Sum Rate Analysis of Non-Orthogonal Multiple Access Systems with Residual Hardware Impairments

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1. Introduction
2. System Model
3. Performance Analysis of NOMA with Hardware Impairments in the High-SNR Regime
4. Numerical Results
5. Conclusion

Table of Contents

1. Introduction
2. System Model
3. Performance Analysis of NOMA with Hardware Impairments in the High-SNR Regime
4. Numerical Results
5. Conclusion

Non-Orthogonal Multiple Access (NOMA)

Requirements for Future Communications Systems, e.g. 5G

- Higher data rates
- Massive connectivity
- Support of heterogeneous quality-of-service (QoS) requirements
- Low latencies

One Important Dilemma

Trade-off between system throughput and user fairness

A Promising Solution

Non-orthogonal multiple access (NOMA)
→ Separating users' signals in the power domain

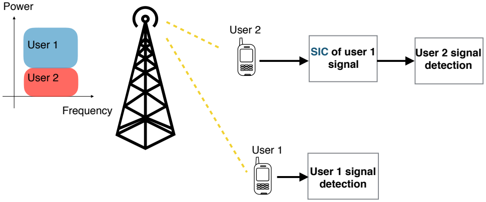
Downlink NOMA

- The received signal at the kth user:

$$y_k = h_k x_{\mathrm{BS}} + z_k = h_k \sum_{j=1}^{K} \sqrt{a_j P} s_j + z_k. \qquad (1)$$

✓ Detection: successive interference cancellation (SIC)

- Power allocation: $|h_1|^2 \leq |h_2|^2 \implies a_1 > a_2$



P: total transmit power at the base station (BS), s_j and a_j: signal and power coefficient of user j, K: number of users, $z_k \sim \mathcal{CN}(0, \sigma_z^2)$: additive white Gaussian noise (AWGN) at the kth user, h_k: channel gain of the kth user.

NOMA Advantages

- + Higher spectral efficiency
- + Supporting larger number of users with diverse QoS requirements
- + Better service for weak users, e.g. the cell-edge users

NOMA Disadvantages

- Higher computational complexity
- Higher inter-cell interference

Motivation

✓ Hardware impairments (HWIs) limit system performance

✓ Existing NOMA system models in literature do not consider HWI

$\rightarrow$ Investigate NOMA performance with HWIs

Table of Contents

1 Introduction

2 System Model

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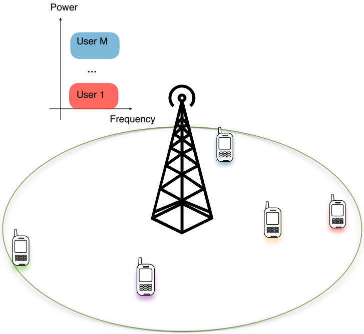
4 Numerical Results

5 Conclusion

System Model

- Single-cell single-carrier downlink scenario
- 2 uniformly distributed users in a cell with radius R
- Total BS transmit power: $P = 1$
- Perfect channel state information (CSI) knowledge at the BS
- Perfect knowledge of the decoding order and power coefficients at the users
- Perfect knowledge of HWI parameters at the BS



Residual Hardware Impairments (HWI) Model

HWI Origins

Nonidealities in real-world implementations, e.g., phase noise, I/Q imbalance, nonlinearities and quantization errors

Residual HWI Model:

- Additive Gaussian distortion, whose variance depends on signal power [1]

→ HWI at the transmitter (BS): $\eta_t \sim \mathcal{CN}(0, \kappa_t P)$

→ HWI at the kth user: $\eta_{r,k} \sim \mathcal{CN}(0, \kappa_{r,k} P |h_k|^2)$

→ κ_t, $\kappa_{r,k}$: transmit and receive HWI parameters

- Stationary within each coherence interval
- Validated in practice [2, 3, 4]

- The received signal at the kth user:

$$y_k = h_k(\sum_{j=1}^{2} \sqrt{a_j} s_j + \eta_t) + \eta_{r,k} + z_k, \qquad (2)$$

