CSE 221

Problem :- 2

Implementation 1 :-

It is a general recursive relation for fibonacci series. The relation is

$$T(n) = T(n-1) + T(n-2)$$

Now,

$T(n-1) = T(n-2)$

$\therefore T(n) = 2T(n-1)$

Now using telescopin method,

$$T(n) = 2T(n-1)$$

$$= 2[2T(n-2)]$$

$$= 2^2 T(n-2)$$

$$= 2^3 T(n-3)$$

Now for k times it will be

$$2^k T(n-k)$$

Now, $n-k=0$

$k=n$

$\therefore$ The relation is $O(2^n)$.

Implementation:

It uses memoization to avoid unnecessary calculations. So a subproblem will be calculated only once. So the relation becomes $O(n)$

Problem 5

1) $T(n) = T(n/2) + n - 1$

Applying master theorem

$a = 1$ $\quad b^k = 2^1 > a$

$b = 2$

$k = 1$

$\therefore$ Relation $= O(n)$

(2)

$$T(n) = T(n-1) + n-1 \quad , \quad T(1) = 0$$

$$T(n-1) = T(n-2) + (n-2) + (n-1)$$

$$= T(n-3) + (n-3) + (n-2) + (n-1)$$

$$= T(n-3) + 1+2+3 \cdots n$$

$$= T(n-3) + \frac{n(n+1)}{2}$$

$\therefore$ Time complexity $= O(n^2)$

(3)

$T(n) = T(n/3) + 2T(n/3) + n$

$= 3T(n/3) + n$

$a = 3$ Now, $b^k = 3^1 = a$

$b = 3$

$k = 1$

$\therefore$ The relation is $O(n \log n)$

(7)

$T(n) = 2T(n/2) + n^2$

$a = 2$ $\qquad$ $b^k = 2^2 > a$

$b = 2$

$k = 2$

$\therefore$ The relation $= \Theta\, O(n^2)$ (Ans)