbrace{\underbrace{(1-\alpha_j)q_t + \alpha_j p_{j,-k,t}(S_{j,-k,t})|_q}_{\text{predictive of } q_{t-\tau}\text{'s}} + \alpha_j v_{j,-k,t}}^{\text{predictive of sector price changes}} + e_{j,k,t}.$$

This decomposition of the signal illustrates the main departure of this paper from models that assume a measure of firms. Since $var(v_{j,-k,t}) \neq 0$, the signal of a firm covaries more with the price changes of its competitors than with the fundamentals of the economy. When there is a

[37] The steady-state information structure is the information structure that emerges in the long-run after a firm continues acquiring information under the solution to its rational inattention problem. Focusing on steady-state information structure allows us to avoid dealing with time-varying or state-dependent impulse response functions or transition dynamics of second-order moments of beliefs. See Afrouzi and Yang (2019) for a precise definition of steady-state information structure.

measure of firms, however, the term $\alpha_j v_{j,-k,t}$ disappears and these two covariances converge to one another. Intuitively, this implies that when α_j is larger, firms in that sector are more informed about their own sector prices than the fundamentals of the economy.

We can go further in our special case and derive a closed-form expression for the Phillips curve of this economy for the case when there is no heterogeneity in the number of competitors across sectors. These assumptions are only made for the illustration of the Phillips curve and I will revert to the general case later in the calibrated model.

Proposition 5. Suppose $\beta = 0$ and $K_j = K, \forall j \in J$ for some $K \in \mathbb{N}$. Then, $\alpha_j = \alpha, \forall j \in J$ and in the stationary equilibrium $\kappa_{j,k,t} = \kappa > 0, \forall j \in J, k \in K$. Moreover, the Phillips curve of this economy is

$$\pi_t = (1-\alpha)\overline{\mathbb{E}_{t-1}^{j,k}[\Delta q_t]} + \alpha\overline{\mathbb{E}_{t-1}^{j,k}[\pi_{j,-k,t}]} + (1-\alpha)(2^{2\kappa}-1)y_t,$$

where $\overline{\mathbb{E}_{t-1}^{j,k}[\Delta q_t]}$ is the average expected growth of nominal demand at $t-1$, which is the sum of inflation and output growth, $\Delta q_t = \pi_t + \Delta y_t$, $\overline{\mathbb{E}_{t-1}^{j,k}[\pi_{j,-k,t}]}$ is the average expectation across firms of their competitors' price changes, and y_t is the output gap.

This Phillips curve crystallizes one of the main conceptual results of this paper. In economies with large micro-level strategic complementarities, it is the firms' average expectation of their own competitors' price changes that drives aggregate inflation rather than their expectations of the growth in aggregate demand.

Moreover, Proposition 4 shows that with endogenous information acquisition, a larger α also implies that firms learn more about the prices of their competitors relative to the aggregate demand. Therefore, when α is large, not only is inflation driven more by firms' expectations of their own competitors' price changes but also firms' expectations are formed under information structures that are more informative about those prices.

Additionally, the slope of the Phillips curve shows how these strategic complementarities and the capacity for processing information interact in affecting monetary non-neutrality in this economy. The higher capacity of processing information makes the Phillips curve steeper, such that in the limit when $\kappa \to \infty$ (which arises endogenously when $\omega \to 0$), the Phillips curve is vertical. In contrast, higher strategic complementarity makes the Phillips curve flatter since firms' higher-order beliefs become more important in their pricing decisions (Woodford, 2003a). I will revisit and decompose the role of each of these two forces later in Section 5.

4.5 Calibration

The model is calibrated to the firm-level survey data from New Zealand at a quarterly frequency, with a discount factor $\beta = 0.96^{1/4}$. A calibration to the US data might also be desirable; however, one of the main objectives of quantifying the model is to examine if it fits the relationship between competition and expectations of firms about aggregate inflation—which I do not target in the calibration but the evidence for it comes from the same survey data for New Zealand.[38] The key and new parameters are the distribution of competitors, $\mathcal{K}$, and the cost of attention ω. I discuss the calibration of these two parameters in the rest of this section. Other parameters are externally calibrated, as presented in Table 4 and discussed in more detail in Appendix H. Finally, in Section 6, I provide robustness checks with respect to the discount factor and the persistence of money growth.

Table 4: Calibration Summary

Parameter	*Description*	*Value*	*Moment Matched*
$\mathcal{K}$	Distribution of K	$\sim \hat{\mathcal{K}}$	Empirical distribution (Fig. 1)
ω	Cost of attention	0.326	Weight on prior in inflation forecasts
η	Elasticity of substitution	12	Elasticity of markups to $1/(1-K_j^{-1})$
$1/(1+\gamma)$	Curvature of production	0.526	Average strategic complementarity
ρ	Persistence of Δq	0.7	Persistence of NGDP growth in NZ
σ_u	Std. Dev. of shock to Δq	0.027	Std. Dev. of NGDP growth in NZ

Notes: the table reports the calibrated values of the parameters for the dynamic model.

Distribution of the number of firms within oligopolies. I match the distribution of K_j in the model, denoted by $\mathcal{K}$, to the empirical distribution of the number of competitors that firms report in the data (Figure 1). As far as I know, there is no data available on how many competitors firms directly face in their market for the US.[39]

Cost of information acquisition. My strategy here is to target the weight that firms put on their priors in their inflation forecasts, which is an indicator of the degree of information rigidity and is similar to the approach in Wiederholt (2015). In particular, the survey follows a subset of firms across different waves and asks them about their yearly inflation forecasts (inflation 12 months ahead) and yearly inflation nowcasts (inflation in the previous 12 months). These horizons collapse for the first and the fourth waves of the survey that were conducted 12 months apart from one another (2013:Q4 to 2014:Q4). Thus, for the subset of firms that are present in both of these surveys, we observe their ex-ante and ex-post beliefs about inflation in that year. Kalman filtering implies that, for any firm i, this revision should be given by

$$\mathbb{E}_{i,t}[\pi_t] = \mathbb{E}_{i,t-4}[\pi_t] + \lambda_i(s_{i,t} - \mathbb{E}_{i,t-4}[s_{i,t}]) \tag{19}$$

[38] In addition, to calibrate the model to the US data, one needs microdata on firms' expectations about inflation to calibrate the cost of attention in the US as well as data on how many competitors firm *directly* face to calibrate the distribution of the number of competitors, none of which are available for the US to the best of my knowledge

[39] It is important to note that the value of K in this model corresponds to direct competitors of a firm that are only a small subset of all the firms that operate in a single SIC classification. Market segmentation, such as spatial constraints for consumers, make the number of firms within a SIC classification not suitable for calibrating this model. For instance, a coffee shop in Manhattan only competes with a small number of coffee shops that are geographically close to it, rather than all coffee shops in the US.

where t is in quarters, λ_i is the Kalman gain of the firm, and $s_{i,t}$ is the signal(s) that the firm has observed within the year. The smaller the λ_i, the more weight firms put on their priors, and information rigidity is larger. Hence, under full information, when $\lambda_i = 1$, all firms should report the realized inflation, and their priors should be irrelevant to their ex-post nowcasts. This approach is similar Coibion and Gorodnichenko (2015) which uses Equation (19) to estimate the degree of information rigidity in professional forecasters' expectations.

A complication here is that we do not observe the true signals of the firms in the data, and thus we cannot include them in the regression; however, one can run the same misspecified regression within data generated by the model under different values of ω and choose the value that generates the same coefficient as in the data. This approach would be valid for the identification of ω as long as the coefficient identified from the misspecified regression moves monotonically with ω. In particular, I run the following regression:

$$\mathbb{E}_{i,t}[\pi_t] = \text{constant} + \delta \mathbb{E}_{i,t-4}[\pi_t] + \text{error} \tag{20}$$

where δ is the coefficient of interest. Since the regression exploits cross-sectional variation, rather than time-series variation as in Coibion and Gorodnichenko (2015), the current value of inflation is absorbed by the constant. Column (1) in Table 5 reports the baseline estimates for this specification. One caveat with this estimate is that there might be inherent heterogeneity among firms in perceiving different long-run inflation rates (Patton and Timmermann, 2010) which might get picked up by their ex-ante forecasts. A question in the survey asks firms about this target, and Column (2) controls for this value.[40]

To identify ω, I simulate the model for a range of ω's, fixing the other parameters at their calibrated values. Figure 3 shows that ω is identified as this regression coefficient increases with ω within the model. I set $\omega = 0.326$ to match the coefficient in Column (2) of Table 5 which yields a more conservative calibration relative to the coefficient in Column (1)—a value of ω matched to Column (1) would imply a higher cost of attention.

Having identified ω, we can compare the degree of information rigidity in this model with estimates of this object in the literature. To do so, one can compare the implied Kalman gain from Equation (19) with the values that have been documented in the literature for professional forecasters. The average firm in this model has a Kalman gain of 0.51 which is *above* the estimated value for Professional Forecasters in the US by Coibion and Gorodnichenko (2015), who find an average Kalman gain of 0.45. Hence, the model implies that firms are more informed about their

[40]The exact question is "What annual percentage rate of change in overall prices do you think the Reserve Bank of New Zealand is trying to achieve?" While this does not necessarily have to comply with firms' beliefs about long-run inflation, a follow-up question verifies that they do. When asked "Do you believe the Reserve Bank of New Zealand can achieve its target?" the overwhelming majority of firms (more than 90 percent) respond yes. See Kumar, Afrouzi, Coibion, and Gorodnichenko (2015) for a detailed discussion.

optimal prices than *professional forecasters are of aggregate inflation.* Nonetheless, firms exhibit large degrees of information rigidity in their inflation forecasts because inflation does not matter much for their decisions and their optimal signals are not as informative of inflation as they are of firms' optimal prices.

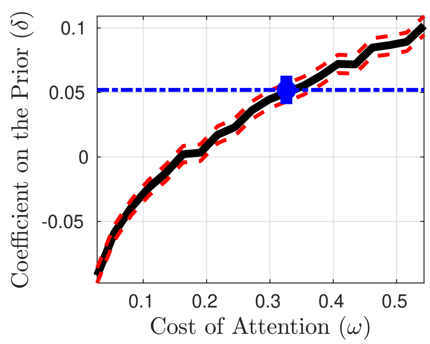

Figure 3: Sensitivity of δ to Cost of Attention (ω)

Notes: the black line shows the predicted value of δ from the regression specified in Equation (20) in model generated data as a function of ω. The blue dot shows the equivalent estimate in the New Zealand data from Table 5.

Table 5: Calibration of Cost of Attention (ω)

	Inflation Nowcast	
	(1)	*(2)*
inflation forecast	0.163	0.052
	(0.011)	(0.008)
perceived target		0.674
		(0.020)
Constant	3.107	0.734
	(0.102)	(0.081)
Observations	1257	1257

Standard errors in parentheses

Notes: the table reports the result of regressing firms' nowcast of yearly inflation on their forecast for the same horizon from a year before. The coefficient on the lagged forecast captures the weight that firms put on their priors and increases with the degree of information rigidity. Column (1) reports the result with no controls. Column (2) controls for the firm's expectation of long-run inflation and gives a more conservative estimate.

4.6 Examining Non-Targeted Moments: Subjective Uncertainty in the Model

Can the calibrated model match the strategic inattention of firms observed in the data? The relevant moments here are the relationship between the subjective uncertainty of firms about aggregate inflation and the number of their competitors: as documented in the previous section, firms' uncertainty in the data is decreasing with the number of their competitors (Table 2). This relationship is not consistent with the benchmark models; a model of oligopolistic competition without rational inattention would predict no correlation between competition and expectation formation about aggregates. However, this relationship emerges endogenously in this model from the interaction between rational inattention and oligopolistic competition.

Figure 4 shows this relationship both in the model (the solid line) and the data (binned scatter plot).[41] The main observation is that the model matches the decline of the subjective uncertainty as a function of the number of competitors well. Both heterogeneity in the number of competitors and endogenous information acquisition are key ingredients for this relationship: the former

[41] I have normalized average uncertainty both in the data and in the model to 1.

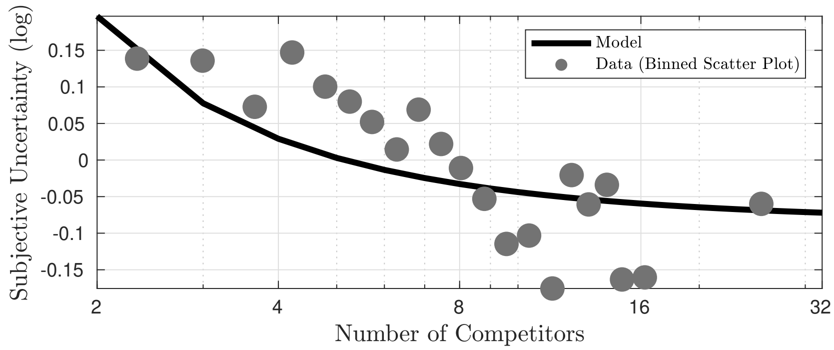

Figure 4: Subjective uncertainty about inflation: Model vs. Data.

Notes: the figure presents the fit of the model for the relationship between firms' (log) subjective uncertainty about inflation and the number of their competitors. The dots show the binned scatter plot of log-subjective uncertainty about aggregate inflation against the number of competitors in the data. The black line depicts this relationship in the calibrated model. The average uncertainty is normalized to one in both the data and the model. This relationship was not targeted in the calibration of the model.

creates the differential incentives for information acquisition, and the latter is essential for the endogenous variation in information acquisition.

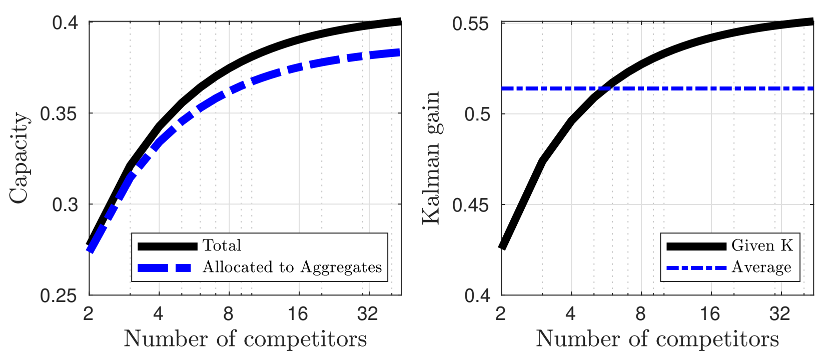

Figure 5: Information Capacity and Kalman Gains for Different Values of K.

Notes: the left panel shows the produced information processing capacity of a firm as a function of the number of competitors within its sector. The right panel shows the model implied true Kalman gains of firms (weight put on the most recent signal by firms) as a function of the number of competitors within a sector. Firms with more competitors acquire more information and have larger Kalman gains. The blue dotted line shows the average Kalman gain of firms weighted by the distribution of the number of competitors in the data.

Figure 5 shows the equilibrium level of firms' information acquisition and their implied Kalman gains as a function of the number of firms' competitors. More competitive firms (1) produce a higher capacity for processing information and (2) allocate more capacity towards aggregate shocks. As a result, more competitive firms have more accurate posteriors about aggregate variables, e.g., inflation.[42]

[42] While information capacity is not observed in the data, in a simplified model, it can be mapped to the persistence of inflation in PPI data. In a discussion of this manuscript, Schoenle (2018) uses this mapping and finds evidence consistent with these predictions in the US PPI data merged with BLS and Census SUBS.

5 Macroeconomic Implications

The main focus of my analysis so far has been the effect of a firm's number of competitors on their information acquisition incentives. But what are the macroeconomic implications of this relationship? In this section, I investigate this mechanism's *aggregate* and *reallocative* consequences for the propagation of monetary policy shocks to inflation and output.

To do so, I consider three measures in the rest of this section. My first measure, following Nakamura and Steinsson (2010), is the variance of output (normalized by its natural level) to assess the degree of monetary non-neutrality of different sectors.[43] My second measure is the cumulative half-life of output and inflation responses (time until the area under the impulse response reaches half of its full cumulative response) to monetary shocks to assess the persistence of the effect of monetary policy on these two variables.[44] The results of these comparisons are discussed in the remainder of this section and summarized in Tables 6 and 7. My third measure, which I use to compare reallocative effects of policy across sectors, is the cumulative response of output (see, e.g., Alvarez, Le Bihan, and Lippi, 2016; Wang and Werning, 2021) defined as the area under the output IRF of sectors with different number of competitors.

5.1 Aggregate Effects: Monopolistic vs. Oligopolistic Competition

Monopolistic competition is the workhorse model for studying aggregate output and inflation dynamics in monetary models. Thus, the first natural exercise is to compare the benchmark calibrated model to a monopolistic competition model, which is nested when $K_j \to \infty$.

Since the goal is to compare the two models quantitatively, it is important to parametrize both models so that the only difference is the strategic attention of oligopolistic firms. To do so, I parameterize the monopolistic competition model to have the *same* average degree of strategic complementarity so that the two models would yield the same impulse response functions in the absence of *strategic inattention*; i.e., the *only* difference in impulse responses come from the different signals that firms choose and utilize under the two settings. Firms in the oligopolistic model pay direct attention to the mistakes of their competitors, but firms in the monopolistic competition model, similar to Woodford (2003a), only pay attention to the fundamental shocks.[45]

[43] Up to a second-order approximation to the household's utility, the variance of output normalized by its natural level is proportional to her welfare loss in consumption equivalent units (Lucas, 2003): $\mathbb{E}\left[\log(Y_t) - \log(\bar{Y})\right] \approx -\frac{1}{2} var\left(\frac{Y_t}{\bar{Y}}\right)$ where $\bar{Y} \equiv \mathbb{E}[Y_t]$ is the natural level of output. Moreover, since we have eliminated the steady-state distortions from imperfect competition, all sectors in the model have the same level of natural output in the model, which I normalize to one going forward. Therefore, for any $j \in J$, we can interpret $Y_{j,t}$ as the output of sector j *relative* to its natural level. Similarly, a lower variance of inflation corresponds to a more muted response of inflation to monetary shocks.

[44] Usually, half-lives are measured as the time until a variable reaches half of its impact response. However, when responses are hump-shaped, as in this model, this can be misleading. To bypass this issue, I report the cumulative half-life.

[45] There is, however, a difference between the shape of signals in the monopolistic competition here with those

Moreover, equating strategic complementarities across the two models is the desirable comparison because we know from a long line of previous papers that higher real rigidities (generated by strategic complementarity here) amplify monetary non-neutrality (Ball and Romer, 1990) (in Section 5.3 below, we offer a more thorough discussion on, and decomposition of, the confounding effect of strategic complementarities). So the comparison of interest is to see how the two models, calibrated to the same level of strategic complementarities, compare in terms of their implications for monetary non-neutrality.

The impulse response functions of output and inflation across these two models are presented in Figure 6. Output response is smaller, and inflation is less persistent in the monopolistic competition model, indicating that firms in the monopolistic competition model are more informed about aggregate monetary shocks. For reference, Figure 6 also presents a comparison in the other direction, where all sectors are duopolies. Consistently, output response is the largest, and inflation is the most persistent in the duopoly model as strategic inattention motives are at their highest level in this economy.

To quantify these differences for output, the first two rows in Table 6 report how output behavior differs across the benchmark and monopolistic competition model. Column (1) reports the variance of output across the two models.[46] Column (2) reports the magnitude of amplification by normalizing the variance of output in the monopolistic competition to 1. Output is 30% more volatile under the benchmark model with strategic inattention. Note that this is independent of strategic complementarity by construction of the comparison, and this amplification is solely due to the strategic inattention of firms to aggregates in the oligopolistic competition model.

In addition to amplifying the variance of output, strategic inattention also increases the persistence of the output response to monetary policy shocks. Column (3) reports the half-life of output response across the two models. While it takes 3.38 quarters for output to reach its half-life in the monopolistic competition model, this duration is 3.93 quarters in the benchmark model—a 16% increase as reported in Column (4).

Finally, the first two rows of Table 7 report how the behavior of inflation is different across these models. Inflation response is smaller and more persistent in the model with strategic inattention. Columns (1) and (2) show that inflation is 6% less volatile compared to the model with monopolistic competition. Furthermore, Column (3) shows that while it takes inflation 3.94 quarters to reach its cumulative half-life in the monopolistic competition model, this number is 4.34 quarters in the benchmark model—a 10% increase as reported in Column (4).

in Woodford (2003a), which assumes $S_{j,k,t} = q_t +$ noise. Here, signals under monopolistic competition are linear functions of innovations to q_t (plus noise), but the exact linear combination still depends on the value of the discount factor and the degree of strategic complementarity.