$s_j \sim \mathcal{CN}(0,1)$: signal of the jth user
a_j: power coefficient of the jth user
$h_k = \frac{g_k}{\sqrt{1+d_k^{\alpha}}}$: channel gain of the kth user, where
$g_k \sim \mathcal{CN}(0,1)$
d_k: distance between the kth user and the BS, and
α: path-loss coefficient
$z_k \sim \mathcal{CN}(0,\sigma_z^2)$, and SNR is defined as $\rho = \frac{1}{\sigma_z^2}$
Based on 3GPP LTE standard requirement:
$\sqrt{\kappa_t}, \sqrt{\kappa_{r,k}} \in [0.08, 0.175]$ [5]
Optimal SIC order with non-ideal hardware based on:
$|h_1|^2/\left(\sigma_z^2 + |h_1|^2(\kappa_t + \kappa_{r,1})\right) < |h_2|^2/\left(\sigma_z^2 + |h_2|^2(\kappa_t + \kappa_{r,2})\right)$

Table of Contents

1. Introduction
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Fixed Power Allocation NOMA (F-NOMA)

Fixed Power Allocation NOMA

- a_k is independent of channel
- Here: $a_1 = 0.8$ and $a_2 = 0.2$

We consider two types:

1. Opportunistic scenario
   $\rightarrow$ Users' data rates are determined according to the channel conditions $(\tilde{R}_k = R_k)$
   $\rightarrow$ Ergodic sum rate
2. QoS-based scenario with predefined target data rates
   $\rightarrow$ $\tilde{R}_k$ based on users' QoS requirements
   $\rightarrow$ Outage probability

R_k and $\tilde{R}_k$: achievable data rate and target data rate of the kth user, respectively.

Type 1: Opportunistic F-NOMA

✓ Performance metric → $\boxed{\text{Ergodic sum rate}}$

$$R_{\text{avg}}=\int_0^{\infty}\sum_{k=1}^{2}\log_2\left(1+\frac{\rho a_k Y}{1+\rho Y\left(\sum_{j<k}a_j+\kappa_t+\kappa_{r,k}\right)}\right)f_y(Y)\mathrm{d}Y, \quad (3)$$

where $f_y(Y)$: probability density function of unordered channel gains.

Required condition: $R_{1\to 2}\geq R_1$ where $R_{1\to 2}=\log_2\left(1+(\rho|h_2|^2a_1)/\left(1+\rho|h_2|^2(a_2+\kappa_t+\kappa_{r,2})\right)\right)$ is the achievable rate for user 1 at user 2. $R_{1\to 2}\geq R_1$ always holds considering the modified SIC order → No outage.

✓ High-SNR behaviour ($\rho\to\infty$):

$$\tilde{R}_{\text{avg}}=\lim_{\rho\to\infty}R_{\text{avg}}=\underbrace{\sum_{k=1}^{2}\log_2\left(1+\frac{a_k}{\sum_{j<k}a_j+\kappa_t+\kappa_{r,k}}\right)}_{\text{ceiling}}. \quad (4)$$

Type 2: QoS-based F-NOMA

Outage events:

$$R_m < \tilde{R}_m \wedge R_{j\to m} < \tilde{R}_j,\ m \in \{1,2\}, j \leq m, \tag{5}$$

where $R_{j\to m}$ is the achievable rate of user j at user m.

✓ Outage probability for user m:

$$\boxed{P_m^{\text{out}} = 1 - P\{\cap_{j=1}^{m} E_{m,j}^c\}} \tag{6}$$

where $E_{m,j}^c$ is the jth component of the complementary set of the outage events of user m, given by

$$E_{m,j}^c \overset{(a)}{=} \{x_m \geq \frac{\phi_j}{\rho(a_j - \phi_j \sum_{i>j} a_i)}\}, \tag{7}$$

where $\phi_j = 2^{\tilde{R}_j} - 1$ and $x_m \triangleq \frac{|h_m|^2}{1+\rho|h_m|^2(\kappa_t+\kappa_{r,m})}$. Step (a) is achieved by assuming $a_j > \phi_j \sum_{i>j} a_i$ and by applying (5).

