Appendix EBS Pollock Model Description

Dynamics

This assessment is based on a statistical age-structured model with the catch equation and population dynamics model as described in D. Fournier and Archibald (1982) and elsewhere (e.g., Hilborn and Walters (1992), Jon T. Schnute and Richards (1995), McAllister and Ianelli (1997)). The catch in numbers at age in year $t(C_{t,a})$ and total catch biomass (Y_t) can be described as:s

$$C_{t,a} = \frac{F_{t,a}}{Z_{t,a}} \left(1 - e^{-Z_{t,a}}\right) N_{t,a}, \qquad\qquad 1 \le t \le T, 1 \le a \le A \qquad (1)$$

$$N_{t+1,a+1} = N_{t,a-1} e^{-Z_{t,a-1}} \qquad\qquad 1 \le t \le T, 1 \le a < A \qquad (2)$$

$$N_{t+1,A} = N_{t,A-1} e^{-Z_{t,A-1}} + N_{t,A} e^{-Z_{t,A}}, \qquad\qquad 1 \le t \le T \qquad (3)$$

$$Z_{t,a} = F_{t,a} + M_{t,a} \qquad (4)$$

$$C_{t,.} = \sum_{a=1}^{A} C_{t,a} \qquad (5)$$

$$p_{t,a} = \frac{C_{t,a}}{C_{t,.}} \qquad (6)$$

where

T	is the number of years,
A	is the number of age classes in the population,
$N_{t,a}$	is the number of fish age a in year t,
$C_{t,a}$	is the catch of age class a in year t,
$p_{t,a}$	is the proportion of the total catch in year t, that is in age class a,
C_t	is the total catch in year t,
w_a	is the mean body weight (kg) of fish in age class a,
Y_t	is the total yield biomass in year t,
$F_{t,a}$	is the instantaneous fishing mortality for age class a, in year t,
$M_{t,a}$	is the instantaneous natural mortality in year t for age class a, and
$Z_{t,a}$	is the instantaneous total mortality for age class a, in year t.

Fishing mortality $(F_{t,a})$ is specified as being semi-separable and non-parametric in form with restrictions on the variability following Butterworth, Ianelli, and Hilborn (2003) :

$$F_{t,a} = s_{t,a} \, \mu^f e^{\epsilon_t}, \qquad\qquad \epsilon_t \sim \mathcal{N}(0, \sigma_E^2) \qquad (7)$$

$$s_{t+1,a} = s_{t,a} \, e^{\gamma_t}, \qquad\qquad \gamma_t \sim \mathcal{N}(0, \sigma_s^2) \qquad (8)$$

where $s_{t,a}$ is the selectivity for age class a in year t, and μ^f is the median fishing mortality rate over time.

If the selectivities $(s_{t,a})$ are constant over time then fishing mortality rate decomposes into an age component and a year component. A curvature penalty on the selectivity coefficients using the squared second-differences to provide smoothness between ages.

Bottom-trawl survey selectivity was set to be asymptotic yet retain the properties desired for the characteristics of this gear. Namely, that the function should allow flexibility in selecting age 1 pollock over time. The functional form of this selectivity was:

$$s_{t,a} = \left[1 + e^{-\alpha_t a - \beta_t}\right]^{-1}, \qquad\qquad a > 1 \qquad\qquad (9)$$

$$s_{t,a} = \mu_s e^{-\delta_t^\mu}, \qquad\qquad a = 1 \qquad\qquad (10)$$

$$\alpha_t = \bar{\alpha} e^{\delta_t^\alpha}, \qquad\qquad (11)$$

$$\beta_t = \bar{\beta} e^{\delta_t^\beta}, \qquad\qquad (12)$$

where the parameters of the selectivity function follow a random walk process as in Dorn et al. (2000):

$$\delta_t^\mu - \delta_{t+1}^\mu \sim \mathcal{N}(0,\ \sigma_{\delta^\mu}^2) \qquad\qquad (13)$$

$$\alpha_t^\mu - \alpha_{t+1}^\mu \sim \mathcal{N}(0,\ \sigma_{\alpha^\mu}^2) \qquad\qquad (14)$$

$$\beta_t^\mu - \beta_{t+1}^\mu \sim \mathcal{N}(0,\ \sigma_{\beta^\mu}^2) \qquad\qquad (15)$$

The parameters to be estimated in this part of the model are thus for t=1982 through to `r thisyr`. The variance terms for these process error parameters were specified to be 0.04.