[46] Magnitudes are small since the variance of innovations to nominal GDP growth is small. The same is true for the US (Nakamura and Steinsson, 2010).

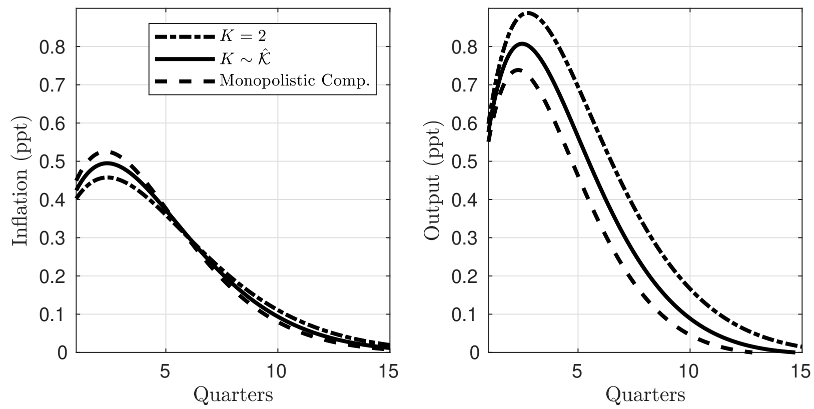

Figure 6: IRFs to a 1% Expansionary Shock

Notes: the figure shows the impulse response functions of output and inflation to a one percent expansionary shock to the growth of nominal demand in three models. The black lines are impulse responses in the benchmark model where the distribution of the number of competitors in the model is calibrated to the empirical distribution in the data (Figure 1). The dashed lines show the impulse responses in the model with monopolistic competition in all sectors. The dash-dotted lines show the impulse responses in the model where all sectors are composed of duopolies.

Table 6: Output and Monetary Non-Neutrality Across Models

Model		*Variance*		*Persistence*	
		var(Y) $^{\times 10^3}$	*amp. factor*	*half-life* ^{qtrs}	*amp. factor*
		(1)	*(2)*	*(3)*	*(4)*
Monopolistic Competition		1.53	1.00	3.38	1.00
Benchmark	$K \sim \hat{\mathcal{K}}$	2.00	1.30	3.93	1.16
2-Competitors	$K = 2$	2.71	1.77	4.38	1.30
4-Competitors	$K = 4$	2.10	1.37	4.00	1.18
8-Competitors	$K = 8$	1.91	1.24	3.86	1.14
16-Competitors	$K = 16$	1.83	1.19	3.79	1.12
32-Competitors	$K = 32$	1.79	1.17	3.77	1.12
∞-Competitors	$K \to \infty$	1.76	1.15	3.74	1.11

Notes: the table presents statistics for monetary non-neutrality across models with different number of competitors at the micro-level. $Var(Y)$ denotes the variance of output multiplied by 10^3. *Half-life* denotes the length of the time that it takes for output to live half of its cumulative response in quarters. *Amp. factor* denotes the factor by which the relevant statistic is larger in the corresponding model relative to the model with monopolistic competition.

5.2 Reallocative Effects and Concentration Multipliers

While the previous section focused on the aggregate implications of the model compared to monopolistic competition, this section studies the heterogeneity of inflation and output responses for sectors with different numbers of competitors. To do so, I present two complementary exercises. First, I show how output volatility differs across sectors based on the number of competitors and compare them to the same monopolistic competition model as before. This exercise focuses on every sector in isolation and has the benefit that its results are independent

Table 7: Inflation Across Models

Model		*Variance*		*Persistence*	
		var(π) $^{\times 10^4}$	*damp. factor*	*half-life* ^{qtrs}	*amp. factor*
		(1)	*(2)*	*(3)*	*(4)*
Monopolistic Competition		9.27	1.00	3.94	1.00
Benchmark	$K \sim \hat{\mathcal{K}}$	8.73	0.94	4.34	1.10
2-Competitors	$K = 2$	8.05	0.87	4.62	1.17
4-Competitors	$K = 4$	8.62	0.93	4.38	1.11
8-Competitors	$K = 8$	8.84	0.95	4.30	1.09
16-Competitors	$K = 16$	8.94	0.96	4.26	1.08
32-Competitors	$K = 32$	8.98	0.97	4.25	1.08
∞-Competitors	$K \to \infty$	9.03	0.97	4.23	1.07

Notes: the table presents statistics for inflation response across models with different number of competitors at the micro-level. $Var(\pi)$ denotes the variance of inflation multiplied by 10^4. *Half-life* denotes the length of the time that it takes for inflation to live half of its cumulative response in quarters. *Damp. factor* (*amp. factor*) denotes the the factor by which the relevant statistic is smaller (larger) in the corresponding model relative to the model with monopolistic competition.

of the distribution of competitors, which might be different across countries. Second, I compare the output response of different sectors to the response of aggregate output in the same economy—which focuses more directly on relative differences within the same economy. In particular, I decompose the *cumulative response of aggregate output* based on the number of competitors across sectors and show that *less competitive* sectors drive a *higher share* of output response and exhibit higher monetary non-neutrality.

Output Volatility Conditional on Number of Competitors. Consider a set of comparisons across sectors at the micro-level with $K \in \{2, 3, 4, \dots\}$ competitors. How do output and inflation responses differ for different values of K? Table 6 reports output volatility and amplification factors relative to the model with monopolistic competition. Monetary non-neutrality is larger, and output response is more persistent in sectors with fewer competitors. The duopoly model has the highest monetary non-neutrality; its output volatility is 77% larger than the monopolistic competition model and a cumulative half-life that is a quarter longer.

Table 7 reports the equivalent results for inflation. The smaller the number of competitors, the more muted is the response of inflation and the longer is its half-life. In the case of a duopoly within all sectors, the variance of inflation is 13% smaller than the model with monopolistic competition, and its half-life is two months (0.68 quarters) longer.

Concentration Multipliers. Since output volatility decreases with competition, output response is distorted in the sense that less competitive sectors drive a higher share of the output response relative to their steady-state market share. A natural exercise is to measure the magnitude of these distortions and calculate how much of the output response comes from less competitive firms.

Formally, let $\mathcal{Y}_k$ denote the average cumulative impulse response of log-output to a one standard deviation monetary policy shock in sectors with k competitors, and let $\mathcal{Y}$ denote the cumulative impulse response of aggregate output:

$$\mathcal{Y}_k \equiv \mathbb{E}^j \left[\frac{\partial}{\partial u_0} \sum_{t=0}^{\infty} \log(Y_{j,t}) \middle| K_j = k \right], \quad \mathcal{Y} \equiv \frac{\partial}{\partial u_0} \sum_{t=0}^{\infty} \log(Y_t) \tag{21}$$

where the expectation operator simply averages responses across j. It is straightforward from Equation (4) to derive the relationship between these aggregate and sectoral responses as

$$\mathcal{Y} \equiv \sum_{k=2}^{\infty} s_k \mathcal{Y}_k \tag{22}$$

where s_k is the steady-state market share of sectors with k competitors (since we have normalized the steady-state output of all sectors to 1, s_k collapses to the fraction of sectors with k competitors). We can now define the *concentration multiplier* of sectors with k competitors as the ratio

$$\mathcal{M}_k \equiv \frac{\mathcal{Y}_k}{\mathcal{Y}} \tag{23}$$

This concentration multiplier has the following properties that clarify the role of heterogeneity. First, it would be constant and equal to one for all k if there was no heterogeneity in output response. However, with heterogeneity, it measures how much the cumulative response of output is on average larger or smaller in a sector with k competitors *relative* to the aggregate output response. Second, it only measures relative differences in the sense that it sums up to one once aggregated across k: $\sum_k s_k \mathcal{M}_k = 1$.

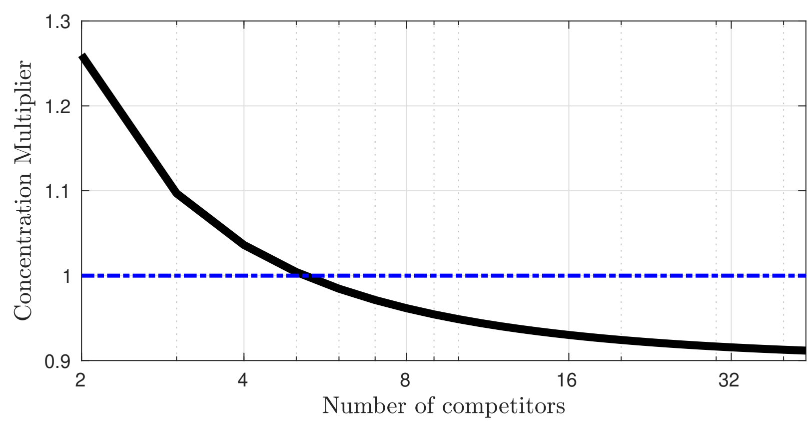

Figure 7: Concentration Multipliers

Notes: the figure shows the *concentration multiplier* as a function of the number of competitors. Concentration multiplier k is defined as the cumulative response of output coming from sectors with k competitors relative to the aggregate cumulative response of output. Less competitive sectors are responsible for a higher share of output response relative to their steady-state market share.

Figure 7 plots these multipliers for different numbers of competitors and shows that less competitive sectors respond more strongly to monetary policy shocks in their output. For instance, sectors with two competitors have a 25% larger output response to monetary policy shocks relative to the aggregate output response.

Thus, in the model, expansionary monetary policy *concentrates* production among *less* competitive firms, increasing the impact of such firms on the economy.

It is important to note that more competitive firms contribute less to output response despite having higher strategic complementarities! Classic papers of information rigidity, such as Woodford (2003b), show that fixing the degree of information rigidity, higher strategic complementarity leads to higher monetary non-neutrality. What is novel here is that endogenous information acquisition *reverses* this result in a calibrated model by adding a new force, strategic inattention. The following section explains and decomposes the roles of each of these forces in the model.

5.3 Inspecting the Mechanism: Strategic Inattention vs. Strategic Complementarities

In the model, the number of competitors affects both the degree of strategic complementarity and the amount of capacity produced by firms. Thus, the degree of monetary non-neutrality across sectors with different K—as reported in the previous section—is the sum of two separate forces. (1) the well-known *real rigidity* channel that alters monetary non-neutrality through changing the degree of strategic complementarity, and (2) the new *strategic inattention* channel that alters the degree of non-neutrality through the information acquisition and utilization of firms.

More importantly for my analysis, in the calibrated model, these two forces work in opposite directions. As discussed in Section 4.6 and shown in Figure 5, firms with more competitors produce higher capacity and allocate a higher amount of attention to learning about aggregates. Since firms with a higher allocated capacity towards aggregate variables learn monetary shocks more precisely, their prices move more swiftly in response to these shocks, which dampens their output response as a result. Hence, monetary non-neutrality decreases with competition through the strategic inattention channel.

On the other hand, the degree of strategic complementarity increases with the number of competitors in the calibrated model, as seen in Equation (13) and Figure 8. Therefore, fixing the capacity of processing information, a larger number of competitors increases monetary non-neutrality by putting a larger weight on firms' higher-order beliefs (This is well-established in the literature of models with information rigidities. see, e.g. Woodford, 2003a; Nimark, 2008; Maćkowiak, Matějka, and Wiederholt, 2018). To verify this mechanism within the model, Figure 9 shows the IRFs of firms' higher-order beliefs to a one percent increase in nominal demand for three different values of K. For any given K, the responses of higher-order beliefs are smaller

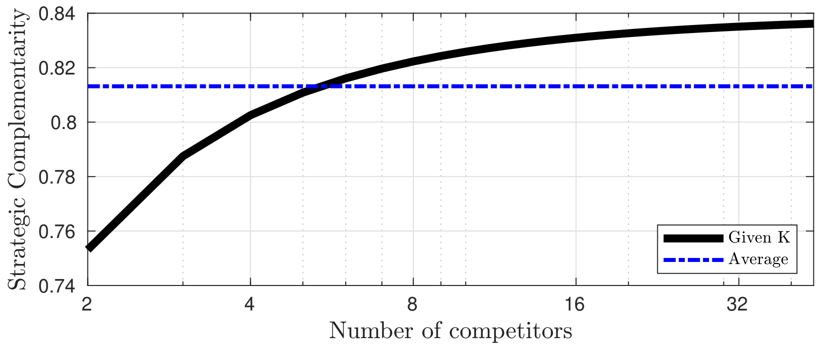

Figure 8: Strategic complementarity as a function of K.

Notes: the figure shows the relationship between the number of competitors within a sector and the degree of strategic complementarity in pricing. Firms with a larger number of competitors have a higher degree of strategic complementarity. The dash-dotted line shows the average degree of strategic complementarity weighted by the empirical distribution of number of competitors in the New Zealand data.

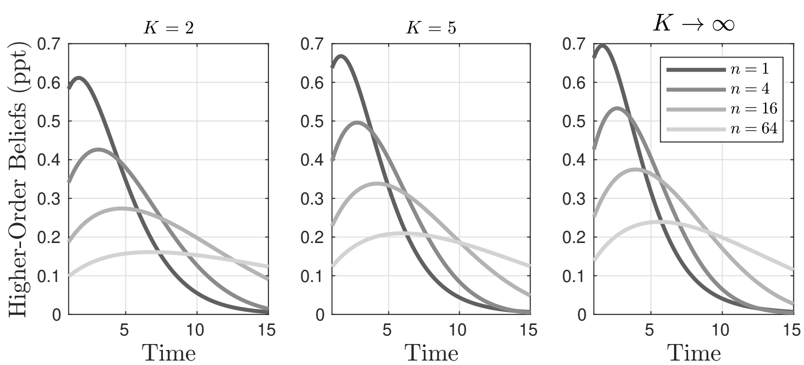

Figure 9: IRFs of Higher-Order Beliefs to a 1% Expansionary Shock

Notes: the figure shows the IRFs of firms' higher-order beliefs to a one percent expansionary shock to the growth of nominal demand across three different models. For any given order (n), firms' n^{th} order beliefs in economies with a larger number of competitors are more responsive to the shock. This is driven by the fact that firms in more competitive economies acquire more information about the aggregate shock.

and more persistent. With larger strategic complementarities, firms put a larger weight on these higher-order beliefs. As a result, their average response also becomes smaller and more persistent, which amplifies monetary non-neutrality. Thus, monetary non-neutrality increases with the number of competitors through the real rigidity channel.

To decompose the effects of these two opposing forces, let us define $\alpha(K)$ to be the degree of strategic complementarity in a model where all sectors have K competitors, and all the other parameters are fixed at their calibrated values. Moreover, let $\sigma_y^2(\alpha(K), K)$ denote the output variance in the model where every sector has K competitors. The first argument captures the effect of the number of competitors on the weight that higher-order beliefs receive in the model (the real rigidity channel), and the second argument captures the effect of the number of competitors on the attention allocation of firms (strategic inattention channel). Then, we can decompose the difference in monetary non-neutrality of the two extreme models ($K = 2$ versus

$K \to \infty$) as

$$\underbrace{\lim_{K\to\infty} \log\left(\frac{\sigma_y^2(\alpha(2),2)}{\sigma_y^2(\alpha(K),K)}\right)}_{\substack{\text{total} \\ \text{percentage change}}} = \underbrace{\lim_{K\to\infty} \log\left(\frac{\sigma_y^2(\alpha(2),2)}{\sigma_y^2(\alpha(2),K)}\right)}_{\substack{\text{percentage change due to} \\ \text{strategic inattention}}} + \underbrace{\lim_{K\to\infty} \log\left(\frac{\sigma_y^2(\alpha(2),K)}{\sigma_y^2(\alpha(K),K)}\right)}_{\substack{\text{percentage change due to} \\ \text{real rigidities}}} \tag{24}$$

Table 8: Decomposition: Strategic Inattention vs. Real Rigidities

	Percentage change in variance of	
	output	*inflation*
	(1)	*(2)*
Total Change (percent)	43.3	-11.5
Due to Str. Inattention (ppt)	85.6	-19.5
Due to Real Rigidities (ppt)	-42.3	8.0

Notes: the table shows the decomposition of the effects of two opposing forces for the change in volatility of output (monetary non-neutrality) and inflation in the model with $K \to \infty$ versus the model with $K = 2$. Firms with a higher number of competitors acquire more information about aggregate shocks, which decreases monetary non-neutrality and increases the volatility of inflation (strategic inattention channel). On the other hand, firms with a larger number of competitors have larger degrees of strategic complementarity in pricing, which increases monetary non-neutrality and decreases the volatility of inflation (real rigidity channel). For calibrated values of the parameters of the model, the strategic inattention channel dominates and monetary non-neutrality is 43.3% larger in the model with $K = 2$ than the model with $K \to \infty$.

Column (1) of Table 8 shows the results of this decomposition. Output variance is 43.3 percent larger with $K = 2$ relative to $K \to \infty$ (percentage difference here is calculated as the log-difference from Table 6). Once decomposed to its two contributing factors, decreasing the number of competitors from $K \to \infty$ to $K = 2$ increases monetary non-neutrality by 85.6 percentage points due to the strategic inattention channel and decreases it by 42.3 percentage points through the real rigidity channel.

We can do a similar decomposition for inflation, whose variance is 11.5 percent smaller in the model with $K = 2$ relative to the model with $K \to \infty$. Column (2) of Table 8 shows that decreasing the number of competitors from $K \to \infty$ to $K = 2$ decreases the variance of inflation by 19.5 percentage points through the strategic inattention channel and increases it by eight percentage points through the real rigidity channel.

6 Discussion and Robustness

6.1 Heterogeneity within sector market shares

In deriving the approximate problem in Section 4.3, I took a second-order approximation to the firms' problem around the symmetric full-information economy, in which all firms within the same sector had the same market share. This approximation makes solving the problem feasible by making problems of all firms within one sector symmetric. However, it ignores potential heterogeneity in market shares *within* sectors.

One important question is how does heterogeneity in market shares affect strategic inattention? For instance, consider a duopoly in which one firm holds almost all the market share where its rival has almost zero market share. Is it the case that the large firm ignores the mistakes of the small firm and allocates all of its attention to the aggregates? The short answer to this question is no. In fact, based on the discussion on the sources of strategic complementarities in the previous section, the opposite is true: the larger firm pays more attention to the mistakes of the smaller firm, and the smaller firm pays almost full attention to the aggregates.

The reasoning behind this argument is that a firm's optimal price is more sensitive to the average price of its competitors at higher levels of market share, an evident observation from the expression of strategic complementarity in Equation (12). Therefore, firms with larger market shares face higher strategic complementarities and are more affected by the mistakes of smaller firms in their sector.

Furthermore, it is worthwhile to note the discontinuity in strategic complementarity as market share tends to one. In an oligopoly where a single firm has an arbitrarily large market share, the mere existence of small rivals motivates the firm to pay a lot of attention to the beliefs of these rivals. In contrast, a monopolist does not have to worry about any threat of small firms stealing its market share. A monopoly with a constant demand elasticity always pays one hundred percent of its attention to shocks to its marginal cost.

While solving the quantitative model without the symmetric market share approximation is not feasible, we can at least investigate how the heterogeneity in market shares would matter by deriving the second-order approximation in a simple model with heterogeneous market shares. Appendix J discusses a simple case with CES preferences and shows that, up to a second-order approximation, the strategic complementarity of any given firm is *equal* to their market share in the steady-state. Therefore, firms with higher market shares have higher strategic complementarities and thus higher incentives to track others' mistakes rather than the aggregate shocks.

Thus, we expect the symmetric market share approximation to be a conservative estimate of the effect of oligopolistic competition on monetary non-neutrality. In non-symmetric cases, larger firms, who contribute *more* to the economy's output, will pay *less* attention to monetary policy shocks, and aggregate output will be more volatile than the symmetric case.

6.2 Alternative discount factor

One of the main mechanisms in attention allocation within the model is the dynamic information acquisition of firms. Forward-looking firms internalize the continuation value of learning about different shocks and incorporate those incentives in their information acquisition. An important force here is that these dynamic incentives shift firms' attention towards more persistent processes because shocks to these processes are longer-lived (Afrouzi and Yang, 2019).