Therefore,

$$P_1^{\text{out}} = 1 - \left(1 - F_{x_{(1)}}(\check{\psi}_1)\right)\left(1 - F_{x_{(2)}}(\check{\psi}_1)\right), \tag{8}$$

$$P_2^{\text{out}} = F_{x_{(1)}}(\check{\psi}_2) \times F_{x_{(2)}}(\check{\psi}_2), \tag{9}$$

where subscript (k) represents unordered variables, $\psi_j \triangleq \frac{\phi_j}{\rho(a_j - \phi_j \sum_{i>j} a_i)}$ and $\check{\psi}_m \triangleq \max\{\psi_1, \ldots, \psi_m\}$. $F_{x_{(k)}}(X)$ denotes cumulative distribution function (CDF) of unordered gains.

To derive $F_{x_{(k)}}(X)$:

First, we have $x_{(k)} = \frac{|h_{(k)}|^2}{1+\rho(\kappa_t+\kappa_{r,(k)})|h_{(k)}|^2}$. Therefore,

$$F_{x_{(k)}}(X) = F_{y_{(k)}}(g(X)), \tag{10}$$

where $y_{(k)} = |h_{(k)}|^2$, $g(X) = \frac{X}{1-c_{(k)}X}$ and $c_{(k)} = \rho(\kappa_t + \kappa_{r,(k)})$. $y_{(1)}$ and $y_{(2)}$ are independent random variables with the same distribution. Therefore, we drop the user index.

Second, from [6]:

$$F_y(Y) = \frac{2}{R^2} \int_0^R (1 - e^{-Y(1+z^{\alpha})}) z \mathrm{d}z. \tag{11}$$

✓ Two approaches to solve (11) in the high-SNR regime →

Approach 1: Power Series

When $\rho \to \infty$: $Y \to 0$
By substituting $e^{-Y} \approx 1 - Y$ and $e^{-Yz^{\alpha}} \approx 1 - Yz^{\alpha}$ in (11):

$$F_y(Y) \approx \frac{1}{\alpha+2}\Big[-2R^{\alpha}Y^2 + (2R^{\alpha}+\alpha+2)Y\Big]. \tag{12}$$

Using (12) and (10):

$$\begin{aligned} P_1^{\text{out}} \approx & 1 - \Bigg(\frac{2R^{\alpha}(\frac{\check{\psi}_1}{1-c_{(1)}\check{\psi}_1})^2-(2R^{\alpha}+\alpha+2)(\frac{\check{\psi}_1}{1-c_{(1)}\check{\psi}_1})+\alpha+2}{(\alpha+2)}\Bigg) \\ & \times\Bigg(\frac{2R^{\alpha}(\frac{\check{\psi}_1}{1-c_{(2)}\check{\psi}_1})^2-(2R^{\alpha}+\alpha+2)(\frac{\check{\psi}_1}{1-c_{(2)}\check{\psi}_1})+\alpha+2}{(\alpha+2)}\Bigg), \end{aligned} \tag{13}$$

$$\begin{aligned} P_2^{\text{out}} \approx & \frac{1}{(\alpha+2)^2}\Big[-2R^{\alpha}(\frac{\check{\psi}_2}{1-c_{(1)}\check{\psi}_2})^2+(2R^{\alpha}+\alpha+2)(\frac{\check{\psi}_2}{1-c_{(1)}\check{\psi}_2})\Big] \\ & \times\Big[-2R^{\alpha}(\frac{\check{\psi}_2}{1-c_{(2)}\check{\psi}_2})^2+(2R^{\alpha}+\alpha+2)(\frac{\check{\psi}_2}{1-c_{(2)}\check{\psi}_2})\Big]. \end{aligned} \tag{14}$$

Approach 2: Gaussian-Chebyshev Quadrature

Using Gaussian-Chebyshev Quadrature technique, $F_y(Y)$ can be approximated as [6]

$$F_y(Y) \approx \frac{1}{R}\sum_{n=1}^{N} \beta_n Y, \tag{15}$$

where $\beta_n = w_n\sqrt{1-\theta_n^2}(\frac{R}{2}\theta_n + \frac{R}{2})c_n$, $w_n = \frac{\pi}{N}$, $\theta_n = \cos(\frac{2n-1}{2N}\pi)$ and $c_n = 1 + (\frac{R}{2}\theta_n + \frac{R}{2})^{\alpha}$.

