Monetary Policy without Commitment[*]

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Abstract

This paper studies the implications of central bank credibility for long-run inflation and inflation dynamics. We introduce central bank lack of commitment into a standard non-linear New Keynesian economy with sticky-price monopolistically competitive firms. Inflation is driven by the interaction of lack of commitment and the economic environment. We show that long-run inflation increases following an unanticipated permanent increase in the labor wedge or decrease in the elasticity of substitution across varieties. In the transition, inflation overshoots and then gradually declines. Quantitatively, the inflation response is large, as is the welfare loss from lack of commitment relative to inflation targeting.

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1 Introduction

Inflation across advanced economies in the aftermath of the COVID-19 pandemic rose to levels not seen since the early 1980s. This has brought about a resurgence of interest in the subject of central bank credibility to the maintenance of low and stable inflation. These recent macroeconomic developments additionally highlight the challenge in applying the most commonly used quantitative macroeconomic models—which assume exogenous central bank reaction functions and inflation targets—for understanding the post-pandemic environment.

In this paper, we approach the analysis of central bank credibility by developing a framework where policy is not exogenous, but is instead dynamically chosen by a central bank that maximizes social welfare in every date. We focus on the implications of the central bank's inability to make ex-ante commitments. Our analysis builds on the seminal work of Barro and Gordon (1983) and Rogoff (1985), who study the implications of the central bank's inflation-output tradeoff for the economy. This work and most of the vast literature that followed it, however, consider simple static settings or linearized dynamic environments.[1] By their construction, these analyses do not inform how central bank credibility impacts long-run inflation or transition dynamics.[2]

We build on this prior work by studying central bank credibility in a standard New Keynesian model.[3] In order to allow for an analysis of long-run inflation and transition dynamics, we do not perform a log-linearization around the zero-inflation steady state but instead examine the fully non-linear model. For tractability, we take a deterministic environment, and we consider the impact of permanent shocks to the economy. The economy is composed of monopolistically competitive firms with sticky prices: in every period, a random fraction of firms have the ability to flexibly choose their price, while the remaining firms are constrained to choosing their previous period's price. Wages are fully flexible and households make consumption, labor, and savings decisions. Firms and households optimize taking into account current economic conditions and policies and their expectations of future economic conditions and policies. As is standard in the literature, we allow for the existence of an exogenous labor wedge that takes the form of a proportional positive or negative tax on labor. This wedge captures statutory taxes on labor and other labor market distortions, such as the pervasiveness of regulation and unionization.

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[1]See, for example, Backus and Driffill (1985), Canzoneri (1985), Cukierman and Meltzer (1986), Athey, Atkeson, and Kehoe (2005), and Halac and Yared (2020, 2022).

[2]Linearized environments are useful for considering transition dynamics around an assumed steady state; however, such a state may not coincide with the actual steady state of the economy, which can only be determined by analyzing the non-linear environment.

[3]See Clarida, Galí, and Gertler (1999), Woodford (2003), and Galí (2015). As we describe next, we consider price stickiness with Calvo pricing (Calvo, 1983), but our results are identical under wage stickiness.

Our economy admits two types of distortions. First, the existence of monopoly power means that absent a sufficiently negative labor wedge, firms underproduce and underhire. To examine the role of this distortion, we assume that the labor wedge is large enough that underproduction arises under flexible prices. Second, the existence of sticky prices generates price dispersion in the goods market (if inflation is non-zero), which causes labor misallocation, with too much labor drawn to the production of low-price varieties and too little to the production of high-price varieties. Our analysis highlights how monopoly distortions and labor misallocation impact the inflation-output tradeoff and guide the conduct of monetary policy.

Before describing our main findings, it is instructive to note that monetary policy in our environment is not neutral in the long run. Different policy paths can lead to a continuum of potential steady states. Comparing across hypothetical steady states, those with higher inflation admit relatively higher price dispersion and labor misallocation, since the divergence in prices between flexible-price firms (which raise prices by more under higher inflation) and sticky-price firms is relatively larger. Moreover, steady states with higher inflation also admit relatively lower monopoly distortions, since overhiring by sticky-price firms is relatively higher.

We study central bank credibility by analyzing the Markov Perfect Competitive Equilibria of our model, where central bank strategies and private sector beliefs are a function of payoff-relevant variables only. In every period, flexible-price firms set prices, the central bank sets the interest rate, and markets open and clear. The central bank lacks commitment and freely chooses the interest rate that maximizes social welfare in every period, taking as given the distribution of prices. Firms setting prices flexibly anticipate that the central bank lacks commitment today and in the future when forming expectations about future policy.

We show that an equilibrium is characterized by two difference equations. The dynamic path of inflation is characterized by an equation that is forward looking; i.e., a non-linear Phillips curve with current inflation being a function of expectations of future inflation. The dynamic path of price dispersion is characterized by an equation that is backward looking; i.e., with current dispersion being a function of past dispersion. These equations yield that there is a unique steady state, which allows us to analyze transition dynamics around such a steady state. The tractable form of the equilibrium owes partly to the timing in our model, where prices are chosen prior to the choice of interest rates. A central bank setting interest rates at time t takes as predetermined the distribution of prices at time t, and therefore also the continuation equilibrium at $t+1$ (since the equilibrium is Markov). Because the central bank cannot change future welfare off the equilibrium path, it optimally chooses an interest

rate that maximizes static welfare conditional on the level of price dispersion.[4] The result is a policy that eliminates the monopoly distortion and sets the labor share to one,[5] and an equilibrium that can be simplified to a system of two equations.

Our main results are as follows. First, we show that long-run inflation is determined by the interaction of the central bank's lack of commitment and the economic environment. Specifically, long-run inflation is higher the higher is the labor wedge and the lower is the elasticity of substitution across varieties. To understand these comparative statics, consider first the incentives of the central bank to cut interest rates off the equilibrium path. A rate cut increases consumption at the cost of increased labor effort, so its marginal benefit is increasing in monopoly distortions (which suppress labor) and decreasing in price dispersion and labor misallocation (which reduce aggregate labor productivity).

Starting from a given steady state, suppose that there is a permanent increase in the labor wedge or a permanent decrease in the elasticity of substitution across varieties (with these changes being unanticipated). A central bank with commitment would be able to respond in a way that preserves inflation stability. However, a central bank without commitment has an incentive to undo the resulting increase in monopoly distortions by cutting interest rates and stimulating output. Flexible-price firms anticipate this and rationally forecast higher future labor demand and higher future real wages (relative to the commitment case), which necessitate higher offsetting prices today. Over time, flexible-price firms thus increase prices, leading to rising price dispersion. The economy converges to a new steady state once the rise in dispersion reduces aggregate productivity sufficiently that the central bank's benefit from cutting interest rates vanishes. Therefore, as a result of the higher labor wedge or lower elasticity of substitution across varieties, price dispersion and inflation are both higher in the new steady state.

Our second main result is a characterization of the transition as the economy moves from an initial steady state to one with higher inflation. We show that the transition features inflation overshooting. Starting from a given steady state, consider a permanent increase in the labor wedge or a permanent decrease in the elasticity of substitution across varieties. Inflation overshooting emerges because of the evolution of central bank incentives as price dispersion rises in the transition towards a higher-inflation steady state. The central bank sees a relatively larger benefit to stimulating output early in the transition when price dispersion and labor misallocation are low. Once price dispersion and labor misallocation rise sufficiently,

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[4]Our results are unchanged if firms and the central bank move simultaneously, as in Barro and Gordon (1983). However, if the central bank sets the interest rate before firms set prices, then off-equilibrium expectations by firms can play an important role in equilibrium selection, complicating the analysis.

[5]In other words, average firm profits (net of labor taxes) across the economy equal zero, with some sticky-price firms making negative profits and all flexible-price firms making positive profits.

it is no longer as worthwhile to stimulate labor to generate additional consumption. Flexible-price firms in turn realize that monetary stimulus will be larger earlier in the transition, so they offset the ensuing higher wage costs with price increases that are also relatively larger earlier in the transition. The implication of these transition dynamics is inflation overshooting.

Our final main result shows that the quantitative magnitudes implied by our model are large. Using standard parameterizations of the New Keynesian model, we evaluate the response of the economy to a permanent increase in the labor wedge or a permanent decrease in the elasticity of substitution across varieties. In both cases, inflation jumps up following the shock and then gradually declines towards a new higher steady-state level. Nominal interest rates jump up and gradually increase to a higher level, while real interest rates jump down and gradually increase back to their original level.[6] Output falls gradually as price dispersion and labor misallocation increase in the transition. Nominal wage inflation rises towards the new steady state, converging to the level of price inflation from below. These dynamics underpin an eventual permanent reduction in the real wage.

We find that small changes in the labor wedge or the elasticity of substitution across varieties have large impacts on the long-run level of inflation and the degree of inflation overshooting. For example, in response to a 0.5 percent increase in the labor wedge starting from a 2-percent-inflation steady state, inflation overshoots to 10.11 percent and eventually converges to 8.76 percent. This overshooting is persistent: it takes 12 months for inflation to decline within 25 basis points of its new steady-state level. We obtain similarly large magnitudes for changes in the elasticity of substitution. Furthermore, we compare the response of the economy under central bank lack of commitment to the response under commitment to inflation targeting, and we find that the welfare loss from lack of commitment is quantitatively large.

Our results have two important implications for the analysis of the post-pandemic inflation in advanced economies. First, our results suggest that changes in the global economy can affect long-run inflation through their interaction with central bank incentives. Consider for instance a decrease in immigration which changes the composition of labor towards more regulated labor sources (higher labor wedge), or a slowdown in globalization which increases the market power of domestic firms (lower elasticity of substitution across varieties). Our model says that if these changes in the global economy are permanent, then they can result in permanently higher inflation. These changes reduce output and consumption and make it more challenging for a central bank without commitment to maintain low inflation. Moreover,

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[6]The higher steady-state level for nominal interest rates reflects the Fisherian effect, which is present in the non-linear New Keynesian model.

under this view, a second implication of our analysis is that the large spike in inflation following the pandemic can be partially understood as a consequence of the private sector anticipating the central bank's response. Firms rationally anticipate that the central bank would be more accommodative earlier in the transition (when the cost of price dispersion is low) relative to later (when the cost of price dispersion is high), resulting in inflation overshooting as the economy moves to a higher-inflation steady state.

Related Literature. Our paper fits into the literature on central bank credibility and reputation pioneered by Barro and Gordon (1983) and Rogoff (1985).[7] As discussed, our departure from this literature is in analyzing the equilibrium of the fully non-linear New Keynesian model. This departure allows us to examine the endogenous dynamic evolution of the central bank's inflation-output tradeoff as well as the quantitative implications of central bank credibility. An approach that considers a Markovian equilibrium under lack of commitment around a linearized (distorted) zero-inflation, zero-dispersion steady state (as in Halac and Yared, 2022, for example) not only features no transition dynamics, but also significantly overestimates the effect of permanent shocks on long-run inflation relative to the non-linear model.[8]

Previous work has considered lack of commitment to monetary policy in non-linear environments. For example, a large number of models of fiscal policy are concerned with the central bank's commitment to not inflating away public debt (e.g., Alvarez, Kehoe, and Neumeyer, 2004; Aguiar, Amador, Farhi, and Gopinath, 2015). Dávila and Schaab (2023) show that lack of commitment to monetary policy has distributional implications in heterogeneous-agent economies.[9] Our departure from these literatures is in considering the dispersion cost that results from price stickiness in standard New Keynesian models, and in examining how this dispersion cost dynamically affects the inflation-output tradeoff for the central bank.

In this regard, we contribute to the literature that studies the central bank's inflation-output tradeoff under lack of commitment in non-linear settings. The focus of this literature has been on determining conditions for equilibrium multiplicity under monetary discretion (see, e.g., Albanesi, Chari, and Christiano, 2003; King and Wolman, 2004; Zandweghe and Wolman, 2019). These considerations do not arise in our setting where we obtain a unique equilibrium. We depart from this work by providing an analytical characterization of the

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[7]See additionally the work cited in Footnote 1.

[8]The reason is that the linearized model underestimates the welfare costs of rising price dispersion due to inflation.

[9]Their model has sticky prices with exogenous costs of price adjustment, as opposed to Calvo pricing as in our framework. As a consequence, there is no price dispersion in their equilibrium.

unique steady state and the transition dynamics of the non-linear model, and by analytically studying how these depend on the economic environment.[10]

Finally, our paper also relates to the literature on optimal commitment policy in the non-linear New Keynesian model (e.g., Benigno and Woodford, 2005; Yun, 2005). In our analysis, we provide a novel recursive representation of the non-linear Phillips curve by defining a new auxiliary variable which can itself be represented recursively. This variable captures the passthrough of real wages to current inflation, holding future inflation expectations fixed. The recursive representations allow us to characterize transition dynamics, and we conjecture that they could be useful in future analyses of the non-linear New Keynesian model.

2   Environment

We consider a standard non-linear New Keynesian model (Clarida, Galí, and Gertler, 1999; Woodford, 2003; Galí, 2015). There is a unit-mass of monopolistically competitive firms that set prices under Calvo-style rigidity (Calvo, 1983): a random fraction of firms in every period have the ability to flexibly choose their price, while the remaining fraction of firms are constrained to choosing the same price as in the previous period. Wages are fully flexible and households make consumption, labor, and savings decisions.[11] Firms and households optimize taking into account current economic conditions and policies and their expectations of future economic conditions and policies.

In every period, flexible-price firms set prices, the central bank sets the interest rate, and markets open and clear. A key feature of our environment is that the central bank lacks commitment and freely chooses the interest rate that maximizes social welfare in every period, taking as given the distribution of prices. Firms setting prices flexibly anticipate that the central bank lacks commitment today and in the future when forming expectations about future policy.

2.1   Households

At every date $t \in \{0, 1, 2, \dots\}$, the representative household chooses its consumption $C_{j,t}$ of each firm variety $j \in [0, 1]$, its labor supply L_t, its holdings B_t of a risk-free nominal government bond that pays interest i_t, and its holdings $s_{j,t}$ of shares of each firm $j \in [0, 1]$. For

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[10]The model of Albanesi, Chari, and Christiano (2003) has sticky prices only within the period, while that of King and Wolman (2004) has prices that are sticky across two periods. The framework in Zandweghe and Wolman (2019) is the closest to ours with Calvo pricing across periods, but their timing is different and their results under lack of commitment are numerical rather than analytical.

[11]Our results are unchanged if we instead consider a sticky wage model.

each j, denote the firm's variety price by $P_{j,t}$, its nominal share price by $P_{j,t}^S$, and its nominal profits by $X_{j,t}$. Then letting W_t denote the nominal wage, the representative household solves the following problem:

$$\max_{C_t, L_t, B_t, (s_{j,t}, C_{j,t})_{j \in [0,1]}} \sum_{t=0}^{\infty} \beta^t \left(\log(C_t) - \frac{L_t^{1+\psi}}{1+\psi} \right) \tag{1}$$

subject to

$$\int_0^1 P_{j,t} C_{j,t} \mathrm{d}j + B_t \leq W_t L_t + (1+i_{t-1})B_{t-1} + \int_0^1 s_{j,t} X_{j,t} \mathrm{d}j + \int_0^1 (s_{j,t-1} - s_{j,t}) P_{j,t}^S \mathrm{d}j - T_t,$$

$$C_t = \left(\int_0^1 C_{j,t}^{1-\sigma^{-1}} \mathrm{d}j \right)^{\frac{1}{1-\sigma^{-1}}}.$$

C_t denotes the aggregate consumption bundle and T_t is a lump sum tax at date t. We have taken $\beta \in (0,1)$ to be the discount factor, $\sigma > 1$ is the elasticity of substitution across varieties, and $\psi > 1$ is the inverse elasticity of labor supply.

The household's optimization yields that the demand for variety j satisfies

$$C_{j,t} = C_t \left(\frac{P_{j,t}}{P_t} \right)^{-\sigma}, \tag{2}$$

where $P_t = \left(\int_0^1 P_{j,t}^{1-\sigma} \mathrm{d}j \right)^{\frac{1}{1-\sigma}}$. Thus, we can write $\int_0^1 P_{j,t} C_{j,t} \mathrm{d}j = P_t C_t$ in the budget constraint in program (1), and this means that households can choose their consumption bundle C_t as a function of P_t before solving the subproblem of choosing the consumption $C_{j,t}$ of each variety $j \in [0,1]$ as a function of $(P_{j,t})_{j \in [0,1]}$. The intratemporal condition is

$$\frac{W_t}{P_t} = C_t L_t^{\psi}. \tag{3}$$

The intertemporal condition is

$$1 = \beta(1+i_t) \frac{P_t C_t}{P_{t+1} C_{t+1}}. \tag{4}$$

The transversality conditions require that for each date t and firm j,[12]

$$\lim_{h \to \infty} \frac{1}{\prod_{\ell=0}^{h}(1+i_{t+\ell})} \mathbb{E}_t^j[X_{j,t+1+h}] = \lim_{h \to \infty} \frac{1}{\prod_{\ell=0}^{h}(1+i_{t+\ell})} i_{t+h} B_{t+h} = 0. \tag{5}$$

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[12]This transversality condition combines a household optimality condition and a no-Ponzi condition in a complete market environment that allows for Arrow-Debreu securities, including securities that pay off at a future date an amount proportional to the profits of any firm conditional on any given history.

