FPT-Problem #13

Chaotic magnetic pendulum

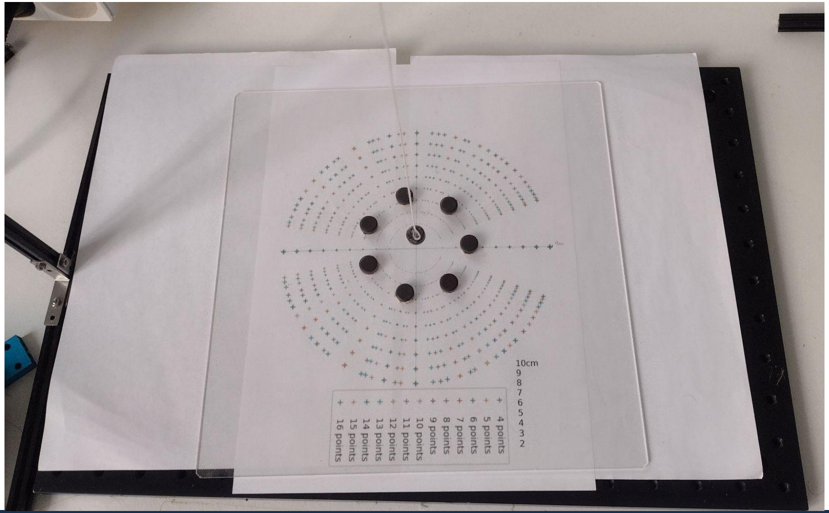
Subject

Consider a pendulum consisting of a magnetic bob attached to a string. If the pendulum is allowed to swing over a structure of permanent magnets, it will display complex motion. Study the pendulum dynamics and its dependence on the number of permanent magnets and their arrangement.

Experiment and goal

A pendulum wire with a magnet at the end
A guide with the different circles and positions of the magnets

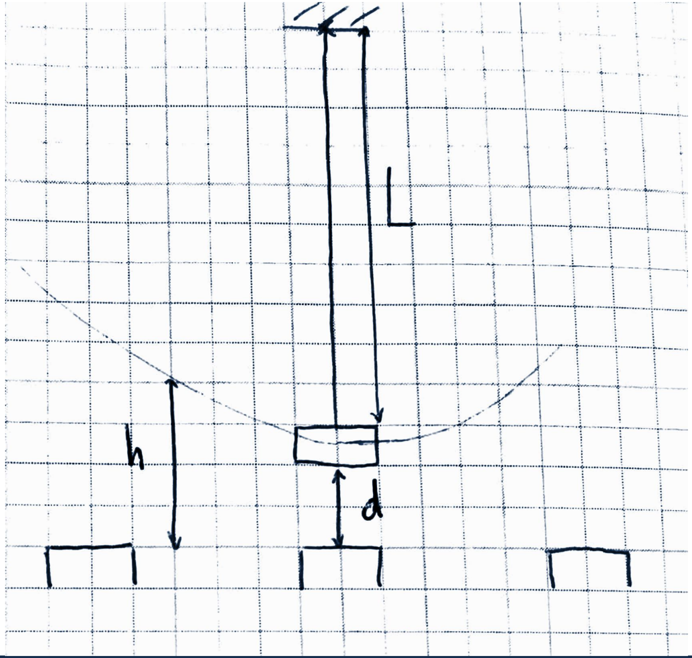
Study:
Dynamic of the pendulum with a circle of magnets



Model the dynamic

$$\ddot{\vec{r}} = -\frac{\dot{r}^2}{(L+d-h)^2}\vec{r} - \frac{g}{L^2}(L+d-h)\vec{r} - \alpha\dot{\vec{r}} + \sum_i \gamma_i \frac{(\vec{r_i} - \vec{r})}{(\sqrt{(x_i - x)^2 + (y_i - y)^2 + h^2}\ )^4}$$