Using (15) and (10):

$$\begin{aligned} P_1^{\text{out}} \approx & (\frac{1}{R}\sum_{n=1}^{N}\beta_n)^2(\frac{\check{\psi}_1}{1-c_{(1)}\check{\psi}_1} \times \frac{\check{\psi}_1}{1-c_{(2)}\check{\psi}_1}) \\ & + \frac{1}{R}\sum_{n=1}^{N}\beta_n(\frac{\check{\psi}_1}{1-c_{(1)}\check{\psi}_1} + \frac{\check{\psi}_1}{1-c_{(2)}\check{\psi}_1}), \end{aligned} \tag{16}$$

$$P_2^{\text{out}} \approx (\frac{1}{R}\sum_{n=1}^{N}\beta_n\frac{\check{\psi}_2}{1-c_{(1)}\check{\psi}_2})(\frac{1}{R}\sum_{n=1}^{N}\beta_n\frac{\check{\psi}_2}{1-c_{(2)}\check{\psi}_2}). \tag{17}$$

Outage probabilities of (13) and (14) can be written as:
$P_1^{\text{out}} \approx \sum_{i=1}^{4} \eta_i \rho^{-i}$ and $P_2^{\text{out}} \approx \sum_{i=2}^{4} \eta_i \rho^{-i}$; $\eta_1 \neq 0$ and $\eta_2 \neq 0$.

❖ Therefore, in NOMA systems with HWIs, the diversity gain of user m is equal to m.

✓ For NOMA with ideal hardware, diversity gain of user m is equal to m [6]

$\Rightarrow$ Reliability of opportunistic F-NOMA w.r.t. HWI effects

Cognitive Radio NOMA (CR-NOMA)

✓ A primary user with high priority, poor channel, and given QoS requirement has to be served
⇒ the power coefficient a_1 is determined.
✓ The remaining power is allocated to the cognitive user

To satisfy $R_1 \geq \tilde{R}_1$:

$$\tilde{a}_1 = \frac{\rho y_1 \phi_1 (1 + \kappa_1) + \phi_1}{\rho y_1 (1 + \phi_1)}, \tag{18}$$

where $y_1 = |h_1|^2$, $\kappa_1 = \kappa_t + \kappa_{r,1}$ and $\phi_1 = 2^{\tilde{R}_1} - 1$. Therefore,

$$a_1 = \min\{\tilde{a}_1, 1\}, \tag{19}$$

$$a_2 = \max\{0, \underbrace{\frac{\rho y_1 (1 - \kappa_1 \phi_1) - \phi_1}{\rho y_1 (1 + \phi_1)}}_{=\tilde{a}_2 = 1 - \tilde{a}_1}\}. \tag{20}$$

✓ Assumption: $1 - \kappa_1 \phi_1 > 0$; $\forall \kappa_1 \in [\kappa_{\min}, \kappa_{\max}]$.
✓ Dynamic power allocation

✓ Outage probability of cognitive radio user:

$$P_2^{\text{out}} = \underbrace{P\{\tilde{a}_2 \leq 0\}}_{Q_1} + \underbrace{P\{\tilde{a}_2 > 0 \wedge R_2 < \tilde{R}_2\}}_{Q2}, \qquad (21)$$

where $y_2 = |h_2|^2$ and $\kappa_2 = \kappa_t + \kappa_{r,2}$.

Q_1:
$$\tilde{a}_2 \leq 0 \iff y_1 \leq \frac{\phi_1}{\rho(1-\kappa_1\phi_1)} \triangleq \epsilon_1. \qquad (22)$$

Q_2:
$$R_2 < \tilde{R}_2 \underset{\rho\to\infty}{\iff} by_1 < a, \qquad (23)$$

where $a \triangleq \frac{\phi_1}{\rho(1+\phi_1)}$, $b \triangleq \frac{1-\kappa_1\phi_1}{1+\phi_1} - \kappa_2\phi_2$ and $\phi_2 = 2^{\tilde{R}_2} - 1$.