The expectation operator $\mathbb{E}_t^j[\cdot]$ operates over firm j's future idiosyncratic shocks, which we will discuss in the next section. No arbitrage for stocks requires $P_{j,t}^S = X_{j,t} + \mathbb{E}_t^j[P_{j,t+1}^S]/(1+i_t)$, which combined with (5) yields

$$P_{j,t}^S = X_{j,t} + \sum_{h=0}^{\infty} \frac{1}{\prod_{\ell=0}^h (1 + i_{t+\ell})} \mathbb{E}_t^j[X_{j,t+1+h}]. \tag{6}$$

Combining (6) with the intertemporal condition for arbitrary horizon $t + h$, we obtain that the nominal share price of firm j satisfies[13]

$$P_{j,t}^S = \sum_{h=0}^{\infty} \beta^h \frac{P_t C_t}{P_{t+h} C_{t+h}} \mathbb{E}_t^j[X_{j,t+h}]. \tag{7}$$

2.2 Firms

A firm selling variety j produces with technology $Y_{j,t} = L_{j,t}$. Given the labor $L_{j,t}$ demanded by each firm $j \in [0, 1]$, in every period t we have $L_t = \int_0^1 L_{j,t} \mathrm{d}j$.

Firms set prices as in Calvo (1983). In every period, a random fraction $1 - \theta \in (0, 1)$ of firms are able to flexibly change their prices; the remaining fraction θ must keep their previous period's price. The exogenous initial distribution of firm prices is given by $\{P_{j,-1}\}_{j \in [0,1]}$. Firms commit to produce enough to meet demand given their price $P_{j,t}$, even if that means making negative profits.

Firms are subject to a proportional payroll tax $\tau \in (0, 1)$, which we refer to as the labor wedge. This wedge captures statutory taxes on labor and other labor market distortions, such as the pervasiveness of regulation and unionization.[14] We make the following assumption:

Assumption 1. *The labor wedge satisfies $\tau > -1/\sigma$.*

This assumption implies that monopoly distortions arise in an economy with flexible prices. As we will discuss in the next section, Assumption 1 guarantees that the labor wedge is sufficiently large that monopoly distortions do not disappear in the steady state of our economy.

Firm profits at any date t satisfy

$$X_{j,t} = P_{j,t} Y_{j,t} - (1 + \tau) W_t L_{j,t},$$

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[13]Note that in equilibrium, all households hold equal portions of all firms, so $\int_0^1 (s_{j,t-1} - s_{j,t}) P_{j,t}^S \mathrm{d}j = 0$.
[14]Under this latter interpretation, $-T_t$ can be thought of as measuring union profits.

which combined with (7) implies

$$P_{j,t}^S = \sum_{h=0}^{\infty} \beta^h \frac{P_t C_t}{P_{t+h} C_{t+h}} \mathbb{E}_t^j \left[P_{j,t+h} Y_{j,t+h} - (1+\tau) W_{t+h} L_{j,t+h} \right]. \tag{8}$$

A firm that resets its price $P_{j,t}$ at time t maximizes its share price $P_{j,t}^S$ given by (8), taking into account that the price $P_{j,t}$ will prevail at date $t+h$ with probability θ^h. The flexible-price firm's problem can thus be represented as follows:

$$\max_{P_t^*} \sum_{h=0}^{\infty} (\beta\theta)^h \frac{P_t C_t}{P_{t+h} C_{t+h}} [P_t^* - (1+\tau) W_{t+h}] C_{t+h} \left(\frac{P_t^*}{P_{t+h}} \right)^{-\sigma}, \tag{9}$$

where we have taken into account that the transversality condition implies that, for all t,

$$\lim_{h\to\infty} (\beta\theta)^h \frac{P_t C_t}{P_{t+h} C_{t+h}} [P_t^* - (1+\tau) W_{t+h}] C_{t+h} \left(\frac{P_t^*}{P_{t+h}} \right)^{-\sigma} = 0. \tag{10}$$

2.3 Government

At every date t, the central bank sets the interest rate i_t to maximize social welfare which is given by (1). We describe the central bank's problem in Section 4.1.

The fiscal authority sets taxes T_t and debt B_t to satisfy its budget constraint:

$$(1 + i_{t-1}) B_{t-1} = B_t + T_t + \tau W_t L_t. \tag{11}$$

The exogenous initial level of government debt is given by B_{-1}.

2.4 Order of Events

The order of events at a date t, given a distribution of prices $P_{j,t-1}$, is as follows:

1. Flexible-price firms choose price $P_{j,t} = P_t^*$. Sticky-price firms choose price $P_{j,t} = P_{j,t-1}$.
2. The central bank chooses monetary policy, i.e. the interest rate i_t.
3. Households choose consumption, labor, and savings $C_t, L_t, B_t, (s_{i,t}, C_{j,t})_{j \in [0,1]}$.
4. The fiscal authority chooses fiscal policy, i.e. taxes T_t and debt B_t.

Observe that monetary policy is chosen after firms have chosen their prices. Hence, if the central bank deviates from equilibrium policy, firms will no longer be optimizing during the period of the deviation off the equilibrium path. The fact that fiscal policy is chosen at the end of the period is for expositional simplicity and without loss given our equilibrium definition which we describe in the next section.

2.5 Equilibrium Definition

Our solution concept is Markov Perfect Competitive Equilibrium (MPCE), implying that households, firms, and the government make decisions as a function of payoff-relevant variables only. Note that because of the presence of lump sum taxes, our economy features Ricardian Equivalence, which means that the level of debt B_t can be treated as payoff irrelevant and can be set to 0 without loss of generality. As such, we assume that at the end of each period, the fiscal authority chooses debt $B_t = 0$ and taxes T_t to balance its budget, both on and off the equilibrium path.[15]

Formally, let Ω_{t-1} correspond to the distribution of prices across firms at date $t-1$. Conditional on Ω_{t-1}, flexible-price firms choose price P_t^* which determines Ω_t. Let Γ denote the corresponding mapping, with $\Omega_t = \Gamma(\Omega_{t-1})$. Given Ω_t, the central bank chooses monetary policy $i_t \equiv \Psi(\Omega_t)$, where Ψ is the central bank's reaction function. Finally, given Ω_t and i_t, households choose consumption, labor, and savings $\left(C_t, L_t, B_t, (s_{j,t}, C_{j,t})_{j\in[0,1]}\right) \equiv \omega(\Omega_t, i_t)$, where ω is the households' reaction function. In equilibrium, $B_t = 0$ and $s_{j,t} = 1$, since households can be identically treated without loss of generality.

An MPCE is a collection of mappings $\{\Gamma, \Psi, \omega\}$ such that, at every date t and given $\{\Gamma, \Psi, \omega\}$, the mapping $\Gamma(\Omega_{t-1})$ satisfies flexible-price firm optimality, $\Psi(\Omega_t)$ maximizes social welfare, and $\omega(\Omega_t, i_t)$ satisfies household optimality.[16]

3 Competitive Equilibrium

Any MPCE is a competitive equilibrium, i.e., it satisfies firm and household optimality given the central bank's policy. In this section, we characterize the conditions that are necessary and sufficient for a sequence of aggregate allocations and prices to constitute a competitive equilibrium given a sequence of policies. We use these conditions to illustrate the non-neutrality of monetary policy in the long run and how long-run inflation is related to price dispersion and monopoly distortions in a hypothetical steady state. These results are useful for our analysis in Section 4 and Section 5, where we study the central bank's problem and characterize equilibrium policy and the unique steady state of the economy.

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[15]The set of continuation MPCE at a date t starting from any two values of debt is the same. Therefore, the fiscal authority at the end of the period is indifferent over all values of B_t and can set debt to zero. Without the Markov restriction, government debt could serve as a payoff-irrelevant coordination device to select among different equilibria.

[16]Observe that an MPCE is a sustainable equilibrium, as defined in Chari and Kehoe (1990).

3.1 Aggregate Production

Define price dispersion $D_t \geq 1$ by[17]

$$D_t = \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\sigma} \mathrm{d}j.$$

Observe that $C_{j,t} = Y_{j,t} = L_{j,t}$, where

$$C_t = Y_t = \left(\int_0^1 Y_{j,t}^{1-\sigma^{-1}} \mathrm{d}j\right)^{\frac{1}{1-\sigma^{-1}}}.$$

Thus, we can write

$$L_t = \int_0^1 L_{j,t}\mathrm{d}j = \int_0^1 C_{j,t}\mathrm{d}j = \int_0^1 C_t \left(\frac{P_{j,t}}{P_t}\right)^{-\sigma} \mathrm{d}j = C_t D_t,$$

which, given $C_t = Y_t$, implies

$$Y_t = \frac{L_t}{D_t}. \tag{12}$$

This relationship shows that conditional on a level of labor L_t, higher price dispersion D_t reduces aggregate production Y_t and thus aggregate consumption C_t. The reason is that households spend too much on low-price varieties and too little on high-price varieties, and therefore too much labor is drawn to the production of low-price varieties and too little to the production of high-price varieties.

Using $C_t = Y_t$, we can rewrite the Euler equation (4) as

$$1 = \beta \frac{1+i_t}{\Pi_{t+1}} \frac{Y_t}{Y_{t+1}}, \tag{13}$$

where Π_{t+1} is the gross level of inflation:

$$\Pi_{t+1} = \frac{P_{t+1}}{P_t}. \tag{14}$$

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[17]We can show that $D_t \geq 1$ with equality only when all prices are almost surely equal. To see this, define the function $f(X) \equiv X^{\frac{\sigma}{\sigma-1}}$ and note that $D_t = \mathbb{E}_j[f((P_{j,t}/P_t)^{1-\sigma})]$, where the expectation is taken according to the Lebesgue measure over $j \in [0,1]$. Now note that $f(\cdot)$ is strictly convex for $\sigma > 1$ and thus, by Jensen's inequality, we have $\mathbb{E}_j[f((P_{j,t}/P_t)^{1-\sigma})] > f(\mathbb{E}_j[(P_{j,t}/P_t)^{1-\sigma}])$, with equality when $P_{j,t}/P_t = 1$ almost surely with respect to the Lebesgue measure. Finally, note that by definition of the aggregate price, $\mathbb{E}_j[(P_{j,t}/P_t)^{1-\sigma}] = 1$, so that $\mathbb{E}_j[f((P_{j,t}/P_t)^{1-\sigma})] > f(1) = 1$.

Using (12), we can rewrite the intratemporal condition (3) as

$$\frac{W_t}{P_t} = D_t^\psi Y_t^{1+\psi}. \tag{15}$$

This relationship shows that the real wage increases with output and with price dispersion. The reason for the latter is that the higher is price dispersion, the more households end up overworking to produce low-price varieties.

To facilitate future discussion, define the labor share μ_t by

$$\mu_t = \frac{W_t L_t}{P_t Y_t}.$$

The labor share is inversely related to monopoly profits and therefore captures the extent of monopoly distortions. Using (12) and (15), we obtain

$$\mu_t = (D_t Y_t)^{1+\psi}. \tag{16}$$

Holding output fixed, greater price dispersion results in higher real wages (to induce overworking on low-price varieties), thus increasing the labor share. Moreover, holding price dispersion fixed, higher output results in higher real wages and higher labor, thus also increasing the labor share.

3.2 Aggregate Price Dynamics

We next derive the dynamics of price dispersion, the Phillips curve, and a transversality condition.

Dispersion Dynamics. Since a random fraction $1 - \theta$ of firms are able to adjust their prices in every period, the price at time t satisfies

$$P_t^{1-\sigma} = (1 - \theta)(P_t^*)^{1-\sigma} + \theta P_{t-1}^{1-\sigma}.$$

Substituting with the definition of gross inflation in (14) and rearranging terms yields

$$\frac{P_t^*}{P_t} = \left(\frac{1 - \theta \Pi_t^{\sigma-1}}{1 - \theta}\right)^{\frac{1}{1-\sigma}}. \tag{17}$$

Intuitively, this relationship says that the larger is the upward price adjustment from P_t to P_t^*, the higher is the level of inflation Π_t.

The dynamics of price dispersion are given by

$$D_t = (1 - \theta) \left(\frac{P_t^*}{P_t} \right)^{-\sigma} + \theta \left(\frac{P_{t-1}}{P_t} \right)^{-\sigma} D_{t-1},$$

or equivalently, substituting with (14) and (17),

$$D_t = (1 - \theta) \left(\frac{1 - \theta \Pi_t^{\sigma-1}}{1 - \theta} \right)^{\frac{\sigma}{\sigma-1}} + \theta \Pi_t^\sigma D_{t-1}. \tag{18}$$

The initial level of dispersion D_{-1} is given by the initial distribution of prices $\{P_{j,-1}\}_{j \in [0,1]}$.

The relationship in (18) is backward looking, with dispersion at t being a positive function of dispersion at $t - 1$. As for the effect of inflation on dispersion, there are two forces at play. On the one hand, higher inflation causes sticky-price firms to be left further behind, which raises dispersion (second term on the right-hand side of (18)). On the other hand, higher inflation causes flexible-price firms to catch up to a higher price level, which reduces dispersion (first term on the right-hand side of (18)). One can show that for non-negative levels of inflation with $\Pi_t \geq 1$, the first force dominates, so higher inflation leads to higher price dispersion.

Phillips Curve. The first-order conditions from the flexible-price firm's problem in (9) yield that at each date t,

$$\frac{P_t^*}{P_t} = \frac{\sigma}{\sigma - 1} \frac{\sum_{h=0}^\infty (\beta\theta)^h \left(\frac{P_{t+h}}{P_t} \right)^\sigma \frac{(1+\tau)W_{t+h}}{P_{t+h}}}{\sum_{h=0}^\infty (\beta\theta)^h \left(\frac{P_{t+h}}{P_t} \right)^{\sigma-1}}.$$

We introduce an auxiliary variable δ_t defined by

$$\delta_t^{-1} \equiv \sum_{h=0}^\infty (\beta\theta)^h \left(\frac{P_{t+h}}{P_t} \right)^{\sigma-1}.$$

This variable allows us to significantly simplify the analysis. Using (14), we can rewrite it recursively as

$$\delta_t^{-1} = 1 + \beta\theta \Pi_{t+1}^{\sigma-1} \delta_{t+1}^{-1}. \tag{19}$$

Then substituting with δ_t^{-1} and (15), the first-order conditions above can be rewritten as

$$\frac{P_t^*}{P_t} = \frac{\sigma}{\sigma - 1} \delta_t \sum_{h=0}^\infty (\beta\theta)^h \left(\frac{P_{t+h}}{P_t} \right)^\sigma (1 + \tau) D_{t+h}^\psi Y_{t+h}^{1+\psi},$$

or, recursively,

$$\frac{P_t^*}{P_t} = \frac{\sigma(1+\tau)}{\sigma-1}\delta_t D_t^\psi Y_t^{1+\psi} + \beta\theta\frac{\delta_t}{\delta_{t+1}}\Pi_{t+1}^\sigma\frac{P_{t+1}^*}{P_{t+1}}.$$

Finally, substituting with (14), (17), and (19) yields the following non-linear Phillips curve:

$$\left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}} = \frac{\sigma(1+\tau)}{\sigma-1}\delta_t D_t^\psi Y_t^{1+\psi} + (1-\delta_t)\Pi_{t+1}\left(\frac{1-\theta\Pi_{t+1}^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}}. \qquad (20)$$

The relationship in (20) is forward looking because flexible-price firms adjust their prices taking into account the path of current and future marginal costs. Specifically, inflation today is increasing in the expectation of real wages today (given by $D_t^\psi Y_t^{1+\psi}$) and in the expectation of future inflation. Observe that δ_t, which captures the sensitivity of current inflation to current real wages, has a useful interpretation of being related to the slope of the Phillips curve.

Transversality Condition. Combining Equation (10) together with (14), (15), and (17), and noting that $P_t C_t \left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}} > 0$, we can rewrite the transversality conditions as requiring, for each date t,

$$\lim_{h\to\infty}\left[\beta\theta\left(\prod_{\ell=1}^h \Pi_{t+\ell}\right)^{\frac{\sigma}{h}}\right]^h\left[\left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}}\frac{1}{\prod_{\ell=1}^h \Pi_{t+\ell}} - (1+\tau)Y_{t+h}^{1+\psi}D_{t+h}^\psi\right] = 0. \qquad (21)$$

Observe that if inflation converges and $\lim_{h\to\infty}\Pi_{t+h} \geq 1$, then this condition can be satisfied in the long run only if $\lim_{h\to\infty}\Pi_{t+h} < (\beta\theta)^{-1/\sigma}$.

3.3 Necessary and Sufficient Conditions

Our analysis thus far leads to a system of equations that must necessarily hold in each period t in a competitive equilibrium. The next lemma shows that these conditions are not only necessary but also sufficient for the construction of a competitive equilibrium.