In this assessment, the random-walk deviation penalty was optionally shifted to the changes in log-selectivity. that is, for the BTS estimates, the process error was applied to the logistic parameters as above, but the lognormal penalty was applied to the resulting selectivities-at-age directly. The extent of this variability was evaluated in the context of the impact on age-specific survey catchability/availability and contrasted with an independent estimate of pollock availability to the bottom trawl survey.

$$ln(s_{t,a}) - ln(s_{t+1,a}) \sim \mathcal{N}(0,\ \sigma_{sel}^2) \qquad\qquad (16)$$

In 2008 the AT survey selectivity approach was modified. As an option, the age one pollock observed in this trawl can be treated as an index and are not considered part of the age composition (which then ranges from age 2-15). This was done to improve some interaction

with the flexible selectivity smoother that is used for this gear and was compared. Additionally, the annual specification of input observation variance terms was allowed for the AT data.

A diagnostic approach to evaluate input variance specifications (via sample size under multinomial assumptions) was added in the 2018 assessment. This method uses residuals from mean ages together with the concept that the sample variance of mean age (from a given annual data set) varies inversely with input sample size. It can be shown that for a given set of input proportions at age (up to the maximum age A) and sample size N_t for year t, an adjustment factor ν for input sample size can be computed when compared with the assessment model predicted proportions at age ($\hat{p}_{ta}$) and model predicted mean age ($\hat{\bar{a}}_t$):

$$\nu = \mathrm{var} \left(r_t^a \sqrt{\frac{N_t}{\kappa_t}} \right)^{-1} \tag{17}$$

$$r_t^a = \bar{a}_t - \hat{\bar{a}}_t \tag{18}$$

$$\kappa_t = \left[\sum_a^A \bar{a}_t - \hat{\bar{a}}_t \right]^{0.5} \tag{19}$$

where r_t^a is the residual of mean age and

$$\hat{\bar{a}}_t = \sum_a^A a \hat{p}_{ta} \tag{20}$$

$$\bar{a}_t = \sum_a^A a p_{ta} \tag{21}$$

Based on previous analyses, we used the above relationship as a diagnostic for evaluating input sample sizes by comparing model predicted mean ages with observed mean ages and the implied 95% confidence bands. This method provided support for modifying the frequency of allowing selectivity changes.

Recruitment

In these analyses, recruitment (R_t) represents numbers of age-1 individuals modeled as a stochastic function of spawning stock biomass.

$$R_t = f(B_{t-1}) \tag{22}$$

with mature spawning biomass during year t was defined as:

$$B_t = \sum_{a=1}^A w_{t,a} \phi_a N_{t,a} \tag{23}$$

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and, ϕ_a is the proportion of mature females at age is as shown in the sub-section titled Natural mortality and maturity at age under "Parameters estimated independently" above.

A reparameterized form for the stock-recruitment relationship following Francis (1992) was used. For the optional Beverton-Holt form (the Ricker form presented in Eq. 12 was adopted for this assessment) we have:

$$R_t = \frac{B_{t-1}e^{\varepsilon_t}}{\alpha + \beta B_{t-1}} \tag{24}$$

where

R_t is recruitment at age 1 in year t,
B_t is the biomass of mature spawning females in year t,
ε_t is the recruitment anomaly for year t, $(\varepsilon_t \sim \mathcal{N}(0, \sigma_R^2)$
α, β are stock recruitment parameters.

Values for the stock-recruitment function parameters and are calculated from the values of (the number of 0-year-olds in the absence of exploitation and recruitment variability) and the steepness of the stock-recruit relationship (h). The steepness is the fraction of R0 to be expected (in the absence of recruitment variability) when the mature biomass is reduced to 20% of its pristine level Francis (1992) , so that:

$$\alpha = \tilde{B}_0 \frac{1-h}{4h} \tag{25}$$

$$\beta = \frac{5h-1}{4hR_0} \tag{26}$$

where $\tilde{B}_0$ is the total egg production (or proxy, e.g., female spawning biomass) in the absence of exploitation (and recruitment variability) expressed as a fraction of R_0.

Some interpretation and further explanation follows. For steepness equal 0.2, then recruits are a linear function of spawning biomass (implying no surplus production). For steepness equal to 1.0, then recruitment is constant for all levels of spawning stock size. A value of $h = 0.9$ implies that at 20% of the unfished spawning stock size will result in an expected value of 90% unfished recruitment level. Steepness of 0.7 is a commonly assumed default value for the Beverton-Holt form (e.g., Kimura (1989)). The prior distribution for steepness used a beta distribution as in Ianelli et al. (2016). The prior on steepness was specified to be a symmetric form of the Beta distribution with $\alpha = \beta = 14.93$ implying a prior mean of 0.5 and CV of 12% (implying that there is about a 14% chance that the steepness is greater than 0.6). This conservative prior is consistent with previous years' application and serves to constrain the stock-recruitment curve from favoring steep slopes (uninformative priors result in F_{MSY} values near an F_{SPR} of about $F_{18\%}$ a value considerably higher than the default

