Appendix EBS Pollock Model Description

Dynamics

This assessment is based on a statistical age-structured model with the catch equation and population dynamics model as described in D. Fournier and Archibald (1982) and elsewhere (e.g., Hilborn and Walters (1992), Jon T. Schnute and Richards (1995), McAllister and Ianelli (1997)). The catch in numbers at age in year $t(C_{t,a})$ and total catch biomass (Y_t) can be described as:s

$$
\begin{aligned}
C_{t,a} &= \frac{F_{t,a}}{Z_{t,a}}\left(1-e^{-Z_{t,a}}\right)N_{t,a}, && 1 \le t \le T, 1 \le a \le A && (1)\\
N_{t+1,a+1} &= N_{t,a-1}e^{-Z_{t,a-1}} && 1 \le t \le T, 1 \le a < A && (2)\\
N_{t+1,A} &= N_{t,A-1}e^{-Z_{t,A-1}} + N_{t,A}e^{-Z_{t,A}}, && 1 \le t \le T && (3)\\
Z_{t,a} &= F_{t,a} + M_{t,a} && && (4)\\
C_{t,.} &= \sum_{a=1}^{A} C_{t,a} && && (5)\\
p_{t,a} &= \frac{C_{t,a}}{C_{t,.}} && && (6)\\
Y_t &= \sum_{a=1}^{A} w_{t,a}C_{t,a} && && (7)\\
& && && (8)
\end{aligned}
$$

where

T	is the number of years,
A	is the number of age classes in the population,
$N_{t,a}$	is the number of fish age a in year t,
$C_{t,a}$	is the catch of age class a in year t,
$p_{t,a}$	is the proportion of the total catch in year t, that is in age class a,
C_t	is the total catch in year t,
w_a	is the mean body weight (kg) of fish in age class a,
Y_t	is the total yield biomass in year t,
$F_{t,a}$	is the instantaneous fishing mortality for age class a, in year t,
$M_{t,a}$	is the instantaneous natural mortality in year t for age class a, and
$Z_{t,a}$	is the instantaneous total mortality for age class a, in year t.

Fishing mortality $(F_{t,a})$ is specified as being semi-separable and non-parametric in form with restrictions on the variability following Butterworth, Ianelli, and Hilborn (2003) :

$$
\begin{aligned}
F_{t,a} &= s_{t,a}\,\mu^f e^{\epsilon_t}, & \epsilon_t &\sim \mathcal{N}(0,\,\sigma_E^2) \qquad (9)\\
s_{t+1,a} &= s_{t,a}\,e^{\gamma_t}, & \gamma_t &\sim \mathcal{N}(0,\,\sigma_s^2) \qquad (10)
\end{aligned}
$$

where $s_{t,a}$ is the selectivity for age class a in year t, and μ^f is the median fishing mortality rate over time.

If the selectivities ($s_{t,a}$) are constant over time then fishing mortality rate decomposes into an age component and a year component. A curvature penalty on the selectivity coefficients using the squared second-differences to provide smoothness between ages.

Bottom-trawl survey selectivity was set to be asymptotic yet retain the properties desired for the characteristics of this gear. Namely, that the function should allow flexibility in selecting age 1 pollock over time. The functional form of this selectivity was:

$$
\begin{aligned}
s_{t,a} &= \left[1 + e^{-\alpha_t a - \beta_t}\right]^{-1}, & a &> 1 \qquad (11)\\
s_{t,a} &= \mu_s e^{-\delta_t^{\mu}}, & a &= 1 \qquad (12)\\
\alpha_t &= \bar{\alpha} e^{\delta_t^{\alpha}}, & & \qquad (13)\\
\beta_t &= \bar{\beta} e^{\delta_t^{\beta}}, & & \qquad (14)
\end{aligned}
$$

where the parameters of the selectivity function follow a random walk process as in Dorn et al. (2000):

$$
\begin{aligned}
\delta_t^{\mu} - \delta_{t+1}^{\mu} &\sim \mathcal{N}(0,\,\sigma_{\delta^{\mu}}^2) \qquad (15)\\
& \qquad (16)\\
\alpha_t^{\mu} - \alpha_{t+1}^{\mu} &\sim \mathcal{N}(0,\,\sigma_{\alpha^{\mu}}^2) \qquad (17)\\
\beta_t^{\mu} - \beta_{t+1}^{\mu} &\sim \mathcal{N}(0,\,\sigma_{\beta^{\mu}}^2) \qquad (18)
\end{aligned}
$$

The parameters to be estimated in this part of the model are thus for t=1982 through to `r thisyr`. The variance terms for these process error parameters were specified to be 0.04.

In this assessment, the random-walk deviation penalty was optionally shifted to the changes in log-selectivity. that is, for the BTS estimates, the process error was applied to the logistic parameters as above, but the lognormal penalty was applied to the resulting selectivities-at-age directly. The extent of this variability was evaluated in the context of the impact on age-specific survey catchability/availability and contrasted with an independent estimate of pollock availability to the bottom trawl survey.

$$ln(s_{t,a}) - ln(s_{t+1,a}) \sim \mathcal{N}(0,\, \sigma^2_{sel}) \tag{19}$$

$$\tag{20}$$

In 2008 the AT survey selectivity approach was modified. As an option, the age one pollock observed in this trawl can be treated as an index and are not considered part of the age composition (which then ranges from age 2-15). This was done to improve some interaction with the flexible selectivity smoother that is used for this gear and was compared. Additionally, the annual specification of input observation variance terms was allowed for the AT data.