Lemma 1. *Given initial price distribution $\{P_{j,-1}\}_{j\in[0,1]}$ and a sequence of policies $\{i_t\}_{t=0}^\infty$, a sequence of allocations and prices $\{L_t, Y_t, D_t, \delta_t, \Pi_t\}_{t=0}^\infty$ is supported by a competitive equilibrium if and only if it satisfies conditions (12), (13), (18), (19), (20), and (21).*

An implication of this lemma is that the dispersion of prices D_{t-1} is a sufficient statistic for the distribution of prices Ω_{t-1}. In other words, the set of continuation MPCE at a date t starting from any two price distributions with the same dispersion is the same. We can

therefore refine our Markov restriction by replacing Ω_{t-1} with D_{t-1} in our consideration of household, firm, and central bank strategies.[18]

3.4   Long Run Monetary Non-Neutrality

A key observation in our environment is that monetary policy is not neutral in the long run. To see why, consider a hypothetical steady state in which $L_t, Y_t, D_t, \delta_t, \Pi_t$ are constant under a constant policy i_t. Equations (18)-(20) can be combined to yield the following steady-state conditions:

$$D = \frac{1 - \theta\Pi^{\sigma-1}}{1 - \theta\Pi^{\sigma}} \left(\frac{1 - \theta\Pi^{\sigma-1}}{1 - \theta} \right)^{\frac{1}{\sigma-1}}, \tag{22}$$

$$\frac{\mu}{D^{1+\psi}} = \frac{\sigma - 1}{\sigma(1 + \tau)} \left(\frac{1 - \theta\Pi^{\sigma-1}}{1 - \theta} \right)^{\frac{1+\psi}{1-\sigma}} \left(\frac{1 - \theta\Pi^{\sigma}}{1 - \theta\Pi^{\sigma-1}} \right)^{\psi} \frac{1 - \beta\theta\Pi^{\sigma}}{1 - \beta\theta\Pi^{\sigma-1}}, \tag{23}$$

where we have taken into account that the steady-state labor share μ satisfies $\mu = (DY)^{1+\psi}$.

The transversality condition (21) requires $\Pi < (\beta\theta)^{-1/\sigma}$. The next lemma considers hypothetical steady states that satisfy the stronger condition $\Pi < \theta^{-1/\sigma}$, as these will be the relevant steady states when we study equilibrium policy in the next section.

Lemma 2. *Given a fixed gross inflation level $\Pi \in [1, \theta^{-1/\sigma})$, there are unique values $\{D, \mu\}$ of price dispersion and labor share that satisfy the steady-state conditions (22)-(23). Moreover, D and μ are both strictly increasing in Π.*

Steady states with higher inflation admit higher price dispersion and lower monopoly distortions (higher labor shares). Higher inflation results in a larger divergence in prices between flexible-price firms (which raise prices by more) and sticky-price firms. This results in higher price dispersion and labor misallocation. At the same time, higher inflation increases overhiring by sticky-price firms. This overhiring increases the labor share and reduces monopoly distortions. We note that there are two subtle competing forces behind this last comparative static. On the one hand, high inflation means that sticky-price firms will be overproducing and overhiring, thus contributing to an increase in the labor share. On the other hand, high inflation means that flexible-price firms will anticipate that in the future they will overproduce and overhire if unable to change their prices. These firms increase their prices to counteract that future possibility, thus contributing to a decrease in the labor share. Because of discounting, however, this second anticipatory force is always dominated by the

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[18]Without the Markov restriction, the distribution of prices could serve as a payoff-irrelevant coordination device to select among different equilibria.

first force through sticky-price firms.

Two observations regarding potential steady states are useful to keep in mind for evaluating the impact of the central bank's lack of commitment in the next sections. First, note that Assumption 1 implies that monopoly distortions are present in a zero-inflation and zero-dispersion steady state (i.e., from (22)-(23), if $\Pi = 1$, then $D = 1$ and $\mu = (\sigma - 1)/[\sigma(1 + \tau)] < 1$). This assumption means that there is a tradeoff between reducing price dispersion and reducing monopoly distortions in this economy, and this guides the dynamic inflation-output tradeoff of the central bank.

Second, note that our environment does not pin down transition dynamics for inflation. Specifically, consider a hypothetical transition from an initial steady state with inflation Π to one with inflation $\Pi' > \Pi$.[19] There are multiple potential transition paths between the steady states which are consistent with the conditions in Lemma 1. One potential transition path admits inflation immediately jumping from Π to Π', with D_t and μ_t (through Y_t) evolving according to (18)-(20). Other transition paths may admit both temporary and permanent changes in inflation. The implication is that economic forces do not directly drive inflation dynamics. Any inflation dynamics that emerge in our model are instead driven by the interaction of economic forces with the central bank's lack of commitment which determines the path of policy.

4 Equilibrium Policy

Thus far, we have defined conditions for a competitive equilibrium and characterized the economy under different hypothetical steady states. We have also illustrated the potential for multiple paths between hypothetical steady states. In this section, we characterize the policy chosen by a central bank without commitment at every date, and we show how this policy path determines the system of equations defining the steady state of the economy and transition dynamics.

4.1 Central Bank Problem

At every date t, the central bank sets the interest rate i_t to maximize social welfare which is given by (1). Given a distribution of prices Ω_t implying dispersion D_t, and substituting with

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[19]The exercise is to take the economy in a given steady state and consider what happens following an unanticipated change in policy that leads the economy on a transition path towards a new steady state.

(12), we can write social welfare at t recursively as follows:

$$V(D_t) = \log(Y_t) - \frac{(D_t Y_t)^{1+\psi}}{1+\psi} + \beta V(D_{t+1}), \tag{24}$$

where we have taken into account that the Markov structure of the equilibrium implies that welfare depends only on the dispersion of prices. Observe that from the perspective of the central bank, the price distribution Ω_t at t, which determines the price level P_t and dispersion D_t, is predetermined. Furthermore, because of the Markov structure, it follows that the price distribution Ω_{t+1} at $t+1$ is also predetermined from the perspective of the central bank at t and is equal to $\Omega_{t+1} = \Gamma(\Omega_t)$, since firms at $t+1$ will set their prices taking as given the prevailing distribution Ω_t. This means that the central bank at date t cannot influence future dispersion D_{t+1} or the value of $V(D_{t+1})$. Moreover, the central bank choosing interest rates at t takes P_{t+1} (and therefore Π_{t+1}) and Y_{t+1} as given. From the Euler equation (13), it follows that the central bank can directly choose Y_t by choosing i_t without affecting future variables (off the equilibrium path).

Taking this into account, the derivative of (24) with respect to Y_t is

$$\frac{1}{Y_t} - D_t^{1+\psi} Y_t^{\psi}. \tag{25}$$

A rate cut by the central bank increases consumption (the first term in (25)) at the cost of increased labor effort (the second term in (25)). The marginal benefit of a rate cut is decreasing in price dispersion D_t which reduces aggregate labor productivity by raising labor misallocation. Moreover, for $Y_t < D_t^{-1}$, the marginal benefit of a rate cut is higher the lower is output Y_t, since lower output (caused by monopoly distortions) is associated with a larger gap between the marginal rate of substitution and the marginal product of labor.

Setting (25) to zero, the central bank's first-order condition yields

$$Y_t = D_t^{-1} . \tag{26}$$

The central bank chooses interest rates to undo all monopoly distortions and to close the gap between the marginal rate of substitution and the marginal product of labor. Using (16), this means that the central bank sets the labor share μ_t to 1.

We make three remarks regarding the central bank's policy. First, note that the central bank does not internalize how firms' anticipation of its policy at t impacts the distribution of prevailing prices at t, which impacts price dispersion D_t. This price distribution is determined by firm decisions made in all periods prior to t. This feature of our dynamic environment

captures the classic commitment problem addressed in the static models of Barro and Gordon (1983) and Rogoff (1985).[20]

Second, note that substitution of the central bank's first-order condition (26) into the Euler equation yields a policy function

$$1 + i_t = \frac{1}{\beta} \Pi_{t+1} Y_{t+1} D_t. \tag{27}$$

This central bank reaction function—which emerges from dynamic optimization on and off the equilibrium path—shares several properties with the exogenous Taylor rules that are typically considered when evaluating quantitative monetary models. In particular, the interest rate is increasing in future expected inflation and in future expected output. It also reacts to the degree of price dispersion today, and this stems from the central bank's marginal benefit of cutting interest rates in (25).[21] Specifically, holding future expectations fixed, higher price dispersion reduces labor productivity by generating labor misallocation. This reduces the benefit to the central bank of stimulating the economy to boost consumption. We will see in the next section that this is important when evaluating the evolution of central bank incentives along the equilibrium path.

Finally, note that the optimal discretionary policy of the central bank—which sets the labor share to 1—is independent of the underlying price-setting model. This means that our analysis can be extended to other environments with sticky prices such as menu-cost models or models with rational inattention, where steady-state inflation and inflation dynamics can be studied under lack of commitment.

4.2 System of Equations

An MPCE is characterized by combining the conditions in Lemma 1 (specifically (18)-(20)) with the central bank's first-order condition (26). This yields a system of two equations defining the dynamics of price dispersion D_t and inflation Π_t:

$$D_t = (1-\theta) \left(\frac{1 - \theta \Pi_t^{\sigma-1}}{1-\theta} \right)^{\frac{\sigma}{\sigma-1}} + \theta \Pi_t^{\sigma} D_{t-1}, \tag{28}$$

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[20]Moreover, note that while (19) and (20) hold along the equilibrium path, they need not hold off the equilibrium path if the central bank deviates, since in that case firms would have chosen their prices without the correct anticipation of policy.

[21]An interesting avenue for future work is to consider how this framework is different from others in addressing issues of potentially unstable inflation expectations and implementation (e.g., Woodford, 2003; Atkeson, Chari, and Kehoe, 2010; Cochrane, 2011; Galí, 2015). As in Atkeson, Chari, and Kehoe (2010), the central bank in our model responds to an off-equilibrium increase in inflation and dispersion by raising interest rates sufficiently that an individual flexible-price firm would actually want to engage in lower price increases off the equilibrium path.

$$\left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}} = \frac{\sigma(1+\tau)}{\sigma-1}\delta_t D_t^{-1} + (1-\delta_t)\Pi_{t+1}\left(\frac{1-\theta\Pi_{t+1}^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}}, \qquad (29)$$

where δ_t is a function of $\{\Pi_{t+h}\}_{h=1}^\infty$ as defined in Equation (19), and where $\{D_t, \Pi_t\}_{t=0}^\infty$ must satisfy the transversality condition in (21) given (26).

5 Main Results

Evaluating the dynamics around the steady state of our economy is challenging given the non-linear nature of the difference equations in (28)-(29). To present our main results, we consider the continuous-time limit of our model.[22] We derive this limit in Appendix A, where we introduce a generalized version of the model for an arbitrary time step $\mathrm{d}t$ and take the limit as $\mathrm{d}t \to 0$. Section 5.1 below describes the system of equations defining an MPCE in the continuous-time limit. We then characterize the steady state of our economy and the transition dynamics around the steady state in Section 5.2 and Section 5.3. We perform comparative statics, highlighting how changes in the economic environment impact the steady state and the transition path between steady states. Finally, in Section 5.4, we explore the quantitative implications of our model.

5.1 Continuous-Time Limit

Let $\lambda \equiv -\log(\theta)$ and $\rho \equiv -\log(\beta)$, and define $\pi_t \equiv \frac{\mathrm{d}}{\mathrm{d}t}\log(P_t)$ as the instantaneous rate of inflation at time t. Using (28)-(29) together with (19), Appendix A shows that the dynamics of price dispersion and inflation in the continuous-time limit of our model are given by

$$\dot{D}_t = \lambda\left(1 - \frac{\sigma-1}{\lambda}\pi_t\right)^{\frac{\sigma}{\sigma-1}} + (\sigma\pi_t - \lambda)D_t, \qquad (30)$$

$$\dot{\pi}_t = -\lambda\frac{\sigma(1+\tau)}{\sigma-1}\left(1 - \frac{\sigma-1}{\lambda}\pi_t\right)^{\frac{\sigma}{\sigma-1}}\frac{\delta_t}{D_t} + (\delta_t - \pi_t)[\lambda - (\sigma-1)\pi_t], \qquad (31)$$

where $\dot{\delta}_t = \delta_t^2 + [(\sigma-1)\pi_t - (\rho+\lambda)]\delta_t$, and where $\{D_t, \pi_t\}_{t=0}^\infty$ must satisfy the continuous-time version of the transversality condition in (21) given (26):

$$\lim_{h\to\infty} e^{[-(\rho+\lambda)+\frac{\sigma}{h}\int_0^h \pi_{t+\ell}\mathrm{d}\ell]h}\left[\left(1 - \frac{\sigma-1}{\lambda}\pi_t\right)^{\frac{1}{1-\sigma}}e^{-\int_0^h \pi_{t+\ell}\mathrm{d}\ell} - \frac{1+\tau}{D_{t+h}}\right] = 0. \qquad (32)$$

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[22]Taking the continuous-time limit is not necessary to perform comparative statics of the steady state, but it does facilitate the analysis of transition dynamics.

5.2   Steady State

Our first main result establishes that there is a unique steady state in which price dispersion D_t and inflation π_t are constant and satisfy the system of equations (30)-(31) together with the transversality condition in (32).[23] We define $D_{ss}(\tau, \sigma)$ and $\pi_{ss}(\tau, \sigma)$ as the values of price dispersion and inflation in the steady state conditional on the labor wedge τ and the elasticity of substitution across varieties σ, and we study their comparative statics. Let us define

$$\overline{\tau}(\sigma) = \begin{cases} \infty & \text{if } \sigma \leq 2 \\ \frac{1}{\sigma^2 - 2\sigma} & \text{otherwise.} \end{cases}$$

We obtain the following result:[24]

Proposition 1. *There is a unique steady state* $\{D_{ss}(\tau, \sigma), \pi_{ss}(\tau, \sigma)\}$. *Moreover,*

1. $D_{ss}(\tau, \sigma)$ *and* $\pi_{ss}(\tau, \sigma)$ *are both strictly increasing in the labor wedge* τ.
2. $D_{ss}(\tau, \sigma)$ *is strictly decreasing in the elasticity of substitution* σ *for* $\tau < \overline{\tau}(\sigma)$, *and* $\pi_{ss}(\tau, \sigma)$ *is strictly decreasing in* σ *for all* τ.

This proposition states that long-run price dispersion and inflation are higher the higher is the labor wedge τ and the lower is the elasticity of substitution across varieties σ (the latter holding for dispersion provided that $\tau < \overline{\tau}(\sigma)$). To understand these comparative statics, consider the incentives of the central bank starting from a given steady state. The central bank chooses a constant interest rate to set the labor share to 1. Any consumption benefit from stimulating output beyond that of the steady state is exactly outweighed by the cost of labor effort needed to do so. Now consider what happens following an unanticipated permanent increase in τ or an unanticipated permanent decrease in σ. A central bank with commitment would be able to respond to these changes in a way that preserves the level of inflation; however, this is not incentive compatible under lack of commitment.

As an illustration, take the limiting case of Assumption 1 and suppose the economy begins in a steady state in which $\tau = -1/\sigma$. From (30)-(31) with $\dot{D}_t = \dot{\pi}_t = \dot{\delta}_t = 0$, the steady state admits zero price dispersion with $D = 1$ and zero inflation with $\pi = 0$, and the labor share satisfies $\mu = 1$. Suppose that τ permanently increases. A central bank with commitment could preserve the levels of price dispersion and inflation by keeping nominal interest rates

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[23]The system of equations (30)-(31) admits two solutions, but only one of them satisfies transversality.
[24]We show in the proof of Proposition 1 that steady-state inflation satisfies $\pi_{ss}(\tau, \sigma) < \lambda/\sigma$, which corresponds to $\Pi_{ss}(\tau, \sigma) < \theta^{-1/\sigma}$ in discrete time, as assumed in Lemma 2.

fixed forever. From Equation (23), the labor share would permanently decrease to satisfy

$$\mu = \frac{\sigma - 1}{\sigma(1 + \tau)}$$

under the new higher level of τ.[25]

For a central bank without commitment, this policy which stabilizes price dispersion and inflation is not incentive compatible. The reason is that it entails a reduction in the labor share, and the central bank has an incentive to undo the increase in monopoly distortions by stimulating output. If firms naively anticipated inflation stability, the central bank's best response would be to surprise markets by cutting interest rates.[26]

Flexible-price firms however are not naive. In equilibrium, they rationally forecast the monetary stimulus and the higher future labor demand and higher future real wages that ensue (relative to the commitment case). They also expect further inflation in the future. These expected future changes necessitate higher offsetting prices today. Over time, sequential price increases by flexible-price firms result in rising price dispersion. Eventually, the rise in dispersion reduces aggregate productivity sufficiently to offset the central bank's benefit from cutting interest rates, leading to a new steady state. Therefore, we obtain that both long-run price dispersion and inflation are higher if the labor wedge τ is higher. Furthermore, observe that steady-state output and the real wage (which are equal to each other) are lower under a higher labor wedge.