In the model, this mechanism dampens monetary non-neutrality. The reason is that monetary policy shocks are more persistent—due to the unit root in nominal demand—than the endogenous mistakes of firms, which are endogenously transitory. Thus, more forward-looking firms allocate more attention to the monetary policy shocks, which attenuates strategic inattention and reduces monetary non-neutrality. Moreover, since the value of $\beta = 0.96^{0.25}$ is very close to 1, the dynamic incentives are very strong in the calibrated model. The rationale behind this calibration is the usual approach in the literature that the households' stochastic discount factor should discount firms' profits because the representative household is the sole shareholder of these firms.

In the firms' problem, β stands for how much firms discount the future value of information because the only dynamic trade-off for firms is in their information acquisition. However, it is unclear whether firms should discount their losses from imperfect information at the market discount rate. Information can be an intangible and, potentially, a non-tradable form of capital and hence not subject to the market discount rate. As an alternative approach, I calibrate β and ω jointly by targeting the same moment in the data (the persistence of forecast errors as in the benchmark calibration) and redo the results for monetary non-neutrality. This joint calibration yields a value of 0.6 for β and 0.217 for ω. An important observation is that the calibrated value for the cost of inattention, in this case, is *smaller* than the value in the benchmark calibration (0.217 versus 0.326) which means that firms face lower costs in acquiring information in this alternative calibration.

Figure A.1b in Appendix A shows the impulse responses of output and inflation to a one percent expansion in nominal aggregate demand for the benchmark model, the model with two competitors in every sector and the model with monopolistic competition. Moreover, Table A.1b in Appendix A reports the statistics for the volatility and persistence of output and inflation in this case.

The main takeaway is that even though the cost of information acquisition, ω, is *smaller* in this calibration than the benchmark, monetary non-neutrality is *larger*. For instance, under this alternative calibration, output volatility is 112% larger in the model with two competitors in every sector relative to the model with monopolistic competition, which is 35 percentage points larger than the analogous factor under the benchmark calibration (77%).

But how can monetary non-neutrality be larger if the cost of information acquisition is smaller? The intuition behind this result is that firms are myopic in information acquisition despite the lower cost of information acquisition, ω. They produce lower capacity than the benchmark calibration, to begin with, and given that capacity, they allocate a higher share of that to the mistakes of their competitors since they are relatively more ignorant of the continuation value of information.

6.3 Lower persistence of nominal demand growth

Many of the parameter values calibrated to the New Zealand data are also consistent with their calibrations for the U.S. One exception is the persistence of the nominal demand growth, ρ. While the value for this parameter is around 0.7 in New Zealand, its value in the US is around a monthly persistence of 0.61 (Mongey, 2021; Midrigan, 2011) (or a quarterly persistence of $0.61^3 = 0.23$). To compare the results for this case, I recalibrate the cost of information acquisition and redo the analysis for monetary non-neutrality for $\rho = 0.23$. Figure A.1a in Appendix A shows the impulse response functions of inflation and output to a one percent expansion in the nominal aggregate demand under this assumption for the benchmark model, the model with two competitors in every sector, and the monopolistic competition model. Moreover, Table A.1a in Appendix A reports the statistics for the volatility and persistence of output and inflation in this case.

The main takeaway is that while the amplification factors are slightly smaller than the case for $\rho = 0.7$, the results are fairly robust. For instance, relative to the model with monopolistic competition, aggregate output is 25% percent more volatile under the benchmark calibration for the distribution of competitors—as opposed to 30% with $\rho = 0.7$.

7 Concluding Remarks

This paper develops a new model that explains how imperfect competition affects firms' information acquisition and expectations. The interaction of these two frictions creates an endogenous correlation between the accuracy of firms' beliefs and the number of their competitors. Oligopolistic firms find it optimal to pay direct attention to the beliefs of their competitors, an incentive that is stronger when they have fewer competitors or higher strategic complementarities in pricing.

The model's implications for monetary non-neutrality and inflation dynamics speak to recently documented trends in rising concentration and market power. These results suggest that with more concentration, monetary policy is more potent and its real effects are stronger. Furthermore, the reallocative effects of strategic inattention imply that this change in potency is not uniform across all firms. These heterogeneous effects introduce new distortions to relative prices that might lead to new sources of misallocation and, more broadly, to efficiency loss, which should be of interest for future research.

Moreover, in tracking their competitors' beliefs, firms ignore aggregate shocks, and, as a result, their beliefs about aggregate variables are more inaccurate and noisy than the beliefs that feed into their prices. Thus, firms' expectations about aggregate variables are no longer the appropriate measures for their decisions once we consider market structures where firms do not price their products relative to all the other firms in the economy and only compete with a few others. These results are informative for surveys that aim to connect firms' expectations to their decisions: under oligopolistic competition, there's a wedge between firms' relevant expectations for their prices and their aggregate inflation expectations. These expectations are relatively unimportant for firms and do not have much impact on their pricing decisions.

Furthermore, the results in this paper have implications for policies that target expectations. In particular, they provide a new perspective on why managing inflation expectations might be less effective than what a model with monopolistic competition would suggest. Managers of oligopolistic firms do not directly care about aggregate inflation and are mainly concerned with how their competitors' prices respond to shocks. Thus, any communication about aggregate variables will be discounted by managers of such firms.

Nevertheless, this result does not necessarily rule out policies that target expectations but rather provides a new view on how those policies should be framed and *which* expectations they should target. An important takeaway from this paper is that for such a policy to be successful, it has to communicate the course of monetary policy to price-setters not in terms of how it will steer the overall prices but in terms of how it will affect their own industry prices. In other words, framing policy in terms of the aggregate variables will not gain as much attention and response from firms as it would if the news about the policy were to reach firms in terms of how their competitors would be affected. How policy can achieve these ends remains a question that deserves more investigation.

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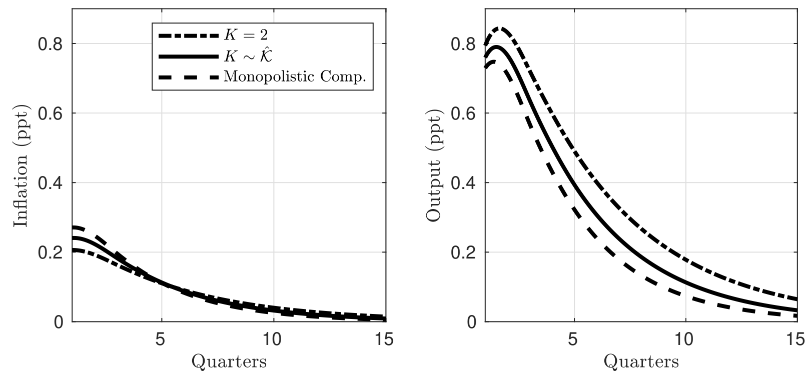
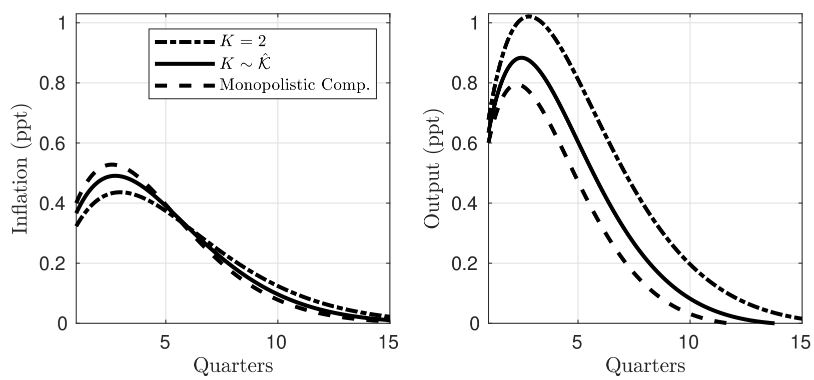
APPENDIX
(FOR ONLINE PUBLICATION)

A Additional Figures and Tables

Figure A.1: Robustness – Overall Effects of Oligopolistic Competition



(a) Alternative persistence for the growth of nominal aggregate demand ($\rho = 0.23$)



(b) Alternative discount rate for information ($\beta = 0.6$)

Notes: the figure shows the impulse response functions of output and inflation to a one percent expansionary shock to the growth of nominal demand in three models with alternative calibration of $\rho = 0.5$. The black lines are impulse responses in the benchmark model where the distribution of the number of competitors in the model is calibrated to the empirical distribution in the data (Figure 1). The dashed lines show the impulse responses in the model with monopolistic competition in all sectors. The dash-dotted lines show the impulse responses in the model where all sectors are composed of duopolies.

Table A.1: Robustness – Output and Inflation Across Models

(a) Alternative persistence for the growth of nominal aggregate demand ($\rho = 0.23$)

		Output				*Inflation*			
		Variance		*Persistence*		*Variance*		*Persistence*	
Model		*var(Y)* $^{\times 10^3}$	*amp. factor*	*half-life* ^{qtrs}	*amp. factor*	*var(π)* $^{\times 10^4}$	*damp. factor*	*half-life* ^{qtrs}	*amp. factor*
		(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*
Monopolistic Competition		1.30	1.00	3.24	1.00	1.63	1.00	3.22	1.00
Benchmark	$K \sim \hat{\mathcal{K}}$	1.62	1.25	3.97	1.23	1.42	0.87	3.91	1.21
2-Competitors	$K = 2$	2.13	1.64	4.61	1.42	1.17	0.72	4.54	1.41
4-Competitors	$K = 4$	1.69	1.31	4.06	1.25	1.37	0.84	4.01	1.25
8-Competitors	$K = 8$	1.55	1.19	3.86	1.19	1.46	0.89	3.83	1.19
16-Competitors	$K = 16$	1.49	1.15	3.78	1.17	1.49	0.92	3.75	1.16
32-Competitors	$K = 32$	1.46	1.13	3.75	1.16	1.51	0.93	3.72	1.16
∞-Competitors	$K \to \infty$	1.43	1.10	3.71	1.15	1.53	0.94	3.68	1.14

(b) Alternative discount rate for information (β=0.6)

		Output				*Inflation*			
		Variance		*Persistence*		*Variance*		*Persistence*	
Model		*var(Y)* $^{\times 10^3}$	*amp. factor*	*half-life* ^{qtrs}	*amp. factor*	*var(π)* $^{\times 10^4}$	*damp. factor*	*half-life* ^{qtrs}	*amp. factor*
		(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*
Monopolistic Competition		1.71	1.00	3.25	1.00	9.31	1.00	3.99	1.00
Benchmark	$K \sim \hat{\mathcal{K}}$	2.34	1.37	3.84	1.18	8.65	0.93	4.44	1.11
2-Competitors	$K = 2$	3.62	2.12	4.41	1.36	7.67	0.82	4.90	1.23
4-Competitors	$K = 4$	2.51	1.47	3.91	1.20	8.50	0.91	4.51	1.13
8-Competitors	$K = 8$	2.18	1.28	3.73	1.15	8.81	0.95	4.38	1.10
16-Competitors	$K = 16$	2.05	1.20	3.66	1.13	8.95	0.96	4.32	1.08
32-Competitors	$K = 32$	1.99	1.17	3.62	1.11	9.01	0.97	4.30	1.08
∞-Competitors	$K \to \infty$	1.94	1.13	3.59	1.10	9.07	0.97	4.27	1.07

Notes: the table presents robustness statistics for output and inflation responses across models with different number of competitors at the micro-level. Panel (a) presents results for an alternative calibration of persistence in the growth of nominal demand ($\rho = 0.23$). Panel (b) presents results for an alternative calibration of discount rate for information ($\beta = 0.6$). $Var(.)$ denotes the variance of output/inflation. *Half-life* denotes the length of the time that it takes for inflation/output to live half of its cumulative response in quarters. *Damp. factor* (*amp. factor*) denotes the the factor by which the relevant statistic is smaller (larger) in the corresponding model relative to the model with monopolistic competition.

B Mutual Information and Data Processing Inequality

In this paper, following the rational inattention literature, I use Shannon's mutual information function for measuring firms' attention. In case of Gaussian variables, this function takes a simple form: if X and Y are two Gaussian random variables, then the mutual information between them is given by $\mathscr{I}(X;Y) = \frac{1}{2}\log_2(\frac{\det(\Sigma_X)}{\det(\Sigma_{X|Y})})$, where $\Sigma_{X|Y}$ is the variance of X conditional on Y. Intuitively, the mutual information is bigger if the Y reveals more information about X, captured by a smaller $\det(\Sigma_{X|Y})$. In the other extreme, where X and Y are independent variables, $\Sigma_{X|Y} = \Sigma_X$ and the mutual information between them is zero, $\mathscr{I}(X;Y) = 0$. In other words, when X is independent of Y, then observing Y does not change the posterior of an agent about X and therefore reveals no information about X.

A result from Information Theory that I use in this Appendix is the *data processing inequality.* The following Lemma proves a weak version of this inequality for completeness.

Lemma B.1. *Let $X \to Y \to Z$ be a Markov chain. Then $\mathscr{I}(X;Y) \geq \mathscr{I}(X;Z)$.*

Proof. The inequality follows immediately from the chain rule of mutual information:[47]

$$\mathscr{I}(X;(Y,Z)) = \mathscr{I}(X;Y) + \mathscr{I}(X;Z|Y) = \mathscr{I}(X;Z) + \mathscr{I}(X;Y|Z)$$

Since $X \perp Z|Y$, we have $\mathscr{I}(X;Z|Y) = 0$. Thus, $\mathscr{I}(X;Y) = \mathscr{I}(X;Z) + \underbrace{\mathscr{I}(X;Y|Z)}_{\geq 0} \geq \mathscr{I}(X;Z)$. ■

C Formalizing the Static Model

This section formalizes the static game in Section 2. The Appendix is organized as follows. Subsection C.1 defines the concept of *richness* for a set of available information, and characterizes such a set. The main idea behind having a rich set of available information is to endow firms with the freedom of choosing their ideal signals given their capacity. Following this, Subsection C.2 proves the optimality of linear pricing strategies given Gaussian signals, and Subsection C.3 proves that when the set of available signals is rich, all firms prefer to see a *single* signal. Subsection C.4 shows that any equilibrium has an equivalent in terms of the joint distribution it implies for prices among the strategies in which all firms observe a single signal. In addition, it derives the conditions that such signals should satisfy. Subsection C.5 shows that the equilibrium is unique given this equivalence relationship. Subsection C.6 derives an intuitive reinterpretation of a firm's attention problem that is discussed in Section 2. Subsection C.7 contains the proofs of propositions and corollaries for the static model.

[47] For a formal definition of the chain rule see Cover and Thomas (2012).

C.1 A Rich Set of Available Information

Conventional rational inattention problems with a single decision-maker assume that the information environment is rich enough that the agent can directly choose the joint distribution of their action with the exogenous random variables that affect their payoff. In games, however, players' endogenous mistakes affect their rivals' payoffs, which leads to an economically novel problem: what does it mean to pay attention to a rival's mistake?

To answer this question, one needs to take a stance on the nature of such mistakes and how agents can pay attention to them. In particular, one way to model such attention strategies is to assume that mistakes are drawn by nature and are objectively accessible for players as sources of information. In the rest of this section, I construct a set of available information for the agents that permits players to pay attention to their rivals' mistakes. I then show that agents can choose their information structure as if they could choose the joint distribution of their action with the actions of their competitors, in a way that is similar to the case with a single decision-maker and an exogenous random variable.

To do so, we start by defining an explicit set of available signals from which agents can choose a subset to observe, subject to the cost of information acquisition. I then show that if this information set is *rich* enough, then choosing a signal from such a set is equivalent to the firms directly choosing the joint distribution of their prices with the prices of their competitors as well as the exogenous shocks in the economy.

Definition. *Let $\mathscr{S}$ be a set of Gaussian signals. We say $\mathscr{S}$ is* rich *if for any mean-zero possibly multivariate Gaussian distribution G, there is a vector of signals in $\mathscr{S}$ that are distributed according to G.*

To characterize a rich information structure, suppose in addition to $q \sim \mathcal{N}(0,1)$ there are countably infinite independent Gaussian noises in the economy, meaning that there is a set

$$\mathscr{B} \equiv \{q, e_1, e_2, \ldots\}$$

where all e_i's are independently and identically distributed according to a standard Normal distribution and are independent of q. Now, define $\mathscr{S}$ as the set of all finite linear combinations of the elements of $\mathscr{B}$ with coefficients in $\mathbb{R}$:

$$\mathscr{S} = \{a_0 q + \sum_{i=1}^{N} a_i e_{\sigma(i)}, N \in \mathbb{N}, (a_i)_{i=0}^{N} \subset \mathbb{R}^{N+1}, (\sigma(i))_{i=1}^{N} \subset \mathbb{N}\}.$$

We let $\mathscr{S}$ denote the set of all available signals in the economy.

Lemma C.1. *$\mathscr{S}$ is rich.*

Proof. Suppose G is a mean-zero Gaussian distribution. Thus, $G = \mathcal{N}(0,\Sigma)$, where $\Sigma \in \mathbb{R}^{N\times N}$ is a positive semi-definite matrix for some $N \in \mathbb{N}$. Since Σ is positive semi-definite, by Spectral theorem there exists $A \in \mathbb{R}^{N\times N}$ such that $\Sigma = A' \times A$. Choose any N elements of $\mathscr{B}$, and let $\mathbf{e}$ be the vector of those elements. Then $\mathbf{e} \sim \mathcal{N}(0, \mathbf{I}_{N\times N})$ where $\mathbf{I}_{N\times N}$ is the N dimensional identity matrix. By definition of $\mathscr{S}$, $S \equiv A'\mathbf{e} \in \mathscr{S}$. Now notice that $\mathbb{E}[S] = 0$, $var(S) = A' var(\mathbf{e}) A = \Sigma$. Hence, $S \sim \mathcal{N}(0, \Sigma) = G$. ■

The following Corollary shows that for any set of signals chosen in $\mathscr{S}$ (e.g., by competitors of a firm) and for any Gaussian distribution that has the distribution of $\mathscr{S}$ as a marginal distribution, there is an element in $\mathscr{S}$ that generates that distribution. In other words, if firms are choosing their signals from the set $\mathscr{S}$, any single firm can freely choose the joint distribution of their price with others' prices by choosing an element in $\mathscr{S}$.

Corollary C.2. *Let S be an N-dimensional vector of distinct signals with non-zero variance whose elements are in $\mathscr{S}$. Let $G = \mathcal{N}(0, \Sigma)$ be the distribution of S. Then for any $N+1$ dimensional Gaussian distribution, $\hat{G}$, one of whose marginals is G, there is at least one signal $\hat{s}$ in $\mathscr{S}$, such that $\hat{S} = (S, \hat{s}) \sim \hat{G}$.*

Proof. Suppose $\hat{G} = \mathcal{N}(0, \hat{\Sigma})$, where $\hat{\Sigma} \in \mathbb{R}^{(N+1)\times(N+1)}$ is a positive semi-definite matrix. Since G is a marginal of $\hat{G}$, without loss of generality, rearrange the vectors and columns of $\hat{\Sigma}$ such that $\hat{\Sigma} = \begin{bmatrix} x & \mathbf{y}' \\ \mathbf{y} & \Sigma \end{bmatrix}$. Since elements of S are distinct with non-zero variance, Σ is invertible and has positive determinant. Also $x > 0$. We can write $\det(\hat{\Sigma}) = \det(x\Sigma)\det(\mathbf{I}_{N\times N} - x^{-1}\Sigma^{-1}\mathbf{y}\mathbf{y}') \geq 0$, which implies $\det(\mathbf{I}_{N\times N} - x^{-1}\Sigma^{-1}\mathbf{y}\mathbf{y}') = 1 - x^{-1}\mathbf{y}'\Sigma^{-1}\mathbf{y} \geq 0 \Leftrightarrow x \geq \mathbf{y}'\Sigma^{-1}\mathbf{y}$, where the equality is given by Sylvester's determinant identity. Now, choose $e_{N+1} \in \mathscr{B}$ such that $e_{N+1} \perp S$. Such an e_{N+1} exists because all the elements of S are finite linear combinations of $\mathscr{B}$ and therefore are only correlated with a finite number of its elements, while $\mathscr{B}$ has countably infinite elements.[48] Let $\hat{s} \equiv \mathbf{y}'\Sigma^{-1}S + \begin{bmatrix} \sqrt{x - \mathbf{y}'\Sigma^{-1}\mathbf{y}} \\ \mathbf{0}_{N\times 1} \end{bmatrix} e_{N+1}$. Notice that $\hat{s} \in \mathscr{S}$ as it is a finite linear combination of the elements of $\mathscr{B}$. Notice that $cov(\hat{s}, S) = \mathbf{y}$ and $var(\hat{s}) = x$. Hence, $(\hat{s}, S) \sim \mathcal{N}(0, \hat{\Sigma})$. ■

C.2 Strategies

Every firm j,k chooses a vector of signals $S_{j,k} \in \mathscr{S}^{n_{j,k}}$, where $n_{j,k} \in \mathbb{N}$ is the number of signals that the firm chooses to observe, and a pricing strategy $p_{j,k} : S_{j,k} \to \mathbb{R}$ that maps the firms signal vector to a price. Thus, the set of firm j,k's pure strategies is

$$\mathscr{A}_{j,k} = \{\varsigma_{j,k} | \varsigma_{j,k} = (S_{j,k} \in \mathscr{S}^{n_{j,k}}, p_{j,k} : S_{j,k} \to \mathbb{R}), n_{j,k} \in \mathbb{N}\}.$$

The set of pure strategies for the game is $\mathscr{A} = \{\varsigma | \varsigma = (\varsigma_{j,k})_{j,k \in J\times K}, \varsigma_{j,k} \in \mathscr{A}_{j,k}, \forall j,k \in J \times K\}$.