A diagnostic approach to evaluate input variance specifications (via sample size under multinomial assumptions) was added in the 2018 assessment. This method uses residuals from mean ages together with the concept that the sample variance of mean age (from a given annual data set) varies inversely with input sample size. It can be shown that for a given set of input proportions at age (up to the maximum age A) and sample size N_t for year t, an adjustment factor ν for input sample size can be computed when compared with the assessment model predicted proportions at age ($\hat{p}_{ta}$) and model predicted mean age ($\hat{\bar{a}}_t$):

$$\nu = \text{var}\left(r^a_t \sqrt{\frac{N_t}{\kappa_t}}\right)^{-1} \tag{21}$$

$$r^a_t = \bar{a}_t - \hat{\bar{a}}_t \tag{22}$$

$$\kappa_t = \left[\sum_a^A \bar{a}_t - \hat{\bar{a}}_t\right]^{0.5} \tag{23}$$

where r^a_t is the residual of mean age and

$$\hat{\bar{a}}_t = \sum_a^A a\hat{p}_{ta} \tag{24}$$

$$\bar{a}_t = \sum_a^A a p_{ta} \tag{25}$$

Based on previous analyses, we used the above relationship as a diagnostic for evaluating input sample sizes by comparing model predicted mean ages with observed mean ages and the implied 95% confidence bands. This method provided support for modifying the frequency of allowing selectivity changes.

Recruitment

In these analyses, recruitment (R_t) represents numbers of age-1 individuals modeled as a stochastic function of spawning stock biomass.

$$R_t = f\left(B_{t-1}\right) \tag{26}$$

with mature spawning biomass during year t was defined as:

$$B_t = \sum_{a=1}^{A} w_{t,a} \phi_a N_{t,a} \tag{27}$$

and, ϕ_a is the proportion of mature females at age is as shown in the sub-section titled Natural mortality and maturity at age under "Parameters estimated independently" above.

A reparameterized form for the stock-recruitment relationship following Francis (1992) was used. For the optional Beverton-Holt form (the Ricker form presented in Eq. 12 was adopted for this assessment) we have:

$$R_t = \frac{B_{t-1} e^{\varepsilon_t}}{\alpha + \beta B_{t-1}} \tag{28}$$

where

R_t	is recruitment at age 1 in year t,
B_t	is the biomass of mature spawning females in year t,
ε_t	is the recruitment anomaly for year t, ($\varepsilon_t \sim \mathcal{N}(0, \sigma_R^2$)
α, β	are stock recruitment parameters.

Values for the stock-recruitment function parameters and are calculated from the values of (the number of 0-year-olds in the absence of exploitation and recruitment variability) and the steepness of the stock-recruit relationship (h). The steepness is the fraction of R0 to be expected (in the absence of recruitment variability) when the mature biomass is reduced to 20% of its pristine level Francis (1992) , so that:

$$\alpha = \tilde{B}_0 \frac{1-h}{4h} \tag{29}$$

$$\beta = \frac{5h-1}{4hR_0} \tag{30}$$

where $\tilde{B}_0$ is the total egg production (or proxy, e.g., female spawning biomass) in the absence of exploitation (and recruitment variability) expressed as a fraction of R_0.

Some interpretation and further explanation follows. For steepness equal 0.2, then recruits are a linear function of spawning biomass (implying no surplus production). For steepness equal to 1.0, then recruitment is constant for all levels of spawning stock size. A value of $h = 0.9$ implies that at 20% of the unfished spawning stock size will result in an expected value of 90% unfished recruitment level. Steepness of 0.7 is a commonly assumed default value for the Beverton-Holt form (e.g., Kimura (1989)). The prior distribution for steepness used a beta distribution as in Ianelli et al. (2016). The prior on steepness was specified to be a symmetric form of the Beta distribution with $\alpha = \beta = 14.93$ implying a prior mean of 0.5 and CV of 12% (implying that there is about a 14% chance that the steepness is greater than 0.6). This conservative prior is consistent with previous years' application and serves to constrain the stock-recruitment curve from favoring steep slopes (uninformative priors result in F_{MSY} values near an F_{SPR} of about $F_{18\%}$ a value considerably higher than the default proxy of $F_{35\%}$). The residual pattern for the post-1977 recruits used in fitting the curve with a more diffuse prior resulted in all estimated recruits being below the curve for stock sizes less than B_{MSY} (except for the 1978 year class). We believe this to be driven primarily by the apparent negative-slope for recruits relative to stock sizes above B_{MSY} and as such, provides a potentially unrealistic estimate of productivity at low stock sizes. This prior was elicited from the rationale that residuals should be reasonably balanced throughout the range of spawning stock sizes. Whereas this is somewhat circular (i.e., using data for prior elicitation), the point here is that residual patterns (typically ignored in these types of models) were qualitatively considered.

In model 16.1 (from the 2019 assessment), a Beverton Holt stock recruitment form was implemented using the prior value of 0.67 for steepness and a CV of 0.17. This resulted in beta distribution parameters (for the prior) at $\alpha = 6.339$ and
$\beta = 4.293$.

The value of σ_R was set at 1.0 to accommodate additional uncertainty in factors affecting recruitment variability.

To have the critical value for the stock-recruitment function (steepness, h) on the same scale for the Ricker model, we begin with the parameterization of Kimura (1989) :

$$R_t = \frac{B_{t-1} e^{\alpha\left(1 - B_{t-1}\frac{R_0}{\psi_0}\right)}}{\psi_0} \tag{31}$$

It can be shown that the Ricker parameter a maps to steepness as:

$$h = \frac{e^{\alpha}}{e^{\alpha} + 4} \tag{32}$$

so that the prior used on h can be implemented in both the Ricker and Beverton-Holt stock-recruitment forms. Here the term ψ_0 represents the equilibrium unfished spawning biomass per-recruit.