The intuition for a shock that permanently reduces the elasticity of substitution σ is similar. In this case we establish the comparative static on long-run price dispersion under a positive upper bound on the labor wedge τ if $\sigma > 2$. The reason is that σ affects the law of motion of dispersion in (30); if $\tau > \overline{\tau}(\sigma)$, in principle dispersion could increase with σ. The comparative static on long-run inflation however is unambiguous: a reduction in σ increases monopoly distortions, and the central bank's policy always leads to an increase in long-run inflation in response.

5.3 Transition Dynamics

Our second main result concerns the transition dynamics between steady states. We study an economy that transitions from an initial steady state to one with higher inflation following

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[25]Analogous logic applies if σ decreases starting from $\tau = -1/\sigma$. More generally, the same reasoning applies with respect to the feasibility and the incentive incompatibility of inflation stabilization beginning from a positive-inflation steady state with $\tau > -1/\sigma$. The only caveat in that case is that changes in σ require transition dynamics in dispersion and the labor share to support inflation stabilization.

[26]Formally, from the central bank's policy function (27), a reduction in Y_{t+1} (due to the reduction in the labor share) holding Π_{t+1} and D_t fixed requires a reduction in i_t.

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an unanticipated permanent shock. We show that inflation overshoots along the transition path to the new steady state.

Proposition 2. *Take the economy at steady state $\{D_{ss}(\tau, \sigma), \pi_{ss}(\tau, \sigma)\}$ at a time t_{ss}.*

1. *Consider the transition to steady state $\{D_{ss}(\tau', \sigma), \pi_{ss}(\tau', \sigma)\}$ following an unantici-pated shock that permanently increases the labor wedge to $\tau' > \tau$. There exists $t' \geq t_{ss}$ such $\pi_t > \pi_{ss}(\tau', \sigma)$ for all $t > t'$.*

2. *Consider the transition to steady state $\{D_{ss}(\tau, \sigma'), \pi_{ss}(\tau, \sigma')\}$ following an unanticipated shock that permanently decreases the elasticity of substitution to $\sigma' < \sigma$ given $\tau < \bar{\tau}(\sigma)$. There exists $t' \geq t_{ss}$ such $\pi_t > \pi_{ss}(\tau, \sigma')$ for all $t > t'$.*

Proposition 2 considers an unanticipated permanent shock that increases the labor wedge τ or reduces the elasticity of substitution across varieties σ. From Proposition 1 we know that long-run price dispersion and inflation increase in response to the shock. What Proposition 2 tells us is that inflation in the transition increases by more than in the long run; that is, transition dynamics involve inflation overshooting. The proof of this result evaluates the three-dimensional phase diagram for price dispersion D_t, inflation π_t, and the auxiliary variable δ_t along a transition where D_t rises towards a higher steady-state level. Below, we provide a heuristic description by considering a two-dimensional representation of the three-dimensional phase diagram, keeping the value of δ_t fixed at its steady-state level. This modified phase diagram is depicted in Figure 1.

The $\dot{\pi}_t = 0$ locus in Figure 1 is a representation of the non-linear Phillips curve in (31). This locus is downward sloping: higher inflation is sustained by lower price dispersion in a steady state, since lower levels of dispersion increase output and real wages, necessitating higher price increases by firms. Inflation increases if dispersion is above the locus (real wages decline, so higher future inflation sustains the current inflation level), and it decreases if dispersion is below the locus (real wages increase, so lower future inflation sustains the current inflation level).

The $\dot{D}_t = 0$ locus in Figure 1 is a representation of the dispersion dynamics equation in (30). This locus is upward sloping: higher inflation is required to sustain higher price dispersion in a steady state, with the main forces being as discussed in our derivation of dispersion dynamics in Section 3.2. Price dispersion increases if inflation is above the locus, and it decreases if inflation is below the locus.

The intersection of the $\dot{\pi}_t = 0$ and $\dot{D}_t = 0$ loci represents the steady state. As depicted in Figure 1, we show that the steady state admits a unique saddle path, and along this saddle path inflation and price dispersion evolve in opposite directions. For intuition, recall the scenario described in the previous section: take an economy with zero inflation ($\pi = 0$) and

FIGURE 1: PHASE DIAGRAM FOR INFLATION AND PRICE DISPERSION
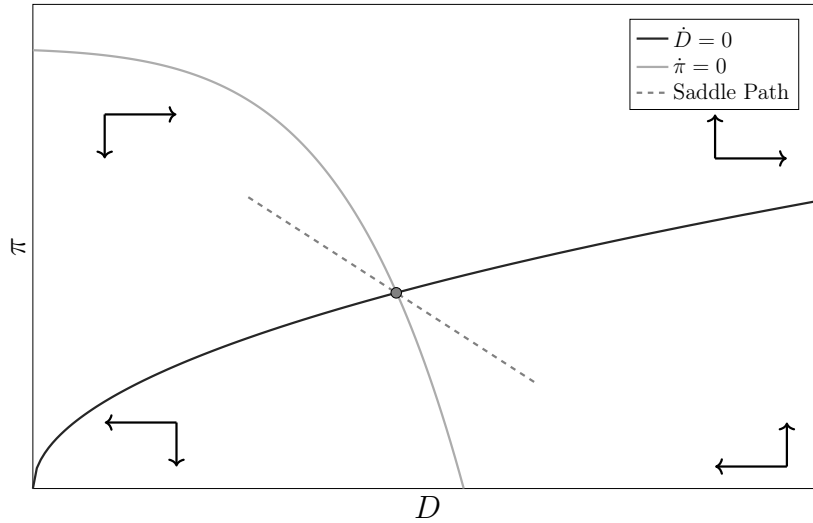
Notes: This figure illustrates the phase diagram of the dynamic economy in the dispersion-inflation (D, π) plane, holding δ fixed. The $\dot{D} = 0$ locus is upward sloping and the $\dot{\pi} = 0$ locus is downward sloping. The intersection point is the steady state when the fixed δ is at its steady-state level. The flows of D and π are depicted by the black arrows in the different regions of the plane. The directions of these flows indicate that any transition dynamics to the steady state should be along a saddle path with a negative slope.

zero dispersion $(D = 1)$, where this dispersion level is strictly below steady-state. While a central bank with commitment could preserve zero inflation and zero dispersion forever, a central bank without commitment would want to take advantage and cut interest rates to stimulate wages and boost output, so as to reduce monopoly distortions. Therefore, the economy must gradually transition to a new steady state with positive inflation and dispersion. Along the transition path, the central bank sees a relatively larger benefit to stimulating output early in the transition when dispersion and labor misallocation are low; once these rise sufficiently, it is no longer as worthwhile to stimulate labor to generate additional consumption. Flexible-price firms in turn realize that monetary stimulus will be larger earlier in the transition, so they offset the ensuing higher wage costs with price increases that are also relatively larger earlier in the transition. The implication is rising price dispersion and declining inflation along the transition path.

To understand the inflation overshooting result in Proposition 2, Figure 2 depicts the response to an unanticipated permanent increase in the labor wedge τ in the phase diagram. This shock does not affect the $\dot{D}_t = 0$ locus but shifts upward the $\dot{\pi}_t = 0$ locus: by (31), a higher level of price dispersion is needed to preserve a given level of inflation so as to offset the higher real wage costs due to the increase in τ. The new steady state following the shock is shown at the crossing point of the two loci, associated with higher inflation and higher

price dispersion than in the initial steady-state crossing point.

FIGURE 2: TRANSITION DYNAMICS FOR UNANTICIPATED INCREASE IN LABOR WEDGE
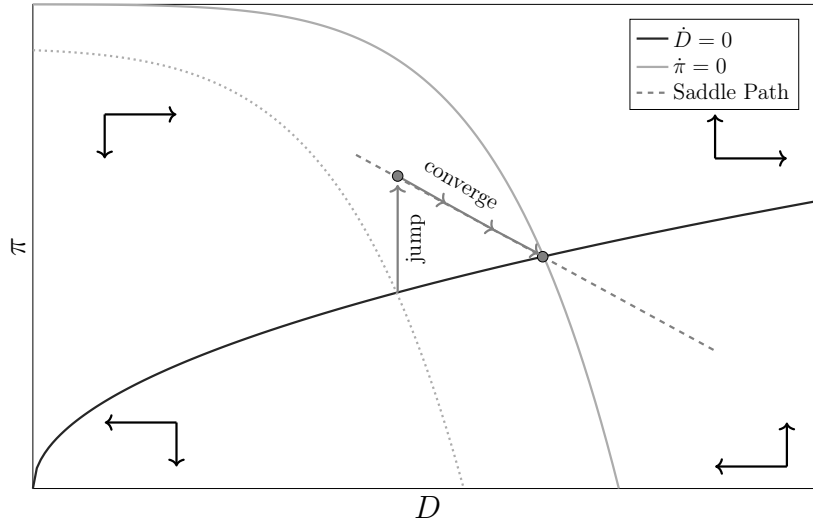
Notes: This figure illustrates the transition dynamics of dispersion and inflation following an unanticipated permanent increase in the labor wedge τ. The shock shifts the $\dot{\pi} = 0$ locus upwards while leaving the $\dot{D} = 0$ locus unchanged. Inflation jumps on impact to move the economy to its new saddle path, after which D increases and π declines towards their new steady-state levels. The transition involves inflation overshooting.

Figure 2 shows that the transition to the new steady state involves inflation overshooting: in the figure, when τ increases, inflation immediately jumps upward and then gradually declines towards its new steady-state level. This overshooting emerges because of the evolution of central bank incentives as price dispersion rises along the transition path. As discussed above, the central bank has a higher incentive to stimulate the economy earlier in the transition when dispersion is low, and this results in high inflation early in the transition. Over time, as dispersion rises, the central bank's incentive to stimulate the economy declines, and so does inflation.

Transition dynamics following an unanticipated permanent decrease in the elasticity of substitution σ are similar to those above (given $\tau < \bar{\tau}(\sigma)$, as stated in Proposition 2). In this case, both loci in Figure 1 shift upward, and inflation overshoots in the transition to the new steady state. Moreover, transition dynamics are analogous if either the labor wedge τ permanently decreases or the elasticity of substitution σ permanently increases. In both cases, starting from a given steady state, inflation overshoots downward and then increases towards a new steady state with lower inflation.

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5.4 Quantitative Implications

In this section, we study the quantitative implications of our model. We use a standard parameterization of the New Keynesian model and simulate a discrete-time economy in which every time period corresponds to a month. We take a discount factor $\beta = (1.02)^{-1/12}$ to target a steady-state annual real interest rate of 2 percent. The probability that a firm has sticky prices is set at $\theta = 0.86$ to target an average duration of price stickiness of 7 months (e.g., Nakamura and Steinsson, 2008). The elasticity of substitution across varieties is set at $\sigma = 7$, in line with previous research on the cost of inflation (e.g., Coibion, Gorodnichenko, and Wieland, 2012). The inverse elasticity of labor supply is set at $\psi = 2.5$, which is in the range of estimates in the literature (e.g., Chetty, Guren, Manoli, and Weber, 2011).[27] Finally, for the labor wedge, we specify $\tau = -0.1427$ to target a steady-state annual inflation rate of 2 percent under central bank lack of commitment.[28] Table 1 summarizes our choice of parameters.

TABLE 1: PARAMETERS

| Parameter	Value	Target
Discount factor, β	$(1.02)^{-1/12}$	2% annual real interest rate
Fraction of sticky-price firms, θ	0.86	Nakamura and Steinsson (2008)
Elasticity of substitution, σ	7	Coibion, Gorodnichenko, and Wieland (2012)
Inverse Frisch elasticity, ψ	2.5	Chetty, Guren, Manoli, and Weber (2011)
Labor wedge, τ	-0.1427	2% annual inflation without commitment

Starting from the steady state of the economy given the parameter values in Table 1, Figure 3 considers the transition to a new steady state following an unanticipated permanent shock that increases the labor wedge τ by 0.5 percent. The figure displays the transition paths of price dispersion, real output, the price inflation rate, the nominal wage growth rate, the nominal interest rate, and the real interest rate, where the monthly values for the latter four variables are represented in annualized form.

In line with our analytical results, Figure 3 shows that inflation overshoots following the shock by immediately jumping up from its initial 2 percent level (not shown in the figure given the scale) and then gradually declining towards its new higher steady-state level. The nominal interest rate jumps up and continues to increase throughout the transition, while

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[27]Observe that this choice has no bearing on our findings under lack of commitment, since ψ does not enter the dynamic equations characterizing our economy. This value only affects the findings under commitment to inflation targeting and the estimates of welfare that we discuss at the end of this section.

[28]This value is uniquely pinned down given the comparative statics in Proposition 1.

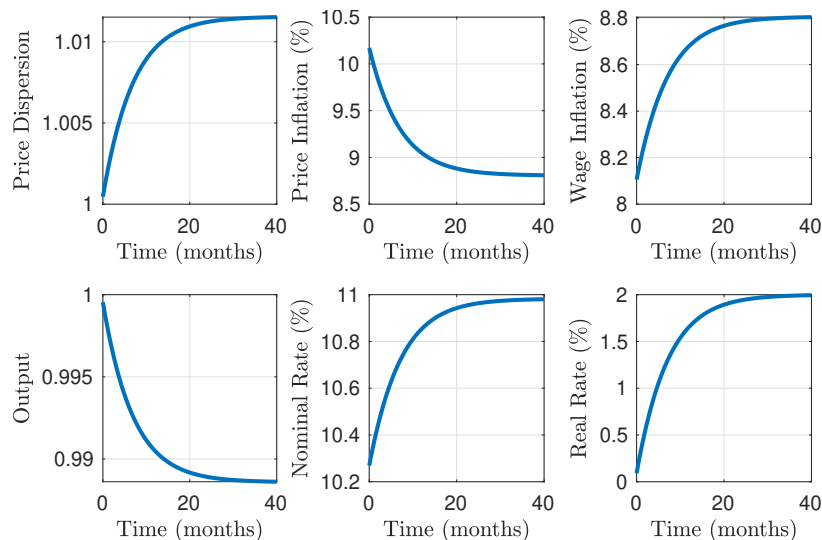Figure 3: Response to Unanticipated Increase in Labor Wedge
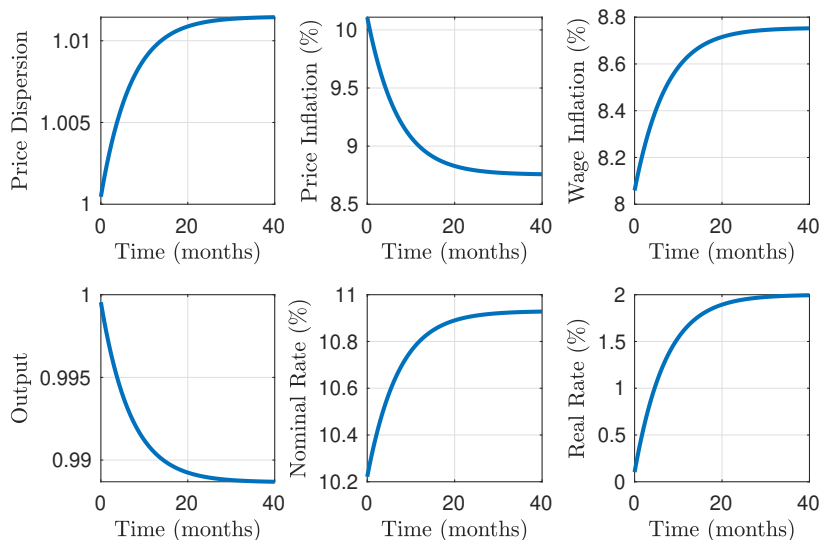
Notes: This figure shows the transition dynamics following an unanticipated permanent increase in the labor wedge from $\tau = -0.1427$ to $\tau = -0.1420$ (a 0.5% increase). Both inflation and interest rates are annualized (annual inflation $= e^{12\pi_t} - 1$). The initial steady state is calibrated to correspond to an annual inflation rate of 2%, not shown in the figure given the scale.

the real interest rate jumps down (since the central bank initially stimulates the economy to weather the shock) and then gradually returns to its original level. Along the transition path, output gradually falls as price dispersion and labor misallocation increase. Nominal wage inflation jumps up initially in tandem with price inflation, and it then gradually converges to a permanently higher level. Note that wage inflation is below price inflation; these dynamics underpin the permanent long-run decline in the real wage.

The quantitative impact of the shock on inflation is large. We find that following a 0.5 percent increase in the labor wedge, the steady-state inflation rate increases from 2 percent to 8.76 percent. Moreover, we find that the shock has a large impact on the degree of inflation overshooting. Following the 0.5-percent labor wedge shock, the inflation rate rises to 10.11 percent on impact, above its new steady-state level of 8.76 percent. This overshooting is persistent: it takes 12 months for the inflation rate to decline within 25 basis points of the new steady-state level.

The dynamics in Figure 3 are markedly different from those that would arise under inflation targeting, namely if the central bank was committed to maintaining a 2 percent inflation level in every period. Under inflation targeting, the central bank would keep real and nominal interest rates fixed so as to preserve the level of inflation. Following the shock to the labor wedge, output would immediately decline and would remain at a permanently lower level. Price dispersion would not change in response to the shock.