[48] In fact, there are countably many elements in $\mathscr{B}$ that are orthogonal to S.

Optimality of linear pricing strategies. Here, I show that in any equilibrium it has to be the case that firms' pricing strategies are linear in their signals.

Lemma C.3. *Take a strategy $\varsigma = (S_{j,k}, p_{j,k})_{j,k \in J\times K} \in \mathscr{A}$. Then if ς is an equilibrium, then $\forall\, j,k \in J\times K$, $p_{j,k} = M'_{j,k} S_{j,k}$ for some $M_{j,k} \in \mathbb{R}^{n_{j,k}}$.*

Proof. A necessary condition for ς to be an equilibrium is that given information acquisition under a strategy $\varsigma \in \mathscr{A}$, all firms' pricing strategies under ς optimize firms' payoffs, i.e., $\forall (j,k) \in (J\times L)$,

$$p_{j,k}(S_{j,k}) = \text{argmin}_{p_{j,k}} \mathbb{E}[(p_{j,k} - (1-\alpha)q - \alpha \frac{1}{K-1} \sum_{l\neq k} p_{j,l}(S_{j,l}))^2 | S_{j,k}].$$

which leads to the following first order condition: $p^*_{j,k}(S_{j,k}) = (1-\tilde{\alpha})\mathbb{E}[q|S_{j,k}] + \tilde{\alpha}\mathbb{E}[p^*_j(S_j)|S_{j,k}]$, where $\tilde{\alpha} \equiv \frac{\alpha + \frac{\alpha}{K-1}}{1+\frac{\alpha}{K-1}} < 1$, and $p^*_j(S_j) \equiv K^{-1}\sum_{k\in K} p^*_{j,k}(S^*_{j,k})$. Iterating this forward, we arrive at

$$p^*_{j,k}(S_{j,k}) = \lim_{M\to\infty} ((1-\tilde{\alpha}) \sum_{m=0}^{M} \tilde{\alpha}^m \mathbb{E}^{(m)}_{j,k}[q] + \tilde{\alpha}^{M+1}\mathbb{E}^{(M+1)}_{j,k}[p^*_j(S_j)])$$

where $\mathbb{E}^{(0)}_{j,k}[q] \equiv \mathbb{E}[q|S_{j,k}]$ is firm j,k's expectation of the fundamental q, and $\forall m \geq 1$,

$$\mathbb{E}^{(m)}_{j,k}[q] = K^{-1} \sum_{l\in K} \mathbb{E}[\mathbb{E}^{(m-1)}_{j,l}[q]|S_{j,k}]$$

is firm j,k's m^{th} order higher order belief of its industry's average expectation of the fundamental. Similarly $\mathbb{E}^{(M+1)}_{j,k}[p^*_j(S_j)]$ is firm j,k's $M+1^{\text{th}}$ order belief of their industry price. Since $\tilde{\alpha} < 1$, the later term in the limit converges to zero (as long as firms' expectations of their own industry prices are not explosive under the strategy ς) and we have:

$$p^*_{j,k}(S_{j,k}) = (1-\tilde{\alpha}) \sum_{m=0}^{\infty} \tilde{\alpha}^m \mathbb{E}^{(m)}_{j,k}[q]. \tag{C.1}$$

Now, it only remains to show that $\mathbb{E}^{(m)}_{j,k}[q]$ is linear in $S_{j,k}$ for all m, which can be shown by induction. Notationwise let $\forall j,k$, let $\Sigma_{q,S_{j,k}} \equiv cov(S_{j,k}, q) = \mathbb{E}[qS'_{j,k}]$. Also given j,k, $\forall l \neq k$, $\Sigma_{S_{j,l},S_{j,k}} = cov(S_{j,k}, S_{j,l}) = \mathbb{E}[S_{j,l}S'_{j,k}]$ and $\Sigma_{S_{j,k}} = var(S_{j,k}) = \mathbb{E}[S_{j,k}S'_{j,k}]$. Now, for $m = 0$, $\mathbb{E}^{(0)}_{j,k}[q] = \mathbb{E}[q|S_{j,k}] = \Sigma_{q,S_{j,k}}\Sigma^{-1}_{S_{j,k}}S_{j,k}$,, which implies that 0^{th} order expectations of firms are linear in their signals. Now suppose $\forall j,l$ $\mathbb{E}^{(m)}_{j,l}[q] = A_{j,l}(m)'S_{j,l}$ for some $A_{j,l}(m) \in \mathbb{R}^{n_{j,l}}$. Thus,

$$\mathbb{E}^{(m+1)}_{j,k}[q] = \underbrace{K^{-1}(A_{j,l}(m) + \sum_{l\neq k} A_{j,l}(m)\Sigma_{S_{j,l},S_{j,k}}\Sigma^{-1}_{S_{j,k}})'}_{\equiv A^{(m+1)}_{j,k} \in \mathbb{R}^{n_{j,k}}} S_{j,k},$$

which shows that the $(m+1)^{\text{th}}$ order expectation is linear in $S_{j,k}$.[49] ■

[49]Here, I have assumed $\Sigma_{S_{j,k}}$ is invertible, which is without loss of generality: if $\Sigma_{j,k}$ is not invertible, since all signals in $S_{j,k}$ are non-zero then it must be the case that $S_{j,k}$ contains co-linear signals. In that case we can exclude the redundant signals without changing the posterior of the firm.

Thus, as long as firms choose Gaussian signals, pricing strategies are linear in those signals in any equilibrium. Furthermore, the proof of Lemma C.3 gives us an expression for the coefficients of these optimal linear strategies, which is characterized in the following Corollary.

Corollary C.4. *If* $\varsigma = (S_{j,k} \in \mathscr{S}^{n_{j,k}}, p_{j,k}(S_{j,k}) = M'_{j,k} S_{j,k})_{j,k \in J\times K} \in \mathscr{A}$ *is an equilibrium, then* $\forall\, j,k \in J \times K$, $M_{j,k} = ((1-\alpha)\Sigma_{q,S_{j,k}}\Sigma^{-1}_{S_{j,k}} + \alpha \frac{1}{K-1}\sum_{l\neq k}\Sigma_{S_{j,l},S_{j,k}}\Sigma^{-1}_{S_{j,k}})'$.

Proof. From the proof of Lemma C.3 that if ς is an equilibrium, then pricing strategies should satisfy the following optimality condition:

$$M_{j,k}S_{j,k} = (1-\alpha)\mathbb{E}[q|S_{j,k}] + \alpha\frac{1}{K-1}\sum_{l\neq k}\mathbb{E}[M'_{j,l}S_{j,l}|S_{j,k}].$$

Thus, $M_{j,k} = ((1-\alpha)\Sigma_{q,S_{j,k}}\Sigma^{-1}_{S_{j,k}} + \alpha \frac{1}{K-1}\sum_{l\neq k}\Sigma_{S_{j,l},S_{j,k}}\Sigma^{-1}_{S_{j,k}})'$. ∎

The results in this section impose a set of necessary conditions on equilibrium strategies. Thus, without loss of generality, we can restrict the set of strategies that we consider to those with linear pricing schemes that satisfy Corollary C.4:

$$\mathscr{A}^* = \{\varsigma \in \mathscr{A} | \varsigma \text{ satisfies Corollary C.4}\}. \tag{C.2}$$

C.3 The Attention Problem of Firms

With the optimal pricing strategies at hand, this section characterizes the information choice problem of the firms.

Consider a strategy $\varsigma \in \mathscr{A}^*$ such that $\varsigma = (S_{j,k} \in \mathscr{S}^{n_{j,k}}, p_{j,k} = M'_{j,k}S_{j,k})_{j,k\in J\times K}$. For ease of notation let $p(\varsigma_{j,k}) \equiv M'_{j,k}S_{j,k}$ denote the optimal price of firm j,k under the given strategy. Also, let $\varsigma_{-(j,k)} \equiv \varsigma \backslash \varsigma_{j,k}$ denote the vector of the strategies for the competitors of firms j,k. Also, for any given firm $j,k \in J \times K$, let $\theta_{j,k}(\varsigma_{-(j,k)}) \equiv (q, (p(\varsigma_{j,l}))_{l\neq k}, (p(\varsigma_{m,n}))_{m\neq j, n\in K})'$ be the augmented vector of the fundamental, the prices of other firms in j,k's industry, and the prices of all other firms in the economy, so that given ς it is optimal for the firm to track a certain linear combination of $\theta_{j,k}(\varsigma_{-(j,l)})$, where the weights are given by a vector $\mathbf{w}$ defined as:

$$\mathbf{w} \equiv (1-\alpha, \underbrace{\frac{\alpha}{K-1}, \dots, \frac{\alpha}{K-1}}_{K-1\,\text{times}}, \underbrace{0,0,0,\dots,0}_{(J-1)\times K\,\text{times}})'.$$

Given this notation observe that firm j,k's problem, as defined in the text, reduces to considering deviations from ς to solve

$$\begin{aligned} \min_{\hat{\varsigma}_{j,k}\in\mathscr{A}_{j,k}} \; & L_{j,k}(\hat{\varsigma}_{j,k}, \varsigma_{-(j,k)}) \equiv \mathbb{E}[(p(\hat{\varsigma}_{j,k}) - \mathbf{w}'\theta_{j,k}(\varsigma_{-(j,k)}))^2 | S(\hat{\varsigma}_{j,k})] \\ s.t. \quad & \mathscr{I}(S(\hat{\varsigma}_{j,k}); \theta_{j,k}(\varsigma_{-(j,k)})) \leq \kappa, \end{aligned} \tag{C.3}$$

where $S(\hat{\varsigma}_{j,k})$ denotes the signals in $\mathscr{S}$ that j,k observes under the strategy $\hat{\varsigma}_{j,k}$ and given the joint distribution of $(S(\hat{\varsigma}_{j,k}), \theta_{j,k}(\varsigma_{-(j,k)}))$, the mutual information is defined in Section B. Given this notation, we can restate the definition of the equilibrium as

Definition. *An equilibrium is a strategy $\varsigma \in \mathscr{A}$ such that $\forall j,k \in J \times K$*

$$\varsigma_{j,k} = argmin_{\varsigma'_{j,k} \in \mathscr{A}_{j,k}} L_{j,k}(\varsigma'_{j,k}, \varsigma_{-(j,k)}) \; s.t. \, \mathscr{I}(S(\varsigma_{j,k}); \theta_{j,k}(\varsigma_{-(j,k)})) \leq \kappa. \tag{C.4}$$

It is worth noting that the solution to this problem is only unique in the implied joint distribution of prices and the fundamental, a result that we prove in Section C.5. However, in terms of strategies, there may be many that generate that unique distribution. To see this, one can define the following relation on the set of deviations of firm j,k, given a strategy $\varsigma \in \mathscr{A}^*$, and note that it is an equivalence.

Definition. *Given a strategy $\varsigma = (\varsigma_{j,k}, \varsigma_{-(j,k)}) \in \mathscr{A}^*$, any two distinct sub-strategies $\{\varsigma^1_{j,k}, \varsigma^2_{j,k}\} \subset \mathscr{A}_{j,k}$ for firm j,k are equivalent under ς (shown by $\varsigma^1_{j,k} \sim_{j,k|\varsigma} \varsigma^2_{j,k}$) if they both yield the same payoff, $L_{j,k}(\varsigma^1_{j,k}, \varsigma_{-(j,k)}) = L_{j,k}(\varsigma^2_{j,k}, \varsigma_{-(j,k)})$, where $L_{j,k}(.,.)$ is defined as in Equation (C.3).*

Note that $\forall j,k \in J \times K$ and $\forall \varsigma \in \mathscr{A}^*$, $\sim_{j,k|\varsigma}$ is an equivalence relation as reflexivity, symmetry and transitivity are trivially satisfied by properties of equality. Therefore, by definition the agent is indifferent between elements of an equivalence class. Now, given $\varsigma = (\varsigma_{j,k}, \varsigma_{-(j,k)}) \in \mathscr{A}^*$, let $[\hat{\varsigma}_{j,k}]_\varsigma \equiv \{\varsigma'_{j,k} \in \mathscr{A}_{j,k} | \varsigma'_{j,k} \sim_{j,k|\varsigma} \hat{\varsigma}_{j,k}\}$.

We can now state the important result that any equilibrium strategy should be such that every firm observes only one signal. The following Lemma shows that for any given strategy, there is always a deviation with a single dimensional signal that requires less attention but yields the same payoff. Therefore, for any strategy of others, the optimal signal choice of a firm is one-dimensional.

Lemma C.5. *For any $j,k \in J \times K$, $\forall \varsigma = (\varsigma_{j,k}, \varsigma_{-(j,k)}) \in \mathscr{A}^*$, $\exists \hat{\varsigma}_{j,k} \in [\varsigma_{j,k}]_\varsigma$ such that the agent observes only one signal under $\hat{\varsigma}_{j,k}$ and $\mathscr{I}(S(\hat{\varsigma}_{j,k}); \theta_{j,k}(\varsigma_{-(j,k)})) \leq \mathscr{I}(S(\varsigma_{j,k}); \theta_{j,k}(\varsigma_{-(j,k)}))$. Moreover, $\hat{\varsigma}_{j,k}$ does not alter the covariance of firm j,k's price with the fundamental and the prices of all other firms in the economy under ς.*

Proof. I prove this lemma by constructing such a strategy. Given $\varsigma \in \mathscr{A}^*$, let $\Sigma_{\varsigma_{j,k}} \equiv var(S(\varsigma_{j,k}))$, $\Sigma_{\theta_{j,k},\varsigma_{j,k}} \equiv cov(\theta_{j,k}(\varsigma_{-(j,k)}), S(\varsigma_{j,k}))$ and $\Sigma_{\theta_{j,k}} \equiv var(\theta_{j,k}(\varsigma_{-(j,k)}))$. Thus,

$$(S(\varsigma_{j,k}), \theta_{j,k}(\varsigma_{-(j,k)})) \sim \mathscr{N}(0, \begin{bmatrix} \Sigma_{\varsigma_{j,k}} & \Sigma'_{\theta_{j,k},\varsigma_{j,k}} \\ \Sigma_{\theta_{j,k},\varsigma_{j,k}} & \Sigma_{\theta_{j,k}} \end{bmatrix}).$$

Moreover, since $\varsigma \in \mathscr{A}^*$, then pricing strategies are linear, and by Corollary C.4

$$p_{j,k}(\varsigma) = \mathbf{w}'\mathbb{E}[\theta_{j,k}(\varsigma_{-(j,k)}) | S(\varsigma_{j,k})] = \mathbf{w}'\Sigma_{\theta_{j,k},\varsigma_{j,k}} \Sigma^{-1}_{\varsigma_{j,k}} S(\varsigma_{j,k})$$

Notice that

$$L_{j,k}(\varsigma_{j,k}, \varsigma_{-(j,k)}) = \mathbf{w}' var(\theta_{j,k}(\varsigma_{-(j,k)}) | S(\varsigma_{j,k}))\mathbf{w} = \mathbf{w}'\Sigma_{\theta_{j,k}}\mathbf{w} - \mathbf{w}'\Sigma_{\theta_{j,k},\varsigma_{j,k}} \Sigma^{-1}_{\varsigma_{j,k}} \Sigma'_{\theta_{j,k},\varsigma_{j,k}}\mathbf{w}.$$

Now, let $\hat{s}_{j,k} \equiv \mathbf{w}'\Sigma_{\theta_{j,k},\varsigma_{j,k}}\Sigma^{-1}_{\varsigma_{j,k}}S(\varsigma_{j,k})$. Clearly, $\hat{s}_{j,k} \in \mathscr{S}$ as it is a finite linear combination of the elements of $S_{j,k}$, and $\mathscr{S}$ is rich. Define $\hat{\varsigma}_{j,k} \equiv (\hat{s}_{j,k}, 1) \in \mathscr{A}_{j,k}$. We have

$$L_{j,k}(\hat{\varsigma}_{j,k}, \varsigma_{-(j,k)}) = \mathbf{w}' var(\theta_{j,k}(\varsigma_{-(j,k)})|\hat{s}_{j,k})\mathbf{w} = L_{j,k}(\varsigma_{j,k}, \varsigma_{-(j,k)}).$$

Thus, $\hat{\varsigma}_{j,k} \in [\varsigma_{j,k}]_{\varsigma}$. Also, observe that $\theta_{j,k}(\varsigma_{-(j,k)}) \perp \hat{s}_{j,k}|S(\varsigma_{j,k})$. Therefore, by the data processing inequality in Lemma B.1, $\mathscr{I}(\hat{s}_{j,k};\theta_{j,k}(\varsigma_{-(j,k)})) \le \mathscr{I}(S(\varsigma_{j,k});\theta_{j,k}(\varsigma_{-(j,k)}))$. Finally, observe that $p_{j,k}(\hat{\varsigma}_{j,k}, \varsigma_{-(j,k)}) = p_{j,k}(\varsigma) = \mathbf{w}'\Sigma_{\theta_{j,k},\varsigma_{j,k}}\Sigma^{-1}_{\varsigma_{j,k}}S(\varsigma_{j,k})$. Thus, the covariance of j,k's price with all the elements of $\theta_{j,k}(\varsigma_{-(j,k)})$ remains unchanged when j,k deviates from $\varsigma_{j,k}$ to $\hat{\varsigma}_{j,k}$. ■

C.4 Equilibrium Signals

Let $\mathscr{E} \equiv \{\varsigma \in \mathscr{A}|\varsigma$ is an equilibrium as stated in Statement (C.4)$\}$ denote the set of equilibria for the game. The following definition states an equivalence relation among the equilibria based on the joint distribution of prices and the fundamental.

