Performance Analysis of NOMA Systems with Residual Hardware Impairments

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***Abstract*—Non-orthogonal multiple access (NOMA) is a promising technique which is considered for future radio access, e.g., 5th generation of mobile networks (5G). The performance of NOMA systems has not been studied considering the effects of hardware non-idealities. However, hardware impairments (HWIs) are known to limit the performance of communications systems in real-world implementations. In this paper, we analyze the performance of several downlink NOMA systems in presence of hardware impairments. First, we show that the achievable system throughput of opportunistic NOMA is limited in high signal-to-noise ratio (SNR) regime and a ceiling is derived in this regard. Then, outage probabilities of quality-of-service (QoS) based and cognitive radio (CR) NOMA are analyzed and diversity gains are discussed for large values of SNR. Finally, simulations are performed which support the analytical results. It can be seen from the analysis that HWI-aware design of NOMA system shows robustness in presence of hardware impairments.**

***Index Terms*—Non-orthogonal multiple access (NOMA), hardware impairments, non-idealities, ergodic sum rate, outage probability.**

I. Introduction

NOMA is capable of improving the system-level performance of cellular mobile communications by serving more than one user in the same time slot and frequency band. Supporting a heterogeneous range of QoS requirements, massive connectivity, and balancing the trade-off between the system throughput and user fairness are key features of NOMA which are also the requirements arising in 5G systems due to existence of applications such as internet of things (IoT).

The performance of several NOMA systems has been analyzed in literature. In [1], the performance of a multi-user single-carrier NOMA system has been investigated in terms of outage probability and ergodic sum rate with perfect channel state information (CSI), for a cellular downlink scenario with randomly deployed users and fixed power allocation. Authors of [2] have studied a two-user single-carrier system with statistical CSI from an information theoretic perspective, where it was proved that NOMA outperforms native time devision multiple access with high probability in terms of both sum rate and individual rates. In addition, the performance of NOMA based on imperfect CSI and second order statistics is analyzed in [3]. However, to the best knowledge of the authors, the effect of transceiver HWI has not been investigated yet.

In this paper, we study the effects of HWIs on the performance of a single-cell two-user downlink NOMA system. To this end, an additive distortion model is adopted to capture the aggregate effect of residual HWIs. First, an opportunistic NOMA system is considered. There, a ceiling is derived which shows that HWIs limit the system throughput in high-SNR regime. For the second and third scenarios, the outage probabilities of QoS-based and CR systems with predefined data rates are investigated. In particular, closed-form expressions are derived for the outage probabilities of each system for large values of SNR, and diversity gains are discussed. The system-level simulations provide numerical results to compare the performance of NOMA and conventional orthogonal multiple access (OMA), and a good match between the analytical and simulation results is observed.

This paper is organized as follows. The residual HWI model is introduced in Section 2. In Section 3, system models of the three NOMA scenarios are discussed. Performance analysis and simulation results are presented in Section 4 and 5, respectively. Finally, conclusions are drawn in Section 6.

Notation: $(.)^*$ and $\mathrm{E}\{.\}$ denote complex conjugation and expected value, respectively.

II. Residual HWI Model

Transceiver HWIs, e.g., amplifier non-linearities, phase noise, and quantization errors are known to fundamentally limit the performance of communications systems [4]. Here, we adopt a well-established HWI model which captures the aggregate impact of residual HWIs, i.e., HWIs which remain after applying appropriate compensation schemes [5].

The residual HWI can be modeled as an additive distortion with zero-mean Gaussian distribution. The Gaussian distribution of HWI is clearly supported by the central limit theorem, and the accuracy of this model is verified by experimental results in [6], [7]. Accordingly, the effect of HWIs on the received signal can be described as

$$r = h(s + \eta_t) + \eta_r + z, \tag{1}$$

where $\eta_t \sim \mathcal{CN}(0, \sigma_t^2)$ and $\eta_r \sim \mathcal{CN}(0, \sigma_r^2)$ represent HWIs at the transmitter and the receiver, respectively. z stands for noise, s is the information symbol, and h indicates the channel.