FIGURE 4: RESPONSE TO UNANTICIPATED DECREASE IN ELASTICITY OF SUBSTITUTION

Notes: This figure shows the transition dynamics following an unanticipated permanent decrease in the elasticity of substitution from $\sigma = 7$ to $\sigma = 6.97$ (a 0.5% decrease). Both inflation and interest rates are annualized (annual inflation $= e^{12\pi_t} - 1$). The initial steady state is calibrated to correspond to an annual inflation rate of 2%, not shown in the figure given the scale.

Figure 4 presents an analogous exercise to that in Figure 3 by considering the impact of an unanticipated permanent shock that decreases the elasticity of substitution σ by 0.5 percent. The economic response is similar as in the case of a positive labor wedge shock, and is in line with our analytical results. Starting from its initial 2 percent level (not shown in the figure given the scale), inflation overshoots on impact when σ decreases, and it then gradually declines to a permanently higher steady-state level. The quantitative impact is large for reasons analogous to those under a labor wedge shock. As for the contrast with the dynamics that would arise under inflation targeting, things are different when the shock is to σ rather than τ. The reason is that the change in the elasticity of substitution σ directly affects the dynamic relationship between price dispersion and inflation. If the central bank was committed to maintaining a 2 percent inflation level in every period, then following a decrease in σ steady-state price dispersion would decline,[29] and the real and nominal interest rates would evolve so as to facilitate the transition of the economy to the lower dispersion level.

The exercises above provide a framework for evaluating the welfare benefits of inflation targeting relative to our central bank's policy under lack of commitment. Given an unanticipated permanent shock, the benefit of inflation targeting over the no-commitment policy

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[29]This is because greater differentiation across varieties means that relative price differences are a less important source of misallocation.

TABLE 2: INFLATION TARGETING VERSUS NO COMMITMENT

Scenario	Welfare under Targeting	Welfare under No Commitment	Welfare Difference
Labor wedge shock	0.981	0.922	0.059
Elasticity of substitution shock	0.981	0.921	0.060

Notes: The table reports welfare expressed in consumption-equivalent terms relative to an identical economy with flexible prices. That is, we compute, right after each shock, how much a household would require in consumption to be indifferent between living in an economy with sticky prices and inflation targeting/no commitment and living in an economy with flexible prices where consumption and its implied labor supply are forever constant at the offered level.

is that it reduces the misallocation cost of long-run price dispersion in the economy. The benefit of the no-commitment policy is that it reduces the short-run and long-run costs of rising monopoly distortions. We compare welfare under each regime in Table 2, where welfare is expressed in consumption-equivalent terms relative to an identical economy with flexible prices. The table considers the two scenarios studied in Figure 3 and Figure 4, namely an unanticipated permanent 0.5 percent increase in the labor wedge and decrease in the elasticity of substitution.

We find that in both scenarios, welfare under inflation targeting is strictly higher than under lack of commitment. Moreover, the welfare gains from inflation targeting are substantial, at about 6 percent in consumption-equivalent terms. In other words, the long-run price dispersion costs under lack of commitment far outweigh the benefits from reducing monopoly distortions, and the high discount factor β implies that these costs enter prominently in the welfare calculation. The analysis suggests that there can be significant benefits to institutions that enhance commitment to inflation targeting.

The large quantitative impact of shocks, both on inflation dynamics and on welfare relative to inflation targeting, is a robust feature of our model. It emerges because the steady-state labor share is relatively insensitive to inflation; much of the positive effect of inflation on the labor share via sticky-price firms is offset by the negative effect via forward-looking flexible-price firms.[30] In fact, note that standard calibrations of the New Keynesian model take high values of β and low values of θ. This means that in response to monetary stimulus, there is a large number $1 - \theta$ of flexible-price firms which raise prices significantly to protect against potentially overhiring in the future, and this puts downward pressure on the labor share.[31] As a result, a central bank without commitment—which seeks to keep the labor share

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[30]See the discussion following Lemma 2 in Section 3.4.

[31]A low value of θ also implies that flexible-price firms place a low probability on the likelihood of not

from declining—ends up increasing inflation substantially in response to a small increase in the labor wedge or a small reduction in the elasticity of substitution.

To see this formally, consider the *long-run* Phillips curve, derived by combining steady-state equations (22) and (23) to express the relationship between the long-run labor share μ and long-run inflation Π:[32]

$$\mu = \frac{\sigma - 1}{\sigma(1 + \tau)} \left[1 + (1 - \beta) \frac{\theta \Pi^{\sigma-1}(\Pi - 1)}{(1 - \theta \Pi^\sigma)(1 - \beta \theta \Pi^{\sigma-1})} \right]. \tag{33}$$

If the discount factor β is close to 1, then the term in brackets on the right-hand side of (33) is relatively insensitive to Π. Thus, in this case, the labor share μ does not respond significantly to inflation and we obtain an almost vertical long-run Phillips curve. As a consequence, small changes in the labor wedge τ or the elasticity of substitution σ require large changes in inflation Π to keep the labor share μ unchanged in (33). This explains the large quantitative magnitudes in our model.

There are several implications that follow from this discussion. First, any changes to parameters or to the underlying price-setting mechanism which result in a flatter long-run Phillips curve would imply smaller quantitative magnitudes in our model. Second, such changes would also imply a lower value of commitment to inflation targeting, since the central bank's lack of commitment would then have a smaller effect on equilibrium inflation and price dispersion. Finally, changes that yield a flatter long-run Phillips curve would also imply meaningful economic benefits from long-run inflation, indicating that inflation targeting at too low an inflation rate would be costly for society.

6   Concluding Remarks

In this paper, we introduced central bank lack of commitment into a standard non-linear New Keynesian model with monopolistically competitive firms and sticky prices. We characterized long-run inflation and studied transition dynamics as the economy responds to an unanticipated permanent shock that increases the labor wedge or decreases the elasticity of substitution across varieties. While a central bank with commitment would be able to keep inflation unchanged, inflation stabilization is not incentive compatible for a central bank without commitment. The private sector anticipates central bank accommodation following the shock, and inflation overshoots before declining to a permanently higher level. These

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being able to adjust prices in the future. While this implies a low anticipatory channel for each individual flexible-price firm, this effect is offset by the fact that a low θ implies a large share of flexible-price firms.

[32]We express these in discrete time for expositional symmetry with the discussion in Section 3.4, but the same point can be made using the continuous-time representation.

effects are quantitatively large, as is the welfare loss from lack of commitment relative to inflation targeting.

Our framework is useful in interpreting the inflationary spike that has befallen advanced economies in the aftermath of the COVID-19 pandemic. Many questions have emerged regarding the causes of this inflation and whether central banks will ultimately be successful in bringing inflation back down to historic levels. Our analysis suggests that the types of shocks that impacted post-pandemic advanced economies—a decrease in immigration which changed the composition of labor towards more regulated labor sources (higher labor wedge) and the supply chain disruptions which increased the market power of domestic firms (lower elasticity of substitution)—do not raise inflation on their own. These shocks raise inflation through their interaction with the central bank's lack of commitment to inflation stabilization. Moreover, inflation can overshoot in response to these shocks as the private sector rationally anticipates the central bank's response. Our results suggest that if these shocks are permanent, then central banks are unlikely to be able to bring inflation back down to historic norms. This is true in our model as long as the central bank operates with full discretion as opposed to a strict commitment to inflation targeting.

Our analysis leaves a number of avenues for future research. First, while we have examined the canonical New Keynesian model with Calvo pricing, our approach can be applied to other models of price setting, such as menu-cost models. The optimal policy of the central bank without commitment—which sets the labor share to 1—is invariant to the details of the underlying price-setting model, and future research can use our approach to explore how inflation dynamics might change under different models. Second, by focusing on the stable steady state, we have ignored broader issues involving equilibrium implementation and off-equilibrium inflation stability. A natural question concerns the extent to which lack of commitment on and off the equilibrium path increases or decreases the scope for off-equilibrium inflation stability in our framework. Finally, our model abstracts from monetary and fiscal interactions by assuming lump sum taxes and Ricardian equivalence. It would be interesting to relax the assumption of Ricardian equivalence and study how central bank lack of commitment interacts with fiscal lack of commitment. Since our analytical framework does not assume a long-run level of debt (as it is not linearized), it can facilitate such an analysis.

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Appendix A Continuous-Time Limit

In this appendix, we solve the discrete-time model for an arbitrary time step of length dt and derive the continuous-time limit as $dt \to 0$. For completeness, we first reiterate the derivations of the discrete-time model for a given dt, where $dt = 1$ corresponds to the derivations in the main text.

Time now runs at increments of dt, so that $t \in T_{dt} \equiv \{0, dt, 2dt, \ldots\}$. Letting $\rho \equiv -\log(\beta)$, the household's problem for a given dt is given by

$$\max_{C_t, L_t, B_t, (s_{j,t}, C_{j,t})_{j \in [0,1]}} \sum_{t \in T_{dt}} e^{-\rho t} \left(\log(C_t) - \frac{L_t^{1+\psi}}{1+\psi} \right) dt$$

subject to

$$\int_0^1 P_{j,t} C_{j,t} dj dt + B_t \leq W_t L_t dt + (1 + i_{t-dt} dt) B_{t-dt} + \int_0^1 s_{j,t} X_{j,t} dt dj + \int_0^1 (s_{j,t-dt} - s_{j,t}) P_{j,t}^S dj - T_t dt,$$

$$C_t = \left(\int_0^1 C_{j,t}^{1-\sigma^{-1}} dj \right)^{\frac{1}{1-\sigma^{-1}}}.$$

Note that this expression of the problem redefines $C_{j,t}$, C_t, L_t, $X_{j,t}$ and T_t as *rates* of consumption, labor supply, profits, and lump-sum taxes per dt.

The implied demand for varieties $j \in [0,1]$, the definition of the aggregate price P_t, the price dispersion measure D_t, and the intratemporal labor supply condition are all identical to those in the main text because they follow from static decisions that are not affected by the time step dt. To reiterate these, we have

$$C_{j,t} = C_t \left(\frac{P_{j,t}}{P_t} \right)^{1-\sigma}, \forall j, \quad P_t = \left(\int_0^1 P_{j,t}^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}, \quad D_t = \int_0^1 \left(\frac{P_{j,t}}{P_t} \right)^{-\sigma} dj, \quad \frac{W_t}{P_t} = C_t L_t^\psi.$$

Moreover, under the assumptions in the main text—in particular, the fact that firms produce with $Y_{j,t} = L_{j,t}$ and always produce enough to meet their demand—we can still use the labor market clearing conditions to derive the aggregate production function of the economy as

$$L_t = \int_0^1 L_{j,t} dj = \int_0^1 C_{j,t} dj = C_t \int_0^1 \left(\frac{P_{j,t}}{P_t} \right)^{-\sigma} dj = C_t D_t \implies C_t = \frac{L_t}{D_t},$$

where D_t is defined as the price dispersion measure similar to the main text.

The Euler equations for nominal bonds and stocks, however, are affected by the time step

and are given by

$$\frac{1}{P_t C_t} = e^{-\rho dt}(1 + i_t dt)\frac{1}{P_{t+dt}C_{t+dt}},$$

$$P_{j,t}^S = X_{j,t}dt + \frac{1}{1 + i_t dt}P_{j,t+dt}^S, \forall j.$$

Rearranging these, we obtain the following expressions:[33]

$$\frac{\dot{P}_t}{P_t} + \frac{\dot{C}_t}{C_t} = i_t - \rho, \qquad \dot{P}_{j,t}^S = i_t P_{j,t}^S - X_{j,t}, \quad \forall j \in [0,1]$$

where for any variable X_t, we define $\dot{X}_t$ as their rate of change over time, i.e., $\dot{X}_t \equiv dX_t/dt$. Integrating the Euler equation for stocks forward and assuming no bubbles gives us the household's valuation of firms at time t:

$$P_{j,t}^S = \int_0^\infty e^{-\int_0^h i_{t+s}ds}X_{j,t+h}dh = \int_0^\infty e^{-\rho h}\frac{P_t C_t}{P_{t+h}C_{t+h}}X_{j,t+h}dh.$$

We will use this valuation to rewrite the optimization problem of a firm that gets the opportunity to reset its price. Before we do so, we have to adjust the frequency of price changes such that the probability of changing prices is independent of the choice of dt. To this end, let θ^{dt} be the probability of not getting the opportunity to adjust the price at an interval of length dt. This defines a consistent distribution of price adjustment frequency for different values of dt such that for any interval length T, the probability of not adjusting the price is θ^T, independent of dt. With $T = 1$, this corresponds to the model in the main text where $dt = 1$. With $dt \to 0$, it corresponds to a Poisson process where the arrival rate of price adjustment opportunities is $\lambda \equiv -\log(\theta)$. We obtain a well-defined limit: under the Poisson arrival rate of λ, the implied distribution of time between price changes is exponential with scale λ. Accordingly, the probability of not adjusting the price in a period of length T is $e^{-\lambda T} = e^{\log(\theta)T} = \theta^T$.

Now, for a given dt, a flexible-price firm's problem for choosing its reset price is given by maximizing the net present value of its profits in the history where it is stuck with the price it chooses at date t:

$$\max_{P_t^*} \sum_{h \in T_{dt}} e^{-(\rho+\lambda)h}\frac{P_t C_t}{P_{t+h}C_{t+h}}[P_t^* - (1+\tau)W_{t+h}]C_t\left(\frac{P_t^*}{P_{t+h}}\right)^{-\sigma} dt.$$

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[33]These expressions follow from dividing the equations above by dt and taking the limit as $dt \to 0$.

The first-order condition for P_t^* is

$$\sum_{h \in T_{\mathrm{dt}}} e^{-(\rho+\lambda)h} \frac{P_t C_t}{P_{t+h} C_{t+h}} C_{t+h} P_{t+h}^{\sigma} \left[P_t^* - \frac{\sigma(1+\tau)}{\sigma-1} W_{t+h} \right] \mathrm{dt} = 0,$$

which, following the main text, can be simplified and rewritten as

$$\frac{P_t^*}{P_t} = \frac{\sigma(1+\tau)}{\sigma-1} \frac{\sum_{h \in T_{\mathrm{dt}}} e^{-(\rho+\lambda)h} \left(\frac{P_{t+h}}{P_t}\right)^{\sigma} \frac{W_{t+h}}{P_{t+h}} \mathrm{dt}}{\sum_{h \in T_{\mathrm{dt}}} e^{-(\rho+\lambda)h} \left(\frac{P_{t+h}}{P_t}\right)^{\sigma-1} \mathrm{dt}}. \tag{A.1}$$

We can again define the auxiliary variable δ_t as the inverse of the denominator in (A.1), which can be written recursively as

$$\delta_t^{-1} \equiv \sum_{h \in T_{\mathrm{dt}}} e^{-(\rho+\lambda)h} \left(\frac{P_{t+h}}{P_t}\right)^{\sigma-1} \mathrm{dt} = \mathrm{dt} + e^{-(\rho+\lambda)\mathrm{dt}} \left(\frac{P_{t+\mathrm{dt}}}{P_t}\right)^{\sigma-1} \delta_{t+\mathrm{dt}}^{-1}. \tag{A.2}$$

Similarly, we can write (A.1) recursively as

$$\frac{P_t^*}{P_t} = \frac{\sigma(1+\tau)}{\sigma-1} \frac{W_t}{P_t} \delta_t \mathrm{dt} + e^{-(\rho+\lambda)\mathrm{dt}} \left(\frac{P_{t+\mathrm{dt}}}{P_t}\right)^{\sigma} \frac{\delta_t}{\delta_{t+\mathrm{dt}}} \frac{P_{t+\mathrm{dt}}^*}{P_{t+\mathrm{dt}}} \tag{A.3}$$

$$= \frac{\sigma(1+\tau)}{\sigma-1} \frac{W_t}{P_t} \delta_t \mathrm{dt} + (1 - \delta_t \mathrm{dt}) \frac{P_{t+\mathrm{dt}}}{P_t} \frac{P_{t+\mathrm{dt}}^*}{P_{t+\mathrm{dt}}},$$

where the second line follows from substituting (A.2) in (A.3).