Definition 3. *Suppose $\{\varsigma_1, \varsigma_2\} \subset \mathscr{E}$ are two equilibria for the game. We call these two equilibria equivalent and write $\varsigma_1 \sim_{\mathscr{E}} \varsigma_2$ if they imply the same joint distribution for prices of firms and the fundamental. Formally, $\varsigma_1 \sim_{\mathscr{E}} \varsigma_2$ if $(q, p_{j,k}(\varsigma_1))_{j,k \in J\times K} \sim G$ implies $(q, p_{j,k}(\varsigma_2))_{j,k\in J\times K} \sim G$. This is trivially an equivalence relation as it satisfies reflexivity, symmetry and transitivity by properties of equality.*

Lemma C.6. *Suppose $\varsigma \in \mathscr{A}$ is an equilibrium and let $\mathscr{A}^{**} \equiv \{\varsigma \in \mathscr{A}|\varsigma = (s_{j,k} \in \mathscr{S}, 1)_{j,k\in J\times K}\}$ denote the the set of all strategies in which (1) each firm sees only one signal and (2) the realization of the signal is the optimal price for the firm under the posterior belief generated by it (in other words, strategies in which the signal is the recommended price for the firm). Then, there exists a strategy $\hat{\varsigma} \in \mathscr{A}^{**}$ that is equivalent to ς: $\hat{\varsigma} \sim_{\mathscr{E}} \varsigma$.*

Proof. The proof is by construction. Since ς is an equilibrium it solves all firms' problems. Start from the first firm in the economy and perform the following iteration process for all firms: from previous section, we know firm $1,1$ has a strategy $\hat{\varsigma}_{1,1} = (s_{1,1} \in \mathscr{S}, 1)$ that is equivalent to $\varsigma_{1,1}$ given ς. Create a new strategy $\varsigma^{1,1} = (\hat{\varsigma}_{1,1}, \varsigma_{-(1,1)})$. We know that $\varsigma^{1,1}$ implies the same joint distribution as ς for the prices of all firms in the economy because we have only changed firm $1,1$'s strategy, and by the previous lemma $\hat{\varsigma}_{1,1}$ does not alter the joint distribution of prices. Now notice that $\varsigma^{1,1}$ is also an equilibrium because (1) firm $1,1$ was indifferent between $\varsigma_{1,1}$ and $\hat{\varsigma}_{1,1}$ and (2) the problem of all other firms has not changed because $1,1$'s price is the same under both strategies. Now, repeat the same thing for firm $1,2$ given $\varsigma^{1,1}$ and so on. At any step given $\varsigma^{j,k}$ repeat the process for $j, k+1$ (or $j+1, 1$ if $k = K$) until the last firm in the economy. At the last step, we have $\varsigma^{J,K} = (\hat{\varsigma}_{j,k})_{j,k\in J\times K}$, which is (1) an equilibrium and (2) implies the same joint distribution among prices and fundamentals as ς. Moreover, notice that $\varsigma^{J,K} \in \mathscr{A}^{**}$. ■

So far, we have shown that any equilibrium has an equivalent in $\mathscr{A}^{**}$. Thus, insofar we are interested in the joint distribution of prices and the fundamental, it suffices to only look at equilibria in this set. The next step is to simplify the problem more but casting it in terms of choosing a joint distribution rather than a particular signal in the set $\mathscr{S}$.

Lemma C.7. *Suppose $\varsigma \in \mathscr{A}^{**}$ is an equilibrium. Then, any deviation for firm j,k in equivalent to choosing a Gaussian joint distribution between their price and $\theta_{j,k}(\varsigma_{-(j,k)})$. Moreover, if two different deviations of j,k imply the same joint distribution, then the firm is indifferent between them.*

Proof. Given ς, let $\Sigma_{\theta_{j,k}}$ be such that $\theta_{j,k}(\varsigma_{-(j,k)}) \sim \mathcal{N}(0, \Sigma_{\theta_{j,k}})$. Notice that $\Sigma_{\theta_{j,k}}$ has to be invertible; otherwise, there must a firm whose signal is perfectly correlated with at least one of the elements in the vector $\theta_{j,k}(\varsigma_{-(j,k)})$. But such perfect correlation violates the capacity constraint of that firm as it would imply that the firm is processing infinite bits of information, which is a contradiction with the assumption that ς is an equilibrium.[50]

From Lemma C.5 we know that it suffices to look at deviations of the form $(s_{j,k} \in \mathscr{S}, 1)$. First, observe that any deviation of the firm j,k generates a Gaussian joint distribution for $(s_{j,k}, \theta_{j,k}(\varsigma_{-(j,k)}))$ as $s_{j,k} \in \mathscr{S}$. Moreover, suppose $G = \mathcal{N}(0, \begin{bmatrix} x & \mathbf{y}' \\ \mathbf{y} & \Sigma_{\theta_{j,k}} \end{bmatrix})$ is a Gaussian distribution. Since $\Sigma_{\theta_{j,k}}$ is invertible, Corollary C.2 implies that there is a signal $s_{j,k} \in \mathscr{S}$, such that $(s_{j,k}, \theta_{j,k}(\varsigma_{-(j,k)})) \sim G$.

To prove the last part, suppose for the two signals $s^1_{j,k}$ and $s^2_{j,k}$ in $\mathscr{S}$, $(s^1_{j,k}, \theta_{j,k}(\varsigma_{-(j,k)}))$ and $(s^2_{j,k}, \theta_{j,k}(\varsigma_{-(j,k)}))$ have the same joint distribution. Then,

$$var(\theta_{j,k}(\varsigma_{-(j,k)})|s^1_{j,k}) = var(\theta_{j,k}(\varsigma_{-(j,k)})|s^2_{j,k})$$

which implies that $L_{j,k}((s^1_{j,k}, 1), \varsigma_{-(j,k)}) = L_{j,k}((s^2_{j,k}, 1), \varsigma_{-(j,k)})$. Moreover, given that the conditional variances under both signals are the same, we have

$$\mathscr{I}(s^1_{j,k}; \theta_{j,k}(\varsigma_{-(j,k)})) = \mathscr{I}(s^2_{j,k}; \theta_{j,k}(\varsigma_{-(j,k)})).$$

Therefore, the firm is indifferent between $s^1_{j,k}$ and $s^2_{j,k}$. ■

Lemma C.7 ensures us that instead of considering all the possible deviations in $\mathscr{S}$, we can simply optimize over all the possible joint distributions that a firm can consider. Moreover, if there is a distribution that solves the firm's problem, then the Lemma implies there is at least one signal in the set of available signals that generates that joint distribution. Given this result, the following Lemma derives the form of the optimal signals:

Lemma C.8. *Suppose $\varsigma = (s^*_{j,k} \in \mathscr{S}, 1) \in \mathscr{A}^{**}$ is an equilibrium, then $\forall j,k \in J \times K$, firm j,k's*

[50]Recall, for any two one dimensional Normal random variables X and Y, $I(X,Y) = -\frac{1}{2}\log_2(1-\rho^2_{X,Y})$, where $\rho_{X,Y}$ is the correlation of X and Y. Notice that $\lim_{\rho^2 \to 1} I(X,Y) \to +\infty$.

optimal signal has the following form:

$$s^*_{j,k} = \lambda \mathbf{w}'\theta_{j,k}(\varsigma_{-(j,k)}) + z_{j,k},\ z_{j,k} \perp \theta_{j,k}(\varsigma_{-(j,k)}),\ var(z_{j,k}) = \lambda(1-\lambda)var(\mathbf{w}'\theta_{j,k}(\varsigma_{-(j,k)})).$$

Proof. For firm $j,k \in J \times K$, let $\Sigma_{\theta_{j,k}}$ denote the covariance matrix of $\theta_{j,k}(\varsigma_{-(j,k)})$. From Lemma C.5, we know that for any equilibrium, there is an equivalent equilibrium among strategies of the form $(s_{j,k} \in \mathscr{S}, 1)$.

Now, for a given $s_{j,k} \in \mathscr{S}$, let $\begin{bmatrix} x^2 & \mathbf{y}' \\ \mathbf{y} & \Sigma_{\theta_{j,k}} \end{bmatrix} \equiv var\big((s_{j,k}, \theta_{j,k}(\varsigma_{-(j,k)}))\big)$. First, recall that for $(s_{j,k} \in \mathscr{S}, 1)$ to be optimal, it has to be the case that $p_{j,k} = \mathbf{w}'\mathbb{E}[\theta_{j,k}(\varsigma_{-(j,k)})|s_{j,k}] = x^{-2}\mathbf{w}'\mathbf{y}s_{j,k}$. Thus,

$$x^2 = \mathbf{w}'\mathbf{y}.$$

Now, given $s_{j,k} \in \mathscr{S}$, the firm's loss in profits is $var(\mathbf{w}'\theta_{j,k}(\varsigma_{-(j,k)})|s_{j,k}) = \mathbf{w}'\Sigma_{\theta_{j,k}}\mathbf{w} - x^{-2}(\mathbf{w}'\mathbf{y})^2$ and the capacity constraint is $\frac{1}{2}\log_2(|\mathbf{I} - x^{-2}\Sigma^{-1}_{\theta_{j,k}}\mathbf{y}\mathbf{y}'|) \geq -\kappa \Leftrightarrow x^{-2}\mathbf{y}'\Sigma^{-1}_{\theta_{j,k}}\mathbf{y} \leq \lambda \equiv 1 - 2^{-2\kappa}$. Moreover, from the previous lemma we know that for any $(x, \mathbf{y})$ such that $\begin{bmatrix} x^2 & \mathbf{y}' \\ \mathbf{y} & \Sigma_{\theta_{j,k}} \end{bmatrix} \geq 0$, there is a signal in $\mathscr{S}$ that creates this joint distribution. Therefore, we let the agent choose $(x, \mathbf{y})$ freely to solve

$$\min_{(x,\mathbf{y})} \mathbf{w}'\Sigma_{\theta_{j,k}}\mathbf{w} - x^{-2}(\mathbf{w}'\mathbf{y})^2\ s.t.\ x^{-2}\mathbf{y}'\Sigma^{-1}_{\theta_{j,k}}\mathbf{y} \leq \lambda$$

The solution can be derived by taking first order conditions, but there is even simpler a way: by Cauchy-Schwarz inequality $x^{-2}(\mathbf{w}'\mathbf{y})^2 = x^{-2}(\Sigma^{\frac{1}{2}}_{\theta_{j,k}}\mathbf{w})'(\Sigma^{-\frac{1}{2}}_{\theta_{j,k}}\mathbf{y}) \leq x^{-2}(\mathbf{w}'\Sigma_{\theta_{j,k}}\mathbf{w})(\mathbf{y}'\Sigma^{-1}_{\theta_{j,k}}\mathbf{y})$. Therefore,

$$\mathbf{w}'\Sigma_{\theta_{j,k}}\mathbf{w} - x^2(\mathbf{w}'\mathbf{y})^2 \geq (\mathbf{w}'\Sigma_{\theta_{j,k}}\mathbf{w})(1 - x^{-2}\mathbf{y}'\Sigma_{\theta_{j,k}}\mathbf{y}) \geq (1-\lambda)\mathbf{w}'\Sigma_{\theta_{j,k}}\mathbf{w},$$

where, the last line uses the capacity constraint. This defines a global lower-bound for the objective of the firm that holds for any choice of $(x, \mathbf{y})$. However, this global minimum is attained if both the Cauchy-Schwarz inequality and the capacity constraint bind. From the properties of the Cauchy-Schwarz inequality, we know it binds if and only if $x^{-1}\Sigma^{-\frac{1}{2}}_{\theta_{j,k}}\mathbf{y} = c_0\Sigma^{\frac{1}{2}}_{\theta_{j,k}}\mathbf{w}$ for some constant c_0. Therefore, there is a unique vector $x^{-1}\mathbf{y}$ that attains the global minimum of the agent's problem given their constraint:$x^{-1}\mathbf{y} = c_0\Sigma_{\theta_{j,k}}\mathbf{w}$. Now, the capacity constraint binds if $c_0 = \sqrt{\frac{\lambda}{\mathbf{w}'\Sigma_{\theta_{j,k}}\mathbf{w}}}$. Together with $x^2 = \mathbf{w}'\mathbf{y}$, this gives us the unique $(x, \mathbf{y})$:$\mathbf{y} = \lambda\Sigma_{\theta_{j,k}}\mathbf{w}$, $x = \sqrt{\lambda\mathbf{w}'\Sigma_{\theta_{j,k}}\mathbf{w}}$. Finally, to find a signal that creates this joint distribution, choose $s^*_{j,k} \in \mathscr{S}$ such that

$$s^*_{j,k} = \lambda\mathbf{w}'\theta_{j,k}(\varsigma_{-(j,k)}) + z_{j,k},\ z_{j,k} \perp \theta_{j,k}(\varsigma_{-(j,k)}),\ var(z_{j,k}) = \lambda(1-\lambda)\mathbf{w}'\Sigma_{\theta_{j,k}}\mathbf{w}.$$

notice that $cov(s^*_{j,k}, \theta_{j,k}(\varsigma_{-(j,k)})) = \lambda\Sigma_{\theta_{j,k}}\mathbf{w}$, and $var(s^*_{j,k}) = \lambda\mathbf{w}'\Sigma_{\theta_{j,k}}\mathbf{w}$. Notice that this implies the equilibrium set of signals are

$$s^*_{j,k} = \lambda(1-\alpha)q + \lambda\alpha\frac{1}{K-1}\sum_{l\neq k} s^*_{j,l} + z_{j,k},\ z_{j,t} \perp (q, s_{m,n})_{(m,n)\neq(j,k)}$$

where $var(z_{j,t}) = \lambda(1-\lambda)var((1-\alpha)q + \alpha\frac{1}{K-1}\sum_{l\neq k} s^*_{j,l})$. ■

C.5 Uniqueness of Equilibrium in the Joint Distribution of Prices

Having specified the equilibrium signals, the following Lemma proves that all equilibria imply the same joint distribution for the vector of all firms' prices along with the fundamental.

Lemma C.9. *Suppose the degree of strategic complementarity is strcitly less than 1, $\alpha \in [0,1)$. Then, the quotient set $\mathscr{E}/\sim_{\mathscr{E}}$ is non-empty and a singleton, i.e., all equilibria of the game are equivalent under the relationship in Definition 3.*

Proof. I show this by directly characterizing the equilibrium. From the previous section, we know that any equilibrium is equivalent to one in strategies of $\mathscr{A}^{**}$. Suppose that $(s^*_{j,k}, 1)_{j,k\in J\times K} \in \mathscr{A}^{**}$ is an equilibrium, and notice that in this equilibrium, every firm sets their price equal to their signal, $p_{j,k} \equiv s^*_{j,k}$. Also, Lemma C.8 showed that in this equilibrium, signals are of the following form:

$$p_{j,k} = \lambda(1-\alpha)q + \lambda\alpha\frac{1}{K-1}\sum_{l\neq k} p_{j,l} + z_{j,k},\ z_{j,k} \perp (q, p_{m,n})_{(m,n)\neq(j,k)}$$

where $var(z_{j,t}) = \lambda(1-\lambda)var((1-\alpha)q + \alpha\frac{1}{K-1}\sum_{l\neq k} p_{j,l})$. Now, we want to find all the joint distributions for $(q, p_{j,k})_{j,k\in J\times K}$ that satisfy this rule. Since all signals are Gaussian, the joint distributions will also be Gaussian.

To derive this distribution, we start by characterizing the covariance of any firm's price with the fundamental. For any industry j, let $p_j \equiv (p_{j,k})_{k\in K}$ and $z_j \equiv (z_{j,k})_{k\in K} \perp q$. Moreover, for ease of notation, in this section, let $\gamma \equiv \frac{1}{K-1}$. Now, the equilibrium condition implies $p_j = \lambda(1-\alpha)\mathbf{1}q + \lambda\alpha\gamma(\mathbf{1}\mathbf{1}' - \mathbf{I})p_j + z_j$ where $\mathbf{1}$ is the unit vector in $\mathbb{R}^K$, and $\mathbf{I}$ is identity matrix in $\mathbb{R}^{K\times K}$ (therefore $\mathbf{1}\mathbf{1}' - \mathbf{I}$ is a matrix with zeros on diagonal and 1's elsewhere). With some algebra it is straightforward to show that $cov(p_j, q) = \frac{\lambda-\lambda\alpha}{1-\lambda\alpha}\mathbf{1}$. Thus, in any equilibrium, the covariance of any firm's price with the fundamental q has be to equal to

$$\delta \equiv \frac{\lambda - \lambda\alpha}{1-\lambda\alpha} \tag{C.5}$$

Next, we show that the prices of any two firms in separate industries are orthogonal conditional on the fundamental. Let p_j be the vector of prices in industry j as defined above. Pick any firm from any other industry $l, m \in J\times K, l \neq j$. Notice that by the equilibrium conditions z_j is orthogonal to $p_{l,m}$. Now, notice that

$$cov(p_j, p_{l,m}) = \lambda(1-\alpha)\mathbf{1}\underbrace{cov(q, p_{l,m})}_{=\delta} + \lambda\alpha\gamma(\mathbf{1}\mathbf{1}' - \mathbf{I})cov(p_j, p_{l,m}) + \underbrace{cov(z_j, p_{l,m})}_{=0}.$$

With some algebra, we get $cov(p_j, p_{l,m}) = \delta^2\mathbf{1} \Rightarrow cov(p_j, p_{l,m}|q) = 0$. Therefore, in any equilibrium, prices of any two firms in two different industries are only correlated through the fundamental. This implies that firms do not pay attention to mistakes of firms in other industries.

Now, we only need to derive the joint distribution of prices within industries. We have $p_j = \mathbf{B}(\lambda(1-\alpha)\mathbf{1}q + z_j)$ where $\mathbf{B} \equiv \frac{1}{1+\alpha\lambda\gamma}\mathbf{I} + \frac{\alpha\lambda\gamma}{(1+\alpha\lambda\gamma)(1-\alpha\lambda)}\mathbf{1}\mathbf{1}'$. This gives $p_j = \delta\mathbf{1}q + \mathbf{B}z_j$, where $\mathbf{B}z_j \perp q$. This corresponds to the decomposition of the prices of firms to parts that are correlated with the fundamental and their mistakes. The vector $\mathbf{B}z_j$ is the vector of firms' mistakes in industry j, and is the same as the vector v_j in the text. Let $\Sigma_{z,j} = cov(z_j, z_j)$ and $\Sigma_{p,j} = cov(p_j, p_j)$. We have $\Sigma_{p,j} = \delta^2\mathbf{1}\mathbf{1}' + \mathbf{B}\Sigma_{z,j}\mathbf{B}'$. Also, since $z_{j,k} \perp p_{j,l\neq k}$, we have $\mathbf{D}_j \equiv cov(p_j, z_j) = \mathbf{B}\Sigma_{z,j}$ where $\mathbf{D}_j$ is a diagonal matrix whose k'th element on the diagonal is $var(z_{j,k})$. From the equilibrium conditions we have

$$
\begin{aligned}
var(z_{j,k}) &= \lambda(1-\lambda)var((1-\alpha)q + \alpha\gamma\sum_{l\neq k} p_{j,l}) \\
&= \lambda(1-\lambda)(1-\alpha)^2 + \lambda(1-\lambda)\alpha^2\gamma^2\mathbf{w}_k'\Sigma_{p,j}\mathbf{w}_k + 2\lambda(1-\lambda)\alpha(1-\alpha)\delta
\end{aligned}
$$

where $\mathbf{w}_k$ is a vector such that $\mathbf{w}_k' p_j = \sum_{l\neq k} p_{j,l}$. This gives K linearly independent equations and K unknowns in terms of the diagonal of $\mathbf{D}_j$. Guess that the unique solution to this is symmetric. After some algebra, we get that the implied distribution for prices is such that

$$
var(p_{j,k}) = \frac{1-\alpha\lambda}{1-\alpha\tilde{\lambda}}\lambda^{-1}\delta^2, \forall j,k;\ cov(p_{j,k}, p_{j,l}) = \frac{1-\alpha\lambda}{1-\alpha\tilde{\lambda}}\frac{\tilde{\lambda}}{\lambda}\delta^2, \forall j,k,l\neq k,
$$

where $\tilde{\lambda} \equiv \frac{\lambda+\alpha\gamma\lambda}{1+\alpha\gamma\lambda}$.

Thus, any equilibrium should have the same distribution of prices and fundamentals derived in this proof, which concludes our proof of existence and uniqueness. ■

C.6 Reinterpretation of a Firm's Attention Problem

This section provides an alternative formulation of the firms' attention problem where they maximize their payoffs by choosing the correlation of their prices with the fundamental and the mistakes of their competitors. The solution to this reformulation is equivalent to the equilibrium characterized in the previous section, but this alternative problem provides further insights about firms' incentives in the mode.