Here, it is assumed that η_t and η_r are independent of signal s, but depend on the channel h and as a result, they are stationary within each coherence period. The practicality of this assumption is theoretically motivated by Bussgang theorem [8]. Therefore, assuming $\mathrm{E}\{ss^*\} = 1$, the HWI variances are [4]

$$\sigma_t^2 = \kappa_t \mathrm{E}\{ss^*\} = \kappa_t, \tag{2}$$

$$\sigma_r^2 = \kappa_r \mathrm{E}\{ss^*\}|h|^2 = \kappa_r |h|^2, \tag{3}$$

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where κ_t and κ_r are HWI parameters of the transmitter and the receiver, respectively. These parameters are determined by the quality of the radio frequency modules. Based on 3rd generation partnership project long-term evolution (3GPP LTE) standard requirement, we assume that $\kappa_t,\ \kappa_r \in [0.0064, 0.0306]$.

III. System Model

In NOMA, several users share the same subband and time slot, and their signals are separated in power domain by performing successive interference cancellation (SIC) at the receivers. Unlike conventional power allocation methods, more power is allocated to the weaker users. Hence, the signal of the weakest user is detected at all receivers and the result is subtracted from the received signal. Here, we consider a single-cell single-carrier downlink scenario where the base station (BS) is located at the center of a disc with radius R, and the locations of users are distributed uniformly. The available bandwidth is 1 Hz, P is the total available power at the BS and s_j is the signal of the jth user with transmission power $a_j P$, where a_j is the power coefficient of user j. The transmitted signal at the BS is given by

$$x_{\mathrm{BS}} = \sum_{j=1}^{M} \sqrt{a_j P} s_j, \tag{4}$$

where M is the number of users utilizing the available bandwidth. In this paper, two randomly selected users perform NOMA since designing NOMA systems with M > 2 imposes tremendous computational complexity. On the other hand, a two-user case is capable of realizing NOMA gains. Hence, the received signal at the kth user is

$$r_k = h_k\Big(\sum_{j=1}^{2} \sqrt{P a_j} s_j + \eta_t\Big) + \eta_{r,k} + z_k, \quad k \in \{1, 2\}, \tag{5}$$

where $z_k \sim \mathcal{CN}(0, \sigma_z^2)$ and $s_j \sim \mathcal{CN}(0, 1)$ are the additive white Gaussian noise and the signal of jth user, respectively. $\eta_t \sim \mathcal{CN}(0, \kappa_t P)$ and $\eta_{r,k} \sim \mathcal{CN}(0, \kappa_{r,k} P |h_k|^2)$ denote residual HWIs at the BS and the kth user, respectively. In addition, $h_k = g_k/\sqrt{1 + d_k^{\alpha}}$ is the channel gain of the kth user where $g_k \sim \mathcal{CN}(0, 1)$ represents a Rayleigh fading coefficient and the denominator denotes the path loss with d_k being the distance between the BS and the kth user, and α being the path loss coefficient. We further assume that, $P = 1$ and $a_1 + a_2 = 1$.

The optimal SIC order is in the order of the increasing channel gain normalized by the noise power. However, taking the effect of HWIs into account influences the optimal decoding order. Since HWIs are modeled as additive distortions with Gaussian distributions, similar to noise, the optimal SIC order becomes in the order of these increasing gains $x_k = |h_k|^2/\big(1 + \rho|h_k|^2(\kappa_t + \kappa_{r,k})\big)$ where $\rho = P/\sigma_z^2$. For the rest of this paper, it is assumed that user 1 comes first in the decoding order. Perfect knowledge of CSI and HWI parameters at the BS, and perfect knowledge of decoding order and CSI at the users are also assumed.

In order to evaluate the performance of NOMA in presence of HWIs, the three following cases are considered.

A. Opportunistic NOMA

In this case, a fixed power allocation with $a_1 = 0.8$ and $a_2 = 0.2$ is adopted. Users' achievable data rates R_k are determined opportunistically according to the channel conditions. Therefore, no outage occurs and system performance is described in terms of ergodic sum rate.

B. QoS-based NOMA

Here, target data rates $\tilde{R}_k$ are determined for users based on their QoS requirements and the same fixed power allocation is adopted. In this case, sum rate is simply sum of the target data rates so, we investigate the outage probability.

C. CR NOMA

In CR NOMA, a primary user with poor channel condition has a high priority to be served based on a QoS requirement or equivalently, a specific target data rate $\tilde{R}_1$ [9]. Several conditions, e.g., improving fairness may cause this high priority. In this case, a strong user, which is referred to as a cognitive user, can be served in the same frequency band to enhance the spectral efficiency. The performance of CR NOMA is evaluated in terms of its outage probability.