Simple gravity pendulum
Fluid friction
Magnetic dipole-dipole interaction

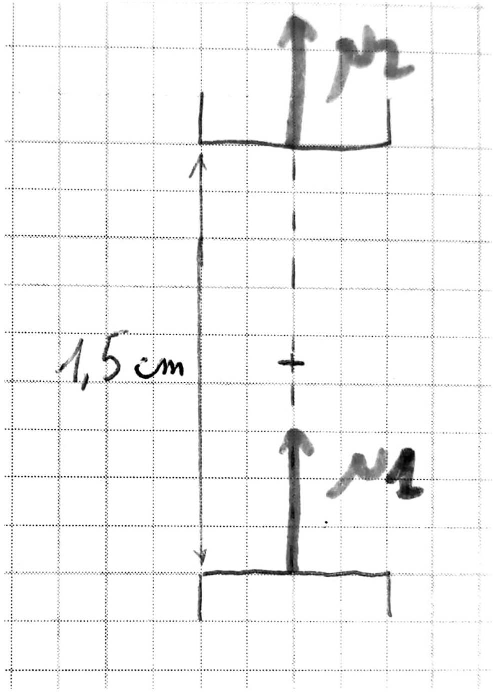


Constants:

Magnetic dipole–dipole interaction
Magnetic field created by a dipole:

$$\vec{B} = (\frac{\mu_0}{4\pi})\frac{3(\vec{\mu}.\vec{u})\vec{u} - \vec{\mu}}{r^3}$$

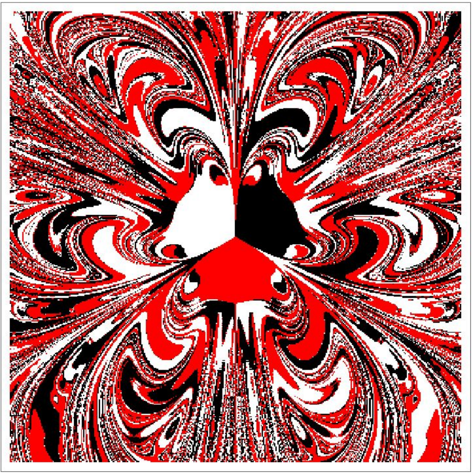
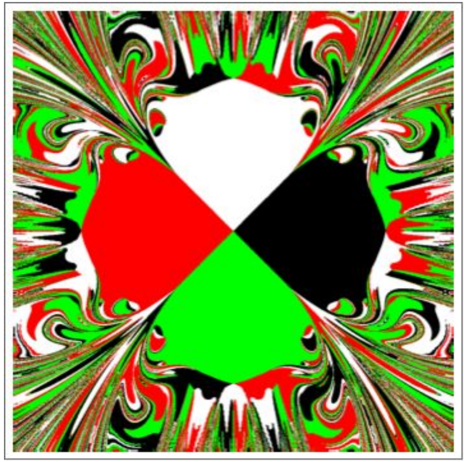
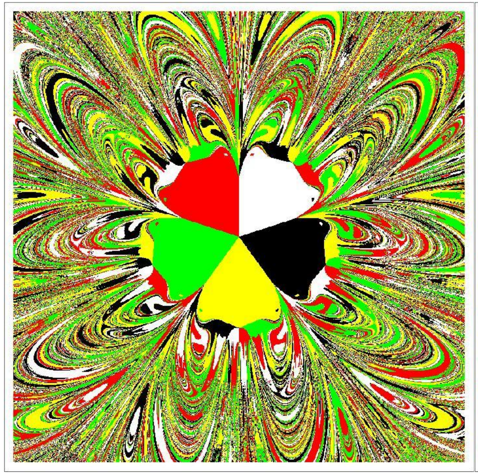
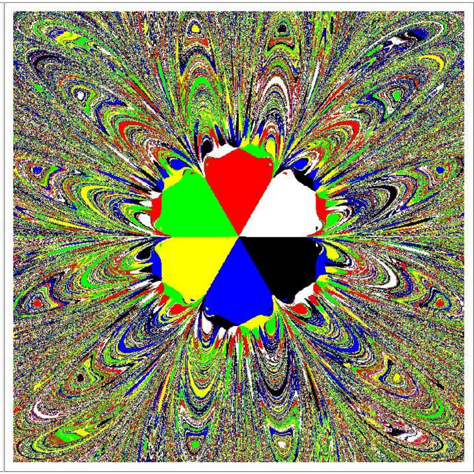
In our case : $$\frac{\mu_0 \mu}{\pi r^3} = B_{middle}$$