Take any firm $j,k \in J \times K$ and suppose all other firms in the economy are playing the equilibrium strategy. Moreover, here I take it as given that the firm does not pay attention to mistakes of firms in other industries ($cov(p_{j,k}, p_{l,m}|q)_{l\neq j} = 0$). Now, take a strategy $\varsigma_{j,-k}$ for other firms and decompose the average price of others under this strategy to its projection on q and the part that is orthogonal to q: $p_{j,-k}(\varsigma_{j,-k}) = \frac{1}{K-1}\sum_{l\neq k} p_{j,l}(\varsigma_{j,l}) = \delta q + v_{j,-k}$. Furthermore, let $\sigma_v^2 \equiv var(v_{j,-k})$ be the variance of the average mistake of other firms in j,k's industry when they play $\varsigma_{j,-k}$. For any $s_{j,k} \in \mathscr{S}$ define $\rho_q(s_{j,k}) \equiv cor(s_{j,k}, q)$, $\rho_v(s_{j,k}) \equiv cor(s_{j,k}, v_{j,-k})$. Notice that firm j,k's loss in profit, given that it observes $s_{j,k}$, is

$$var((1-\alpha)q+\alpha p_{j,-k}|s_{j,k}) \quad = \quad (1-\alpha+\alpha\delta)^2 var(q+\frac{\alpha}{1-\alpha(1-\delta)}v_{j,-k}|s_{j,k}).$$

With some algebra, it is straightforward to show that the variance in the second part of the above equation is given by

$$var(q+\frac{\alpha}{1-\alpha(1-\delta)}v_{j,-k}|s_{j,k}) \quad = \quad 1+(\frac{\alpha}{1-\alpha(1-\delta)})^2\sigma_v^2-(\rho_q(s_{j,k})+\frac{\alpha\sigma_v}{1-\alpha(1-\delta)}\rho_v(s_{j,k}))^2.$$

Now, to derive the information constraint in terms of the two correlation terms, we have

$$\mathscr{I}(s_{j,k};(q,p^*_{j,-k}))\le\kappa \Leftrightarrow \frac{1}{2}\log_2(\frac{var(s_j)}{var(s_{j,k}|(q,p^*_{j,-k}))})\le\kappa$$

Notice that $\frac{var(s_j|(q,p^*_{j,-k}))}{var(s_j)}=1-(\rho_q(s_j)^2+\rho_v(s_j)^2)$. Thus, the information constraint becomes $\rho_q^2(s_j)+\rho_v^2(s_j)\le\lambda\equiv 1-2^{-2\kappa}$. So, j,k's problem reduces to

$$\max_{\rho_q,\rho_v}(\rho_q(s_{j,k})+\frac{\alpha\sigma_v}{1-\alpha(1-\delta)}\rho_v(s_{j,k}))^2 \quad s.t. \quad \rho_q(s_{j,k})^2+\rho_v(s_{j,k})^2\le\lambda.$$

C.7 Proofs of Propositions for the Static Model

This section includes the proofs of Proposition 1, Proposition 2, and Corollary 3. The proofs and derivations for Section 4 are included in Appendix G.

Proof of Proposition 1.

1. First, observe that the correlation between the firm's price and the fundamental is given by

$$\rho_q^{*2}=\frac{cov(p_{j,k},q)^2}{var(p_{j,k})}=\frac{K-1+\alpha\delta}{K-1+\alpha\lambda}\lambda.$$

   Moreover, notice that $\delta=\frac{1-\alpha}{1-\alpha\lambda}\lambda<\lambda$ as long as $\lambda>0$ and $\alpha>0$. This implies directly that $\rho_q^{*2}<\lambda$. Thus, the correlation between the firm's price and the mistakes of its competitors is strictly positive: $\rho_v^{*2}=\lambda-\rho_q^{*2}>0$, meaning that firms pay attention to the mistakes of their competitors.

2. Differentiating the correlation ρ_q^* with respect to the number of competitors K, we have

$$\frac{\partial\rho_q^{*2}}{\partial K}\frac{1}{\rho_q^{*2}}=\frac{\alpha(\lambda-\delta)}{(K-1+\alpha\lambda)(K-1+\alpha\delta)}>0$$

   Also, with respect to α:

$$\frac{\partial\rho_q^{*2}}{\partial\alpha}\frac{1}{\rho_q^{*2}}=\frac{(K-1)(\delta-\lambda)+(K-1+\alpha\lambda)\alpha\frac{\partial\delta}{\partial\alpha}}{(K-1+\alpha\delta)(K-1+\alpha\lambda)}<0.$$

   The inequality comes from $\delta-\lambda<0$ and $\frac{\partial\delta}{\partial\alpha}=\delta\frac{\lambda-1}{(1-\alpha)(1-\alpha\lambda)}<0$.

3. Shown in the proof of Lemma C.9.

Proof of Proposition 2.

First, observe that the aggregate price is given by

$$p \equiv J^{-1}K^{-1}\sum_{j,k\in J\times K} p_{j,k} = \delta q + \frac{1}{JK}\sum_{j,k\in J\times K} v_{j,k}$$

Since J is large and $v_{j,k}$'s are independent across industries, this average mistake across all the firms in the economy converges to zero by the law of large numbers as $J\to\infty$. Therefore, $p=\delta q$. Moreover, $\mathbb{E}^{j,k}[p_{j,-k}] = \frac{cov(s_{j,k},p_{j,-k})}{var(p_{j,k})}s_{j,k} = \tilde{\lambda}p_{j,k}$ and $\mathbb{E}^{j,k}[p] = \frac{cov(s_{j,k},p)}{var(p_{j,k})}p_{j,k} = \frac{1-\alpha\tilde{\lambda}}{1-\alpha\lambda}\lambda p_{j,k}$ where $\tilde{\lambda} = \frac{\lambda(K-1)+\alpha\lambda}{K-1+\alpha\lambda} > \lambda$ is defined as in the proof of Lemma C.9. So, $\overline{\mathbb{E}^{j,k}[p_{j,-k}]} = \tilde{\lambda}p$, $\overline{\mathbb{E}^{j,k}[p]} = \frac{1-\alpha\tilde{\lambda}}{1-\alpha\lambda}\lambda p$. Therefore,

$$cov(\overline{\mathbb{E}^{j,k}[p_{j,-k}]},p) \quad = \quad \tilde{\lambda}var(p) > \frac{1-\alpha\tilde{\lambda}}{1-\alpha\lambda}\lambda var(p) = cov(\overline{\mathbb{E}^{j,k}[p]},p).$$

Also, if $K\to\infty$ then $\tilde{\lambda}\to\lambda$ and $cov(\overline{\mathbb{E}^{j,k}[p]},p)\to cov(\overline{\mathbb{E}^{j,k}[p_{j,-k}]},p)$.

Proof of Corollary 3.

Conditional on realization of the aggregate price $|p-\overline{\mathbb{E}^{j,k}[p]}| = (1-\frac{1-\alpha\tilde{\lambda}}{1-\alpha\lambda}\lambda)|p| > (1-\tilde{\lambda})|p| = |p-\overline{\mathbb{E}^{j,k}[p_{j,-k}]}|$.

D Available Information in the Dynamic Model

The set of available signals in the dynamic model is an extension of the set defined in Appendix C.1. The key notion in this extension is that nature draws new shocks every period, and the set of the available information in the economy expands to incorporate these new realizations. To capture this evolution, I define a signal structure as a sequence of sets $(\mathscr{S}^t)_{t=-\infty}^{\infty}$ where $\mathscr{S}^{t-s}\subset\mathscr{S}^t, \forall s\geq 0$. Here, $\mathscr{S}^t$ denotes the set of available signals at time t, and it contains all the previous sets of signals that were available in previous periods.

To construct the signal structure, suppose that every period, in addition to the shock to the nominal demand, the nature draws countably infinite uncorrelated standard normal noises. Similar to Appendix C.1, let $\mathscr{S}_t$ be the set of all finite linear combinations of these uncorrelated noises along with the newest innovation to q_t. Now, define $\mathscr{S}^t = \{\sum_{s=0}^{\infty} a_\tau e_{t-\tau} | \forall\tau\geq 0, a_\tau\in\mathbb{R}, e_{t-\tau}\in\mathscr{S}_{t-\tau}\}, \forall t$. First, for all t, $q_t\in\mathscr{S}^t$, as it is a linear combination of all $u_{t-\tau}$'s and $u_{t-\tau}\in\mathscr{S}_{t-\tau}, \forall\tau\geq 0$. This implies that perfect information is available about the fundamentals of the economy, but also arbitarily precise information is also available when information is costly.

E Derivations for the Dynamic Model

E.1 Solution to Household's Problem (5)

Let $\beta^t \varphi_{1,t}$ and $\beta^t \varphi_{2,t}$ be the Lagrange multipliers on household's budget and aggregation constraints, respectively. For ease of notation let $\mathscr{C}_{j,t} \equiv (C_{j,1,t}, \ldots, C_{j,K_j,t})$ be the vector of household's consumption from firms in industry $j \in J$, so that $C_{j,t} \equiv \Phi_j(\mathscr{C}_{j,t})$ where $\Phi_j(.)$ is an aggregator function that is homogenous of degree one and at least thrice differentiable in its arguments (note that this embeds the CES aggregator in the main text as well as the Kimball aggregator discussed in Appendix F). Moreover, for less crowded notation, I drop subscript j for ϕ_j and K_j whenever the industry index is implied from context. First, I derive the demand of the household for different goods. The first order condition with respect to $C_{j,k,t}$ is

$$P_{j,k,t} = \frac{1}{J}\frac{\varphi_{2,t}}{\varphi_{1,t}} C_t \frac{\Phi_k(\mathscr{C}_{j,t})}{\Phi(\mathscr{C}_{j,t})} \tag{E.1}$$

where $\Phi_k(\mathscr{C}_{j,t}) \equiv \frac{\partial \Phi(\mathscr{C}_{j,t})}{\partial C_{j,k,t}}$. Given these optimality conditions, we can show that total sales in the economy is proportional to aggregate output:

$$\sum_{(j,k)\in J\times K} P_{j,k,t} C_{j,k,t} = \frac{1}{J}\frac{\varphi_{2,t}}{\varphi_{1,t}} C_t \sum_{j\in J} \underbrace{\sum_{k\in K} \frac{\Phi_k(\mathscr{C}_{j,t})}{\Phi(\mathscr{C}_{j,t})} C_{j,k,t}}_{=1,\forall j\in J} = \frac{\varphi_{2,t}}{\varphi_{1,t}} C_t$$

where the equality under curly bracket is from Euler theorem for homogeneous function $\Phi(.)$. Therefore, $P_t \equiv \frac{\varphi_{2,t}}{\varphi_{1,t}}$ is the price of the aggregate consumption basket C_t and we can write $Q_t = P_t C_t$ as the nominal demand of the household for the aggregate consumption good. Now, for the particular case of the CES function in the main case, Equation (E.1) becomes:

$$P_{j,k,t} = (JK_j)^{-1} Q_t C_{j,k,t}^{-\eta^{-1}} C_{j,t}^{\eta^{-1}-1} \Rightarrow \sum_{k\in K_j} P_{j,k,t}^{1-\eta} = (JK_j)^{\eta-1} K_j Q_t^{1-\eta} C_{j,t}^{\eta-1} \tag{E.2}$$

where the right hand side follows from raising the left hand side to the power of $1-\eta$ and summing over k. Now, raising the right hand side to the power of $-\eta$ and dividing it by the left hand side gives the demand curve in the text:

$$C_{j,k,t} = Q_t \mathscr{D}(P_{j,k,t}, P_{j,-k,t}), \quad \mathscr{D}(P_{j,k,t}, P_{j,-k,t}) \equiv \frac{1}{J} \frac{P_{j,k,t}^{-}\eta}{\sum_{k\in K_j} P_{j,k,t}^{1-\eta}} \tag{E.3}$$

Now, for a general Φ: from Equation (E.1), $\mathscr{P}_{j,t} \equiv (P_{j,1,t}, \ldots, P_{j,K,t}) = \nabla \log(\Phi(\frac{\mathscr{C}_{j,t}}{J^{-1}P_t C_t}))$. I need to show that this function is invertible to prove that a demand function exists. For ease of notation, define function $f: \mathbb{R}^K \to \mathbb{R}^K$ such that $f(\mathbf{x}) \equiv \nabla \log(\Phi(\mathbf{x}))$. Notice that $f(.)$ is homogeneous of degree -1, and the m,n'th element of its Jacobian, denoted by matrix $\mathscr{J}^f(\mathbf{x})$, is given by $\mathscr{J}^f_{m,n}(\mathbf{x}) \equiv \frac{\partial}{\partial x_n} \frac{\Phi_m(\mathbf{x})}{\Phi(\mathbf{x})} = \frac{\Phi_{m,n}(\mathbf{x})}{\Phi(\mathbf{x})} - \frac{\Phi_n(\mathbf{x})}{\Phi(\mathbf{x})}\frac{\Phi_m(\mathbf{x})}{\Phi(\mathbf{x})}$. Let $\mathbf{1}$ be the unit vector in $\mathbb{R}^K$. Since $\Phi(.)$ is symmetric along its arguments, for any $k \in (1, \ldots, K)$, $\Phi_1(\mathbf{1}) = \Phi_k(\mathbf{1})$, $\Phi_{11}(\mathbf{1}) = \Phi_{kk}(\mathbf{1}) < 0$. Since $\Phi(.)$ is homogeneous of degree 1, by Euler's theorem we have $\Phi(\mathbf{1}) = \sum_{k\in K} \Phi_k(\mathbf{1}) = K\Phi_1(\mathbf{1})$. Also, since

$\Phi_k(.)$ is homogeneous of degree zero.[51] Similarly we have $0 = 0 \times \Phi_k(\mathbf{1}) = \sum_{l \in K} \Phi_{kl}(\mathbf{1})$. So, for any $l \neq k$, $\Phi_{kl}(\mathbf{1}) = -\frac{1}{K-1}\Phi_{11}(\mathbf{1}) > 0$. This last equation implies that $\mathscr{J}^f(\mathbf{1})$ is an invertible matrix.[52] Therefore, by inverse function theorem $f(.)$ is invertible in an open neighborhood around $\mathbf{1}$, and therefore any symmetric point $\mathbf{x} = x.\mathbf{1}$ such that $x > 1$. We can write $\frac{\mathscr{C}_{j,t}}{J^{-1}P_t C_t} = f^{-1}(\mathscr{P}_{j,t})$. It is straight forward to show that $f^{-1}(.)$ is homogeneous of degree -1 because $f(\mathbf{x})$ is homogeneous of degree -1: for any $\mathbf{x} \in \mathbb{R}^K$, $f^{-1}(a\mathbf{x}) = f^{-1}(af(f^{-1}(\mathbf{x})) = f^{-1}(f(a^{-1}f^{-1}(\mathbf{x})) = a^{-1}f^{-1}(\mathbf{x})$. Now, $C_{j,k,t} = J^{-1}P_t C_t f_k^{-1}(\mathscr{P}_{j,t})$, where $f_k^{-1}(\mathbf{x})$ is the k'th element of the vector $f^{-1}(\mathscr{P}_{j,t})$. Finally, since $f(.)$ is symmetric across its arguments, so is $f^{-1}(\mathscr{P}_{j,t})$, meaning that $f_k^{-1}(\mathscr{P}_{j,t}) = f_1^{-1}(\sigma_{k,1}(\mathscr{P}_{j,t}))$, where $\sigma_{k,1}(\mathscr{P}_{j,t})$ is a permutation that changes the places of the first and k'th element of the vector $\mathscr{P}_{j,t}$. Now, to get the notation in the text let $(P_{j,k,t}, P_{j,-k,t}) \equiv \sigma_{k,1}(\mathscr{P}_{j,t})$ and $\mathscr{D}(\mathbf{x}) \equiv J^{-1}f_1^{-1}(\mathbf{x})$, which gives us the notation in the text: $C_{j,k,t} = P_t C_t \mathscr{D}(P_{j,k,t}, P_{j,-k,t})$, where $\mathscr{D}(.,.)$ is homogeneous of degree -1. Finally, the optimality conditions of the household's problem with respect to B_t, C_t and L_t are straight forward and are given by $P_t C_t = \beta(1+i_t)\mathbb{E}_t^f[P_{t+1}C_{t+1}]$ and $P_t C_t = W_t$.

E.2 Quadratic Approximation to Firms' Profits

Define a firm's revenue net of its production costs at a given time as

$$\Pi(P_{j,k,t}, P_{j,-k,t}, Q_t) = P_{j,k,t} Q_t \mathscr{D}(P_{j,k,t}, P_{j,-k,t}) - (1-\bar{s}_j) Q_t^{2+\gamma} \mathscr{D}(P_{j,k,t}, P_{j,-k,t})^{1+\gamma} \tag{E.4}$$

Now for any given set of signals over time that firm j,k could choose to see, its profit maximization problem is

$$\max_{(P_{j,k,t}:S_{j,k}^t \to \mathbb{R})_{t=0}^\infty} \mathbb{E}[\sum_{t=0}^\infty \beta^t Q_t^{-1} \Pi(P_{j,k,t}, P_{j,-k,t}, Q_t) | S_{j,k}^{-1}] = \max_{(P_{j,k,t}:S_{j,k}^t \to \mathbb{R})_{t=0}^\infty} \mathbb{E}[\sum_{t=0}^\infty \beta^t \Pi(\frac{P_{j,k,t}}{Q_t}, \frac{P_{j,-k,t}}{Q_t}, 1) | S_{j,k}^{-1}].$$

where the second equality follows from the fact that the profit function is homogeneous of degree 1 as $\mathscr{D}(.,.)$ is homogeneous of degree -1. Now, let small letters denote logs of corresponding variables so that $p_{j,k,t} - q_t \equiv \ln(P_{j,k,t}/Q_t)$ and $p_{j,-k,t} - q_t \equiv \ln(P_{j,-k,t}/Q_t)$ and define the loss function of the firm from mispricing at a given time as

$$L(p_{j,k,t} - q_t, p_{j,-k,t} - q_t) \equiv \Pi(\frac{P_{j,k,t}^*}{Q_t}, \frac{P_{j,-k,t}}{Q_t}, 1) - \Pi(\frac{P_{j,k,t}}{Q_t}, \frac{P_{j,-k,t}}{Q_t}, 1),$$

where $P_{j,k,t}^* = \text{argmax}_x \Pi(x, P_{j,-k,t}, Q_t)$ is the firms' optimal price for the particular realizations of Q_t and $P_{j,-k,t}$. Now note that

$$\min_{(p_{j,k,t}:S_{j,k}^t \to \mathbb{R})_{t=0}^\infty} \mathbb{E}[\sum_{t=0}^\infty \beta^t L(p_{j,k,t} - q_t, p_{j,-k,t} - q_t) | S_{j,k}^{-1}]$$

[51] Follows from homogeneity of $\Phi(\mathbf{x})$. Notice that $\Phi(a\mathbf{x}) = a\Phi(\mathbf{x})$. Differentiate with respect to k'th argument to get $\Phi_k(a\mathbf{x}) = \Phi_k(\mathbf{x})$.

[52] With some algebra, we can show that $\mathscr{J}^f(\mathbf{1}) = \frac{\Phi_{11}(\mathbf{1})}{K-1}\mathbf{I} - \frac{\Phi_{11}(\mathbf{1})+K^{-1}}{K(K-1)}\mathbf{1}\mathbf{1}'$, meaning that $\mathscr{J}^f(\mathbf{1})$ is a symmetric matrix whose diagonal elements are strictly different than its off-diagonal elements. Hence, it is invertible.

has the same solution as profit maximization problem of the firm. Moreover, recall from the main text that in the symmetric equilibrium of the full-information economy $\frac{P_{j,k,t}}{Q_t} = \frac{P_{j,-k,t}}{Q_t} = 1$. Taking a second-order approximation to the net present value of firm's losses at a given time around the symmetric full-information equilibrium, we arrive at:

$$\sum_{t=0}^{\infty} \beta^t L(p_{j,k,t} - q_t, p_{j,-k,t} - q_t) \approx \underbrace{-\frac{1}{2}\Pi_{11}(1,1,1)}_{>0} \sum_{t=0}^{\infty} \beta^t (p_{j,k,t} - p^*_{j,k,t})^2,$$

where $p^*_{j,k,t}$ is such that $\Pi_1(\exp(p^*_{j,k,t})/Q_t, P_{j,-k,t}/Q_t, 1) = 0$, meaning that

$$p^*_{j,k,t} = q_t + \underbrace{\left(1 + \frac{\Pi_{13}(1,1,1)}{\Pi_{11}(1,1,1)}\right)}_{\text{strategic complementarity} = \alpha_j} \times \frac{1}{K_j - 1} \sum_{l \neq k} (p_{j,l,t} - q_t) \tag{E.5}$$

$$= (1 - \alpha_j) q_t + \alpha_j p_{j,-k,t} \tag{E.6}$$

It is straightforward to calculate the derivatives $\Pi_{11}(1,1,1)$ and $\Pi_{13}(1,1,1)$ as

$$\Pi_{11}(1,1,1) = -\mathtt{rs}_j(\varepsilon^{\varepsilon}_{j,D} + (1+\gamma)(\varepsilon^j_D - 1)) \tag{E.7}$$

$$\Pi_{13}(1,1,1) = \mathtt{rs}_j(\varepsilon^j_D - 1) \tag{E.8}$$

where $\mathtt{rs}_j \equiv \mathscr{D}(1,1) = (JK_j)^{-1}$ is the revenue share (or relative size) of the firm in the symmetric full-information equilibrium, ε^j_D is the demand elasticity and $\varepsilon^{\varepsilon}_{j,D}$ is the superelasticity of demand for a firm in sector j in the full-information symmetric equilibrium. Note that this gives a general expression for strategic complementarity as:

$$\alpha_j = 1 - \frac{\varepsilon^j_D - 1}{\varepsilon^{\varepsilon}_{j,D} + (1+\gamma)(\varepsilon^j_D - 1)} \tag{E.9}$$

Thus, note that we can write Π_{11} as:

$$\Pi_{11} = -\mathtt{rs}_j \frac{\varepsilon^j_D - 1}{1 - \alpha_j} \tag{E.10}$$

and the firm's objective for its attention problem is therefore given by

$$\max_{\{\kappa_{j,k,t}, S_{j,k,t}, p_{j,k,t}(S^t_{j,k})\}_{t\geq 0}} -\mathtt{rs}_j \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left(\underbrace{\frac{1}{2} \frac{\varepsilon^j_D - 1}{1 - \alpha_j} (p_{j,k,t}(S^t_{j,k}) - p^*_{j,k,t})^2}_{\text{loss from mispricing}} + \underbrace{(1 - \mathtt{s}_j)\omega\kappa_{j,k,t}}_{\text{cost of capacity}} | S^{-1}_{j,k} \right)\right] \tag{E.11}$$

Dividing the objective by $1 - s_j = \frac{\varepsilon_D^j - 1}{\varepsilon_D^j}$ gives us the objective in the main text where

$$B_j = \frac{\varepsilon_D^j}{1-\alpha_j} = \frac{\eta - (\eta-1)K_j^{-1}}{(1-(1-\eta^{-1})K_j^{-1})(\frac{1+\gamma}{1+\gamma\eta(1-(1-\eta^{-1})K_j^{-1})^2})} = \frac{\eta + \gamma(\eta - (\eta-1)K_j^{-1})^2}{1+\gamma} \tag{E.12}$$

where the equalities follow from the expression for demand elasticities and strategic complementarities in the main text.