For optimum SIC order : $\underbrace{\frac{y_1}{1+\rho y_1\kappa_1}}_{x_1} < \underbrace{\frac{y_2}{1+\rho y_2\kappa_2}}_{x_2}$

✓ Based on SIC order assumption: $x_1 < x_2 \underset{\rho\to\infty}{\Rightarrow} \kappa_2 < \kappa_1$,
or equivalently $\Delta\kappa \triangleq \kappa_2 - \kappa_1 < 0$

$\Rightarrow \Delta\kappa > 0$ is irrelevant and we consider these cases:

1. $b < 0$
2. $b > 0$

Case 1: $b < 0$:

$x_1 < x_2$ can be reformulated as $y_2 > \frac{y_1}{1-\rho y_1 \Delta\kappa} \underset{\rho\to\infty}{\to} 1/(\rho|\Delta\kappa|)$.

$$\begin{aligned} P_2^{\text{out}} =& P\{y_2 > 1/(\rho|\Delta\kappa|)\} \times \Big(P\{y_1 \le \epsilon_1\} + P\{y_1 > \epsilon_1\}\Big) \\ =& P\{y_2 > 0^+\} \to 1 \end{aligned} \tag{24}$$

✓ Reason: $b < 0$ is valid only for large values of target data rates

Case 2: $b > 0$

$$\begin{aligned} P_2^{\text{out}} \overset{\text{(b)}}{=}& P\{y_1 \le \epsilon_1, y_2 > 0^+\} + P\{\epsilon_1 < y_1 < a/b, y_2 > 0^+\} \\ \approx& F_{y_1}(a/b). \end{aligned} \tag{25}$$

Step (b) is obtained since $a/b > \epsilon_1$ for $b > 0$. To make the problem tractable, we assume $y_1 < y_2$. Therefore, order statistics can be utilized:

$$P_2^{\text{out}} \approx 2F_y(a/b) - F_y^2(a/b). \tag{26}$$

Since a/b includes $1/\rho \to 0^+ \Rightarrow$ Case 2 provides a lower outage probability.

❖ To achieve the best possible NOMA gains:
It is necessary to design the system parameters such that
$\Delta\kappa < 0$ and $b < 0$

Using (15), we have

$$P_2^{\text{out}} \approx \frac{2a}{Rb}\sum_{n=1}^{N}\beta_n - \left(\frac{a}{Rb}\sum_{n=1}^{N}\beta_n\right)^2. \tag{27}$$

- Diversity gain of the cognitive user is one
- The performance of the stronger user depends on the channel condition of the poor user and is limited
- Diversity gain of cognitive user with ideal hardware is one [7]
  $\Rightarrow$ Robustness of NOMA against HWI

Table of Contents

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5. Conclusion

Numerical Results

Simulation parameters:

- Path-loss coefficient $\alpha = 3$
- Cell radius $R = 1000$ m
- User distance d_k is between 50 m and 1000 m
- A path-loss of $\beta = 20$ dB at $d_0 = 50$ m is assumed.
- For Gaussian-Chebyshev quadrature approach: $N = 2$

Conventional orthogonal multiple access (OMA):

- Random scheduling
- Resources are divided equally among two users.
- Ceiling of opportunistic OMA can be achieved by following the same steps as with NOMA.

Opportunistic F-NOMA

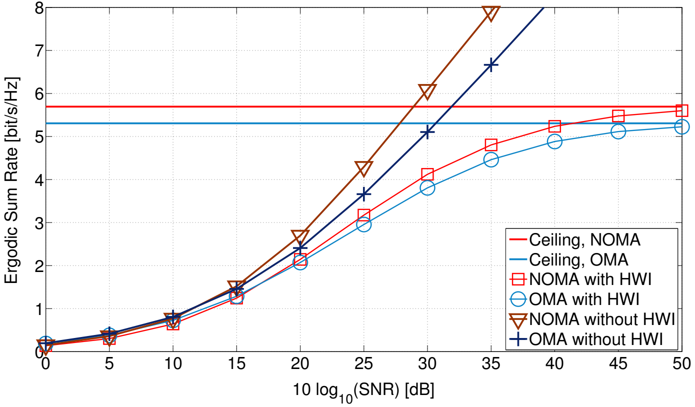

Figure: Ergodic sum rate vs. SNR for $\kappa_t = 0.01$, $\kappa_{r,1} = 0.01$ and $\kappa_{r,2} = 0.03$.