Next, we can derive the aggregate price as

$$P_t^{1-\sigma} = \int_0^1 P_{i,t}^{1-\sigma} \mathrm{d}j = e^{-\lambda \mathrm{dt}} (P_t^*)^{1-\sigma} + (1 - e^{-\lambda \mathrm{dt}}) P_{t-\mathrm{dt}}^{1-\sigma},$$

where we have used the property that the set of firms with sticky prices are a random sample of the population at each instant. This equation implies the following relationship between relative reset price and gross inflation rate:

$$1 = (1 - e^{-\lambda \mathrm{dt}}) \left(\frac{P_t^*}{P_t}\right)^{1-\sigma} + e^{-\lambda \mathrm{dt}} \left(\frac{P_t}{P_{t-\mathrm{dt}}}\right)^{\sigma-1}.$$

Defining $\pi_t \equiv \frac{1}{\mathrm{dt}} \log(P_t/P_{t-\mathrm{dt}})$ as the rate of inflation at time t, we can rewrite this equation as

$$\frac{P_t^*}{P_t} = \left[\frac{1 - e^{[(\sigma-1)\pi_t - \lambda]\mathrm{dt}}}{1 - e^{-\lambda \mathrm{dt}}} \right]^{\frac{1}{1-\sigma}},$$

which is the equivalent of Equation (17) in the main text once we set $dt = 1$ and plug $\theta = e^{-\lambda}$. Moreover, using this equation, combined with the intratemporal labor supply condition and the aggregate production function $C_t = Y_t = L_t/D_t$, Equations (A.2) and (A.3) become

$$\delta_t^{-1} = dt + e^{[(\sigma-1)\pi_{t+dt}-(\rho+\lambda)]dt}\delta_{t+dt}^{-1}, \tag{A.4}$$

$$\left[\frac{1 - e^{[(\sigma-1)\pi_t-\lambda]dt}}{1 - e^{-\lambda dt}}\right]^{\frac{1}{1-\sigma}} = \frac{\sigma(1+\tau)}{\sigma - 1}Y_t^{1+\psi}D_t^\psi\delta_t dt + (1 - \delta_t dt)e^{\pi_{t+dt}dt}\left[\frac{1 - e^{[(\sigma-1)\pi_{t+dt}-\lambda]dt}}{1 - e^{-\lambda dt}}\right]^{\frac{1}{1-\sigma}}, \tag{A.5}$$

which are the equivalents of Equations (19) and (20) in the main text, respectively.

We next write the equation for the price dispersion dynamics. By random selection of price-setters at any given t, we can write this equation as

$$D_t = \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\sigma} dj = (1 - e^{-\lambda dt})\left(\frac{P_t^*}{P_t}\right)^{-\sigma} + e^{-\lambda dt}\left(\frac{P_t}{P_{t-dt}}\right)^\sigma \int_0^1 \left(\frac{P_{i,t-dt}}{P_{t-dt}}\right)^{-\sigma} dj$$

$$= (1 - e^{-\lambda dt})\left[\frac{1 - e^{[(\sigma-1)\pi_t-\lambda]dt}}{1 - e^{-\lambda dt}}\right]^{\frac{\sigma}{\sigma-1}} + e^{\sigma\pi_t dt-\lambda dt}D_{t-dt}. \tag{A.6}$$

Finally, we can write the central bank's problem with a general time step as follows:

$$V(\Omega_t) = \max_{D_t, L_t}\left\{\log(D_t) - \frac{L_t^{1+\psi}}{1+\psi} + e^{-\rho dt}V(\Omega_{t+dt})\right\} \quad \text{subject to} \quad Y_t = \frac{L_t}{D_t},$$

which gives the same optimal policy as in the main text, $Y_t = 1/D_t$. This policy implies that the real wage from the intratemporal labor supply condition is given by

$$\frac{W_t}{P_t} = Y_t L_t^\psi = Y_t^{1+\psi}D_t^\psi = \frac{1}{D_t}.$$

Plugging this optimal policy into Equation (A.5) and taking the limit as $dt \to 0$ in Equations (A.4) to (A.6), we obtain the continuous-time analogs of the equations that characterize D_t, π_t and δ_t, as presented in the main text:

$$\dot{D}_t = \lambda\left(1 - \frac{\sigma-1}{\lambda}\pi_t\right)^{\frac{\sigma}{\sigma-1}} + (\sigma\pi_t - \lambda)D_t,$$

$$\dot{\pi}_t = -\lambda\frac{\sigma(1+\tau)}{\sigma-1}\left(1 - \frac{\sigma-1}{\lambda}\pi_t\right)^{\frac{\sigma}{\sigma-1}}\frac{\delta_t}{D_t} + (\delta_t - \pi_t)[\lambda - (\sigma-1)\pi_t],$$

$$\dot{\delta}_t = \delta_t^2 + [(\sigma-1)\pi_t - (\rho+\lambda)]\delta_t.$$

Appendix B   Proofs

B.1   Proof of Lemma 1

Take an initial price distribution $\{P_{j,-1}\}_{j\in[0,1]}$ and a sequence of policies $\{i_t\}_{t=0}^{\infty}$. The arguments in the text show that if a sequence of allocations and prices $\{L_t, Y_t, D_t, \delta_t, \Pi_t\}_{t=0}^{\infty}$ is supported by a competitive equilibrium, then it satisfies conditions (12), (13), (18), (19), (20), and (21). This proves the necessity claim.

To prove the sufficiency claim, suppose that a sequence $\{L_t, Y_t, D_t, \delta_t, \Pi_t\}_{t=0}^{\infty}$ satisfies conditions (12), (13), (18), (19), (20), and (21) given $\{P_{j,-1}\}_{j\in[0,1]}$ and $\{i_t\}_{t=0}^{\infty}$. The set $\{P_{j,-1}\}_{j\in[0,1]}$ defines P_{-1}, and we can define $P_t = \Pi_{t-1} P_{t-1}$ recursively. Let $P_{j,t} = P_{j,t-1}$ if firm j cannot change prices at t, and $P_{j,t} = P_t^*$ if the firm can change prices at t, where P_t^* is given by (17). Define W_t according to (15) and let $B_t = 0$ at all dates with T_t chosen to satisfy (11). Letting $C_t = Y_t$, define $C_{j,t}$ according to (2), and let $Y_{j,t} = L_{j,t} = C_{j,t}$. Additionally, let

$$X_{j,t} = [P_{j,t} - (1+\tau)W_t]C_t \left(\frac{P_{j,t}}{P_t}\right)^{-\sigma},$$

define $P_{j,t}^S$ according to (6), and let $s_{j,t} = 1$ so that the representative household holds a share of every firm $j \in [0,1]$. The household's problem (1) is concave and yields a unique solution. It can be verified that the values of $\left\{C_t, L_t, B_t, (s_{j,t}, C_{j,t})_{j\in[0,1]}\right\}_{t=0}^{\infty}$ satisfy all optimality conditions of the household's problem, with the transversality condition being verified below. The firm's problem (9) is concave and yields a unique solution. It can be verified that the values of $\{P_t^*, Y_{j,t}, L_{j,t}\}_{t=0}^{\infty}$ satisfy all optimality conditions of the firm's problem. Therefore, we conclude that the sequence $\{L_t, Y_t, D_t, \delta_t, \Pi_t\}_{t=0}^{\infty}$ supports a competitive equilibrium.

We next verify the transversality condition. Consider the date-t price of an Arrow-Debreu security that pays a coupon equal to firm j's profits at date $t+h$ for $h > 0$. There are three cases to consider. First, suppose the firm's price has always been sticky. Then the probability of arriving at such a history at $t+h$ from the perspective of date t is θ^h and the price that the firm is charging at $t+h$ is $P_{j,-1}$. Appealing to the intertemporal condition, we can write the limiting price of the Arrow-Debreu security at date t as $h \to \infty$ as

$$\lim_{h\to\infty} \beta^h \theta^h \frac{P_t C_t}{P_{t+h} C_{t+h}} [P_{j,-1} - (1+\tau) W_{t+h}] C_{t+h} \left(\frac{P_{j,-1}}{P_{t+h}}\right)^{-\sigma} = 0, \tag{B.1}$$

where transversality requires that this price go to zero.

Second, suppose the firm's price has been sticky since date ℓ for $0 \le \ell \le t$. Then the probability of arriving at such a history at $t+h$ from the perspective of date t is θ^h and the

price that the firm is charging at $t + h$ is P_ℓ^*. The transversality condition in this case is

$$\lim_{h \to 0} \beta^h \theta^h \frac{P_t C_t}{P_{t+h} C_{t+h}} \left[P_\ell^* - (1 + \tau) W_{t+h} \right] C_{t+h} \left(\frac{P_\ell^*}{P_{t+h}} \right)^{-\sigma} = 0. \tag{B.2}$$

Finally, suppose the firm's price has been sticky since date $\ell > t$. Then the probability of arriving at such a history at $t + h$ from the perspective of date t is $(1 - \theta) \theta^{t+h-\ell}$ and the price that the firm is charging at $t + h$ is P_ℓ^*. The transversality condition in this case is

$$\lim_{h \to 0} \beta^h (1 - \theta) \theta^{t+h-\ell} \frac{P_t C_t}{P_{t+h} C_{t+h}} \left[P_\ell^* - (1 + \tau) W_{t+h} \right] C_{t+h} \left(\frac{P_l^*}{P_{t+h}} \right)^{-\sigma} = 0. \tag{B.3}$$

To verify that (B.2) and (B.3) are satisfied, note that we can multiply (B.2) by $\beta^{-\ell} \theta^{-\ell} P_\ell C_\ell / P_t C_t$ without changing its limit as $h \to \infty$, which means that satisfaction of (B.2) is equivalent to

$$\lim_{h \to 0} \beta^{h-\ell} \theta^{h-\ell} \frac{P_\ell C_\ell}{P_{t+h} C_{t+h}} \left[P_\ell^* - (1 + \tau) W_{t+h} \right] C_{t+h} \left(\frac{P_\ell^*}{P_{t+h}} \right)^{-\sigma} = 0. \tag{B.4}$$

Similarly, we can multiply (B.3) by $(1 - \theta)^{-1} \theta^{-t} P_\ell C_\ell / P_t C_t$ without changing its limit as $h \to \infty$, which means that satisfaction of (B.3) is also equivalent to (B.4). Moreover, observe that given (14), (15), and (17), and noting that $P_t C_t \left(\frac{1 - \theta \Pi_t^{\sigma-1}}{1-\theta} \right)^{-\sigma} > 0$, it follows that satisfaction of (21) implies satisfaction of (B.4). Hence, (B.2) and (B.3) are both satisfied.

We are left to verify that (B.1) is also satisfied. We can multiply (B.1) by $P_{j,-1}^\sigma / P_t C_t$ without changing its limit as $h \to \infty$, which means that satisfaction of (B.1) is equivalent to

$$\lim_{h \to \infty} \beta^h \theta^h P_h^\sigma \left[\left(\frac{P_{j,-1}}{P_{-1}} \right) \frac{P_{-1}}{P_h} - (1 + \tau) \frac{W_h}{P_h} \right] = 0.$$

Under the constructed equilibrium, this limit can be rewritten as

$$\lim_{h \to \infty} \left[\beta \theta \left(\prod_{\ell=0}^{h} \Pi_\ell \right)^{\frac{\sigma}{h}} \right]^h \left[\left(\frac{P_{j,-1}}{P_{-1}} \right) \frac{1}{\prod_{\ell=0}^{h} \Pi_\ell} - (1 + \tau) Y_h^\psi D_h^{1+\psi} \right] = 0. \tag{B.5}$$

There are two possible cases. Suppose first that $\lim_{h \to \infty} \left[\beta \theta \left(\prod_{\ell=0}^{h} \Pi_\ell \right)^{\frac{\sigma}{h}} \right]^h = 0$. Then note that by (B.4) for $\ell = 0$, the second bracket stays finite as $h \to \infty$. Hence, in this case, (B.5) and thus (B.1) are satisfied.

Suppose next that $\lim_{h \to \infty} \left[\beta \theta \left(\prod_{\ell=0}^{h} \Pi_\ell \right)^{\frac{\sigma}{h}} \right]^h \neq 0$. Then satisfaction of (B.4) for $\ell = 0$

implies

$$\lim_{h \to \infty} \left[\frac{P_0^*}{P_0} \frac{1}{\prod_{\ell=0}^h \Pi_\ell} - (1+\tau)Y_h^\psi D_h^{1+\psi} \right] = 0.$$

It follows that if (B.5) is not satisfied, then we must have

$$\lim_{h \to \infty} \left[\frac{P_0^*}{P_0} \frac{1}{\prod_{\ell=0}^h \Pi_\ell} - (1+\tau)Y_h^\psi D_h^{1+\psi} - \left(\frac{P_{j,-1}}{P_{-1}}\right) \frac{1}{\prod_{\ell=0}^h \Pi_\ell} - (1+\tau)Y_h^\psi D_h^{1+\psi} \right] \neq 0,$$

or, equivalently,

$$\lim_{h \to \infty} \left\{ \left[\frac{P_0^*}{P_0} - \left(\frac{P_{j,-1}}{P_{-1}}\right) \frac{P_0}{P_1} \right] \frac{1}{\prod_{\ell=0}^h \Pi_\ell} \right\} \neq 0.$$

But this means that $\frac{1}{\prod_{\ell=0}^h \Pi_\ell}$ does not approach zero as $h \to \infty$, which contradicts the assumption that $\lim_{h\to\infty} \left[\beta\theta \left(\prod_{\ell=0}^h \Pi_\ell\right)^{\frac{\sigma}{h}} \right]^h \neq 0$. Hence, (B.5) and thus (B.1) are satisfied.

B.2 Proof of Lemma 2

Consider first price dispersion D. Equation (22) defines D as a function of Π in the steady state. Differentiating this equation yields

$$\begin{aligned}
\frac{\partial}{\partial \Pi} D &= \theta\sigma D \Pi^{\sigma-2} \left(-\frac{1}{1-\theta\Pi^{\sigma-1}} + \frac{\Pi}{1-\theta\Pi^\sigma} \right) \\
&= \theta\sigma D \Pi^{\sigma-2} \frac{\Pi-1}{(1-\theta\Pi^{\sigma-1})(1-\theta\Pi^\sigma)}.
\end{aligned}$$

This expression is strictly positive for $\Pi \in (1, \theta^{-1/\sigma})$, including D itself (which is a function of Π per Equation (22)). Thus, D is strictly increasing in Π for $\Pi \in [1, \theta^{-1/\sigma})$.

Consider next the labor share μ. Raising Equation (22) to the power of $1 + \psi$ and substituting in Equation (23) yields

$$\begin{aligned}
\mu &= \frac{\sigma-1}{\sigma(1+\tau)} \frac{1-\theta\Pi^{\sigma-1}}{1-\theta\Pi^\sigma} \frac{1-\beta\theta\Pi^\sigma}{1-\beta\theta\Pi^{\sigma-1}} \\
&= \frac{\sigma-1}{\sigma(1+\tau)} \left[1 + \frac{(1-\beta)\theta\Pi^{\sigma-1}(\Pi-1)}{(1-\theta\Pi^\sigma)(1-\beta\theta\Pi^{\sigma-1})} \right].
\end{aligned}$$

Note that the fraction inside the brackets is strictly positive for $\Pi \in (1, \theta^{-1/\sigma})$ and is equal to zero for $\Pi = 1$. Thus, $\mu \geq (\sigma-1)/[\sigma(1+\tau)]$, with equality only when $\Pi = 1$. Differentiating

this equation yields

$$\frac{\partial}{\partial \Pi} \mu = \left[\mu - \frac{\sigma - 1}{\sigma(1+\tau)}\right]\left[\frac{\sigma-1}{\Pi} + \frac{1}{\Pi - 1} + \frac{\sigma\theta\Pi^{\sigma-1}}{1 - \theta\Pi^{\sigma}} + \frac{(\sigma-1)\beta\theta\Pi^{\sigma-2}}{1 - \beta\theta\Pi^{\sigma-1}}\right].$$

This expression is strictly positive for $\Pi \in (1, \theta^{-1/\sigma})$. Thus, μ is strictly increasing in Π for $\Pi \in [1, \theta^{-1/\sigma})$.

B.3   Proof of Proposition 1

Uniqueness.   In the steady-state, $\dot{D}_t = \dot{\pi}_t = \dot{\delta}_t = 0$. Setting these to zero and dropping the time subscript, we obtain the following system of equations:

$$(\delta - \pi)[\lambda - (\sigma - 1)\pi] = \lambda\frac{\sigma(1+\tau)}{\sigma - 1}\left(1 - \frac{\sigma-1}{\lambda}\pi\right)^{\frac{\sigma}{\sigma-1}}\frac{\delta}{D}, \tag{B.6}$$

$$(\lambda - \sigma\pi)D = \lambda\left(1 - \frac{\sigma-1}{\lambda}\pi\right)^{\frac{\sigma}{\sigma-1}}, \tag{B.7}$$

$$\delta = \rho + \lambda - (\sigma - 1)\pi. \tag{B.8}$$

Substituting the last two equations into the first one yields

$$(\rho + \lambda - \sigma\pi)[\lambda - (\sigma - 1)\pi] = \frac{\sigma(1+\tau)}{\sigma - 1}(\lambda - \sigma\pi)[\rho + \lambda - (\sigma - 1)\pi],$$

which can be rewritten as

$$\frac{\rho(\sigma - 1)}{1 + \sigma\tau}\pi = (\lambda - \sigma\pi)[\rho + \lambda - (\sigma - 1)\pi]. \tag{B.9}$$

Since this is a quadratic equation, there are at most two steady-state values of π that solve it. Rather than solving for these roots explicitly, observe that the left-hand side of the equation is a linear increasing function of π, while the right-hand side has two zeros, one at $\pi = \frac{\lambda}{\sigma}$ and another at $\pi = \frac{\rho+\lambda}{\sigma-1}$. Since $\frac{\lambda}{\sigma} < \frac{\rho+\lambda}{\sigma-1}$, we need to consider three regions:

1. $\pi < \frac{\lambda}{\sigma}$: In this region, the right-hand side of (B.9) is positive. The two sides intersect at a point where both are positive, so the quadratic has at least one root $\pi \in (0, \frac{\lambda}{\sigma})$.