Here, we define $\kappa_k = \kappa_t + \kappa_{r,k}$. To satisfy the condition on the target data rate of the primary user ($R_1 = \tilde{R}_1$), we have

$$\tilde{a}_1 = \frac{\rho y_1 \phi_1 (1 + \kappa_1) + \phi_1}{\rho y_1 (1 + \phi_1)}, \tag{6}$$

where $y_1 = |h_1|^2$ and $\phi_1 = 2^{\tilde{R}_1} - 1$. Therefore, $a_1 = \min\{\tilde{a}_1, 1\}$. The rest of the power of the BS can be used by the stronger user according to $a_2 = \max\{0, \tilde{a}_2\}$ where $\tilde{a}_2 = 1 - \tilde{a}_1$.

IV. Performance Analysis

A. Opportunistic NOMA

The ergodic sum rate of opportunistic NOMA is as follows

$$R_{\mathrm{avg}} = \int_0^{\infty} \sum_{k=1}^{2} \log_2\Big(1 + \frac{\rho a_k y}{1 + \rho y (\sum_{j<k} a_j + \kappa_t + \kappa_{r,k})}\Big) f_y(Y) \mathrm{d}y, \tag{7}$$

where $f_y(Y)$ stands for the probability density function (PDF) of $|h_k|^2$. Note that the achievable rate of user k is $R_k = \log_2\Big(1 + \frac{a_k |h_k|^2}{\sigma_z^2 + |h_k|^2 (\sum_{j<k} a_j + \kappa_t + \kappa_{r,k})}\Big)$, conditioned on $R_{1\to 2} \geq R_1$, where $R_{1\to 2} = \log_2\Big(1 + (\rho |h_2|^2 a_1)/\big(1 + \rho |h_2|^2 (a_2 + \kappa_t + \kappa_{r,2})\big)\Big)$ denotes the achievable rate for user 1 at user 2. It can be shown that this condition always holds if the SIC order is obtained based on x_k.

When $\rho \longrightarrow \infty$, by using L' Hopital's rule, we can write

$$\tilde{R}_{\mathrm{avg,NOMA}} = \sum_{k=1}^{2} \log_2\Big(1 + \frac{a_k}{\sum_{j<k} a_j + \kappa_t + \kappa_{r,k}}\Big). \tag{8}$$

It can be seen that the performance of NOMA is saturated by the ceiling of (8) when the effect of HWIs is considered. Higher values of κ_t and $\kappa_{r,k}$ result in a lower ceiling on the sum rate at high-SNR values.

The ergodic sum rate of OMA at high-SNR regime can be similarly derived.

B. QoS-based NOMA

In this section, two unordered users with $\kappa_{r,(1)}$ and $\kappa_{r,(2)}$ are considered. After computing users' gains, $x_{(k)} = \frac{|h_{(k)}|^2}{1+\rho(\kappa_t+\kappa_{r,(k)})|h_{(k)}|^2}$, for determining the SIC order, HWI parameter of the receiver of the weaker user is shown by $\kappa_{r,1}$ which can be equal to either $\kappa_{r,(1)}$ or $\kappa_{r,(2)}$. Here, the outage events are defined as $E_{k,j} = \{R_{j\to k} < \tilde{R}_j\}$; $k \in \{1,2\}, j \le k$. Therefore, the outage probability at the kth user can be expressed as

$$\mathrm{P}_k^{\text{out}} = 1 - \mathrm{P}\{\cap_{j=1}^{k} E_{k,j}^c\}, \tag{9}$$

where $E_{k,j}^c$ is the complementary set of $E_{k,j}$, given by

$$E_{k,j}^c \overset{\text{(a)}}{=} \{x_k \ge \frac{\phi_j}{\rho(a_j - \phi_j \sum_{i>j} a_i)}\}, \tag{10}$$

where $\phi_j = 2^{\tilde{R}_j} - 1$. Step (a) is achieved by assuming $a_j > \phi_j \sum_{i>j} a_i$. This condition reduces to $a_1 > \phi_1 a_2$ for $M = 2$, and it is equivalent to $\tilde{R}_1 \le 2.32$ bit per channel use for the fixed power allocation that we adopt.