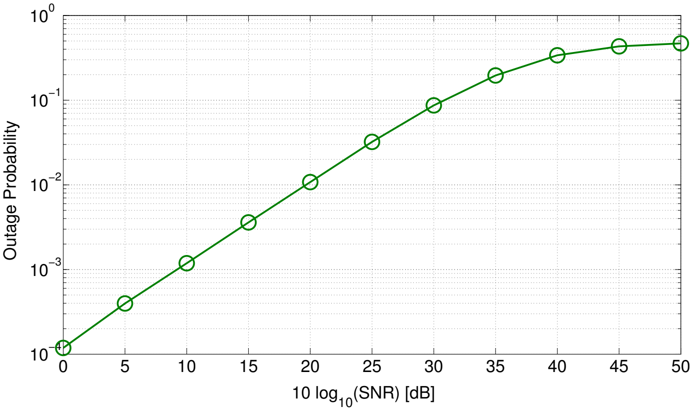

Figure: Outage probability vs. SNR for imperfect hardware and HWI-unaware SIC order, $\kappa_t = 0.01$, $\kappa_{r,(1)} = 0.03$ and $\kappa_{r,(2)} = 0.01$.

✓ (3) is achievable $\iff \kappa_{r,2} - \kappa_{r,1} \leq \frac{1}{\rho}(\frac{1}{|h_1|^2} - \frac{1}{|h_2|^2}) \rightarrow$ It is vital to consider HWI parameters in the SIC order.

✓ Outage is not negligible for an HWI-unaware system design.

QoS-based F-NOMA

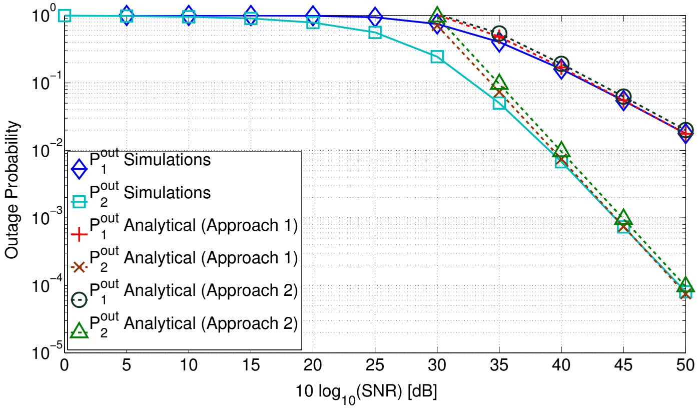

Figure: Outage probability vs. SNR for $\kappa_t = 0.01$, $\kappa_{r,(1)} = 0.01$, $\kappa_{r,(2)} = 0.03$ and $\tilde{R}_1 = \tilde{R}_2 = 2$.

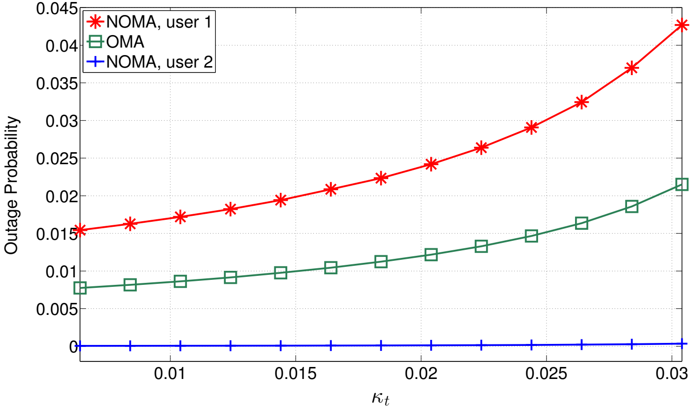

Figure: Outage probability vs. κ_t for SNR=50 dB, $\tilde{R}_1 = \tilde{R}_2 = 2$, $\kappa_{r,(1)} = 0.01$ and $\kappa_{r,(2)} = 0.03$.

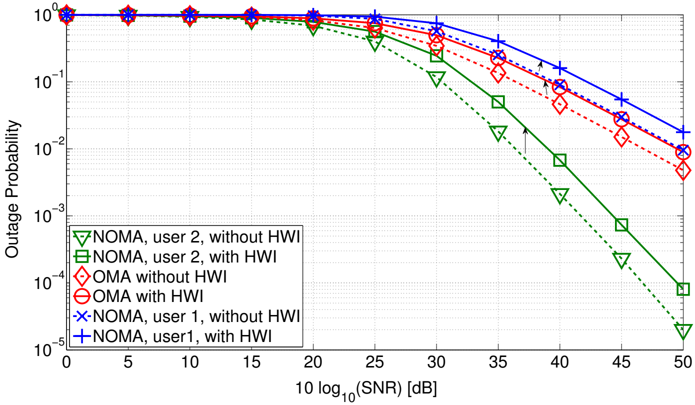

Figure: Outage probability vs. SNR for $\kappa_t = 0.01$, $\kappa_{r,(1)} = 0.01$, $\kappa_{r,(2)} = 0.03$ and $\tilde{R}_1 = \tilde{R}_2 = 2$.