2. $\frac{\lambda}{\sigma} \leq \pi \leq \frac{\rho+\lambda}{\sigma-1}$: In this region, the right-hand side of (B.9) is negative while the left-hand side is strictly positive. Thus, there cannot be a solution here.

3. $\pi > \frac{\rho+\lambda}{\sigma-1}$: In this region, the right-hand side of (B.9) is positive and grows quadratically from 0, whereas the left-hand side grows linearly from a positive number. The two sides intersect at a point where both are positive, so the quadratic has at least one root

$\pi \in (\frac{\rho + \lambda}{\sigma - 1}, \infty)$.

Since a quadratic cannot have more than two roots, we conclude that the roots found in the first and third regions above are unique within their regions.

Finally, note that the root $\pi > \frac{\rho + \lambda}{\sigma - 1}$ violates the natural bound on inflation implied by sticky prices $\pi < \frac{\lambda}{\sigma - 1}$ and thus cannot be a steady state. Therefore, the unique steady state is the one found in the first region, $\pi \in (0, \frac{\lambda}{\sigma})$.

Comparative Statics. It follows from the proof of uniqueness above that steady-state inflation $\pi_{ss}(\tau, \sigma)$ solves

$$\frac{\rho(\sigma - 1)}{1 + \sigma\tau}\pi_{ss}(\tau, \sigma) = (\lambda - \sigma\pi_{ss}(\tau, \sigma))[\rho + \lambda - (\sigma - 1)\pi_{ss}(\tau, \sigma)], \qquad (B.10)$$

where the value of $\pi_{ss}(\tau, \sigma)$ is the root of this quadratic equation in the interval $(0, \frac{\lambda}{\sigma})$. Given this value, we can then derive steady-state price dispersion $D_{ss}(\tau, \sigma)$ using Equation (B.7):

$$D_{ss}(\tau, \sigma) = \frac{\lambda}{\lambda - \sigma\pi_{ss}(\tau, \sigma)} \left(1 - \frac{\sigma - 1}{\lambda}\pi_{ss}(\tau, \sigma)\right)^{\frac{\sigma}{\sigma - 1}}. \qquad (B.11)$$

<u>Part 1.</u> Consider first $\pi_{ss}(\tau, \sigma)$. Differentiating (B.10) with respect to τ yields

$$\left[\frac{\sigma}{\lambda - \sigma\pi_{ss}(\tau, \sigma)} + \frac{\sigma - 1}{\rho + \lambda - (\sigma - 1)\pi_{ss}(\tau, \sigma)} + \frac{1}{\pi_{ss}(\tau, \sigma)}\right] \frac{\partial}{\partial \tau}\pi_{ss}(\tau, \sigma) = \frac{\sigma}{1 + \sigma\tau}.$$

All the terms in the bracket on the left-hand side are positive given $\pi_{ss}(\tau, \sigma) \in (0, \frac{\lambda}{\sigma})$. The right-hand side is also positive by Assumption 1. Thus, $\frac{\partial}{\partial \tau}\pi_{ss}(\tau, \sigma) > 0$ and $\pi_{ss}(\tau, \sigma)$ is strictly increasing in τ.

Consider next $D_{ss}(\tau, \sigma)$. From (B.11), we see that $D_{ss}(\tau, \sigma)$ depends on τ only through $\pi_{ss}(\tau, \sigma)$. Thus,

$$\begin{aligned}
\frac{\partial}{\partial \tau}D_{ss}(\tau, \sigma) &= \frac{\partial}{\partial \pi_{ss}(\tau, \sigma)}D_{ss}(\tau, \sigma) \times \frac{\partial}{\partial \tau}\pi_{ss}(\tau, \sigma) \\
&= \frac{\sigma D_{ss}(\tau, \sigma)\pi_{ss}(\tau, \sigma)}{(\lambda - \sigma\pi_{ss}(\tau, \sigma))[\lambda - (\sigma - 1)\pi_{ss}(\tau, \sigma)]}\frac{\partial}{\partial \tau}\pi_{ss}(\tau, \sigma).
\end{aligned}$$

All the terms involved are positive given $\pi_{ss}(\tau, \sigma) \in (0, \frac{\lambda}{\sigma})$. Thus, $\frac{\partial}{\partial \tau}D_{ss}(\tau, \sigma) > 0$ and $D_{ss}(\tau, \sigma)$ is strictly increasing in τ.

Part 2. Consider first $\pi_{ss}(\tau, \sigma)$. Differentiating (B.10) with respect to σ yields

$$
\left[\frac{\sigma - 1}{\rho + \lambda - (\sigma - 1)\pi_{ss}(\tau, \sigma)} + \frac{\sigma}{\lambda - \sigma\pi_{ss}(\tau, \sigma)} + \frac{1}{\pi_{ss}(\tau, \sigma)} \right] \frac{\partial}{\partial \sigma} \pi_{ss}(\tau, \sigma) \tag{B.12}
$$
$$
= - \left[\frac{1 + \tau}{(\sigma - 1)(1 + \sigma\tau)} + \frac{\pi_{ss}(\tau, \sigma)}{\lambda - \sigma\pi_{ss}(\tau, \sigma)} + \frac{\pi_{ss}(\tau, \sigma)}{\rho + \lambda - (\sigma - 1)\pi_{ss}(\tau, \sigma)} \right].
$$

Using $\pi_{ss}(\tau, \sigma) \in (0, \frac{\lambda}{\sigma})$ and Assumption 1, we can conclude that all the terms inside the brackets on both sides are positive. Thus, by the negative sign on the right-hand side, $\frac{\partial}{\partial \sigma} \pi_{ss}(\tau, \sigma) < 0$ and $\pi_{ss}(\tau, \sigma)$ is strictly decreasing in σ.

Consider next $D_{ss}(\tau, \sigma)$. Observe that $D_{ss}(\tau, \sigma)$ depends on σ both directly through aggregation, and indirectly through $\pi_{ss}(\tau, \sigma)$ as the central bank's optimal policy changes $\pi_{ss}(\sigma, \tau)$ when σ varies. Accordingly, we will investigate the total derivative of $D_{ss}(\tau, \sigma)$ by decomposing it into these direct and indirect effects of σ:

$$
\frac{\partial}{\partial \sigma} D_{ss}(\tau, \sigma) = \frac{\partial}{\partial \sigma} D_{ss}(\tau, \sigma)\Big|_{\pi_{ss}(\tau, \sigma)} + \frac{\partial}{\partial \pi_{ss}(\tau, \sigma)} D_{ss}(\tau, \sigma)\Big|_{\sigma} \times \frac{\partial}{\partial \sigma} \pi_{ss}(\tau, \sigma). \tag{B.13}
$$

To derive the first term on the right-hand side, we use (B.11) to obtain

$$
\frac{\partial}{\partial \sigma} D_{ss}(\tau, \sigma)\Big|_{\pi_{ss}(\tau, \sigma)} = \frac{D_{ss}(\tau, \sigma)}{(\sigma - 1)^2} \left(1 - \frac{1}{1 - \frac{\sigma - 1}{\lambda}\pi_{ss}(\tau, \sigma)} - \log\left(1 - \frac{\sigma - 1}{\lambda}\pi_{ss}(\tau, \sigma) \right) \right)
$$
$$
+ D_{ss}(\tau, \sigma) \left[\frac{\pi_{ss}(\tau, \sigma)}{\lambda - \sigma\pi_{ss}(\tau, \sigma)} - \frac{\pi_{ss}(\tau, \sigma)}{\lambda - (\sigma - 1)\pi_{ss}(\tau, \sigma)} \right].
$$

As for the partial derivative of $D_{ss}(\tau, \sigma)$ with respect to $\pi_{ss}(\tau, \sigma)$, holding σ fixed, we use (B.11) to obtain

$$
\frac{\partial}{\partial \pi_{ss}(\tau, \sigma)} D_{ss}(\tau, \sigma)\Big|_{\sigma} = D_{ss}(\tau, \sigma) \left[\frac{\sigma}{\lambda - \sigma\pi_{ss}(\tau, \sigma)} - \frac{\sigma}{\lambda - (\sigma - 1)\pi_{ss}(\tau, \sigma)} \right].
$$

Substituting these into (B.13) yields

$$
\frac{\partial}{\partial \sigma} D_{ss}(\tau, \sigma) = \frac{D_{ss}(\tau, \sigma)}{(\sigma - 1)^2} \overbrace{\left(1 - \frac{1}{1 - \frac{\sigma - 1}{\lambda}\pi_{ss}(\tau, \sigma)} - \log\left(1 - \frac{\sigma - 1}{\lambda}\pi_{ss}(\tau, \sigma) \right) \right)}^{\text{①} < 0}
$$
$$
+ D_{ss}(\tau, \sigma) \underbrace{\left(\sigma\frac{\partial}{\partial \sigma}\pi_{ss}(\tau, \sigma) + \pi_{ss}(\tau, \sigma) \right)}_{\text{②}} \underbrace{\left[\frac{1}{\lambda - \sigma\pi_{ss}(\tau, \sigma)} - \frac{1}{\lambda - (\sigma - 1)\pi_{ss}(\tau, \sigma)} \right]}_{\text{③} > 0}.
$$

It is straightforward to show that ① is strictly negative for $\pi_{ss}(\tau, \sigma) \in (0, \frac{\lambda}{\sigma})$.[34] Moreover, ③ is strictly positive for $\pi_{ss}(\tau, \sigma) \in (0, \frac{\lambda}{\sigma})$. Thus, a sufficient condition for $\frac{\partial}{\partial \sigma} D_{ss}(\tau, \sigma)$ to be strictly negative is that ② is negative. We next show that this holds under $\tau < \bar{\tau}(\sigma)$. Using (B.12), we have

$$
②= -\frac{\dfrac{\sigma(1+\tau)}{(\sigma-1)(1+\sigma\tau)} + \dfrac{\sigma\pi_{ss}(\tau,\sigma)}{\lambda - \sigma\pi_{ss}(\tau,\sigma)} + \dfrac{\sigma\pi_{ss}(\tau,\sigma)}{\rho + \lambda - (\sigma-1)\pi_{ss}(\tau,\sigma)}}{\dfrac{\sigma-1}{\rho + \lambda - (\sigma-1)\pi_{ss}(\tau,\sigma)} + \dfrac{\sigma}{\lambda - \sigma\pi_{ss}(\tau,\sigma)} + \dfrac{1}{\pi_{ss}(\tau,\sigma)}} + \pi_{ss}(\tau,\sigma)
$$

$$
= \frac{-\dfrac{\sigma(1+\tau)}{(\sigma-1)(1+\sigma\tau)} - \dfrac{\pi_{ss}(\tau,\sigma)}{\rho + \lambda - (\sigma-1)\pi_{ss}(\tau,\sigma)} + 1}{\dfrac{\sigma-1}{\rho + \lambda - (\sigma-1)\pi_{ss}(\tau,\sigma)} + \dfrac{\sigma}{\lambda - \sigma\pi_{ss}(\tau,\sigma)} + \dfrac{1}{\pi_{ss}(\tau,\sigma)}}.
$$

The denominator is positive for $\pi_{ss}(\tau, \sigma)$ in $(0, \frac{\lambda}{\sigma})$. We show that the numerator is negative for $\tau < \bar{\tau}(\sigma)$. To see this, note that the fraction involving $\pi_{ss}(\tau, \sigma)$ is negative, so it is sufficient to show that

$$
-\frac{\sigma(1+\tau)}{(\sigma-1)(1+\sigma\tau)} + 1 < 0 \iff (\sigma - 2)\sigma\tau < 1.
$$

Now note that under $\tau < \bar{\tau}(\sigma)$ and Assumption 1, we have

$$
1 < \sigma < 2 \implies (\sigma - 2)\sigma\tau < (2 - \sigma) < 1,
$$

$$
\sigma \geq 2 \implies (\sigma - 2)\sigma\tau < (\sigma - 2)\sigma\bar{\tau}(\sigma) = (\sigma - 2)\sigma\frac{1}{\sigma(\sigma - 2)} = 1.
$$

Hence, given $\tau < \bar{\tau}(\sigma)$ and $\sigma > 1$, we obtain ② < 0. It follows that $\frac{\partial}{\partial \sigma} D_{ss}(\tau, \sigma) < 0$ and $D_{ss}(\tau, \sigma)$ is strictly decreasing in σ for all $\tau < \bar{\tau}(\sigma)$.

B.4   Proof of Proposition 2

To prove this proposition, we will rely on the Stable Manifold and the Hartman-Grobman theorems (Perko, 2001, pages 107 and 120, respectively). These two theorems relate the dynamics of a non-linear dynamical system to its local linearized dynamics around a fixed point (in our case, the unique steady state). To make use of their predictions, we rewrite our dynamical system involving the variables π_t, D_t and δ_t in the following form. Let

---

[34]To see this, note that $\pi_{ss}(\tau, \sigma) \in (0, \frac{\lambda}{\sigma})$ implies that $1 - \frac{\sigma-1}{\lambda}\pi_{ss}(\tau, \sigma) \in (\frac{1}{\sigma}, 1)$. Moreover, note that the function $f(x) \equiv 1 - 1/x - \log(x)$ is strictly increasing in $x \in (0, 1)$ (as $f'(x) = 1/x^2 - 1/x > 0, x \in (0, 1)$), so that $\forall x \in (\frac{1}{\sigma}, 1) : f(x) < f(1) = 0$.

$X_t \equiv (\pi_t, D_t, \delta_t)$. Then the non-linear dynamical system implied by the model can be characterized by a function $f : \mathbb{R}^3 \to \mathbb{R}^3$ defined as

$$\dot{X}_t = f(X_t) \equiv \begin{bmatrix} -\lambda \frac{\sigma(1+\tau)}{\sigma-1}\left(1 - \frac{\sigma-1}{\lambda}\pi_t\right)^{\frac{\sigma}{\sigma-1}}\frac{\delta_t}{D_t} + (\delta_t - \pi_t)[\lambda - (\sigma-1)\pi_t] \\ \lambda\left(1 - \frac{\sigma-1}{\lambda}\pi_t\right)^{\frac{\sigma}{\sigma-1}} + (\sigma\pi_t - \lambda)D_t \\ \delta_t^2 + [(\sigma-1)\pi_t - (\rho+\lambda)]\delta_t \end{bmatrix},$$

where the unique steady state that we characterized is a fixed point of this system.

Note that $f(\cdot)$ is a smooth function; importantly, it is continuously differentiable, which implies that the flows of the system are also continuous. In order to understand the dynamics of the system and how the transition to a new steady state happens, we need to first characterize the nature of the unique steady state for the above system. To do this, we can apply the Hartman-Grobman theorem, which states that if the eigenvalues of the Jacobian of the function f evaluated at the fixed point have non-zero real parts, then there exists a neighborhood N around the fixed point of the system where the flows of the non-linear system are topologically conjugate to the flows of the linearized system. We will apply this theorem in the following way. First, we will show that the fixed point is a saddle point of the linearized system. Then verifying the assumptions of the Hartman-Grobman theorem, we will conclude from topological conjugacy that the steady state is also a saddle point of the non-linear system.