By defining $\psi_j \triangleq \frac{\phi_j}{\rho(a_j - \phi_j \sum_{i>j} a_i)}$ and $\check{\psi}_k \triangleq \max\{\psi_1, \cdots \psi_k\}$, and considering that $x_{(k)}$ are independent random variables, the outage probabilities can be stated as

$$\begin{aligned}\mathrm{P}_1^{\text{out}} &= 1 - \mathrm{P}\{x_1 \ge \check{\psi}_1\}\\ &= 1 - \mathrm{P}\{x_{(1)} \ge \check{\psi}_1, x_{(2)} \ge \check{\psi}_1\}\\ &= 1 - \big(1 - F_{x_{(1)}}(\check{\psi}_1)\big)\big(1 - F_{x_{(2)}}(\check{\psi}_1)\big),\end{aligned} \tag{11}$$

$$P_2^{\text{out}} = F_{x_{(1)}}(\check{\psi}_2) \times F_{x_{(2)}}(\check{\psi}_2), \tag{12}$$

where $F_{x_{(k)}}(X)$ stands for cumulative distribution function (CDF) of unordered gains $x_{(k)}$, and we have

$$F_{x_{(k)}}(X) = F_{y_{(k)}}(g(X)), \tag{13}$$

where $y_{(k)} = |h_{(k)}|^2$, $g(X) = \frac{X}{1-c_{(k)}X}$ and $c_{(k)} = \rho(\kappa_t + \kappa_{r,(k)})$. Furthermore, $y_{(1)}$ and $y_{(2)}$ are independent random variables with the same distribution. Therefore, we drop the user index. The CDF of channel gains is given by [1]:

$$F_y(Y) = \frac{2}{R^2}\int_0^R (1 - e^{-Y(1+z^\alpha)}) z \mathrm{d}z. \tag{14}$$

The elementary function of this integral can be approximated using Gaussian-Chebyshev quadrature and when $\rho \to \infty$,

$$F_y(Y) \approx \frac{1}{R}\sum_{n=1}^{N} \beta_n Y, \tag{15}$$

where $\beta_n = w_n\sqrt{1-\theta_n^2}(\frac{R}{2}\theta_n + \frac{R}{2})c_n$, $w_n = \frac{\pi}{N}$, $\theta_n = \cos(\frac{2n-1}{2N}\pi)$ and $c_n = 1 + (\frac{R}{2}\theta_n + \frac{R}{2})^\alpha$. Therefore,

$$\begin{aligned}P_1^{\text{out}} \approx &(\frac{1}{R}\sum_{n=1}^{N}\beta_n)^2(\frac{\check{\psi}_1}{1-c_{(1)}\check{\psi}_1} \times \frac{\check{\psi}_1}{1-c_{(2)}\check{\psi}_1})\\ &+ \frac{1}{R}\sum_{n=1}^{N}\beta_n(\frac{\check{\psi}_1}{1-c_{(1)}\check{\psi}_1} + \frac{\check{\psi}_1}{1-c_{(2)}\check{\psi}_1}),\end{aligned} \tag{16}$$

$$\begin{aligned}P_2^{out} \approx &\frac{1}{(\alpha+2)^2}\big[-2R^\alpha(\frac{\check{\psi}_2}{1-c_{(1)}\check{\psi}_2})^2 + (2R^\alpha + \alpha + 2)(\frac{\check{\psi}_2}{1-c_{(1)}\check{\psi}_2})\big]\\ &\times\big[-2R^\alpha(\frac{\check{\psi}_2}{1-c_{(2)}\check{\psi}_2})^2 + (2R^\alpha + \alpha + 2)(\frac{\check{\psi}_2}{1-c_{(2)}\check{\psi}_2})\big].\end{aligned} \tag{17}$$

Outage probabilities of (16) and (17) can be written as $P_1^{\text{out}} \approx \sum_{i=1}^{4}\eta_i\rho^{-i}$ and $P_2^{\text{out}} \approx \sum_{i=2}^{4}\eta_i\rho^{-i}$; $\eta_1 \ne 0$ and $\eta_2 \ne 0$. Therefore, in NOMA systems with HWIs, the diversity gain of user k is equal to k which is similar for NOMA systems adopting ideal hardware [1]. Consequently, QOS-based NOMA is showing reliability in presence of HWIs.