CR-NOMA

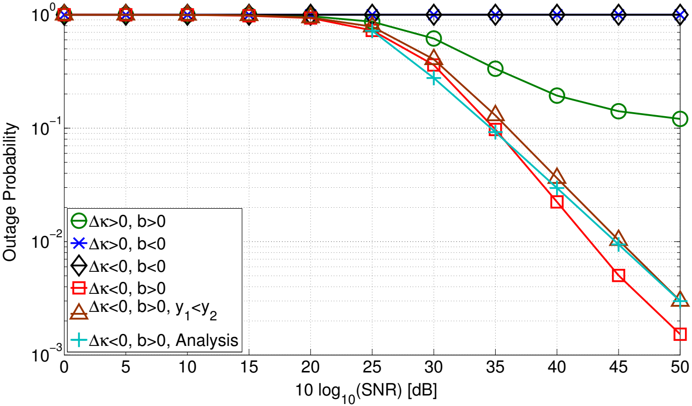

Figure: Outage probability of cognitive radio user vs. SNR for $\kappa_t = 0.01$.
$\tilde{R}_1 = \tilde{R}_2 = 2$ for $b > 0$,
$\tilde{R}_1 = \tilde{R}_2 = 3$ for $b < 0$,
$\kappa_{r,1} = 0.01$ and $\kappa_{r,2} = 0.03$ for $\Delta\kappa > 0$,
$\kappa_{r,1} = 0.03$ and $\kappa_{r,2} = 0.01$ for $\Delta\kappa < 0$.

Table of Contents

1 Introduction

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5 Conclusion

Conclusion

- Sum rate performance of the opportunistic NOMA is limited by the derived **ceiling** in the high-SNR regime.
- Outages occurring in the opportunistic NOMA for **HWI-unaware design** are not negligible.
- **Diversity gains** of the weak and strong users in **QoS-based F-NOMA** are one and two, respectively.
- **Diversity gain** of the strong user in the **CR-NOMA** is one for a well-designed system.
- A user with **smaller HWI parameter should be selected as the cognitive radio user**.
- NOMA shows **robustness** in presence of HWI, in a **HWI-aware system design**.

References I

[1] S. Tim,
RF Imperfections in High-rate Wireless Systems, 1st ed. Dordrecht, The Netherlands: Springer, 2008.

[2] C. Studer, M. Wenk and A. Burg,
"MIMO Transmission with Residual Transmit-RF Impairments," International ITG Workshop on Smart Antennas (WSA), Bremen, 2010, pp. 189–196.

[3] P. Zetterberg,
"Experimental Investigation of TDD Reciprocity-based Zero-forcing Transmit Precoding," EURASIP Journal on Advances in Signal Processing, 2011.

[4] M. Wenk,
"MIMO-OFDM Testbed: Challenges, Implementations, and Measurement Results," (Microelectronics). Konstanz, Germany: Hartung-Gorre, 2010.

References II

[5] E. Björnson, J. Hoydis, and M. Kountouris,
"Massive MIMO Systems with Non-Ideal Hardware: Energy Efficieny, Estimation and Capacity Limits," IEEE Transactions on Information Theory, vol. 60, no. 11, pp. 7112–7139, 2014.

[6] Z. Ding, Z. Yang, P. Fan, and H. V. Poor,
"On the Performance of Non-orthogonal Multiple Access in 5G Systems with Randomly Deployed Users," IEEE Signal Processing Letters, vol. 21, no. 12, pp. 1501–1505, 2014.

[7] Z. Ding, P. Fan, and H. V. Poor
"Impact of User Pairing on 5G Non-orthogonal Multiple Access Downlink Transmissions," IEEE Transactions on Vehicular Technology, vol. 65, no. 8, pp. 6010–6023, 2016.

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