To show that the steady state is a saddle point of the linearized system, we first need to compute the Jacobian of f at the steady state. Letting $X_{ss} = (\pi_{ss}, D_{ss}, \delta_{ss})$ denote the steady state under a certain set of parameters, note that

$$0 = \dot{X}_{ss} = f(X_{ss}) \implies \begin{cases} \frac{\rho(\sigma-1)}{1+\sigma\tau}\pi_{ss} = (\lambda - \sigma\pi_{ss})[\rho + \lambda - (\sigma-1)\pi_{ss}] \\ D_{ss} = \frac{\lambda}{\lambda-\sigma\pi_{ss}}\left(1 - \frac{\sigma-1}{\lambda}\pi_{ss}\right)^{\frac{\sigma}{\sigma-1}} \\ \delta_{ss} = \rho + \lambda - (\sigma-1)\pi_{ss} \end{cases}, \qquad \text{(B.14)}$$

and, letting $\mathbf{D}f$ denote the Jacobian of f evaluated at X_{ss}, we have

$$\mathbf{D}f = \begin{bmatrix} \frac{\partial}{\partial\pi}f_1 & \frac{\partial}{\partial D}f_1 & \frac{\partial}{\partial\delta}f_1 \\ \frac{\partial}{\partial\pi}f_2 & \frac{\partial}{\partial D}f_2 & \frac{\partial}{\partial\delta}f_2 \\ \frac{\partial}{\partial\pi}f_3 & \frac{\partial}{\partial D}f_3 & \frac{\partial}{\partial\delta}f_3 \end{bmatrix},$$

where all the partial derivatives are evaluated at X_{ss} and are given by

$$\frac{\partial}{\partial \pi} f_1 = \frac{\sigma^2(1+\tau)}{\sigma-1}\left(1 - \frac{\sigma-1}{\lambda}\pi_{ss}\right)^{\frac{1}{\sigma-1}}\frac{\delta_{ss}}{D_{ss}} - [\lambda - (\sigma-1)\pi_{ss}] - (\sigma-1)(\delta_{ss} - \pi_{ss})$$

$$= \rho - \pi_{ss}, \qquad\qquad\qquad \text{(using Equation (B.14))}$$

$$\frac{\partial}{\partial D} f_1 = \lambda\frac{\sigma(1+\tau)}{\sigma-1}\left(1 - \frac{\sigma-1}{\lambda}\pi_{ss}\right)^{\frac{\sigma}{\sigma-1}}\frac{\delta_{ss}}{D_{ss}^2} = \frac{\rho\pi_{ss} + (\lambda - \sigma\pi_{ss})\delta_{ss}}{D_{ss}},$$

$$\frac{\partial}{\partial \delta} f_1 = -\frac{1+\sigma\tau}{\sigma-1}(\lambda - \sigma\pi_{ss}) + \pi_{ss} = \frac{\pi_{ss}}{\delta_{ss}}[\lambda - (\sigma-1)\pi_{ss}],$$

$$\frac{\partial}{\partial \pi} f_2 = -\sigma\left(1 - \frac{\sigma-1}{\lambda}\pi_{ss}\right)^{\frac{1}{\sigma-1}} + \sigma D_{ss} = \sigma D_{ss}\frac{\pi_{ss}}{\lambda - (\sigma-1)\pi_{ss}},$$

$$\frac{\partial}{\partial D} f_2 = \sigma\pi_{ss} - \lambda,$$

$$\frac{\partial}{\partial \delta} f_2 = 0,$$

$$\frac{\partial}{\partial \pi} f_3 = (\sigma-1)\delta_{ss},$$

$$\frac{\partial}{\partial D} f_3 = 0,$$

$$\frac{\partial}{\partial \delta} f_3 = 2\delta_{ss} + (\sigma-1)\pi_{ss} - (\rho + \lambda) = \delta_{ss}.$$

To show that the Hartman-Grobman theorem applies, we need to show that X_{ss} is a hyperbolic fixed point—i.e., all the eigenvalues of $\mathbf{D}f$ have non-zero real parts. To calculate the eigenvalues of $\mathbf{D}f$, we need to compute the roots of its characteristic polynomial:

$$\det(\mathbf{D}f - \eta\mathbf{I}) = 0,$$

where any η that solves this polynomial is an eigenvalue of the Jacobian. The characteristic polynomial is given by:

$$\det(\mathbf{D}f - \eta\mathbf{I}) =$$
$$(\tfrac{\partial}{\partial \pi} f_1 - \eta)(\tfrac{\partial}{\partial D} f_2 - \eta)(\tfrac{\partial}{\partial \delta} f_3 - \eta) - \tfrac{\partial}{\partial D} f_1\tfrac{\partial}{\partial \pi} f_2\left(\tfrac{\partial}{\partial \delta} f_3 - \eta\right) - \tfrac{\partial}{\partial \delta} f_1\tfrac{\partial}{\partial \pi} f_3\left(\tfrac{\partial}{\partial D} f_2 - \eta\right),$$

where we have used $\frac{\partial}{\partial \delta} f_2 = \frac{\partial}{\partial D} f_3 = 0$. Plugging in the derived values for other partial derivatives, we obtain the following cubic polynomial:

$$\det(\mathbf{D}f - \eta\mathbf{I})$$
$$= (\rho - \pi_{ss} - \eta)(\sigma\pi_{ss} - \lambda - \eta)(\delta_{ss} - \eta) - \sigma\pi_{ss}(\rho + \lambda - \sigma\pi_{ss})(\delta_{ss} - \eta)$$
$$- (\sigma-1)\pi_{ss}(\lambda - (\sigma-1)\pi_{ss})(\sigma\pi_{ss} - \lambda - \eta).$$

We now need to compute the roots of this cubic equation. One could use the general formula for roots of a cubic but that requires some tedious algebra. An easier path is to guess and

verify that one of the roots is ρ.[35] To verify this guess observe that at $\eta = \rho$,

$$\det\left(\mathbf{D}f - \rho\mathbf{I}\right) = (\sigma - 1)\pi_{ss}(\sigma\pi_{ss} - \lambda - \rho)(\delta_{ss} - \rho) - (\sigma - 1)\pi_{ss}(\delta_{ss} - \rho)(\sigma\pi_{ss} - \lambda - \rho) = 0.$$

Thus, the characteristic polynomial is divisible by $\rho - \eta$. Using this fact, we can factorize the characteristic polynomial as

$$\det(\mathbf{D}f - \eta\mathbf{I}) = (\rho - \eta)\left[\eta^2 - \rho\eta - (\rho + \lambda)\lambda + \sigma(\sigma - 1)\pi_{ss}^2\right],$$

where the rest of the eigenvalues are the roots of the quadratic equation $\eta^2 - \rho\eta - (\rho + \lambda)\lambda + \sigma(\sigma - 1)\pi_{ss}^2 = 0$. Therefore, the eigenvalues of the Jacobian at the steady state are

$$\eta = \begin{cases} \eta_1 \equiv \rho \\ \eta_2 \equiv \frac{\rho}{2} + \sqrt{\left(\frac{\rho}{2} + \lambda\right)^2 - \sigma(\sigma - 1)\pi_{ss}^2} \\ \eta_3 \equiv \frac{\rho}{2} - \sqrt{\left(\frac{\rho}{2} + \lambda\right)^2 - \sigma(\sigma - 1)\pi_{ss}^2} \end{cases}.$$

We can make the following observations about these eigenvalues. First, all of them are real. To see this, we just need to confirm that the term inside the square root is always positive. This follows from $\rho > 0$ and the fact that $\pi_{ss} \in (0, \lambda/\sigma)$ under Assumption 1:

$$\left(\frac{\rho}{2} + \lambda\right)^2 - \sigma(\sigma - 1)\pi_{ss}^2 > \lambda^2 - \sigma^2\pi_{ss}^2 = (\lambda - \sigma\pi_{ss})(\lambda + \sigma\pi_{ss}) > 0.$$

A second observation is that the first two eigenvalues are strictly positive (which is straightforward to confirm from the observation above) and the third one is negative. To verify the latter, note that

$$\frac{\rho}{2} - \sqrt{\left(\frac{\rho}{2} + \lambda\right)^2 - \sigma(\sigma - 1)\pi_{ss}^2} < 0 \iff \left(\frac{\rho}{2}\right)^2 < \left(\frac{\rho}{2} + \lambda\right)^2 - \sigma(\sigma - 1)\pi_{ss}^2$$

$$\iff 0 < \lambda^2 + \rho\lambda - \sigma(\sigma - 1)\pi_{ss}^2,$$

and the last inequality holds since

$$\lambda^2 + \rho\lambda - \sigma(\sigma - 1)\pi_{ss}^2 > \lambda^2 - \sigma^2\pi_{ss}^2 = (\lambda - \sigma\pi_{ss})(\lambda + \sigma\pi_{ss}) > 0.$$

Therefore, the Jacobian $\mathbf{D}f$ has two strictly positive eigenvalues and one strictly negative

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[35]There is an economic intuition for this guess. We know that at $\rho = 0$, the Phillips curve of the economy is fully vertical, which implies that $\rho = 0$ is a bifurcation point for the system. So the behavior of the system should switch at $\rho = 0$, making it reasonable to guess that ρ is one of its eigenvalues.

eigenvalue. This implies that the fixed point X_{ss} is a hyperbolic fixed point and is a saddle point for the linearized dynamical system. Thus, the Hartman-Grobman theorem applies and we can conclude that the fixed point is also a saddle point for the non-linear system.

Since all eigenvalues are distinct, the three eigenvectors associated with them are linearly independent and span $\mathbb{R}^3$. Thus, these eigenvalues imply that the dynamics of the linearized system are stable along the eigenspace spanned by the negative eigenvalue (which is one-dimensional as we show below) and unstable along the eigenspace associated with the two positive eigenvalues. Now, to study the convergence of the non-linear dynamics, we appeal to the Stable Manifold Theorem. When applied to our setting, this theorem states that in an open neighborhood around the fixed point X_{ss} where the function f is continuously differentiable (which is the case for our system), there exists a one-dimensional differentiable manifold S tangent to the stable subspace of the linear system such that for all $t \geq 0$, $X \in S$,

$$\lim_{t \to \infty} \phi_t(X) = X_{ss},$$

where $\phi_t(X)$ denotes the flow of the non-linear system starting from X at time $t = 0$ (i.e., $\phi_0(X) = X$) and evolves according to the non-linear dynamics. Therefore, we have established that in an open neighborhood N of the fixed point X_{ss}, the non-linear dynamics converge to the fixed point X_{ss} along a stable manifold S that is one-dimensional and tangent to the one-dimensional eigenspace of the linearized system at the fixed point. It then suffices to characterize the direction of convergence along the stable eigenspace of the linearized system. To this end, consider the linear dynamics around the fixed point X_{ss}:

$$\dot{X}_t = \mathbf{D}f\left(X_t - X_{ss}\right).$$

Let $\psi_t(X)$ denote the flow of this linearized system starting from some $X \in \mathbb{R}^3$. Since the eigenvectors of $\mathbf{D}f$ are linearly independent, we can write this flow as

$$\psi_t(X) = \alpha_{1,X}(t)\mathbf{v}_1 + \alpha_{2,X}(t)\mathbf{v}_2 + \alpha_{3,X}(t)\mathbf{v}_3,$$

where $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are eigenvectors of $\mathbf{D}f$ that correspond to eigenvalues η_1, η_2, and η_3 respectively. Furthermore, since $\psi_0(X) = X$, $\alpha_{i,X}(0)$ for $i = 1, 2, 3$ are given by the projection of X on the eigenvectors of $\mathbf{D}f$. Also, note that since $\psi_t(X_{ss}) = X_{ss}$, $\alpha_{i,X_{ss}}(t)$ is constant over time, and we use $\bar{\alpha}_i$ to refer to it. Plugging this decomposition into the linearized system

yields

$$\sum_{i=1}^{3} \dot{\alpha}_{i,X}(t)\mathbf{v}_i = \mathbf{D}f \sum_{i=1}^{3}(\alpha_{i,X}(t) - \bar{\alpha}_i)\mathbf{v}_i = \sum_{i=1}^{3} \eta_i(\alpha_{i,X}(t) - \bar{\alpha}_i)\mathbf{v}_i.$$

Therefore, for $i = 1, 2, 3$,

$$\dot{\alpha}_{i,X}(t) = \eta_i(\alpha_{i,X}(t) - \bar{\alpha}_i) \implies \alpha_{i,X}(t) - \bar{\alpha}_i = (\alpha_{i,X}(0) - \bar{\alpha}_i)e^{\eta_i t},$$

which implies

$$\psi_t(X) = X_{ss} + \sum_{i=1}^{3}(\alpha_{i,X}(0) - \bar{\alpha}_i)e^{\eta_i t}\mathbf{v}_i.$$

Note that since the $\mathbf{v}_i$'s are linearly independent, $\psi_t(X)$ is convergent if and only if $\alpha_{1,X}(0) - \bar{\alpha}_1 = \alpha_{2,X}(0) - \bar{\alpha}_2 = 0$ (since $\eta_1 > 0$ and $\eta_2 > 0$). This identifies the stable eigenspace of the linearized system as the span of $\mathbf{v}_3$ shifted to cross X_{ss}; that is,

$$\lim_{t \to \infty} \psi_t(X) = X_{ss} \iff X \in X_{ss} + \mathrm{span}(\mathbf{v}_3)$$

$$\iff \psi_t(X) - X_{ss} = ke^{\eta_3 t}\mathbf{v}_3 \quad \text{for some } k \in \mathbb{R}.$$

Given that $\mathbf{v}_3 = (v_{3,1}, v_{3,2}, v_{3,3})$ is an eigenvector associated with the negative eigenvalue η_3, and normalizing $v_{3,1} = 1$, we have

$$\frac{\partial}{\partial \pi}f_2 + \left(\frac{\partial}{\partial D}f_2 - \eta_3\right)v_{3,2} = 0 \implies v_{3,2} = \frac{\frac{\partial}{\partial \pi}f_2}{\eta_3 - \frac{\partial}{\partial D}f_2},$$

$$\frac{\partial}{\partial \pi}f_3 + \left(\frac{\partial}{\partial \delta}f_3 - \eta_3\right)v_{3,3} = 0 \implies v_{3,3} = \frac{\frac{\partial}{\partial \pi}f_3}{\eta_3 - \frac{\partial}{\partial \delta}f_3}.$$

For a given $k \in \mathbb{R}$, let $\psi_t(X) - X_{ss} = (D_t^L - D_{ss}, \pi_t^L - \pi_{ss}, \delta_t^L - \delta_{ss})$ denote the flow of the linearized system towards the steady state. We show that along the transition path, if D_t^L converges to D_{ss} from below, then π_t^L converges to π_{ss} from above and vice versa. To see this, note that

$$\frac{\pi_t^L - \pi_{ss}}{D_t^L - D_{ss}} = \frac{v_{3,1}}{v_{3,2}} = \frac{\eta_3 - \frac{\partial}{\partial D}f_2}{\frac{\partial}{\partial \pi}f_2} = \frac{\eta_3 - \sigma\pi_{ss} + \lambda}{\sigma D_{ss}\pi_{ss}}[\lambda - (\sigma - 1)\pi_{ss}].$$

In the expression above, $\sigma D_{ss}\pi_{ss} > 0$ and $\lambda - (\sigma - 1)\pi_{ss} > 0$ as $\pi_{ss} \in (0, \lambda/\sigma)$. Thus, to conclude that the ratio has a negative sign, we need to show that $\eta_3 - \sigma\pi_{ss} + \lambda < 0$. To see

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that this is indeed the case, note that

$$\eta_3 + \lambda - \sigma\pi_{ss} < 0 \iff \lambda - \sigma\pi_{ss} + \frac{\rho}{2} < \sqrt{\left(\frac{\rho}{2} + \lambda\right)^2 - \sigma(\sigma - 1)\pi_{ss}^2}$$

$$\iff \left(\frac{\rho}{2} + \lambda\right)^2 + \sigma^2\pi_{ss}^2 - (2\lambda + \rho)\sigma\pi_{ss} < \left(\frac{\rho}{2} + \lambda\right)^2 - \sigma(\sigma - 1)\pi_{ss}^2$$

$$\iff 2\sigma\pi_{ss} - 2\lambda - \rho - \pi_{ss} < 0,$$

and the last inequality holds since $\pi_{ss} \in (0, \lambda/\sigma)$. Hence, linearized dynamics are such that

$$\kappa \equiv \frac{\pi_t^L - \pi_{ss}}{D_t^L - D_{ss}} < 0.$$

Finally, let $\phi_t(X) - X_{ss} = (D_t - D_{ss}, \pi_t - \pi_{ss}, \delta_t - \delta_{ss})$ denote the flow of the non-linear system starting from an X on the one-dimensional stable manifold so that $\lim_{t\to\infty} \phi_t(X) = X_{ss}$. Since the stable manifold is tangent to the stable subspace of the linearized system, for sufficiently small $\varepsilon > 0$ such that $\varepsilon + \kappa < 0$, there exists $\bar{t} \geq 0$ such that for all $t > \bar{t}$,

$$\frac{\pi_t - \pi_{ss}}{D_t - D_{ss}} \in (\kappa - \varepsilon, \kappa + \varepsilon) \implies \frac{\pi_t - \pi_{ss}}{D_t - D_{ss}} < 0.$$

Hence, there exists $\bar{t} \geq 0$ such that, after time $\bar{t}$, if D_t of the non-linear system converges to D_{ss} from below, then π_t of the non-linear system converges to π_{ss} from above and vice versa.

To conclude the proof of Proposition 2, consider a change in the parameters of the model that leads to an increase in D_{ss}, as is the case in both parts 1 and 2 of the proposition. First note that since our non-linear system is continuously differentiable, D_t (along with π_t and δ_t) have continuous paths along the transition. Moreover, since D_t is backward-looking, it is also continuous at $t = 0$ (i.e., $\lim_{t\to 0} D_t = D_0$, unlike π_t and δ_t which jump to the stable manifold to accommodate convergence to the steady state). Thus, it has to be that conditional on converging to the new steady state, D_t is a continuous function of time with $D_0 < D_{ss} = \lim_{t\to\infty} D_t$.

If along the transition path D_t never crosses D_{ss}, then $D_t - D_{ss} < 0$ for all t. This means that there exists $\bar{t} \geq 0$ such that $\pi_t - \pi_{ss} > 0$ for all $t > \bar{t}$.

Suppose instead that D_t crosses D_{ss} along the transition path to possibly converge to D_{ss} from above. If this was possible, then there would be two paths for convergence starting from D_{ss}: one that increases and then converges back to D_{ss} from above, and another that starts at D_{ss} and stays at D_{ss} forever. However, in this case, the equilibrium cannot be Markov. Therefore, the only possibility of convergence in a Markov equilibrium is that D_t converges to D_{ss} from below, and thus π_t converges to π_{ss} from above.