C. CR NOMA

The outage probability of the cognitive radio user is

$$P_2^{\text{out}} = \mathrm{P}\{\tilde{a}_2 \le 0\} + \mathrm{P}\{\tilde{a}_2 > 0, R_2 < \tilde{R}_2\}, \tag{18}$$

where $y_2 = |h_2|^2$ and $\phi_2 = 2^{\tilde{R}_2} - 1$. Here, it is assumed that the target data rate of the primary user satisfies $1 - \kappa_1\phi_1 > 0$ for all values of κ_1 because otherwise, the outage probability of user 2 always approaches 1 since $\tilde{a}_2 < 0$. Note that the decoding order imposes more constraints on (18). Clearly, $\tilde{a}_2 \le 0 \iff y_1 \le \frac{\phi_1}{\rho(1-\kappa_1\phi_1)} \triangleq \epsilon_1$, and $R_2 < \tilde{R}_2 \underset{\rho\to\infty}{\iff} by_1 < a$, where $a \triangleq \frac{\phi_1}{\rho(1+\phi_1)}$ and $b \triangleq \frac{1-\kappa_1\phi_1}{1+\phi_1} - \kappa_2\phi_2$. Moreover, since we analyze the performance in high-SNR regime, $\kappa_2 < \kappa_1$ must hold based on optimum SIC order condition ($x_1 < x_2$), and $\kappa_1 < \kappa_2$ is irrelevant for our analysis. This implies that for achieving NOMA gains, a user with better HWI parameter should be selected for pairing with the primary user in high-SNR regime. Hence, we consider the following cases.

b<0: The SIC order constraint can be reformulated as $y_2 > \frac{y_1}{1-\rho y_1 \Delta\kappa} \to 1/(\rho\Delta\kappa)$ in high-SNR regime. In addition, since y_1 and y_2 are independent variables, and $by_1 < a$, the outage probability of the cognitive radio user can be written as

$$\begin{aligned}P_2^{\text{out}} =& \mathrm{P}\{y_2 > 1/(\rho\Delta\kappa)\} \times \Big(\mathrm{P}\{y_1 \le \epsilon_1\} + \mathrm{P}\{y_1 > \epsilon_1\}\Big)\\ =& \mathrm{P}\{y_2 > 1/(\rho\Delta\kappa)\}.\end{aligned} \tag{19}$$

When $\rho \to \infty$, (19) reduces to $\mathrm{P}\{y_2 > 0^+\} \to 1$. The reason behind this high outage probability is that $b < 0$ only holds for very large values of target data rates.

b>0: Similarly, outage probability can be derived as

$$\begin{aligned}P_2^{\text{out}} \overset{\text{(b)}}{=}& P\{y_1 \le \epsilon_1, y_2 > 0^+\} + P\{\epsilon_1 < y_1 < a/b, y_2 > 0^+\}\\ \approx& F_{y_1}(a/b),\end{aligned} \tag{20}$$

where step (b) in (20) is due to the fact that $a/b > \epsilon_1$ always holds for $b > 0$. To make the problem tractable, we show that cases with $y_1 < y_2$ have the most contribution in determining $F_{y_1}(a/b)$. Considering $\kappa_2 < \kappa_1$, if $y_1 < y_2$ then $y_2 \in (y_1, \infty)$. On the other hand, if $y_1 > y_2$ then $y_2 \in (0, y_1)$. Considering the length of both intervals, it can be seen that the contribution of the latter cases to the CDF is limited, and as a result assuming $y_1 < y_2$ provides a good approximation of the CDF in (20). To this end, order statistics can be utilized and we have

$$P_2^{\text{out}} \approx 2F_y(a/b) - F_y^2(a/b). \tag{21}$$

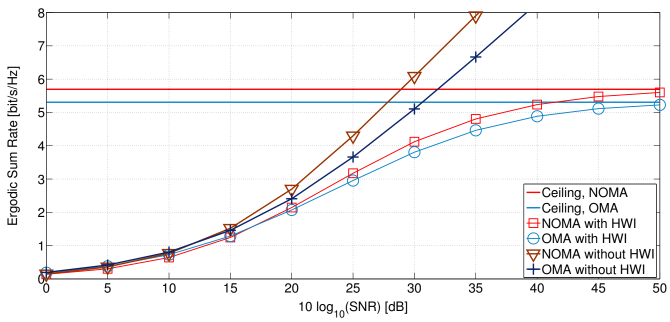

Fig. 1. Ergodic sum rate vs. SNR for $\kappa_t = 0.01$, $\kappa_{r,1} = 0.01$ and $\kappa_{r,2} = 0.03$.