proxy of $F_{35\%}$). The residual pattern for the post-1977 recruits used in fitting the curve with a more diffuse prior resulted in all estimated recruits being below the curve for stock sizes less than B_{MSY} (except for the 1978 year class). We believe this to be driven primarily by the apparent negative-slope for recruits relative to stock sizes above B_{MSY} and as such, provides a potentially unrealistic estimate of productivity at low stock sizes. This prior was elicited from the rationale that residuals should be reasonably balanced throughout the range of spawning stock sizes. Whereas this is somewhat circular (i.e., using data for prior elicitation), the point here is that residual patterns (typically ignored in these types of models) were qualitatively considered.

In model 16.1 (from the 2019 assessment), a Beverton Holt stock recruitment form was implemented using the prior value of 0.67 for steepness and a CV of 0.17. This resulted in beta distribution parameters (for the prior) at $\alpha = 6.339$ and
$\beta = 4.293$.

The value of σ_R was set at 1.0 to accommodate additional uncertainty in factors affecting recruitment variability.

To have the critical value for the stock-recruitment function (steepness, h) on the same scale for the Ricker model, we begin with the parameterization of Kimura (1989) :

$$R_t = \frac{B_{t-1} e^{\alpha\left(1 - B_{t-1}\frac{R_0}{\psi_0}\right)}}{\psi_0} \tag{27}$$

It can be shown that the Ricker parameter a maps to steepness as:

$$h = \frac{e^\alpha}{e^\alpha + 4} \tag{28}$$

so that the prior used on h can be implemented in both the Ricker and Beverton-Holt stock-recruitment forms. Here the term ψ_0 represents the equilibrium unfished spawning biomass per-recruit.

Diagnostics

In 2006 a replay feature was added where the time series of recruitment estimates from a particular model is used to compute the subsequent abundance expectation had no fishing occurred. These recruitments are adjusted from the original estimates by the ratio of the expected recruitment given spawning biomass (with and without fishing) and the estimated stock-recruitment curve. I.e., the recruitment under no fishing is modified as:

$$R'_t = \hat{R}_t \frac{f(B'_{t-1})}{f(B_{t-1})} \tag{29}$$

where R_t is the original recruitment estimate in year t with B'_{t-1} and B_{t-1} representing the stock-recruitment function given spawning biomass under no fishing and under the estimated fishing intensity, respectively.

The assessment model code allows retrospective analyses (e.g., Parma (1993), and J. N. Ianelli and Fournier (1998)). This was designed to assist in specifying how spawning biomass patterns (and uncertainty) have changed due to new data. The retrospective approach simply uses the current model to evaluate how it may change over time with the addition of new data based on the evolution of data collected over the past several years.

To provide a diagnostic for diversity in the age structure of the spawning biomass, we computed the Shannon entropy in exponential form. Following Jost (2006), and substituting the population's mature age structure in place of species, this provides an estimate of the effective number of ages. This is computed for each year as a measure of different age groups contributing to the spawning population as:

$$H_i = \exp(-\sum p_a \cdot \log(p_a)) \tag{30}$$

where p_a is the proportion of mature fish biomass-at-age a in year i.

Parameter estimation

The objective function was simply the sum of the negative log-likelihood function and logs of the prior distributions. To fit large numbers of parameters in nonlinear models it is useful to be able to estimate certain parameters in different stages. The ability to estimate stages is also important in using robust likelihood functions since it is often undesirable to use robust objective functions when models are far from a solution. Consequently, in the early stages of estimation we use the following log- likelihood function for the survey and fishery catch at age data (in numbers):

$$nll(i) = n \sum_{t,a} p_{ta} \ln \hat{p}_{ta} \tag{31}$$

$$p_{ta} = \frac{O_{ta}}{\sum_a O_{ta}} \qquad \hat{p}_{ta} = \frac{\hat{C}_{ta}}{\sum_a \hat{C}_{ta}} \tag{32}$$

$$\mathbf{C} = \mathbf{CE} \tag{33}$$

$$\mathbf{E} = \begin{matrix} b_{1,1} & b_{1,2} & \ldots & b_{1,15} \\ b_{2,1} & b_{2,2} & & b_{2,15} \\ \vdots & & \ddots & \vdots \\ b_{15,1} & b_{15,2} & \ldots & b_{15,15} \end{matrix} \tag{34}$$

where A, and T, represent the number of age classes and years, respectively, n is the sample size, and represent the observed and predicted numbers at age in the catch. The elements bi,j represent ageing mis-classification proportions are based on independent agreement rates between otolith age readers. For the models presented this year, the option for including aging errors was re-evaluated.