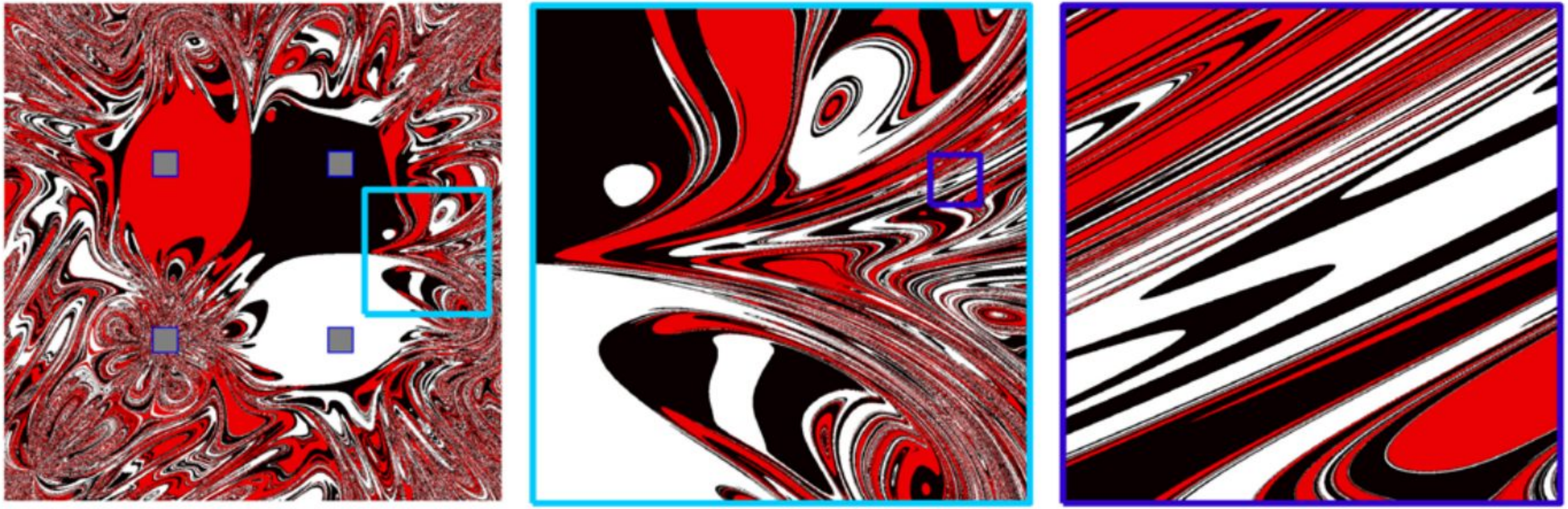
Fluid friction coefficient is given by the logarithm of the ratio between 2 successive maxima for the simple pendulum