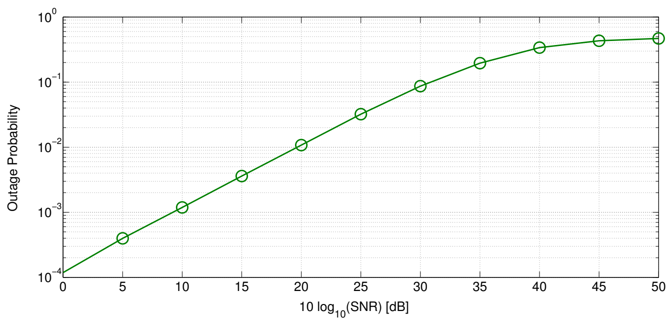

Fig. 2. Outage probability vs. SNR for imperfect hardware and HWI-unaware SIC order ($\frac{|h_k|^2}{\sigma_z^2}$), $\kappa_t = 0.01$, $\kappa_{r,(1)} = 0.03$ and $\kappa_{r,(2)} = 0.01$.

Since a/b includes $1/\rho \longrightarrow 0^+$, this case provides a lower outage probability, and it can be concluded that to achieve the best possible NOMA gains in high-SNR regime, the system parameters should be designed such that $\kappa_1 > \kappa_2$ and $b > 0$.

By using (15), outage probability can be approximated as

$$P_2^{\text{out}} \approx \frac{2a}{Rb}\sum_{n=1}^{N}\beta_n - (\frac{a}{Rb}\sum_{n=1}^{N}\beta_n)^2. \tag{22}$$

It can be seen from (22) that diversity gain of the cognitive user is one, and since diversity gain of cognitive user with ideal hardware is one [9], robustness of NOMA against HWIs is concluded. Note that the performance of the stronger user in CR NOMA depends on the channel condition of the poor user and hence, is limited in comparison with QoS-based NOMA.

V. Simulation Results

The following assumptions are considered in performing the simulations. The path loss coefficient is $\alpha = 3$, cell radius is considered to be $R = 1000$ m, and the distance between the kth user and the BS which is denoted by d_k is between 50 m and 1000 m. Moreover, a path loss of $\beta = 20$ dB at $d_0 = 50$ m is assumed, and so, path loss is obtained as $\frac{\beta}{(\frac{d}{d_0})^\alpha}$. In Gaussian-Chebyshev quadrature approximation, $N = 2$.

A. Opportunistic NOMA

The ergodic sum rate of NOMA and OMA vs. SNR $= \rho = \frac{P}{\sigma_z^2}$ considering ideal and non-ideal hardware is shown in Fig. 1. For OMA, we consider that resources are divided

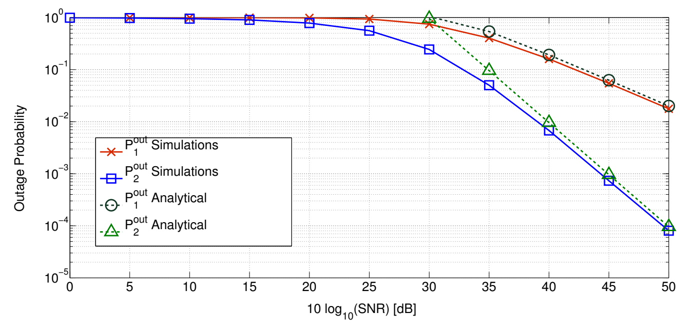

Fig. 3. Outage probability vs. SNR for $\kappa_t = 0.01$, $\kappa_{r,(1)} = 0.01$, $\kappa_{r,(2)} = 0.03$ and $\tilde{R}_1 = \tilde{R}_2 = 2$.

equally among two randomly deployed users. In case of ideal hardware, NOMA outperforms OMA in the high-SNR regime, e.g., a gain of ≈ 1.3 bit/s/Hz in ergodic sum rate is achieved at $\rho = 35$ dB. However, NOMA does not show any improvement for small SNR values.