F Strategic Complementarity under Kimball Demand

In the paper's main text, I consider a nested CES aggregator and derive the strategic complementarities under the demand system implied by that aggregator. An alternative approach in the literature is using the Kimball aggregator but mainly used with monopolistic competition. In this section, I derive the demand functions of firms given this aggregator in an *oligopolistic* setting for comparison.

The Kimball aggregator assumes that the function $\Phi(C_{j,1,t},\ldots,C_{j,K,t})$ is implicitly defined by

$$1 = K^{-1}\sum_{k\in K} f(\frac{KC_{j,k,t}}{\Phi(C_{j,1,t},\ldots,C_{j,K,t})}), \tag{F.1}$$

where $f(.)$ is at least thrice differentiable, and $f(1)=1$ (so that $\Phi(1,\ldots,1)=K$). Observe that this coincides with the CES aggregator when $f(x) = x^{\frac{\eta-1}{\eta}}$. To derive the demand functions, recall that the first order conditions of the household's problem are $P_{j,k,t} = J^{-1}Q_t\frac{\frac{\partial}{\partial C_{j,k,t}}C_{j,t}}{C_{j,t}}, \forall j,k$where $C_{j,t} = \Phi(C_{j,1,t},\ldots,C_{j,K,t})$. Implicit differentiation of Equation (F.1) gives

$$P_{j,k,t} = J^{-1}Q_t\frac{f'(\frac{KC_{j,k,t}}{C_{j,t}})}{\sum_{l\in K} C_{j,l,t}f'(\frac{KC_{j,l,t}}{C_{j,t}})}, \forall j,k. \tag{F.2}$$

To invert these functions and get the demand for every firm in terms of their competitors' prices, guess that there exists a function $F:\mathbb{R}^K \to \mathbb{R}$ such that $\frac{\sum_{l\in K} C_{j,l,t}f'(\frac{KC_{j,l,t}}{C_{j,t}})}{J^{-1}Q_t} = F(P_{j,1,t},\ldots,P_{j,K,t})$. I verify this guess by plugging in this guess to Equation (F.2), which implies the function $F(.)$ is implicitly defined by $1 = K^{-1}\sum_{k\in K} f(f'^{-1}(P_{j,l,t}F(P_{j,1,t},\ldots,P_{j,K,t})))$. Note that this is consistent with the guess and $F(.)$ only depends on the vector of these prices. It is straight forward to show that $F(.)$ is symmetric across its arguments and homogeneous of degree -1.[53] Now, given these derivations, we can derive the demand function of firm j,k as a function of the aggregate demand, its own price and the prices of its competitors. Similar to the main text we can write

[53] Symmetry is obvious to show. To see homogeneity, differentiate the implicit function that defines $F(.)$ with respect to each of its arguments and sum up those equations to get that for any $X = (x_1,\ldots,x_K)\in\mathbb{R}^K$, $-F(X) = \sum_{k\in K} x_k\frac{\partial}{\partial x_k}F(X)$. Now, notice that for any $a\in\mathbb{R}, X\in\mathbb{R}^K$, $\frac{\partial aF(aX)}{\partial a} = 0$. Thus, for any $X\in\mathbb{R}^K$, $aF(aX)$ is independent of a, and in particular $aF(aX) = F(X) \Rightarrow F(aX) = a^{-1}F(X)$.

this as

$$C_{j,k,t} = J^{-1} Q_t D(P_{j,k,t}, P_{j,-k,t}),\ D(P_{j,k,t}, P_{j,-k,t}) \equiv \frac{f'^{-1}(P_{j,k,t} F(P_{j,1,t}, \dots, P_{j,K,t}))}{\sum_{l \in K} P_{j,l,t} f'^{-1}(P_{j,l,t} F(P_{j,1,t}, \dots, P_{j,K,t}))}$$

In the spirit of the CES aggregator I define $\eta \equiv -\frac{f'(1)}{f''(1)}$ as the inverse of the elasticity of $f'(x)$ at $x = 1$, and assume $\eta > 1$. It is straightforward to show that η is the elasticity of substitution between industry goods around a symmetric point. Moreover, the elasticity of demand for every firm around a symmetric point is $\eta - (\eta - 1)K^{-1}$ similar to the case of a CES aggregator. Also, define $\zeta(x) \equiv \frac{\partial \log(-\frac{\partial \log(f'(x))}{\partial \log(x)})}{\partial \log(x)}$ as the elasticity of the elasticity of $f'(x)$:$\zeta(x) = \frac{f'''(x)}{f''(x)} x - \frac{f''(x)}{f'(x)} x + 1$. For notational ease let $\zeta \equiv \zeta(1)$ and assume $\zeta \geq 0$ ($\zeta = 0$ corresponds to the case of CES aggregator). These assumptions ($\eta > 1$ and $\zeta \geq 0$ are sufficient for weak strategic complementarity, $\alpha \in [0,1)$). While the usual approach in the literature is to assume $K \to \infty$ and look at super elasticities in this limit, a part of my main results revolve around the finiteness of the number of competitors and the fact that the degree of strategic complementarity is decreasing in K. Therefore, I derive the degree of strategic complementarity for any finite K. With some intense algebra we get $\alpha = \frac{\zeta(K-2)+(1-\eta^{-1})^2}{\zeta(K-2)+(1-\eta^{-1})K} \in [0,1)$. This imbeds the CES aggregator when $\zeta = 0$, in which case $\alpha = (1-\eta^{-1})K^{-1}$.

G Proofs of Propositions for the Dynamic Model

Proof of Proposition 4.

The optimality of one signal at any given time follows directly from Lemma 1 in Afrouzi and Yang (2019). Here, I include an adaptation of that proof for the special case of $\beta = 0$ that builds on the result in Lemma (C.8) for the dynamic case. Many arguments in the proof are similar and are omitted to avoid repetition. At a given time t, let $(S^{t-1}_{j,k})_{(j,k) \in J \times K}$ denote the signals that all firms have received until time $t-1$, and are born with at time t. In particular, for any j, k, $S^{t-1}_{j,k} = (\dots, S_{j,k,t-3}, S_{j,k,t-2}, S_{j,k,t-1})$,where $\forall \tau \geq 1$, $S_{j,k,t-\tau} \subset \mathscr{S}^{t-\tau}$. This implies that (1) $S_{j,k,t-\tau}$ only contains information that were available at time $t-\tau$, and therefore are available at time t, and (2) $S_{j,k,t-\tau}$ is available for all other firms in the economy in case they find it desirable to learn about it.

Given this initial signal structure, pick a strategy profile for all firms at time t:$\varsigma_t = (S_{j,k,t} \subset \mathscr{S}^t, p_{j,k,t} : S^t_{j,k,t} \to \mathbb{R})_{(j,k) \in J \times K}$, where $S^t_{j,k,t} = (S^{t-1}_{j,k,t}, S_{j,k,t})$. First, similar to the static case, we can show that in any equilibrium strategy $p_{j,k,t}(S^t_{j,k})$ is linear in the vector $S^t_{j,k}$. This result follows with an argument similar to Lemma (C.3). Given this, let $p_{j,k,t}(S^t_{j,k}) = \sum_{\tau=0}^{\infty} \delta^{\tau}_{j,k,t} S_{j,k,t-\tau}$ denote the pricing strategy for any $(j,k) \in J \times K$. This is without loss of generality because the equilibrium has to be among such strategies. Notice that due to linearity and definition of $\mathscr{S}^t$, $p_{j,k,t}(S^t_{j,k}) \in \mathscr{S}^t$, $\forall (j,k) \in J \times K$. Now, pick a particular firm j,k and let $\varsigma_{-(j,k),t}$ denote the signals and pricing strategies that ς_t implies for all other firms in the economy except for j,k. Similar to

Subsection C.3 let $\theta_{j,k,t}(\varsigma_{-(j,k),t}) \equiv (q, (p_{j,l,t}(S^t_{j,l}))_{l\neq k}, (p_{m,n,t}(S^t_{m,n}))_{m\neq j, n\in K})'$ be the augmented vector of the fundamental, the prices of other firms in j,k's industry, and the prices of all other firms in the economy. Now, define $\mathbf{w} = (1-\alpha, \underbrace{\frac{\alpha}{K-1}, \dots, \frac{\alpha}{K-1}}_{K-1\,\text{times}}, \underbrace{0,0,0,\dots,0}_{(J-1)\times K\,\text{times}})'$. For the remainder of the proof fix the capacity of the firm at $\kappa \geq 0$. I will show that the result holds for any such κ and hence is true also under the optimal κ. Since $\beta = 0$, fixing the capacity at some $\kappa \geq 0$, firm j,k's signal choice problem is

$$\min_{S_{j,k,t}\subset\mathscr{S}^t} var(\mathbf{w}'\theta_{j,k,t}(\varsigma_{-(j,k),t})|S^t_{j,k})$$
$$s.t. \quad \mathscr{I}(S_{j,k,t}, \theta_{j,k,t}(\varsigma_{-(j,k),t})|S^{t-1}_{j,k}) \leq \kappa.$$

To show that a single signal solves this problem, suppose not, so that $S_{j,k,t}$ contains more than one signal. Then, we know that $p_{j,k,t}(S^t_{j,k}) = \mathbf{w}'\mathbb{E}[\theta_{j,k,t}(\varsigma_{-(j,k),t})|S^t_{j,k}]$. Notice that I am assuming signals are such that these expectations exist. If not, then the problem of the firm is not well-defined as the objective does not have a finite value. To get around this issue, for now assume that the initial signal structure of the game is such that expectations and variances are finite. Since both $\theta_{j,k,t}(\varsigma_{-(j,k),t})$ and $S^t_{j,k}$ are Gaussian, $p_{j,k,t}(S^t_{j,k}) = \sum \delta^t_{j,k,\tau} S_{j,k,t-\tau}$ by Kalman filtering. Here for any $S_{j,k,t-\tau}$ that is not a singleton, let $\delta^t_{j,k,\tau}$ be a vector of the appropriate size that is implied by Kalman filtering. Therefore, by definition of $\mathscr{S}^t$, $p_{j,k,t}(S^t_{j,k}) \in \mathscr{S}^t$, meaning that there is a signal in $\mathscr{S}^t$ that directly tells firm j,k what their price would be under $S^t_{j,k}$ and $\varsigma_{-(j,k),t}$. Let $\hat{S}^t_{j,k} \equiv (S^{t-1}_{j,k}, p_{j,k}(S^t_{j,k}))$ and observe that by definition of $p_{j,k,t}(S^t_{j,k})$, $var(\mathbf{w}'\theta_{j,k,t}(\varsigma_{-(j,k),t})|S^t_{j,k}) = var(\mathbf{w}'\theta_{j,k,t}(\varsigma_{-(j,k),t})|\hat{S}^t_{j,k})$. Therefore, we have found a single signal that implies the same loss for firm j,k under $S^t_{j,k}$. Now, we just need to show that it is feasible, which is straight forward from data processing inequality: since $p_{j,k,t}(S^t_{j,k})$ is a function $S^t_{j,k}$, we have

$$\mathscr{I}(p_{j,k,t}(S^t_{j,k}), \theta_{j,k,t}(\varsigma_{-(j,k),t})|S^{t-1}_{j,k}) \leq \mathscr{I}(S_{j,k,t}, \theta_{j,k,t}(\varsigma_{-(j,k),t})|S^{t-1}_{j,k}) \leq \kappa.$$

which concludes the proof for sufficiency of one signal. Now, given $S^{t-1}_{j,k}$ and $\theta_{j,k,t}(\varsigma_{-(j,k),t})$ let $\Sigma_{j,k,t|t-1} \equiv var(\theta_{j,k,t}(\varsigma_{-(j,k),t})|S^{t-1}_{j,k})$. Without loss of generality assume $\Sigma_{j,k,t|t-1}$ is invertible. If not, then there are elements in $\theta_{j,k,t}(\varsigma_{-(j,k),t})$ that are colinear conditional on $S^{t-1}_{j,k}$, in which case knowing about one completely reveal the other; this means we can reduce $\theta_{j,k,t}(\varsigma_{-(j,k),t})$ to its orthogonal elements without limiting the signal choice of the agent. Now, for any non-zero singleton $S_{j,k,t} \in \mathscr{S}^t$, it is straight forward to show that

$$\mathscr{I}(S_{j,k,t}, \theta_{j,k,t}(\varsigma_{-(j,k),t})|S^{t-1}_{j,k}) = \frac{1}{2}\log(1 - \mathbf{z}'_t\Sigma^{-1}_{j,k,t|t-1}\mathbf{z}_t),$$

where $\mathbf{z}_t \equiv \frac{cov(S_{j,k,t}, \theta_{j,k,t}(\varsigma_{-(j,k),t})|S^{t-1}_{j,k})}{\sqrt{var(S_{j,k,t}|S^{t-1}_{j,k})}}$. The capacity constraint of the agent becomes $\mathbf{z}'_t\Sigma^{-1}_{j,k,t|t-1}\mathbf{z}_t \leq$

$\lambda \equiv 1 - 2^{-2\kappa}$. Moreover, notice that the loss of the firm becomes

$$var(\mathbf{w}'\theta_{j,k,t}(\varsigma_{-(j,k),t})|S_{j,k}^{t-1}, S_{j,k,t}) = \mathbf{w}'\Sigma_{j,k,t|t-1}\mathbf{w} - (\mathbf{w}'\mathbf{z}_t)^2.$$

This means that the agent can directly choose $\mathbf{z}_t$ as long as there is a signal in $\mathscr{S}^t$ that induces that covariance. I first characterize the $\mathbf{z}_t$ that solves this problem and then show that such a signal exists. Notice that minimizing the loss is equivalent to maximizing $(\mathbf{w}'\mathbf{z}_t)^2$. The firm's problem is $\max_{\mathbf{z}_t}(\mathbf{w}'\mathbf{z}_t)^2 \; s.t. \, \mathbf{z}_t'\Sigma_{j,k,t|t-1}^{-1}\mathbf{z}_t \leq \lambda$. By Cauchy-Schwarz inequality we know $(\mathbf{w}'\mathbf{z}_t)^2 \leq (\mathbf{w}'\Sigma_{j,k,t|t-1}\mathbf{w})(\mathbf{z}_t'\Sigma_{j,k,t|t-1}^{-1}\mathbf{z}_t) \leq \lambda\mathbf{w}'\Sigma_{j,k,t|t-1}\mathbf{w}$, where the second inequality follows from the capacity constraint. Observe that $\mathbf{z}_t^* = \sqrt{\frac{\lambda}{\mathbf{w}'\Sigma_{j,k,t|t-1}\mathbf{w}}}\Sigma_{j,k,t|t-1}\mathbf{w}$ achieves this upper-bar. The properties of the Cauchy-Schwarz inequality imply that this is the only vector that achieves this upper-bar. Hence, $\mathbf{z}_t^*$ is the unique solution to the firm's problem.[54] Now, I just need to show that a signal exists in $\mathscr{S}^t$ that implies this $\mathbf{z}_t^*$. To see this, let $S_{j,k,t}^* = (1-\alpha)q_t + \alpha\frac{1}{K-1}\sum_{l\neq k} p_{j,l,t}(S_{j,l}^t) + e_{j,k,t}$. It is straight forward to show that this these signals imply $\mathbf{z}_t^*$.

Proof of Proposition 5.

The independence of strategic complementarity α_j from j follows from the symmetry in the number of competitors across industries. Moreover, in the stationary equilibrium capacity is time-invariant because it only depends on the underlying parameters and the variances of subjective beliefs, which are constant under the steady-state Kalman filter. Symmetric equilibrium also implies that optimal capacities are also symmetric across all firms; so $\kappa_{j,k,t} = \kappa \geq 0$. To see that $\kappa > 0$, suppose that in the equilibrium $\kappa = 0$. Then firms are not acquiring any information about the prices of their competitors and the monetary policy shocks. But monetary policy shocks have a unit root which under the assumption that $\kappa = 0$ implies that firms' uncertainty about their optimal price, which is proportional to their losses from imperfect information, is growing linearly over time and exceeds any finite upper-bound. Now, consider an information acquisitoin strategy that sets $\kappa = \epsilon > 0$. It follows that firms' losses under this strategy is bounded above by $\mathscr{O}(\frac{1}{\epsilon})$ which dominates $\kappa = 0$. Thus, in the stationary equilibrium, $\kappa > 0$.