Sample size values were revised and are shown in the main document. Strictly speaking, the amount of data collected for this fishery indicates higher values might be warranted. However, the standard multinomial sampling process is not robust to violations of assumptions (D. A. Fournier et al. (1990)). Consequently, as the model fit approached a solution, we invoke a robust likelihood function which fit proportions at age as:

$$\prod_{a=1}^{A}\prod_{t=1}^{T}\left[\left(\exp\left(-\frac{(p_{ta}-\hat{p}_{ta})^2}{2\left(\eta_{ta}+0.1/A\right)\tau_t^2}\right)+0.01\right)\times\frac{1}{\sqrt{2\pi\left(\eta_{ta}+0.1/A\right)\tau_t}}\right] \tag{35}$$

Taking the logarithm we obtain the log-likelihood function for the age composition data:

$$nll(i)=-0.5\sum_{a=1}^{A}\sum_{t=1}^{T}\ln 2\pi\left(\eta_{ta}+0.1/A\right)-\sum_{t}^{T}A\ln\tau_t+\sum_{a=1}^{A}\sum_{t=1}^{T}\ln\left\{\exp\left(-\frac{(p_{ta}-\hat{p}_{ta})^2}{(2\eta_{ta}+0.1/A)\tau_t^2}\right)+0.01\right\}$$
$$\tag{36}$$

where

$$\eta_{ta}=p_{ta}(1-p_{ta}) \tag{37}$$

and

$$\tau_t^2=1/n_t \tag{38}$$

which gives the variance for p_{ta}

$$(\eta_{ta}+0.1/A)\tau_t^2 \tag{39}$$

Completing the estimation in this fashion reduces the model sensitivity to data that would otherwise be considered outliers.

Within the model, predicted survey abundance accounted for within-year mortality since surveys occur during the middle of the year. As in previous years, we assumed that removals by the survey were insignificant (i.e., the mortality of pollock caused by the survey was considered insignificant). Consequently, a set of analogous catchability and selectivity terms were estimated for fitting the survey observations as:

$$\hat{N}_{ta}^s = e^{-0.5Z_{ta}} N_{ta} q_t^s s_{ta}^S \tag{40}$$

where the superscript s indexes the type of survey (AT or BTS). For the option to use the survey predictions in biomass terms instead of just abundance, the above was modified to include observed survey biomass weights-at-age:

$$\hat{N}_{ta}^s = e^{-0.5Z_{ta}} w_{ta} N_{ta} q_t^s s_{ta}^S \tag{41}$$

For the AVO index, the values for selectivity were assumed to be the same as for the AT survey and the mean weights at age over time was also assumed to be equal to the values estimated for the AT survey.

For these analyses we chose to keep survey catchabilities constant over time (though they are estimated separately for the AVO index and for the AT and bottom trawl surveys). The contribution to the negative log-likelihood function (ignoring constants) from the surveys is given by either the lognormal distribution:

$$nll(i) = \sum_t \frac{\ln(u_t^s/\hat{N}_t^s)^2}{2\sigma_{s,t}^2} \tag{42}$$

where u_t^s is the total (numerical abundance or optionally biomass) estimate with variance $\sigma_{s,t}$ from survey s in year t or optionally, the normal distribution can be selected:

$$nll(i) = \sum_t \frac{(u_t^s - \hat{N}_t^s)^2}{2\sigma_{s,t}^2}. \tag{43}$$

The AT survey and AVO index is modeled using a lognormal distribution whereas for the BTS survey, a multivariate log-normal distribution was applied.

For model configurations in which the BTS data are corrected for estimated efficiency, a multivariate lognormal distribution was used. For the negative- log likelihood component this was modeled as

$$nll_i = 0.5 \mathbf{X} \Sigma^{-1} \mathbf{X}' \tag{44}$$

where is a vector of observed minus model predicted values for this index and Σ is the estimated covariance matrix provided from the method provided in Kotwicki, Ianelli, and Punt (2014). For the VAST estimates, the supplied covariance matrix was used in the same way.