For imperfect hardware, the SIC order is based on the so far introduced gains x_k and thus, no outage occurs. Based on results of Fig. 1, a degradation of ≈ 2.2 bit/s/Hz and more than 3 bit/s/Hz in sum rate can be observed at $\rho = 35$ dB for OMA and NOMA, respectively. In addition, the terms that we derived as ceilings for the ergodic sum rate in high-SNR regime are depicted in Fig. 1. The ceiling values confirm that NOMA outperforms OMA for high values of SNR. For example, there is a gain of ≈ 0.4 bit/s/Hz for NOMA at $\rho = 40$ dB.

As discussed before, the new criterion on the decoding order prevents the occurrence of $R_{1\to 2} < R_1$ for the systems with imperfect hardware. However, for the conventional decoding order based on $\frac{|h_k|^2}{\sigma_z^2}$, data rates of the users in (7) are achievable in NOMA systems if and only if $\kappa_{r,2} - \kappa_{r,1} \leq \frac{1}{\rho}(\frac{1}{|h_1|^2} - \frac{1}{|h_2|^2})$. Otherwise, NOMA system might be in outage. This outage probability vs. SNR is shown in Fig. 2. As can be seen, this outage reaches one for high values of SNR because when $\rho \longrightarrow \infty$, $\frac{1}{\rho}(\frac{1}{|h_1|^2} - \frac{1}{|h_2|^2}) \longrightarrow 0$. Clearly, these high values of outage probabilities in Fig. 2 are not negligible for a HWI-unaware system design and it is important to modify the conventional SIC order for implementations in practice.

B. QoS-based NOMA

In this part, analytical results of IV-B are verified by numerical simulations. In Fig. 3, $\tilde{R}_1 = \tilde{R}_2 = 2$, and hence, the outage probability of the stronger user is lower than that of the user with poor channel condition for all SNRs. Furthermore, a good match between the analytical and simulation results can be observed. Note that these analytical estimations are derived only for large values of SNR. That is why they do not match the simulation results for low values of SNR.

In Fig. 4, outage probabilities of NOMA and OMA are compared for perfect and imperfect hardware. The degradation caused by imperfect hardware is not significant, e.g., the outage probabilities are increased by $\approx 4 \times 10^{-3}$, 10^{-2} and 6×10^{-5} at $\rho = 50$ dB for the OMA user, the weak and the strong users of NOMA, respectively. Moreover, from Fig. 4

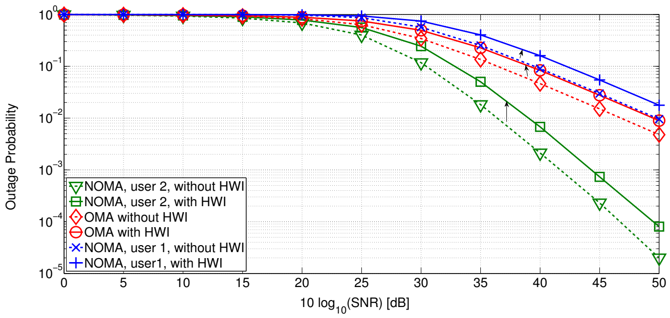

Fig. 4. Outage probability vs. SNR for $\kappa_t = 0.01$, $\kappa_{r,(1)} = 0.01$, $\kappa_{r,(2)} = 0.03$ and $\tilde{R}_1 = \tilde{R}_2 = 2$.

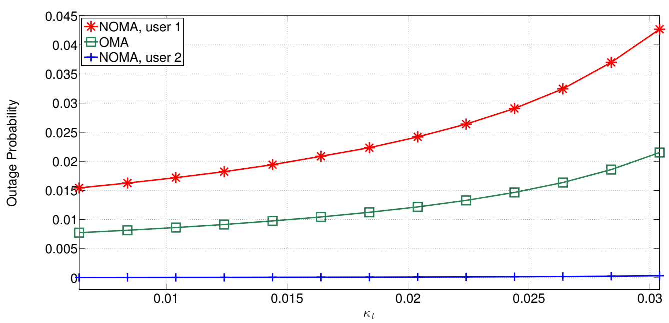

Fig. 5. Outage probability vs. κ_t for SNR=50 dB, $\tilde{R}_1 = \tilde{R}_2 = 2$, $\kappa_{r,(1)} = 0.01$ and $\kappa_{r,(2)} = 0.03$.

and as it was analytically shown, the diversity gains remain unchanged in presence of HWIs which implies the robustness of NOMA against HWIs.