$$\alpha = 0.044 s^{-1}, |\gamma_i| = 9.42 10^{-6} m^4 s^{-2}$$

Basins of attraction

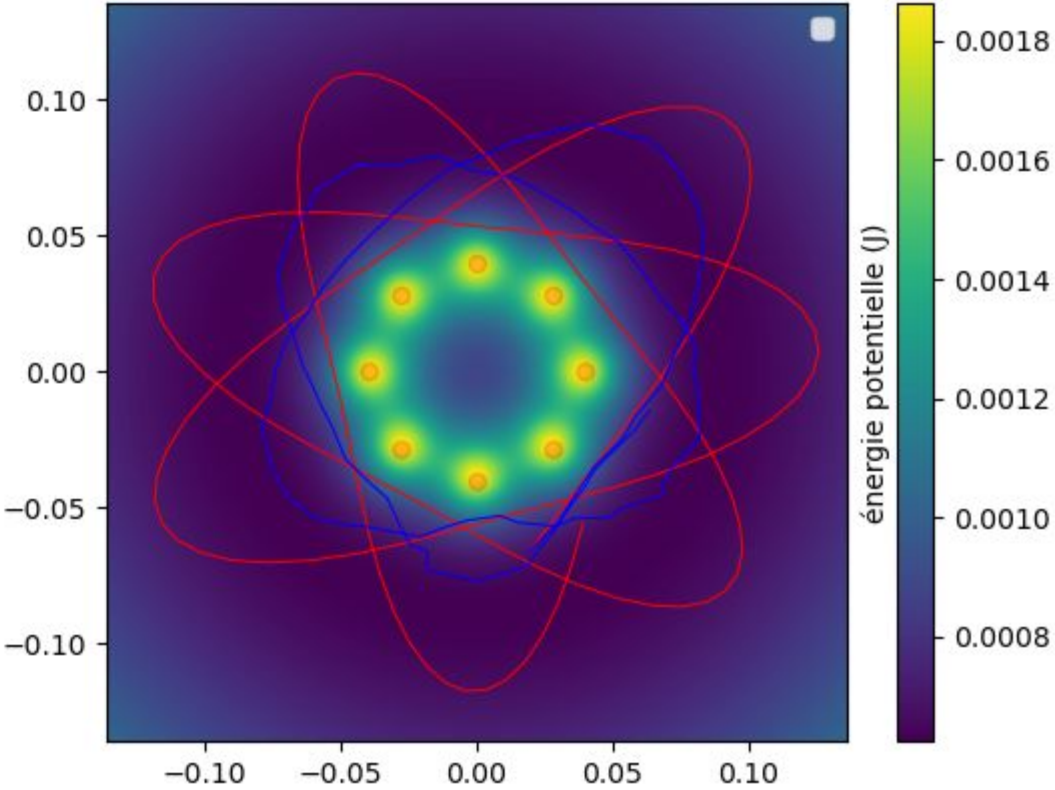
Basins of mixed attraction & repulsion



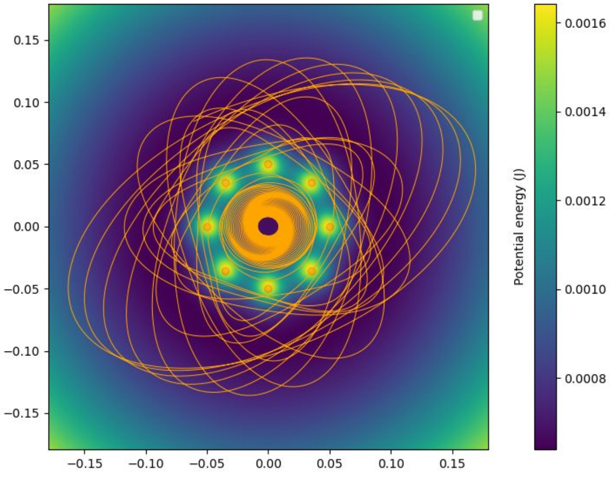
Experiment vs Simulation:

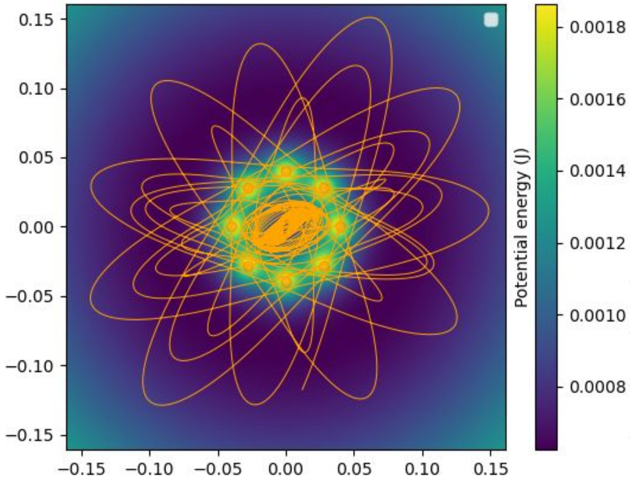
Doesn't fit perfectly
Allow us to model some phenomenon.