Now, from the proof of Proposition 4, recall that in the equilibrium, for all $(j,k) \in J \times K$, $p_{j,k,t}(S_{j,k}^t) = \mathbf{w}'\mathbb{E}[\theta_{j,k,t}(\varsigma_{-(j,k),t})|S_{j,k}^t]$ where $S_{j,k}^t = (S_{j,k}^{t-1}, S_{j,k,t})$ and

$$S_{j,k,t} = (1-\alpha)q_t + \alpha\frac{1}{K-1}\sum_{l\neq k} p_{j,l,t}(S_{j,l}^t) + e_{j,k,t}$$

From Kalman filtering

$$\mathbf{w}'\mathbb{E}[\theta_{j,k,t}(\varsigma_{-(j,k),t})|S_{j,k}^t] \quad = \quad \mathbb{E}[\mathbf{w}'\theta_{j,k,t}(\varsigma_{-(j,k),t})|S_{j,k}^{t-1}]$$

[54] This solution can also be obtained by applying the Kuhn-Tucker conditions.

$$+ \quad \frac{\mathbf{w}' cov(S_{j,k,t}, \theta_{j,k,t}(\varsigma_{-(j,k),t}))}{var(S_{j,k,t}|S^{t-1}_{j,k})}(S_{j,k,t} - \mathbb{E}[S_{j,k,t}|S^{t-1}_{j,k}]).$$

Notice from the proof of Proposition 4 that $\frac{\mathbf{w}' cov(S_{j,k,t}, \theta_{j,k,t}(\varsigma_{-(j,k),t}))}{var(S_{j,k,t}|S^{t-1}_{j,k})} = \frac{\lambda}{\mathbf{w}'\Sigma_{j,k,t|t-1}\mathbf{w}}\mathbf{w}'\Sigma_{j,k,t|t-1}\mathbf{w} = \lambda$. Thus, using $p_{j,k,t}$ as shorthand for $p_{j,k,t}(S^t_{j,k})$, $p_{j,k,t} = (1-\lambda)\mathbb{E}[S_{j,k,t}|S^{t-1}_{j,k}] + \lambda S_{j,k,t}$. Finally, notice that $p_{j,k,t-1} = \mathbb{E}[S_{j,k,t-1}|S^{t-1}_{j,k}]$. Subtract this from both sides of the above equation to get $\pi_{j,k,t} \equiv p_{j,k,t} - p_{j,k,t-1} = (1-\lambda)\mathbb{E}[\Delta S_{j,k,t}|S^{t-1}_{j,k}] + \lambda(S_{j,k,t} - p_{j,k,t-1})$, where $\Delta S_{j,k,t} = S_{j,k,t} - S_{j,k,t-1}$. Subtract $\lambda\pi_{j,k,t}$ from both sides and divide by $(1-\lambda)$ to get $\pi_{j,k,t} = \mathbb{E}[\Delta S_{j,k,t}|S^{t-1}_{j,k}] + \frac{\lambda}{1-\lambda}(S_{j,k,t} - p_{j,k,t})$. Averaging this equation over all firms gives us the Phillips curve:

$$\overline{\mathbb{E}^{j,k}_{t-1}[\Delta S_{j,k,t}]} \equiv \frac{1}{JK}\sum_{(j,k)\in J\times K}\mathbb{E}[\Delta S_{j,k,t}|S^{t-1}_{j,k}] = (1-\alpha)\overline{\mathbb{E}^{j,k}_{t-1}[\Delta q_t]} + \alpha\overline{\mathbb{E}^{j,k}_{t-1}[\pi_{j,-k,t}]}.$$

where $\pi_{j,-k,t} \equiv \frac{1}{K-1}\sum_{l\neq k}(p_{j,l,t} - p_{j,l,t-1})$ is the average price change of all others in industry j except k. Moreover,

$$\frac{1}{JK}\sum_{(j,k)\in J\times K}(S_{j,k,t} - p_{j,k,t}) = (1-\alpha)q_t + \underbrace{\frac{\alpha}{JK}\sum_{(j,k)\in J\times K}\frac{1}{K-1}\sum_{l\neq k}p_{j,l,t} - \frac{1}{JK}\sum_{(j,k)\in J\times K}p_{j,k,t}}_{=\frac{\alpha-1}{JK}\sum_{(j,k)\in J\times K}p_{j,k,t}}.$$

The last term asymptotically converges to zero as $J\to\infty$ as mistakes are orthogonal across sectors—$e_{j,k,t}\perp p_{m,l,t}, \forall m\neq j$. Now, define $p_t \equiv \frac{1}{JK}\sum_{(j,k)\in J\times K}p_{j,k,t}$, and recall that $q_t = p_t + y_t$. Therefore, $\frac{1}{JK}\sum_{(j,k)\in J\times K}(S_{j,k,t} - p_{j,k,t}) = (1-\alpha)y_t$. Finally, define aggregate inflation as the average price change in the economy, $\pi_t \equiv \frac{1}{JK}\sum_{(j,k)\in J\times K}\pi_{j,k,t}$. Plugging these into the expression above we get

$$\pi_t = (1-\alpha)\overline{\mathbb{E}^{j,k}_{t-1}[\Delta q_t]} + \alpha\overline{\mathbb{E}^{j,k}_{t-1}[\pi_{j,-k,t}]} + (1-\alpha)\frac{\lambda}{1-\lambda}y_t.$$

Finally, notice that $\frac{\lambda}{1-\lambda} = \frac{1-2^{-2\kappa}}{2^{-2\kappa}} = 2^{2\kappa} - 1$.

H Calibration Details

This section discusses the calibration of several model parameters in detail.

Elasticity of substitution. A usual approach in monopolistic competition models is to choose η to match an average markup given by $\frac{\eta}{\eta-1}$. In the oligopolistic competition model, markups depend on the number of competitors and in the steady-state are given by

$$\mu_j = 1 + \frac{1}{(\eta-1)(1-K_j^{-1})} \tag{H.1}$$

where K_j is the number of competitors in j. The survey elicits firms' markups by asking the following question: *"Considering your main product line or main line of services in the domestic market, by what margin does your sales price exceed your operating costs (i.e., the cost material inputs plus wage costs but not overheads and depreciation)? Please report your current margin as* *well as the historical or average margin for the firm."* The average markup reported by firms in the sample is 1.3 and varies from 1.1 to 1.6. These values are in the plausible range of markups measured in the literature for the US. Given this measure of markups, I run the analogous regression to Equation (H.1) and set $\eta = 12$ to match the coefficient on $\frac{1}{1-K_j^{-1}}$ in the regression. Table H.1 reports the result of this regression. This value is well in line with the values used in the literature for the US.

Table H.1: Calibration of η

	Average markup			
	(1)		*(2)*	
$1/(1-K_j^{-1})$	0.107	(0.016)	0.123	(0.017)
Manufacturing			0.040	(0.007)
Professional and Financial Services			0.169	(0.007)
Trade			0.027	(0.007)
Constant	1.205	(0.018)	1.106	(0.020)
Observations	3152		3152	

Standard errors in parentheses

Notes: the table reports the result of regressing the average markups of firms on $1/(1-K_j^{-1})$. The coefficient on this statistic is $1/(\eta-1)$ in the model.

Curvature of the production function. Given the empirical distribution of the number of firms, $\mathcal{K}$, and the elasticity of substitution, $\eta = 12$, I set $\gamma = 0.9$ to match the average degree of strategic complementarity $\bar{\alpha} = 0.8$ from Table 1. Given this value, the elasticity of output to labor in the model is 0.52. This is consistent with calibrations of this parameter for the U.S. if we were to calibrate it to the labor share of income in the U.S. data (see e.g. Bilal, Engbom, Mongey, and Violante, 2019, where the targeted value for the U.S. is 0.518).[55]

Persistence and variance of shocks to nominal demand. I calibrate $\rho = 0.7$ to match the persistence of the growth of nominal GDP in New Zealand for post-1991 data.[56] Nonetheless, the model is not very sensitive to this parameter in this range and I present results for an alternative value of ρ in Section 6.

Given the quarterly persistence, I then set $\sigma_u = 0.027$ to match the unconditional variance of quarterly nominal GDP growth.[57] Nonetheless, since monetary policy shocks are the only shocks in the model, the standard deviation of all variables – including endogenous non-fundamental

[55]Although we have not explicitly modeled capital, one could think of the production function of firms as one with constant returns to scale in capital and labor, where capital is exogenously fixed.

[56]This coefficient is obtained by regressing the annual log-growth of nominal GDP in New Zealand on one lag where I obtain a yearly persistence of 0.25. I then convert this to the quarterly persistence through $\rho = 0.25^{1/4}$. I restrict the time series to post 1991 data to be consistent with New Zealand's shift in monetary policy towards inflation targeting in that time frame.

[57]The unconditional variance is given by $\frac{\sigma_u^2}{1-\rho^2}$ which is 0.0014 in the data.

shocks – are scaled by the standard deviation of the innovations to nominal demand. Accordingly, in my counterfactual comparisons I will mainly focus on numbers relative to a benchmark so that the reported relative numbers are independent of this scale.[58]

I Symmetric Stationary Equilibrium and Solution Method

To characterize the equilibrium, I will use decomposition of firms' prices to their correlated parts with the fundamental shocks and mistakes as defined in the main text. I start with the fundamental q_t itself. Notice that since q_t has a unit root and is Gaussian, it can be decomposed to its random walk components: $q_t = \sum_{n=0}^{\infty} \psi_q^n \tilde{u}_{t-n}$, where $\tilde{u}_{t-n} = \sum_{\tau=0}^{\infty} u_{t-n-\tau}$, and $(\psi_q^n)_{n=0}^{\infty}$ is a summable sequence as Δq_t is stationary and $\Delta q_t = \sum_{n=0}^{\infty} \psi_q^n u_{t-n}$. In the case of $\beta = 0$, following Proposition 4 we know that given an initial signal structure for the game $(S_{j,k}^{-1})_{(j,k)\in J\times K}$, the equilibrium signals and pricing strategies are

$$\begin{aligned} S_{j,k,t} &= (1-\alpha)q_t + \alpha\frac{1}{K-1}\sum_{l\neq k} p_{j,k,t}(S_{j,k}^t) + e_{j,k,t}, \\ p_{j,k,t}(S_{j,k}^t) &= \mathbb{E}[(1-\alpha)q_t + \alpha\frac{1}{K-1}\sum_{l\neq k} p_{j,l,t}(S_{j,l}^t)|S_{j,k}^t] \\ &= \sum_{\tau=0}^{\infty} \delta_{j,k,t}^{\tau} S_{j,k,t-\tau}, \forall (j,k)\in J\times K, t \forall t \geq 0. \end{aligned}$$

However, these signals take a slightly different form when $\beta > 0$. Firms still receive one signal every period but they put different weights on the shocks. In particular, they realize that shocks to q are more persistent and taking the continuation value of knowing these shocks into account focus more of their attention on the aggregate shocks. To solve for the optimal form of the signal, we follow the solution method outlined in Afrouzi and Yang (2019) for deriving the optimal weights on each of these shocks.

To characterize the equilibrium, given the form for the optimal signal, I do a similar decomposition analogous to the one in the static model. Given the pricing strategies of firms at time t, decompose their price to its correlated parts with the fundamental and parts that are orthogonal to it over time: $p_{j,k,t}(S_{j,k}^t) = \sum_{n=0}^{\infty}(a_{j,k,t}^n \tilde{u}_{t-n} + b_{j,k,t}^n v_{j,k,t-n})$.Here, $\sum_{n=0}^{\infty} b_{j,k,t}^n v_{j,k,t-n}$ is the part of j,k's price at time t that is orthogonal to all these random walk components (mistake of firm j,k at time t). Moreover, $v_{j,k,t-n}$ is the innovation to j,k's price at time t that was drawn at time $t-n$. In other words, I have also decomposed the mistake of the firm over time. This decomposition is necessary because other firms follow all these mistakes, but they can only do so after it was

[58] This is due to potential concerns in matching the unconditional volatility. Calibrating the standard deviation needs to be done on the part of nominal demand that is driven by monetary policy shocks. In the US one can calibrate this variance by projecting nominal demand on known monetary policy shock series, such as Romer and Romer (2004) shocks, and fitting an AR(1) to the predicted series (See, for instance, Midrigan (2011)). For the case of New Zealand, however, this becomes a complication since, as far as I know, there is no unanimously agreed upon series for monetary shocks.

drawn at a certain point in time, in the sense that no firm can pay attention to future mistakes of their competitors as they have not been made yet. Before proceeding with characterization, I define the stationary symmetric equilibrium.

Definition 4. *Given an initial information structure* $(S_{j,k}^{-1})_{(j,k)\in J\times K}$*, suppose a strategy profile* $(S_{j,k,t}\in\mathscr{S}^t, p_{j,k,t}: S_{j,k}^t\to\mathbb{R})_{k\in K,t\geq 0}$ *is an equilibrium for the game. We call this a symmetric steady state equilibrium if the pricing strategies of firms is independent of time,* $t\geq 0$*, and identity,* $k\in K$*. Formally,* $\exists\{(a^n)_{n=0}^{\infty},(b^n)_{n=0}^{\infty}\}$*, such that* $\forall t\geq 0, \forall (j,k)\in J\times K$*,* $p_{j,k,t}=\sum_{n=0}^{\infty}(a^n\tilde{u}_{t-n}+b^n v_{j,k,t-n})$*.*

To characterize the equilibrium, notice that we not only need to find the sequences $(a^n,b^n)_{n=0}^{\infty}$, but also the joint distribution of $v_{j,k,t-n}$'s across the industries. To see this, take firm j,k and suppose all other firms are setting their prices according to $p_{j,k,t}=\sum_{n=0}^{\infty}(a^n\tilde{u}_{t-n}+b^n v_{j,k,t-n})$. Then, firm j,k's optimal signals are given by the solution method outlined n Afrouzi and Yang (2019). In the special case of $\beta=0$ this form is given by

$$S_{j,k,t} = \sum_{n=0}^{\infty}\left[((1-\alpha)\psi_q^n+\alpha a^n)\tilde{u}_{t-n}+\alpha b^n\frac{1}{K-1}\sum_{l\neq k}v_{j,l,t-n}+e_{j,k,t}\right],$$

where by properties of the equilibrium $e_{j,k,t}$ is the rational inattention error and is orthogonal to $\tilde{u}_{t-n}$ and $v_{j,l,t-n}$, $\forall n\geq 0, \forall l\neq k$. Using the joint distributions of errors $(v_{j,k,t-n})_{k\in K}$, by Kalman filtering, the firm would choose to set their price according to

$$p_{j,k,t} = \sum_{n=0}^{\infty}\delta^n S_{j,k,t-n}=\sum_{n=0}^{\infty}(\tilde{a}_n\tilde{u}_{t-n}+\tilde{b}_n\frac{1}{K-1}\sum_{l\neq k}v_{j,k,t-n}+\tilde{c}_n e_{j,k,t-n})$$

for some sequences $(\tilde{a}_n,\tilde{b}_n,\tilde{c}_n)$. But in the equilibrium, $p_{j,k,t}=\sum_{n=0}^{\infty}(a^n\tilde{u}_{t-n}+b^n v_{j,k,t-n})$. This implies, $a^n=\tilde{a}^n$, $b^n v_{j,k,t-n}=\tilde{b}_n\frac{1}{K-1}\sum_{l\neq k}v_{j,l,t-n}+\tilde{c}_n e_{j,k,t-n}$, where $e_{j,k,t-n}\perp v_{j,l,t-n}, \forall l\neq k$. Using the second equation we can characterize the joint distribution of $(v_{j,k,t-n})_{k\in K}, \forall n\geq 0$. This joint distribution is itself a fixed point and should be consistent with the Kalman filtering behavior of the firm that gave us $(\tilde{a}_n,\tilde{b}_n,\tilde{c}_n)_{n=0}^{\infty}$ in the first place. Finally, notice that underneath all these expressions we assume that these processes are stationary meaning that the tails of all these sequences should go to zero. Otherwise, the problems of the firms are not well-defined and do not converge. I verify this computationally, by truncating all these sequences such that $\forall n\geq\bar{T}\in\mathbb{N}, a^n=b^n=0$ where $\bar{T}$ is large, solving the problem computationally, and checking whether the sequences go to zero up to a computational tolerance before reaching $\bar{T}$. In my code I set $\bar{T}=100$. The economic interpretation for this truncation is that all real effects of monetary policy should disappear within 100 quarters. Such truncations are the standard approach in the literature for solving dynamic imperfect information models.

The following algorithm illustrates my method for solving the problem.

Algorithm 1. *Characterizing a symmetric stationary equilibrium:*

1. Start with an initial guess for $(a^n,b^n)_{n=0}^{\bar{T}-1}$, and solve for a representative firm j,k's optimal

signal using the method in Afrouzi and Yang (2019) (for the the case of $\beta = 0$ set $S_{j,k,t} = \sum_{n=0}^{\bar{T}-1}\left[((1-\alpha)\psi_q^n + \alpha a^n)\tilde{u}_{t-n} + \alpha b^n \frac{1}{K-1}\sum_{l\neq k} v_{j,l,t-n} + e_{j,k,t}\right]$).

2. Using Kalman filtering, given the set of signals implied by previous step, form the best pricing response of a firm and truncate it. Formally, find coefficients $(\tilde{a}_n, \tilde{b}_n, \tilde{c}_n)_{n=0}^{\bar{T}-1}$ such that $p_{j,k,t} \approx \sum_{n=0}^{\bar{T}-1}(\tilde{a}_n \tilde{u}_{t-n} + \tilde{b}_n \frac{1}{K-1}\sum_{l\neq k} v_{j,l,t-n} + \tilde{c}_n e_{k,t-n})$.
3. $\forall n \in \{0,\ldots,\bar{T}-1\}$, update $a^n = \tilde{a}^n$, and b^n such that $b_n v_{k,t-n} = \tilde{b}_n \frac{1}{K-1}\sum_{l\neq k} v_{j,l,t-n} + \tilde{c}_n e_{k,t-n}$, using $e_{k,t} \perp v_{-k,t}$, and the symmetry of the distribution of $(v_{j,k,t})_{k\in K}$.
4. Iterate until convergence of the sequence $(a^n, b^n)_{n=0}^{\bar{T}-1}$.

J A Static Model with Heterogeneous Market Shares

Consider the household's demand with CES aggregator from Equation 5 with the following modification:

$$C_t = \prod_{j\in J}\left[\left(\sum_{k\in K_j} \bar{m}_{j,k}^{\frac{1}{\eta}} C_{j,k,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}\right]^{J^{-1}} \tag{J.1}$$

where now $\bar{m}_{j,k}$ captures the taste of the consumer for the product of firm k in industry j. Moreover, $\forall j$ we normalize $\sum_k \bar{m}_{j,k} = 1$ so that that these tastes are relative. It is straight forward to show that $\bar{m}_{j,k}$ shows up as a demand shifter in firm j,k' demand

$$C_{j,k,t} = P_t C_t \frac{\bar{m}_{j,k} P_{j,k,t}^{-\eta}}{\sum_l \bar{m}_{j,l} P_{j,l,t}^{1-\eta}} \tag{J.2}$$

On the firm side, this implies that the elasticity of demand for firm j,k at time t is given by

$$\varepsilon_{j,k,t} = \eta - (\eta-1)\frac{\bar{m}_{j,k} P_{j,k,t}^{1-\eta}}{\sum_l \bar{m}_{j,l} P_{j,l,t}^{1-\eta}} \tag{J.3}$$

On the firm side, assume constant returns to scale in production ($\gamma = 0$) and that there is a subsidy for every firm such that it sets their steady state price equal to the aggregate marginal cost given their optimal markup (so that there is no price dispersion in the steady state). Then the approximate problem of the firm, as in Equation 15, is given by

$$\max_{\{\kappa_{j,k,t}, S_{j,k,t}, p_{j,k,t}(S_{j,k}^t)\}_{t\geq 0}} -\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\left(\underbrace{\eta(p_{j,k,t}(S_{j,k}^t) - p_{j,k,t}^*)^2}_{\text{loss from mispricing}} + \underbrace{\omega\kappa_{j,k,t}}_{\text{cost of capacity}} | S_{j,k}^{-1}\right)\right] \tag{J.4}$$

$$\begin{aligned} s.t.\quad & p_{j,k,t}^* \equiv (1-\alpha_{j,k})q_t - \alpha_{j,k} p_{j,-k,t}(S_{j,-k,t}) \\ & \mathscr{I}\left(S_{j,k,t}, (q_\tau, p_{l,m,\tau}(S_{l,m}^\tau))_{0\leq\tau\leq t}^{(l,m)\neq(j,k)}\right) \leq \kappa_{j,k,t} \\ & S_{j,k}^t = S_{j,k}^{t-1} \cup S_{j,k,t}, \quad S_{j,k}^{-1} \text{ given.} \end{aligned} \tag{J.5}$$

where we have already imposed that in the case of $\gamma = 0$, the curvature of the profit function is uniquely determined by the elasticity of substitution ($B_j = \eta$). The only major difference to this problem is that now, with heterogeneity in market shares, there is also heterogeneity in the degree of strategic complementarity within industries. In fact, in this case, the degree of strategic complementarity for every firm is proportional to their steady-state market share:

$$\alpha_{j,k} = (1 - \eta^{-1})\bar{m}_{j,k} \tag{J.6}$$

Note that, here, $\bar{m}_{j,k}$ is simply the market share of firm k in industry j in the steady-state, and we can study the impact of heterogeneity in market shares on the attention allocation of firms. Finally, to make this case even simpler, assume that $\eta \to \infty$.[59] Then, taking a second-order approximation around this steady state, it follows from Equation (12) that the ideal price of firm j, k is given by

$$p^*_{j,k,t} = (1 - \bar{m}_{j,k})q_t + \bar{m}_{j,k}\frac{\sum_{l \neq k}\bar{m}_{j,l}p_{j,l,t}}{\sum_{l \neq k}\bar{m}_{j,l}} \tag{J.7}$$

This representation also shows that higher market share leads to higher strategic complementarity and hence magnifies the degree of strategic inattention.

[59]In this hypothetical example, having $\eta \to \infty$ means that firms' profit functions are infinitely concave and that the benefit of information is arbitrarily large given a fixed ω. Therefore, for a fixed ω firms will acquire almost perfect information. To resolve this, we assume that ω is also proportional to η so that the ratio stays constant as $\eta \to \infty$.