In order to further investigate the impact of HWIs on outage probability, in Fig. 5, P^{out} for NOMA and OMA vs. $\kappa_{r,t}$ is shown at SNR=50 dB. Note that for small values of SNR, outage probability is almost one so, $\rho = 50$ dB is considered here. The results indicate that the performance of the stronger user is significantly more robust with respect to changes in HWI parameters than the weaker NOMA user and OMA user. For example, the difference between the maximum and minimum outage probabilities for OMA, weak and strong NOMA users are ≈ 0.014, 0.028 and 10^{-4}, respectively. It is worth mentioning that the changes in κ_t is more destructive than the same change in $\kappa_{r,(k)}$ (the figure is not included due to limited space).

C. CR NOMA

Fig. 6 shows the outage probability of the cognitive radio user vs SNR. For $b < 0$ and $b > 0$, $\tilde{R}_1 = \tilde{R}_2 = 3$ and $\tilde{R}_1 = \tilde{R}_2 = 2$, respectively. In addition, assuming $\Delta\kappa = \kappa_2 - \kappa_1$, for positive values of $\Delta\kappa$, $\kappa_{r,1} = 0.01$ and $\kappa_{r,2} = 0.03$, and for negative values of $\Delta\kappa$, $\kappa_{r,1} = 0.03$ and $\kappa_{r,2} = 0.01$. As can be seen, the outage probability is always equal to one, for the curves with $b < 0$. The reason is the large target data rates which result in negative values of b as discussed before.

Furthermore, if the system parameters are selected such that $\Delta\kappa > 0$ and $b > 0$, satisfying $x_1 < x_2$ requires very poor channels for the primary user and very strong channels

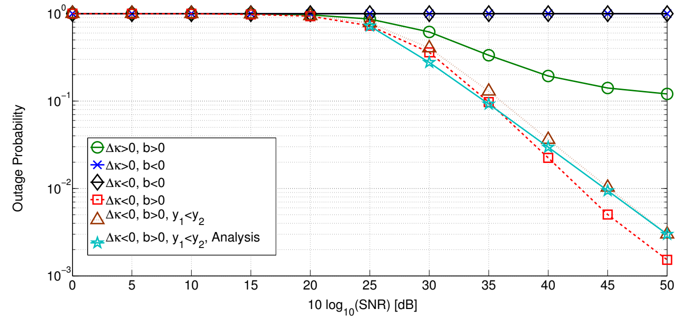

Fig. 6. Outage probability of cognitive radio user vs. SNR.

for the cognitive radio user. Additionally, performance of the cognitive radio user depends on the channel condition and HWI parameter of the primary user. Therefore as expected from analysis, in spite of $b > 0$ the outage probability values are still large even in high-SNR region, e.g., ≈ 0.1 at $\rho = 50$ dB.

To this end, designing the system parameters satisfying $\kappa_1 > \kappa_2$ and $b > 0$ is supposed to provide the best outage performance based on analysis which is also confirmed by the results of Fig. 6. As can be observed, the curve marked with triangles in Fig. 6 shows the outage probability of (21) which is clearly a good approximation of the outage provided in (20) shown by the square marked curve. In addition, the analytical results of (22) perfectly match the simulation results in high-SNR regime. Clearly, the diversity gain of the cognitive radio user is equal to one which emphasizes the robustness of NOMA against HWIs in high-SNR regime.

VI. Conclusion

In this paper, we analyze the the performance of NOMA in presence of HWIs while a HWI-aware design is considered. Particularly, HWI parameters are taken into the account in determining the optimum SIC order. It was shown that ergodic sum rate of the opportunistic NOMA is limited by the derived ceiling in the high-SNR regime. Furthermore, it was concluded from simulations that outages occurring in the opportunistic NOMA for HWI-unaware design are not negligible. For QoS-based NOMA, closed form expressions of outage probabilities were derived, and it was shown that diversity gains of the weak and strong users are one and two, respectively. Moreover, we studied the outage probability of the cognitive user in CR NOMA. It was shown that in order to achieve good outage probabilities, a user with smaller HWI parameter should be selected as the cognitive radio user and target data rates should be selected so that $b > 0$ holds. For such a system design, diversity gain of the strong user in the CR NOMA is one. Consequently, NOMA shows robustness in presence of HWI in a HWI-aware system design.

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