The contribution to the negative log-likelihood function for the observed total catch biomass $(C_b^{obs}, \hat{C}_b)$ by the fishery is given by

$$nll_i = 0.5 \sum_t \frac{\ln(C_b^{obs}/\hat{C}_b)^2}{2\sigma_{C_b,t}^2} \tag{45}$$

where $\sigma_{C_b,t}$ is pre-specified (set to 0.05) reflecting the accuracy of the overall observed catch in biomass. Similarly, the contribution of prior distributions (in negative log-density) to the log-likelihood function include $\lambda_\varepsilon \sum_t \varepsilon_t^2 + \lambda_\gamma \sum_{ta} \gamma^2 + \lambda_\delta \sum_t \delta_t^2$ where the size of the 's represent prior assumptions about the variances of these random variables. Most of these parameters are associated with year-to- year and age specific deviations in selectivity coefficients. For a presentation of this type of Bayesian approach to modeling errors-in- variables, the reader is referred to J. T. Schnute (1994). To facilitate estimating such a large number of parameters, automatic differentiation software extended from Greiwank and Corliss (1991) and developed into C++ class libraries was used. This software provided the derivative calculations needed for finding the posterior mode via a quasi-Newton function minimization routine (e.g., Press et al. (1992)). The model implementation language (ADModel Builder) gave simple and rapid access to these routines and provided the ability estimate the variance-covariance matrix for all dependent and independent parameters of interest.

Uncertainty in mean body mass

The approach we use to solve for F_{MSY} and related quantities (e.g., B_{MSY} MSY) within a general integrated model context was shown in S. B. Ianelli James N. and Williamson (2001). In 2007 this was modified to include uncertainty in weight-at-age as an explicit part of the uncertainty for F_{MSY} calculations. This involved estimating a vector of parameters (w_{ta}^{future}) on current (`r thisyr`) and future mean weights for each age i, $i=$ (1, 2,…,15), given actual observed mean and variances in weight-at-age over the period 1991-`r thisyr-1`. The values of based on available data and (if this option is selected) estimates the parameters subject to the natural constraint:

$$w_{ta}^{future} \sim \mathcal{N}(\bar{w}_a, \sigma_{w_a}^2) \tag{46}$$

Note that this converges to the mean values over the time series of data (no other likelihood component within the model is affected by future mean weights-at-age) while retaining the natural uncertainty that can propagate through estimates of F_{MSY} uncertainty. This latter point is essentially a requirement of the Tier 1 categorization.

Subsequently, this method was refined to account for current-year survey data and both cohort and year effects. The model for this is:

$$\hat{w}_{ta} = \bar{w}_a e^{\upsilon_t} \qquad\qquad a = 1,\, t \geq 1964 \qquad (47)$$

$$\hat{w}_{ta} = \hat{w}_{t-1,a-1} + \Delta_a e^{\psi_t} \qquad\qquad a > 1,\, t > 1964 \qquad (48)$$

$$\Delta_a = \bar{w}_{a+1} - \bar{w}_a \qquad\qquad a < A \qquad (49)$$

$$\bar{w}_a = \alpha \left\{ L_1 + (L_2 - L_1)\left(\frac{1 - K^{a-1}}{1 - K^{A-1}}\right) \right\}^3 \qquad (50)$$

where the fixed effects parameters are $L_1, L_2, K,$ and α while the random effects parameters are υ_t and ψ_t.

Tier 1 projections

Tier 1 projections were calculated two ways. First, for `r nextyr` and `r nextyr+1` ABC and OFL levels, the harmonic mean F_{MSY} value was computed and the analogous harvest rate (u_{HM}^-) applied to the estimated geometric mean fishable biomass at B_{MSY} :

$$ABC_t = B_{GM,t}^f \hat{u}_{HM} \zeta_t \qquad (51)$$

$$B_{GM,t}^f = e^{\ln \hat{B}_t^f - 0.5\sigma_{Bf}^2} \qquad (52)$$

$$u_{HM,t}^f = e^{\ln \hat{u}_{MSY,t} - 0.5\sigma_{u_{MSY}}^2} \qquad (53)$$

$$\zeta_t = \frac{B_t/B_{MSY} - 0.05}{1 - 0.05} \qquad\qquad B_t < B_{MSY} \qquad (54)$$

$$\zeta_t = 1.0 \qquad\qquad B_t \geq B_{MSY} \qquad (55)$$

where $\hat{B}_t^f$ is the point estimate of the fishable biomass defined (for a given year): $\sum_a N_a s_{ta} w_{ta}$ with N_{ta}, s_{ta}, and w_{ta} the estimated population numbers (begin year), selectivity and weights-at-age, respectively. B_{MSY} and B_t are the point estimates spawning biomass levels at equilibrium F_{MSY} and in year t (at time of spawning). For these projections, catch must be specified (or solved for if in the current year when $B_t < B_{MSY}$). For longer term projections a form of operating model (as has been presented for the evaluation of $B_{20\%}$) with feedback (via future catch specifications) using the control rule and assessment model would be required.

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