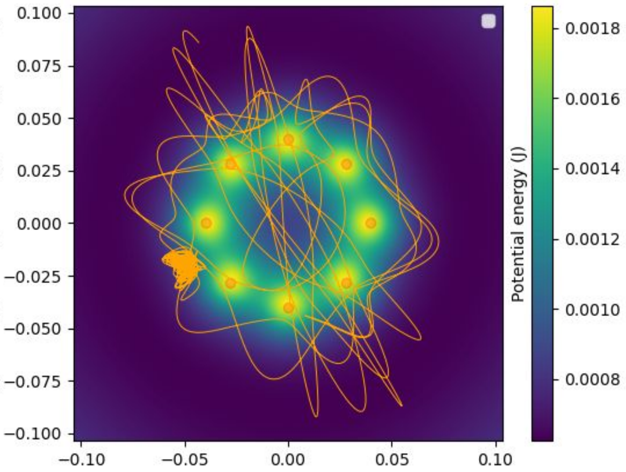
<u>Difficulty</u>:
Finding a good step



First results:





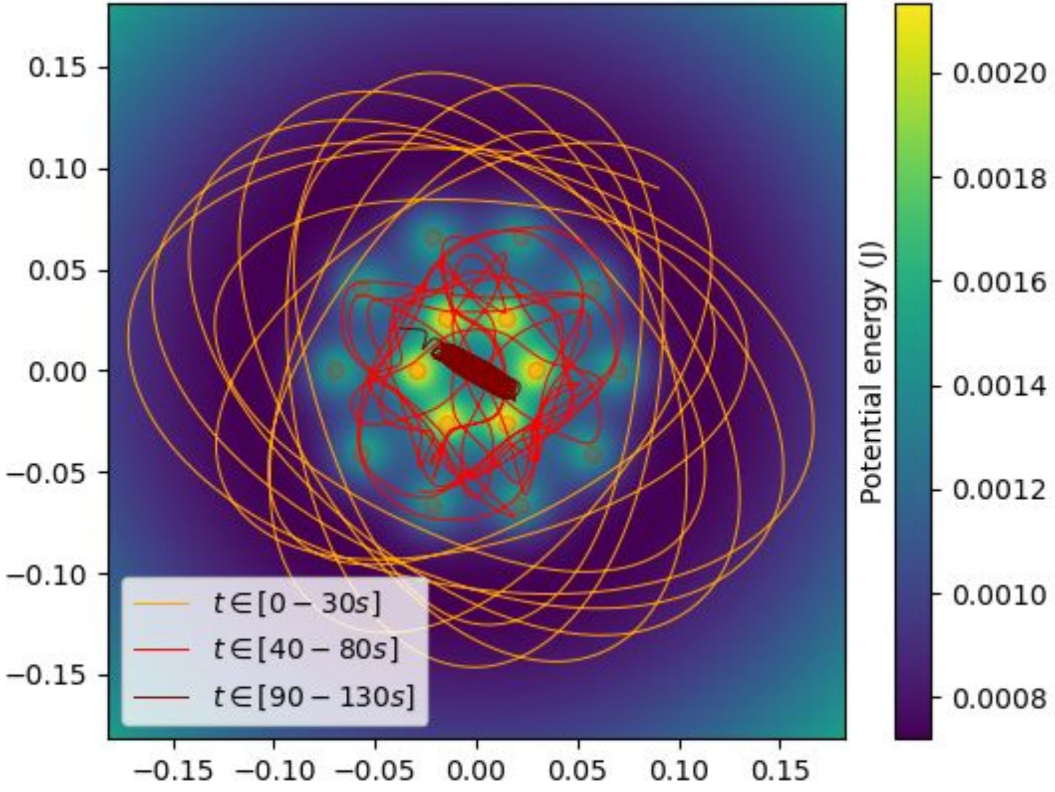


3 kinds of trajectories
2 different kinds of ending

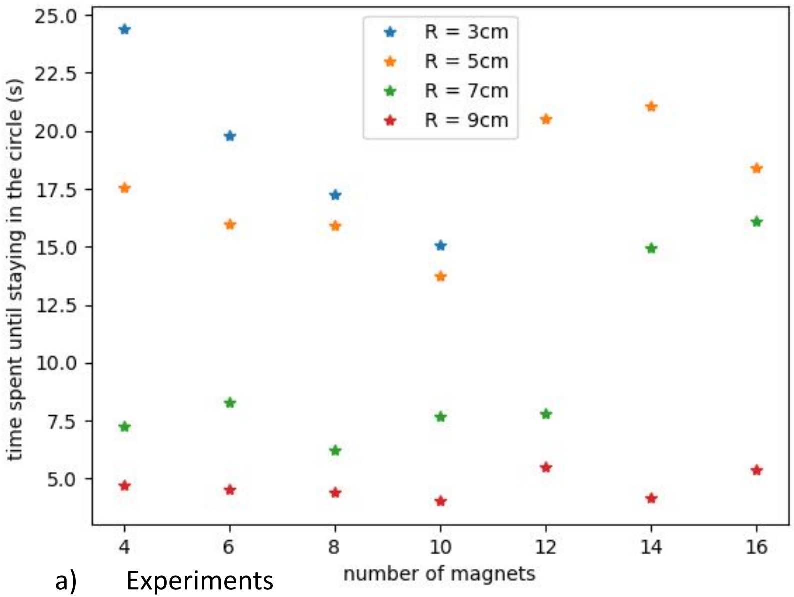
First results:

Set up : Two circles of magnets

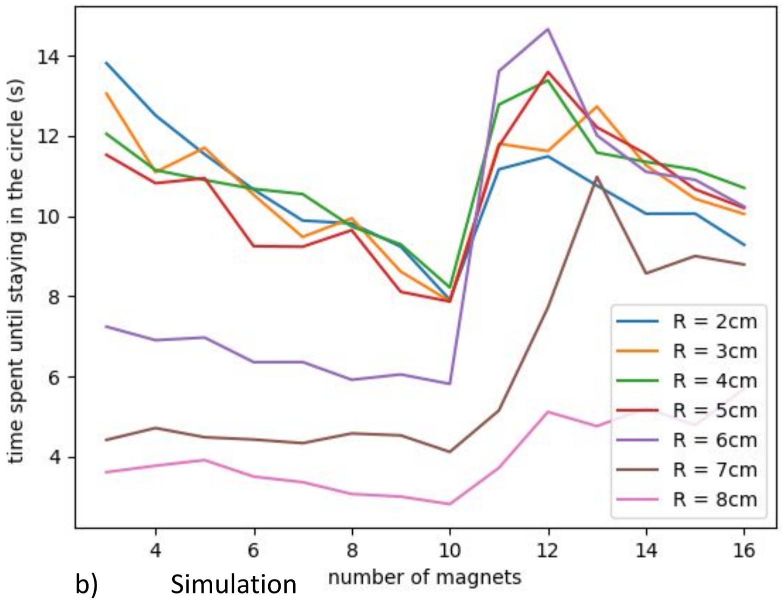
The trajectory is restreint by 3 ways



Changing the number of magnets



a) Experiments



b) Simulation

Big circles depend few of numbers of magnets
Small circles : few magnets pendulum without many magnetic interactions
Many magnets it's hard for the pendulum to be confined du to the potential barrier of the circle
A minimum for 10 magnets

Conclusion / Opening

Model / find constants / Simulate
Find differents kind of trajectories fitting with the experiments
Relations between the trajectories and the the circles (Radius, number of magnets)

We can confined the chaos of the trajectory with geometric patterns of repulsive magnets

New experiments:

- Asymmetrical arrangement
- Arrangement of repulsive and attractive magnets to confined more the pendulum